INTERDEPENDENCY BETWEEN STOCK MARKET INDICES AND SILVER, CRUDE OIL, NATURAL GAS IN INDIA

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***Abstract*—This study investigates the time-varying volatility and correlations between natural gas, crude oil, silver and Indian stock prices using different multivariate generalized autoregressive conditional heteroskedasticity (MGARCH) models considering symmetry and asymmetry. We analyze optimal portfolio weights and hedging ratios through pair trading between stocks, silver and natural gas, crude oil energy commodity futures for forecasting potential market risk exposure and to determine portfolio diversification benefits. Results shows that VARMA-DCC-GARCH model shows more efficiency than CCC model with asymmetry in time-varying correlations estimation. Results reveal that there is significant cointegration between natural gas, crude oil, silver and stock prices.**

***Index Terms*—volatility, correlation, multivariate, autoregressive, heteroskedasticity, cointegration**

I. INTRODUCTION

There is increased demand for oil, natural gas and silver in emerging countries. This has caused the rise in prices which has motivated the policy makers to understand the relationship between these commodities and the stock markets which impacts the overall economy of the country. Natural gas supplies 22% of total energy used worldwide and releases lower level of CO2 compared to oil and coal which makes it environmental friendly and is the critical source of energy for transportation. Nearly 85% of the total imports are met through natural gas in India. Among all the precious metals, silver is more affordable and has been preferred choice of masses for bullion investment. The potential of prices going higher domestically look high due to good demands and industrial usage. So, for long term investors, it is a good choice to invest in silver.

In this study, we analyse empirical properties of natural gas, crude oil, silver and stock price volatilities using MGARCH models with constant conditional correlation (CCC) and dynamic conditional correlation (DCC) which examine the interactions and spillovers between oil, natural gas, silver and stock prices. AR-GARCH (1,1) and VARMA-GARCH (1,1) specifications are used to derive conditional variance. Finally, optimal portfolio design and hedge ratios are examined using conditional covariances between the stock and energy commodity and silver markets.

II. LITERATURE REVIEW

There are many studies investigating the link between oil and stock indices. Most of them found negative impact of oil price shocks on international stock market returns (Chiou and Lee, 2009; Jones and Kaul, 1996; Lee and Chiou, 2011; Narayan and Narayan, 2010; Park and Ratti, 2008; Sadorsky, 1999). The dynamic volatility spillover between oil and stock markets are of increasing interest to the construction of optimal risky portfolios and hedge ratios in financial risk management because of the recent uncertainty in oil prices. Many studies have focused on using various MGARCH models to examine the dynamic correlation, portfolio design, and hedging effectiveness of commodity-equity portfolios. S.Venkata Behera, Chinmaya (2009) examined Stock prices and its relation with crude oil prices, analyzes that whether the crude oil prices have great authority over Indian Stock market prices or not. Studies suggested that oil prices had a negative effect on all stock markets except during the 2008 global financial crisis (GFC).

DCC approach to investigate the relationship between the S&P 500 index and commodity prices including oil, copper, gold, and silver by Choi and Hammoudeh (2010) demonstrated evidence of increasing correlations between all the commodities but a decreasing correlation with the S&P 500 index. They implied that commodity futures play an important role in portfolio diversification benefits and risk reduction in the stock markets.Chkili (2016) explored the dynamic correlation and hedging effectiveness between gold and stock markets of the BRICS countries using the asymmetric DCC-MGARCH model.

Melvin and Sultan (1990) analyzed the interrelationship between gold silver with the perspective that variations in oil price act as prominent determinants of instability in gold prices. They examined that hike in oil prices generates high income for the countries who are exporting oil. At the same time, gold constitutes a important portion of their portfolios, this maximize the demand for gold and results to higher gold prices.W Mensi(2017) found increases in the correlations among the markets in the aftermath of the 2008–2009 GFC and the oil, gold, energy.

More recently, Zhang et al. (2017) employed the volatility threshold dynamic conditional correlations (VT-DCC) approach to investigate the spillover effect of stock market volatility index for crude oil and natural gas markets during 1999–2015 and found evidence of similarities in the correlation dynamics between crude oil and stock volatility series. Overall, it is apparent that the previous empirical studies provide strong evidence of low and positive correlations between commodity and developed equity markets, implying that commodity futures offer diversification benefits and play an important role in reducing equity risk in an investment portfolio.

III. Methodology

A. Multivariate GARCH modeling

To specify time-varying variances and covariances, MGARCH has become the popular tool. There are several MGARCH models available. For DCC and CCC models, the conditional variance is assumed to be VARMA-GARCH(1,1) to include asymmetric GARCH effect. This is called as VARMA-AGARCH model. VARMA-AGARCH is used to estimate the mean and conditional variance of daily returns of natural gas, crude oil, silver and stock prices.

$$r_{it} = m_{i0}+\sum_{j=1}^{3} m_{ij}r_{jt-1}+\epsilon_{it}, \epsilon_{it}|\Omega_{it-1} \sim N(0,h_{it}), i=1,2,3 \tag{1}$$

$$\epsilon_{it} = v_{it}h_{it}^{1/2}, v_{it} \sim N(0,1) \tag{2}$$

$$h_{it} = c_{it}+\sum_{j=1}^{3} \alpha_{ij}\epsilon_{jt-1}^2+\sum_{j=1}^{3} \beta_{ij}h_{jt-1}+d_i\epsilon_{it-1}^2 I(\epsilon_{it-1}) \tag{3}$$

B. Dynamic conditional correlation

DCC model has been used to deal with time-varying correlations. GARCH parameters are tested before the correlations are estimated. The correlations are estimated as follows:

$$H_t = D_t R_t D_t \tag{4}$$

H_t is a 3×3 conditional covariance matrix, R_t is a conditional correlation matrix, and D_t is a diagonal matrix with time-varying standard deviations on the diagonal.

$$D_t = diag(h_{11t}^{1/2}, ..., h_{33t}^{1/2}), \tag{5}$$

$$R_t = diag(p_{11t}^{-1/2}, ..., p_{33t}^{-1/2}) A_t diag(p_{11t}^{-1/2}, ..., p_{33t}^{-1/2}), \tag{6}$$

At is the symmetric positive definite matrix.

$$A_t = (1-\Theta_1-\Theta_2)\overline{A} + \Theta_1(\xi_{t-1}\xi\,'_{t-1}) + \Theta_2 A_{t-1}, \tag{7}$$

where $\overline{A}$ is the 3×3 unconditional correlation matrix of the standardized residuals ξ_t. The parameters Θ_1 and Θ_2 are non-negative with a sum of less than unity. The correlation estimator is:

$$\rho_{i,j,t} = \frac{P_{i,j,t}}{\sqrt{P_{i,i,t}P_{j,j,t}}}, \tag{8}$$

In the CCC model, R_t = R and R_{ij} = ρ_{ij}. In the diagonal MGARCH model, ρ_{ij} = 0 for all i and j, h_{ij} = 0 for all i not equal to j. To compute an unconditional covariance matrix, we use standardized residuals of MGARCH diagonal model.

IV. Data

In this study, we consider daily data of natural gas futures(NTG), crude oil futures(OIL) and silver futures from Multi Commodity Exchange of India Limited. Natural gas prices are denoted in Indian rupees per million British thermal units, OIL process are denoted in Indian rupees per barrel. Silver prices are denoted in Indian rupees per kilogram. The proxy for the Indian stock market Nifty 50 index (N50) is obtained from National Stock Exchange of India. We use daily data from July 10 2006 to April 07 2020. The choice of daily data gives detailed picture of energy commodities, silver and financial markets.

