LINE

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October 2022

***Problem Statement* -Let P=(-1, 0), Q = (0, 0) and R = (3, 3 $\sqrt{3}$) be three point. The equation of the bisector of the $\angle(\mathbf{PQR})$**

1 Solution

Given $\mathbf{P} = \begin{pmatrix} -1 \\ 0 \end{pmatrix}$, $\mathbf{Q} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}$ and $\mathbf{R} = \begin{pmatrix} 3 \\ -3\sqrt{3} \end{pmatrix}$

Let $\mathbf{M}$ be a point on the bisector. $\mathbf{M}$ is equidistant from both the lines $\mathbf{PQ}$ and $\mathbf{QR}$

Equation of the $\mathbf{PQ}$ is given by

$$\mathbf{n_1}^\top \mathbf{x} = 0 \tag{1}$$

$$\mathbf{PQ} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \tag{2}$$

$$\mathbf{n_1} = \begin{pmatrix} 0 \\ -1 \end{pmatrix} \tag{3}$$

$$\mathbf{n_1}^\top = \begin{pmatrix} 0 & -1 \end{pmatrix} \tag{4}$$

$$\mathbf{n_1}^\top \mathbf{x} = \begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x} = 0 \tag{5}$$

$$\|\mathbf{n_1}\| = \sqrt{\mathbf{n_1}^\top \mathbf{n_1}} \tag{6}$$

$$= \sqrt{(0)^2 + (-1)^2} = 1 \tag{7}$$

Equation of the $\mathbf{QR}$ is given by

$$\mathbf{n_2}^\top \mathbf{x} = 0 \tag{8}$$

$$\mathbf{QR} = \begin{pmatrix} 3 \\ 3\sqrt{3} \end{pmatrix} \tag{9}$$

$$\mathbf{n_2} = \begin{pmatrix} 3\sqrt{3} \\ -3 \end{pmatrix} \tag{10}$$

$$\mathbf{n_2}^\top = \begin{pmatrix} 3\sqrt{3} & -3 \end{pmatrix} \tag{11}$$

$$\mathbf{n_2}^\top \mathbf{x} = \begin{pmatrix} 3\sqrt{3} & -3 \end{pmatrix} \mathbf{x} = 0 \tag{12}$$

$$\|\mathbf{n_2}\| = \sqrt{\mathbf{n_2}^\top \mathbf{n_2}} \tag{13}$$

$$= \sqrt{(3\sqrt{3})^2 + (-3)^2} = 6 \tag{14}$$

As $\mathbf{M}$ is equidistant from $\mathbf{PQ}$ and $\mathbf{QR}$

$$\frac{|\mathbf{n_1}^\top \mathbf{x}|}{\|\mathbf{n_1}\|} = \frac{|\mathbf{n_2}^\top \mathbf{x}|}{\|\mathbf{n_2}\|} \tag{15}$$

$$\frac{\begin{pmatrix} 0 & -1 \end{pmatrix} \mathbf{x}}{1} = \frac{\begin{pmatrix} 3\sqrt{3} & -3 \end{pmatrix} \mathbf{x}}{6} \tag{16}$$

By solving the above expression we get,

$$\begin{pmatrix} \sqrt{3} & 1 \end{pmatrix} \mathbf{x} = 0 \tag{17}$$

$$and \begin{pmatrix} \sqrt{3} & -1 \end{pmatrix} \mathbf{x} = 0 \tag{18}$$

Equations (17) and (18) represent the equations of internal and external angular bisectors of $\angle(\mathbf{PQR})$.

Hence the required equation of the internal angular bisector of the $\angle(\mathbf{PQR})$ is

$$\boxed{\begin{pmatrix} \sqrt{3} & 1 \end{pmatrix} \mathbf{x} = 0} \tag{19}$$

The generalised form of the above equation is given by

$$\sqrt{3}x + y = 0 \tag{20}$$

2 Construction

Symbol	**Value**	**Description**
P	(-1,0)	point P
Q	(0,0)	point Q
R	($3\sqrt{3}$,3)	point R
M	(-0.7,1.21)	point M

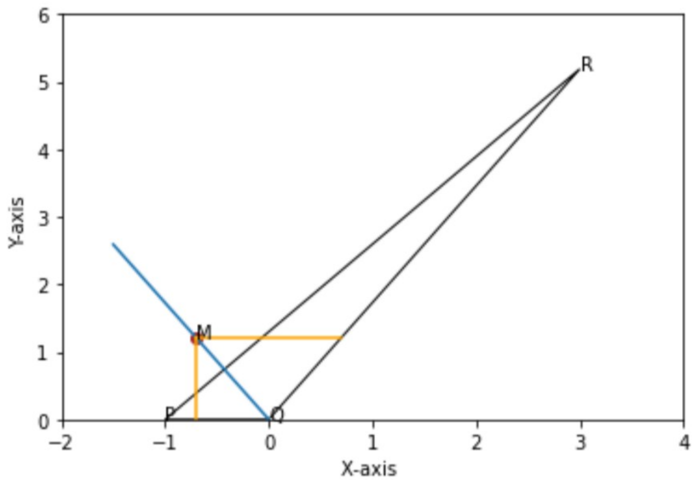

Figure 1: Bisector

The above construction is realized by executing the following code.

https://raw.githubusercontent.com /BavyaVemulapalli/FWC-IITH/main/Line/Code/line.py

3 Conclusion

Hence, the equation of the bisector of angle PQR was found using matrices.