Empirical Project Overview

TIØ4317 Empirical and Quantitative methods in Finance

Problem statement

Can stock market returns, as measured by the S&P 500 index, be explained using time series data on nonfarm payrolls and the federal funds rate?

Empirical analysis

Data	ARIMAX	GARCH	Conclusion
▪ Sourced from the US Federal Reserve Economic Data ▪ Two exogenous explaining variables: Federal funds rate and nonfarm payroll employment ▪ One explained variable: The S&P 500 index ▪ 10 years of monthly time series data	▪ Good framework for financial time series data ▪ Indicates high impact of statistical significance from payroll employment ▪ Indicates low impact of statistical insignificance from federal funds rate	▪ Good framework for modelling time-varying volatility ▪ Indicates the same results as ARIMAX model, however with *higher confidence*. ▪ Indicates a high degree of volatility persistence	▪ A 1% change in payroll employment gives 1.5-2% increase in MoM S&P 500 returns ▪ The FFR(1) data needs to be preprocessed better ▪ The GARCH model is preferred due to superior volatility modelling

Note: (1) Federal funds rate

Problem statement – what do I want to research?

Can stock market returns, as measured by the S&P 500 index, be explained using time series data on nonfarm payrolls and the federal funds rate?

This report investigates whether expectations to macroeconomic variables such as employment growth and interest rates can serve as explaining variables of market behavior. Interest rates and employment are two leading macroeconomic variables, as employment is a strong indicator of the current state of the economy and the federal funds interest rate is a tool for adjusting future activity.

Since direct data on market expectations (e.g., futures-implied rate expectations or labor forecasts) is difficult to obtain and quantify, the analysis uses the actual month-over-month changes in nonfarm payrolls and the federal funds rate as proxies for market expectations. This approximation can be justified by the relatively short forecast horizon and the tendency of market expectations to be fairly precise with near-term outcomes.

This problem delves into the world of statistical models for time series explanation and forecasting and aims to make use of several key concepts from the curriculum in TIØ4317.

Data | I have sourced three datasets from the US Federal Reserve Economic Data (FRED)

SP500
(S&P 500)

- Indexed data
- Daily frequency
- Apr 2015 - Apr 2025

Dataset on the value of the S&P 500 index, an index consisting of the 500 most important equities in the US

FEDFUNDS
(Federal funds deposit rate)

- Percentage data
- Monthly frequency
- Jan 1957 - Apr 2025

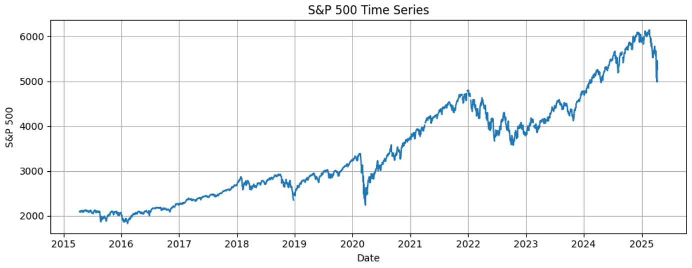
Dataset on the value of the US federal funds deposit rate, as decided by the Federal Reserve

PAYEMS
(Non-farm employment numbers)

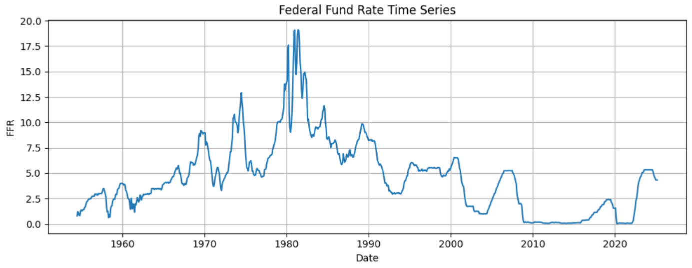
- Absolute data
- Monthly frequency
- Jan 1939 - Apr 2025

Dataset on the number of employed Americans, ex. The more seasonal agricultural sector

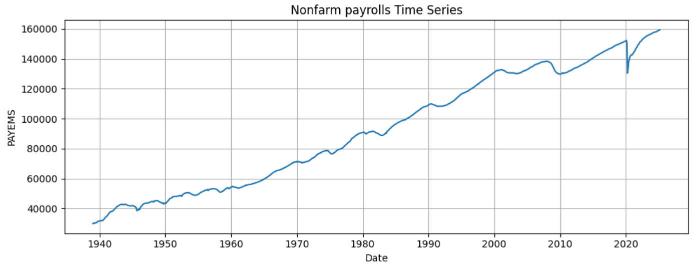
SP500 Plot



FEDFUNDS Plot



PAYEMS Plot



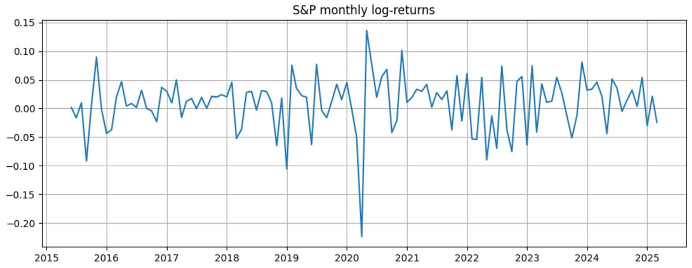
Data | To be compatible for statistical modelling, the data had to be transformed and standardized

