The max-min-hill-climbing algorithm

Michael Bauer

M.Sc. Comp. Science

June 1, 2014

Algorithm 1 $\overline{MMPC}$ Algorithm

procedure $\overline{MMPC}$ $(T, \mathcal{D})$
  Input: target variable T; data $\mathcal{D}$
  Output: the parents and children of T in any Bayesian network faithfully representing the data distribution
  %Phase I: Forward
  $\mathbf{CPC} = \emptyset$
  repeat
    $\langle F, assocF \rangle = MaxMinHeuristic(T; \mathbf{CPC})$
    if $assocF \neq 0$ **then**
      $\mathbf{CPC} = \mathbf{CPC} \cup F$
    end if
  until CPC has not changed

  %Phase II: Backward
  for all $X \in \mathbf{CPC}$ **do**
    if $\exists \mathbf{S} \subseteq \mathbf{CPC}$, s.t. $Ind(X; T|\mathbf{S})$ **then**
      $\mathbf{CPC} = \mathbf{CPC} \setminus \{X\}$
    end if
  end for

  return CPC
end procedure

procedure MaxMinHeuristic$(T, \mathbf{CPC})$
  Input: target variable T; subset of variables **CPC**
  Output: the maximum over all variables of the minimum association with T relative to **CPC**, and the variable that achieves the maximum
  $assocF = \max_{X \in \mathcal{V}} MinAssoc(X; T|\mathbf{CPC})$
  $F = \arg\max_{X \in \mathcal{V}} MinAssoc(X; T|\mathbf{CPC})$
  return $\langle F, assocF \rangle$
end procedure

Reminder (Conditional Independence)

Two variables X and Y are conditionally independent given $\mathbf{Z}$ w.r.t a probability distribution P, denoted as $Ind_p(X; Y|\mathbf{Z})$, if $\forall x, y, \mathbf{z}$, where $P(\mathbf{Z} = \mathbf{z}) > 0$,

$$P(X, Y|\mathbf{Z}) = P(X|\mathbf{Z}) * P(Y|\mathbf{Z}), \tag{1}$$

where $P(X, Y|\mathbf{Z}) = P(X \cap Y|\mathbf{Z})$.

It is equivalent

$$Ind(X; T|\mathbf{Z}) \Longleftrightarrow (Assoc(X; T|\mathbf{Z}) = 0), \tag{2}$$

where $Assoc(X; T|\mathbf{Z})$ is the strength association (dependency) of X and T given $\mathbf{Z}$.

Reminder (Conditional Independence)

Two variables X and Y are conditionally independent given $\mathbf{Z}$ w.r.t a probability distribution P, denoted as $Ind_p(X;Y|\mathbf{Z})$, if $\forall x, y, \mathbf{z}$, where $P(\mathbf{Z}=\mathbf{z}) > 0$,

$$P(X,Y|\mathbf{Z}) = P(X|\mathbf{Z}) * P(Y|\mathbf{Z}), \tag{1}$$

where $P(X,Y|\mathbf{Z}) = P(X \cap Y|\mathbf{Z})$.
It is equivalent

$$Ind(X;T|\mathbf{Z}) \iff (Assoc(X;T|\mathbf{Z}) = 0), \tag{2}$$

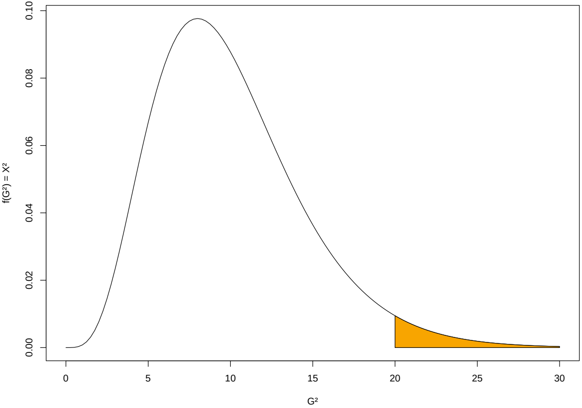
where $Assoc(X;T|\mathbf{Z})$ is the strength association (dependency) of X and T given $\mathbf{Z}$.

Definition

We calculate the G^2 value under the nullhypothesis of the conditional independence of $Ind_P(X_i, X_j|\mathbf{X}_k)$ holding. Then, the G^2 value is defined as:

$$G^2 := 2 * \sum_{a,b,c} S_{ijk}^{ab\mathbf{c}} * ln\left(\frac{S_{ijk}^{ab\mathbf{c}} * S_k^{\mathbf{c}}}{S_{ik}^{a\mathbf{c}} * S_{jk}^{b\mathbf{c}}}\right), \tag{3}$$

where S^{abc} is the number of times in the data where $X_i = a$, $X_j = b$ and $\mathbf{X}_k = \mathbf{c}$. We define in a similar fashion S^{ac}, S^{bc} and S^c, respectively.



