Probabilistic logic and statistical inference

STATISTICAL THINKING IN PYTHON (PART 1)

Justin Bois
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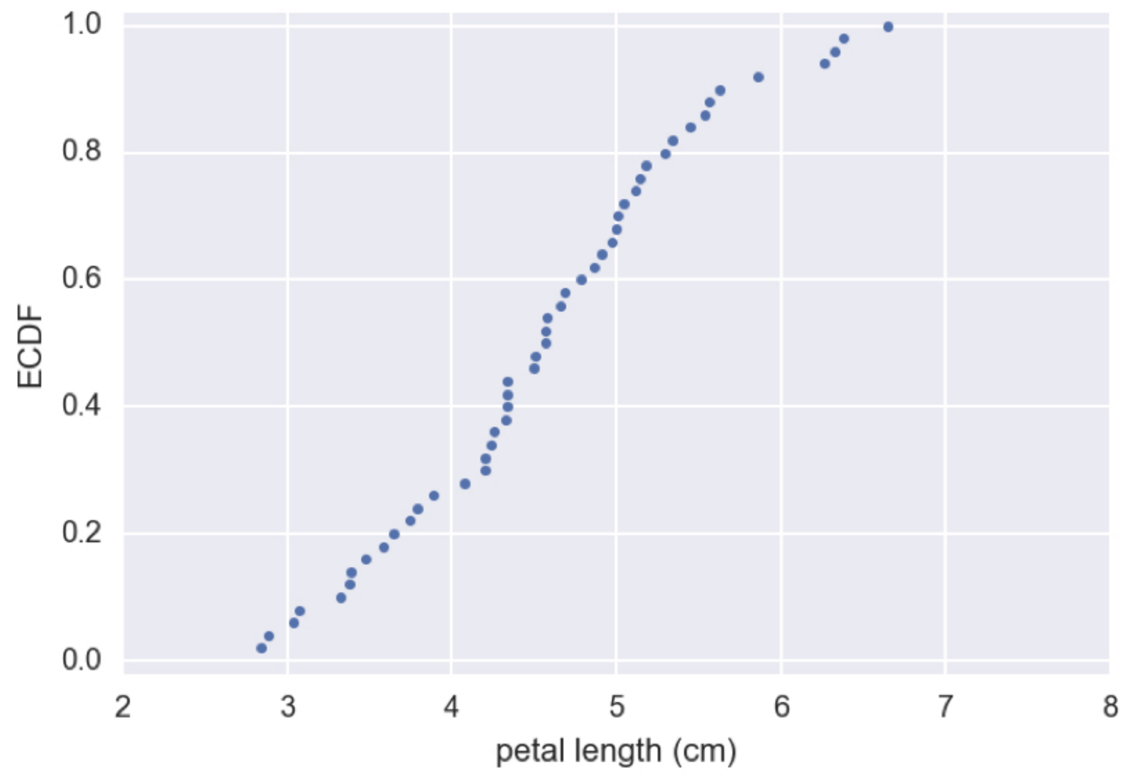
50 measurements of petal length



