The max-min-hill-climbing algorithm

Michael Bauer

M.Sc. Comp. Science

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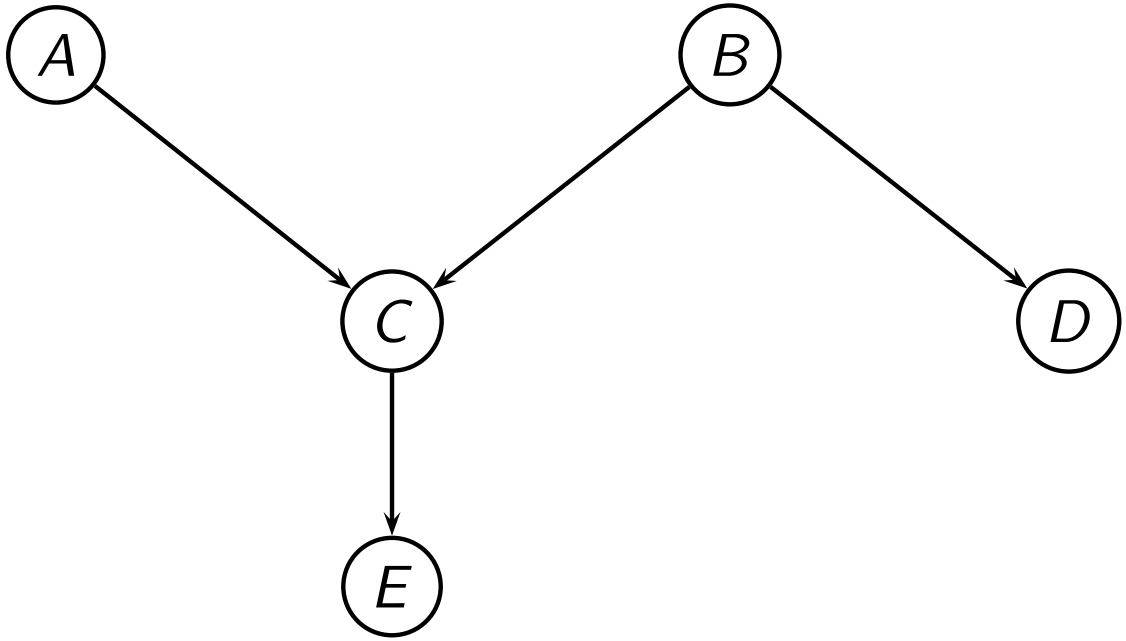
Definition

A Bayesian Network is a directed acyclic graph (DAG) whose nodes are random variables and edges represent conditional dependencies. If two random variables are connected they are said to be dependent. If there is no connection they are said to be conditional independent.
For instance, we say: "A and B are conditional independent given C".



- directed edges
- free of cycles
- random variable is represented as a node
- edges encode dependencies

Predicting the effect of missense mutations on protein function: analysis with Bayesian networks

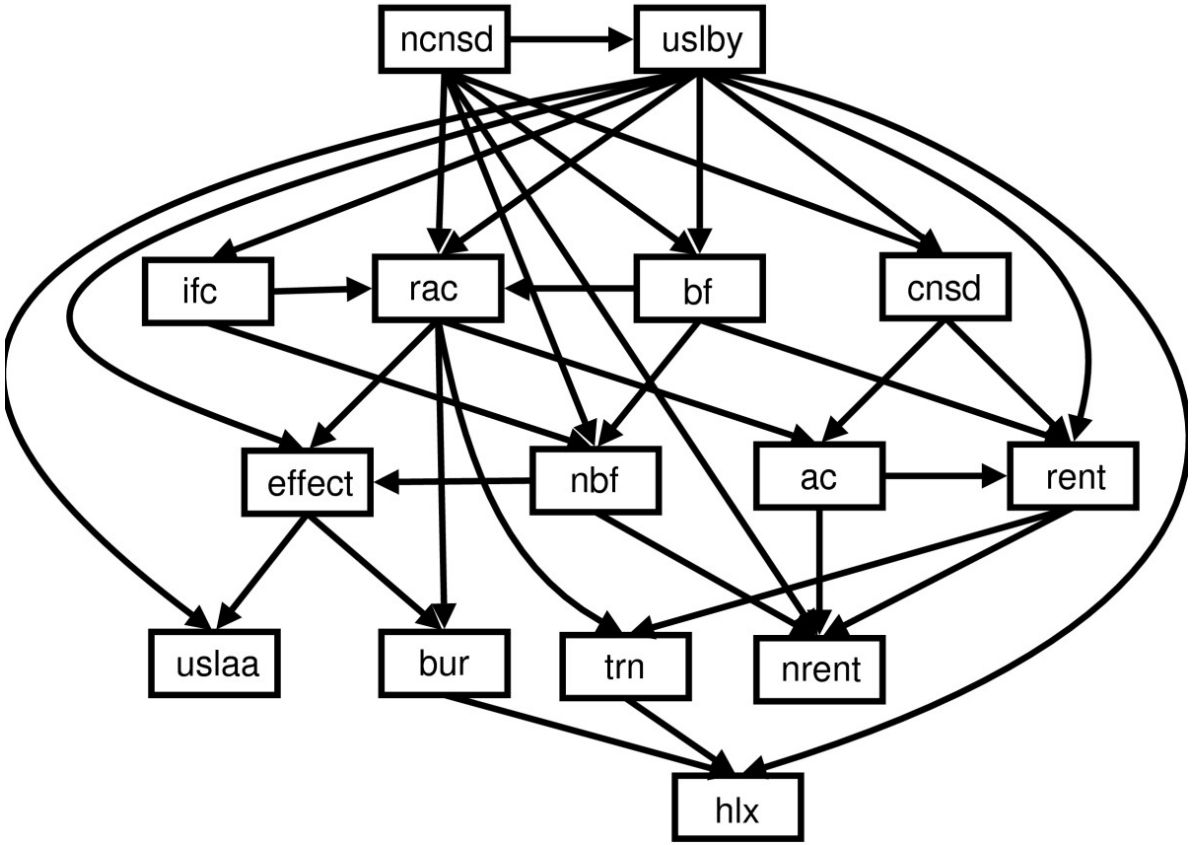


Figure: http://www.biomedcentral.com/1471-2105/7/405/figure/F2?highres=y (by Chris J Needham1, James R Bradford, Andrew J Bulpitt, Matthew A Care and David R Westhead)

Bayesian Networks in sports and medicine

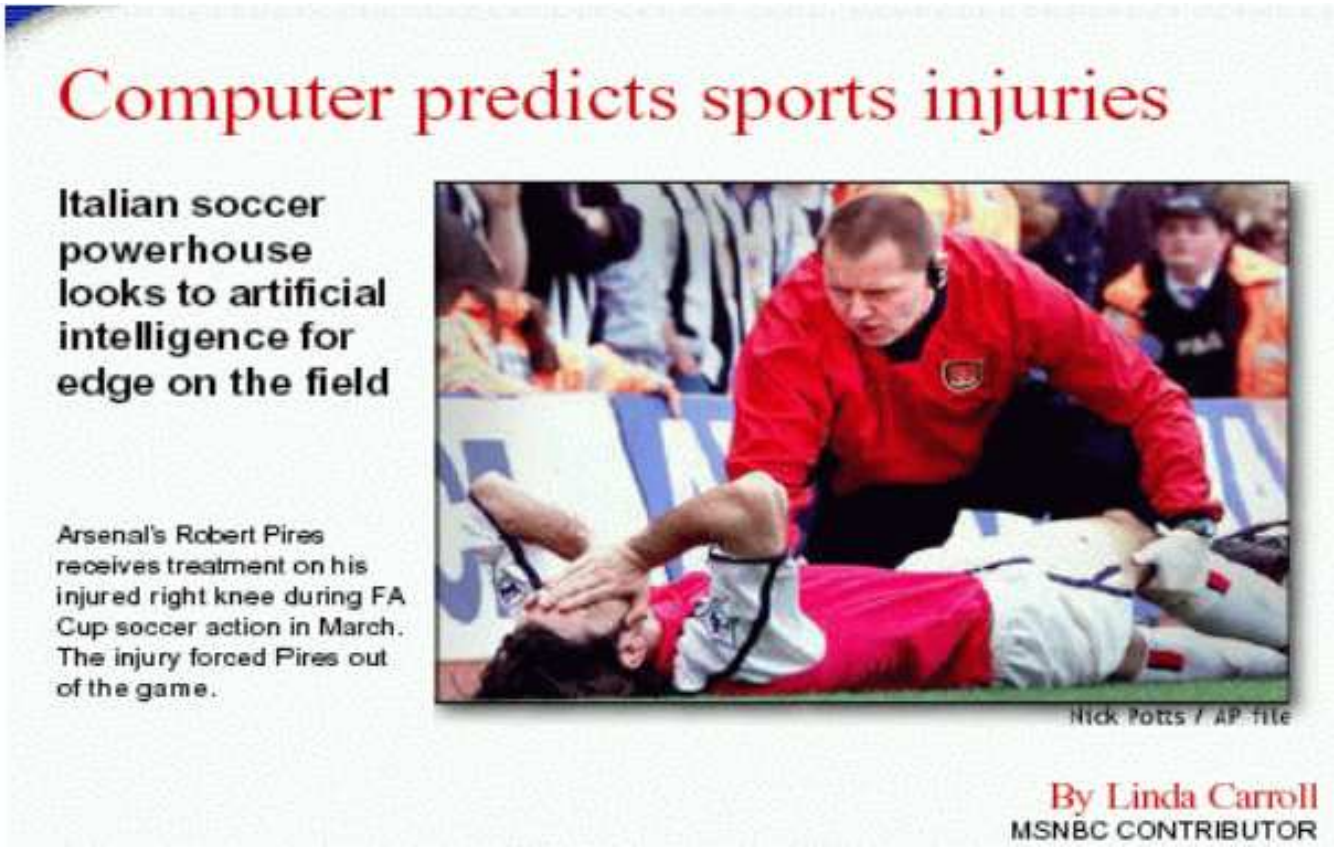
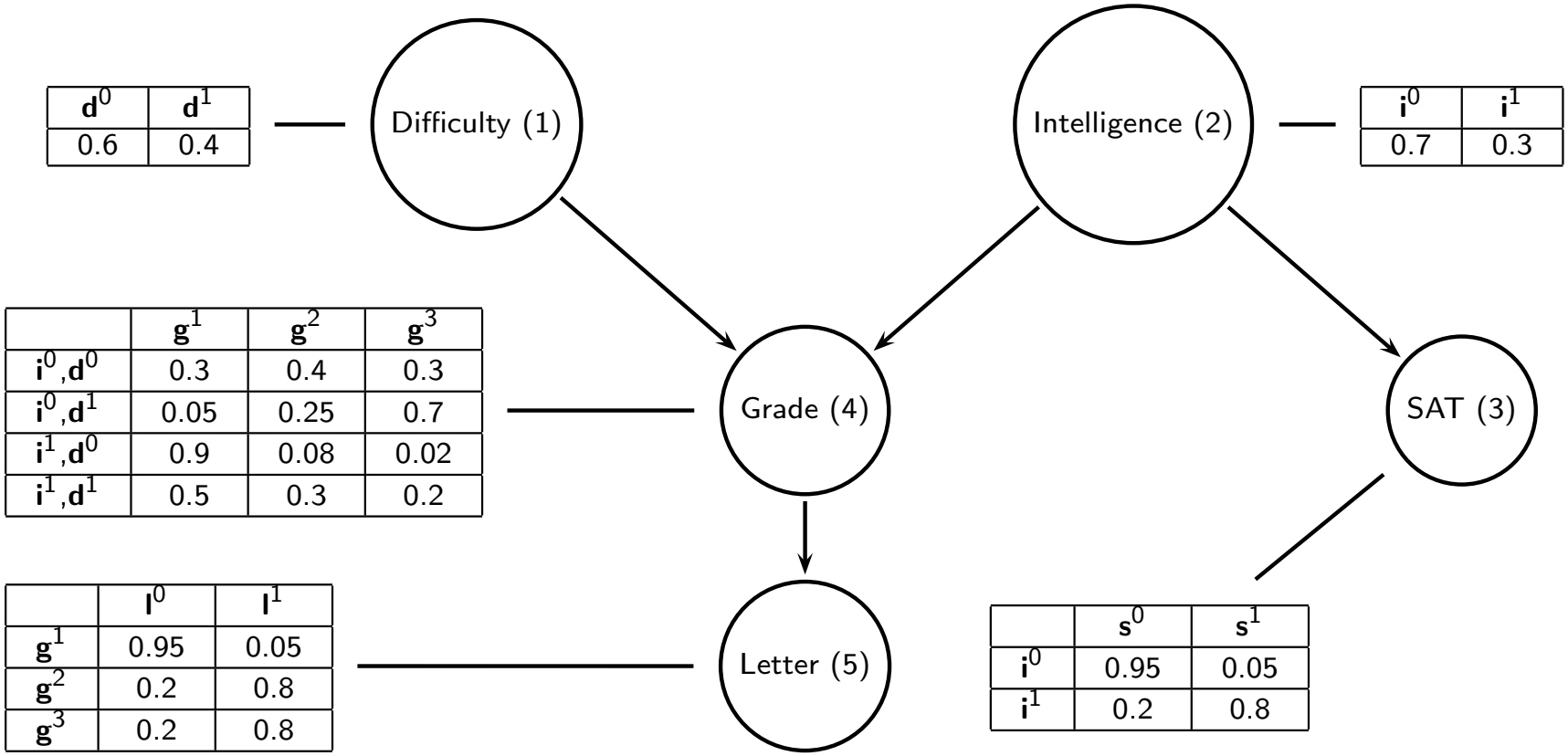


