Named Discrete Distribution Application Solutions

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Exercises

For each of the problems below, ***1)*** define a random variable that will help you answer the question, ***2)*** state the distribution and parameters of that random variable; ***3)*** determine the expected value and variance of that random variable, and ***4)*** use that random variable to answer the question.

We will demonstrate using 1.1.

1. The T-6 training aircraft is used during UPT. Suppose that on each training sortie, aircraft return with a maintenance-related failure at a rate of 1 per 100 sorties.

   a) Find the probability of no maintenance failures in 15 sorties.

X: the number of maintenance failures in 15 sorties.

$X \sim \textsf{Bin}(n=15,p=0.01)$

$\mbox{E}(X)=15*0.01=0.15$ and $\mbox{Var}(X)=15*0.01*0.99=0.1485$.

$\mbox{P}(\mbox{No maintenance failures})=\mbox{P}(X=0)={15\choose 0}0.01^0(1-0.01)^{15}=0.99^{15}$

```
0.99^15
```

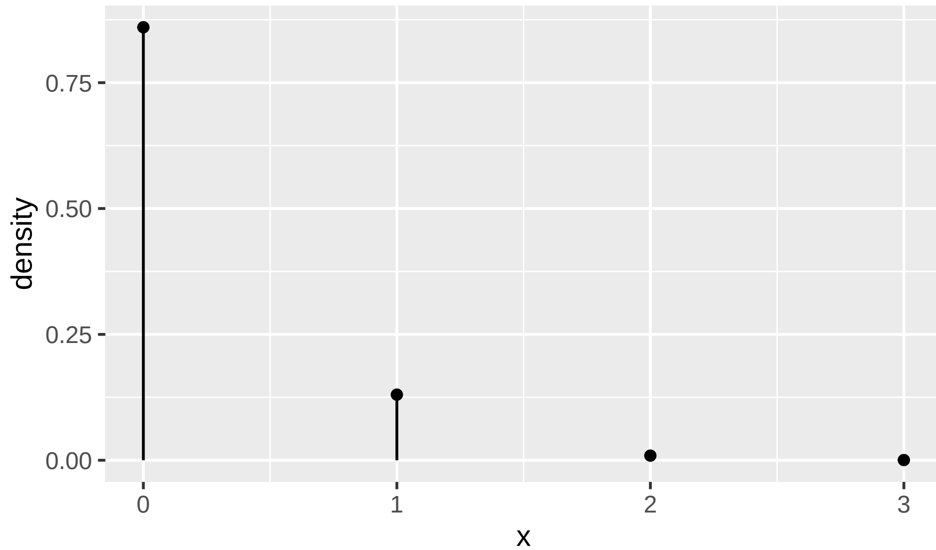
```
## [1] 0.8600584
```

```
## or
dbinom(0,15,0.01)
```

```
## [1] 0.8600584
```

This probability makes sense, since the expected value is fairly low. Because, on average, only 0.15 failures would occur every 15 trials, 0 failures would be a very common result. Graphically, the pmf looks like this:

```
gf_dist("binom",size=15,prob=0.01)
```



b) Find the probability of at least two maintenance failures in 15 sorties.

We can use the same X as above. Now, we are looking for $P(X \geq 2)$. This is equivalent to finding $1 - P(X \leq 1)$:

```
## Directly
1-(0.99^15 + 15*0.01*0.99^14)
```

```
## [1] 0.009629773
```

```
## or, using R
sum(dbinom(2:15,15,0.01))
```

```
## [1] 0.009629773
```

```
## or
1-sum(dbinom(0:1,15,0.01))
```

```
## [1] 0.009629773
```

```
## or
1-pbinom(1,15,0.01)
```

```
## [1] 0.009629773
```

```
## or
pbinom(1,15,0.01,lower.tail = F)
```

```
## [1] 0.009629773
```

c) Find the probability of at least 30 successful (no mx failures) sorties before the first failure.

X: the number of maintenance failures out of 30 sorties.

$X \sim \textsf{Binom}(n = 30, p = 0.01)$, and $\mathrm{E}(X) = 0.3$ and $\mathrm{Var}(X) = 0.297$.

$\mathrm{P}(0 \text{ failures}) = \mathrm{P}(X = 0) = 0.99^{30}$

```
0.99^30
```

```
## [1] 0.7397004
```

```
##or
dbinom(0,30,0.01)
```

```
## [1] 0.7397004
```

Using negative binomial, which was not in the reading but you can research:

Y: the number of successful sorties before the first failure.

