Modélisation du transport d'un polluant

Projet d'introduction à la recherche

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Introduction

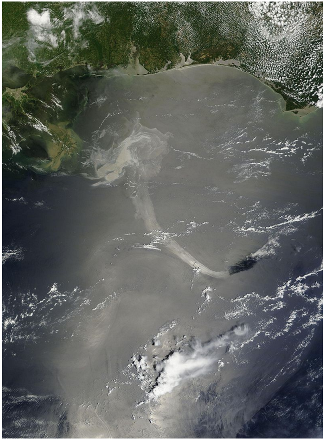


FIGURE 1: Explosion de *Deepwater Horizon, 20/04/2010*

Equation de transport

$$\boxed{\partial_t c + u \cdot \nabla c = 0}$$

c(x,t) : concentration en polluant à la position $x \in (0,1)^2 = \Omega$ à l'instant $t \in [0, T]$
u : champ de vitesse, dépend *a priori* de x et de t

Champs de vitesse étudiés

Champ de vitesse uniforme

$$\begin{cases} u_x = ||u|| \cos(\theta) \\ u_y = ||u|| \sin(\theta) \end{cases}$$

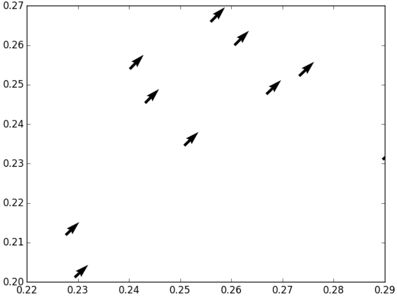


FIGURE 2: Écoulement constant avec un angle θ

Champs de vitesse étudiés

Écoulements cellulaires

$$\psi(x, y) = \sin(2\pi x)\sin(2\pi y) + \theta_0 \cos(2\pi\theta_1 x)\cos(2\pi\theta_2 y)$$
$$\theta_0 \in [0, 2.5] \text{ et } (\theta_1, \theta_2) \in [0.5, 4]^2$$

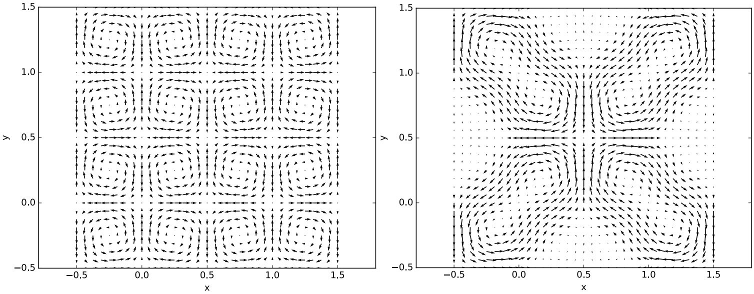


FIGURE 3: Champs de vitesse pour deux jeux de paramètres

Champs de vitesse étudiés

Champ de vitesse de Lamb-Oseen

$$\mathbf{V}(r,\theta,t) = \frac{\Gamma}{2\pi r}\left(1 - \exp\left(\frac{-r^2}{4\nu t + r_c^2}\right)\right)\mathbf{u}_\theta$$

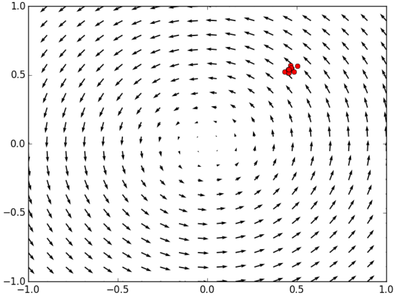


FIGURE 4: Écoulement Lamb-Oseen

Approche eulerienne

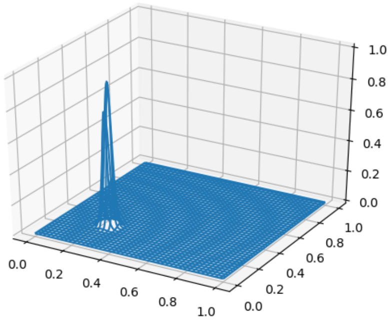


FIGURE 5: Condition initiale...

