The max-min-hill-climbing algorithm

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M.Sc. Comp. Science

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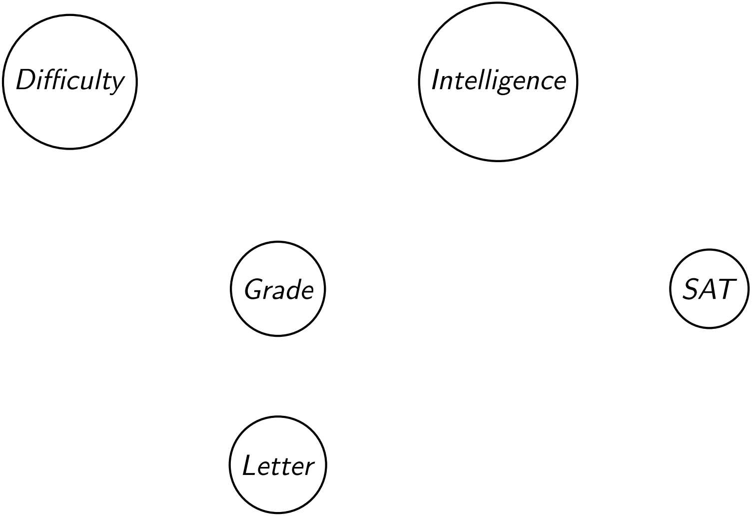
Difficulty
Intelligence
Grade
SAT
Letter

Definition (Conditional Independence)

Two variables X and Y are conditionally independent given $\mathbf{Z}$ w.r.t a probability distribution P, denoted as $Ind_p(X;Y|\mathbf{Z})$, if $\forall x, y, \mathbf{z}$, where $P(\mathbf{Z} = \mathbf{z}) > 0$,

$$P(X, Y|\mathbf{Z}) = P(X|\mathbf{Z}) * P(Y|\mathbf{Z}), \quad (1)$$

where $P(X, Y|\mathbf{Z}) = P(X \cap Y|\mathbf{Z})$.

It is equivalent

$$Ind(X;Y|\mathbf{Z}) \iff (Assoc(X;Y|\mathbf{Z}) = 0), \quad (2)$$

where $Assoc(X;Y|\mathbf{Z})$ is the strength association (dependency) of X and Y given $\mathbf{Z}$.

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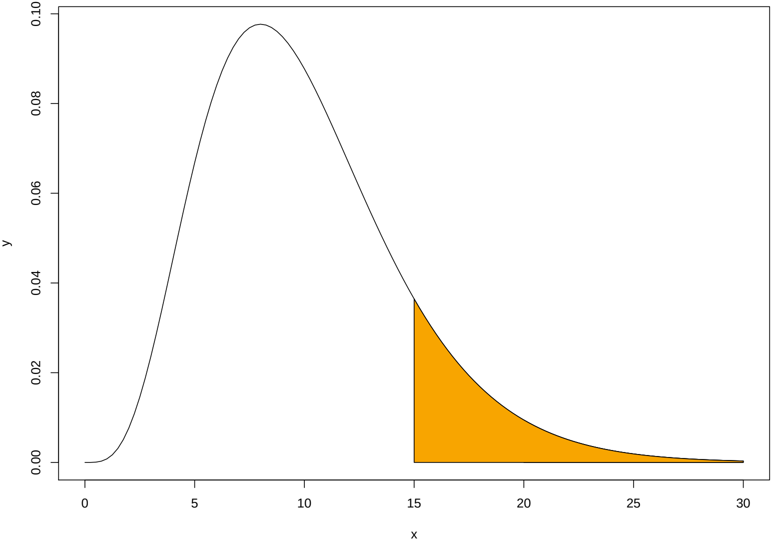
$$Ind(X; Y|\mathbf{Z}) \Longleftrightarrow (Assoc(X; Y|\mathbf{Z}) = 0), \tag{2}$$

where $Assoc(X; Y|\mathbf{Z})$ is the strength association (dependency) of X and Y given $\mathbf{Z}$.

