Risky Business: The Choice of Entrepreneurial Risk under Incomplete Markets

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Abstract

This paper studies how the uninsurable nature of entrepreneurial risk reduces entrepreneurial activity and affects aggregate output, productivity, and the distribution of wealth. I model the occupational choice of individuals who can choose to become workers or entrepreneurs. Individuals that choose to be entrepreneurs also choose how risky a business to start, with higher-risk businesses leading to higher expected productivity. My model features two distinct financial frictions. First, a missing market for entrepreneurial risk prevents entrepreneurs from insuring themselves against their income risk and the risk of business failure. Second, borrowing constraints limit the size of an entrepreneur's business. I contribute to a literature on financial frictions and entrepreneurship by studying the missing market for entrepreneurial risk and contrasting its effects with borrowing constraints, which have been extensively studied. I calibrate the strength of these two financial frictions using micro data on new U.S. firms from the Kauffman Firm Survey. I find that completing the missing market for entrepreneurial risk improves aggregate productivity by 5.3%, which is substantially more than the 2.9% increase that results from relaxing the borrowing constraints. I also find that completing the missing market for entrepreneurial risk reduces the share of wealth held by the wealthiest 1% by more than half. In a policy experiment, I show that a partial insurance scheme for unsuccessful entrepreneurs can increase aggregate productivity and output by encouraging entrepreneurs to start riskier businesses.

Keywords: Entrepreneurship, Risk-Taking, Financial Frictions, Aggregate Productivity, Wealth Inequality
JEL Codes: E20, D31, G32, J24, L26

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1 Introduction

Entrepreneurs play a crucial role in determining aggregate output, productivity, and the distribution of wealth by actively managing and owning private businesses. Decisions that entrepreneurs make directly affect their businesses' output and productivity (Smith et al., 2019). These decisions have aggregate implications, as Asker et al. (2015) estimate that privately held U.S. firms account for 69% of private sector employment and 59% of aggregate sales. The entrepreneurs who own these private businesses are disproportionately wealthy. In the 2004 Survey of Consumer Finances[1] only 11% of households contain at least one entrepreneur but these households own 40% of U.S. wealth and constitute 65% of the wealthiest 1% of households. Understanding how entrepreneurs decide to start, run, and growth their businesses is therefore vital for understanding aggregate economic outcomes.

In this paper, I study the quantitative importance of two distinct financial frictions, a missing market for entrepreneurial risk and borrowing constraints, for aggregate output, productivity, and the distribution of wealth. The missing market for entrepreneurial risk prevents entrepreneurs from insuring themselves against fluctuations in entrepreneurial income or the risk of business failure, while borrowing constraints limit the amount of debt entrepreneurs can raise in order to invest in capital. This paper examines how these financial frictions distort individuals' decisions to start new businesses, their decisions to pursue riskier or safer business ideas, and their investment decisions. I then investigate how these distorted decisions impact aggregate economic outcomes.

This paper makes two main contributions. First, I measure the losses to aggregate productivity and output from the missing market for risk in the U.S. economy. A large literature[2] has studied how financial frictions distort entrepreneurial decisions and impact aggregate productivity and output. While previous work has focused on borrowing constraints, little attention has been paid to how a missing market for risk can distort entrepreneurial decisions. My paper addresses this gap, focusing on the missing market for risk and contrasting its effects with borrowing constraints. I find that the missing market for entrepreneurial risk reduces aggregate productivity by substantially more than the reduction that comes from the borrowing constraints.

Second, I study how the missing market for entrepreneurial risk contributes to wealth inequality. I build on quantitative work by Cagetti and De Nardi (2006) who show that a model of entrepreneurship with both borrowing constraints and a luxury bequest motive can generate the high concentration of wealth observed in U.S. data. My paper uses a similar calibration

[1]The Kauffman Firm Survey data I study in this paper consists of new businesses started in 2004. To be consistent, I use the 2004 wave of the Survey of Consumer Finances to characterize the distribution of wealth.

[2]See for example Buera (2009), Buera et al. (2011), Moll (2014), and Midrigan and Xu (2014), or the excellent review Buera et al. (2015).

strategy, but differs in how it models uninsurable entrepreneurial risk along two dimensions. First, entrepreneurs are able to choose how risky a business to start, with higher risk businesses generating higher expected returns. Second, in Cagetti and De Nardi (2006) individuals make occupational decisions after productivity shocks are realized so uninsurable entrepreneurial risk has no direct impact on the choice of occupation. By contrast, in my model occupational decisions are made before productivity shocks are realized. This means that uninsurable entrepreneurial risk will discourage risk-averse individuals from selecting into entrepreneurship. I find that the missing market for risk is a major contributor to the high concentration of wealth at the top of the distribution.

To study the aggregate effects of these two financial frictions, I build a dynamic equilibrium model of entrepreneurship. Individuals have two types of ability, entrepreneurial and worker. Each period, they choose to be an entrepreneur or a worker. When an individual chooses to become an entrepreneur they also decide how risky a business to start, with higher risk businesses resulting in higher expected productivity. A missing market for risk prevents individuals from insuring against shocks to their abilities or their businesses' productivity. Entrepreneurs also face borrowing constraints that limit the amount of capital they can invest in. Wealth helps individuals overcome both of these financial frictions, as it allows them to self-insure against the risks they face and self-finance a large capital stock. Consequently, wealth plays an important role in an individual's decision to become an entrepreneur and in their decision of how risky a business to start.

I use micro data from the Kauffman Firm Survey to quantify the importance of the two financial frictions. In the calibrated model, both financial frictions play an important role distorting individual choices. The missing market for risk discourages entrepreneurs from starting high risk businesses and individuals from becoming entrepreneurs. If entrepreneurs had access to complete insurance markets they could start a business with the highest expected productivity and then fully insure themselves against the resulting risks. Faced with the missing market, some entrepreneurs will choose lower expected productivity businesses because these businesses have a lower probability of failure and generate more certain income. Similarly, some individuals may choose to be workers, even though their expected income as entrepreneurs is higher, because entrepreneurial income is more volatile than workers' income. Wealth helps individuals self-insure against idiosyncratic shocks, as wealthy individuals are able to use their wealth to smooth out their consumption. As a consequence, wealthier individuals are more likely to become entrepreneurs and are more likely to start higher risk businesses.

Borrowing constraints limit the size of many entrepreneurs' businesses and discourage individuals from becoming entrepreneurs. In the absence of borrowing constraints, a wealth-poor individual with high entrepreneurial ability might rent a large stock of capital in order to operate a large scale business. However, borrowing constraints prevent these wealth-poor entrepreneurs from renting as much capital as they would like, forcing them to operate inefficiently small businesses. These smaller businesses generate less income for their entrepreneurs. If this reduced entrepreneurial income falls below what they could earn as a worker, some wealth-poor individuals will choose to be workers despite their high entrepreneurial ability.

The main quantitative analysis I perform is to remove each financial friction and compare the resulting steady state equilibria. I first complete the missing market for risk by introducing a full set of state-contingent assets. Each individual is able to buy or sell securities at actuarially fair prices that pay off based on their individual abilities and their businesses' productivity. I then relax the borrowing constraints, allowing entrepreneurs to invest in any amount of capital regardless of their personal net-worth.

I find that completing the missing market for entrepreneurial risk improves aggregate productivity and output by 5.3%. The state-contingent assets allow individuals to transfer resources from the state of the world where their business is successful to the state of the world where their business is unsuccessful. As a direct consequence, all entrepreneurs choose to run businesses with the highest expected productivity and use the state-contingent assets to insure themselves against the resulting risks.

Relaxing borrowing constraints increases aggregate productivity by 2.9% and aggregate output by 5.5%. Entrepreneurs that were previously constrained to operate inefficiently small businesses are now able to invest in much more capital. Some wealth-poor individuals with high entrepreneurial ability switch from being workers to being entrepreneurs. Without the constraint, they can now run larger businesses and therefore generate more income as entrepreneurs. As a consequence of the increased investment, the aggregate capital stock increases by 10.7% and so the response of aggregate output is larger than the response of aggregate productivity.

Completing the missing market for risk also substantially reduces wealth inequality. The share of wealth held by the top 1% falls from 23% down to 10% and the wealth Gini falls from 0.84 down to 0.78. This decrease results because in the benchmark economy, wealthier entrepreneurs choose to start higher risk businesses. These high-risk businesses earn higher expected returns on average, leading to faster wealth accumulation by the wealthier entrepreneurs. When all entrepreneurs are able to insure against business risk, they all choose to start the highest expected productivity businesses, removing this difference in the rates of return. This means that completing the missing market for risk leads to a rare equity-efficiency win-win, with both an increase in aggregate productivity and a reduction in wealth inequality.

Finally, I demonstrate that these quantitative results have implications for public policy by showing that a government can increase aggregate output with a simple partial insurance scheme for entrepreneurs. Given the large increases in aggregate output and productivity from completing the missing market, a natural policy implication is for governments to provide more insurance to entrepreneurs. However, any public insurance scheme may run into problems with adverse selection if it encourages low ability individuals to choose to become entrepreneurs, not because they expect to make substantial income as entrepreneurs, but because the insurance payouts would be larger than the wages they would earn in the labour market. I study a simple insurance scheme where governments can only observe an entrepreneur's income. I find that when the insurance scheme pays out moderate benefits it encourages more entrepreneurs to start higher risk businesses leading to higher aggregate productivity. Aggregate output increases because few low-ability individuals choose to become entrepreneurs. At higher levels of benefits, adverse selection overwhelms the positive benefits of higher risk taking and aggregate output decreases.

