

# Named Continuous Distribution Applications

YOUR NAME

14 September, 2020

## Exercises

For problems 1-3 below, *1)* define a random variable that will help you answer the question, *2)* state the distribution and parameters of that random variable; *3)* determine the expected value and variance of that random variable, and *4)* use that random variable to answer the question.

1. On a given Saturday, suppose vehicles arrive at the USAFA North Gate according to a Poisson process at a rate of 40 arrivals per hour.

- a) Find the probability no vehicles arrive in 10 minutes.
- b) Find the probability that at least 5 minutes will pass before the next arrival.
- c) Find the probability that the next vehicle will arrive between 2 and 10 minutes from now.
- d) Find the probability that at least 7 minutes will pass before the next arrival, given that 2 minute have already passed. Compare this answer to part (b). This is an example of the memoryless property of the exponential distribution.
- e) Fill in the blank. There is a probability of 90% that the next vehicle will arrive within \_\_\_\_ minutes. This value is known as the 90% percentile of the random variable.
- f) Use the function `stripplot()` to visualize the arrival of 30 vehicles using a random sample from the appropriate exponential distribution.

2. Suppose time until computer errors on the F-35 follows a Gamma distribution with mean 20 hours and variance 10.

- a) Find the probability that 20 hours pass without a computer error.
- b) Find the probability that 45 hours pass without a computer error, given that 25 hours have already passed. Does the memoryless property apply to the Gamma distribution?
- c) Find  $a$  and  $b$ : There is a 95% probability time until next computer error will be between  $a$  and  $b$ . (note: technically, there are many answers to this question, but find  $a$  and  $b$  such that each tail has equal probability.)

3. Suppose PFT scores in the cadet wing follow a normal distribution with mean 330 and standard deviation 50.

- a) Find the probability a randomly selected cadet has a PFT score higher than 450.
- b) Find the probability a randomly selected cadet has a PFT score within 2 standard deviations of the mean.
- c) Find  $a$  and  $b$  such that 90% of PFT scores will be between  $a$  and  $b$ .
- d) Find the probability a randomly selected cadet has a PFT score higher than 450 given he/she is among the top 10% of cadets.

4. Let  $X \sim \text{Beta}(\alpha = 1, \beta = 1)$ . Show that  $X \sim \text{Unif}(0, 1)$ . Hint: write out the beta distribution pdf where  $\alpha = 1$  and  $\beta = 1$ .

5. When using R to calculate probabilities related to the gamma distribution, we often use `pgamma`. Recall that `pgamma` is equivalent to the cdf of the gamma distribution. If  $X \sim \text{Gamma}(\alpha, \lambda)$ , then

$$P(X \leq x) = \text{pgamma}(x, \alpha, \lambda)$$

The `dgamma` function exists in R too. In plain language, explain what `dgamma` returns. I'm not looking for the definition found in R documentation. I'm looking for a simple description of what that function returns. Is the output of `dgamma` useful? If so, how?

6. Advanced Question. You may have heard of the 68-95-99.7 rule. This is a helpful rule of thumb that says if a population has a normal distribution, then 68% of the data will be within one standard deviation of the mean, 95% of the data will be within two standard deviations and 99.7% of the data will be within three standard deviations. Create a function in R that has two inputs (a mean and a standard deviation). It should return a vector with three elements: the probability that a randomly selected observation from the normal distribution with the inputted mean and standard deviation lies within one, two and three standard deviations. Test this function with several values of  $\mu$  and  $\sigma$ . You should get the same answer each time.

7. Derive the mean of a general uniform distribution,  $U(a, b)$ .