Probability Rules Applications Solutions

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Exercises

1. Let A, B and C be events such that $P(A) = 0.5$, $P(B) = 0.3$, and $P(C) = 0.4$. Also, we know that $P(A \cap B) = 0.2$, $P(B \cap C) = 0.12$, $P(A \cap C) = 0.1$, and $P(A \cap B \cap C) = 0.05$. Find the following:

a) $P(A \cup B)$
$$P(A \cup B) = P(A) + P(B) - P(A \cap B) = 0.5 + 0.3 - 0.2 = 0.6$$

b) $P(A \cup B \cup C)$
$$P(A \cup B \cup C) = P(A) + P(B) + P(C) - P(A \cap B) - P(A \cap C) - P(B \cap C) + P(A \cap B \cap C)$$
$$= 0.5 + 0.3 + 0.4 - 0.2 - 0.12 - 0.1 + 0.05 = 0.83$$

c) $P(B' \cap C')$
$$P(B' \cap C') = P((B \cup C)') = 1 - P(B \cup C) = 1 - [P(B) + P(C) - P(B \cap C)]$$
$$= 1 - (0.3 + 0.4 - 0.12) = 0.42$$

d) $P(A \cup (B \cap C))$
$$P(A \cup (B \cap C)) = P(A) + P(B \cap C) - P(A \cap B \cap C) = 0.5 + 0.12 - 0.05 = 0.57$$

e) $P((A \cup B \cup C) \cap (A \cap B \cap C)')$
$$P((A \cup B \cup C) \cap (A \cap B \cap C)') = P(A \cup B \cup C) - P(A \cap B \cap C) = 0.83 - 0.05 = 0.78$$

f) Advanced: Find $P(A|B)$, the probability of A given we know B has occurred.
$$P(A|B) = \frac{P(A \cap B)}{P(B)} = \frac{0.2}{0.3} = 0.667$$

2. Consider the example of the family in the reading. What is the probability that the family has at least one boy?
$$P(\text{at least one boy}) = 1 - P(\text{no boys}) = 1 - P(\text{GGG}) = 1 - \frac{1}{8} = 0.875$$

3. The Birthday Problem Revisted.

a) Suppose there are $n = 20$ people in a classroom. My birthday is April 3rd. What is the probability that at least one other person shares my birthday? Assume only 365 days in a year and assume that all birthdays are equally likely.

$$\text{P(at least one other person shares my bday)} = 1 - \text{P(no one else has my bday)} = 1 - \frac{364^{20}}{365} = 0.0534$$

b) In R, find the probability that at least one other person shares my birthday for each value of n from 1 to 80. Plot these probabilities with n on the x-axis and probability on the y-axis. At what value of n would the probability be at least 50%?

Generalizing,

$$\text{P(at least one other person shares my bday)} = 1 - \text{P(no one else has my bday)} = 1 - \frac{364^{n}}{365}$$
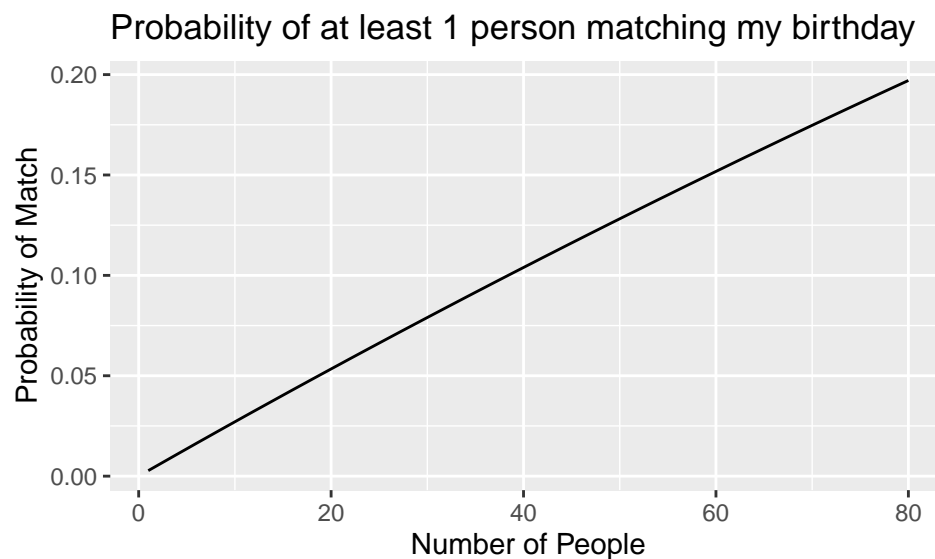
```
n<-1:80
mybday<-function(x) 1-(364/365)^x
mybday <- Vectorize(mybday)
```

