Optimization

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***Problem Statement* - It is given that at x=1, the function $x^4 - 62x^2 + ax + 9$ attains its maximum value, on the interval [0,2]. Find the value of a.**

1 Solution

Given function is,

$$f(x) = x^4 - 62x^2 + ax + 9 \tag{1}$$

1.1 Calculation of Maxima using normal differentiation

Differentiating above Eq(1), we get,

$$\nabla f(x) = 4x^3 - 124x + a$$

f attains its maximum value on the interval [0,2] at x=1.

$$\implies \nabla f(1) = 0$$

$$\implies a = 120$$

1.2 Calculation of Maxima using gradient ascent algorithm

Maxima of the above equation (1), can be calculated from the following expression,
To find,

$$\max_x f(x) \tag{2}$$

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{3}$$

Taking $x_0 = 0.5, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Maxima} = 68}$$

$$\boxed{\text{Maxima Point} = 1}$$

$$\boxed{\text{a} = 120}$$

2 Construction

1. At first, the given function has been differentiated and it is solved by setting f'(x) equal to zero. By using x values, f(x) values are calculated.

2. Later, the given function f(x) is solved by gradient ascent algorithm to find maxima and the point at which f(x) is maximum.

3. Maxima and related points are,

Maxima point, Max=(1 , 68)

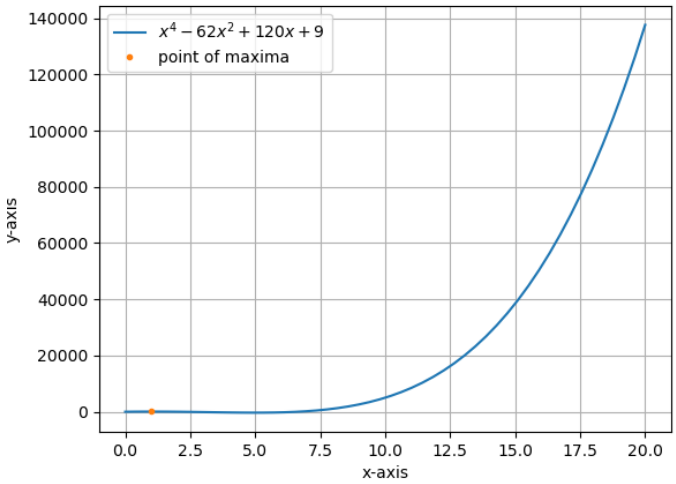

Figure 1: Graph