(fig:1) plots the daily evaluation of OIL,NTG, SILVER and N50 indexes during the sample period and (fig:4) plots the volatility from VARMA-GARCH DCC model in each of them. A closer examination of this figure reveals that all markets decreased after 2008 and 2009, corresponding to the GFC period.The energy markets further exhibited a second important decline between mid-2014 and 2015.The closer examination of all plots shows that there is sudden variation in markets starting from mid of first quarter of 2020 (this decline is caused due to COVID outbreak worldwide).

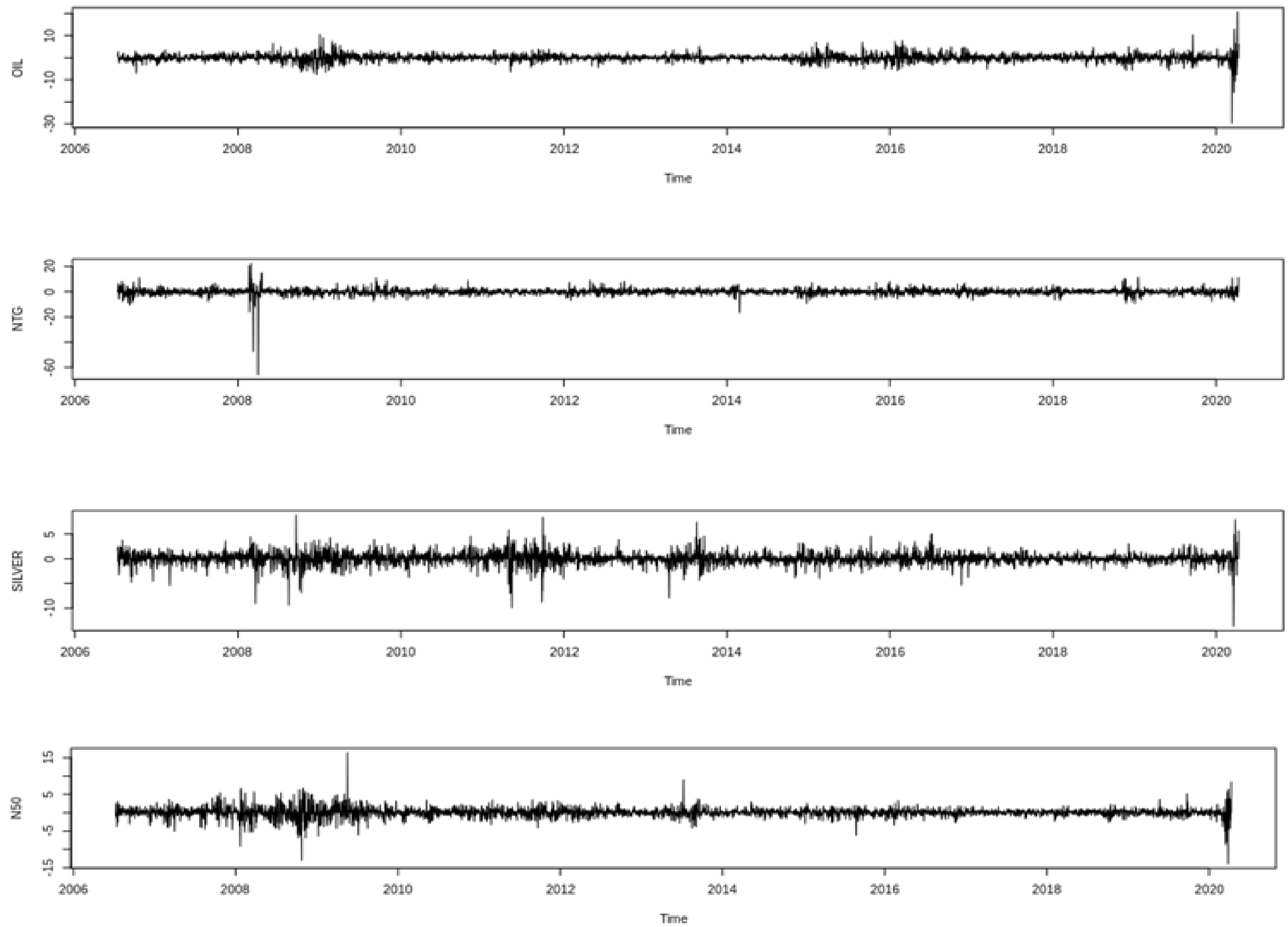

Fig. 1: Time-variations in daily NTG, OIL, SILVER and N50.

V. ANALYSIS

Inoder to calculate the daily returns of stock ,oil, natural gas and silver we consider the difference in the logarithmic values of two consecutive prices: $r_{i,t} = ln(P_{i,t}/P_{i,t-1}) \times 100$,where r_{it} denotes the continuously compounded percentage daily returns for index i at time t , while P_{it} denotes the price of index i at time t .

(fig:5) shows descriptive statistics for the daily returns of OIL, NTG, SILVER AND N50. The mean values are insignificantly different fr om zero while the standard deviations are larger than the mean values for each series. Asymmetry of the probability distribution of a random variable about its mean is measured by skewness value. Skewness is negative for all series indicating the data are negatively skewed or skewed left, meaning that the left tail is longer. Oil and NTG are highly skewed as their values are less than -1. For all return series the kurtosis values exceed three,indicating the leptokurtic distribution which shows heavy tails on either side, giving a sign of large outliers. The Jarque and Bera (1987) test results reject the null hypothesis of normality in the returns.Any deviation from normal distribution with expected skewness of 0 and an expected excess kurtosis of 0 increases the JB statistic. According to the Ljung-Box test Q (20) results, we provide evidence for serial correlations at 1% significant level for residuals in OIL, NTG, SILVER and N50. Whereas in the case of silver with a p value 0.7482 we don't have enough evidence to reject the null hypothesis that the residuals are not autocorrelated. We use unit root tests, such as augmented Dickey and Fuller(ADF)(1979) test, Phillips and Perron(PP)(1988) test and the Kwiatkowski et al.(KPSS) (1992) test to check the stationarity of the time series variables. The results indicate that all index return series are stationary at the conventional 1% level of significance.

We perform 3 cointegration tests to examine the long-run relationships among all the returns. In Johansen cointegration test (fig:7) the first hypothesis,r=0, tests for the presence of cointegration. Since the test statistic exceeds the 1% level significantly (7539.68 $>$ 55.43), we have strong evidence to reject the null hypothesis of no cointegration. In oder to confirm the findings of Johansen test we carry out Phillips-Ouliaris cointegration test (fig:7). A small p-value (typically 0.05) and high test statistics indicate strong evidence against the null hypothesis, so we reject the null hypothesis. Finally,