Section 2 provides evidence on entrepreneurial risk taking and funding sources from the Kauffman Firm Survey that motivates some of the modelling choices in this paper. I present my dynamic model of entrepreneurship in section 3. Section 4 explains how I quantify the strength of the two financial frictions using the micro data. I report the results of the quantitative analysis where I remove the two financial frictions in section 5. Section 6 studies the effects of a simple government policy that provides partial insurance to entrepreneurs. Section 7 concludes.

Related Literature

My paper contributes to a large literature studying how financial frictions distort entrepreneurial decisions and the impacts on aggregate productivity and output. My paper studies how the missing market for risk, a previously understudied financial friction distorts entrepreneurial decisions. Previous work in this area has focused almost exclusively on borrowing constraints. For example, papers such as Buera (2009), Buera et al. (2011), Moll (2014), Midrigan and Xu (2014), and Castro and Sevcik (2016), study how borrowing constraints impact entrepreneurial decisions, including whether to start a business, which sector to start businesses in, whether to adopt more capital-intensive technology, or how much to invest in human capital.

My paper also contributes to a literature studying the determinants of the distribution of wealth. Two recent surveys of this literature, De Nardi and Fella (2017) and Benhabib and Bisin (2018), both consider differences in the earned rates of return to be key drivers of the high concentration of wealth among the wealthiest. In my model, entrepreneur's endogenous choice of business risk generates a previously unstudied channel for explaining the high concentration of wealth. Wealthier entrepreneurs choose higher risk businesses that on average earn them higher returns. This is consistent with empirical evidence from Scandinavian countries. Both Bach et al. (2020) and Fagereng et al. (2020) show that wealthier individuals do in fact earn persistently higher rates of return on their assets. See also Peter (2021), which looks at how ownership institutions across countries lead to different levels of wealth inequality with a model of entrepreneurship.

This paper is closely related to two papers that study entrepreneur's choice of risk. First, Choi (2017) uses U.S. Census Bureau data to provide evidence that individuals who had higher paying jobs prior to starting a business take larger risks, as measured by higher exit rates, more dispersion in growth, and faster average growth conditional on survival. In a quantitative model, he demonstrates the importance of entrepreneur's labour market options for encouraging entrepreneurial risk taking. By contrast, my paper considers how wealth, rather than an entrepreneur's labour market opportunities, encourages risk taking and measures how aggregate productivity and output would change with complete insurance markets. My paper is also distinct from Choi (2017) in that I study the implications of entrepreneurial risk choice for the distribution of wealth.

Second, Vereshchagina and Hopenhayn (2009) study how wealth impacts both the choice to become an entrepreneur and the choice of business risk. In their model, wealth-poor entrepreneurs start riskier businesses due to a non-concavity in the value function created by the insurance value of becoming a worker in the future. I find the opposite result with wealth-poor individuals starting safer businesses. The key difference between these models is that in my model higher risk businesses deliver higher expected productivity while in the Vereshchagina and Hopenhayn (2009) model all businesses have the same expected productivity.

More broadly, my paper is related to a large literature on the decision to become an entrepreneur. In my model individuals decide to become an entrepreneur based on their abilities and two financial frictions. These two financial frictions correspond to two different strands of literature studying the decision to start a business. First, papers such as Kihlstrom and Laffont (1979) and Cressy (2000) argue that uninsurable entrepreneurial risk is a major determinant of the decision to become an entrepreneur. Second, a literature starting with Evans and Jovanovic (1989) considers how borrowing constraints will influence the decision to become an entrepreneur, see also Quadrini (2000), Gentry and Hubbard (2004), and Hurst and Lusardi (2004). My paper incorporates both of these mechanisms of occupational selection and quantify their relative importance for aggregate outcomes.

Empirical work documenting that entrepreneurs face a high degree of uninsurable idiosyncratic risk motivate the focus in my paper on the missing market for entrepreneurial risk. Entrepreneurship is risky both because of the risk of business failure and the volatile nature of entrepreneurial income. Decker et al. (2014) find that most new businesses exit within the first ten years. DeBacker et al. (2023) and Boar et al. (2022) document the high volatility of entrepreneurial earnings and returns, while Castro et al. (2015) measures the importance of idiosyncratic shocks relative to aggregate shocks.

Finally, this paper is related to a large literature on government policy for entrepreneurs. In particular, this paper's policy analysis complements the empirical results in Hombert et al. (2020), who study a reform in France that extended unemployment insurance to self-employed individuals. They find that the reform led to more new businesses that had higher productivity than incumbents. My paper's key policy implication is that providing unemployment insurance to entrepreneurs can increase risk taking and therefore aggregate productivity.

Quantitative work on government policy for entrepreneurs tends to focus on a single financial friction. Meh (2005), Brüggemann (2021), and Guvenen et al. (2023) all study the effects of different government taxes on entrepreneurs given the existence of borrowing constraints. By contrast, Panousi and Reis (2019) considers optimal taxation with uninsurable capital income risk. The quantitative results in my paper suggests that entrepreneurial policies should not ignore either of these frictions, as the missing market for risk and borrowing constraints are both quantitatively important.

2 Stylized Facts

In this section I document key stylized facts about entrepreneurship from the confidential version of the Kauffman Firm Survey. These facts motivate the modelling choices in the model in section 3.

2.1 Kauffman Firm Survey Data

Table 1: Kauffman Firm Survey Summary Statistics

	Mean	Percentiles			Number of
		10^{th}	50^{th}	90^{th}	Firms
Number of Employees in Year 1	2.0	0	0	5	4,823
Number of Employees in Year 8	5.2	0	1	10	2,000
Number of Entrepreneurs	1.4	1	1	2	4,928
Cumulative Investment Over Years 1-8	480.5	67	277	762	3,488

Summary statistics for the firms in the Kauffman Firm Survey. The number of firms in each row varies depending on data availability. Cumulative investment is in thousands of dollars and included all sources of equity and debt.

The Kauffman Firm Survey is a single cohort panel of 4,928 new U.S. firms. All firms are founded in the year 2004, and the survey follows them until they exit or until 2011. It was designed to provide a representative sample of all new businesses started in 2004.[3] The firms

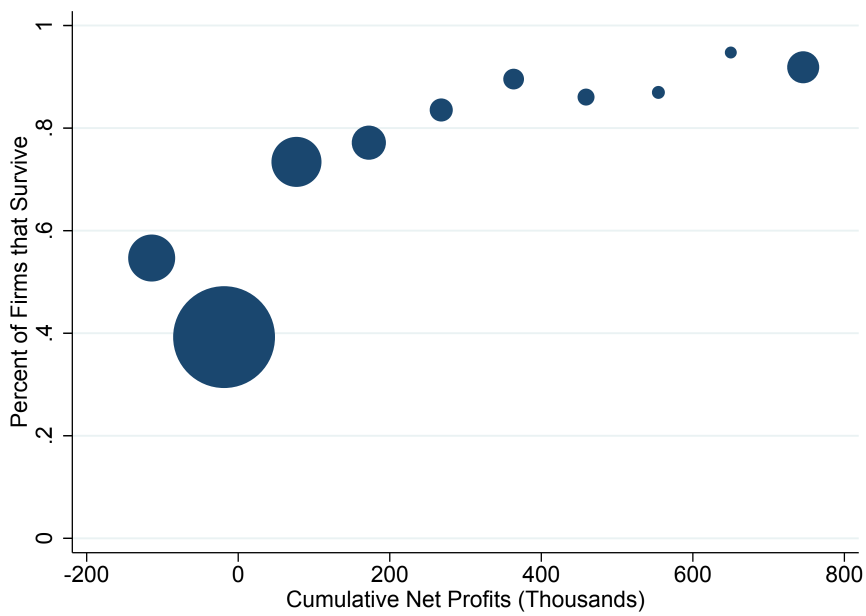
[3] The sampling frame is taken from the Dun and Bradstreet U.S. Business database.

include businesses that were independently founded, purchased from an existing business or purchased as a franchise, and exclude any inherited businesses, any non-profits, and businesses that were started as a branch or subsidiary of an existing business. For each firm, information for up to 10 owners is provided.

These firms are highly heterogeneous. As table 1 shows, the majority of firms are non-employers in the first year, though many go on to hire at least one worker later. Most firms are owned and operated by a single entrepreneur, though a small proportion have multiple owners. The distribution of total investment is highly skewed with the mean level of investment almost twice as large as the median level of investment.