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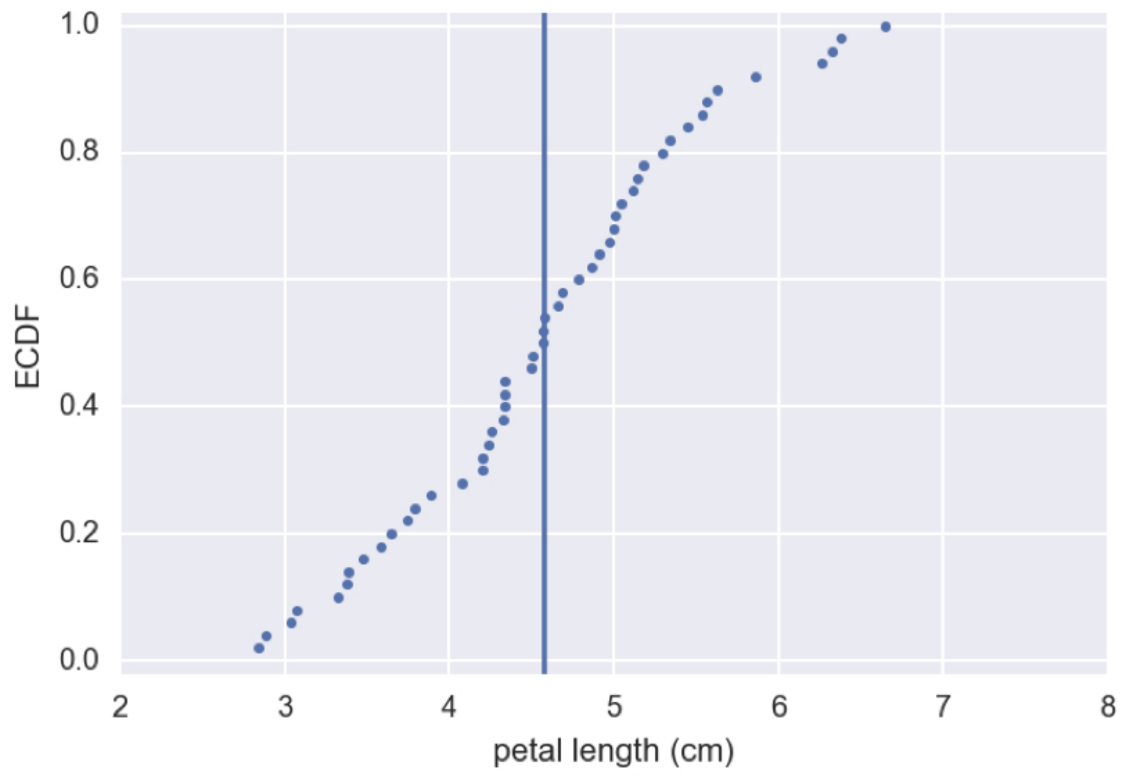