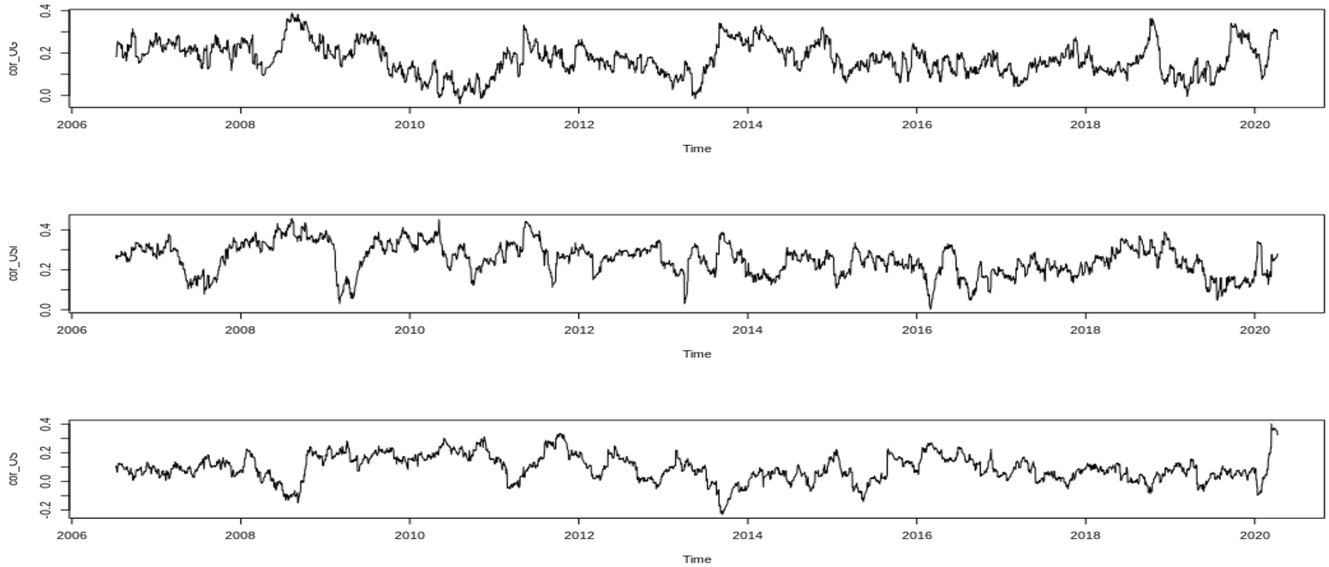

Fig. 2: Conditional Correlation between OIL AND NTG, OIL AND SILVER , OIL AND N50

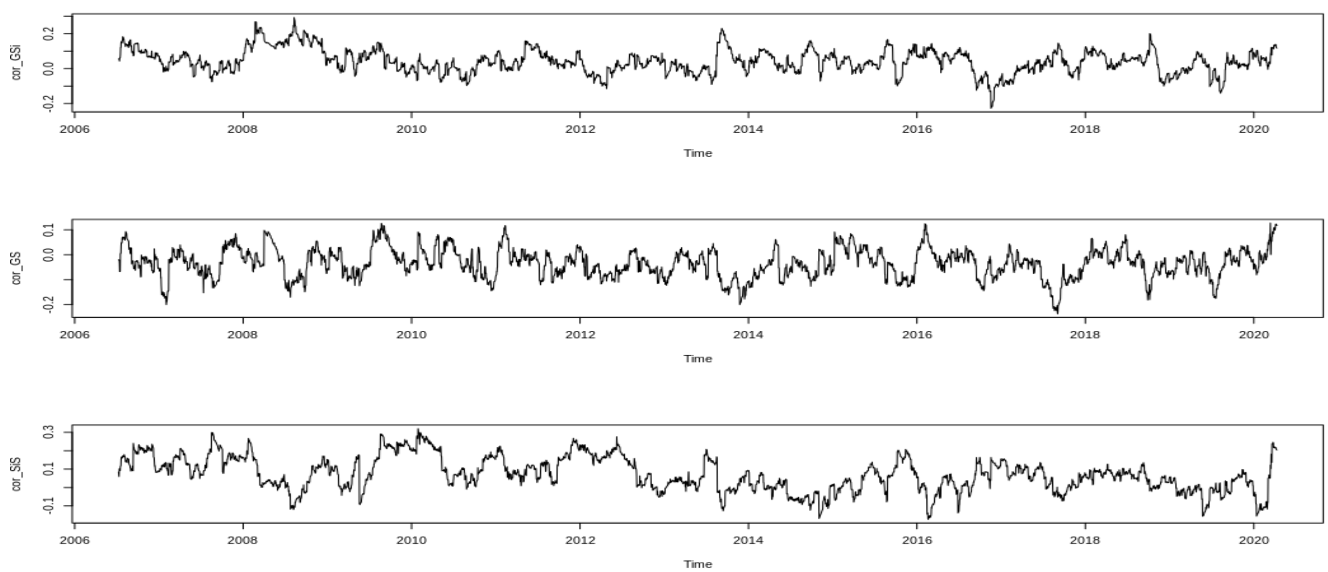

Fig. 3: Conditional Correlation between NTG AND SILVER, NTG AND N50 , SILVER AND N50

Table. 2 shows the Engle and Granger (1987) cointegration tests results (fig:6) which also collaborates the results of the first two tests of cointegration since the test statistics is significant and p value is small.

Our findings have shown significant relationship between energy and stock prices (see Sadorsky, 1999; Park and Ratti, 2008; Kilian and Park, 2009). Though it is inapposite to some other studies. For instance, Kandir et al. (2013) found no long-run relationship between natural gas prices, real GDP, real exchange rates, and stock prices in Turkey. Market efficiency which necessitates all relevant information to be reflected in the stock prices and the large amount of data subjected to study enables us to find the presence of cointegration between natural gas prices, oil prices, silver prices and stock prices.

VI. Empirical Results

We estimate the MGARCH modeling and report the results of AR(1)-GARCH (1,1) model and the VARMA (1,1)-GARCH (1,1) models.

The mean model equations of NTG, OIL, SILVER and N50 indicate persistence in the returns.The coefficients of the N50, OIL, NTG, SILVER equations are positive and significant for with asymmetry and without asymmetry for all the models. The Arch (alpha) terms represent the estimated coefficients for GARCH volatility that measure short-term persistence

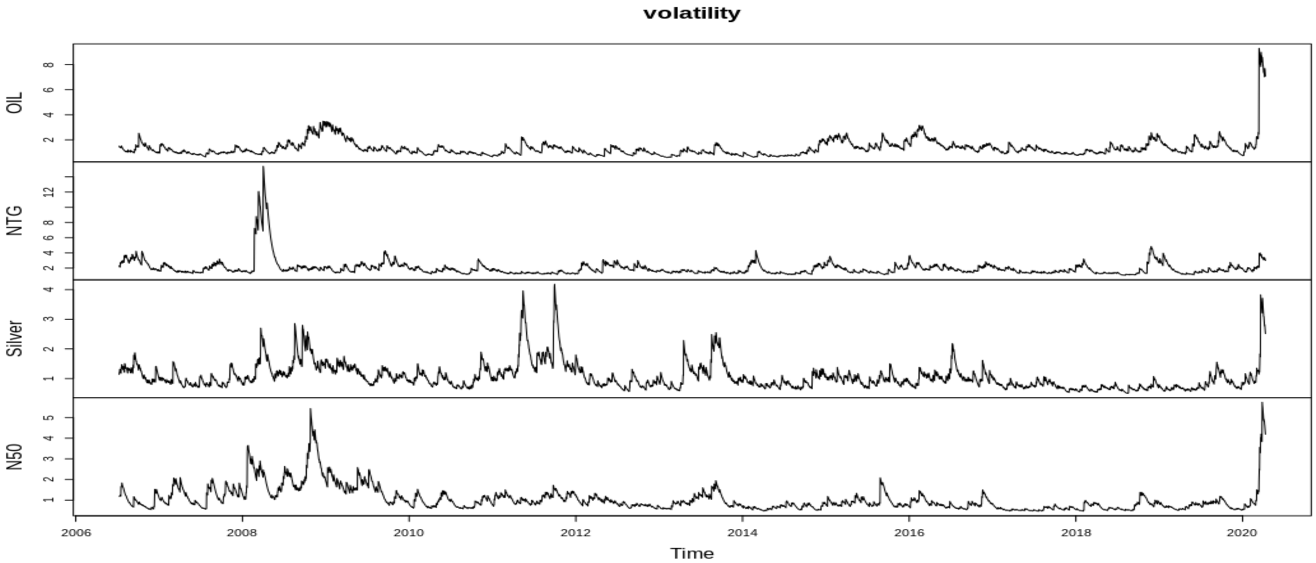

Fig. 4: volatility of OIL, GAS,SILVER and STOCKS from VARMA-GARCH-DCC model

	OIL	NTG	N50	SILVER
Mean	-0.04	-0.08	0.03	0.02
Standard Deviation	1.52	2.31	1.19	1.16
Minimum	-29.68	-65.78	-13.9	-13.66
Maximum	20.67	22.25	16.33	8.82
Skewness	-1.1	-5.91	-0.26	-0.85
Kurtosis	42.37	171.77	21.14	13.1
Jarque-Bera	376909.1	6206648	93602.68	36519.1
ARCH-LM(10)	312.16	22.43	409.95	472.44
Q(20)	138.4***	51.065***	79.989***	14.591
ADF	-48.5165***	-46.9206***	-48.924 ***	45.7054***
PP	-61.2814***	-63.4265***	-69.6561***	-58.3026***
KPSS	0.0695	0.0401	0.0609	0.057

