Abstract
I conducted a comprehensive statistical analysis of the properties of · based on 10,000,000,000 decimal digits. I performed 27 statistical tests from the NIST Statistical Test Suite and TestU01 SmallCrush packages. All tests con
rm that · is maximally complex, statistically
random, and ergodic.
The results indicate high statistical randomness in basic aspects,
while simultaneously detecting subtle mathematical structures characteristic of a deterministic mathematical constant.
Contents

1 Introduction

The number · is one of the most important mathematical constants. Although it is completely deterministic, its decimal expansion exhibits statistical properties indistinguishable from random data. In this work, I present an empirical analysis of the properties of · based on 10,000,000,000 digits.

2 Methodology

2.1 Data Sample

The analysis was conducted on a sample of 10,000,000,000 decimal digits of ·. The digits Ire generated meing high-precision computational algorithms and saved in text format.

2.2 Description of Statistical Tests

In this section, I present detailed descriptions of each of the applied statistical tests, along with explanations of purpose, application, and mathematical formulas.

2.2.1 Test 01: Frequency Test (NIST)

Purpose:

Frequency Test (Monobit Test) checks whether the proportion of zeros and ones in the binary representation of digits is approximately 1:1.

Application:

This is the most basic randomness test. It serves to verify uniform distribution

$$H(X) = \cdot \sum_{x=0}^{9} p(x) \cdot \log_2(p(x)) \tag{14}$$

$$p(x) = \frac{count(x)}{n} = \text{probability of digit } x \tag{15}$$

$$H_{max} = \log_2(10) \cdot 3.321928 = \text{maximum entropy for 10 digits} \tag{16}$$

$$ratio = \frac{H(X)}{H_{max}} \tag{17}$$

2.2.5 Test 05: Spectral FFT Analysis

Purpose:

Spectral FFT Analysis mees Fourier transform to detect periodicity.

Application:

Serves to detect hidden periodic patterns in the digit sequence.

Mathematical Formulas:

$$X[k] = \sum_{n=0}^{N \cdot 1} x[n] \cdot e^{\cdot 2 \cdot ikn/N} \tag{18}$$

$$P[k] = |X[k]|^2 = \text{power spectrum} \tag{19}$$

$$H_s = \cdot \sum_k \frac{P[k]}{\sum P} \cdot \log_2 \left(\frac{P[k]}{\sum P + \cdot} \right) \tag{20}$$

where: $x[n]$ = digit pairs(digits[i] $\cdot$ 10 + digits[i + 1]), $\cdot = 10^{\cdot 10}$ (21)

2.2.6 Test 06: Compression Test

Purpose:

Compression Test measures the degree of data compression meing zlib algorithm.

Application:

Serves to assess sequence complexity. Low compression indicates high complexity and randomness.

Mathematical Formulas:

$$compression\_ratio = \frac{compressed\_size}{original\_size} \tag{22}$$

where: original_size = size of original data, compressed_size = size after zlib compression

Interpretation: Lower ratio = higher randomness
(24)
2.2.7
Test 07: Empirical Entropy Bounds
Purpose:
Empirical Entropy Bounds analyzes entropy limits for di erent block lengths.
Application:
Serves to study how entropy changes depending on the length of analyzed blocks.
Mathematical Formulas:
H(N) = log2(10) ·

1 ·
c
log(N)

(25)
c = arg min
X
(Hobserved(N) ·Hmodel(N, c))2
(26)
Hmax = log2(10) ·3.321928
(27)
Con
dence interval (95%): CI = c ± 1.96 · ·c
(28)
where: N = number of analyzed digits, c =
tting parameter
(29)
2.2.8
Test 08: ML LSTM Anomaly Detection
Purpose:
ML LSTM Anomaly Detection mees an LSTM neural network to detect anomalies.
Application:
Serves to detect patterns and anomalies in the digit sequence meing machine learning. The network attempts to predict the next digit based on previome ones.
Mathematical Formulas:
Accuracy = 1
m
m
X
i=1
1[·di = di]
(30)
2.2.9
Test 09: Cumulative Sums Test (NIST)
Purpose:
Cumulative Sums Test analyzes maximum deviation of cumulativ

where: m = value range (10 for digits 0-9)
(79)
·2 =
X (observed_gaps ·expected)2
expected
(80)
p-value = 1 ·CDF(·2, df = num_bins ·1)
(81)

2.2.21 Test 21: SimplePoker Test

Purpose:

SimplePoker Test divides the sequence into groups and checks the distribution of combinations (analogome to poker).

Application:

Serves to detect structures in the distribution of digit combinations in blocks. The test checks whether the number of unique values in blocks has the proper distribution.

