

Named Continuous Distribution Applications

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Exercises

For problems 1-3 below, **1)** define a random variable that will help you answer the question, **2)** state the distribution and parameters of that random variable; **3)** determine the expected value and variance of that random variable, and **4)** use that random variable to answer the question.

1. On a given Saturday, suppose vehicles arrive at the USAFA North Gate according to a Poisson process at a rate of 40 arrivals per hour.

- a) Find the probability no vehicles arrive in 10 minutes.
- b) Find the probability that at least 5 minutes will pass before the next arrival.
- c) Find the probability that the next vehicle will arrive between 2 and 10 minutes from now.
- d) Find the probability that at least 7 minutes will pass before the next arrival, given that 2 minute have already passed. Compare this answer to part (b). This is an example of the memoryless property of the exponential distribution.
- e) Fill in the blank. There is a probability of 90% that the next vehicle will arrive within ____ minutes. This value is known as the 90% percentile of the random variable.

2. Suppose time until computer errors on the F-35 follows a Gamma distribution with mean 20 hours and variance 10.

- a) Find the probability that 20 hours pass without a computer error.
- b) Find the probability that 45 hours pass without a computer error, given that 25 hours have already passed. Does the memoryless property apply to the Gamma distribution?
- c) Find a and b : There is a 95% probability time until next computer error will be between a and b . (note: technically, there are many answers to this question, but find a and b such that each tail has equal probability.)

3. Suppose PFT scores in the cadet wing follow a normal distribution with mean 330 and standard deviation 50.

- a) Find the probability a randomly selected cadet has a PFT score higher than 450.
- b) Find the probability a randomly selected cadet has a PFT score within 2 standard deviations of the mean.
- c) Find a and b such that 90% of PFT scores will be between a and b .

- d) Find the probability a randomly selected cadet has a PFT score higher than 450 given he/she is among the top 10% of cadets.
4. Let $X \sim \text{Beta}(\alpha = 1, \beta = 1)$. Show that $X \sim \text{Unif}(0, 1)$. Hint: write out the beta distribution pdf where $\alpha = 1$ and $\beta = 1$.
5. When using **R** to calculate probabilities related to the gamma distribution, we often use **pgamma**. Recall that **pgamma** is equivalent to the cdf of the gamma distribution. If $X \sim \text{Gamma}(\alpha, \lambda)$, then

$$P(X \leq x) = \text{pgamma}(x, \alpha, \lambda)$$

The **dgamma** function exists in **R** too. In plain language, explain what **dgamma** returns. I'm not looking for the definition found in **R** documentation. I'm looking for a simple description of what that function returns. Is the output of **dgamma** useful? If so, how?

6. Advanced Question. You may have heard of the 68-95-99.7 rule. This is a helpful rule of thumb that says if a population has a normal distribution, then 68% of the data will be within one standard deviation of the mean, 95% of the data will be within two standard deviations and 99.7% of the data will be within three standard deviations. Create a function in **R** that has two inputs (a mean and a standard deviation). It should return a vector with three elements: the probability that a randomly selected observation from the normal distribution with the inputted mean and standard deviation lies within one, two and three standard deviations. Test this function with several values of μ and σ . You should get the same answer each time.