Fig. 5: Descriptive statistics of daily returns

and volatility clustering.All variables show evidence of short-term persistence. Moreover, the VARMA (1,1)-GARCH (1,1) model measures the short-term volatility spillovers between the variables. If coefficients from the VARMA-GARCH are statistically significant, it means that there exists a volatility spillover. We find evidence of short-term spillovers between OIL, NTG,SILVER and N50 at 1% significance level.

A significant GARCH term (beta) indicates long term volatility persistence. We find evidence of own long-term persistence, which is greater than the estimated coefficient of own short-term persistence. For instance, the coefficient of [Silver].beta1 is greater than that of [Silver].alpha1 presented in the AR (1)-GARCH (1,1) models in fig:8.We find similar results in VARMA (1,1)-GARCH(1,1) models in fig:9, fig:10, fig:11 and fig:12 also.Hence, we find evidence of long-term volatility spillovers from OIL to NTG, OIL to SILVER,OIL to N50, NTG to SILVER, NTG to N50 and SILVER to N50. Determinants of the equity markets or herd behavior in the financial markets could be the reasons for the persistence of volatility spillovers. If the leverage coefficient (Gamma) is negative and significant , it indicates the presence of an asymmetric behavior. Positive and significant gamma coefficients in GJR-GARCH models of OIL and N50 for both DCC and CCC models with asymmetry in AR(1)-GARCH(1,1) and VARMA

	WITHOUT TREND					WITH TREND				
		1pct	5pct	10pct	test-statistic		1pct	5pct	10pct	test-statistic
OIL∼NTG	**tau1**	-2.58	-1.95	-1.62	-47.1794	**tau3**	**-3.96**	-3.41	-3.12	-47.1905
						phi2	**6.09**	4.68	4.03	742.318
						phi3	**8.27**	6.25	5.34	1113.475
NTG∼N50	**tau1**	-2.58	-1.95	-1.62	-46.9949	**tau3**	-3.96	-3.41	-3.12	-47.0046
						phi2	6.09	4.68	4.03	736.4823
						phi3	8.27	6.25	5.34	1104.721
OIL∼N50	**tau1**	-2.58	-1.95	-1.62	-48.9181	**tau3**	-3.96	-3.41	-3.12	-48.9084
						phi2	6.09	4.68	4.03	797.3482
						phi3	8.27	6.25	5.34	1196.022
NTG∼SILVER	**tau1**	-2.58	-1.955	-1.62	-47.10899	**tau3**	-3.966	-3.411	-3.12	-47.1207
						phi2	6.09	4.68	4.03	740.1242
						phi3	8.27	6.25	5.34	1110.184
OIL∼SILVER	**tau1**	-2.58	-1.95	-1.62	-48.6744	**tau3**	-3.96	-3.41	-3.12	-48.6646
						phi2	6.09	4.68	4.03	789.4182
						phi3	8.27	6.25	5.34	1184.127
SILVER∼N50	**tau1**	-2.58	-1.95	-1.62		**tau3**	-3.96	-3.41	-3.12	-46.0527
						phi2	6.09	4.68	4.03	706.9575
						phi3	8.27	6.25	5.34	1060.436

Fig. 6: Engle and Granger cointegration test

(1,1)-GARCH(1,1) indicate that negative shocks will increase the volatility more than positive shocks. We are unable to find a positive and a statistically significant leverage effect for NTG and SILVER as evident from the coefficients.This leverage effect is attributable to arbitrage and heterogenous and asymmetric information in the market.

fig:2 and fig:3 presents the conditional correlation obtained from the VARMA-DCC-GARCH model from July 2006 to November 2020. The correlation coefficients are not constant but vary tremendously with time in pair-wise comparisons. Note that the correlations indicate the benefit of diversifying and investing in the Indian stock and energy futures markets. To be precise, the pair of OIL-N50 shows more volatility and higher correlation coefficients than the NTG-N50 and SILVER-N50 pairs ,indicating that there is little benefit of portfolio diversification between the Indian stock and oil futures markets. Indeed, the DCCs revealed that investors adjust their portfolio structure accordingly. It is also important to compute the optimal portfolio weights and the time-varying hedge ratios, which are of concern to investors and portfolio managers.

The constant conditional correlations (fig:8, fig:11and fig:12) from the AR-GARCH model with and without asymmetry are very similar to the VARMA-GARCH model with and without asymmetry. C1,C2,C3,C4,C5,C6 have similar values in all CCC models.The negative correlation between NTG and N50 gives a gives a thumps up to investment in NTG as it assures low portfolio risk. The DCC model shows that DCC(A) and DCC(B) are positive and statistically significant at the 1% level. As the sum of the values of CCC(A) and DCC(B) is less than one, it indicates that dynamic conditional correlations are mean reverting.The AIC ,BIC ,SBC and Hannan-Quinn information criterion show that the DCC model is better than the CCC model for both with asymmetry and without asymmetry.

VII. Optimal portfolio designs and dynamic hedging ratios:

Finding from the models suggest that volatility link across the Indian stock and energy markets are an important element for more efficient diversification of portfolios and risk management.Accurate estimation of the temporal covariance matrix helps in building an optimal portfolio by making risk management and portfolio allocation decisions. We compute optimal portfolio weights and hedge ratios for designing optimal hedging strategies. This will help to manage the risks in the Indian stock , energy markets and Bullion market more efficiently. Investors hedge one investment by making a trade in another. We construct a portfolio of Indian stocks and two energy indices Oil and NTG,and bullion index silver for the minimization of the risks without reducing expected returns. We can show how a portfolio investor seeks to hedge the exposure to the stock price movements by investing in India's energy market and Bullion market.As per the Kroner and Ng

```
#######################
# Johansen-Procedure #
#######################
Test type: trace statistic , with linear trend
Eigenvalues (lambda):
[1] 0.3357770 0.3191643 0.3074143 0.2891769
Values of teststatistic and critical values of test:
            test 10pct  5pct  1pct
r <= 3 | 1713.14  6.50  8.18 11.65
r <= 2 | 3556.74 15.66 17.95 23.52
r <= 1 | 5486.21 28.71 31.52 37.22
r = 0  | 7539.68 45.23 48.28 55.43

#######################
Phillips-Ouliaris Cointegration Test
#######################
data:  dat
Phillips-Ouliaris demeaned = -5172.6, Truncation lag parameter
The value of the test statistic is: 21227.92
```

Fig. 7: Johansen and Phillips-Ouliari cointegration test Result

(1998), the portfolio weight of the commodity futures holdings is given by:

$$w_t^C = \frac{h_t^S - h_t^{CS}}{h_t^C - 2h_t^{CS} + h_t^S}, with w_t^C = \begin{cases} 0, w_t^C < 0 \\ w_t^C, 0 \leq w_t^C \leq 1 \\ 1, w_t^C \end{cases} \tag{9}$$

where h_t^C , h_t^S , and h_t^{CS} are the conditional volatility of each commodity returns, the conditional volatility of N50 returns, and the conditional covariance between commodities, silver and N50 returns at time t respectively. From the budget constraint, the optimal weight of the stock is equal to 1 - w_t^C . For each energy/silver−stock pair, all information needed to compute w_t^C is obtained from the VARMA (1,1)GARCH(1,1)-DCC model.