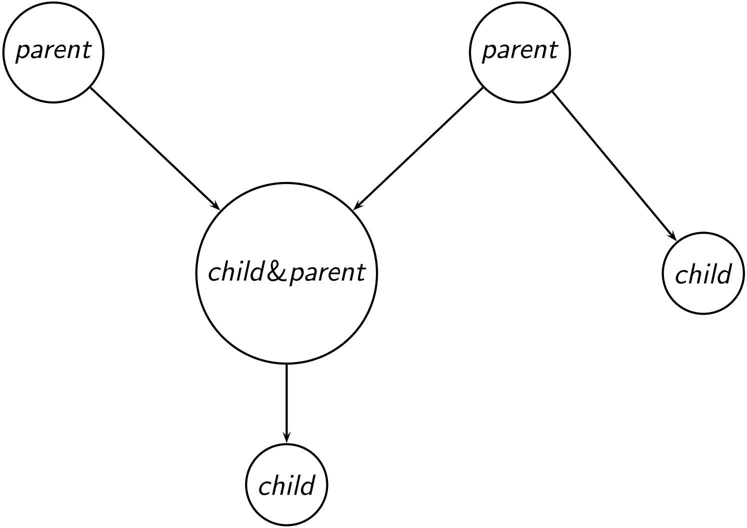
Definition

We calculate the G^2 value under the nullhypothesis of the conditional independence of $Ind_P(X_i, X_j | \mathbf{X}_k)$ holding. Then, the G^2 value is defined as:

$$G^2 := 2 * \sum_{a,b,c} S_{ijk}^{ab\mathbf{c}} * ln \left(\frac{S_{ijk}^{ab\mathbf{c}} * S_k^{\mathbf{c}}}{S_{ik}^{a\mathbf{c}} * S_{jk}^{b\mathbf{c}}} \right) \tag{3}$$


0.10
0.08
0.06
0.04
0.02
0.00
y
0
5
10
15
20
25
30
x

Bayesian Networks



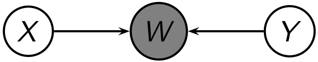
Collider

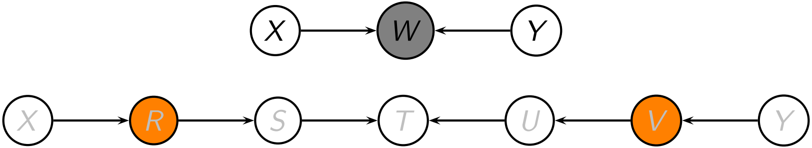
Blocked paths

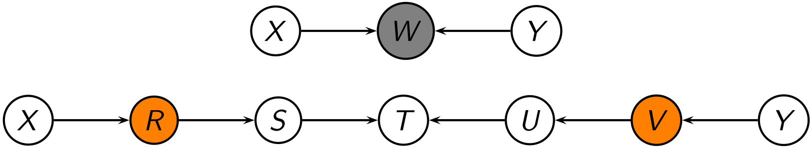
Define four sets

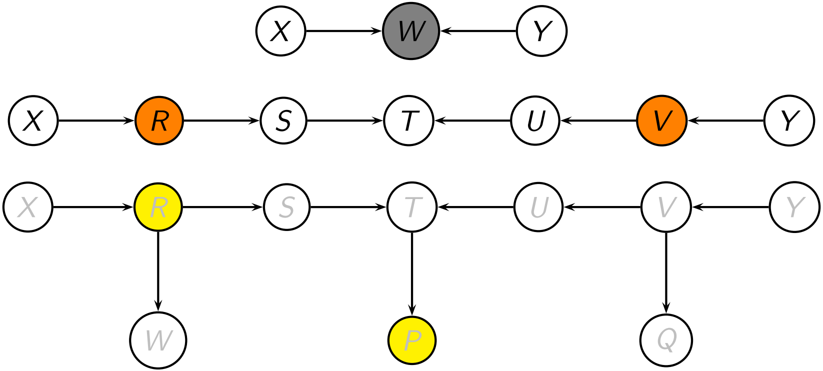
Let A_1, A_2 and A_3 denote sets with:

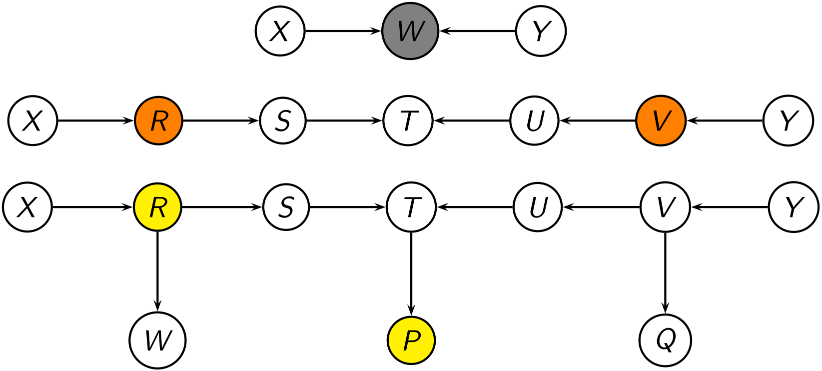
$$\begin{aligned} A_1 &= \{W\} \\ A_2 &= \{R, V\} \\ A_3 &= \{R, P\} \end{aligned}$$


X
W
Y


X
W
Y
X
R
S
T
U
V
Y


X
W
Y
X
R
S
T
U
V
Y


X
W
Y
X
R
S
T
U
V
Y
X
R
S
T
U
V
Y
W
P
Q


X
W
Y
X
R
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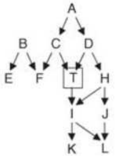
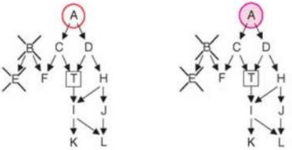
V
Y
X
R
S
T
U
V
Y
W
P
Q



(a) Generating Network

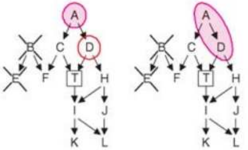


(b) (c)

Iteration 1

(b) A is selected. B & E are removed since $Ind(T;B|\{\})$ and $Ind(T;E|\{\})$.

(c) **CPC** $= \{A\}$

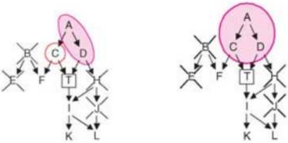


(d) (e)

Iteration 2

(d) D is selected.

(e) **CPC** $= \{A, D\}$

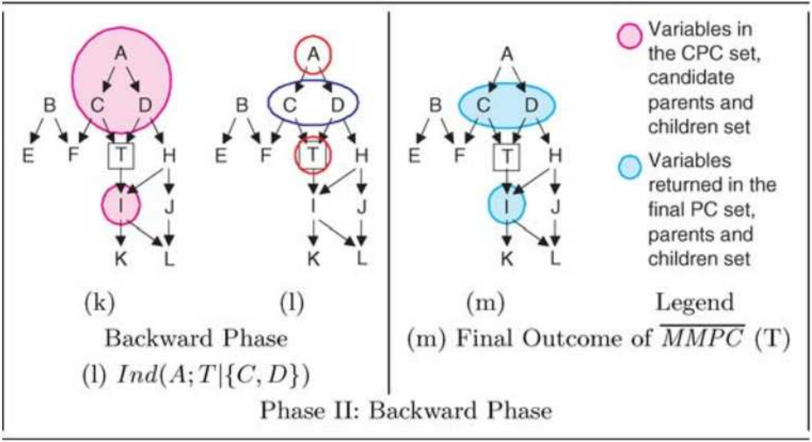


(f) (g)

Iteration 3

(f) C is selected. H & J are removed since $Ind(T;H|\{D\})$ and $Ind(T;J|\{D\})$.

(g) **CPC** $= \{A, C, D\}$


A
B
C
D
E
F
T
H
I
J
K
L
Variables in the CPC set, candidate parents and children set
Variables returned in the final PC set, parents and children set
(k)
(l)
(m)
Legend
Backward Phase
(l) $Ind(A; T|\{C, D\})$
(m) Final Outcome of $\overline{MMPC}$ (T)
Phase II: Backward Phase

Thanks for your attention!