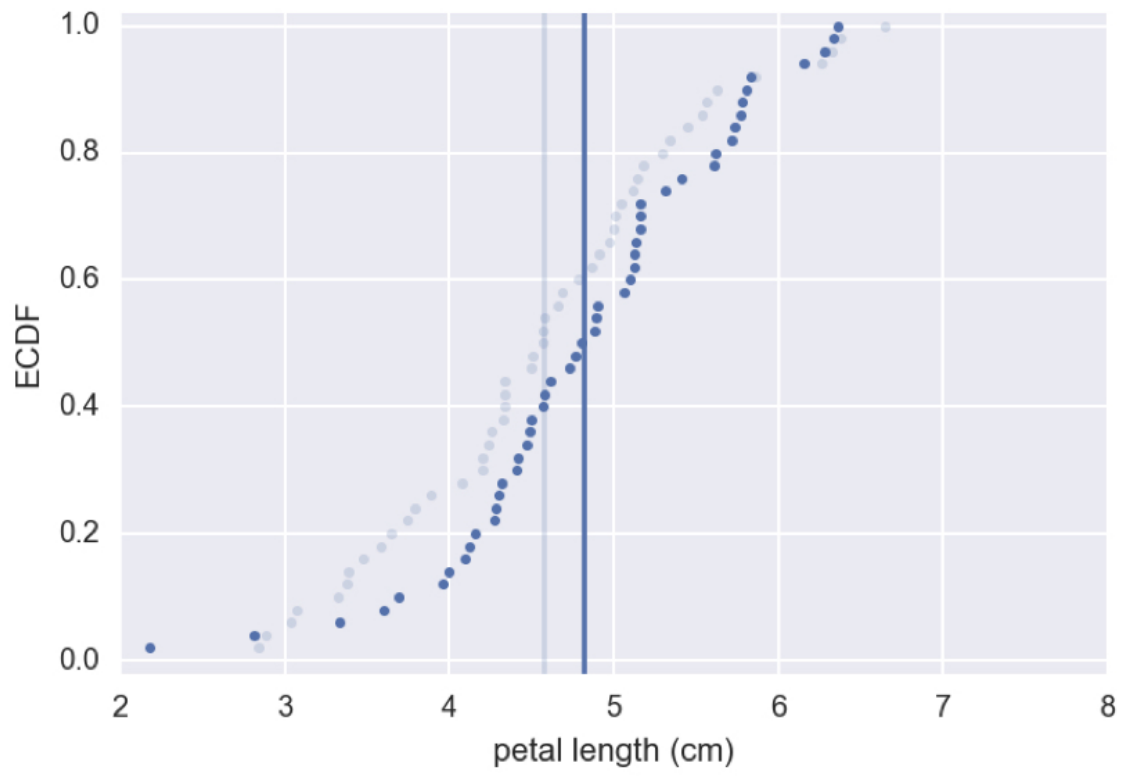
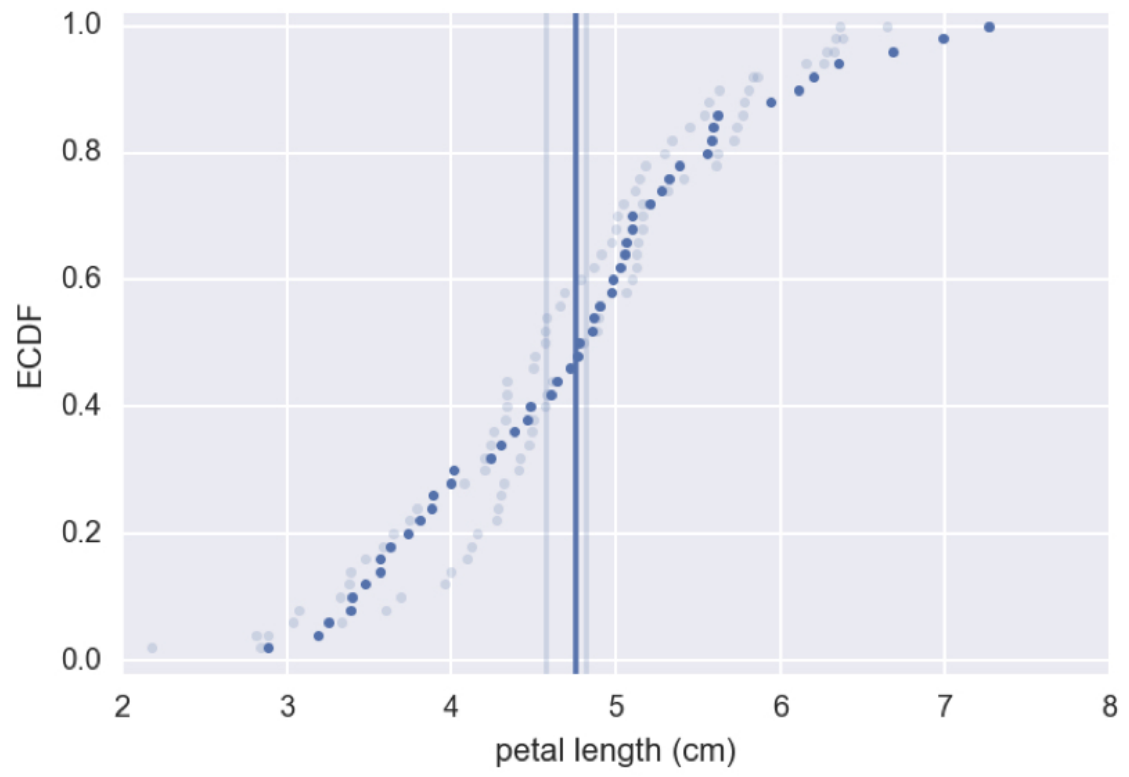