Approche eulerienne

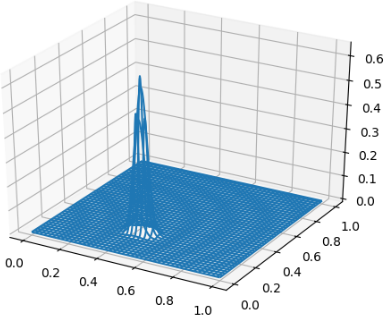


Figure 6: ... et après un certain temps

Approche eulerienne

Méthode des volumes finis

Approche eulerienne

Modèle réduit

Approche lagrangienne

$X(\xi, t) \in \mathbb{R}^{n\times 2}$ désigne les positions, à l'instant $t \in [0, T]$, des particules qui étaient initialement aux positions $\xi \in \mathbb{R}^{n\times 2}$

$$\begin{cases} \partial_t X = v(X(\xi, t), t) \\ X(\xi, 0) = \xi \end{cases}$$

Hypothèses

$v \in \mathcal{C}^0(\mathbb{R}^{n\times 2} \times \mathbb{R}) \bigcap W^{1,\infty}(\mathbb{R}^{n\times 2} \times \mathbb{R})$

Théorème de Cauchy-Lipschitz

Existence et unicité d'une solution locale pour des temps arbitraires

Conséquence

Le transport est à vitesse finie

Approche lagrangienne
Résolution numérique

Schéma de Crank-Nicholson :

$$X^{(k+1)} = X^{(k)} + \frac{\Delta t}{2}(v(X^{(k)}, t^k) + v(X^{(k+1)}, t^{k+1}))$$

Algorithme du point fixe :

$$\left\{ \begin{array}{c} X_0^{(k+1)} = X^{(k)} \\ X_1^{(k+1)} = X^{(k)} + \Delta t v(X^{(k)}, t^k) \\ X_{r+1}^{(k+1)} = X^{(k)} + \frac{\Delta t}{2}(v(X^{(k)}, t^k) + v(X_r^{(k+1)}, t^{k+1})) \end{array} \right.$$

Approche eulerienne

Champ de vitesse uniforme

16 simulations, $\theta \in [0, \frac{\pi}{2}]$

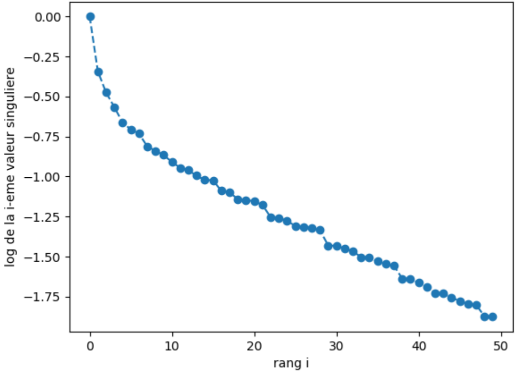
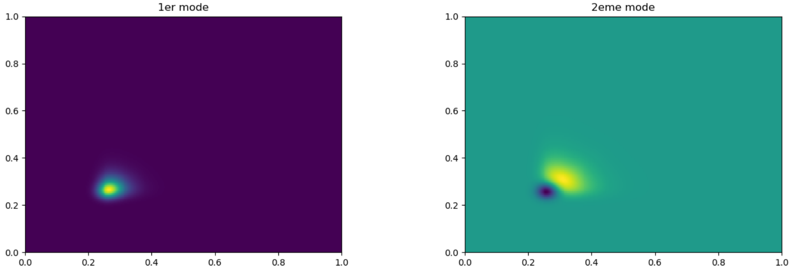
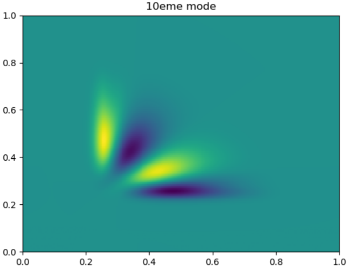


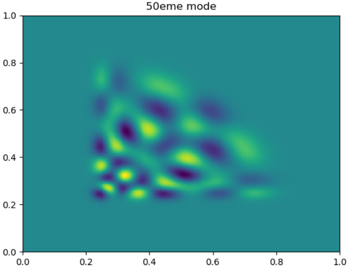
FIGURE 7: Tracé du log des valeurs singulières en fonction du rang

Approche eulerienne
Champ de vitesse uniforme

Rôle des modes propres :




10eme mode
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
0.6
0.8
1.0


50eme mode
1.0
0.8
0.6
0.4
0.2
0.0
0.0
0.2
0.4
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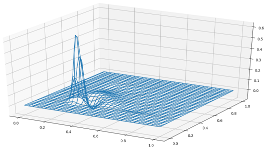
1.0