2.2 Idiosyncratic Risk

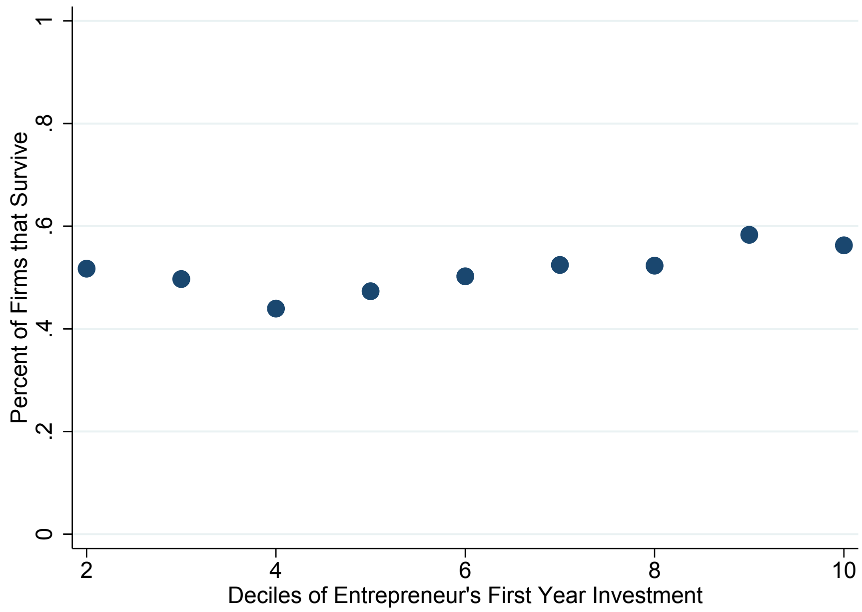
Figure 1: Firm Survival to Year 8 by Cumulative Profits



The 8-year survival rates of firms in ten bins of cumulative net profits. Cumulative profits have been winsorized between the 5th and 95th percentiles and then firms are sorted into the ten equally-sized bins between these percentiles. The size of each circle is proportional to the number of observations in that bin.

To evaluate the degree of risk that entrepreneurs in the Kauffman Firm Survey face, I examine the survival rates and cumulative profits over the eight year panel. I calculate the survival rate as the number of firms either operating in the final sample year or that have been

Figure 2: Firm Survival to Year 8 by Initial Investment



Survival rates of firms grouped into deciles based on the total amount invested by the entrepreneurs in the first three years of operation. This measure excludes equity invested by individuals who do not have an active management role in the firm.

merged or sold in a previous year, divided by the total number of firms with a known status in the final year. Without information about the final sale price, it is difficult to assess whether firms that are merged or sold constitute a successful or unsuccessful outcome for entrepreneurs. I include these firms in survival in order to provide a conservative estimate of the likelihood of undesirable outcomes. See appendix A.1 for additional details on the calculation of survival.

45% of the firms in the Kauffman Firm Survey shut down all operations by the end of their eighth year. In figure 1, I graph firm survival against the absolute amount of profits earned by these new firms over the sample. On the x-axis, I calculate the cumulative amount of profits for each firm over all the years that they exist in the sample. I then place firms into 10 equally sized bins and calculate the survival rate of these firms, which is plotted on the y-axis. The size of each circle represents the number of firms in each profit bin. This figure shows that the firms that earn low or negative profits are much less likely to survive. A significant fraction of these entrepreneurs therefore have started a business, lost money, and then chosen to exit, which is consistent with the idea that entrepreneurs starting a new business face a high degree of idiosyncratic risk.

These firm exits are not confined to low-investment firms. Figure 2 shows that exit is common across the distribution of entrepreneur's own investment, the absolute amount of their own money the entrepreneur is investing in their own business. It is not just low-investment

Table 2: Firm Survival to Year 8 By Industry

Industry (2 digit NAICs Code)	Percent Surviving in Year 8	Number of Firms
Retail Trade (44)	42.9	238
Retail Trade (45)	43.7	229
Transportation and Warehousing (48)	45.3	86
Construction (23)	46.4	306
Finance and Insurance (52)	51.2	164
Other Services (except Public Administration) (81)	51.4	389
Health Care and Social Assistance (62)	52.1	94
Administrative, Support, Waste Management, and Remediation Services (56)	52.6	310
Manufacturing (32)	54.5	123
Accommodation and Food Services (72)	54.5	88
Information (51)	55.6	144
Real Estate and Rental and Leasing (53)	56.9	160
Wholesale Trade (42)	57.4	188
Arts, Entertainment, and Recreation (71)	57.6	92
Professional, Scientific, and Technical Services (54)	58.4	1,058
Manufacturing (33)	63.5	425

firms, which may have been started to provide temporary self-employment[4] that exit. While firms in the top decile of initial entrepreneurial investments have slightly higher survival rates than lower-investment firms, 42% of these firms have exited by the end of the eighth year.

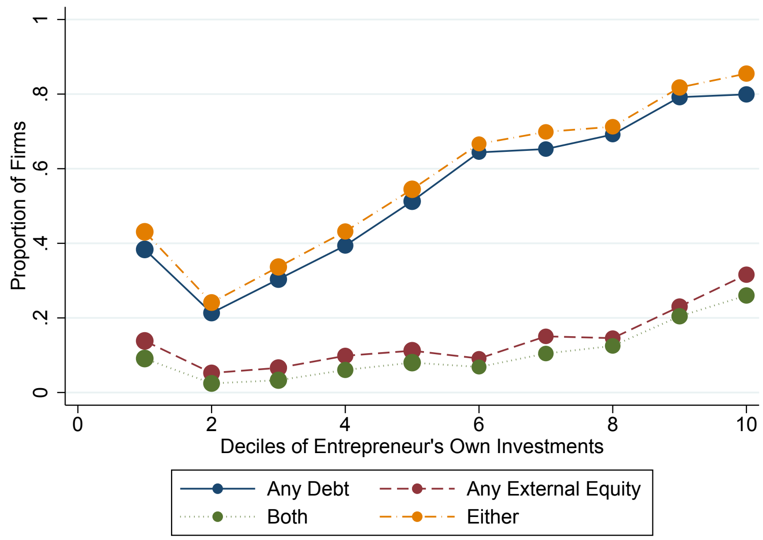
Firm exits are also not confined to a specific set of industries. Table 2 shows the proportion of firms that survive to the end of the sample according to their 2-digit NAICs code. Dropping industries with fewer than 50 observations, survival rates range from 43% of firms in the case of Retail Trade (44), to 64% in Manufacturing (33).

2.3 Sources of Funding

How do new entrepreneurs raise funds to start their businesses? Entrepreneurs invest their own money in 89% of these firms. 15% of new firms raise some external equity from sources beyond the actively managing entrepreneurs. Outside investors and other companies are common sources, though some is also provided by the entrepreneurs' families. 53% of the firms are funded with some debt. Much of this is personal debt, taken out in the entrepreneurs' names,

[4]See Galindo da Fonseca (2022), who documents different patterns of entrepreneurial activity based on employment status before starting a business.

Figure 3: The Proportion of Firms with External Sources of Funding



The proportion of firms that have raised some debt, some external equity, both or either across the distribution of entrepreneur's initial investment. "Both" means that a firm has both debt and external equity. "Either" means that a firm has either some debt or some external equity. Firms are sorted into deciles based on the total amount of their entrepreneur's own investment in the firm in the first three years of operations.

rather than debt that is owed by the business. 25% of firms raising only personal debt, 7% of firms raise only business debt, and 21% of firms raise both. See table 8 in appendix A.2 for additional details on the sources of debt and external equity.

Entrepreneurs are more likely to raise some funds form other sources if they invest more of their own money in the business. Figure 3 shows the proportion of firms that raise any debt or any external equity for each of the ten deciles of initial investment, defined as the amount of their own money that the entrepreneurs invest in the business within the first three years. On the intensive margin, entrepreneurs who invest more of their own money are also likely to raise larger amounts of money conditional on raising some debt or equity, as can be seen in appendix figures 17 and 18.

These figures suggest that both on the extensive and on the intensive margin, entrepreneurs who invest more of their own money raise more external funds. These patterns are consistent with the idea that wealth-poor entrepreneurs are unable to borrow to finance their business. They motivate the modelling of borrowing constraints as a collateral constraint, which allows entrepreneurs to borrow more only as they invest more.

3 Dynamic Model

In order to quantify the relative importance of the missing market for entrepreneurial risk and borrowing constraints, I build a general equilibrium model where individuals choose whether to be workers or entrepreneurs and entrepreneurs choose the riskiness of the businesses they start.

A simplified static model of these two financial frictions is available in appendix B for the interested reader.

3.1 Environment

There are a unit measure of agents. Each agent faces a constant probability $(1-\psi)$ of dying every period and has preferences given by:

$$U = \sum_{t=0}^{\infty} (\psi\beta)^t \frac{c^{1-\sigma}}{1-\sigma} \tag{1}$$

Where β is the discount factor and σ is the coefficient of relative risk aversion.

Each agent has two types of ability, their ability as a worker (h^W) and their ability as an entrepreneur (h^E). At the beginning of each period, an agent will choose whether to operate as a worker or as an entrepreneur for that period. If the agent chooses to be a worker, they will supply labour inelastically and earn wh^W, where w is the common wage.

The first period an agent decides to be an entrepreneur, they start a business by choosing how risky a business to start from a menu of risky options $x \in \{x_1, x_2, ..., x_{n_x}\}$, and investing in a capital stock k. Once chosen, the business's riskiness x is fixed. In a future period, if an entrepreneur wants to change the riskiness of their business they must shut down their business and liquidate their capital stock before they are able to select a new level of business risk.

After all agents make an occupational choice decisions and all new entrepreneurs decide on the riskiness of their business, all agents receive shocks to both their ability as a worker h^W and their ability as an entrepreneur h^E. While both types of ability are partially persistent, neither are perfectly so, and so agents face idiosyncratic income risk from choosing either occupation.