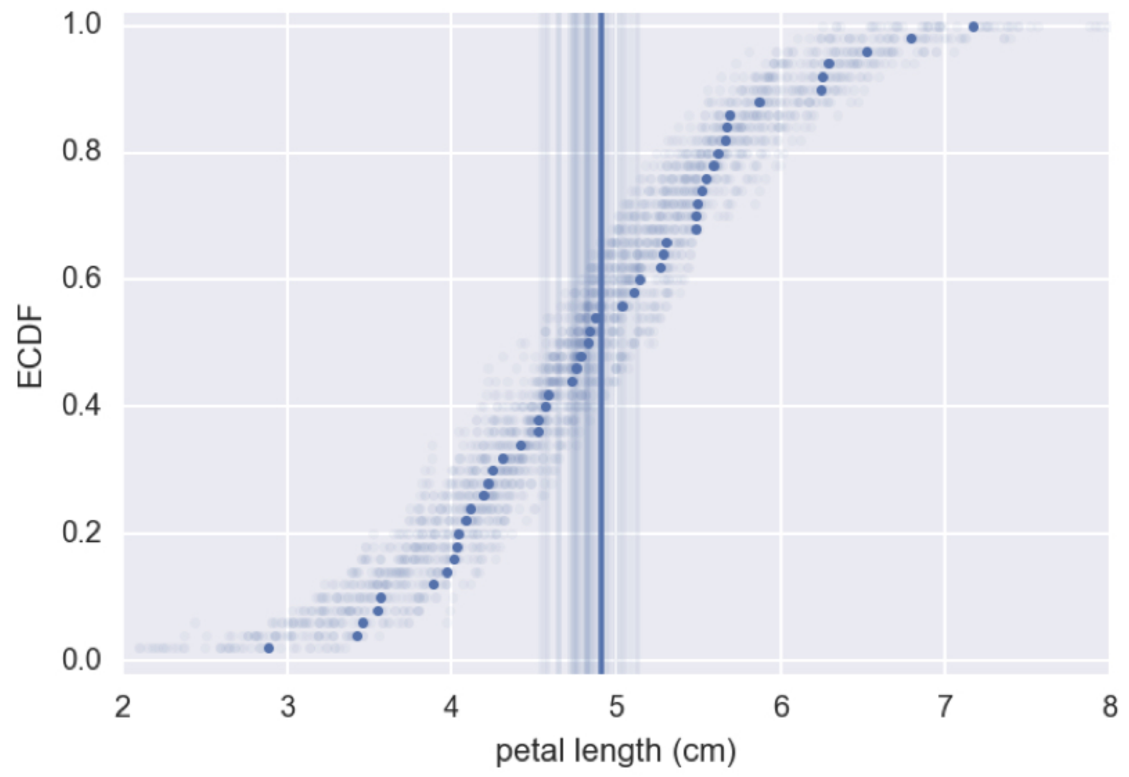
50 measurements of petal length



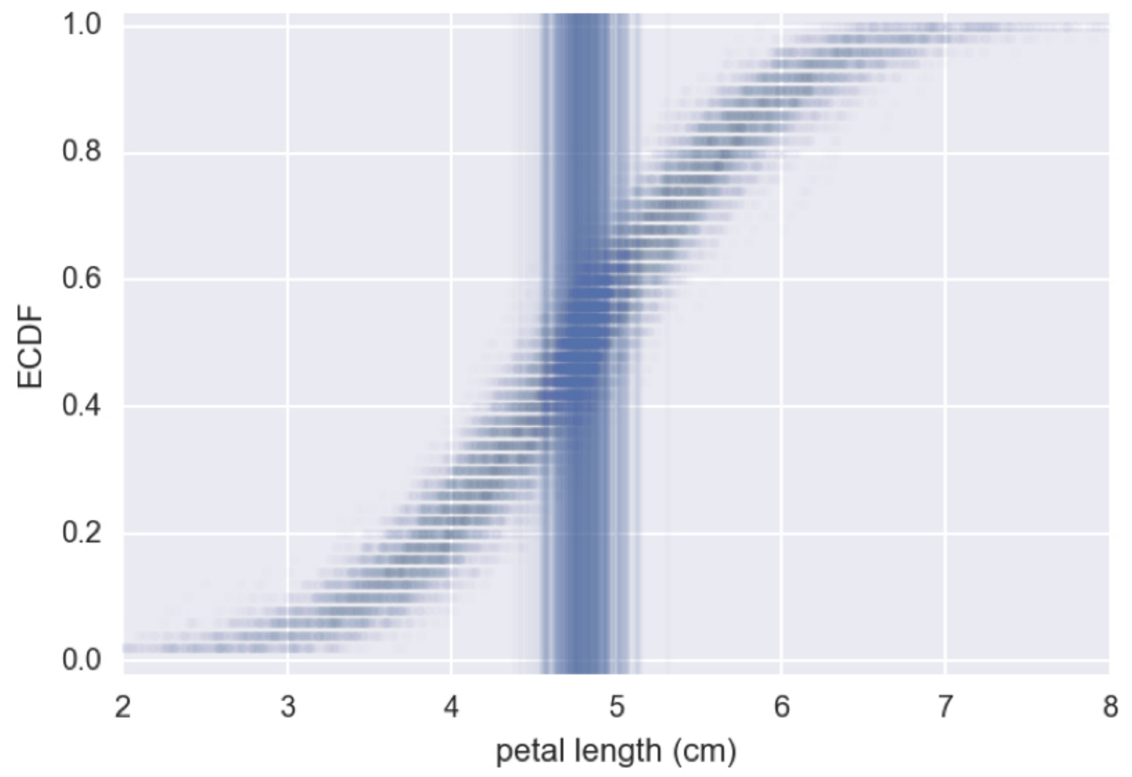
50 measurements of petal length



50 measurements of petal length



Repeats of 50 measurements of petal length



Let's practice!

STATISTICAL THINKING IN PYTHON (PART 1)

Random number generators and hacker statistics

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Hacker statistics

- Uses simulated repeated measurements to compute probabilities.

