Optimization Advanced

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***Problem Statement* - If $\frac{dy}{dx} = x(x-1)^2(x-3)^3$, show that $x = 0$ gives a maximum value to y and $x = 3$ gives a minimum.**

1 Solution

Given function is,

$$\nabla f(x) = x(x-1)^2(x-3)^3 \tag{1}$$

1.1 Calculation of Maxima using gradient ascent algorithm

To find:

$$\max_x f(x) \tag{2}$$

Maxima of the above equation (1), can be calculated from the following expression,

$$x_{n+1} = x_n + \alpha \nabla f(x_n) \tag{3}$$

Taking $x_0 = 0, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Maxima} = 0}$$

$$\boxed{\text{Maxima Point} = 1}$$

1.2 Calculation of Minima using gradient descent algorithm

To find:

$$\min_x f(x) \tag{4}$$

Minima of the above equation (1), can be calculated from the following expression,

$$x_{n+1} = x_n - \alpha \nabla f(x_n) \tag{5}$$

Taking $x_0 = 3, \alpha = 0.001$ and precision = 0.00000001, values obtained using python are:

$$\boxed{\text{Minima} = 0}$$

$$\boxed{\text{Minima Point} = 3}$$

2 Plot to find maxima and minima of the function

Plot of the function $\frac{dy}{dx} = x(x-1)^2(x-3)^3$ is shown in the figure 1.

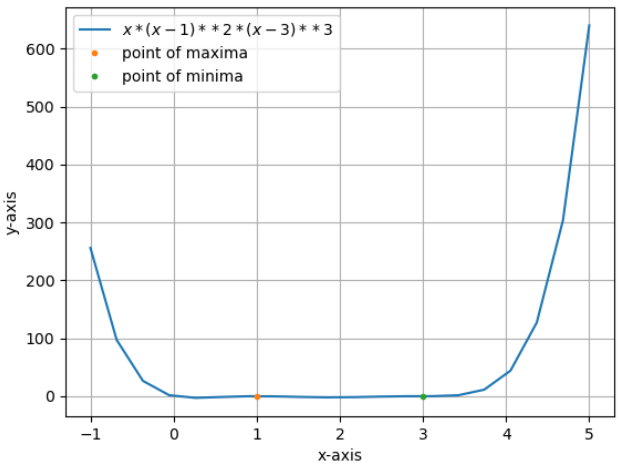

Figure 1: Plot of df(x) to find Maxima and Minima

3 Conclusion

1. At first, the given function has been differentiated and it is solved by setting f'(x) equal to zero. By using x values, f(x) values are calculated.

2. Later, the given function f(x) is solved by gradient ascent algorithm to find maxima and the point at which f(x) is maximum.

3. Then, the given function f(x) is solved by gradient descent algorithm to find minima and the point at which f(x) is is minimum.

4. Maxima and Minima and related points are,

Maxima point, Max=(1 , -2) and

Minima point, Min=(3 , -2)