Conditional Probability Applications

YOUR NAME

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Exercises

1. Consider: A, B and C are events such that $\mathrm{P}(A) = 0.5$, $\mathrm{P}(B) = 0.3$, $\mathrm{P}(C) = 0.4$, $\mathrm{P}(A \cap B) = 0.2$, $\mathrm{P}(B \cap C) = 0.12$, $\mathrm{P}(A \cap C) = 0.1$, and $\mathrm{P}(A \cap B \cap C) = 0.05$.

   a. Are A and B independent?

   b. Are B and C independent?

2. Suppose I have a biased coin (the probability I flip a heads is 0.6). I flip that coin twice. Assume that the coin is memoryless (flips are independent of one another).

   a. What is the probability that the second flip results in heads?

   b. What is the probability that the second flip results in heads, given the first also resulted in heads?

   c. What is the probability both flips result in heads?

   d. What is the probability exactly one coin flip results in heads?

   e. Now assume I flip the coin five times. What is the probability the result is 5 heads?

   f. What is the probability the result is exactly 2 heads (out of 5 flips)?

3. Suppose there are three assistants working at a company: Moe, Larry and Curly. All three assist with a filing process. Only one filing assistant is needed at a time. Moe assists 60% of the time, Larry assists 30% of the time and Curly assists the remaining 10% of the time. Occasionally, they make errors (misfiles); Moe has a misfile rate of 0.01, Larry has a misfile rate of 0.025, and Curly has a rate of 0.05. Suppose a misfile was discovered, but it is unknown who was on schedule when it occurred. Who is most likely to have committed the misfile? Calculate the probabilities for each of the three assistants.

4. You are playing a game where there are two coins. One coin is fair and the other comes up *heads* 80% of the time. One coin is flipped 3 times and the result is three *heads*, what is the probability that the coin flipped is the fair coin? You will need to make an assumption about the probability of either coin being selected.

   a. Use Bayes formula to solve this problem.

   b. Use simulation to solve this problem.