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The np.random module

- Suite of functions based on random number generation
- `np.random.random()` : draw a number between 0 and 1

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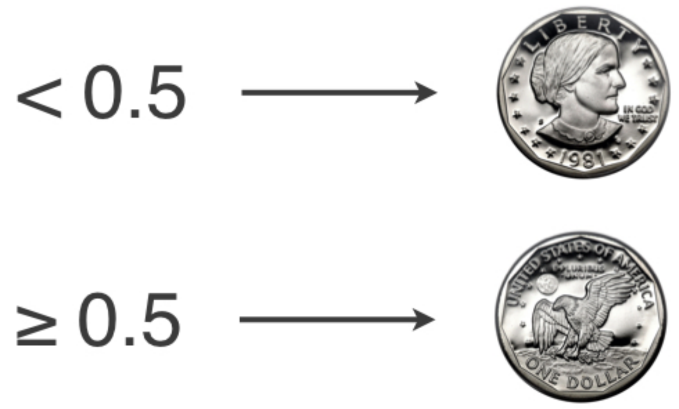

Bernoulli trial

- An experiment that has two options, "success" (True) and "failure" (False).

Random number seed

- Integer fed into random number generating algorithm
- Manually seed random number generator if you need reproducibility
- Specified using `np.random.seed()`

Simulating 4 coin flips

```python
import numpy as np
np.random.seed(42)
random_numbers = np.random.random(size=4)
random_numbers
```

```
array([ 0.37454012,  0.95071431,  0.73199394,  0.59865848])
```

```python
heads = random_numbers < 0.5
heads
```

```
array([ True, False, False, False], dtype=bool)
```

```python
np.sum(heads)
```

```
1
```

Simulating 4 coin flips

```python
n_all_heads = 0  # Initialize number of 4-heads trials
for _ in range(10000):
         heads = np.random.random(size=4) < 0.5
         n_heads = np.sum(heads)
         if n_heads == 4:
             n_all_heads += 1

n_all_heads / 10000
```

```
0.0621
```

Hacker stats probabilities

- Determine how to simulate data
- Simulate many many times
- Probability is approximately fraction of trials with the outcome of interest

Let's practice!

STATISTICAL THINKING IN PYTHON (PART 1)

Probability distributions and stories: The Binomial distribution

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Probability mass function (PMF)

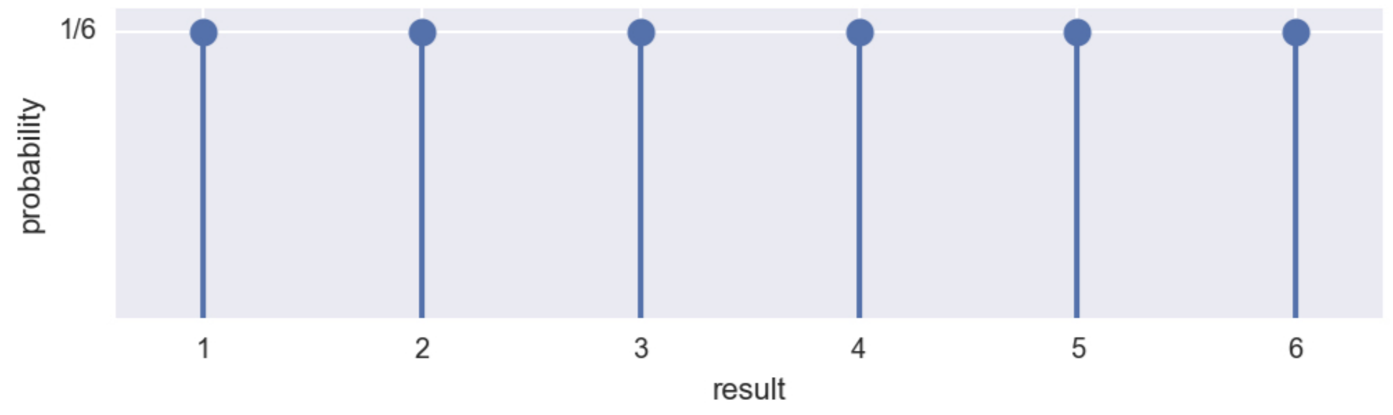
- The set of probabilities of discrete outcomes

Discrete Uniform PMF

Tabular

⚀	⚁	⚂	⚃	⚄	⚅
1/6	1/6	1/6	1/6	1/6	1/6

Graphical



Probability distribution

- A mathematical description of outcomes

Discrete Uniform distribution: the story

The outcome of rolling a single fair die is

- Discrete
- Uniformly distributed.

