An Algorithm for Minimizing the Mumford-Shah Functional

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Outline

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1. Related Work
2. The Primal-Dual Algorithm
3. Demo

Related work

- T. Pock and D. Cremers and H. Bischof and A. Chambolle, An Algorithm for Minimizing the Piecewise Smooth Mumford-Shah Functional, ICCV, 2009.

Outline

Primal-Dual Algorithm

Algorithm

Choose $(x^0, y^0) \in C \times K$ *and let* $\bar{x}^0 = x^0$. *We choose* $\tau, \sigma > 0$. *Then, we let for each* $n \geq 0$

$$\begin{cases} y^{n+1} = \Pi_K(y^n + \sigma A \bar{x}^n) \\ x^{n+1} = \Pi_C(x^n - \tau A^* y^{n+1}) \\ \bar{x}^{n+1} = 2x^{n+1} - x^n. \end{cases}$$

The Projection onto C

$$C = \{x \in X : x(i,j,k) \in [0,1], x(i,j,1) = 1, x(i,j,M) = 0\} \subseteq X.$$

Algorithm (Clipping)

$$x^{n+1} = \min\{0, \max\{0, x^n\}\}.$$

The Projection onto K

$$K = \{y = (y^1, y^2, y^3)^T \in Y :$$

$$y^3(i,j,k) \geq \frac{y^1(i,j,k)^2 + y^2(i,j,k)^2}{4} - \lambda(\frac{k}{M} - f(i,j))^2, \quad (1)$$

$$\left| \sum_{k_1 \leq k \leq k_2} (y^1(i,j,k), y^2(i,j,k))^T \right| \leq \nu\} \quad (2)$$

Boyle-Dykstra Algorithm

Algorithm ([pami11])

Choose u_i^k, v_i^k *and initialize* $u_p^0 = u_{cur}$ *and* $v_i^0 = 0$ *for all* $i = 1, 2, ..., p.$

$$
\begin{aligned}
u_0^k &= u_p^{k-1}, \\
u_i^k &= \Pi_i(u_{i-1}^k - v_i^{k-1}), \quad i = 1, 2, ..., p, \\
v_i^k &= u_i^i - (u_{i-1}^k - v_i^{k-1}), \quad i = 1, 2, ..., p.
\end{aligned}
$$

Minimization with Lagrange-Multipliers

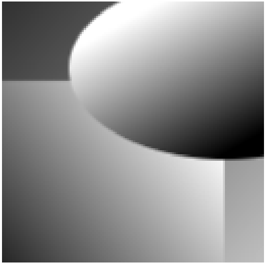
Algorithm

Choose $(x^0, y^0, \lambda^0, p^0) \in C \times K_p \times \mathbb{R}^{2\times N\times N\times M} \times \mathbb{R}^{2\times N\times N\times M}$ *and let* $\bar{x}^0 = x^0, \bar{\lambda}^0 = \lambda^0$. *We choose* $\tau_x = \frac{1}{6}, \tau_\lambda = \frac{1}{2+k_2-k_1}, \sigma_y = \frac{1}{3+M}, \sigma_p = 1$. *Then, we let for each* $n \geq 0$

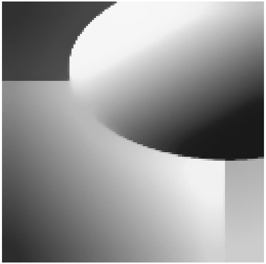
$$\begin{cases} y^{n+1} = \Pi_{K_p}(y^n + \sigma_y(A\bar{x}^n + \tilde{y})) \\ p^{n+1}_{k_1,k_2} = \Pi_{||\cdot||_2 \leq \nu}(p^n_{k_1,k_2} + \sigma_p \bar{\lambda}^n_{k_1,k_2}) \\ x^{n+1} = \Pi_C(x^n - \tau_x A^* y^{n+1}) \\ \lambda^{n+1}_{k_1,k_2} = \lambda^n_{k_1,k_2} - \tau_\lambda(p^{n+1}_{k_1,k_2} - \sum_{k_1 \leq k \leq k_2}(y_1(i,j,k), y_2(i,j,k))^T) \\ \bar{x}^{n+1} = 2x^{n+1} - x^n \\ \bar{\lambda}^{n+1}_{k_1,k_2} = 2\lambda^{n+1}_{k_1,k_2} - \lambda^n_{k_1,k_2}. \end{cases} \tag{3}$$