The hedge ratios of Kroner and Sultan (1993) is considered to minimize the risk of this portfolio (Indian stock index and energy commodity/silver index). By how much should a long position (buy) of one dollar in the stock market be hedged by a short position (sell) of β dollar in the energy commodity/silver markets, where β at time t is given by:

$$\beta_t = \frac{h_t^{CS}}{h_t^C} \tag{10}$$

Values of the optimal weight and hedge ratios of the energy commodity/silver markets based on the VARMA (1,1)/-GARCH (1,1)/-DCC model with asymmetry is summarized in the fig:13. As shown in fig:13, the N50/SILVER pair shows a lower average weight value than the N50/NTG and N50/OIL pair, indicating that investors hold more Indian stock proportions in the stock/-oil and stock/-ntg portfolio than their counterpart.The average optimal hedge ratio of natural gas is lower than that of oil and Silver.More precisely, the hedge ratio value of the N50/OIL pair is 0.11, meaning that a dollar long position in the stock market should be hedged by shorting (selling) a ten-cent investment in the oil market. However, the hedge ratio value of the N50/NTG pair is 0.03(refer fig:13 and (fig:14)), suggesting that a one-cent investment in the natural gas market can minimize risk for investors with stock holdings.

Findings suggest that investors prefer Silver and NTG over Oil to hedge their positions against unfavorable stock price movements, indicating that energy commodities and bullion commodities are the perfect assets for reducing portfolio

AR(1)-GARCH(1,1)	Without Asymmetry		With Asymmetry	
	DCC	CCC	DCC	CCC
AR(1) Mean Equation				
OIL-constant	0.017465	0.017462	-0.009871	-0.00907
OIL-ar1	0.211416	0.211414	0.211347	0.211348
NTG-constant	-0.079	-0.078998	-0.050894	-0.050885
NTG-ar1	0.189374	0.189374	0.19029	0.190293
SILVER-constant	0.010336	0.010336	0.01359	0.013584
SILVER-ar1	0.206319	0.206319	0.205784	0.205785
N50-constant	0.058012	0.05804	0.034086	0.034087
N50-ar1	0.039309	0.039324	0.049732	0.049721
GARCH variance equation				
[OIL].omega	0.012591	0.012591	0.015177	0.015177
[OIL].alpha1	0.066361	0.066362	0.032642	0.032643
[OIL].beta1	0.932637	0.932638	0.933908	0.933908
[OIL].gamma1			0.060491	0.060491
[NTG].omega	0.054205	0.054205	0.04867	0.048668
[NTG].alpha1	0.066593	0.066593	0.082321	0.082323
[NTG].beta1	0.924451	0.924451	0.928908	0.928909
[NTG].gamma1			-0.037425	-0.03743
[Silver].omega	0.013187	0.013187	0.013119	0.013119
[Silver].alpha1	0.068359	0.068359	0.073076	0.073077
[Silver].beta1	0.924705	0.924705	0.924802	0.9248
[Silver].gamma1			-0.009292	-0.009288
[N50].omega	0.006436	0.006431	0.007291	0.00729
[N50].alpha1	0.059686	0.059662	0.019069	0.019069
[N50].beta1	0.938785	0.938808	0.939727	0.939729
[N50].gamma1			0.077612	0.077606
[Joint]dcca1	0.011483		0.011434	
[Joint]dccb1	0.97285		0.9734	
[Joint]C1		0.18425		0.185835
[Joint]C2		0.265132		0.264832
[Joint]C3		0.111501		0.101939
[Joint]C4		-0.002311		-0.003578
[Joint]C5		-0.056208		-0.053638
[Joint]C6		0.056259		0.057696
Akaike	12.742	12.777	12.705	12.739
Bayes	12.778	12.811	12.746	12.778
Shibata	12.742	12.777	12.705	12.739
Hannan-Quinn	12.754	12.789	12.719	12.753
Log L	-31960.05	-32051.54	-31863.67	-31951.94

Fig. 8: AR (1)-multivariate GARCH(1,1) models.

VARMA(1,1)-GARCH(1,1) DCC With Asymmetry				
Residual covariance matrix(Sigma)	OIL	NTG	SILVER	N50
OIL	2.2513065	0.4931048	0.4555206	0.3143396
NTG	0.4931048	5.2463488	0.2013588	0.0515632
SILVER	0.4555206	0.2013588	1.2910626	0.162467
N50	0.3143396	0.0515632	0.162467	1.4004603
VAR coefficients(Phi)	OIL	NTG	SILVER	N50
OIL	-0.3377124	0.04437887	0.20172927	0.23435869
NTG	-0.07453392	0.04723526	0.29699724	0.48081678
SILVER	0.05018831	0.0164426	0.03488401	0.08706928
N50	0.0368522	-0.13464538	0.24723451	-0.20677363
VMA coefficients(Theta)				
	OIL	NTG	SILVER	N50
OIL	-0.49967192	0.0239454792	0.2128676	0.2530071
NTG	-0.08492142	-0.060992936	0.3070383	0.49215918
SILVER	0.04227033	-0.000896214	-0.1550715	0.08725132
N50	0.02749755	-0.12404384	0.2486535	-0.22084611
	OIL	NTG	SILVER	N50
The constant vector	-0.05719377	-0.09839578	0.02306444	0.02490024

GARCH variance equation	
[OIL].omega	0.015173
[OIL].alpha1	0.032541
[OIL].beta1	0.933992
[OIL].gamma1	0.060085
[NTG].omega	0.04879
[NTG].alpha1	0.082672
[NTG].beta1	0.928661
[NTG].gamma1	-0.037791
[Silver].omega	0.013158
[Silver].alpha1	0.073134
[Silver].beta1	0.9248
[Silver].gamma1	-0.009474
[N50].omega	0.007333
[N50].alpha1	0.017348
[N50].beta1	0.939655
[N50].gamma1	0.081833
[Joint]dcca1	0.011255
[Joint]dccb1	0.973901
Akaike	12.7
Bayes	12.746
Shibata	12.699
Hannan-Quinn	12.716
Log L	-31846.19

Fig. 9: DCC-VARMA (1,1)–multivariate GARCH(1,1) models with asymmetry

VARMA(1,1)-GARCH(1,1) DCC Without Asymmetry				
Residual covariance matrix(Sigma)	OIL	NTG	SILVER	N50
OIL	2.2513065	0.4931048	0.4555206	0.3143396
NTG	0.4931048	5.2463488	0.2013588	0.0515632
SILVER	0.4555206	0.2013588	1.2910626	0.162467
N50	0.3143396	0.0515632	0.162467	1.4004603
VAR coefficients(Phi)	OIL	NTG	SILVER	N50
OIL	-0.3377124	0.04437887	0.20172927	0.23435869
NTG	-0.07453392	0.04723526	0.29699724	0.48081678
SILVER	0.05018831	0.0164426	0.03488401	0.08706928
N50	0.0368522	-0.13464538	0.24723451	-0.20677363
VMA coefficients(Theta)				
	OIL	NTG	SILVER	N50
OIL	-0.49967192	0.0239454792	0.2128676	0.2530071
NTG	-0.08492142	-0.060992936	0.3070383	0.49215918
SILVER	0.04227033	-0.000896214	-0.1550715	0.08725132
N50	0.02749755	-0.12404384	0.2486535	-0.22084611
	OIL	NTG	SILVER	N50
The constant vector	-0.05719377	-0.09839578	0.02306444	0.02490024
GARCH variance equation				
[OIL].omega	0.01255			
[OIL].alpha1	0.066484			
[OIL].beta1	0.932516			
[NTG].omega	0.054328			
[NTG].alpha1	0.066743			
[NTG].beta1	0.924261			
[Silver].omega	0.013221			
[Silver].alpha1	0.068313			
[Silver].beta1	0.924714			
[N50].omega	0.0064			
[N50].alpha1	0.059482			
[N50].beta1	0.938979			
[Joint]dcca1	0.011306			
[Joint]dccb1	0.973415			
Akaike	12.737			
Bayes	12.779			
Shibata	12.737			
Hannan-Quinn	12.752			
Log L	-31944.52			