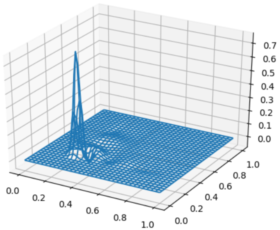
Approche eulerienne

Champ de vitesse uniforme

Reconstruction d'une solution fine



Condition initiale, 10 modes

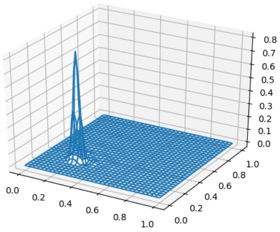


Condition initiale, 20 modes

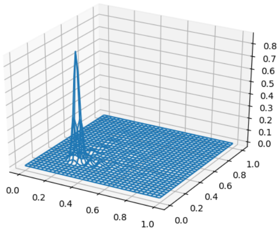
Approche eulerienne

Champ de vitesse uniforme

Reconstruction d'une solution fine



Condition initiale, 50 modes

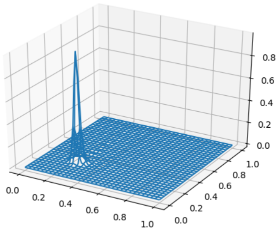


Condition initiale, 100 modes

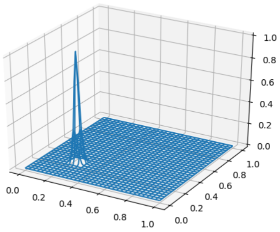
Approche eulerienne

Champ de vitesse uniforme

Reconstruction d'une solution fine



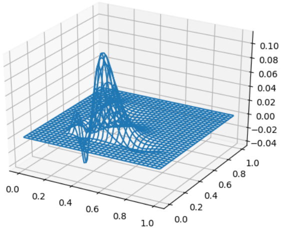
Condition initiale, 200 modes



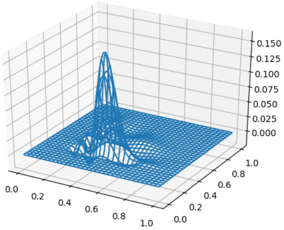
Condition initiale, 500 modes

Approche eulerienne
Champ de vitesse uniforme

Reconstruction d'une solution fine



A mi-parcours, 10 modes

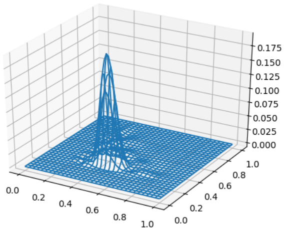


A mi-parcours, 20 modes

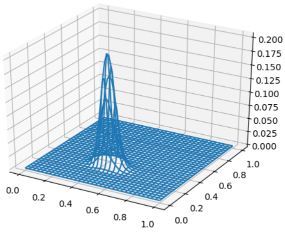
Approche eulerienne

Champ de vitesse uniforme

Reconstruction d'une solution fine



A mi-parcours, 50 modes

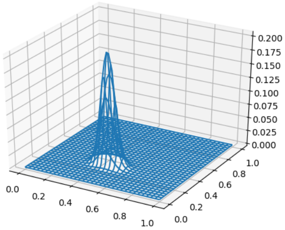


A mi-parcours, 100 modes

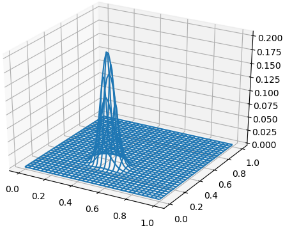
Approche eulerienne

Champ de vitesse uniforme

Reconstruction d'une solution fine



A mi-parcours, 200 modes



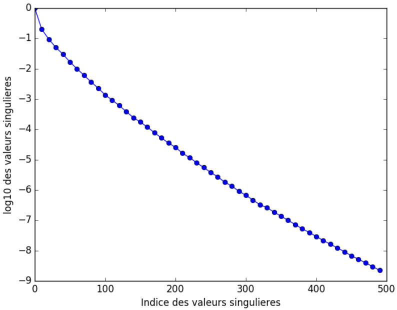
A mi-parcours, 500 modes

Approche eulerienne

Ecoulements cellulaires

64 simulations en volumes finis

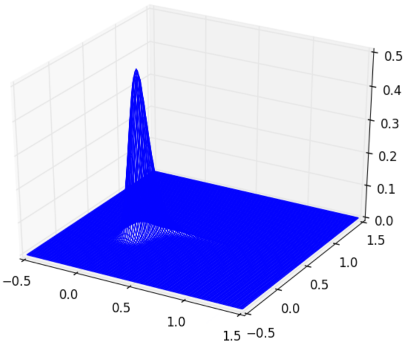
- On se place sur $[-0.5, 1.5] \times [-0.5, 1.5]$
- Maillage de 2^7 mailles de côté
- $(x_0, y_0, \theta_0) \in [0.25, 0.75] \times [0.25, 0.75] \times [0, 2.5]$ tirés uniformément



