Introduction

A watermark is a digitally added text used to copyright an image and prevent it from unrightful copy or misappropriation (cf. fig.1). This project aims at the complete removal of such watermarks.

We chose to use theoretical results on blind source separation in order to create an efficient watermark-erasing program. We first considered the simplest case of linear watermarks, and then more realistic ones which were not necessarily linear. In each case, we assumed that we had a set of three images composed of a target image with a watermark to erase and two copies of another image, one with the same watermark and the other without any.





Fig. 1: Examples of watermarked images¹

I - Theory and Mathematical tools

The observed information, contained in a random vector \vec{x} , is the image of an unknown random vector \vec{s} by an unknown linear transformation A, i.e. $\vec{x} = A\vec{s}$. The question is, can we find \vec{s} using only \vec{x} ?

According to Comon's therom, if the problem satisfies the following conditions:

- the components of \vec{s} are independent
- at most, one of \vec{s} components is gaussian
- A is a time independent invertible matrix

then, any matrix B such that the vector $B\vec{x}$ has independent components satisfies that $B\vec{x} \simeq \vec{s}$, meaning that $B\vec{x}$ is the image of \vec{s} under a permutation and a homothety.

To characterize the independence of the components of a vector, we can use the **Mutual penalized Information** J:

$$J(\vec{y}) = \int_{\vec{t}} p_{\vec{y}}(\vec{t}) \ln \left(\frac{p_{\vec{y}}(\vec{t})}{\prod_{i} p_{y_{i}}(t_{i})} \right) d\vec{t} + \lambda \sum_{i} \left(\operatorname{Var}(y_{i}) - 1 \right)^{2}$$

$$\tag{1}$$

where $\vec{y} = (y_i)_i$ is a random vector, and λ is the penalty setting.

J satisfies the following properties:

- $J \ge 0$
- $J(\vec{y}) = 0 \Leftrightarrow \vec{y}$ has independent components, and $Var(y_i) = 1 \forall i$

Now we can reformulate the problem as follows: find B such that $J(B\vec{x}) = 0$ which is equivalent to find min_B $J(B\vec{x})$.

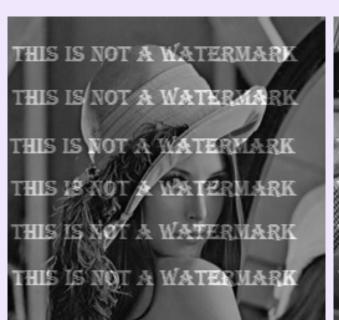
J does not only help us find a vector with independent components from \vec{x} but also a bounded one.

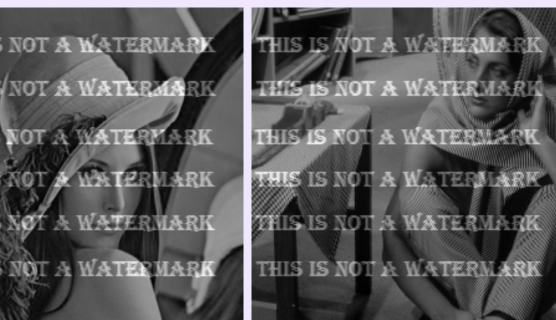
To minimize J over B we use the gradient descent algorithm: $B_{n+1} = B_n - \mu \frac{\partial J}{\partial B}(B_n)$, where μ is the step parameter.

Finally, we chose to impose $\mathbb{E}[y_i] = 0$ at all times in our algorithm for mathematical simplicity.

II - Case of a linear watermark

At first, we used our algorithm in a case that matched the theoretical framework, that is to say a watermark obtained by linear combination of a text on plain background with the original image:





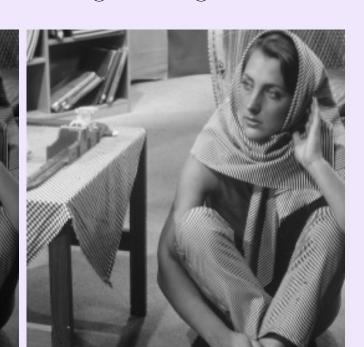
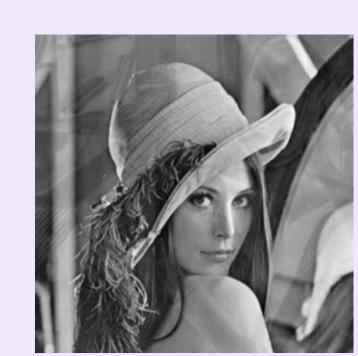


Fig. 2: Input of two images with a linear watermark and one without

As expected, the algorithm returns both original images along with the image of the watermark:



THIS IS NOT A WATERMARK THIS IS NOT A WATERMARK



Fig. 3: Output of the program after 5000 iterations and $\mu = 0.01$

III - Case of a common (non-linear) watermark

We generalized the code to colored images working separately on each of the three RGB colors. The case of a linear watermark are not that simple. This is why we also tried to remove a watermark made by a specialized website².

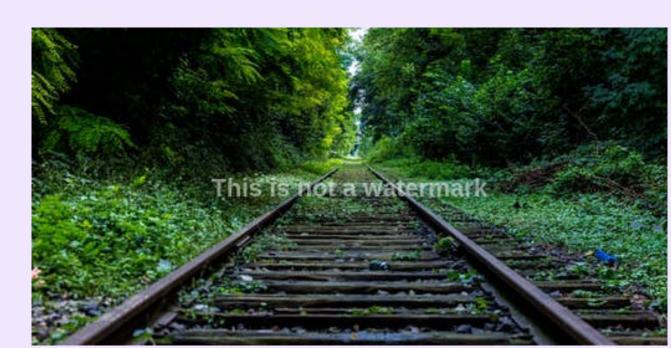






Fig. 4: Input of two images with a non-linear watermark and one without

As the modification of the images we obtained that way were not necessarily linear, we couldn't expect to erase the watermark only by using blind source separation. The returned values are nevertheless interesting.



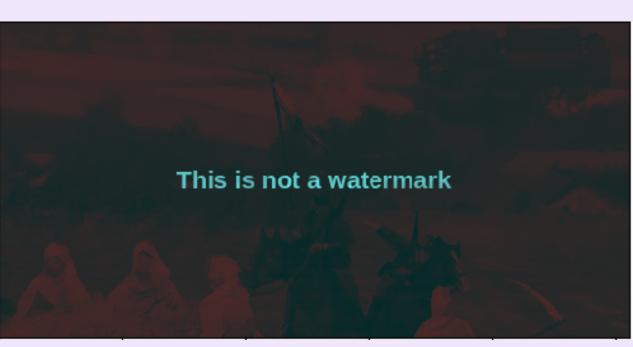




Fig. 5: Result of the program

The watermark barely changed and is still visible. However, the algorithm manages to isolate the letters of the watermark on a plain background.

Thus, it becomes rather easy to identify the pixels of the images that correspond to the letters. This allows to determine the kind of transformation which was applied by comparing the pixels of the image we possess with and without a watermark. Obviously, an important drawback of this method is that it is not systematic and is possibly inconclusive when the transformation is too complicated. The results were very satisfying in our case where transformations turned out to be affine.

Linear watermarking can easily be dealt with using blind source separation, but more complex and common watermarks with non-linear transformations need a complementary treatment to obtain satisfying results. What needs to be kept in mind is that the application fields of the blind source separation method are numerous and do not stop at image processing - cryptography or sound treatment for instance can also benefit from it.



Fig. 6: Final result