The max-min-hill-climbing algorithm

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M.Sc. Comp. Science

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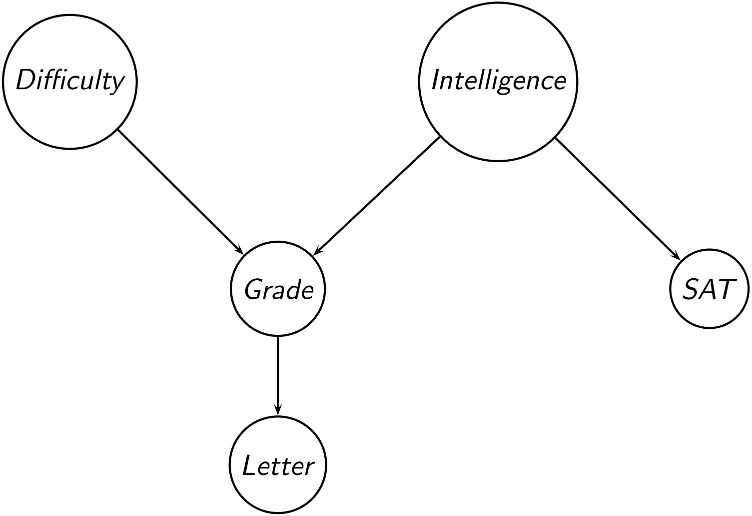
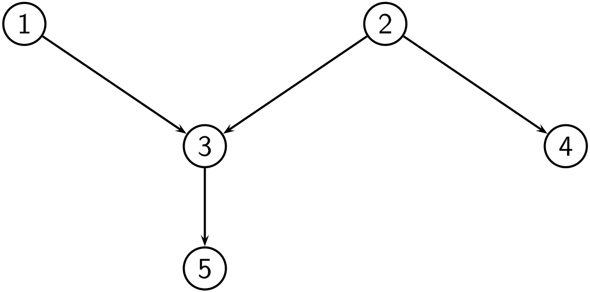
Learning a graph and its structure

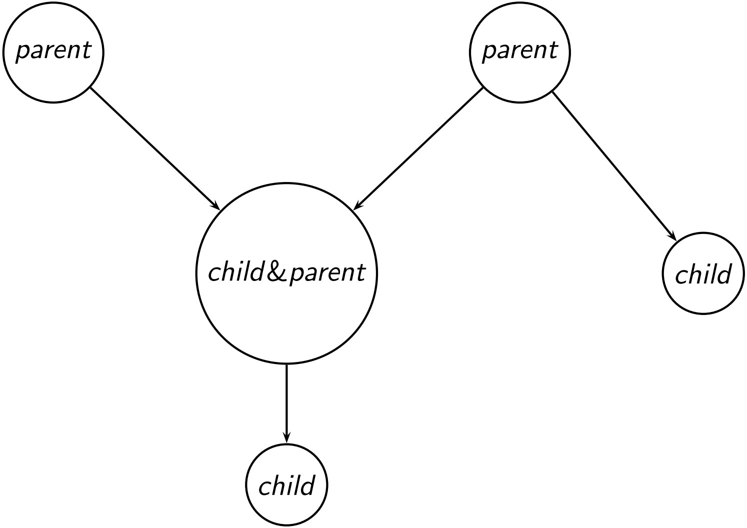

Difficulty
Intelligence
Grade
SAT
Letter

Graph theory - Part I

Directed Acyclic Graph



Bayesian Network



Probability theory

Reminder

Definition (Independence)

Let A, B denote random variables. Then A and B are independent iff

$$P(A \cap B) = P(A) * P(B) \tag{1}$$

Definition (Conditional Probability)

Let A, B denote random variables and $P(B) > 0$. The probability of A given B is defined as:

$$P(A|B) = \frac{P(A \cap B)}{P(B)} \tag{2}$$

Combining these approaches

Definition (Conditional Independence)

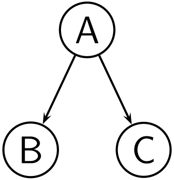
Two variables X and Y are conditionally independent given $\mathbf{Z}$ w.r.t a probability distribution P, denoted as $Ind_p(X; Y|\mathbf{Z})$, if $\forall x, y, \mathbf{z}$, where $P(\mathbf{Z} = \mathbf{z}) > 0$,

$$P(X, Y|\mathbf{Z}) = P(X|\mathbf{Z}) * P(Y|\mathbf{Z}) \tag{3}$$

where $P(X, Y|\mathbf{Z}) = P(X \cap Y|\mathbf{Z})$.