0.10
0.08
0.06
0.04
0.02
0.00
f(G²) = X²
0
5
10
15
20
25
30
G²


0.10
0.08
0.06
0.04
0.02
0.00
f(G²) = X²
0
5
10
15
20
25
30
G²

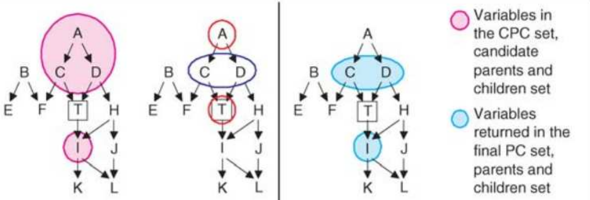
Algorithm 1 $\overline{MMPC}$ Algorithm

procedure $\overline{MMPC}$ $(T,\mathcal{D})$
  Input: target variable T; data $\mathcal{D}$
  Output: the parents and children of T in any Bayesian network faithfully representing the data distribution
  %Phase I: Forward
  $\mathbf{CPC} = \emptyset$
  repeat
    $\langle F, assocF \rangle = MaxMinHeuristic(T; \mathbf{CPC})$
    if $assocF \neq 0$ **then**
      $\mathbf{CPC} = \mathbf{CPC} \cup F$
    end if
  until CPC has not changed

  %Phase II: Backward
  for all $X \in \mathbf{CPC}$ **do**
    if $\exists \mathbf{S} \subseteq \mathbf{CPC}$, s.t. $Ind(X; T|\mathbf{S})$ **then**
      $\mathbf{CPC} = \mathbf{CPC} \setminus \{X\}$
    end if
  end for

  return CPC
end procedure

procedure MaxMinHeuristic$(T,\mathbf{CPC})$
  Input: target variable T; subset of variables **CPC**
  Output: the maximum over all variables of the minimum association with T relative to **CPC**, and the variable that achieves the maximum
  $assocF = \max_{X \in \mathcal{V}} MinAssoc(X; T|\mathbf{CPC})$
  $F = \arg\max_{X \in \mathcal{V}} MinAssoc(X; T|\mathbf{CPC})$
  return $\langle F, assocF \rangle$



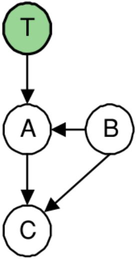
(k) (l) (m) Legend

Backward Phase

(l) $Ind(A;T|\{C,D\})$

(m) Final Outcome of $\overline{MMPC}$ (T)

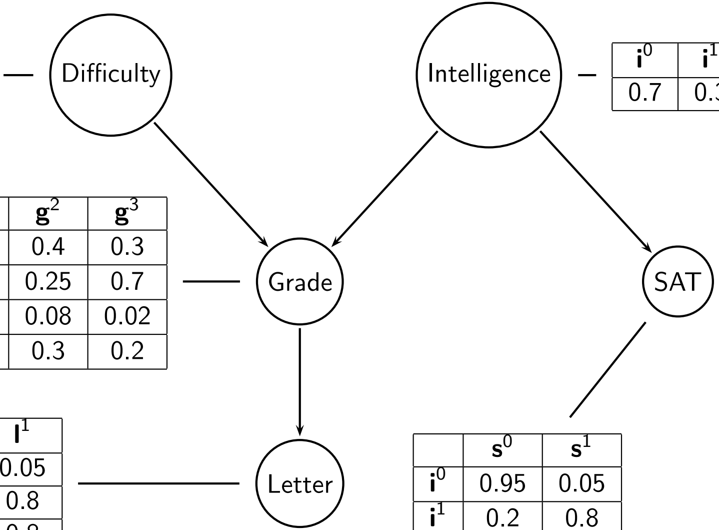
Phase II: Backward Phase


T
A
B
C

Algorithm 2 Algorithm *MMPC*

procedure MMPC(T,$\mathcal{D}$)
    $\mathbf{CPC} = \overline{MMPC}(T, \mathcal{D})$
    for every variable $X \in \mathbf{CPC}$ **do**
        if $T \notin \overline{MMPC}(X, \mathcal{D})$ **then**
            $\mathbf{CPC} = \mathbf{CPC} \setminus X$
        end if
    end for