Figure: http://www-ekp.physik.uni-karlsruhe.de/županc/WS1011/docs/Datenanalyse2010_3.pdf



d^0	d^1
0.6	0.4

i^0	i^1
0.7	0.3

	g^1	g^2	g^3
i^0,d^0	0.3	0.4	0.3
i^0,d^1	0.05	0.25	0.7
i^1,d^0	0.9	0.08	0.02
i^1,d^1	0.5	0.3	0.2

	l^0	l^1
g^1	0.95	0.05
g^2	0.2	0.8
g^3	0.2	0.8

	s^0	s^1
i^0	0.95	0.05
i^1	0.2	0.8

```
> head(dataFrame)
     diff int SAT gra let
[1,]    1   2   2   1   2
[2,]    1   1   1   1   2
[3,]    1   1   1   3   1
[4,]    2   2   2   2   2
[5,]    1   1   1   1   2
[6,]    2   1   1   3   1
```

Figure: The data we observe from following the rules above.

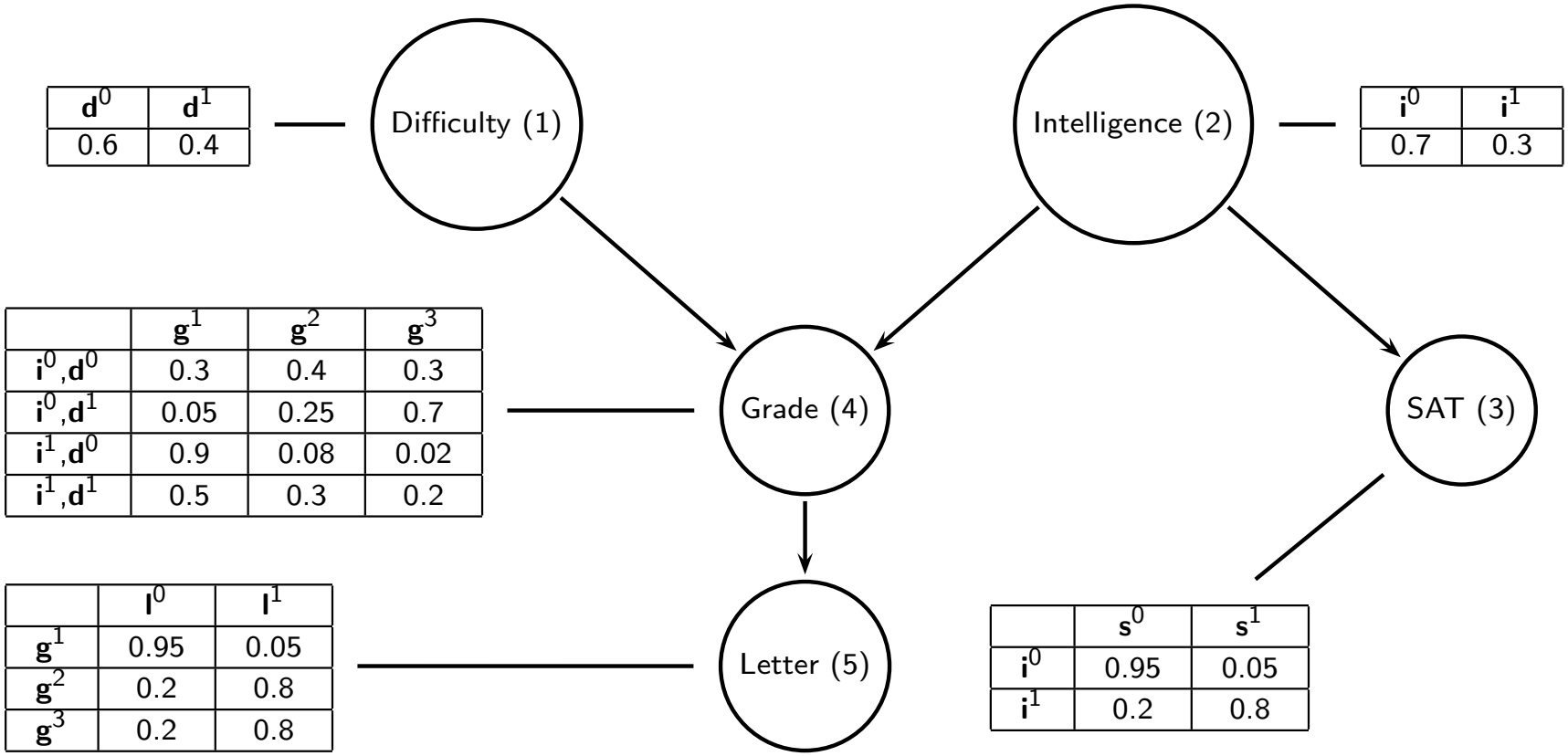


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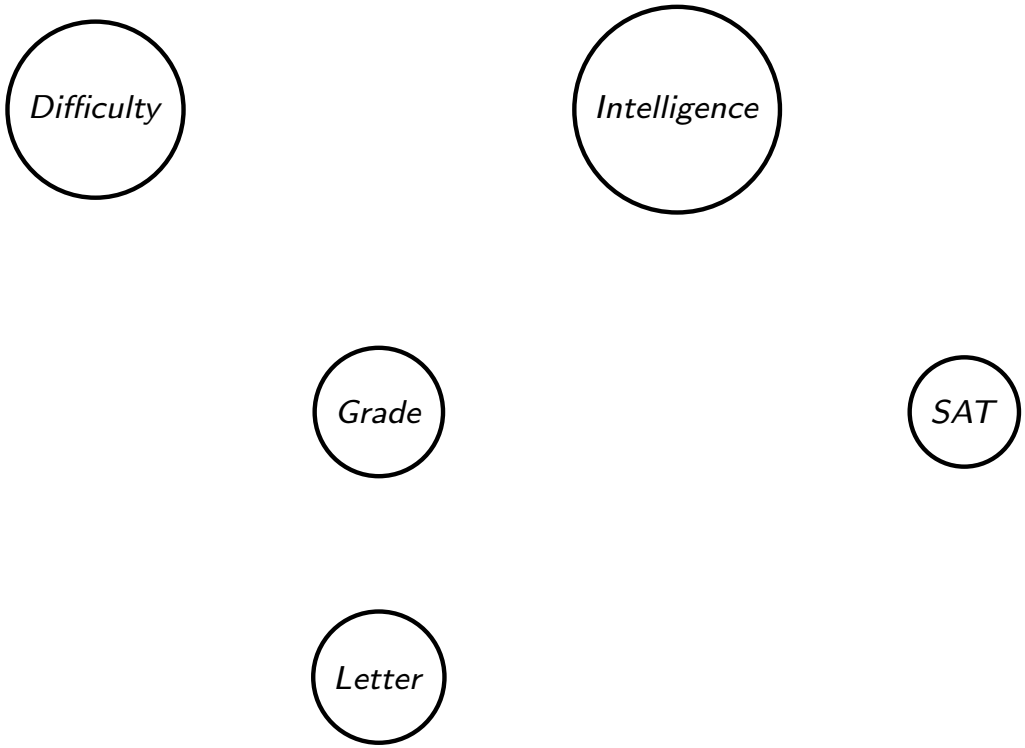
Figure: The data we observe from following the rules above.