Outline

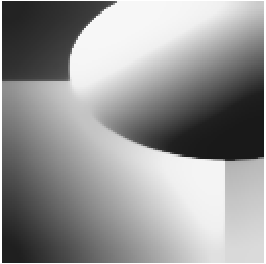
Synthetic Image (Size: 128 x 128)



Synthetic Image
Size: 128 x 128
grayscale

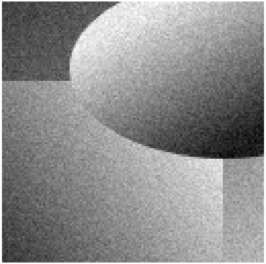


Boyle-Dysktra
Level: 16
Memory Used: 1.33 GB
Runtime: 4505 s
Iterations: 1000

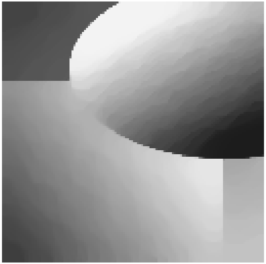


Lagrange-Multipliers
Level: 16
Memory Used: 0.155 GB
Runtime: 10.39 s
Iterations: 1000

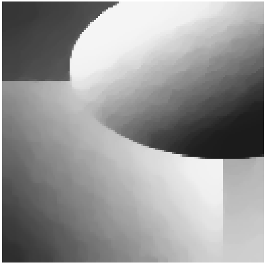
Synthetic Image With Gaussian Noise (Size: 128 x 128)



Noisy Image
Size: 128 x 128
grayscale



Boyle-Dysktra
Level: 16
Memory Used: 1.33 GB
Runtime: 4495 s
Iterations: 1000



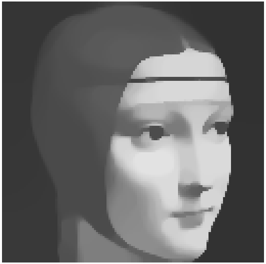
Lagrange-Multipliers
Level: 16
Memory Used: 0.155 GB
Runtime: 10.47 s
Iterations: 1000

La dama con l'ermellino (Size: 128 x 128)

La dama Image

Size: 128 x 128

grayscale



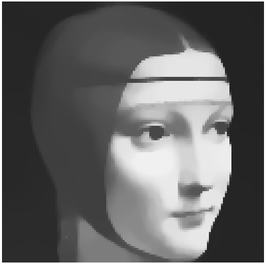
Boyle-Dysktra

Level: 16

Memory Used: 1.33 GB

Runtime: 4495 s

Iterations: 1000



Lagrange-Multipliers

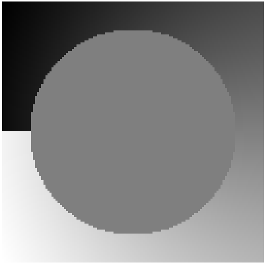
Level: 16

Memory Used: 0.155 GB

Runtime: 10.42 s

Iterations: 1000

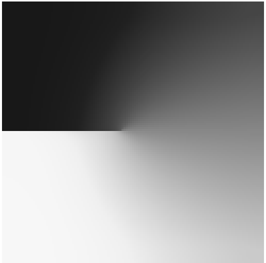
Crack Tip Inpainting (Size: 128 x 128)



Crack Tip Problem

Size: 128 x 128

grayscale



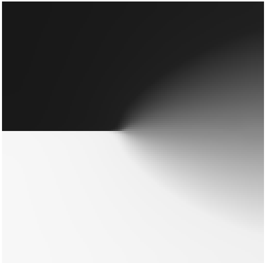
Boyle-Dysktra

Level: 16

Memory Used: 1.33 GB

Runtime: 4501 s

Iterations: 1000



Lagrange-Multipliers

Level: 16

Memory Used: 0.155 GB

Runtime: 10.49 s

Iterations: 1000

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- Prof. Dr. Daniel Cremers
- Evgeny Strekalovskiy
- Thomas Moellenhoff

Bibliography I

[iccv09] T. Pock and D. Cremers and H. Bischof and A. Chambolle, An Algorithm for Minimizing the Piecewise Smooth Mumford-Shah Functional, iccv, 2009.

[pami11] D. Cremers and K. Kolev Multiview Stereo and Silhouette Consistency via Convex Functionals over Convex Domains, pami, 2011.