Graph theory - Part II

Markow Condition



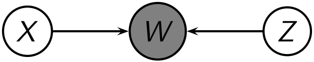
Collider

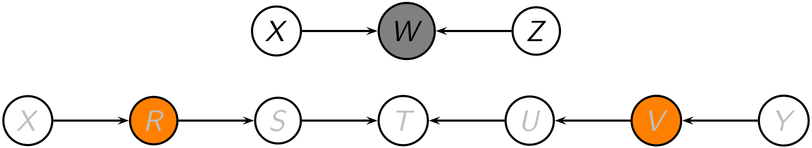
Blocked paths and d-seperation (Examples)

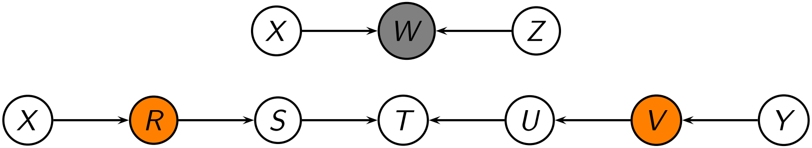
Define three sets

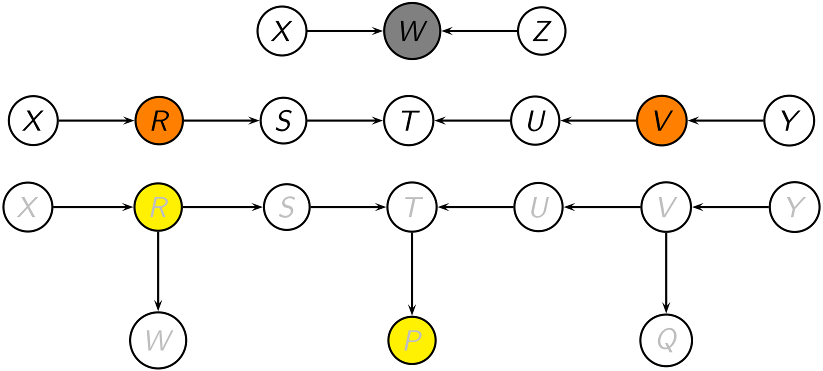
Let Z_1, Z_2 and Z_3 denote sets with:

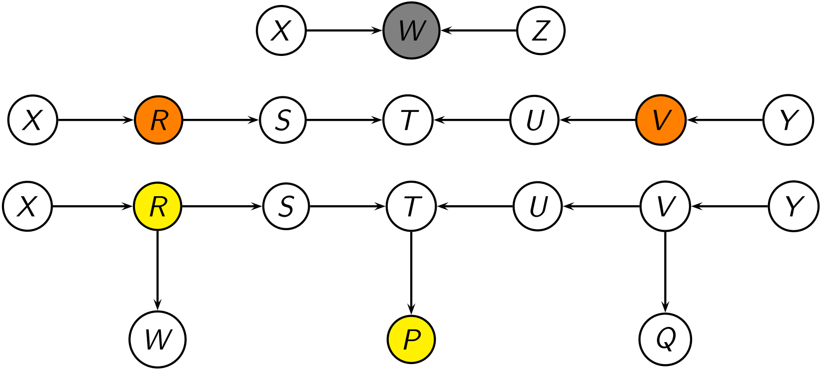
$$\begin{aligned} Z_1 &= \{W\} \\ Z_2 &= \{R, V\} \\ Z_3 &= \{R, P\} \end{aligned}$$