In addition, entrepreneurs also receive a project productivity shock (z) for their business. z is drawn from a distribution that depends on the riskiness of their business (x). Higher x businesses have higher expected z but also more dispersed z. The productivity of a business depends on both the firm-specific productivity shock as well as the entrepreneur's entrepreneurial ability h^E. Once they received their shocks, entrepreneurs hire an amount of labour n at wage rate w and produce output y according to:

$$y = (zh^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma \tag{2}$$

Note how business productivity (z) and entrepreneurial ability (h^E) enter multiplicatively. This means that better entrepreneurs are able both to boost the productivity of successful projects and also to improve business outcomes when businesses are unsuccessful.

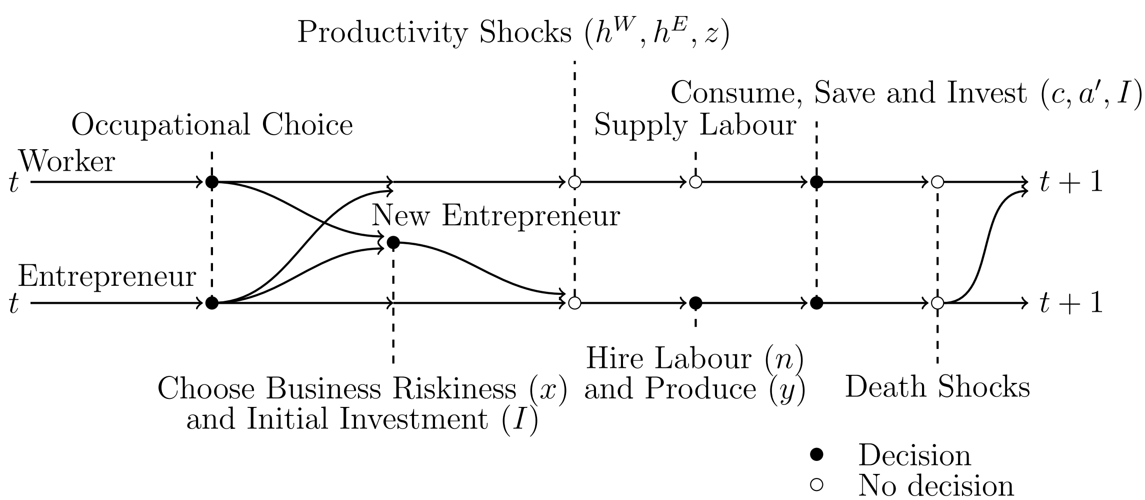
After entrepreneurs produce and pay their employees, all agents make a consumption, savings, and investment decision. The model has two assets. Only entrepreneurs can invest in capital k, which depreciates at rate δ. Capital is also illiquid, so that liquidating one unit of capital produces only $\frac{1}{\chi} < 1$ units of consumption.[5] Given an investment of I, an entrepreneur's capital stock k evolves according to:

$$k' = \begin{cases} k(1-\delta) + I & \text{if } I \geq 0 \\ k(1-\delta) + \chi I & \text{if } I < 0 \end{cases} \tag{3}$$

All agents can save in a liquid financial asset a, that pays a constant $1 + r^A$ in all states of the world. In addition, for each unit of capital that an entrepreneur invests in, they can borrow an addition $\phi \in [0, 1]$ units of a. Thus the borrowing constraint is given by:

$$a \geq -\phi k \tag{4}$$

Figure 4: Timing in the Dynamic Model



After their consumption, savings, and investment decisions, agents will die with probability $(1 - \psi)$. If they were an entrepreneur, their invested capital stock is liquidated. All agents that

[5]See Tan (2018) for empirical evidence on capital liquidation costs.

die are immediately replaced by a descendant who inherits the full value of their liquidated assets. Figure 4 summarizes the timing in the model.

3.2 Agent's Problems

Worker's Problem

A worker makes a consumption-savings decision, and, at the beginning of the next period, will choose between being a worker (V^W) and becoming a new entrepreneur (V^{NE}):

$$V^W(a, h^W, h^E) = \max_{a',c} \frac{c^{1-\sigma}}{1-\sigma} + \psi\beta \max\left\{\mathbb{E}\left[V^W(a', h^{W\prime}, h^{E\prime})\right], V^{NE}(a', h^{W\prime}, h^{E\prime})\right\} \tag{5}$$

s.t.

$$a' + c = wh^W + (1 + r^a)a$$

$$a' \geq \underline{a}$$

New Entrepreneur's Problem

An agent that has decided to start a new business chooses the riskiness of their business x. Given their current financial assets a, they also choose how much to invest in the business I and how much to borrow or save in the financial assets $\tilde{a}$. They will then operate as an entrepreneur later this period with a capital stock $\tilde{k}$.

$$V^{NE}(a, h^W_{-1}, h^E_{-1}) = \max_{\tilde{a},I,x} \mathbb{E}\left[V^E(\tilde{a}, \tilde{k}, h^W, h^E, z, x)\right] \tag{6}$$

s.t.

$$\tilde{a} = a - I$$

$$\tilde{a} \geq -\phi\tilde{k}$$

$$\tilde{k} = I$$

Entrepreneur's Problem

An entrepreneur that has a business will choose an amount of labour n to hire, consumption c, savings $(I + a')$, and investment I. At the beginning of the next period, they will choose between shutting down their business to become a worker (V^W), shutting down their business to start a new business (V^{NE}) and continuing to operate the same business (V^E).

$$V^E(a,k,h^W,h^E,z,x) \quad = \quad \max_{n,c,a',I} \frac{c^{1-\sigma}}{1-\sigma} \;+\; \psi\beta \max \left\{ \begin{array}{c} \mathbb{E}\left[V^W(a',h^{W\prime},h^{E\prime})\right], \\ \\ V^{NE}(a',h^W,h^E), \\ \\ \mathbb{E}\left[V^E(a',k',h^{W\prime},h^{E\prime},z',x)\right] \end{array} \right\} \tag{7}$$

s.t.

$$c + a' + I = (zh^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn + (1+r^a)a$$

$$a' \geq \underline{a} - \phi k'$$

$$k' = \begin{cases} k(1-\delta) + I & \text{if } I \geq 0 \\ k(1-\delta) + \chi I & \text{if } I < 0 \end{cases}$$

Note that if the entrepreneur decides to start a new business, they cannot use the same capital from their current business. They must first fully liquidate it and then invest in a new capital stock for the new business.

3.3 Equilibrium

An equilibrium is a set of value functions $\{V^W, V^{NE}, V^E\}$, occupational choices, a set of policy functions $\{c^W, a'^W, \tilde{a}^{NE}, I^{NE}, x^{NE}, c^E, a'^E, I^E, n^E\}$, a distribution of agents $\{\Gamma^E(a,k,h^W,h^E,z,x), \Gamma^W(a,h^W,h^E)\}$, and a price w such that

1. The policy functions solve the individual's problems given by (5), (6) and (7).
2. All markets clear:
   - Labour

$$\int h d\Gamma^W(a,h^W,h^E) = \int n^E d\Gamma^E(a,k,h^W,h^E,z,x)$$

   - Final Goods

$$\begin{aligned} \int (c+a')d\Gamma^W(a,h^W,h^E) + \int (c+a'+I)d\Gamma^E(a,k,h^W,h^E,z,x) = \\ \int \left((zh^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma + (1+r^A)a\right) d\Gamma^E(a,k,h^W,h^E,z,x) \\ + \int ((1+r^A)a)d\Gamma^W(a,h^W,h^E) \end{aligned}$$

3. The distribution of agents is stationary

$$\Gamma^E(a, k, h^W, h^E, z, x) = \Gamma^{E\prime}(a, k, h^W, h^E, z, x)$$

$$\Gamma^W(a, h^W, h^E) = \Gamma^{W\prime}(a, h^W, h^E)$$

Note that I assume the economy is a small open economy, and so the financial asset can be in net positive or negative supply.

4 Calibration

I calibrate this model to match micro data on new firm dynamics and wealth inequality by using the Kauffman Firm Survey and the 2004 Survey of Consumer Finances. I use the 2004 wave of the Survey of Consumer Finances as it represents the distribution of wealth in the year that all of the Kauffman Firm Survey firms were founded. For a set of parameters that are difficult to identify using my data, I rely on commonly used values in the literature.

Parameterization of the Ability and Productivity Processes

I parameterize both the worker and entrepreneurial ability processes with AR(1) processes:

$$\log(h^{i\prime}) = \rho_{h^i} \log(h^i) + \epsilon_{h^i}$$

where $\epsilon_{h^i} \sim N\left(\frac{\mu_{h^i}}{1-\rho_{h^i}}, \sigma^2_{h^i}\right)$ for $i \in \{E, W\}$. I assume that the two types of abilities are uncorrelated.[6]

In order to keep the problem computationally tractable, the entrepreneurs choose between two project types, a relatively safe project (x_1) and a relatively risky project (x_2). Each project type receives productivity shocks z the follows an AR(1) process:

$$\log(z') = \rho_{z|x_i} \log(z) + \epsilon_{z|x_i}$$

$$\epsilon_{z|x_i} \sim N\left(\frac{\mu_{z|x_i}}{1-\rho_{z|x_i}}, \sigma^2_{\epsilon_z|x_i}\right)$$

for $i \in \{1, 2\}$.

I use the Rowenhorst method to discretize these four AR(1) processes $\{h^W, h^E, z_{x_1}, z_{x_2}\}$.

