

Named Discrete Distribution Applications

YOUR NAME

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Exercises

For each of the problems below, **1)** define a random variable that will help you answer the question, **2)** state the distribution and parameters of that random variable; **3)** determine the expected value and variance of that random variable, and **4)** use that random variable to answer the question.

We will demonstrate using 1a and 1b.

1. The T-6 training aircraft is used during UPT. Suppose that on each training sortie, aircraft return with a maintenance-related failure at a rate of 1 per 100 sorties.

a. Find the probability of no maintenance failures in 15 sorties.

X : the number of maintenance failures in 15 sorties.

$X \sim \text{Bin}(n = 15, p = 0.01)$

$E(X) = 15 * 0.01 = 0.15$ and $\text{Var}(X) = 15 * 0.01 * 0.99 = 0.1485$.

$P(\text{No maintenance failures}) = P(X = 0) = \binom{15}{0} 0.01^0 (1 - 0.01)^{15} = 0.99^{15}$

```
0.99^15
```

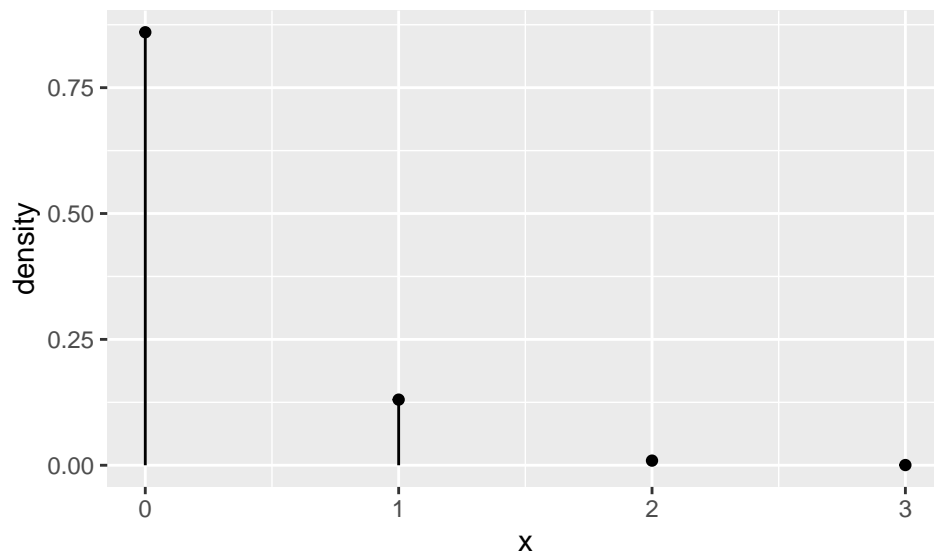
```
## [1] 0.8600584
```

```
## or  
dbinom(0,15,0.01)
```

```
## [1] 0.8600584
```

This probability makes sense, since the expected value is fairly low. Because, on average, only 0.15 failures would occur every 15 trials, 0 failures would be a very common result. Graphically, the pmf looks like this:

```
gf_dist("binom",size=15,prob=0.01)
```



b. Find the probability of at least two maintenance failures in 15 sorties.

We can use the same X as above. Now, we are looking for $P(X \geq 2)$. This is equivalent to finding $1 - P(X \leq 1)$:

```
## Directly
1-(0.99^15 + 15*0.01*0.99^14)
```

```
## [1] 0.009629773
```

```
## or, using R
sum(dbinom(2:15,15,0.01))
```

```
## [1] 0.009629773
```

```
## or
1-sum(dbinom(0:1,15,0.01))
```

```
## [1] 0.009629773
```

```
## or
1-pbinom(1,15,0.01)
```

```
## [1] 0.009629773
```

```
## or
pbinom(1,15,0.01,lower.tail = F)
```

```
## [1] 0.009629773
```

c. Find the probability of at least 30 successful (no mx failures) sorties before the first failure.

- d. Find the probability of at least 50 successful sorties before the third failure.
2. On a given Saturday, suppose vehicles arrive at the USAFA North Gate according to a Poisson process at a rate of 40 arrivals per hour.
- a. Find the probability no vehicles arrive in 10 minutes.
 - b. Find the probability at least 50 vehicles arrive in an hour.
 - c. Find the probability that at least 5 minutes will pass before the next arrival.
3. Suppose there are 12 male and 7 female cadets in a classroom. I select 5 completely at random (without replacement).
- a. Find the probability I select no female cadets.
 - b. Find the probability I select more than 2 female cadets.