Estimation Methods Notes

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Exercises

1. In the Notes, we found that if we take a sample from the uniform distribution $\mathsf{Unif}(0, \theta)$, the method of moments estimate of θ is $\hat{\theta}_{MoM} = 2\bar{x}$. Suppose our sample consists of the following values:

$$0.2 \quad 0.9 \quad 1.9 \quad 2.2 \quad 4.7 \quad 5.1$$

a) What is $\hat{\theta}_{MoM}$ for this sample?

b) What is an wrong with this estimate?

c) Show that this estimator is unbiased.

d) ADVANCED: Use simulation in `R` to find out how often the method of moment estimator is less the maximum observed value, ($\hat{\theta}_{MoM} < \max x$). Report an answer for various sizes of samples. You can just pick an arbitrary value for θ when you sample from the uniform. However, the minimum must be 0.

2. Let $x_1, x_2, ..., x_n$ be a simple random sample from an exponentially distributed population with parameter λ. Find $\hat{\lambda}_{MoM}$.

3. Let $x_1, x_2, ..., x_n$ be an iid random sample from an exponentially distributed population with parameter λ. Find $\hat{\lambda}_{MLE}$.

4. It is mathematically difficult to determine if the estimators found in questions 2 and 3 are unbiased. Since the sample mean is in the denominator; mathematically we may have to work with the joint pdf. So instead, use simulation to get an sense of whether the method of moments estimator for the exponential distribution is unbaised.

5. Find a maximum likelihood estimator for θ when $X \sim \mathsf{Unif}(0, \theta)$. Compare this to the method of moments estimator we found. Hint: Do not take the derivative of the likelihood function.