Multivariate Expectation Applications

YOUR NAME

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Exercises

1. Let X and Y be continuous random variables with joint pmf:

$$f_{X,Y}(x, y) = x + y$$

where $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

a. Find $\mathrm{E}(X)$ and $\mathrm{E}(Y)$.

b. Find $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$.

c. Find $\mathrm{Cov}(X, Y)$ and ρ. Are X and Y independent?

d. Find $\mathrm{Var}(3X + 2Y)$.

2. Optional - not difficult but does have small Calc III idea. Let X and Y be continuous random variables with joint pmf:

$$f_{X,Y}(x, y) = 1$$

where $0 \leq x \leq 1$ and $0 \leq y \leq 2x$.

a. Find $\mathrm{E}(X)$ and $\mathrm{E}(Y)$.

b. Find $\mathrm{Var}(X)$ and $\mathrm{Var}(Y)$.

c. Find $\mathrm{Cov}(X, Y)$ and ρ. Are X and Y independent?

d. Find $\mathrm{Var}\left(\frac{X}{2} + 2Y\right)$.

3. Suppose X and Y are *independent* random variables. Show that $\mathrm{E}(XY) = \mathrm{E}(X)\mathrm{E}(Y)$.
4. You are playing a game with a friend. Each of you roll a fair sided die and record the result.

a. Write the joint probability mass function.

b. Find the expected value of the product of your score and your friends score.

c. Verify the previous part using simulation.

d. Using simulation, find the expected value of the maximum number on the two roles.

5. A miner is trapped in a mine containing three doors. The first door leads to a tunnel that takes him to safety after two hours of travel. The second door leads to a tunnel that returns him to the mine after three hours of travel. The third door leads to a tunnel that returns him to his mine after five hours. Assuming that the miner is at all times equally likely to choose any one of the doors, yes a bad assumption but it makes for a nice problem, what is the expected length of time until the miner reaches safety?

6. ADVANCED: Let $X_1, X_2, ..., X_n$ be independent, identically distributed random variables. (This is often abbreviated as “iid”). Each X_i has mean μ and variance σ^2 (i.e., for all i, $\mathrm{E}(X_i) = \mu$ and $\mathrm{Var}(X_i) = \sigma^2$).

Let $S = X_1 + X_2 + ... + X_n = \sum_{i=1}^{n} X_i$. And let $\bar{X} = \frac{\sum_{i=1}^{n} X_i}{n}$.

Find $\mathrm{E}(S)$, $\mathrm{Var}(S)$, $\mathrm{E}(\bar{X})$ and $\mathrm{Var}(\bar{X})$.