CONIC

VEMULAPALLI BAVYA SRI

October 2022

Problem Statement **- Find the area of the region bounded by the ellipse** $\frac{x^2}{16} + \frac{y^2}{9} = 1$

1 Solution

Given,

$$\frac{x^2}{16} + \frac{y^2}{9} = 1 \tag{1}$$

The standard equation of the conics is given as

$$\mathbf{x}^\top \mathbf{V}\mathbf{x} + 2\mathbf{u}^\top \mathbf{x} + f = 0 \tag{2}$$

By Comparing equations (1) and (2) we get,

$$\mathbf{V} = \begin{pmatrix} 9 & 0 \\ 0 & 16 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, f = -144 \tag{3}$$

Comparing eq (1) with the general form of ellipse,

$$a = 4, b = 3 \tag{4}$$

The Vertices of the ellipse are given as

$$\mathbf{A} = \begin{pmatrix} 4 \\ 0 \end{pmatrix} \tag{5}$$

Area bounded by the ellipse is given by,

$$Area = 4 \times areaof\boldsymbol{OAB} \tag{6}$$

$$= 4 \times \int_0^a f(x)\ dx \tag{7}$$

$$= 4 \times \int_0^4 \frac{3}{4}\sqrt{16 - x^2}\ dx \tag{8}$$

By solving eq (8) we get the required area as

$$\boxed{Area = 12\pi\ units} \tag{9}$$

Symbol	Value	Description
O	(0,0)	Origin
A	(4,0)	point A
B	(0,3)	point B
C	(-4,0)	point C
D	(0,-3)	point D

2 Construction

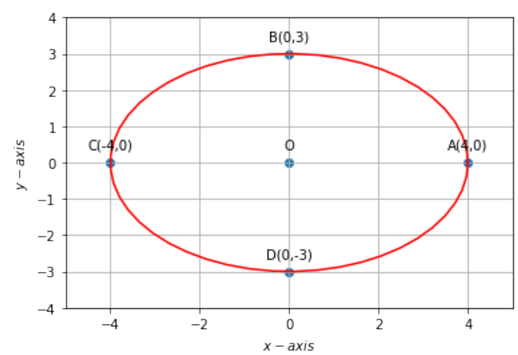

Figure 1: Bisector

The above construction is realized by executing the following code.

https://raw.githubusercontent.com /BavyaVemulapalli/FWC-IITH/main /Conic/Code/conic.py

3 Conclusion

Hence, the area of the region bounded by the given ellipse was found.