Binomial distribution: the story

- The number r of successes in n Bernoulli trials with probability p of success, is Binomially distributed
- The number r of heads in 4 coin flips with probability 0.5 of heads, is Binomially distributed

Sampling from the Binomial distribution

```
np.random.binomial(4, 0.5)
```

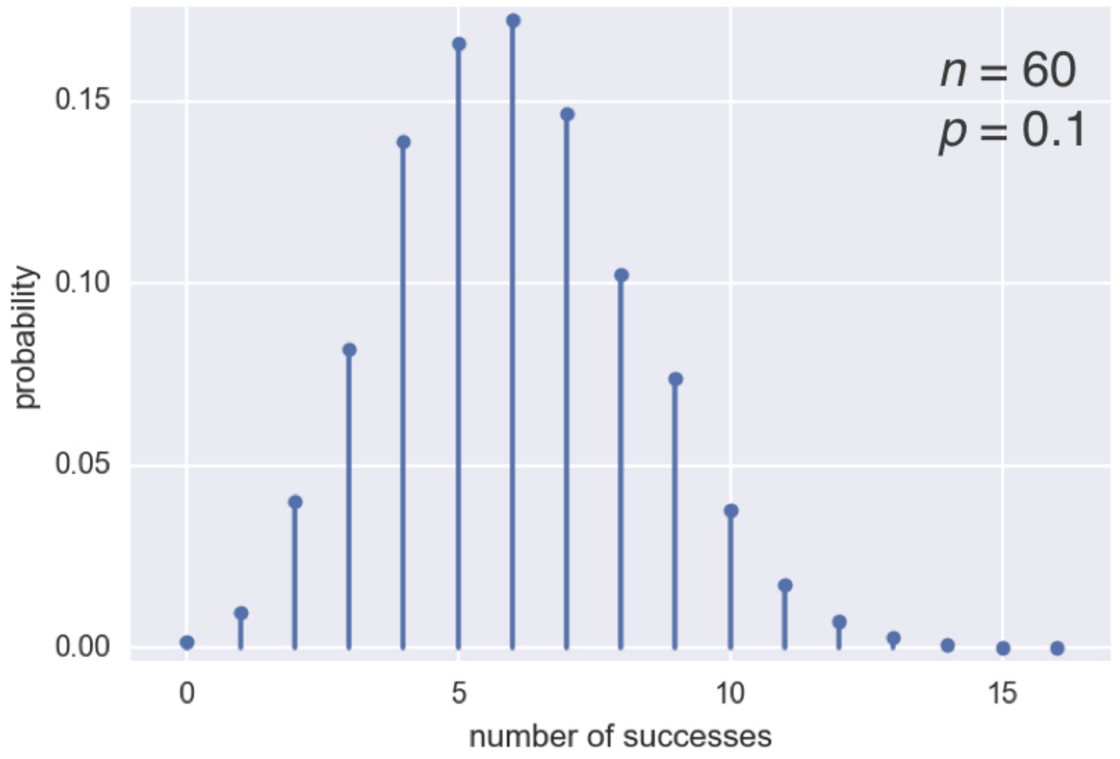
```
2
```

```
np.random.binomial(4, 0.5, size=10)
```

```
array([4, 3, 2, 1, 1, 0, 3, 2, 3, 0])
```