Empty graph without any edges

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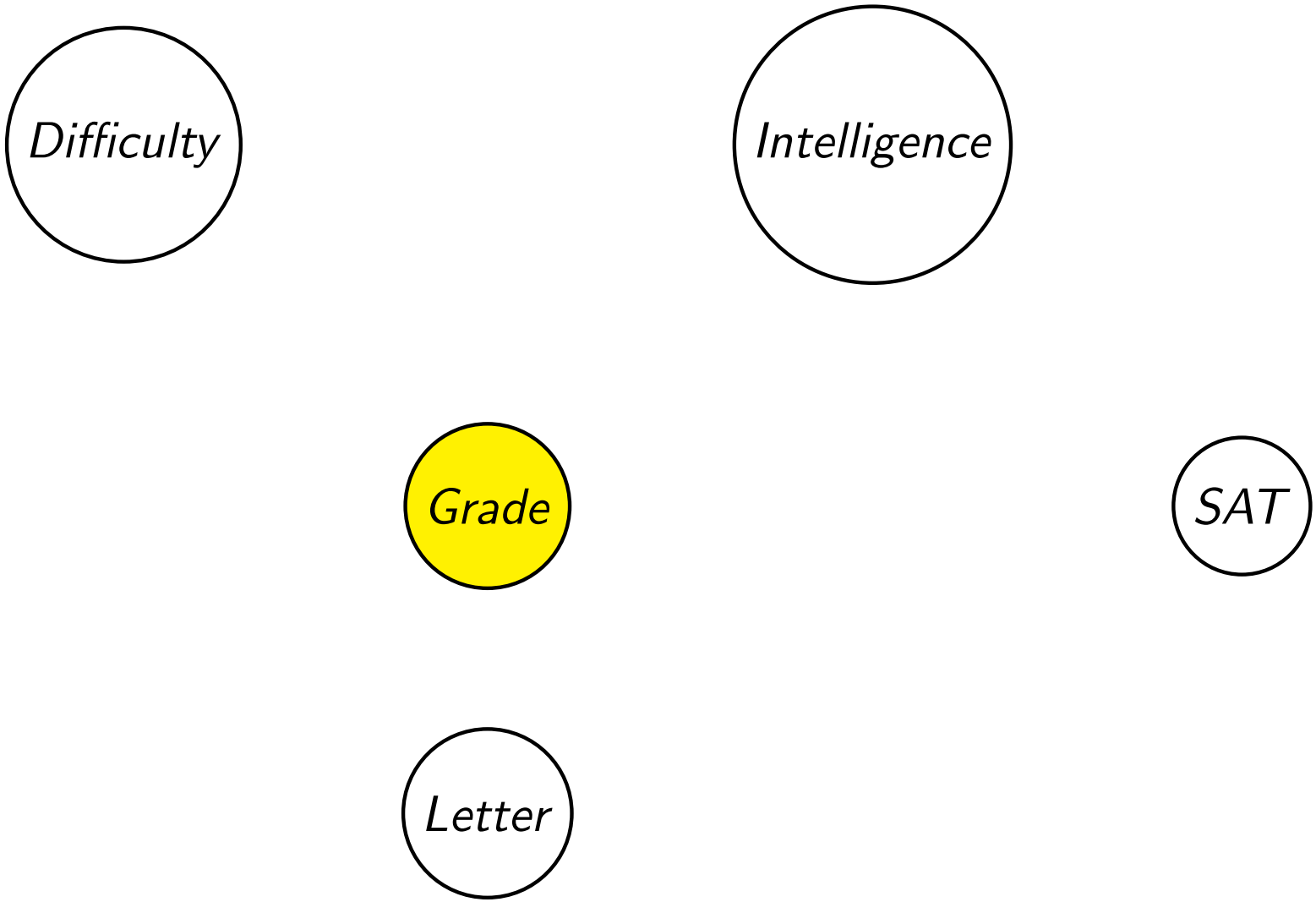
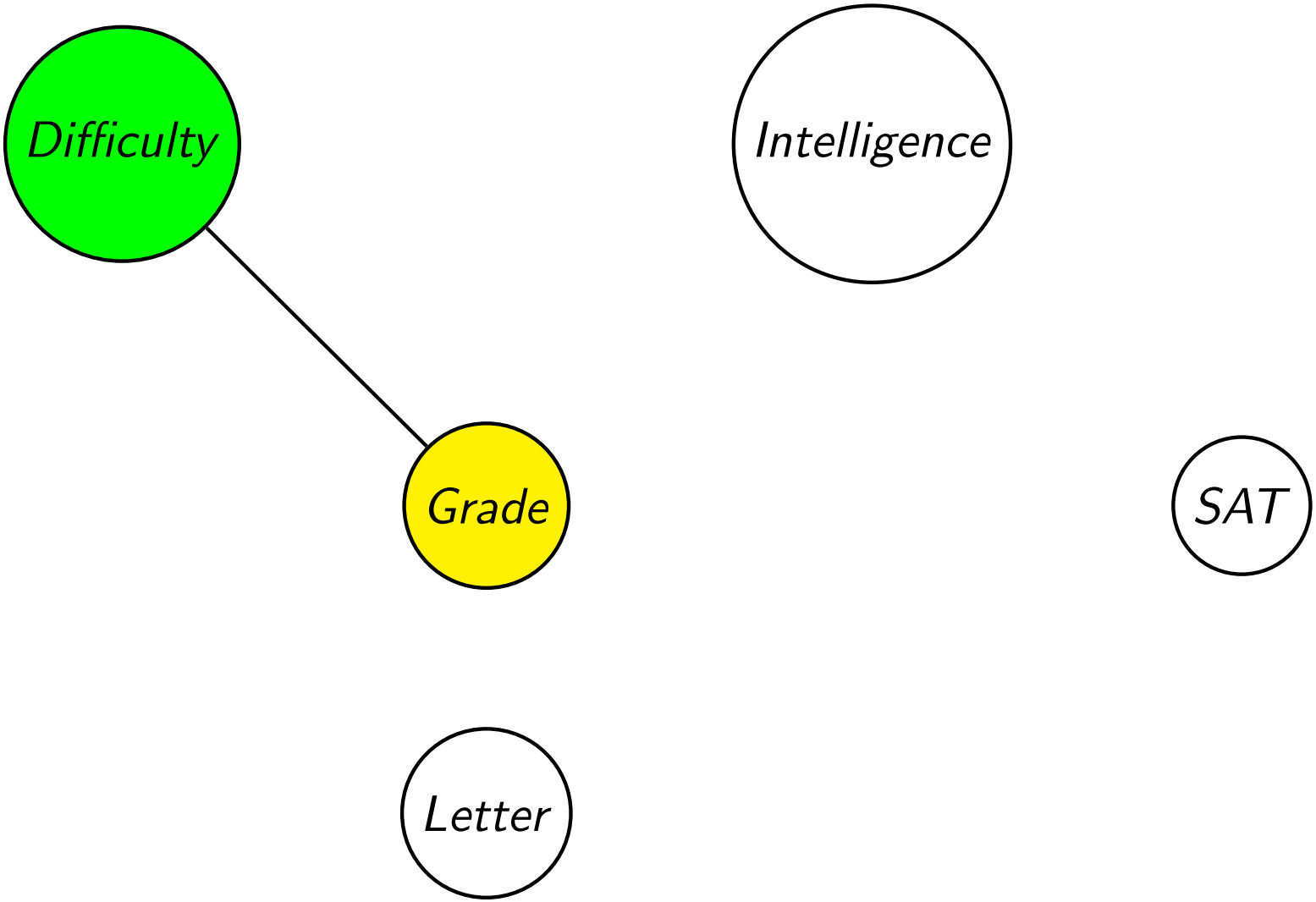
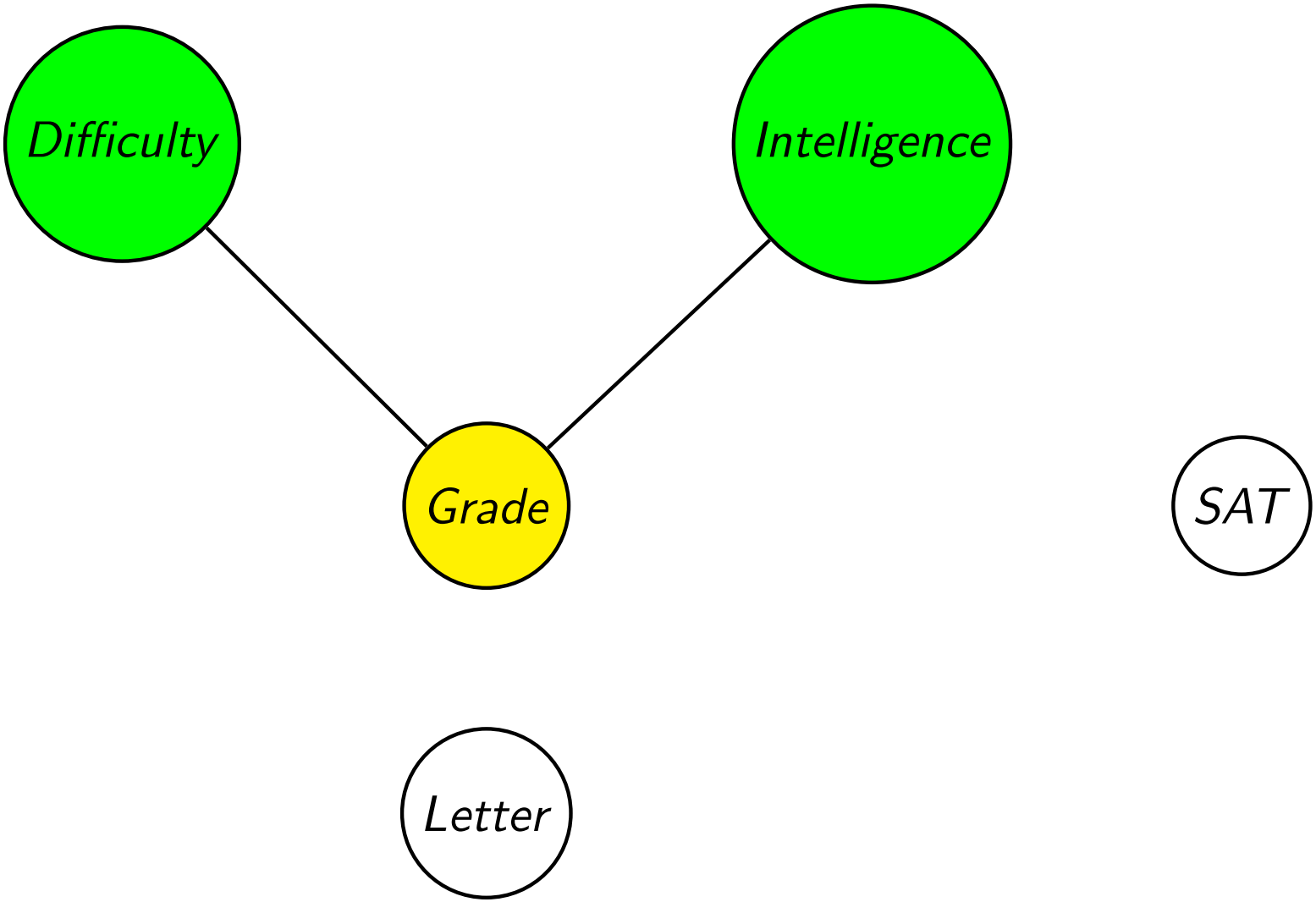
One iteration for the "Grade" node