Fig. 10: DCC-VARMA (1,1)–multivariate GARCH(1,1) models without asymmetry

VARMA(1,1)-GARCH(1,1) CCC With Asymmetry				
Residuals cov-matrix	**OIL**	**NTG**	**SILVER**	**N50**
OIL	2.2513065	0.4931048	0.4555206	0.3143396
NTG	0.4931048	5.2463488	0.2013588	0.0515632
SILVER	0.4555206	0.2013588	1.2910626	0.162467
N50	0.3143396	0.0515632	0.162467	1.4004603
AR coefficient matrix	**OIL**	**NTG**	**SILVER**	**N50**
OIL	-0.3377	0.0444	0.2017	0.2344
NTG	-0.0745	0.0472	0.297	0.4808
SILVER	0.0502	0.0164	0.0349	0.0871
N50	0.0369	-0.1346	0.2472	-0.2068
MA coefficient matrix	**OIL**	**NTG**	**SILVER**	**N50**
OIL	-0.4997	0.023945	0.213	0.253
NTG	-0.0849	-0.060993	0.307	0.4922
SILVER	0.0423	-0.000896	-0.155	0.0873
N50	0.0275	-0.124044	0.249	-0.2208
Constant	**OIL**	**NTG**	**SILVER**	**N50**
	-0.05719377	-0.09839578	0.02306444	0.02490024

GARCH variance equation	
[OIL].omega	0.015547
[OIL].alpha1	0.031915
[OIL].beta1	0.93571
[OIL].gamma1	0.058631
[NTG].omega	0.048168
[NTG].alpha1	0.080308
[NTG].beta1	0.929058
[NTG].gamma1	-0.033812
[Silver].omega	0.013269
[Silver].alpha1	0.073087
[Silver].beta1	0.924909
[Silver].gamma1	-0.009763
[N50].omega	0.007204
[N50].alpha1	0.019259
[N50].beta1	0.940537
[N50].gamma1	0.075558
[Joint]C1	0.18832
[Joint]C2	0.262879
[Joint]C3	0.101625
[Joint]C4	-0.004874
[Joint]C5	-0.053658
[Joint]C6	0.057496
Akaike	12.747
Bayes	12.776
Shibata	12.747
Hannan-Quinn	12.757
Log L	-31979.74

Fig. 11: CCC-VARMA (1,1)–multivariate GARCH(1,1) models with asymmetry

VARMA(1,1)-GARCH(1,1) CCC Without Asymmetry				
Residuals cov-matrix	**OIL**	**NTG**	**SILVER**	**N50**
OIL	2.2513065	0.4931048	0.4555206	0.3143396
NTG	0.4931048	5.2463488	0.2013588	0.0515632
SILVER	0.4555206	0.2013588	1.2910626	0.162467
N50	0.3143396	0.0515632	0.162467	1.4004603
AR coefficient matrix	**OIL**	**NTG**	**SILVER**	**N50**
OIL	-0.3377	0.0444	0.2017	0.2344
NTG	-0.0745	0.0472	0.297	0.4808
SILVER	0.0502	0.0164	0.0349	0.0871
N50	0.0369	-0.1346	0.2472	-0.2068
MA coefficient matrix	**OIL**	**NTG**	**SILVER**	**N50**
OIL	-0.4997	0.023945	0.213	0.253
NTG	-0.0849	-0.060993	0.307	0.4922
SILVER	0.0423	-0.000896	-0.155	0.0873
N50	0.0275	-0.124044	0.249	-0.2208
Constant	**OIL**	**NTG**	**SILVER**	**N50**
	-0.05719377	-0.09839578	0.02306444	0.02490024
GARCH variance equation				
[OIL].omega	0.012249			
[OIL].alpha1	0.064216			
[OIL].beta1	0.934784			
[NTG].omega	0.053534			
[NTG].alpha1	0.066536			
[NTG].beta1	0.924839			
[Silver].omega	0.013308			
[Silver].alpha1	0.06804			
[Silver].beta1	0.924875			
[N50].omega	0.006292			
[N50].alpha1	0.058555			
[N50].beta1	0.939924			
[Joint]C1	0.186844			
[Joint]C2	0.26336			
[Joint]C3	0.11147			
[Joint]C4	-0.003682			
[Joint]C5	-0.055854			
[Joint]C6	0.056414			
Akaike	12.787			
Bayes	12.81			
Shibata	12.786			
Hannan-Quinn	12.795			
Log L	-32082.56			

Fig. 12: CCC-VARMA (1,1)–multivariate GARCH(1,1) models without asymmetry

	Optimal portfolio weights				Hedging ratios			
	Mean	Std.Dev	Max	Min	Mean	Std.Dev	Max	Min
N50/OIL	0.57	0.14	0.9	-2.23	0.11	0.18	2.14	-0.42
N50/NTG	0.65	0.11	1.12	0.24	-0.03	0.08	0.6	-0.38
N50/SILVER	0.51	0.14	0.87	-4.87	0.09	0.14	1.24	-0.26

Fig. 13: Optimal portfolio weights and Hedge ratios of OIL, GAS, SILVER

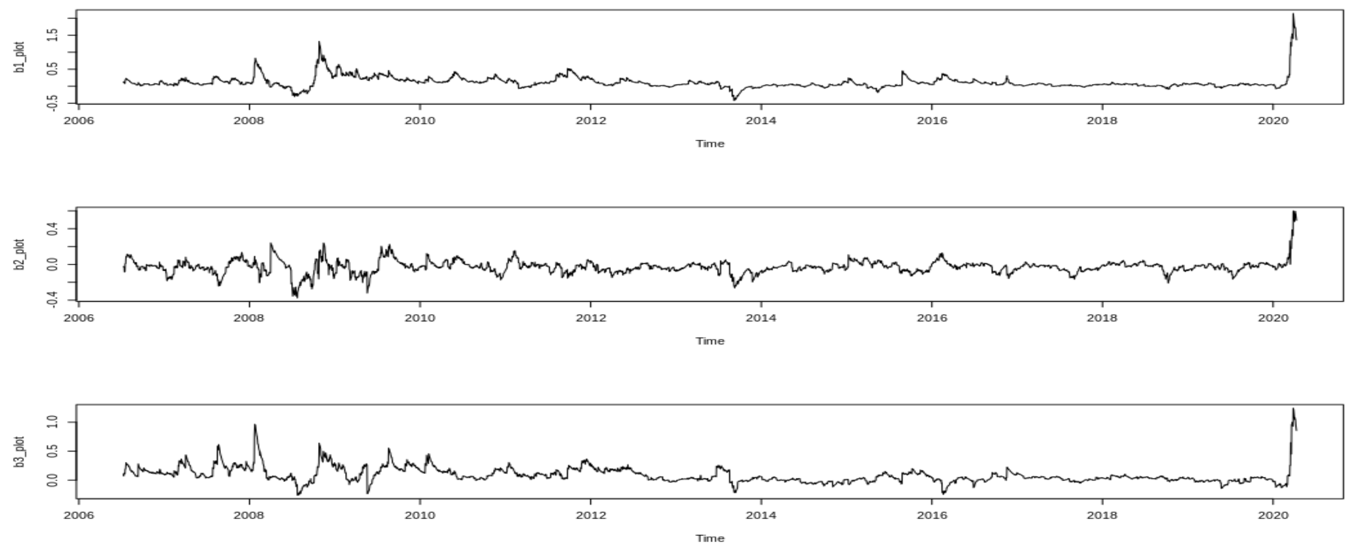

Fig. 14: Time-varying hedging ratios. b1:Hedge of N50 with OIL, b2:Hedge of N50 with NTG, b3:Hedge of N50 with Silver.

risk.When we consider the energy assets, investment in Natural Gas is more secure than Oil.

