

Optimization Advanced

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Problem Statement - If $\frac{dy}{dx} = x(x-1)^2(x-3)^3$, show that $x = 0$ gives a maximum value to y and $x = 3$ gives a minimum.

1 Solution

Given function is,

$$\nabla f(x) = x(x-1)^2(x-3)^3 \quad (1)$$

1.1 Calculation of Maxima using gradient ascent algorithm

To find:

$$\max_x f(x) \quad (2)$$

Maxima of the above equation (1), can be calculated from the following expression,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \quad (3)$$

Taking $x_0 = 0, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\text{Maxima} = 0$$

$$\text{Maxima Point} = 1$$

1.2 Calculation of Minima using gradient descent algorithm

To find:

$$\min_x f(x) \quad (4)$$

Minima of the above equation (1), can be calculated from the following expression,

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \quad (5)$$

Taking $x_0 = 3, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\text{Minima} = 0$$

$$\text{Minima Point} = 3$$

2 Plot to find maxima and minima of the function

Plot of the function $\frac{(x^2-7x+6)}{(x-10)}$ is shown in the figure 1.

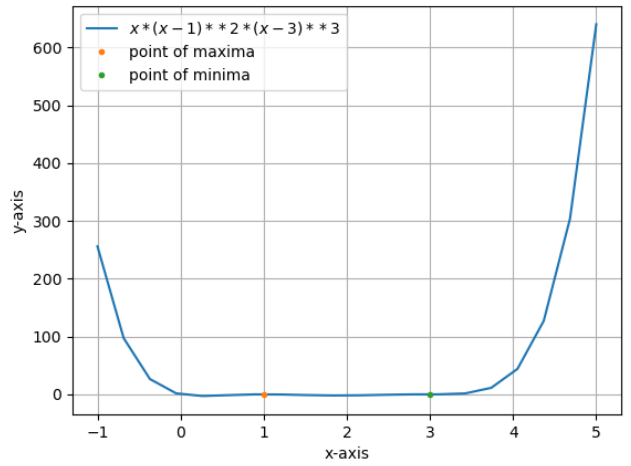


Figure 1: Plot of $df(x)$ to find Maxima and Minima

3 Conclusion

Maxima and Minima and related points are,

$$\text{Maxima point, Max} = (1, -2)$$

and

$$\text{Minima point, Min} = (3, -2)$$