

Confidence Interval Applications

YOUR NAME

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Exercises

1. Chronic illness

In 2013, the Pew Research Foundation reported that “45% of U.S. adults report that they live with one or more chronic conditions”.¹ However, this value was based on a sample, so it may not be a perfect estimate for the population parameter of interest on its own. The study reported a standard error of about 1.2%, and a normal model may reasonably be used in this setting.

- a. Create a 95% confidence interval for the proportion of U.S. adults who live with one or more chronic conditions. Also interpret the confidence interval in the context of the study.
- b. Create a 99% confidence interval for the proportion of U.S. adults who live with one or more chronic conditions. Also interpret the confidence interval in the context of the study.
- c. Identify each of the following statements as true or false. Provide an explanation to justify each of your answers.
- d. We can say with certainty that the confidence interval from part a contains the true percentage of U.S. adults who suffer from a chronic illness.
- ii. If we repeated this study 1,000 times and constructed a 95% confidence interval for each study, then approximately 950 of those confidence intervals would contain the true fraction of U.S. adults who suffer from chronic illnesses.
- iii. The poll provides statistically significant evidence (at the $\alpha = 0.05$ level) that the percentage of U.S. adults who suffer from chronic illnesses is not 50%.
- iv. Since the standard error is 1.2%, only 1.2% of people in the study communicated uncertainty about their answer.
- v. Suppose the researchers had formed a one-sided hypothesis, they believed that the true proportion is less than 50%. We could find an equivalent one-sided 95% confidence interval by taking the upper bound of our two-sided 95% confidence interval.

2. Vegetarian college students

Suppose that 8% of college students are vegetarians. Determine if the following statements are true or false, and explain your reasoning.

- a. The distribution of the sample proportions of vegetarians in random samples of size 60 is approximately normal since $n \geq 30$.

¹<http://pewinternet.org/Reports/2013/The-Diagnosis-Difference.aspx> The Diagnosis Difference. November 26, 2013. Pew Research.

- b. The distribution of the sample proportions of vegetarian college students in random samples of size 50 is right skewed.
- c. A random sample of 125 college students where 12% are vegetarians would be considered unusual.
- d. A random sample of 250 college students where 12% are vegetarians would be considered unusual.
- e. The standard error would be reduced by one-half if we increased the sample size from 125 to 250.
- f. A 99% confidence interval will be wider than a 95% confidence interval because to have a higher confidence level requires a wider interval.

3. Orange tabbies

Suppose that 90% of orange tabby cats are male. Determine if the following statements are true or false, and explain your reasoning.

- a. The distribution of sample proportions of random samples of size 30 is left skewed.
- b. Using a sample size that is 4 times as large will reduce the standard error of the sample proportion by one-half.
- c. The distribution of sample proportions of random samples of size 140 is approximately normal.

4. Working backwards

A 90% confidence interval for a population mean is (65,77). The population distribution is approximately normal and the population standard deviation is unknown. This confidence interval is based on a simple random sample of 25 observations. Calculate the sample mean, the margin of error, and the sample standard deviation.

5. Find the p-value

An independent random sample is selected from an approximately normal population with an unknown standard deviation. Find the p-value for the given set of hypotheses and T test statistic. Also determine if the null hypothesis would be rejected at $\alpha = 0.05$.

- a. $H_A : \mu > \mu_0, n = 11, T = 1.91$
- b. $H_A : \mu < \mu_0, n = 17, T = -3.45$
- c. $H_A : \mu \neq \mu_0, n = 7, T = 0.83$
- d. $H_A : \mu > \mu_0, n = 28, T = 2.13$

6. Sleep habits of New Yorkers

New York is known as “the city that never sleeps”. A random sample of 25 New Yorkers were asked how much sleep they get per night. Statistical summaries of these data are shown below. Do these data provide strong evidence that New Yorkers sleep more or less than 8 hours a night on average?

n	\bar{x}	s	min	max
25	7.73	0.77	6.17	9.78

- a. Write the hypotheses in symbols and in words.
- b. Check conditions, then calculate the test statistic, T , and the associated degrees of freedom.
- c. Find and interpret the p-value in this context.
- d. What is the conclusion of the hypothesis test?

- e. Construct a 95% confidence interval that corresponded to this hypothesis test, would you expect 8 hours to be in the interval?

7. Vegetarian college students II

From problem 2 part c, suppose that it has been reported that 8% of college students are vegetarians. We think USAFA is not typical because of their fitness and health awareness, we think there are more vegetarians. We collect a random sample of 125 cadets and find 12% claimed they are vegetarians. Is there enough evidence to claim that USAFA cadets are different?

- a. Use `binom.test()` to conduct the hypothesis test and find a confidence interval.
- b. Use `prop.test()` with `correct=FALSE` to conduct the hypothesis test and find a confidence interval.
- c. Use `prop.test()` with `correct=TRUE` to conduct the hypothesis test and find a confidence interval.
- d. Which test should you use?