$Y \sim \textsf{NegBin}(n = 1, p = 0.01)$, and $\mathrm{E}(X) = 99$ and $\mathrm{Var}(X) = 9900$.

$\mathrm{P}(\text{at least 30 successes before first failure}) = \mathrm{P}(Y \geq 30)$

```
1-pnbinom(29,1,0.01)
```

```
## [1] 0.7397004
```

d) Find the probability of at least 50 successful sorties before the third failure.

Using a binomial random variable, we have 52 trials and need at least 50 to be a success. The random variable is X the number of succesful sorties out of 52.

```
1-pbinom(49,52,.99)
```

```
## [1] 0.9846474
```

Or using a negative binomial, let

Y: the number of successful sorties before the third failure.

$Y \sim \textsf{NegBin}(n = 3, p = 0.01)$, and $\mathrm{E}(X) = 297$ and $\mathrm{Var}(X) = 29700$.

$\mathrm{P}(\text{at least 50 successes before 3rd failure}) = \mathrm{P}(Y \geq 50)$

```
1-pnbinom(49,3,0.01)
```

```
## [1] 0.9846474
```

Notice if the question had been exactly 50 successful sorties before the 3 failure, that is a different question. Then we could use either:

```r
dbinom(50,52,.99)*.01
```

```
## [1] 0.000802238
```

The 0.01 is because the last trial is a failure.

Or

```r
dnbinom(50,3,0.01)
```

```
## [1] 0.000802238
```

2. On a given Saturday, suppose vehicles arrive at the USAFA North Gate according to a Poisson process at a rate of 40 arrivals per hour.

a) Find the probability no vehicles arrive in 10 minutes.

X: number of vehicles that arrive in 10 minutes

$X \sim \textsf{Pois}(\lambda = 40/6 = 6.67)$ and $\mathrm{E}(X) = \mathrm{Var}(X) = 6.67$.

$\mathrm{P}(\text{no arrivals in 10 minutes}) = \mathrm{P}(X = 0) = \frac{6.67^0 e^{-6.67}}{0!} = e^{-6.67}$

```r
exp(-40/6)
```

```
## [1] 0.001272634
```

```r
##or
dpois(0,40/6)
```

```
## [1] 0.001272634
```

b) Find the probability at least 50 vehicles arrive in an hour.

X: number of vehicles that arrive in an hour

$X \sim \textsf{Pois}(\lambda = 40)$ and $\mathrm{E}(X) = \mathrm{Var}(X) = 40$.

$\mathrm{P}(\text{at least 50 arrivals in 1 hour}) = \mathrm{P}(X \geq 50)$

```r
1-ppois(49,40)
```

```
## [1] 0.07033507
```

c) Find the probability that at least 5 minutes will pass before the next arrival.

X: number of vehicles that arrive in 5 minutes

$X \sim \textsf{Pois}(\lambda = 40/12 = 3.33)$ and $\mathrm{E}(X) = \mathrm{Var}(X) = 3.33$.

$\mathrm{P}(\text{no arrivals in 5 minutes}) = \mathrm{P}(X = 0) = \frac{3.33^0 e^{-3.33}}{0!} = e^{-3.33}$

```r
exp(-40/12)
```

```
## [1] 0.03567399
```

```r
##or
dpois(0,40/12)
```

```
## [1] 0.03567399
```

3. Suppose there are 12 male and 7 female cadets in a classroom. I select 5 completely at random (without replacement).

a) Find the probability I select no female cadets.

X: number of female cadets selected out of sample of size 5

$X \sim \mathsf{Hypergeom}(m = 7, n = 12, k = 5)$ and $\mathrm{E}(X) = 1.842$ and $\mathrm{Var}(X) = 0.905$.

$$\mathrm{P}(\text{no female cadets selected}) = \mathrm{P}(X = 0) = \frac{\binom{7}{0}\binom{12}{5}}{\binom{19}{5}}$$

```r
choose(12,5)/choose(19,5)
```

```
## [1] 0.06811146
```

```r
##or
dhyper(0,7,12,5)
```

```
## [1] 0.06811146
```

b) Find the probability I select more than 2 female cadets.

Using the same random variable:

$$\mathrm{P}(\text{more than 2 female}) = \mathrm{P}(X > 2) = 1 - \mathrm{P}(X \leq 2)$$

```r
1-phyper(2,7,12,5)
```

```
## [1] 0.2365841
```

```r
##or
sum(dhyper(3:5,7,12,5))
```

```
## [1] 0.2365841
```