The Binomial PMF

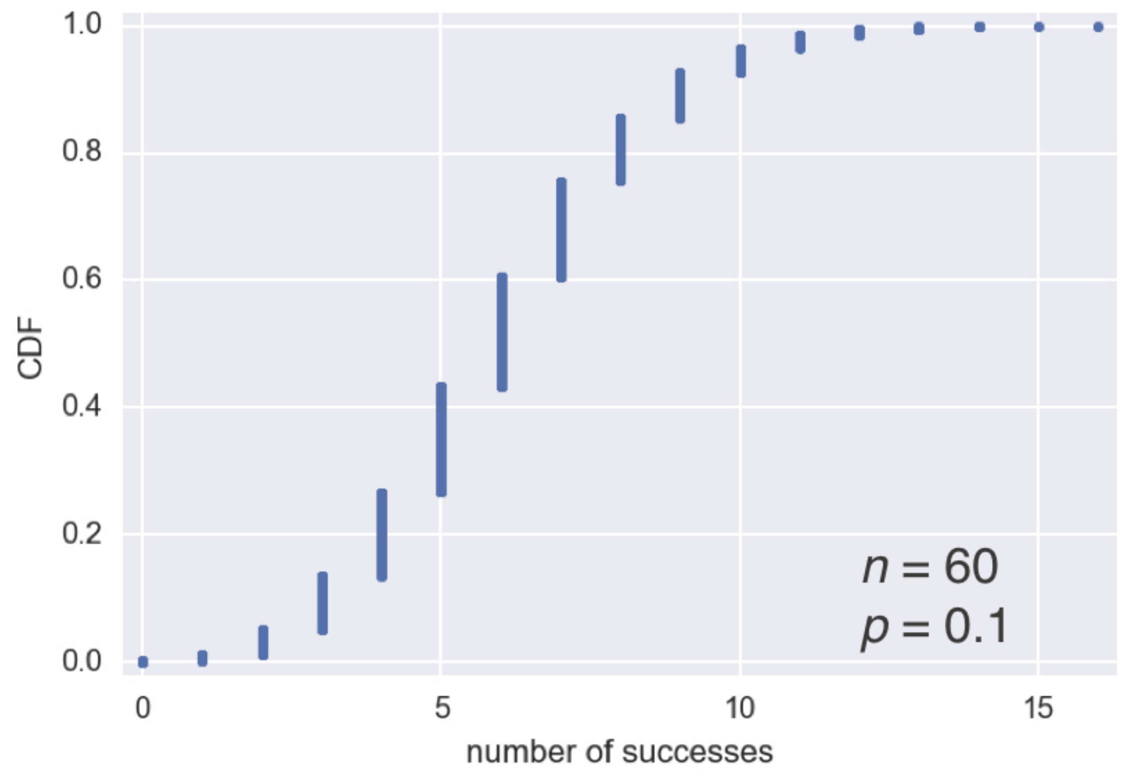
```
samples = np.random.binomial(60, 0.1, size=10000)
n = 60
p = 0.1
```



The Binomial CDF

```python
import matplotlib.pyplot as plt
import seaborn as sns
sns.set()
x, y = ecdf(samples)
_ = plt.plot(x, y, marker='.', linestyle='none')
plt.margins(0.02)
_ = plt.xlabel('number of successes')
_ = plt.ylabel('CDF')
plt.show()
```

The Binomial CDF



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STATISTICAL THINKING IN PYTHON (PART 1)

Poisson processes and the Poisson distribution

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Poisson process

- The timing of the next event is completely independent of when the previous event happened