    return CPC
end procedure



$\mathbf{d}^0$	$\mathbf{d}^1$
0.6	0.4

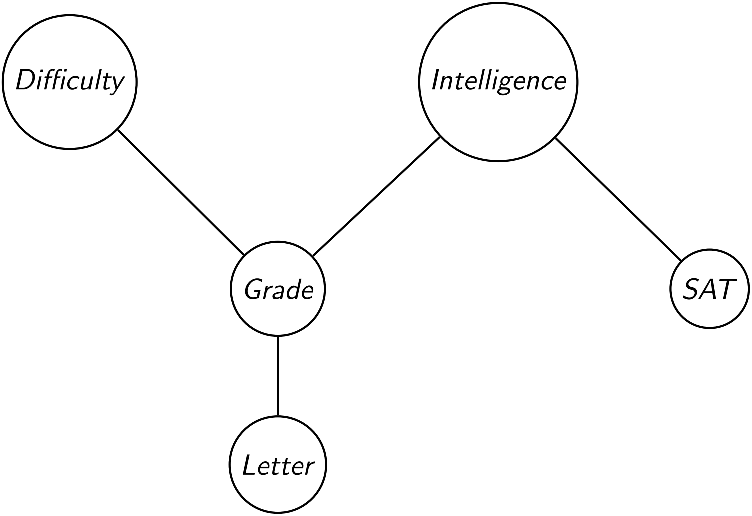
$\mathbf{i}^0$	$\mathbf{i}^1$
0.7	0.3

	$\mathbf{g}^1$	$\mathbf{g}^2$	$\mathbf{g}^3$
$\mathbf{i}^0$,$\mathbf{d}^0$	0.3	0.4	0.3
$\mathbf{i}^0$,$\mathbf{d}^1$	0.05	0.25	0.7
$\mathbf{i}^1$,$\mathbf{d}^0$	0.9	0.08	0.02
$\mathbf{i}^1$,$\mathbf{d}^1$	0.5	0.3	0.2

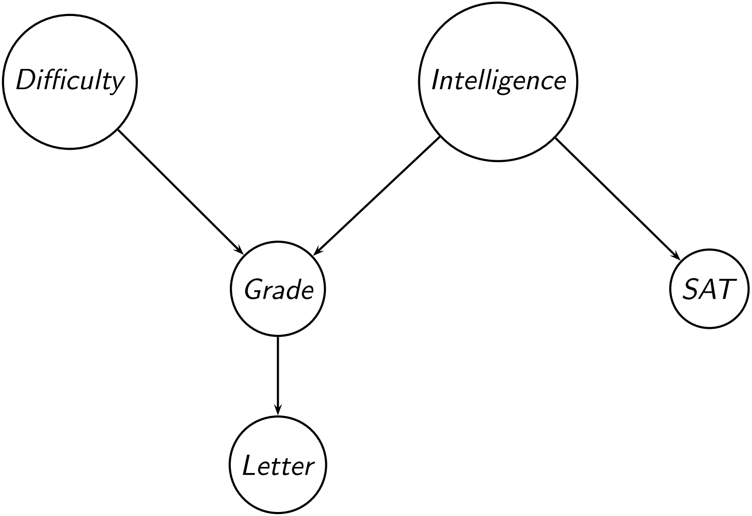
	$\mathbf{l}^0$	$\mathbf{l}^1$
$\mathbf{g}^1$	0.95	0.05
$\mathbf{g}^2$	0.2	0.8
$\mathbf{g}^3$	0.2	0.8

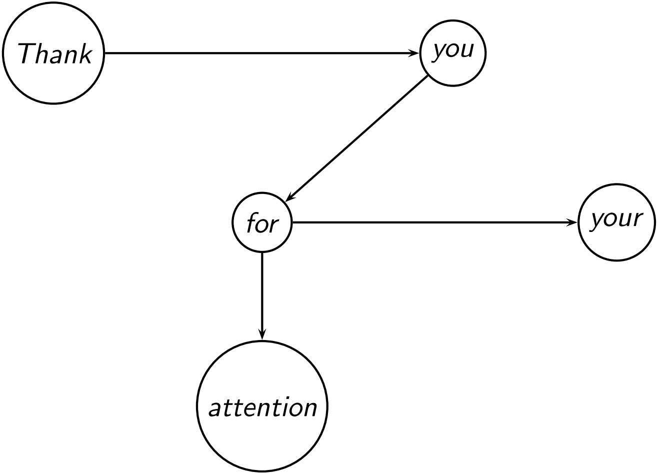
	$\mathbf{s}^0$	$\mathbf{s}^1$
$\mathbf{i}^0$	0.95	0.05
$\mathbf{i}^1$	0.2	0.8

How this is working until now:



The last goal:




Thank
you
for
your
attention