Deriving the equation of Circle with given parameters Using Matrices

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Contents

1 Problem statement

Intercept on the line y=x by the circle $x^2 + y^2 - 2x = 0$ is AB. Equation of the circle on AB as a diameter is.

2 Considerations

The input parameters are the lengths r, c and angle θ.

Symbol	Description
$x^2 + y^2 - 2x = 0$	Given circle
y=x	Diameter of the required Circle
$\mathbf{C} = \begin{pmatrix} x \\ y \end{pmatrix}$	Center of the required Circle

3 Plotting the Circle with center and radius

Plot of the Circle with center $\mathbf{C} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix}$ and radius r=0.707 is shown in figure 1, .

4 Solution

As per given data, equations of diameter line and circle are

$$y = x \tag{4.0.1}$$

$$x^2 + y^2 - 2x = 0 \tag{4.0.2}$$

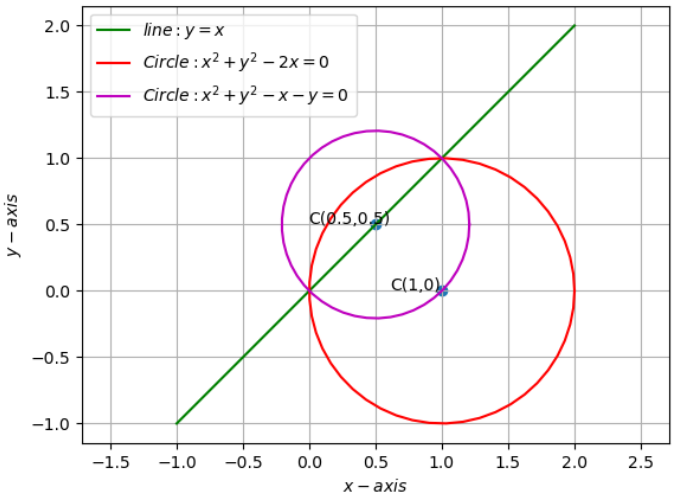

Figure 1: Circle with radius r=0.707 and center C($\frac{1}{2}, \frac{1}{2}$)

The above equations can be written in matrix form as,

$$\begin{pmatrix} 1 \\ -1 \end{pmatrix} \mathbf{x} = 0 \tag{4.0.3}$$

$$\mathbf{x}^T \mathbf{V} \mathbf{x} + 2 \begin{pmatrix} -1 & 0 \end{pmatrix} \mathbf{x} = 0 \tag{4.0.4}$$

On solving above equations, we get two end points of diameter,

$$\mathbf{A} = \begin{pmatrix} 0 \\ 0 \end{pmatrix}, \mathbf{B} = \begin{pmatrix} 1 \\ 1 \end{pmatrix} \tag{4.0.5}$$

4.1 Calculation of center of the Circle

From Point A and B, we can find the center of the required circle as,

$$\mathbf{C} = \frac{\mathbf{A} + \mathbf{B}}{2} = \begin{pmatrix} \frac{1}{2} \\ \frac{1}{2} \end{pmatrix} \tag{4.1.1}$$

4.2 Calculation of radius of the Circle

As per the given data, y=x is the diameter of the required circle and it is also calculated that the Points A and B are end points of the diameter,
Let r be the radius of circle,

$$r = \frac{\|\mathbf{A} - \mathbf{B}\|}{2} = \frac{\sqrt{2}}{2} = 0.707 \tag{4.2.1}$$

4.3 Deriving equation for Circle in matrix form

The equation of circle in matrix form is,

$$\mathbf{x}^T\mathbf{V}\mathbf{x} + 2\mathbf{u}^T\mathbf{x} + f = 0 \tag{4.3.1}$$

where,

$$\mathbf{V} = \mathbf{I} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}, \mathbf{u} = \begin{pmatrix} -\frac{1}{2} \\ -\frac{1}{2} \end{pmatrix}, f = ||\mathbf{u}||^2 - r^2 = \tfrac{1}{2} - \tfrac{1}{2} = 0$$

$$\implies \mathbf{x}^T\mathbf{I}\mathbf{x} + 2\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}\mathbf{x} = 0 \tag{4.3.2}$$

$$\implies \mathbf{x}^T\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}\mathbf{x} + 2\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}\mathbf{x} = 0 \tag{4.3.3}$$

Therefore, the circle equation can be written as

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}\mathbf{x} = 0 \tag{4.3.4}$$

5 Conclusion

1. At first, end points of diameter (**A** *and* **B**) have been found from the given data.

2. Radius of the center has been calculated from **A** *and* **B**.

3. Matrix equation for $\mathbf{V}, \mathbf{u}, \mathbf{u^T}$ and f has been derived.

4. Finally, the Circle equation has been derived as,

$$\mathbf{x}^T\mathbf{x} + 2\begin{pmatrix} -\frac{1}{2} & -\frac{1}{2} \end{pmatrix}\mathbf{x} = 0$$