Additional Hypothesis Test Applications Solutions

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Exercises

1. Golf balls

Repeat the analysis of the golf ball problem from earlier this semester.

a. Load the data and tally the data into a table. The data is in `golf_balls.csv`.

```
golf_balls <- read_csv("data/golf_balls.csv")
```

```
head(golf_balls)
```

```
## # A tibble: 6 x 1
##   number
##    <dbl>
## 1      3
## 2      2
## 3      1
## 4      4
## 5      4
## 6      3
```

```
tally(~number,data=golf_balls)
```

```
## number
##   1   2   3   4
## 137 138 107 104
```

b. Using the function `chisq.test` conduct a hypothesis test of equally likely distribution of balls. You may have to read the help menu.

```
chisq.test(tally(~number,data=golf_balls),p=c(.25,.25,.25,.25))
```

```
##
##  Chi-squared test for given probabilities
##
## data:  tally(~number, data = golf_balls)
## X-squared = 8.4691, df = 3, p-value = 0.03725
```

c. Repeat part b. but assume balls with the numbers 1 and 2 occur 30% of the time and balls with 3 and 4 occur 20%.

```
chisq.test(tally(~number,data=golf_balls),p=c(.3,.3,.2,.2))
```

```
##
##  Chi-squared test for given probabilities
##
## data:  tally(~number, data = golf_balls)
## X-squared = 2.4122, df = 3, p-value = 0.4914
```

2. Bootstrap hypothesis testing

Repeat the analysis of the MLB data from the lesson but this time generate a bootstrap distribution of the F statistic.

First, read in the data.

```
mlb_obp <- read_csv("data/mlb_obp.csv")
```

Convert `position` to a factor.

```
mlb_obp <- mlb_obp %>%
  mutate(position=as.factor(position))
```

Summarize the data.

```
favstats(obp~position,data=mlb_obp)
```

```
##   position   min      Q1 median      Q3   max      mean         sd   n missing
## 1        C 0.219 0.30000 0.3180 0.35700 0.405 0.3226154 0.04513175  39       0
## 2       DH 0.287 0.31625 0.3525 0.36950 0.412 0.3477857 0.03603669  14       0
## 3       IF 0.174 0.30800 0.3270 0.35275 0.437 0.3315260 0.03709504 154       0
## 4       OF 0.265 0.31475 0.3345 0.35300 0.411 0.3342500 0.02944394 120       0
```

We need a function to resample the data, we will use the `resample()` from the `mosaic` package.

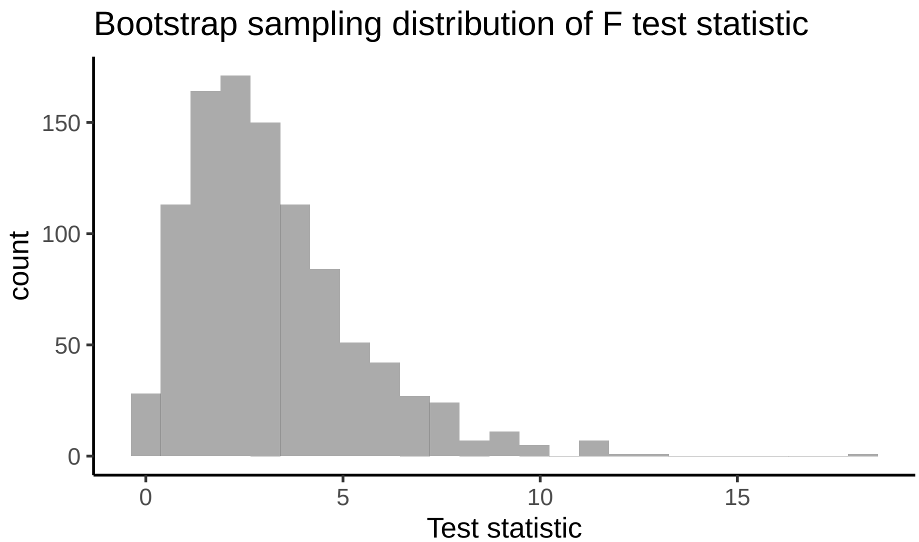
```
library(broom)
```

```
f_boot <- function(x){
  aov(obp~position,data=resample(x)) %>%
  tidy() %>%
  summarize(stat=meansq[1]/meansq[2]) %>%
  pull()
}
```

```
set.seed(541)
results<-do(1000)*f_boot(mlb_obp)
```

Let's plot our sampling distribution.

```r
results %>%
  gf_histogram(~f_boot) %>%
  gf_theme(theme_classic()) %>%
  gf_labs(title="Bootstrap sampling distribution of F test statistic",
          x="Test statistic")
```



Now the confidence interval for the F-statistic is:

```r
cdata(~f_boot,data=results)
```

```
##          lower    upper central.p
## 2.5% 0.3546682 8.724895      0.95
```

We are 95% confident that the F statistic is in the interval $(0.35, 8.72)$ which includes 1 so we fail to reject the null hypothesis of equal means. Remember under the null hypothesis the ratio of the variance between means to the pooled variance within categories should be 1.