Dynamic hedge ratios across the Indian stock and energy/silver markets are displayed in fig:14. Variability is following the order of hedge ratio. Over the sample data taken from 2006 to 2020 we observe significantly time-varying hedge ratios. It indicates the investors risk-switching behavior.

Investors adjust their portfolio structure and hedging positions frequently according to the Indian stock and energy/silver futures market conditions. We can also compute risk measures(fig:15, fig:16, fig:17) such as (Standard Deviation (StD), Value at Risk (VaR), Expected Loss (EL), Expected Loss Deviation (ELD), Expected Shortfall (ES), Shortfall Deviation Risk (SDR), Expectile Value at Risk (EVaR), Deviation Expectile Value at Risk (DEVaR), Entropic (ENT), Deviation Entropic (DENT), Maximum Loss (ML)) from empirical data. Silver shows very less value for risk measures. NTG is better than OIL to reduce the portfolio risk according to the risk measures except Entropic (ENT), Deviation Entropic (DENT), Maximum Loss (ML)) from empirical data.

We use VARMA(1,1)-GARCH(1,1)-DCC with asymmetry, the best estimated model with comparatively lowest AIC, BIC to produce 20 forecasts for the covariance or correlation matrix between OIL and N50,NTG and N50,Silver and N50.In Fig19 and Fig20 we display the forecast along with the 30 last in-sample estimates of correlation. The volatility(sigma) forecast for OIL,NTG,Silver and N50 is displayed in fig:20.

VIII. Conclusions and Implications

By virtue of being more affordable, silver has been a preferred choice among all precious metals for billion investment. As the energy commodity prices have low or negative correlations with equity assets dynamics of them are an important element for investors in diversifying their stock portfolios. We investigate time-varying volatilities and dynamic conditional correlations between crude oil, natural gas, silver and stock prices in India using various MGARCH models (with and

	1%	5%
StD	0.00920777	0.00920777
VaR	0.10670483	-0.02610251
EL	19.99994991	19.99994991
ELD	0.05162642	0.05162642
ES	0.09725054	0.03918168
SDR	0.10824215	0.04480819
EVaR	0.08932692	0.04388779
DEVaR	0.09212050	0.04606415
ENT	0.42236312	0.42236312
DENT	0.41705581	0.41705581
ML	0.50746070	0.50746070

Fig. 15: Risk measure of NTG on STOCK

	1%	5%
StD	0.1370905	0.1370905
VaR	0.2610052	0.1941790
EL	19.9999499	19.9999499
ELD	0.1976400	0.1976400
ES	0.2668294	0.1838993
SDR	0.2726505	0.2111549
EVaR	0.2079391	0.1844143
DEVaR	0.2387400	0.2054430
ENT	0.3932351	0.3932351
DENT	0.3924628	0.3924628
ML	0.4479834	0.4479834

Fig. 16: Risk measure of OIL on STOCK

	1%	5%
StD	1.205151e-01	1.205151e-01
VaR	2.681980e-01	5.914231e-02
EL	-1.999995e+01	-1.999995e+01
ELD	1.139714e-01	1.139714e-01
ES	2.519129e-01	1.698356e-01
SDR	2.565235e-01	1.885547e-01
EVaR	2.134246e-01	1.386252e-01
DEVaR	2.235647e-01	1.553728e-01
ENT	-1.492119e-02	-1.492119e-02
DENT	1.866355e-04	1.866355e-04
ML	-1.946069e-01	-1.946069e-01