Check our function.

```
mybday(20)
```

```
## [1] 0.05339153
```

```
gf_line(mybday(n)~ n,
        xlab="Number of People",
        ylab="Probability of Match",
        title="Probability of at least 1 person matching my birthday")
```



```
prob <- mybday(1:300)
n<-1:300
which(prob>= .5)
```

```
##  [1] 253 254 255 256 257 258 259 260 261 262 263 264 265 266 267 268 269 270 271
## [20] 272 273 274 275 276 277 278 279 280 281 282 283 284 285 286 287 288 289 290
## [39] 291 292 293 294 295 296 297 298 299 300
```

So 253 people.

4. Thinking of the cards again. Answer the following questions:

a) Define two events that are mutually exclusive.

The first card drawn is red.
The first card drawn is black.

b) Define two events that are independent.

The first card drawn is black.
The first card drawn is a face card.

c) Define an event and its complement.

The first card drawn is less than 5.
The first card drawn is equal to or more than 5.

5. Consider the license plate example from the reading.

a) What is the probability that a license plate contains **exactly** one "B"?

```r
#fourth spot
num4<-10*10*10*1*25*25

#fifth spot
num5<-10*10*10*25*1*25

#sixth spot
num6<-10*10*10*25*25*1

denom<-10*10*10*26*26*26

(num4+num5+num6)/denom
```

```
## [1] 0.1066796
```

b) What is the probability that a license plate contains **at least one** "B"?

$$1 - P(\text{no B's})$$

```r
num0<-10*10*10*25*25*25
1-num0/denom
```

[1] 0.1110036

6. Consider party example in the reading.

a) Suppose 8 people showed up to the party dressed as zombies. What is the probability that all three awards are won by people dressed as zombies?

$$\frac{8 \cdot 7 \cdot 6}{25 \cdot 24 \cdot 23}$$

```
(8*7*6)/(25*24*23)
```

[1] 0.02434783

b) What is the probability that zombies win "most creative" and "funniest" but not "scariest"?

$$\frac{8 \cdot 17 \cdot 7}{25 \cdot 24 \cdot 23}$$

```
(8*17*7)/(25*24*23)
```

[1] 0.06898551

7. Consider the cards example from the reading.

a) How many ways can we obtain a "full house" (3 of one number and 2 of another)?

There are $_6P_2 = 6 \cdot 5 = 30$ "full house" scenarios (three 1's and two 2's, three 1's and two 3's, etc.).

b) Consider all of the distinct outcomes of the dice rolls. Is each outcome equally likely? Explain why or why not.

No. There is only one way to obtain the outcome $\{1, 1, 1, 1, 1\}$. However, there are 5 ways to obtain four 1's and one 2.

c) What is the probability of drawing a "four of a kind" (four cards of the same value)?

$$\text{P(4 of a kind)} = \frac{\binom{13}{1}\binom{4}{4}\binom{48}{1}}{\binom{52}{5}}$$

```
(13*1*48)/choose(52,5)
```

[1] 0.000240096

8. Advanced Question: Consider rolling 5 dice. What is the **probability** of a pour resulting in a full house?

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First pick the value for the three of a kind, there are 6. Then pick the value from the remaining 5 for the two of a kind. This is actually a permuation. There are 30 distinct "flavors" of full house (three 1's & two 2's, three 1's & two 3's, etc.). In the reading we did this as

$$\binom{6}{1} \times \binom{5}{1}$$

We now have the 5 dice. We have to select three to have the same value and the order doesn't matter since they are the same value. Thus we multiple by $\binom{5}{3}$. Divide this by the total distinct ways the dice could have landed (assuming order matters).

$$\text{P(full house)} = \frac{30 \times \frac{5!}{3!2!}}{6^5}$$

$$\text{P(full house)} = \frac{\binom{6}{1} \times \binom{5}{1} \times \binom{5}{3}}{6^5}$$

```
30*10/(6^5)
```

```
## [1] 0.03858025
```

File Creation Information

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