# The max-min-hill-climbing algorithm

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M.Sc. Comp. Science

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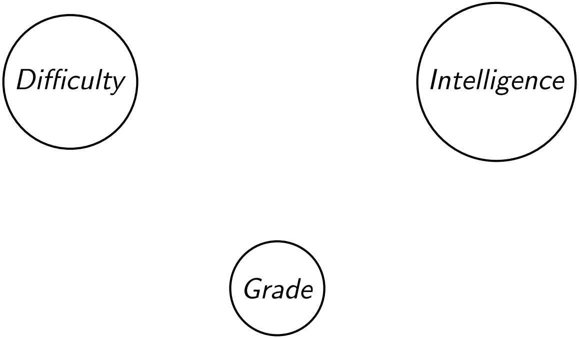
Difficulty
Intelligence
Grade

### Definition (Conditional Independence)

Two variables $X$ and $Y$ are conditionally independent given $\mathbf{Z}$ w.r.t a probability distribution P, denoted as $Ind_p(X; Y|\mathbf{Z})$, if $\forall x, y, \mathbf{z}$, where $P(\mathbf{Z} = \mathbf{z}) > 0$,

$$P(X, Y|\mathbf{Z}) = P(X|\mathbf{Z}) * P(Y|\mathbf{Z}), \quad (1)$$

where $P(X, Y|\mathbf{Z}) = P(X \cap Y|\mathbf{Z})$.

It is equivalent

$$Ind(X; Y|\mathbf{Z}) \iff (Assoc(X; Y|\mathbf{Z}) = 0), \quad (2)$$

where $Assoc(X; Y|\mathbf{Z})$ is the strength association (dependency) of $X$ and $Y$ given $\mathbf{Z}$.

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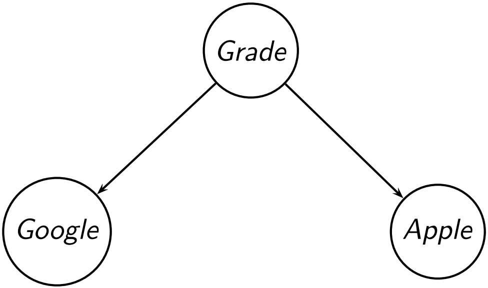
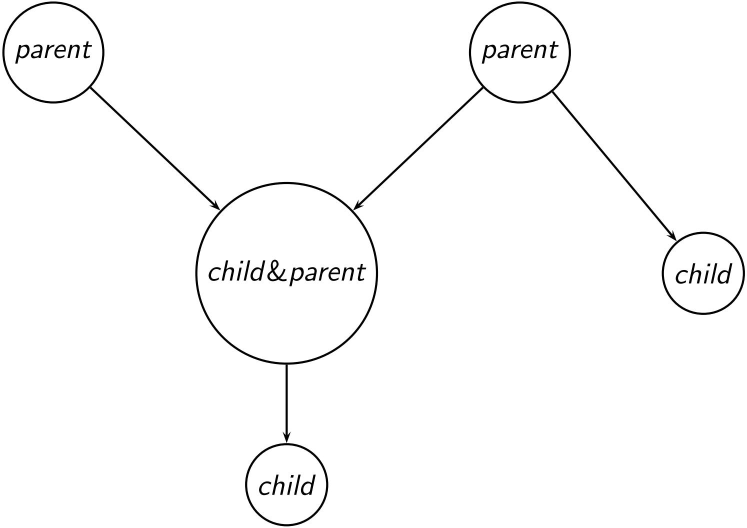