SP500
(S&P 500)

Transformation steps:
- Downsampling to monthly frequency
- Applying log-returns

Log-returns have been applied to address stationarity and normality in returns

Transformed SP500 Plot

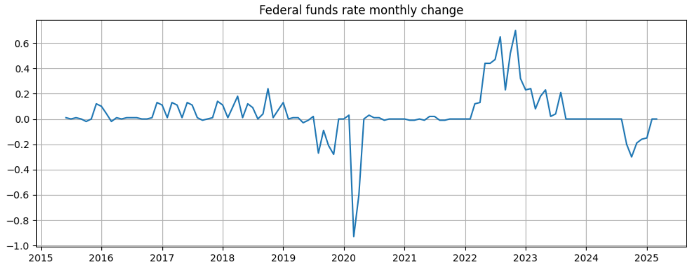


FEDFUNDS
(Federal funds deposit rate)

Transformation steps:
- Applying first differences

To better capture the effects of monetary policy shocks, the month-over-month **first difference** has been used

Transformed FEDFUNDS Plot

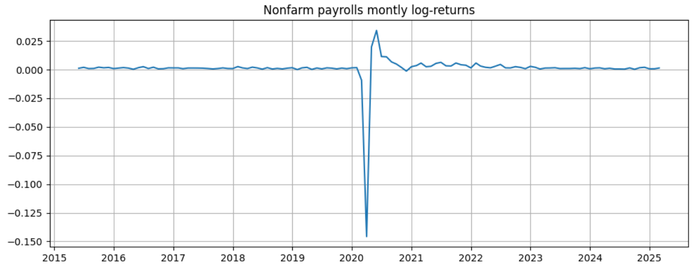


PAYEMS
(Non-farm employment numbers)

Transformation steps:
- Applying log-returns

As with the SP500 dataset, **log-returns** have been applied to address stationarity and normality in returns

Transformed PAYEMS Plot



Empirical analysis | Using two models for statistical explanation

ARIMAX

The AutoRegressive Integraged Moving Average with Exogenous variables (ARIMAX(p, d, q)) model is a framework for modelling financial time series, building on the ARMA(p, q) model by incorporating differencing and exogenous variables.

Well suited for the task because of three main reasons:

- Captures autocorrelation patterns
- Accounts for residual non-stationarity not captured by the log-transformation
- Explicitly incorporates exogenous variables, i.e. FEDFUNDS and PAYEMS

GARCH

The Generalized AutoRegressive Conditional Heteroskedasticity (GARCH(p, q)) model is a framework for modelling time varying volatility in financial time series.

Well suited for the task because of two main reasons:

- Captures volatility clustering and time-varying risk
- Better inference on exogenous variables due to separation of drift and volatility modelling

Empirical analysis | The ARIMAX model

General formula

$$\boldsymbol{\phi(L)(1-L)^d * y_t = \mu + \theta(L)u_t + \beta * X_t}$$

μ is the drift of the process
L are the lag variables such that L^i is equivalent to y_{t-i}

			Explanation
AR	Autoregression $\phi(L)y_t$	y_t is the target variable	Captures the relationship between the current value and its lag values (preceding values)
I	Differencing $(1-L)^d$		Removes non-stationarity in the time series by computing the difference of degree d
MA	Moving Average $\theta(L)u_t$	$u_t \sim N(0, \sigma^2)$ is the white noise term	Captures the impact of past errors (shocks) on the current value
X	Exogenous variables $\beta * X_t$	X_t are the exogenous variables with coefficients β	Captures the impact of external variables (i.e. macroeconomic variables) on the target

Empirical analysis | The ARIMAX model – model selection and results

Information criterion

AIC: best order (2,0,2)

$$AIC = \ln(\hat{\sigma}^2) + \frac{2k}{T}$$

AIC penalizes the model complexity somewhat, but not as hard as BIC

BIC: best order (0,0,0)

$$BIC = \ln(\hat{\sigma}^2) + \ln(T) * \frac{k}{T}$$

BIC penalizes model complexity harsher than AIC

The final model was selected using the **AIC criterion**, as I deem the model complexity of (2,0,2) to be acceptable

Results from ARIMAX(2,0,2)

	coef	std err	z	P>|z|	[0.025	0.975]
PAYEMS	1.6507	0.381	4.336	0.000	0.905	2.397
FEDFUNDS	-0.0186	0.019	-0.974	0.330	-0.056	0.019
ar.L1	1.2773	0.215	5.954	0.000	0.857	1.698
ar.L2	-0.3927	0.217	-1.812	0.070	-0.817	0.032
ma.L1	-1.5874	0.149	-10.643	0.000	-1.880	-1.295
ma.L2	0.7359	0.164	4.496	0.000	0.415	1.057
sigma2	0.0015	0.000	7.207	0.000	0.001	0.002