[6]This assumption is also made by Cagetti and De Nardi (2006) and Brüggemann (2021). Allub and Erosa (2019) calibrate a model of entrepreneurs and workers to Brazilian data on transition rates across occupations and differences in earnings. They find a low correlation of 0.145.

Externally Chosen Parameters

A period in the model is a year. I fix ten parameters to commonly used values in the literature. I set the capital share α to $\frac{1}{3}$ and the interest rate r^A to 4% per year. I set the decreasing returns to scale parameter γ to 0.85 as in Midrigan and Xu (2014) and the depreciation rate to 6% per year. The time discount factor is set such that $\beta = \frac{1}{1+r^A}$. I set the coefficient of relative risk aversion $\sigma = 1.50$.

I set the probability of death $(1-\psi)$ to $\frac{1}{40}$ so that the expected working lifetime is 40 years. The persistence of the labour income process ρ_{ϵ_h} is set to to 0.9, which is in the range of empirical estimates according to Guvenen (2007). The standard deviation of the innovation σ_{ϵ_h} is set to 0.2.

Internally Calibrated Parameters

Table 3: Benchmark Calibration

Target	Data	Model	Parameter	Value
Exogenously Calibrated				
40 Year Working Lifespan			$1-\psi$	0.03
Capital Share			α	0.33
Interest Rate			r^A	0.04
Entrepreneurial Share			$1-\gamma$	0.12
Depreciation			δ	0.06
Discount Factor			β	0.96
Coefficient of Relative Risk Aversion			σ	1.50
Average Labour Ability			μ_{h^W}	1.00
Dispersion of Labour Ability			σ_{h^W}	0.20
Persistence of Labour Ability			ρ_{h^W}	0.90
Endogenously Calibrated				
Ratio of Average Debt to Average Equity	1.35	1.32	ϕ	0.61
Proportion of Entrepreneurs	0.11	0.11	χ	1.08
Wealth Ratio of Entrepreneurs to Workers	7.14	7.15	μ_{h^E}	-2.42
Wealth Gini of Entrepreneurs	0.75	0.60	σ_{h^E}	1.66
Wealth Gini	0.79	0.84	ρ_{h^E}	0.95
Proportion of Entrepreneurs in Wealthiest 1%	0.74	0.68	$\mu_{z\|x_1}$	2.30
Relative Employment (High-to-Low Investment)	4.12	3.48	$\mu_{z\|x_2}$	-3.73
Autocorrelation of Employment (Low Investment)	0.63	0.70	$\sigma_{z\|x_1}$	0.52
Autocorrelation of Employment (High Investment)	0.56	0.56	$\sigma_{z\|x_2}$	5.37
Survival Rate (Low Investment)	0.53	0.48	$\rho_{z\|x_1}$	0.51
Survival Rate (High Investment)	0.61	0.58	$\rho_{z\|x_2}$	0.94

In order to discipline the remaining eleven parameters, I jointly calibrate them to eleven data moments. Given that the aim of this paper is to quantify the importance of financial frictions, I use moments characterizing firm dynamics from the Kauffman Firm Survey. As wealth plays such an important role in allowing individuals to mitigate the financial frictions, I also use moments from the Survey of Consumer Finances to discipline the calibration.

In the model, I simulate an eight year panel of new firms, corresponding to the Kauffman Firm Survey's eight year panel. In the data, I observe the ratio of debt and equity that these new firms start with, and use this ratio to pin down the tightness of the borrowing constraint (ϕ).

The degree of idiosyncratic entrepreneurial risk is vital to this model. Obviously, it is not possible to directly observe the distribution of risk each entrepreneur is drawing from. However, the distribution of firm outcomes is informative about the nature of entrepreneurial risk that all entrepreneurs are facing. The model predicts that wealthier individuals will select higher risk projects and invest more in them. Therefore, I separate firms both in the model simulation and in the data into high and low categories and compute moments separately for these two groups. I define high-investment as firms within the top 10% of owner's own initial investment over the first three years.

To discipline the size and persistence of productivity shocks, I target the survival rates and the auto-correlation of employment for both these high-investment and low-investment firms. In order to pin down the differences in average productivity between the low-risk and high risk project, I target the ratio of average employment in the high-investment firms to the average employment in the low-investment firms at the eighth year of the panel, conditional on survival.

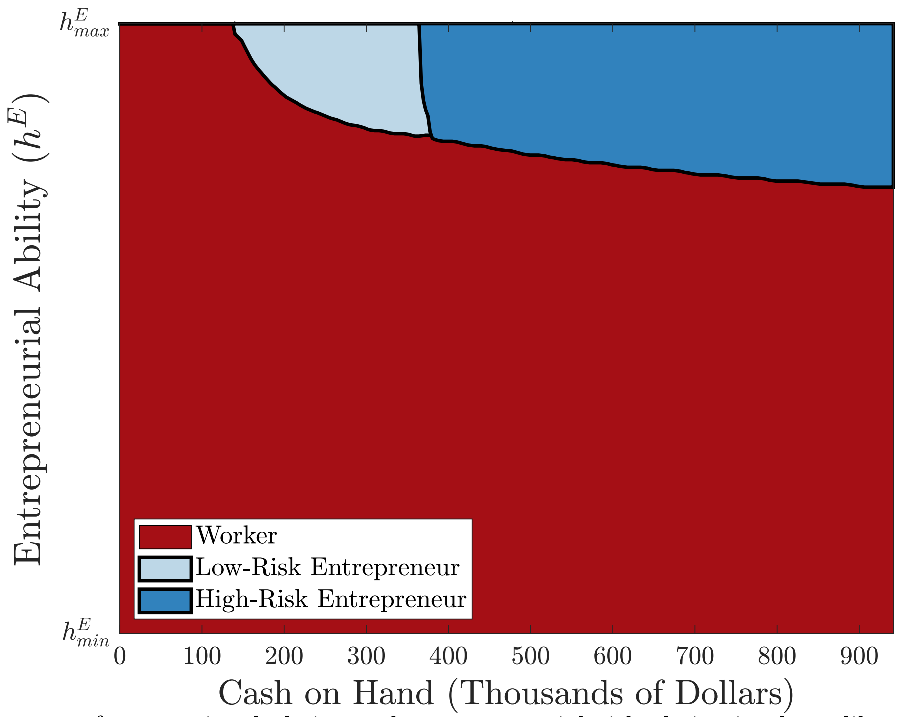
Wealth plays a key role in the model in generating selection into entrepreneurship. I therefore target the ratio of average wealth of entrepreneurs to workers, the proportion of entrepreneurs in the economy, and the proportion of entrepreneurs in the wealthiest 1%. Following Cagetti and De Nardi (2006), I target the wealth Gini, and since this model is a model of heterogeneous entrepreneurs, I also target the wealth Gini of entrepreneurs.

4.1 Model Outcomes

In this subsection, I describe how wealth impacts entrepreneurial choices and outcomes in the calibrated economy.

Each period, based on their two abilities and their current level of wealth, individuals in the model must choose whether to be a worker or an entrepreneur and if they become an entrepreneur their level of business risk (x). Figure 5 illustrates these choices in the model. For a worker with the median worker ability (h^W), the graph shows their occupational choice depending on their level of entrepreneurial productivity (h^E) and their current level of cash on hand, which is equal to their wealth plus their current labour income.

Figure 5: Patterns of Occupational Selection and Risk Choice in the Model



Patterns of occupational choice and entrepreneurial risk choice in the calibrated model. Given a worker with a particular worker ability (h^W), the graph shows how the worker would select their occupation depending on their current level of entrepreneurial productivity (h^E) and their current cash on hand ($a(1+r^A)+wh^W$).

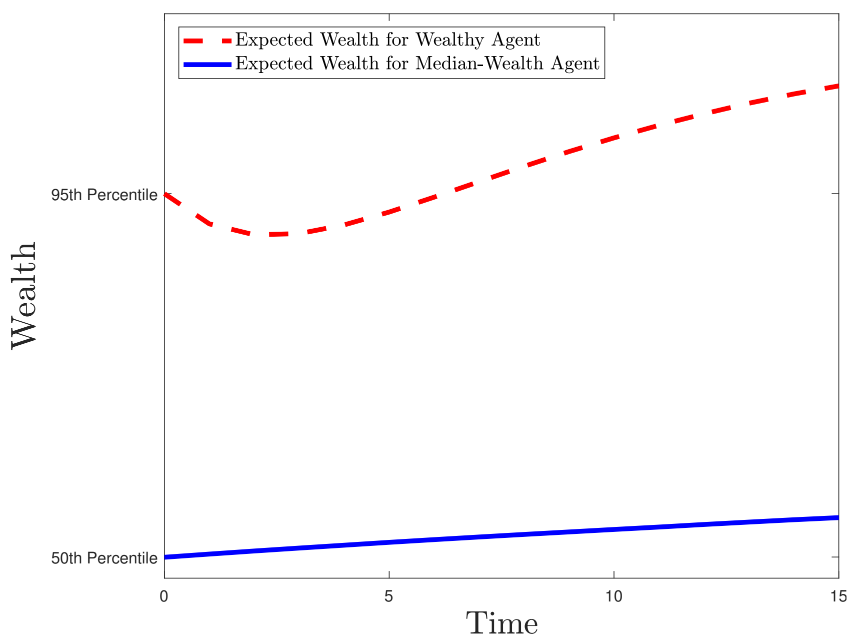
Individuals with low entrepreneurial ability (h^E) will become workers regardless of their wealth. By contrast, individuals with high entrepreneurial ability will choose depending on their level of wealth. If the agent has little wealth, they will choose to be a worker. They will do so because both financial frictions reduce the value of entrepreneurship. The borrowing constraint limits the scale of the business a poor entrepreneur can operate, which limits their entrepreneurial income. Additionally, if the business fails, the poor agent will earn very low income that period and consume very little without access to insurance.

