

Multivariate Expectation Applications

YOUR NAME

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Exercises

1. Let X and Y be continuous random variables with joint pmf:

$$f_{X,Y}(x,y) = x + y$$

where $0 \leq x \leq 1$ and $0 \leq y \leq 1$.

- a) Find $E(X)$ and $E(Y)$.
- b) Find $\text{Var}(X)$ and $\text{Var}(Y)$.
- c) Find $\text{Cov}(X, Y)$ and ρ . Are X and Y independent?
- d) Find $\text{Var}(3X + 2Y)$.

2. Let X and Y be continuous random variables with joint pmf:

$$f_{X,Y}(x,y) = 1$$

where $0 \leq x \leq 1$ and $0 \leq y \leq 2x$.

- a) Find $E(X)$ and $E(Y)$.
- b) Find $\text{Var}(X)$ and $\text{Var}(Y)$.
- c) Find $\text{Cov}(X, Y)$ and ρ . Are X and Y independent?
- d) Find $\text{Var}\left(\frac{X}{2} + 2Y\right)$.

3. Suppose X and Y are *independent* random variables. Show that $E(XY) = E(X)E(Y)$.

4. ADVANCED: Let X_1, X_2, \dots, X_n be independent, identically distributed random variables. (This is often abbreviated as “iid”). Each X_i has mean μ and variance σ^2 (i.e., for all i , $E(X_i) = \mu$ and $\text{Var}(X_i) = \sigma^2$).

Let $S = X_1 + X_2 + \dots + X_n = \sum_{i=1}^n X_i$. And let $\bar{X} = \frac{\sum_{i=1}^n X_i}{n}$.

Find $E(S)$, $\text{Var}(S)$, $E(\bar{X})$ and $\text{Var}(\bar{X})$.