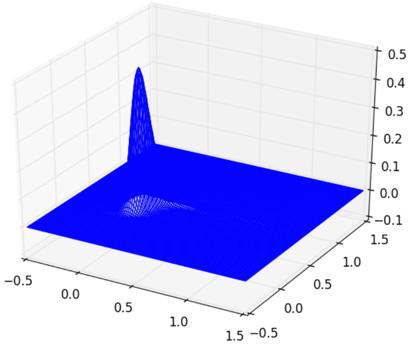
Approche eulerienne

Ecoulements cellulaires

Reconstruction "in-range" :



Snapshot à mi-parcours, 200 modes

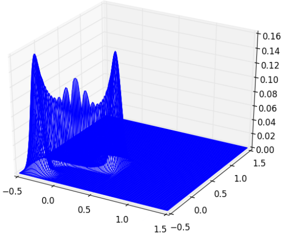


A mi-parcours, 200 modes

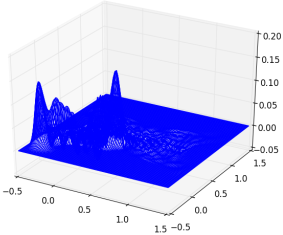
Approche eulerienne

Ecoulements cellulaires

Reconstruction "out of range" :



Snapshot à mi-parcours, 500 modes



A mi-parcours, 200 modes

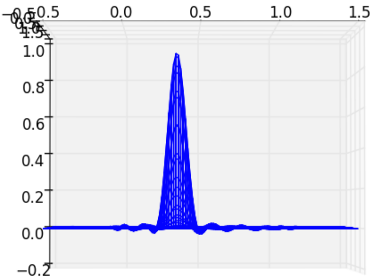
Approche eulerienne

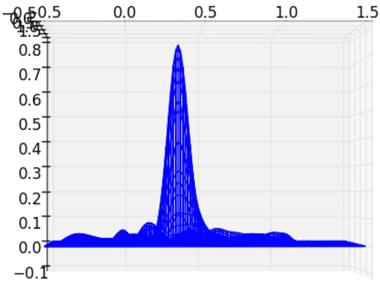
Ecoulements cellulaires

Prise en en compte de la positivité de la concentration

Recherche des coefficients $(a_j)_j$ tels que :

$$(a_j)_j = \text{argmin}\|c_* - \sum_j a_j\varphi_j\|_{L^2} \text{ s.c. } \sum_j a_j\varphi_j \geq 0$$