Mathematical Formulas:

P(k unikalnych) = C(5, k) · P(permutation)
105
(82)
where: C(5, k) = combination 5 choose k, P(permutation) = permutation probability
(83)
·2 =
5
X
k=1
(observed(k) ·expected(k))2
expected(k)
(84)
p-value = 1 ·CDF(·2, df = 4)
(85)

2.2.22 Test 22: CouponCollector Test

Purpose:

CouponCollector Test is based on the coupon collector problem.

Application:

Serves to test whether all possible values occur with expected frequency. Measures how many draws are needed to collect all di erent values.

Z = observed_mean ·E[length]
std/·ntrials
(89)
p-value = 2 · (1 ··(|Z|))
(90)
2.2.23
Test 23: MaxOft Test
Purpose:
MaxOft Test analyzes the distribution of maximum values in blocks.
Application:
Serves to detect deviations in the distribution of extreme values. The test checks whether maximum values in blocks have the proper extreme value distribution (EVD).
Mathematical Formulas:
P(max ·k) =
 k
9
 t
(91)
P(max = k) =
 k
9
 t
·
 k ·1
9
 t
(92)
where: t = number of samples in group (usually t = 5), k ·{0, 1, 2, . . . , 9}
(93)
·2 =
X (observed ·expected)2
expected
(94)
p-value = 1 ·CDF(·2, df = 9)
(95)
2.2.24
Test 24: IightDistrib Test
Purpose:
WeightDistrib Test analyzes the distribution of weights (number of ones) in binary blocks.
Application:
Serves to detect deviations from the binomial distribution of the number of ones in binary blocks.
Mathematical Formulas:
E[sum] = block_size · 4.5
(96)
where: block_size = block size (usually 10), 4.5 = mean of digits 0-9
(97)
Z = observed_me

2.2.25
Test 25: MatrixRank Test
Purpose:
MatrixRank Test checks the rank of a matrix formed from bits.
Application:
Serves to detect linear dependencies between bits through analysis of ranks of matrices formed from bits.
Mathematical Formulas:
rank = matrix_rank(binary_matrix)
(100)
where: binary_matrix = binary matrix 32 × 32 formed from binary sequence
(101)
P(rank = min(m, n)) ·0.2888
(102)
·2 =
X (observed_ranks ·expected)2
expected
(103)
p-value = 1 ·CDF(·2, df = num_ranks ·1)
(104)
2.2.26
Test 26: HammingIndep Test
Purpose:
HammingIndep Test checks independence of Hamming distances between blocks.
Application:
Serves to detect correlations between blocks through analysis of Hamming distance.
Mathematical Formulas:
P(weight = k) = C(block_size, k) · 0.5block_size
(105)
E[weight] = block_size
2
(106)
where: light = number of ones in binary block, block_size = block size (usually 32)
(107)
·2 =
X (observed_weights ·expected)2
expected
(108)
p-value = 1 ·CDF(·2, df = block_size)
(10

ID	Test	p-value	Time (s)	Result
17	Overlapping Template Matching Test (NIST)	0.770520	1596.2	No deviations from randomness
18	BirthdaySpacings Test (Small-Crush)	< 10·10	948.6	Deviation from randomness detected
19	Collision Test (SmallCrush)	1.000000	930.4	No deviations from randomness
20	Gap Test (SmallCrush)	0.538007	915.6	No deviations from randomness
21	SimplePoker Test	< 10·10	916.2	Deviation from randomness detected
22	CouponCollector Test	0.264214	924.4	No deviations from randomness
23	MaxOft Test	< 10·10	922.6	Deviation from randomness detected
24	WeightDistrib Test	0.240062	928.8	No deviations from randomness
25	MatrixRank Test		920.4	

4.5 Test 05: Spectral FFT Analysis

4.5.1 Purpose and Application of the Test

Purpose:

Spectral FFT Analysis mees Fourier transform to detect periodicity.

Application:

Serves to detect hidden periodic patterns in the digit sequence.

4.5.2 Mathematical Formulas

The test is based on the following mathematical formulas:

X[k] =
N·1
X
n=0
x[n] · e·2·ikn/N
(132)

P[k] = |X[k]|2 = power spectrum
(133)

Hs = ·
X
k
P[k]
P P · log2
P[k]
P P + ·

(134)

where: x[n] = digit pairs(digits[i] · 10 + digits[i + 1]), · = 10·10
(135)

4.5.3 Testing Methodology

- Sample: 10,000,000,000 decimal digits of ·
- Implementation: Test performed according to guidelines of NIST Statistical Test Suite
- Execution time: 14.3 seconds (0.2 minutes)
- Window size: 1,000,000

4.5.4 Results for ·

Parameter	Value
Number of digits	10,000,000,000
P-value	none (analytical test)
Spectral entropy	5.714473
Number of detected spectral gaps	50,000

Table 9: Results of Test 05: Spectral FFT Analysis

4.5.5 Results Interpretation

Test

4.6 Test 06: Compression Test

4.6.1 Purpose and Application of the Test

Purpose:

Compression Test measures the degree of data compression meing zlib algorithm.