Grade
Google
Apple

# Bayesian Networks

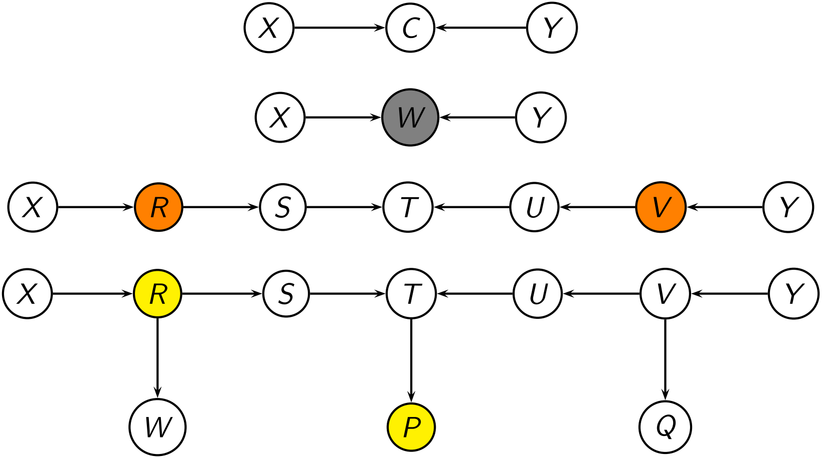


# Blocked paths & d-seperation

**Define three sets**

Let $A_1$, $A_2$ and $A_3$ denote sets with:

$$\begin{aligned} A_1 &= \{W\} \\ A_2 &= \{R, V\} \\ A_3 &= \{R, P\} \end{aligned}$$


X
C
Y
X
W
Y
X
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T
U
V
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X
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U
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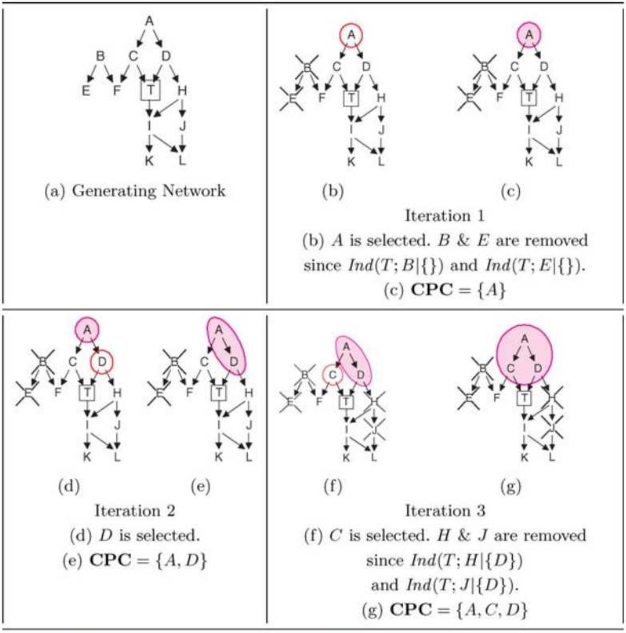
Y
W
P
Q




(a) Generating Network

(b) (c)

Iteration 1

(b) $A$ is selected. $B$ & $E$ are removed since $Ind(T; B|\{\})$ and $Ind(T; E|\{\})$.

(c) **CPC** $= \{A\}$

(d) (e)

Iteration 2

(d) $D$ is selected.

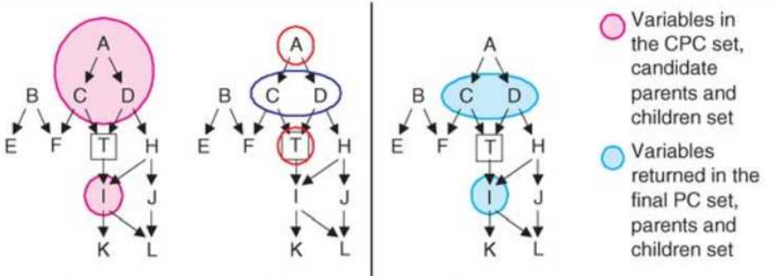
(e) **CPC** $= \{A, D\}$

(f) (g)

Iteration 3

(f) $C$ is selected. $H$ & $J$ are removed since $Ind(T; H|\{D\})$ and $Ind(T; J|\{D\})$.

(g) **CPC** $= \{A, C, D\}$




(k) (l) (m) Legend

Backward Phase

(l) $Ind(A; T | \{C, D\})$

(m) Final Outcome of $\overline{MMPC}$ (T)

Phase II: Backward Phase

# Thanks for your attention!