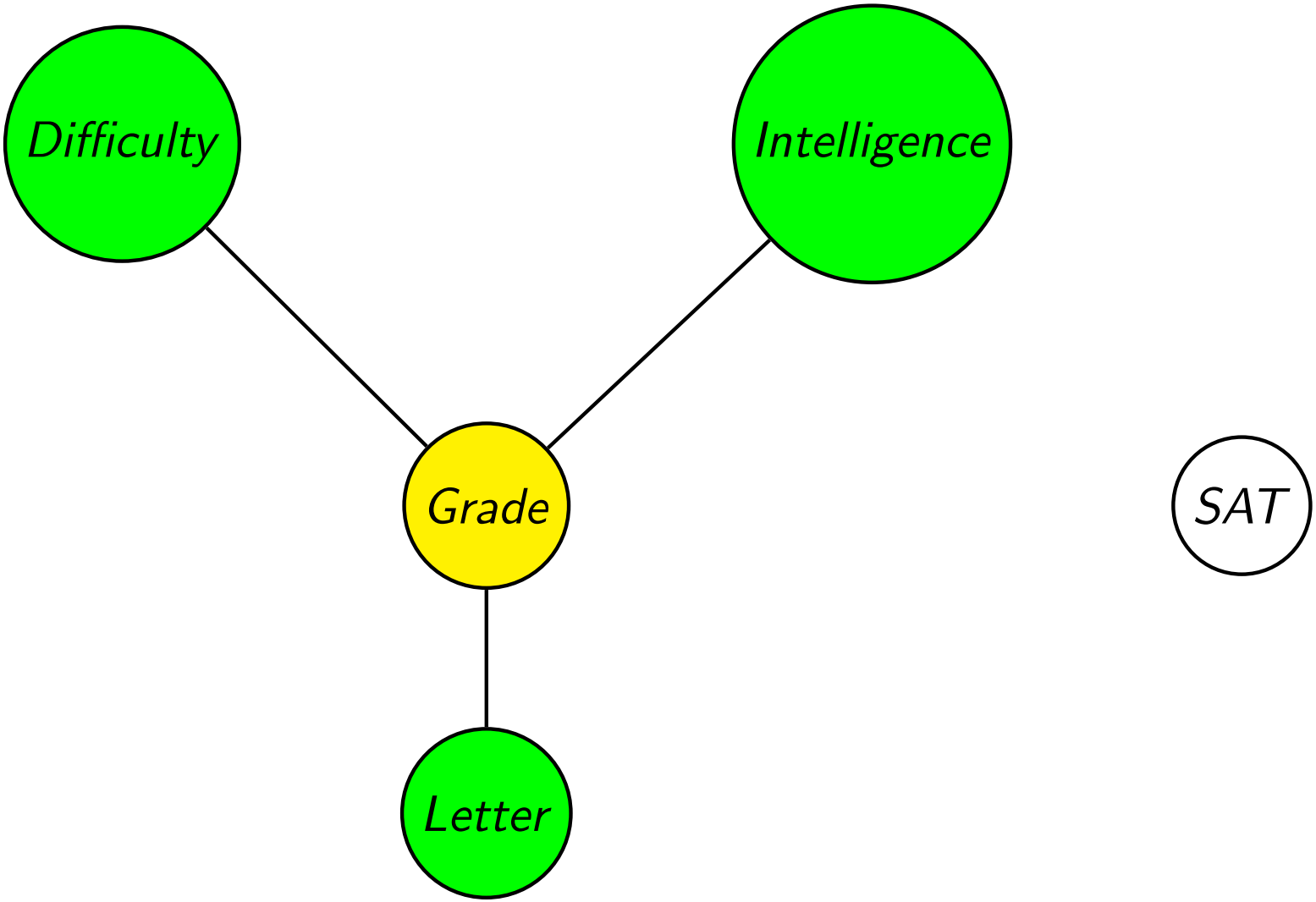
One iteration for the "Grade" node



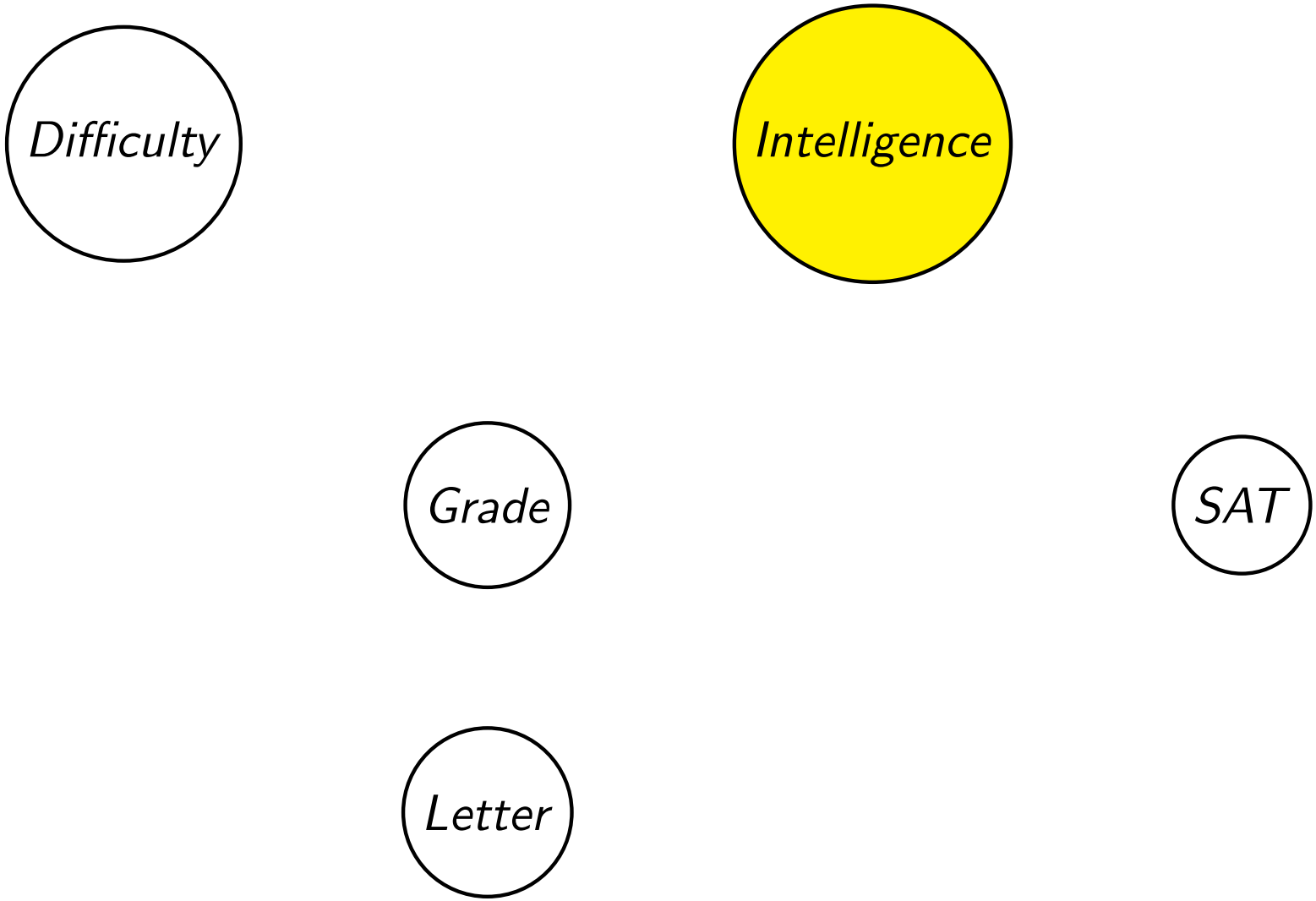
One iteration for the "Grade" node

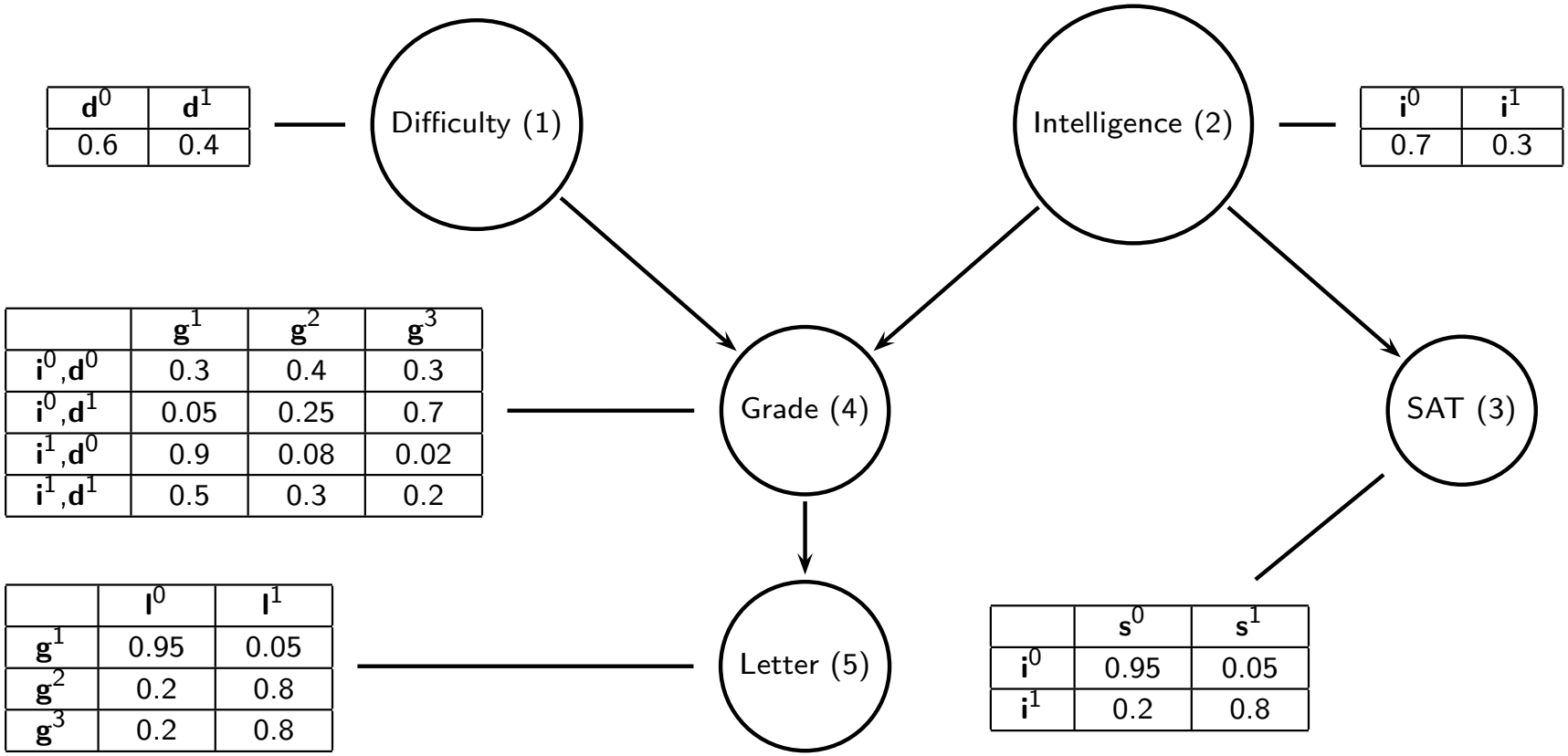


All parents or children are found



Start new iteration