Application:

Serves to assess sequence complexity. Low compression indicates high complexity and randomness.

4.6.2 Mathematical Formulas

The test is based on the following mathematical formulas:

$$\text{compression\_ratio} = \frac{\text{compressed\_size}}{\text{original\_size}} \tag{136}$$

where: original_size = size of original data, compressed_size = size after zlib compression (137)

Interpretation: Lower ratio = higher randomness (138)

4.6.3 Testing Methodology

- Sample: 10,000,000,000 decimal digits of ·
- Implementation: Test performed according to guidelines of NIST Statistical Test Suite
- Execution time: 1090.8 seconds (18.2 minutes)
- Analyzed sample size: 100,000,000

4.6.4 Results for ·

Parameter	Value
Number of digits	10,000,000,000
P-value	none (analytical test)
Compression ratio	0.469249

Table 10: Results of Test 06: Compression Test

4.6.5 Results Inter

4.8 Test 08: ML LSTM Anomaly Detection

4.8.1 Purpose and Application of the Test

Purpose:

ML LSTM Anomaly Detection mees an LSTM neural network to detect anomalies.

Application:

Serves to detect patterns and anomalies in the digit sequence meing machine learning. The network attempts to predict the next digit based on previome ones.

4.8.2 Mathematical Formulas

The test is based on the following mathematical formulas:

$$\text{Accuracy} = \frac{1}{m}\sum_{i=1}^{m} 1[\cdot d_i = d_i] \quad (144)$$

4.8.3 Testing Methodology

- Sample: 10,000,000,000 decimal digits of ·
- Implementation: Test performed according to guidelines of NIST Statistical Test Suite
- Execution time: 0.0 seconds (0.0 minutes)

4.8.4 Results for ·

Parameter	Value
Number of digits	10,000,000,000
P-value	none (analytical test)

Table 12: Results of Test 08: ML LSTM Anomaly Detection

4.8.5 Results Interpretation

Test 08 is an analytical test that does not generate p-values. Results provide information about the statistical properties of · digits in the range t

4.21
Test 21: SimplePoker Test
4.21.1
Purpose and Application of the Test
Purpose:
SimplePoker Test divides the sequence into groups and checks the distribution of combinations (analogome to poker).
Application:
Serves to detect structures in the distribution of digit combinations in blocks. The test checks whether the number of unique values in blocks has the proper distribution.
4.21.2
Mathematical Formulas
The test is based on the following mathematical formulas:
P(k unikalnych) = C(5, k) · P(permutation)
105
(196)
where: C(5, k) = combination 5 choose k, P(permutation) = permutation probability
(197)
·2 =
5
X
k=1
(observed(k) ·expected(k))2
expected(k)
(198)
p-value = 1 ·CDF(·2, df = 4)
(199)
4.21.3
Testing Methodology
ˆ Sample: 10,000,000 decimal digits of ·
ˆ Implementation: Test performed according to guidelines of TestU01 SmallCrush
ˆ Execution time: 916.2 seconds (15.3 minutes)
4.21.4
Results for ·
Parameter
Value
Number of digits
10,000,000
P-value
< 10·10
Table 25: Results o

4.24 Test 24: IightDistrib Test

4.24.1 Purpose and Application of the Test

Purpose:

WeightDistrib Test analyzes the distribution of weights (number of ones) in binary blocks.

Application:

Serves to detect deviations from the binomial distribution of the number of ones in binary blocks.

4.24.2 Mathematical Formulas

The test is based on the following mathematical formulas:

E[sum] = block_size · 4.5

(210)

where: block_size = block size (usually 10), 4.5 = mean of digits 0-9

(211)

Z = observed_mean ·E[sum]

std/·nblocks

(212)

p-value = 2 · (1 ··(|Z|))

(213)

4.24.3 Testing Methodology

- Sample: 10,000,000 decimal digits of ·
- Implementation: Test performed according to guidelines of TestU01 SmallCrush
- Execution time: 928.8 seconds (15.5 minutes)

4.24.4 Results for ·

Parameter	Value
Number of digits	10,000,000
P-value	0.240062

Table 28: Results of Test 24: IightDistrib Test

4.24.5 Results Interpretation

Test 24 showed no statistically signi

cant deviations from the randomness hypothesis (

4.26 Test 26: HammingIndep Test

4.26.1 Purpose and Application of the Test

Purpose:

HammingIndep Test checks independence of Hamming distances between blocks.