Fig. 17: Risk measure of SILVER on STOCK

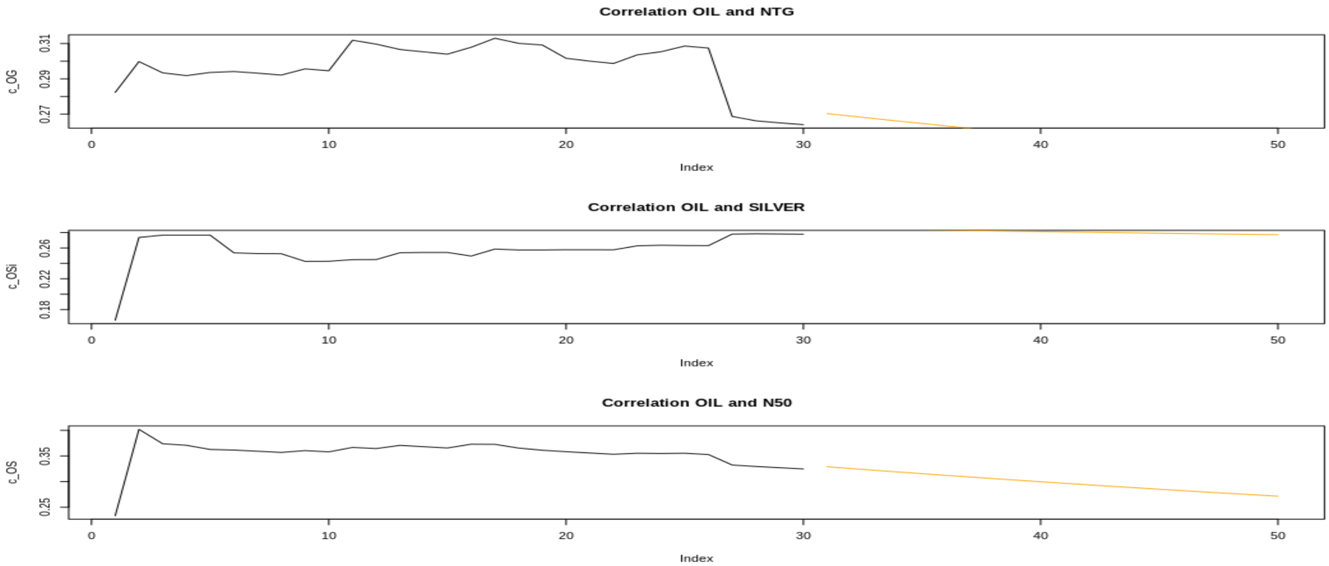

Fig. 18: Correlation forecast between OIL and NTG,SILVER,N50

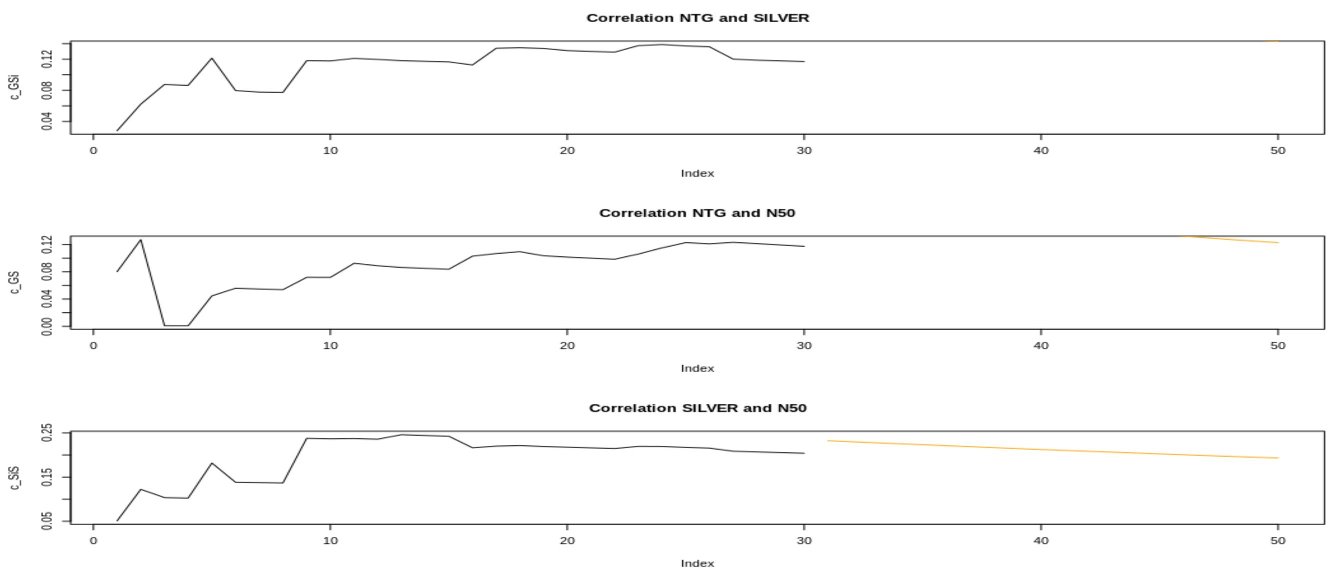

Fig. 19: Correlation forecast between NTG and SILVER, N50; SILVER and N50

without asymmetry) in this paper. In addition to that we analyze the optimal weights and hedge ratios for portfolios to minimize exposure to portfolio risk with respect to portfolio management. we also investigated the risk measure on stocks by investing each of crude oil(fig:16), natural gas((fig:15) and silver(fig:17).

Our empirical results suggest that there is long-run cointegration between energy futures,silver and the stock market in India. By analysing various multivariate GARCH models we find that the DCC model performs better than the CCC model with asymmetry in estimating time-varying correlations.Furthermore, the comparison of optimal portfolio weights and hedging ratios between stock-oil,stock-ntg, stock-silver futures pairs suggests that in optimal portfolios, stocks outweigh commodity futures, and that the stock market risks can be hedged with relatively lower costs. We also forecast the correlation that might occur in the next 20 days(starting from april 8 2020 as we considered data till april 07 2020) between OIL, NTG, SILVER and N50 as shown in fig:18 and fig:19.

The study shows two important implications for portfolio investors dealing with the Indian stock market ,OIL,NTG and Silver while forecasting portfolio risk exposures.First, the superiority of DCC models over the CCC models indicates mean

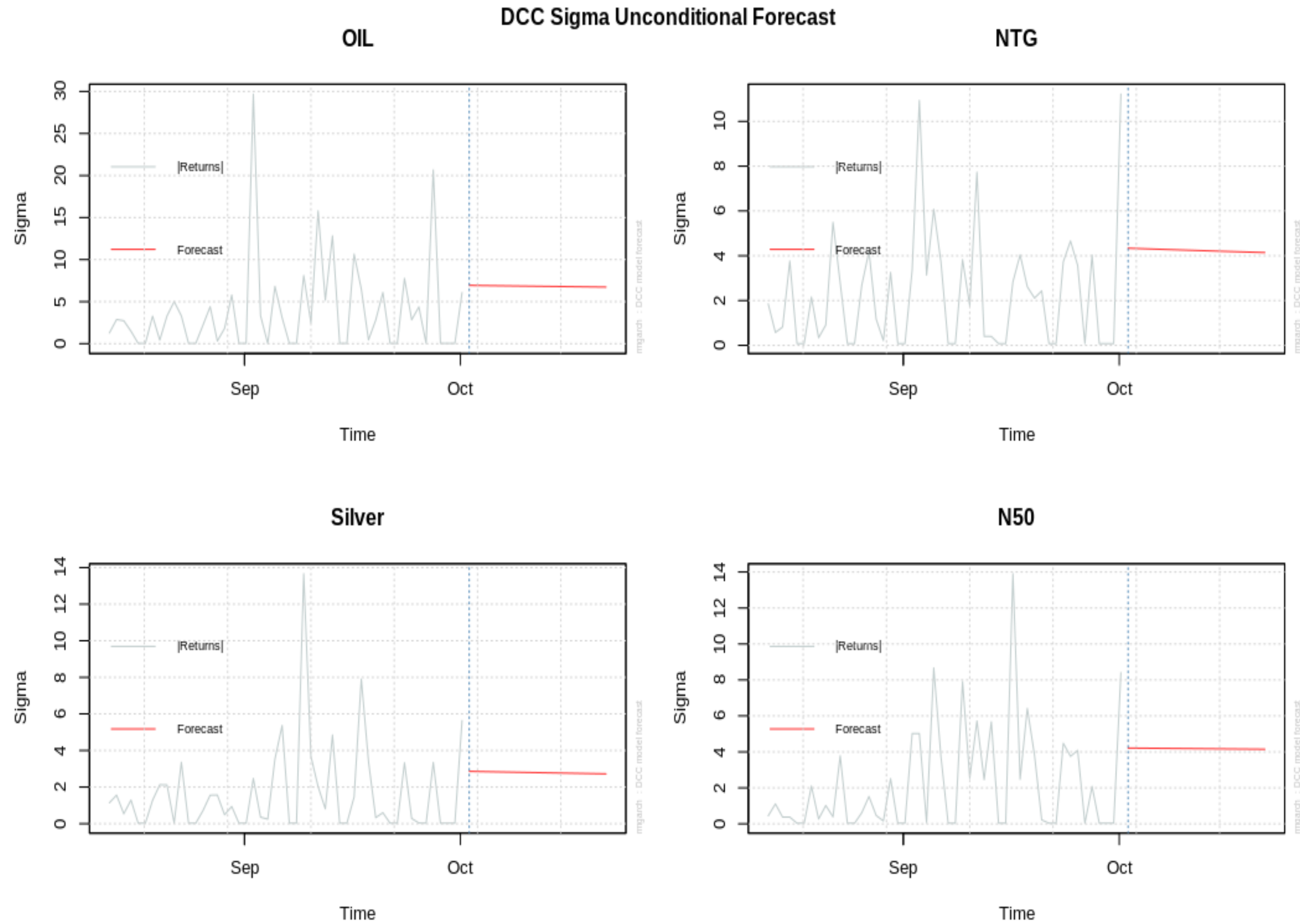

Fig. 20: DCC volatility(sigma) unconditional forecast

reverting tendencies and persistence of asymmetric effects in OIL and N50 stock markets but no asymmetric effects in the NTG, Silver market.In other words, the presence of asymmetric effect or “leverage effect” explains that an unexpected drop in asset prices (bad news) increases volatility more than an unexpected increase in asset prices (good news) with an equal force. In designing decoupling strategies to protect against market contagion risks this findings are important for portfolio investors.Second,we construct synthetic portfolio of stock and oil,NTG,silver by considering optimal portfolio weights and the hedging ratios of these portfolios.Study shows that average hedge ratio for oil is relatively higher than that of natural gas and silver.In sum, a long position (buy) of one dollar in the stock market should be hedged with a short selling of 10-cent in the oil futures or investing either 3-cent in the natural gas futures Or 9 cents in Silver which is cheaper than oil futures. This demonstrates that investors investing in Indian stock market choose to invest in natural gas and silver over oil to hedge their positions against critical and abrupt movement in stock prices,indicating that energy commodities and bullion are the perfect assets for reducing portfolio risk.

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