Individuals with high entrepreneurial ability and moderate wealth will choose to be entrepreneurs operating the low risk project. Moderate wealth allows them to self-finance a sufficient amount of capital. However, the inability to insure against business risk means that they will choose the low-risk project. Even though the high-risk project has higher expected productivity, it's higher level of risk makes it unattractive. A bad productivity shock would leave the entrepreneur with low income for at least one period, and if the shock was low enough that they decided to exit, the entrepreneur would have to liquidate their capital stock, leading to a loss of wealth.

Agents with high entrepreneurial ability and high wealth will choose to be entrepreneurs operating the high risk project. Since they have sufficient wealth to self-insure any bad productivity shocks, they choose the higher risk project with higher expected productivity.

Given these choices, wealthier individuals accumulate wealth faster than poor individuals with the same abilities. Figure 6 illustrates this dynamic for two individuals born with identical entrepreneurial and worker abilities. Both individuals are born at time 1 as workers with the maximum entrepreneurial and worker ability. However, they are endowed with different initial levels of wealth, one born with an inheritance equal to the median level of wealth in the economy and one born with an inheritance equal to the 95^{th} percentile of wealth in the economy.

Figure 6: Wealth Accumulation for Individuals with Different Initial Wealth

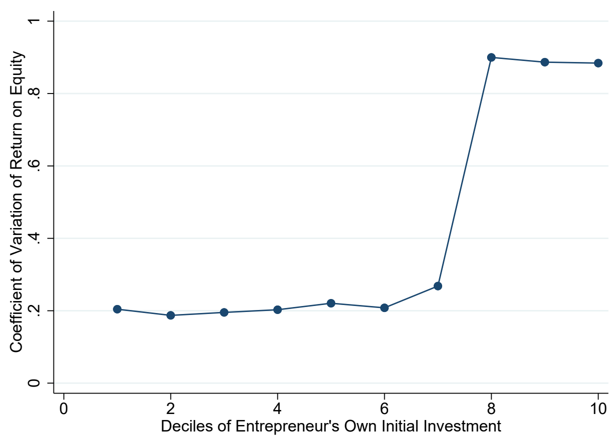


Graph shows the patterns of wealth accumulation for two agents. One is born with the median level of wealth in the economy and one born with an inheritance equal to the 95^{th} percentile of wealth in the economy. The two lines correspond to the average level of wealth given all the possible shocks to their abilities and business productivities that they could receive.

The wealthier agent immediately starts a high risk business. They have some probability of receiving initial productivity shocks low enough that they will shut down their business, liquidate their capital and start a new high risk business next period. This process is costly, and so on average the wealth of the wealthy agent declines briefly. As soon as they start a business that turns out to be successful, they tend to stay in business, receiving high productivity and earning high returns on their wealth. The wealthy agent then saves a high proportion of their entrepreneurial income for two reasons. First, because the project is still risky, the agent wants to engage in precautionary savings in order to self-insure against the project's failure. Second, because of the borrowing constraint, the entrepreneur wants to save more in order to operate a larger business next period and earn an even higher income. As a consequence the wealth of the wealthy agent accumulates wealth quickly.

The agent born with median wealth initially chooses to be a worker, and saves up to become an entrepreneur in the future. Depending on their shocks, they will likely start a business after a few periods of saving. However, when they choose to do so they will start the low-risk business. Starting the high-risk business is simply too risky, as low productivity shocks would induce them to liquidate their capital and exit after having earned little, or even lost money, in that period. As time progresses, these entrepreneurs continue to save. Some of the luckier individual eventually go on to accumulate enough wealth to start the high risk business, but many remain with the low risk business.

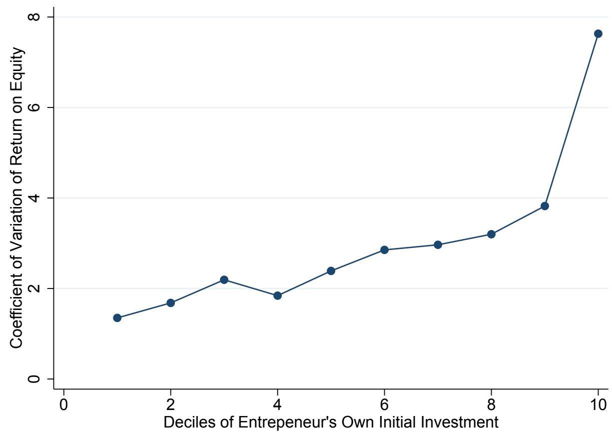
Figure 7: Dispersion in Outcomes and Wealth: Model



The average path of wealth depicted in figure 6 masks a great deal of heterogeneity. Some of the entrepreneurs who start riskier businesses create very successful businesses, while other entrepreneurs end up with unsuccessful businesses. Comparing dispersion in outcomes across different groups of entrepreneurs is one way to infer the degree of risk each group has taken. To illustrate this, figure 7 shows one measure of dispersion for different groups of entrepreneurs defined by their initial wealth. For each entrepreneur, an average return on equity is computed over each year the new firm operates up to a maximum of eight years of operation. Then the coefficient of variation[7] of that average return on equity is plotted for each of the deciles of initial wealth. As wealthier entrepreneurs start higher risk projects in the model, the dispersion in outcomes is much larger for the wealthier entrepreneurs.

[7]The coefficient of variation is the standard deviation divided by the mean.

Figure 8: Dispersion in Outcomes and Wealth: Data



This figure shows the coefficient of variation of the firm's average return on equity over the 8 year sample for the ten deciles of the entrepreneur's own initial investments.

Unfortunately, the model's prediction in figure 7 cannot be directly tested in the Kauffman Firm Survey data. This is because the Kauffman Firm Survey does not include information about the entrepreneur's initial wealth. It is however possible to construct a proxy for the initial entrepreneur's wealth, specifically the amount of the entrepreneurs' own money that they are investing in the business, excluding equity or debt they raise from other sources. [8] Figure 8 plots the coefficient of variation for the average return on equity against deciles of entrepreneur's own initial investment.

Qualitatively, the data shows a similar pattern as in the model, with higher wealth or higher investment firms earning more dispersed returns on their equity. However, the measured coefficient of variation of the average returns on equity in the data is much higher than in the model. Appendix A.3 describes in more detail how these statistics are calculated and discusses the sources of measurement error which may explain the discrepancy.

5 Quantitative Analysis

In this section I measure the quantitative significance of the two financial frictions, the missing market for entrepreneurial risk and entrepreneurial borrowing constraints, for aggregate economic outcomes. To do so, I remove first one, then the other, and finally both from the model in order to compare the resulting equilibria to the benchmark economy with both financial frictions.

First, I complete this missing market by introducing state-contingent assets. In the benchmark economy, there is a single risk-free asset. In the complete markets economy, each agent

[8] See table 8 for details about the various sources of external funding.

can purchase assets that pay off in future states of the world based on their individual realizations of h^W, h^E and z. An asset a_i pays off $1 + r^A$ in the state of the world $i \in H^W \times H^E \times Z$. Each of these assets is sold at an actuarially fair price q_i by competitive risk-neutral financial intermediaries.

$$q(h^{W\prime}, h^{E\prime}, z') = \text{Prob}(h^{W\prime}, h^{E\prime}, z' | h^W, h^E, z, x) \tag{8}$$

In order to separate the impact of the missing market for entrepreneurial risk from the impact of the borrowing constraint, I initially keep the borrowing constraint in the economy unchanged. In the benchmark economy the borrowing constraint requires that $a \geq -\phi k$. In the complete markets economy, the borrowing constraint requires that:

$$\sum_i q_i a_i \geq -\phi k \tag{9}$$

Thus an entrepreneur with the same net worth in both economies can invest in the same maximum amount of capital.

In this complete markets economy, I assume that these financial intermediaries have perfect information over the abilities of potential entrepreneurs and the expected productivity of these projects. This experiment therefore measures an upper bound on the potential benefits of providing additional insurance to entrepreneurs, as access to this degree of information is unrealistic. However, understanding the size of the potential gains helps us understand which financial frictions public policy should aim to mitigate. In section 6, I consider the policy implications of these frictions without requiring that a government have perfect information about private individuals.

Second, I relax the collateral constraint that limits entrepreneurial investment by setting $\phi = 1$, so that entrepreneurs can borrow against the full value of their capital. Note that while this relaxes the entrepreneur's borrowing constraint for investment in capital, workers and entrepreneurs are still unable to borrow using unsecured debt. This experiment therefore is focused on borrowing constraints specifically that prevent entrepreneurial investment, as has been the subject of study in a large literature (Buera et al., 2015), rather than more general borrowing constraints that apply to individuals for consumption.