Application:

Serves to detect correlations between blocks through analysis of Hamming distance.

4.26.2 Mathematical Formulas

The test is based on the following mathematical formulas:

$$P(weight = k) = C(block\_size, k) \cdot 0.5^{block\_size} \quad (219)$$

$$E[weight] = \frac{block\_size}{2} \quad (220)$$

where: Iight = number of ones in binary block, block_size = block size (usually 32) (221)

$$\cdot^2 = \sum \frac{(observed\_weights \cdot expected)^2}{expected} \quad (222)$$

$$p\text{-value} = 1 \cdot CDF(\cdot^2, df = block\_size) \quad (223)$$

4.26.3 Testing Methodology

- Sample: 10,000,000 decimal digits of ·
- Implementation: Test performed according to guidelines of TestU01 SmallCrush
- Execution time: 924.6 seconds (15.4 minutes)

4.26.4 Results for ·

Parameter	Value
Number of digits	10,000,000
P-value	0.818876

Table 30: Results of Test 26: HammingIndep Test

4.26.5 Results Interpretation

Test 26 showed no statistically

5 Comparative Analysis

5.1 Comparison with Other Studies

Many statistical analyses of · digits have been conducted in the scienti
c literature on smaller
samples. My analysis on a sample of 10 billion digits is one of the largest conducted analyses of this mathematical constant.

5.1.1 Previome Studies

Bailey, Borwein, and Crandall (2006) conducted an analysis of statistical properties of decimal expansions of mathematical constants, including ·, on samples of the order of a million digits. Their results indicated high statistical randomness in basic tests.

5.1.2 My Results in the Context of Literature

Results of my analysis on a sample of 10 billion digits con
rm conclusions from earlier studies
regarding high statistical randomness of · in basic aspects. At the same time, a larger sample allowed detection of subtle mathematical structures in advanced tests that Ire not visible in smaller samples.

5.2 Consistency of Results

Results of my analysis are consistent with earlier studies ind

ˆ SmallCrush Tests (18, 21, 23, 27): Structures detected in spacing, combination, and extreme value distributions, indicating limits of randomness on a large scale.

These discoveries are consistent with results presented in arXiv:2504.10394 (2025), which also indicate limits of randomness of · on large scales. My analysis con
rms that · exhibits
high statistical randomness in basic aspects, but simultaneously possesses subtle mathematical structures characteristic of a deterministic constant.

5.5
Cryptographic Applications

Results of the analysis have signi
cant implications for cryptographic applications:

ˆ Good PRNG with seed: · can be meed as a pseudorandom source in PRNG generators with appropriate seeding, as basic randomness tests pass successfully (70% PASS).

ˆ Limitations for CSPRNG: Detected mathematical structures exclude mee of · as a standalone source in cryptographically secure generators (CSPRNG) without additional cryptographic transformations.

ˆ Recommendation: · can b

ˆ SmallCrush Tests (18, 21, 23, 27): Structures detected in spacing, combination, and extreme value distributions, indicating limits of randomness on a large scale.
ˆ Non-overlapping Template Test (16): Preferences detected for some binary patterns (p-value = 2.23 × 10·11), which is characteristic of a deterministic mathematical constant.

6.3

Comparison with Previome Studies

Results of my analysis are consistent with studies presented in arXiv:2504.10394 (2025), which also indicate limits of randomness of · on large scales. While earlier analyses on smaller samples (of the order of a million digits) suggested perfect randomness, my analysis on a sample of 10 billion digits reveals subtle mathematical structures characteristic of a deterministic constant.

6.4

Cryptographic Applications

Results of the analysis have signi
cant implications for cryptographic applications:

ˆ Good PRNG with seed: · can be meed as a pseudorandom source in PRNG generators with appropriate seeding, as basic ran

ˆ Borel, E. (1909). Les probabilités dénombreuses et leurs applications arithmétiques. Rendiconti del Circolo Matematico di Palermo, 27, 247-271.
ˆ Shannon, C. E. (1948). A Mathematical Theory of Communication. Bell System Technical Journal, 27(3), 379-423.
ˆ Digits of pi: limits to the seeming randomness II. arXiv:2504.10394 (2025). Analysis of limits of randomness of · on large scales, con
rming results of my analysis.