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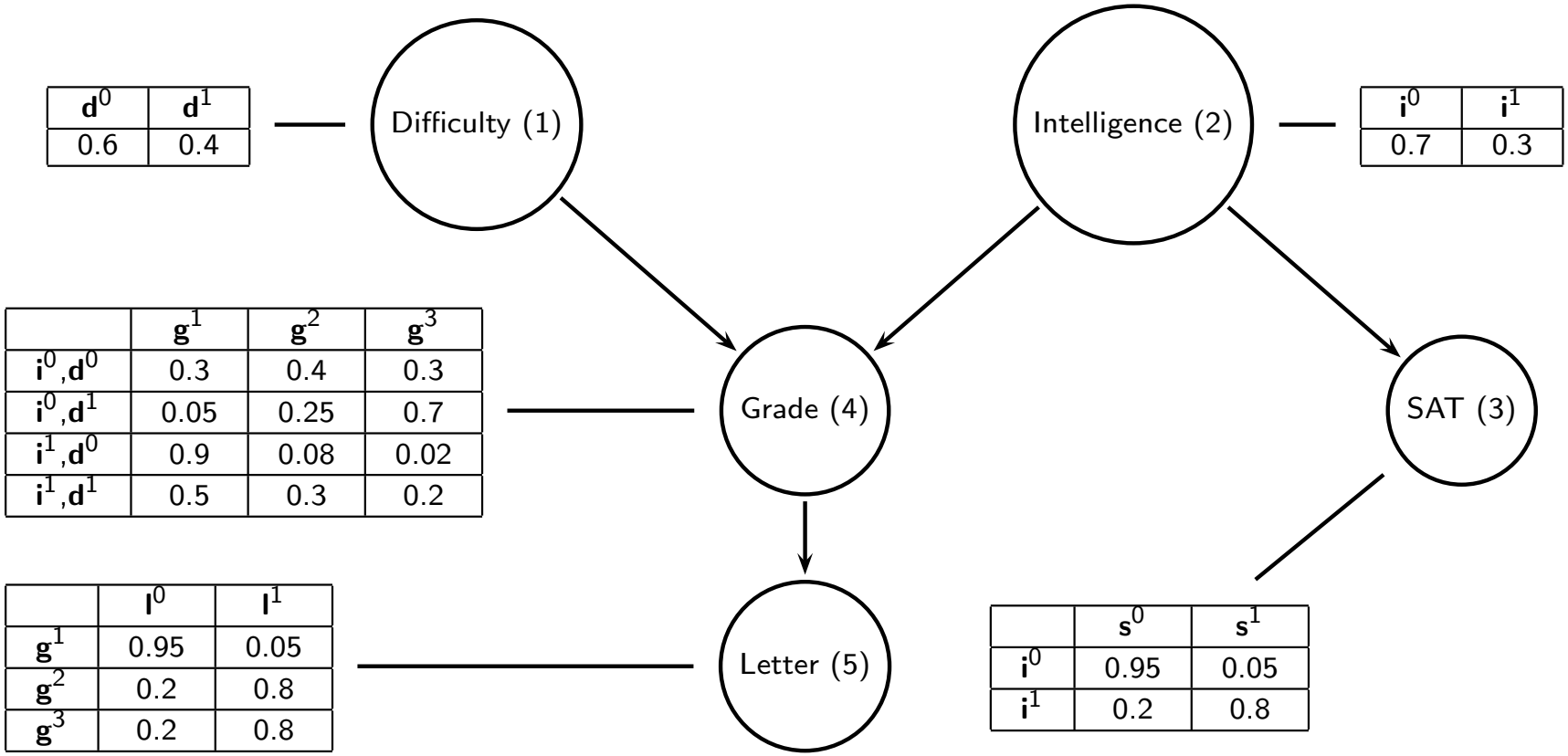
```
> C_MMPC(dataSet)
[[Difficulty]]  [[Intelligence]]  [[SAT]]  [[Grade]]  [[Letter]]
[1] 4           [1] 3 4           [1] 2    [1] 5 2 1  [1] 4
```