I then compare the steady-states of four economies. The benchmark economy with both financial frictions, a complete markets economy where state-contingent assets complete the missing market for risk but entrepreneurs still face borrowing constraints ($\phi < 1$), the relaxed borrowing constraints economy where entrepreneurs can borrow the full value of their capital ($\phi = 1$) but there are no state-contingent assets, and finally a neither friction economy with both a complete market for risk and unrestricted capital investment ($\phi = 1$). Table 4 reports the changes in aggregate output, wages, capital stock, and productivity in these four economies relative to the benchmark economy.

Table 4: Change in Aggregate Outcomes

	Output	Wages	Capital	Productivity
Benchmark Economy	0.0	0.0	0.0	0.0
Complete Markets	5.3	5.8	0.9	5.3
Relaxed Borrowing Constraints	5.5	6.4	10.8	2.9
Neither Friction	14.1	15.1	28.2	6.6

All numbers are percent changes relative to the benchmark economy.

In the complete markets economy aggregate productivity is 5.3% higher than in the benchmark economy. Access to state-contingent insurance encourages almost all entrepreneurs to now operate the high risk project, as shown in table 5. With more high-risk projects getting started, the distribution of firm-level productivities in the economy improves and so aggregate productivity rises. In addition, investment increases as some wealthy entrepreneurs choose to invest more, given now that they can now directly insure against the investment risk.

Table 5: Occupational and Risk Choices

	Percentage of		Entrepreneurs operating	
	Workers	Entrepreneurs	Low Risk	High Risk
Benchmark Economy	88.6	11.4	6.1	5.3
Complete Markets	87.6	12.4	0.2	12.2
Relaxed Borrowing Constraints	87.2	12.8	5.8	7.0
Neither Friction	87.1	12.9	0.0	12.9

As existing entrepreneurs operate more productive firms and with higher investment, aggregate labour demand increases substantially. In response, the wage rate rises by 5.8% in order to clear the labour market. As the wage rate increases, less-productive entrepreneurs face additional costs and a rising opportunity cost of exiting and entering the labour market. As a result, less-productive entrepreneurs are effectively crowded out by more-productive entrepreneurs through the equilibrium in the labour market. Installed capital in the economy rises by 0.9% as wealthy entrepreneurs increase how much they invest. Overall, aggregate output rises by 5.3%.

In the relaxed borrowing constraints economy aggregate productivity increases by 2.9%. While most of the gains in the complete markets economy come from the increase in high-risk projects started, more of the gains in the relaxed borrowing constraints economy come from improving selection. Borrowing constraints are a major barrier for low-wealth high-ability individuals from starting a business. Relaxing borrowing constraints also allows formerly constrained entrepreneurs to invest in more capital, leading to an increase in the aggregate capital stock of 10.7%. Wages rise by 6.4% to clear the labour market. Overall, aggregate output increases by 5.5%.

In the economy with neither friction, aggregate productivity increases by 6.6% as all entrepreneurs switch to the higher risk project and occupational selection improves. Entrepreneurs invest far more, leading to an aggregate capital stock 28.2% higher than the benchmark economy. Aggregate output increases by 14.1%.

Table 6: Wealth Inequality

	Wealth Share of Top				Wealth	Wealth Gini of	Entrepreneur to Worker
	1%	5%	10%	20%	Gini	Entrepreneurs	Wealth Ratio
Benchmark	23	62	78	90	0.84	0.60	7.15
Contingent Claims	10	36	60	87	0.78	0.38	5.15
No Borrowing Constraint	21	60	80	90	0.84	0.66	6.23
Frictionless	8	33	56	83	0.76	0.31	5.24

Table 6 reports statistics about the distribution of wealth in the four economies. Completing the missing market substantially reduces wealth inequality relative to the benchmark economy. The share of wealth owned by the wealthiest 1% declines from 23% down to 10% and the wealth Gini declines from 0.84 to 0.78. This substantial decline in inequality occurs mainly between entrepreneurs as completing the missing market for risk eliminates an important source of rate of return heterogeneity. As illustrated by figure 6, in the benchmark economy wealthier entrepreneurs earn higher rates of return on their wealth because they start higher risk businesses. By contrast under complete markets, almost all entrepreneurs now choose high-risk projects. As a consequence the wealth Gini of entrepreneurs declines from 0.60 down to 0.38. This means that completing the missing market for risk delivers a rare equity-efficiency win-win, where both aggregate productivity is higher and wealth inequality is low.

By contrast, relaxing the borrowing constraints has little effect on wealth inequality. While the share of wealth owned by the wealthiest 1% declines from 23% to 21%, the wealth Gini remains unchanged at 0.84. The wealth Gini of entrepreneurs increases modestly from 0.60 to 0.66 as the lack of borrowing constraints encourages entrepreneurs to increase their leverage. As a consequence, entrepreneurs that receive good productivity shocks on highly leveraged businesses accumulate wealth much faster, and those that receive bad shocks lose wealth faster.

Moving from the benchmark economy to the economy with neither friction reduces wealth inequality, which demonstrates the dominant role that the endogenous choice of project risk plays in determining the distribution of wealth in the economy. The wealth Gini declines from 0.84 to 0.76 and the share of wealth owned by the wealthiest 1% declines from 23% down to 8%.

In subsections 5.1, 5.2, and 5.3 I illustrate how these changes in aggregate quantities are related to the decisions of individuals under the different financial frictions. Throughout these subsections, I keep the wage constant at the benchmark economy's equilibrium level, in order to directly show the impact of removing the frictions.

5.1 Completing the Missing Market for Risk

Completing the missing market for risk could increase output in the economy in three distinct ways. First, with the ability to insure against entrepreneurial risk, more entrepreneurs could choose the high risk project, rather than the low risk project. As the higher-risk project has a higher expected productivity, this would boost productivity and aggregate output. Secondly, individuals with higher entrepreneurial ability could be more likely to select into entrepreneurship. Third, conditional on wealth, unconstrained entrepreneurs could invest more in their businesses.

Figure 9: Comparing Occupational Choice and Endogenous Risk Choice

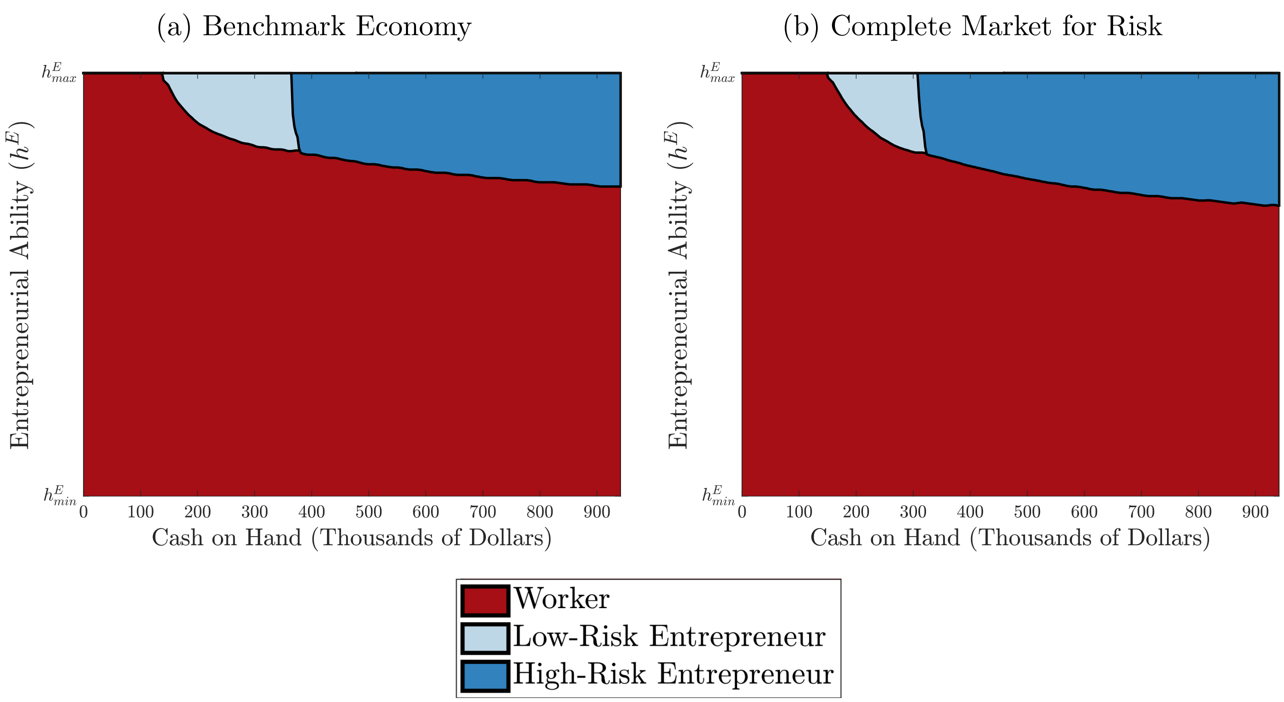
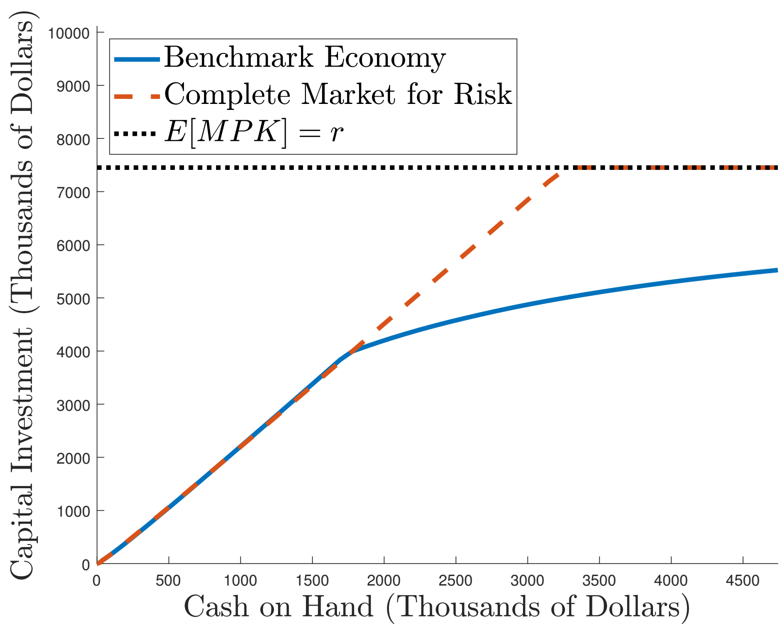