Approche lagrangienne

Champ de vitesse uniforme

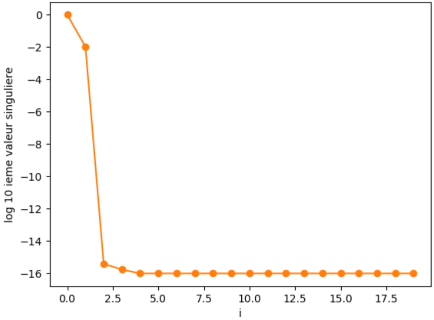


FIGURE 8: log des valeurs singulières pour 8000 particules

Approche lagrangienne
Écoulements cellulaires

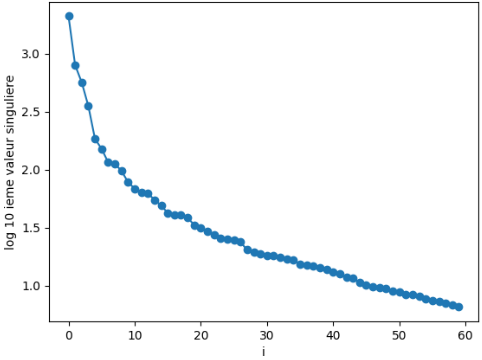


FIGURE 9: log des valeurs singulières pour 500 particules

Approche lagrangienne

Écoulements cellulaires

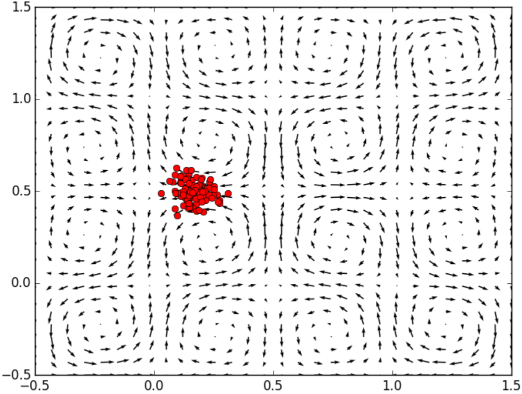


FIGURE 10: simulation pour 100 particules, $\theta_0 = \theta_1 = \theta_2 = 0.5$

Approche lagrangienne
Écoulements cellulaires

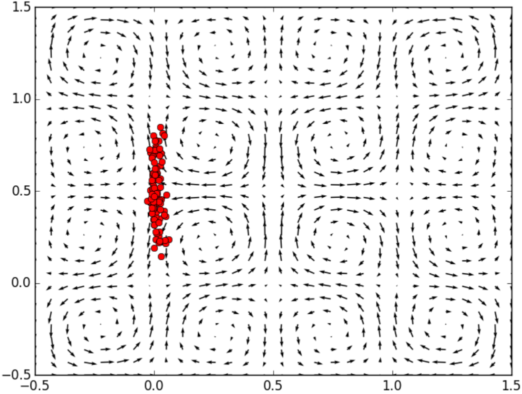


FIGURE 11: simulation pour 100 particules, $\theta_0 = \theta_1 = \theta_2 = 0.5$

Approche lagrangienne
Écoulements cellulaires

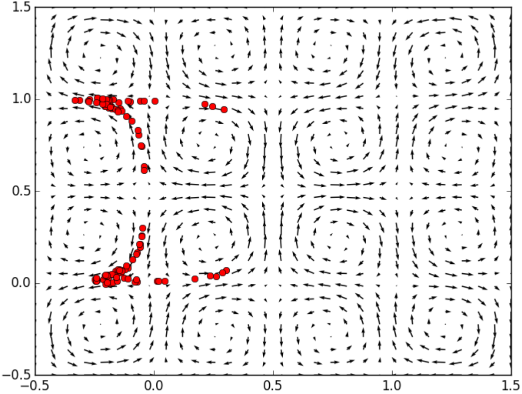


FIGURE 12: simulation pour 100 particules, $\theta_0 = \theta_1 = \theta_2 = 0.5$

Approche lagrangienne

Écoulements cellulaires

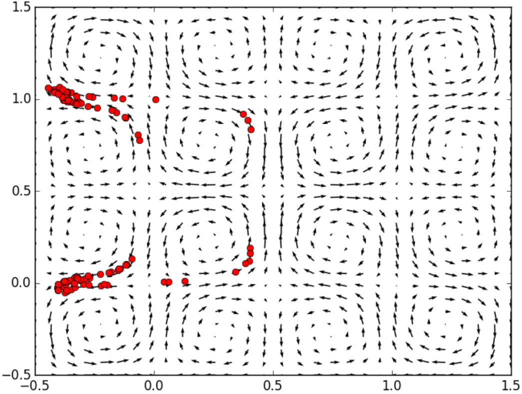


FIGURE 13: simulation pour 100 particules, $\theta_0 = \theta_1 = \theta_2 = 0.5$

Approche lagrangienne

Champ de vitesse de Lamb-Oseen

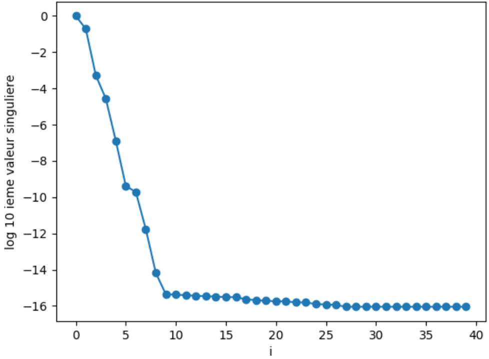


FIGURE 14: log des valeurs singulières pour 100 particules

For Further Reading I

A. Author.
Handbook of Everything.
Some Press, 1990.

S. Someone.
On this and that.
Journal of This and That, 2(1) :50–100, 2000.