X
W
Z


X
W
Z
X
R
S
T
U
V
Y


X
W
Z
X
R
S
T
U
V
Y


X
W
Z
X
R
S
T
U
V
Y
X
R
S
T
U
V
Y
W
P
Q


X
W
Z
X
R
S
T
U
V
Y
X
R
S
T
U
V
Y
W
P
Q

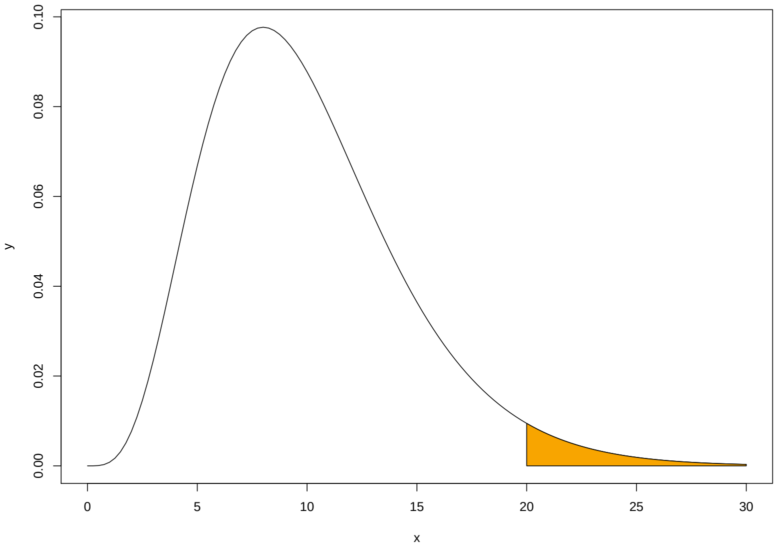
Statistics

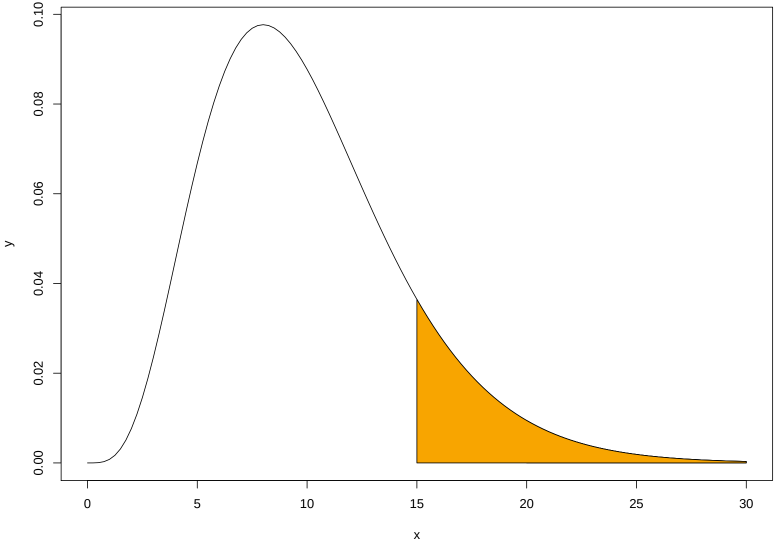
The G^2 value

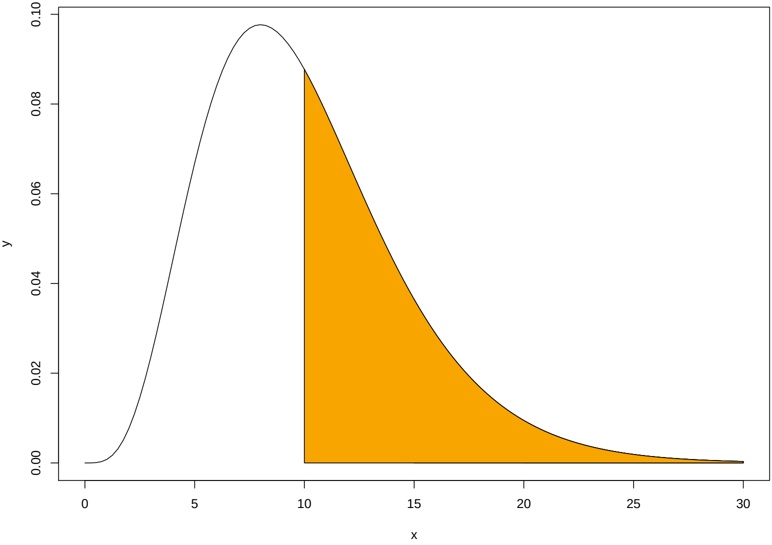
Definition

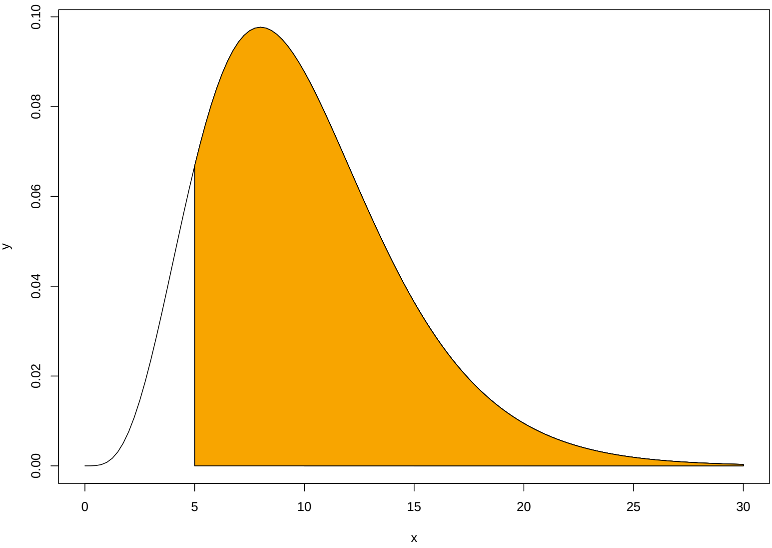
We calculate the G^2 value under the nullhypothesis of the conditional independence of $Ind(X_i, X_j|\mathbf{X}_k$ holding. Then, the G^2 value is defined as:

$$G^2 := 2 * \sum_{a,b,c} S_{ijk}^{ab\mathbf{c}} * ln\left(\frac{S_{ijk}^{ab\mathbf{c}} * S_k^{\mathbf{c}}}{S_{ik}^{a\mathbf{c}} * S_{jk}^{b\mathbf{c}}}\right) \tag{4}$$


0.00
0.02
0.04
0.06
0.08
0.10
y
0
5
10
15
20
25
30
x


0.10
0.08
0.06
0.04
0.02
0.00
y
0
5
10
15
20
25
30
x