Ljung-Box (L1) (Q):	0.01	Jarque-Bera (JB):	4.32
Prob(Q):	0.91	Prob(JB):	0.12
Heteroskedasticity (H):	2.14	Skew:	-0.46
Prob(H) (two-sided):	0.02	Kurtosis:	3.19

- PAYEMS has a positive and statistically significant impact ($p < 0.01$) on S&P 500 returns, with a 1% increase in PAYEMS leading to a 1.65% in stock returns, ceteris paribus
- FEDFUNDS has a negative impact on stock returns, however the results are not statistically significant ($p = 0.33$)

Empirical analysis | The GARCH model

General formula

$$y_t = \mu + \epsilon_t$$

$$\epsilon_t = \sqrt{\sigma_t^2} * z_t$$

$$\sigma_t^2 = \omega + \alpha * \epsilon_{t-1}^2 + \beta * \sigma_{t-1}^2$$

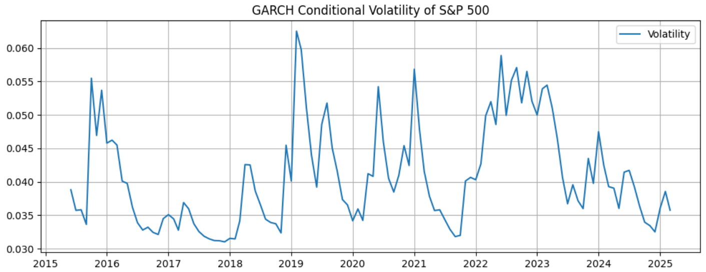
Explanation

ϵ_t is the residual at time t
$u_t \sim N(0,1)$ is the white noise term, standard normal dist.
μ is the constant drift of the process
σ^2 is the conditional volatility of the process at time t

ω	Constant volatility	Represents the long-term average volatility of the share price
α	Impact of past shocks	Captures the impact of recent shocks to the share price, as given by the last residual
β	Impact of past volatility	Captures the impact of recent volatility in the share price

Empirical analysis | The GARCH model – results

Conditional volatility



The conditional volatility estimated by the GARCH model implies a high degree of volatility clustering and heteroskedasticity, bolstering the need for a GARCH model to accurately model the SP500 development

Results from GARCH(1,1)

Mean Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
Const	8.0089e-03	3.082e-03	2.598	9.370e-03	[1.967e-03,1.405e-02]
PAYEMS	1.9267	0.407	4.733	2.207e-06	[1.129, 2.725]
FEDFUNDS	-0.0178	2.512e-02	-0.707	0.480	[-6.699e-02,3.149e-02]

Volatility Model

	coef	std err	t	P>|t|	95.0% Conf. Int.
omega	3.7228e-04	2.063e-04	1.804	7.116e-02	[-3.208e-05,7.766e-04]
alpha[1]	0.1960	0.153	1.279	0.201	[-0.104, 0.496]
beta[1]	0.5914	0.178	3.327	8.780e-04	[0.243, 0.940]

- PAYEMS has a positive and statistically significant impact ($p < 0.01$) on S&P 500 returns, with a 1% increase in PAYEMS leading to a 1.93% in stock returns, ceteris paribus
- FEDFUNDS has a negative impact on stock returns, however the results are not statistically significant ($p = 0.48$)
- Past volatility is persistent and explains nearly all present volatility

Conclusion | PAYEMS shows significant impact, FEDFUNDS needs better modelling to accurately capture known effect

PAYEMS impact

Both statistical models indicate a strong and statistically significant ($p < 0.05$) impact of change in payroll numbers on S&P 500 performance, namely a 1% change in payroll employment gives a 1.5-2.0% increase in returns. This empirical finding is rooted in the real world, as employment is a strong indicator of overall activity in the economy and thus also listed company performance.

High impact, high statistical significance

FEDFUNDS impact

Both statistical models indicate a vague negative impact, however statistically insignificant. It is widely known that the federal funds rate is highly impactful on markets, thus indicating that there are issues with how the data is presented to the model. I.e., the interest rate path is communicated by the fed well in advance, and changes are often priced into markets well before the same month

Low impact, low statistical significance

Model selection

Although the ARIMAX model is able to capture the impact of the exogenous variables with high confidence, there are strong signs of heteroskedasticity in the data which implies that stronger volatility modeling is required. Evidently, the GARCH model is able to better isolate the true effect of the exogenous variables, further strengthening the credibility of the results from ARIMAX.

GARCH required to accurately model volatility

References & LLM disclaimer

On LLMs

I have used LLMs throughout the report to explain me complex topics and debug my code, as I find that both can be very time-consuming activities, and as such I see that my work has become much more efficient by utilising LLMs such as ChatGPT.
I find that using LLMs for university assignments can be a two-sided sword, as it can both empower the user to focus on the more important tasks at hand and also give the user a way around actually learning and implementing the tasks.
Therefore, I have tried to contain LLM usage to the tasks that provide me the lowest marginal benefit compared to the time I spend on them.

References

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