

Probability Rules Applications

Lt Col Ken Horton

Professor Bradley Warner

27 May, 2020

Exercises

1. Let A , B and C be events such that $P(A) = 0.5$, $P(B) = 0.3$, and $P(C) = 0.4$. Also, we know that $P(A \cap B) = 0.2$, $P(B \cap C) = 0.12$, $P(A \cap C) = 0.1$, and $P(A \cap B \cap C) = 0.05$. Find the following:

- a) $P(A \cup B)$
- b) $P(A \cup B \cup C)$
- c) $P(B' \cap C')$
- d) $P(A \cup (B \cap C))$
- e) $P((A \cup B \cup C) \cap (A \cap B \cap C)')$
- f) Advanced: Find $P(A|B)$, the probability of A given we know B has occurred.

2. Consider the example of the family in the reading. What is the probability that the family has at least one boy?

3. The Birthday Problem Revisted.

- a) Suppose there are $n = 20$ people in a classroom. My birthday is April 3rd. What is the probability that at least one other person shares my birthday? Assume only 365 days in a year and assume that all birthdays are equally likely.
- b) In R, find the probability that at least one other person shares my birthday for each value of n from 1 to 80. Plot these probabilities with n on the x -axis and probability on the y -axis. At what value of n would the probability be at least 95%?

4. Thinking of the cards again. Answer the following questions:

- a) Define two events that are mutually exclusive.
- b) Define two events that are independent.
- c) Define an event and its complement.

5. Consider the license plate example from the reading.

- a) What is the probability that a license plate contains **exactly** one “B”?

- b) What is the probability that a license plate contains **at least one** “B”?
6. Consider party example in the reading.
- a) Suppose 8 people showed up to the party dressed as zombies. What is the probability that all three awards are won by people dressed as zombies?
 - b) What is the probability that zombies win “most creative” and “funniest” but not “scariest”?
7. Consider the cards example from the reading.
- a) How many ways can we obtain a “two pairs” (2 of one number, 2 of another, and the final different)?
 - b) Consider all of the distinct outcomes of the dice rolls. Is each outcome equally likely? Explain why or why not.
 - c) What is the probability of drawing a “four of a kind” (four cards of the same value)?
8. Advanced Question: Consider rolling 5 dice. What is the **probability** of a pour resulting in a full house?