3. Test of variance

We have not performed a test of variance so we will create our own.

a. Using the MLB from the lesson, subset on `IF` and `OF`.

```r
mlb_prob3 <- mlb_obp %>%
  filter(position=="IF"|position=="OF") %>%
  droplevels()
```

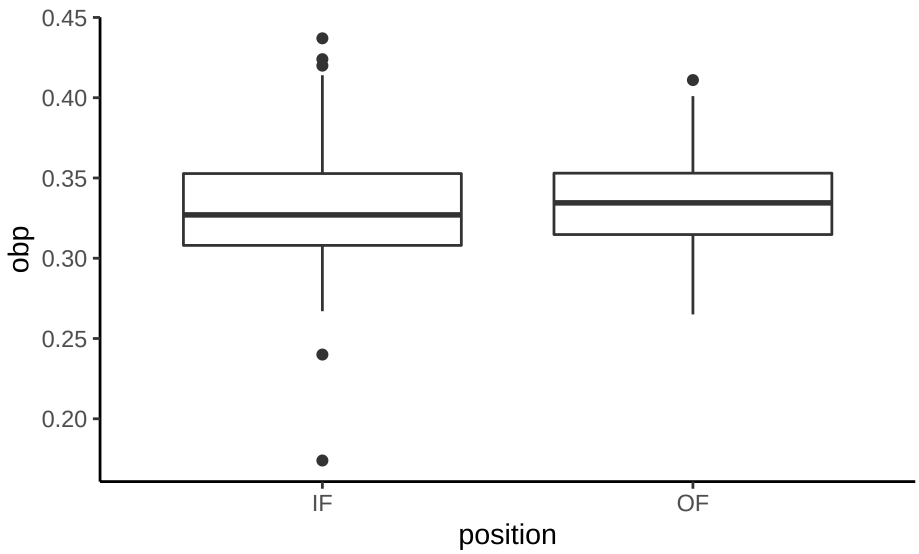
```r
summary(mlb_prob3)
```

```
##  position      obp
##  IF:154   Min.   :0.1740
##  OF:120   1st Qu.:0.3100
##           Median :0.3310
##           Mean   :0.3327
##           3rd Qu.:0.3530
##           Max.   :0.4370
```

The function `droplevels()` gets rid of `C` and `DH` in the factor levels.

b. Create a side-by-side boxplot.

```
mlb_prob3 %>%
  gf_boxplot(obp~position) %>%
  gf_theme(theme_classic())
```



The hypotheses are:
H_0: $\sigma^2_{IF} = \sigma^2 OF$. There is no difference in the variance of on base percentage for infielders and outfielders.
H_A: $\sigma^2_{IF} \neq \sigma^2_{OF}$. There is a difference in variances.

c. Use the differences in sample standard deviations as your test statistic. Using a permutation test, find the p-value and discuss your decision.

```
mlb_prob3 %>%
  group_by(position) %>%
  summarize(stat=sd(obp))
```

```
## `summarise()` ungrouping output (override with `.groups` argument)
```

```
## # A tibble: 2 x 2
##   position   stat
##   <fct>     <dbl>
## 1 IF       0.0371
## 2 OF       0.0294
```

```
obs <- mlb_prob3 %>%
  summarize(stat=sd(obp[position=="IF"])-sd(obp[position=="OF"])) %>%
  pull()
obs
```