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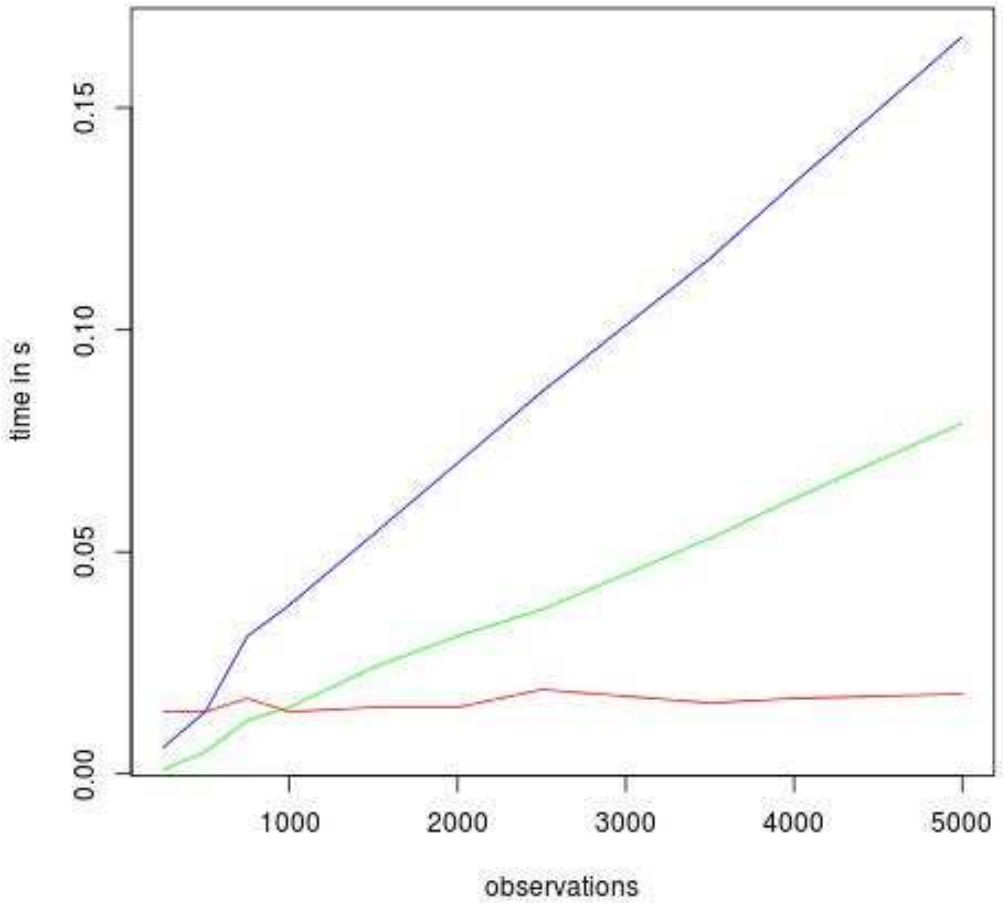
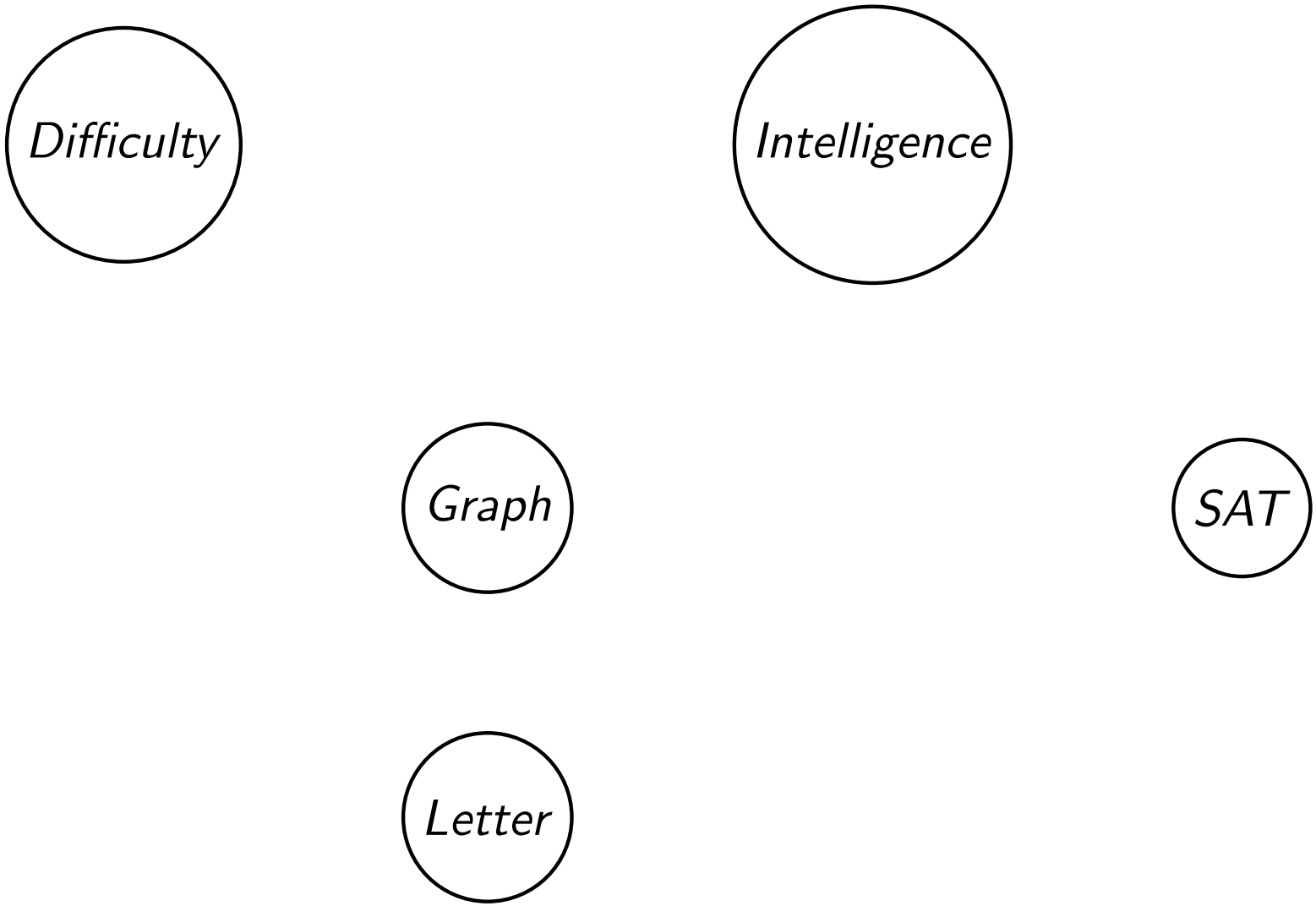
The benchmark for this algorithm

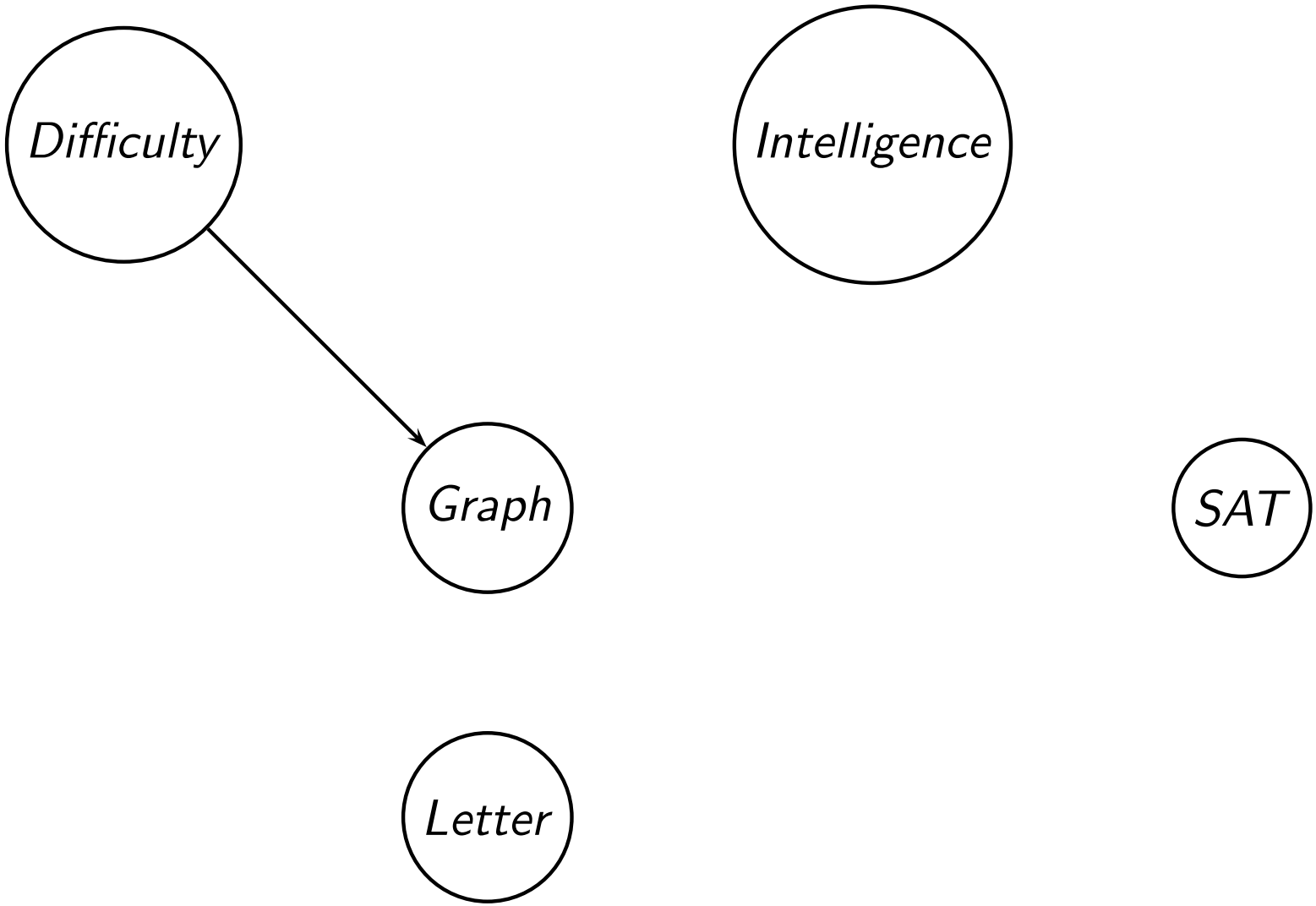


nobs	R	C	bnlearn
250	0.006	0.001	0.014
500	0.014	0.005	0.014
750	0.031	0.012	0.017
1000	0.038	0.015	0.014
1500	0.054	0.024	0.015
2500	0.086	0.037	0.019
5000	0.166	0.079	0.018