0.10
0.08
0.06
0.04
0.02
0.00
y
0
5
10
15
20
25
30
x


0.00
0.02
0.04
0.06
0.08
0.10
y
0
5
10
15
20
25
30
x

Computational properties

Remark to MMHC

- A hybrid algorithm
  - Greedy Algorithm

Remark to MMHC

- A hybrid algorithm
  - Greedy Algorithm
  - Constrained-based

Remark to MMHC

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  - Greedy Algorithm
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- np-hard

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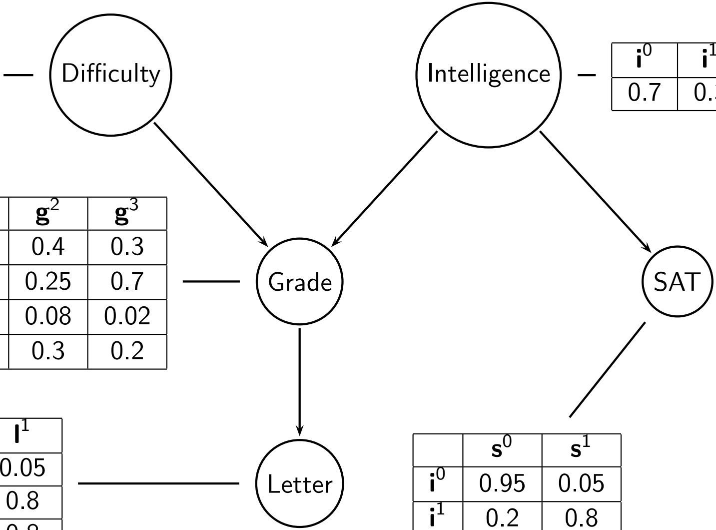

d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

	g^1	g^2	g^3
i^0,d^0	0.3	0.4	0.3
i^0,d^1	0.05	0.25	0.7
i^1,d^0	0.9	0.08	0.02
i^1,d^1	0.5	0.3	0.2

	l^0	l^1
g^1	0.95	0.05
g^2	0.2	0.8
g^3	0.2	0.8

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8