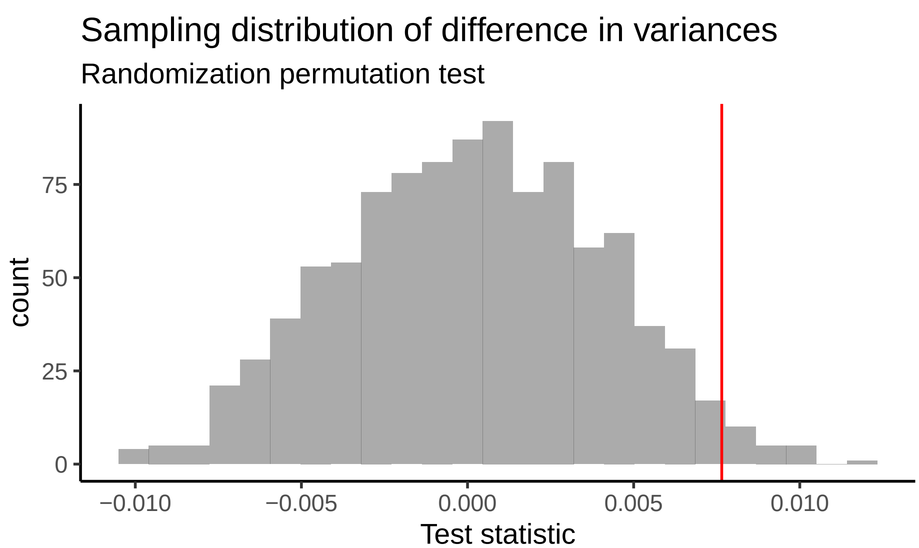
```
## [1] 0.007651101
```

Let's write a function to shuffle the position.

```
perm_stat <- function(x){
  x %>%
  mutate(position=shuffle(position)) %>%
  summarize(stat=sd(obp[position=="IF"])-sd(obp[position=="OF"])) %>%
  pull()
}
```

```
set.seed(443)
results<-do(1000)*perm_stat(mlb_prob3)
```

```
results %>%
  gf_histogram(~perm_stat) %>%
  gf_vline(xintercept=obs,color="red") %>%
  gf_theme(theme_classic()) %>%
  gf_labs(title="Sampling distribution of difference in variances",
          subtitle="Randomization permutation test",
          x="Test statistic")
```



The p-value is

```
2*prop1(~(perm_stat>=obs),data=results)
```

```
##   prop_TRUE
## 0.04395604
```

This is a two sided test since we did not know in advance which variance would be larger. We reject the hypothesis of equal variance but the p-value is too close to the significance level. The conclusion is suspect. We need more data.

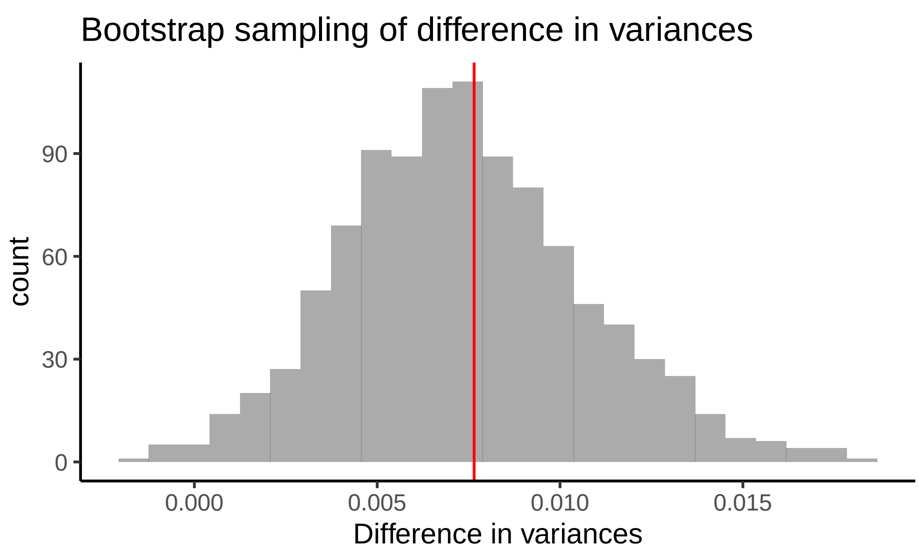
d. Create a bootstrap distribution of the differences in sample standard deviations, and report a 95% confidence interval. Compare with part c.

Let's write a function.

```
var_stat <- function(x){
  resample(x) %>%
  summarize(stat=sd(obp[position=="IF"])-sd(obp[position=="OF"])) %>%
  pull()
}
```

```
set.seed(827)
results<-do(1000)*var_stat(mlb_prob3)
```

```
results %>%
  gf_histogram(~var_stat) %>%
  gf_vline(xintercept=obs,color="red")%>%
  gf_theme(theme_classic()) %>%
  gf_labs(title="Bootstrap sampling of difference in variances",
          x="Difference in variances")
```



File Creation Information

- File creation date: 2020-10-28
- Windows version: Windows 10 x64 (build 18362)
- R version 3.6.3 (2020-02-29)
- `mosaic` package version: 1.7.0
- `tidyverse` package version: 1.3.0
- `openintro` package version: 2.0.0
- `broom` package version: 0.7.0