Bayesian Dirichlet equivalent uniform (BDeu) score



Adding an edge



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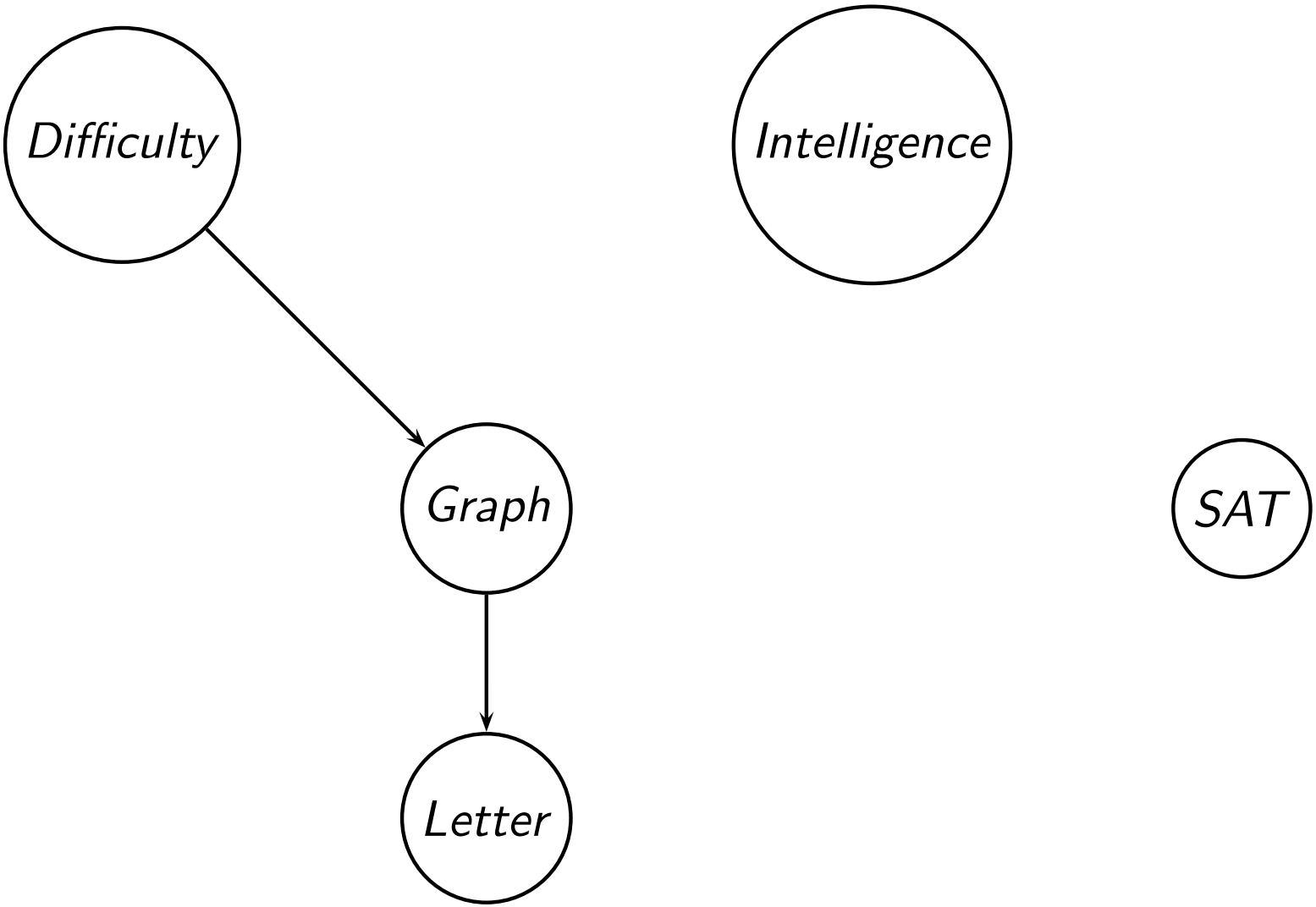
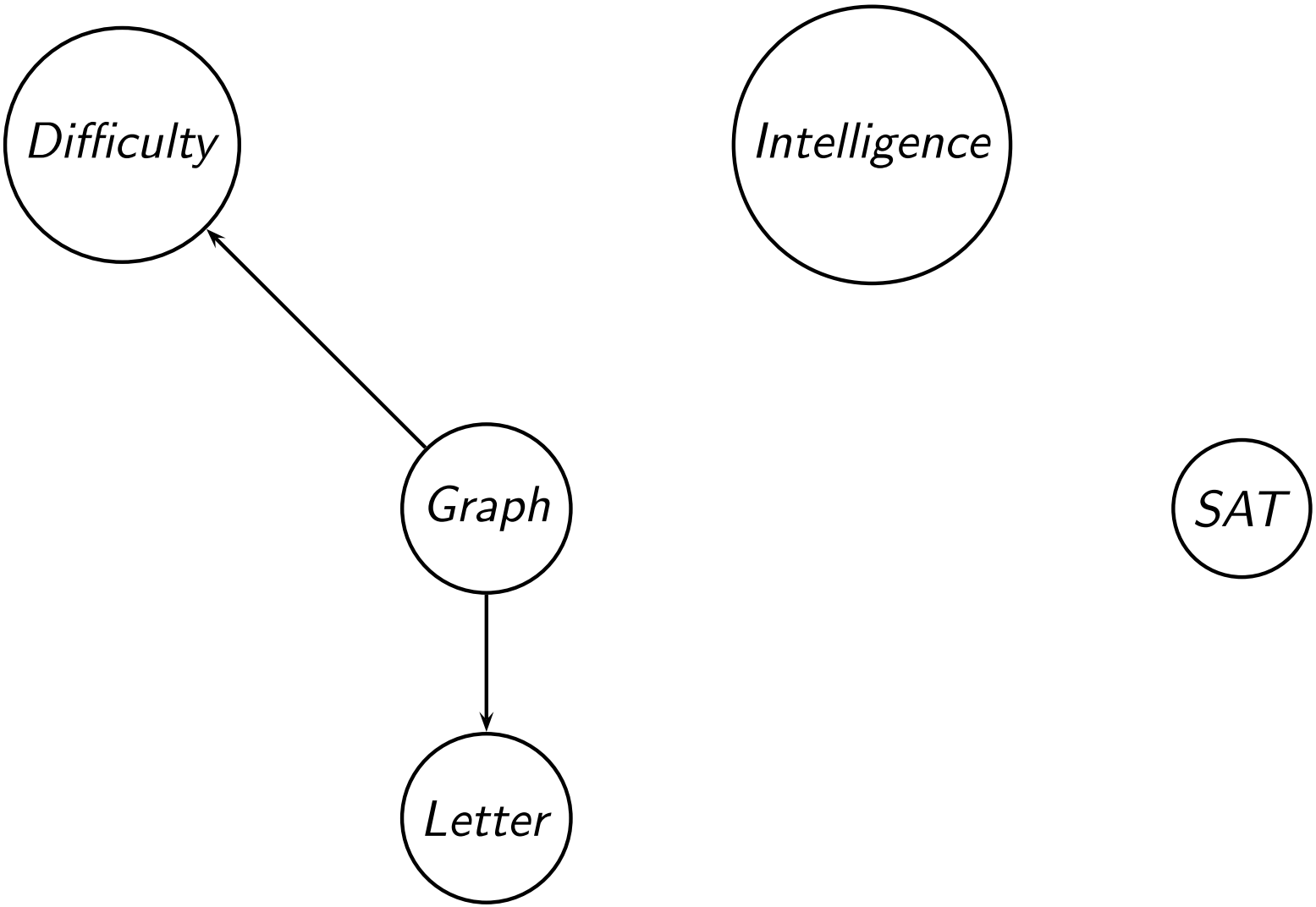


Also possible: reverse and delete



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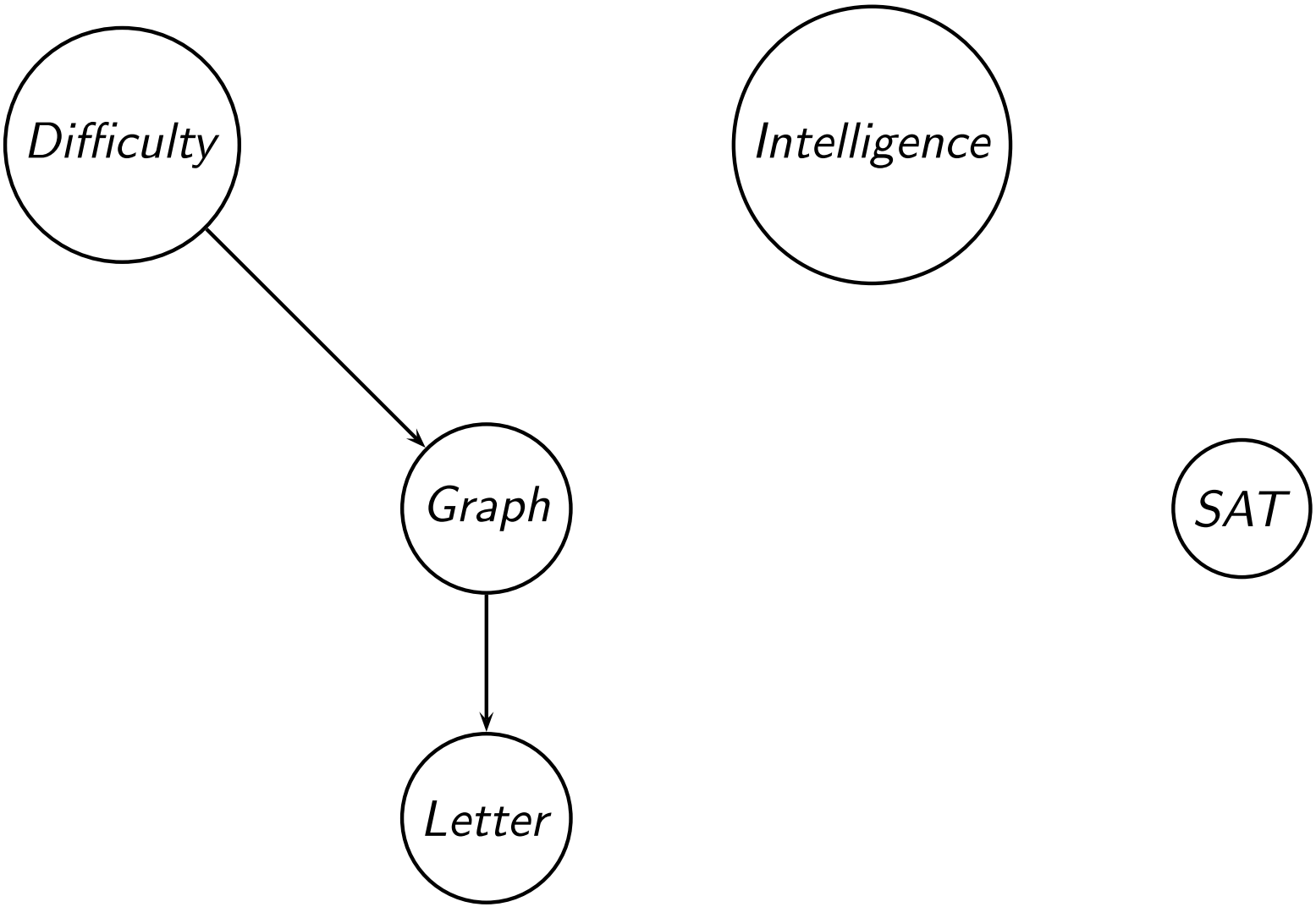