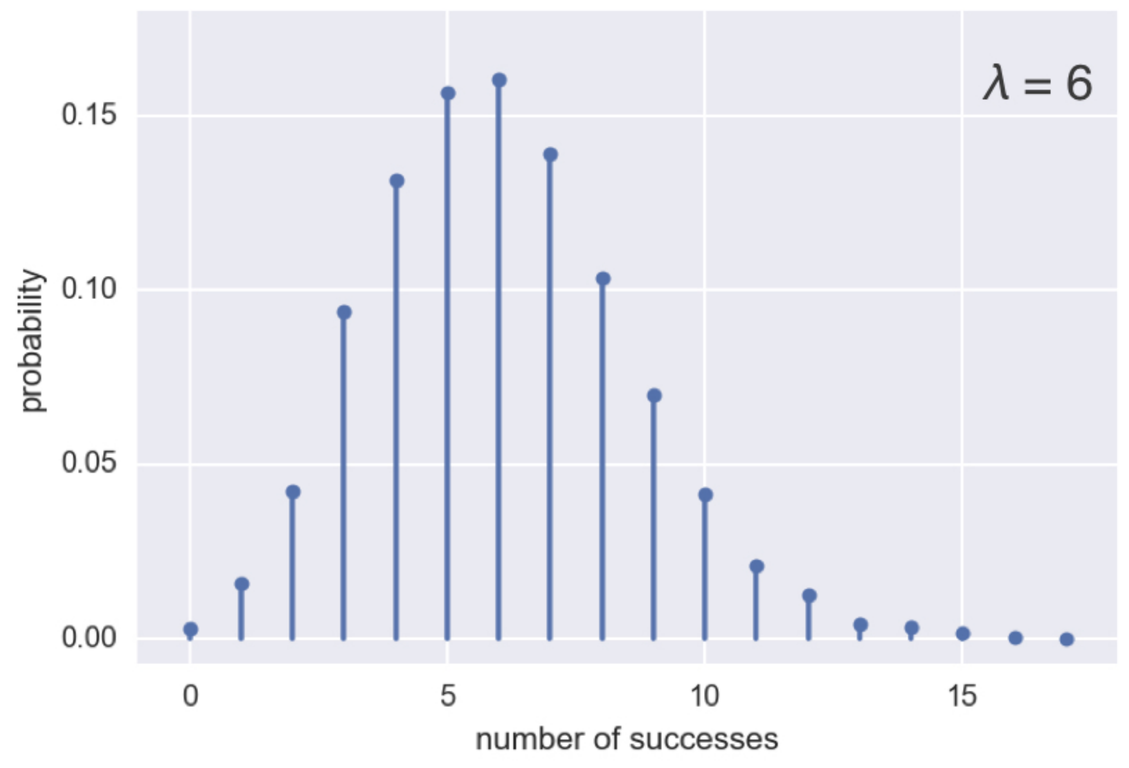
Examples of Poisson processes

- Natural births in a given hospital
- Hit on a website during a given hour
- Meteor strikes
- Molecular collisions in a gas
- Aviation incidents
- Buses in Poissonville

Poisson distribution

- The number r of arrivals of a Poisson process in a given time interval with average rate of ? arrivals per interval is Poisson distributed.
- The number r of hits on a website in one hour with an average hit rate of 6 hits per hour is Poisson distributed.

Poisson PMF



$\lambda = 6$
probability
number of successes
0.00
0.05
0.10
0.15
0
5
10
15

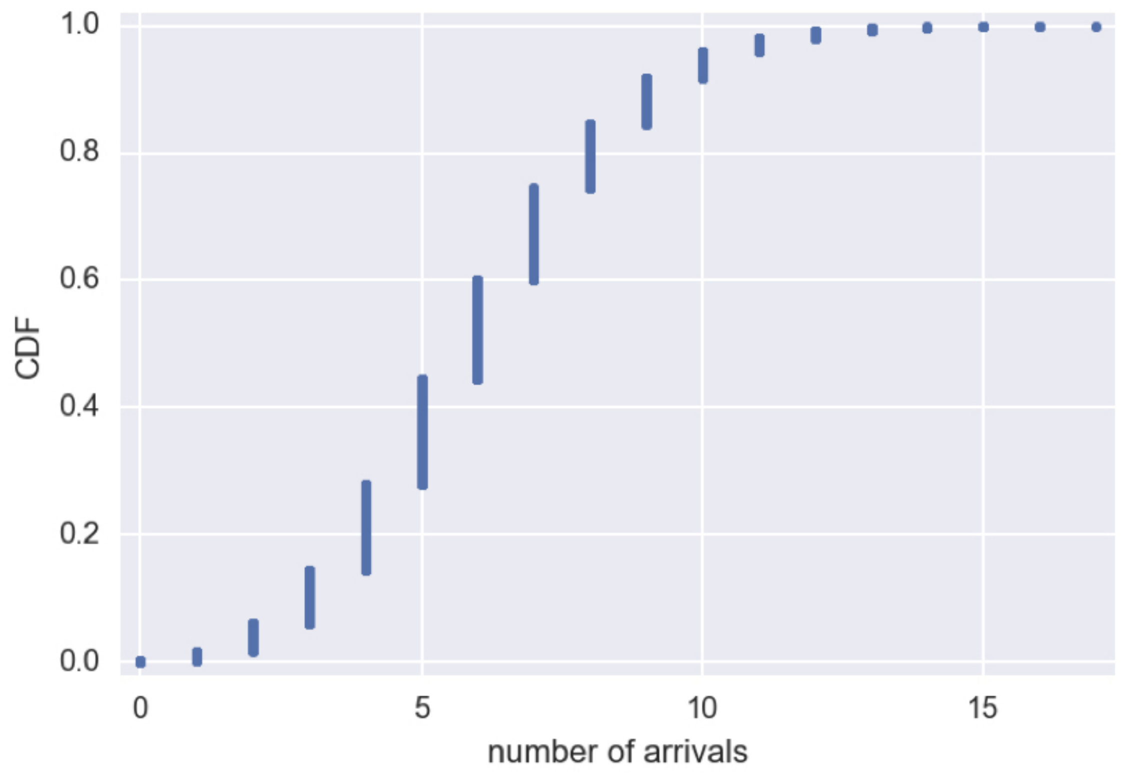
Poisson Distribution

- Limit of the Binomial distribution for low probability of success and large number of trials.
- That is, for rare events.

The Poisson CDF

```
samples = np.random.poisson(6, size=10000)
x, y = ecdf(samples)
_ = plt.plot(x, y, marker='.', linestyle='none')
plt.margins(0.02)
_ = plt.xlabel('number of successes')
_ = plt.ylabel('CDF')
plt.show()
```

The Poisson CDF



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