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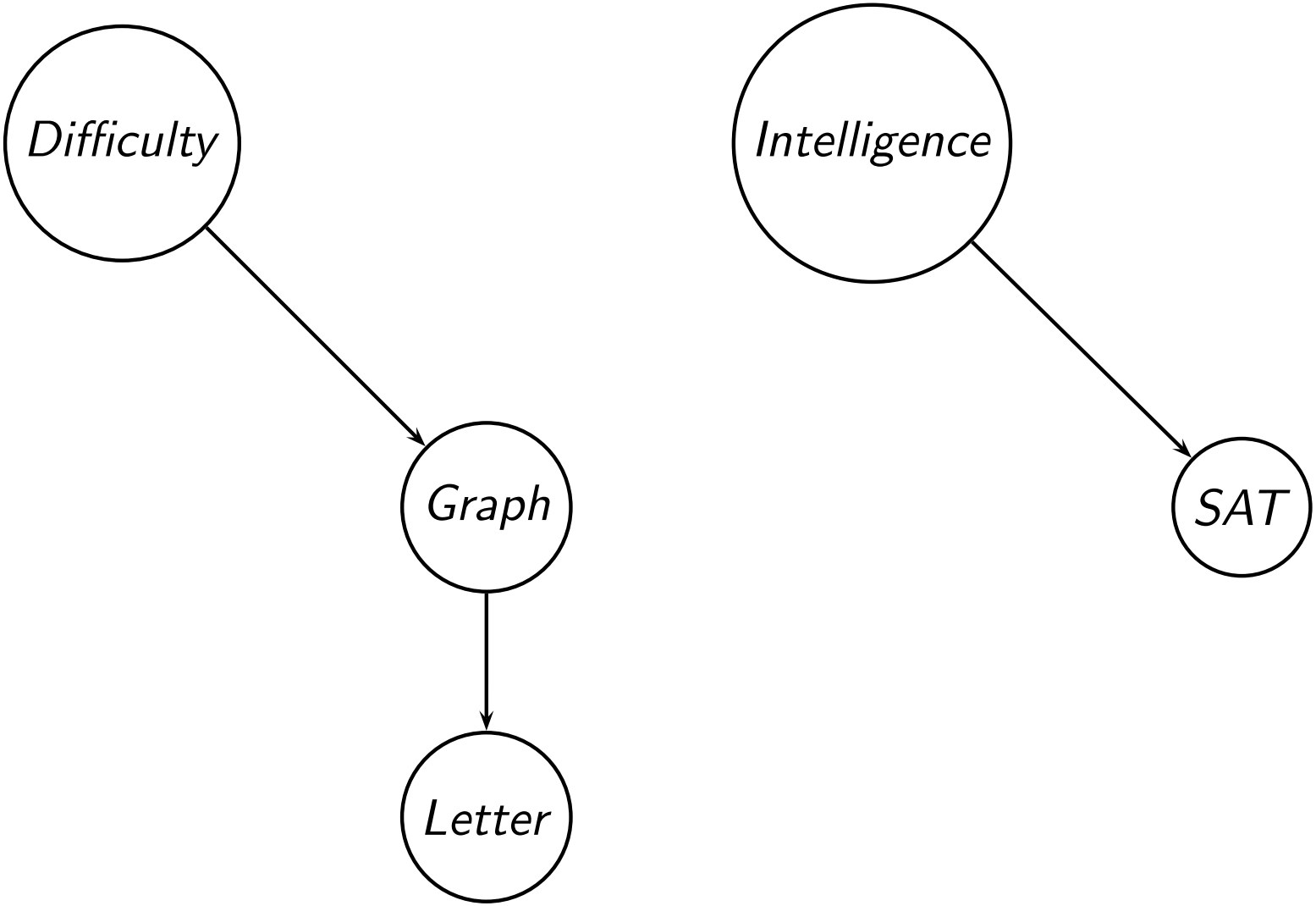

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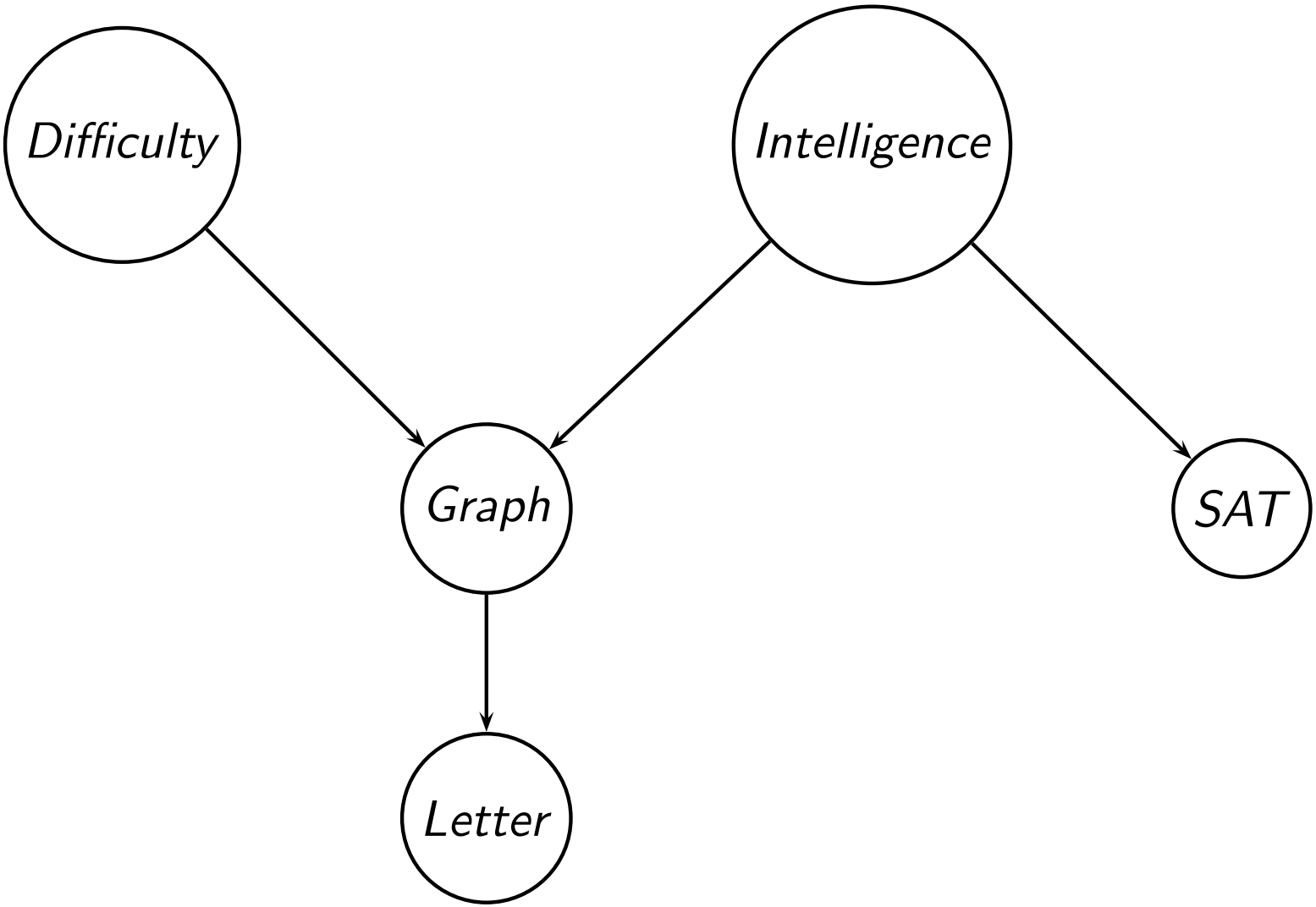
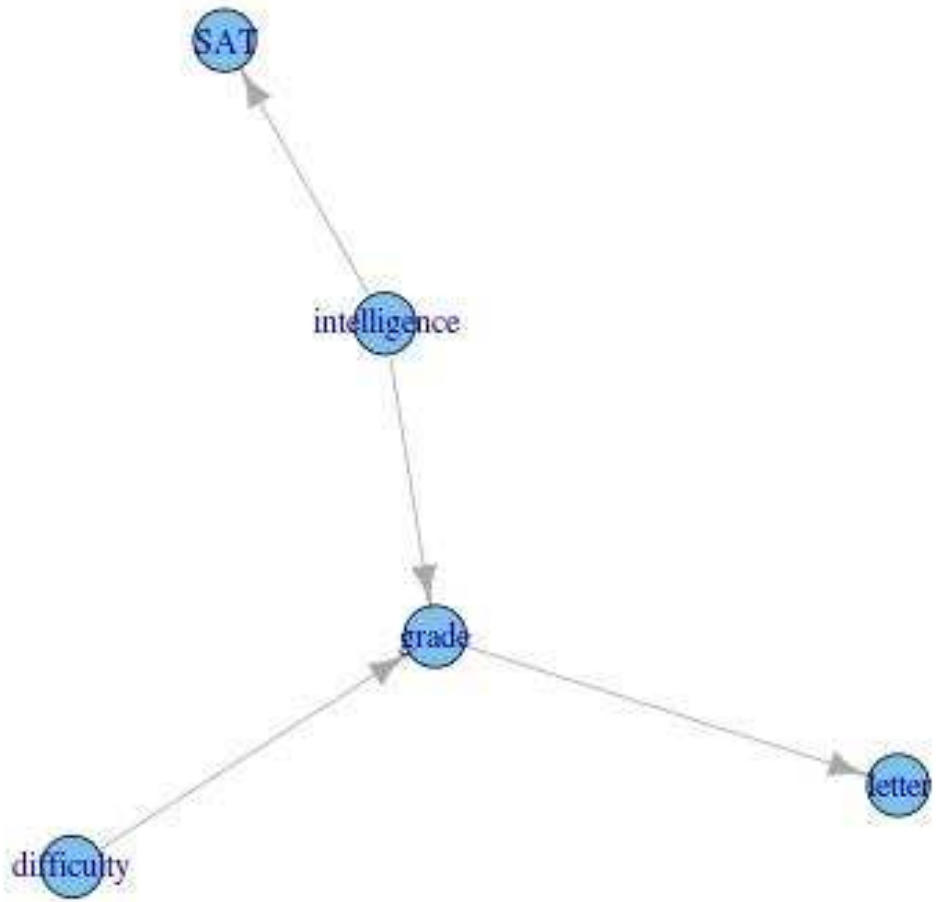


Output of my programme



Thanks for your attention!