Figure 9 compares the patterns of occupational choice and endogenous risk choice between the benchmark economy and the economy with a complete market for risk. In the economy with state-contingent assets, more entrepreneurs choose the riskier project with higher expected productivity. They then insure themselves against the substantial risks associated with this using the state-contingent assets. Due to the borrowing constraints and the possibility of hitting the unsecured borrowing constraint, some entrepreneurs still choose to start the low-risk project in partial equilibrium.[9] There is little change in the wealth that makes individuals indifferent between being workers or entrepreneurs, which suggests that the missing market for entrepreneurial risk does not play a major role in distorting individuals' occupational choices.

Figure 10: Capital Investment for the Highest Ability Entrepreneur



Capital investment for the highest ability entrepreneur in the benchmark economy with both frictions (solid blue line) and in the economy with contingent claims (dashed orange).

Figure 10 shows the amount of capital invested by the most productive entrepreneur in the economy in the high risk project. In the benchmark economy, even when the entrepreneur is wealthy enough to be unconstrained by the borrowing constraint they invest less than the level at which the expected marginal product of capital equals the real interest rate. They under-invest because of the substantial capital investment risks they face. With the introduction of state-contingent assets, entrepreneurs insure themselves against bad shocks, and so choose to invest more in their businesses. In both economies, relatively wealth-poor individuals are constrained by their personal wealth and borrow up to the borrowing limit.

[9]In general equilibrium, a higher wage will make many of these low-risk businesses less profitable. Therefore, the equilibrium with complete markets has only 0.2% of the population running low risk businesses as can be seen in table 5.

5.2 Relaxing The Borrowing Constraints

Relaxing entrepreneurs' borrowing constraints likewise could increases output through the same three channels as the introduction of state-contingent assets.

Figure 11: Comparing Occupational Choice and Endogenous Risk Choice

(a) Benchmark Economy (b) Relaxed Borrowing Constraints

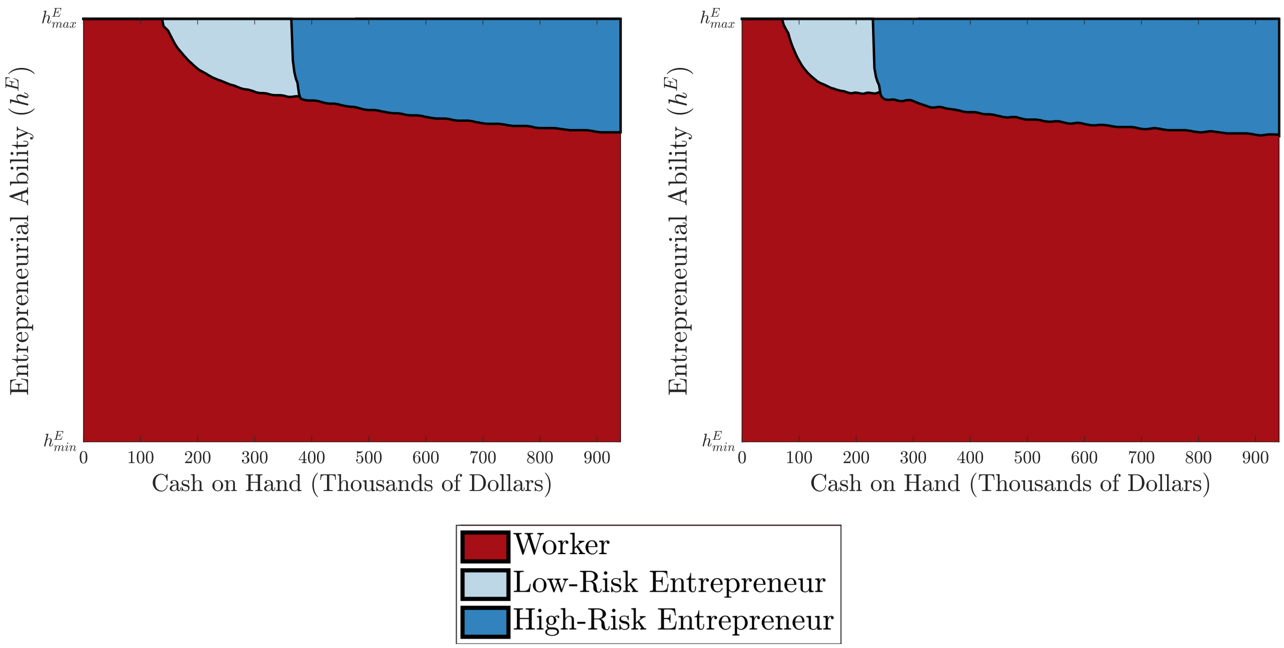
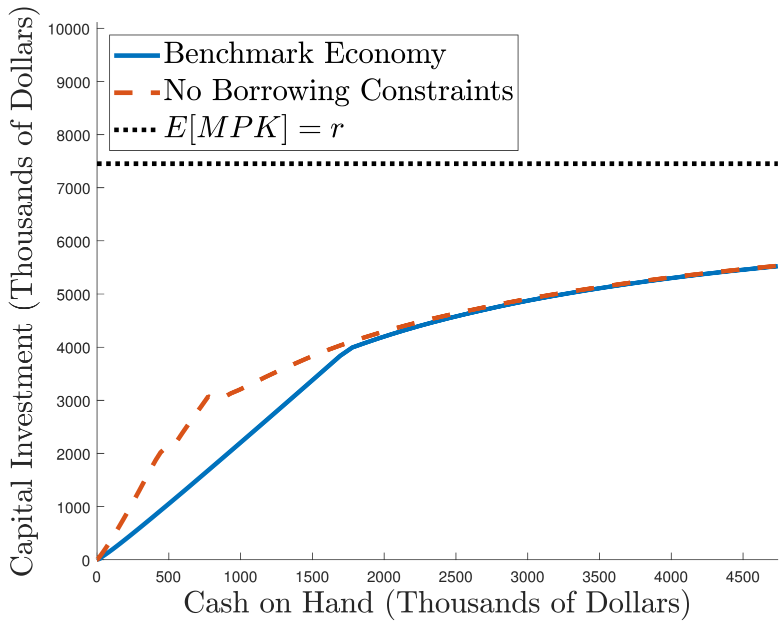


Figure 11 shows the pattern of occupational choice and risk choice in economies with and without borrowing constraints. Without borrowing constraints, many more wealth-poor individuals with high entrepreneurial ability switch from being workers to being entrepreneurs. They do so because they are now able to operate much larger scale businesses and so generate far more entrepreneurial income. Note that while the shift in the graphical region might appear small, given the large mass of individuals with low levels of wealth, many more individuals are now willing to become entrepreneurs. In partial equilibrium, the proportion of entrepreneurs would more than double to 24%.

Relaxing the borrowing constraints also induced some moderately-wealthy entrepreneurs to switch from the low risk project to the high risk project. With borrowing constraints, if a moderately-wealthy entrepreneur starts a high risk project, receives a bad productivity shock and looses wealth, either through consuming out of their savings or because they must liquidate their capital stock, the next business they start must be operated at an even smaller scale because of their limited wealth. Without borrowing constraints, this negative effect on serial entrepreneurship doesn't take place, as entrepreneurs are able to operate subsequent businesses at any scale. As a consequence, removing the borrowing constraints encourages lower wealth entrepreneurs to start the high-risk project.

Figure 12: Capital Investment for the Highest Ability Entrepreneur



Capital investment for the highest ability entrepreneur in the benchmark economy with both frictions (solid blue line) and in the economy with no borrowing constraints ($\phi = 1$) (dashed orange).

Figure 12 shows the investment patterns for the highest productivity entrepreneur in the benchmark and unrestricted investment economies. Wealth-poor individuals in the unrestricted investment economy are able to invest more than those in the benchmark economy because of the lack of borrowing constraints. However, because of the substantial capital investment risk, they still invest less than the level at which the expected marginal product of capital equals the real interest rate.

5.3 Neither Financial Friction

Figure 13 compares the pattern of occupational choice and risk choice in an economy with both financial frictions to an economy with neither. Due to the availability of state-contingent assets, fewer entrepreneurs will choose to start the low-risk business. The lack of entrepreneurial borrowing constraints means that poorer individuals are also more willing to become entrepreneurs. Despite these absence of these two frictions, wealth still plays a role in determining both occupational selection and the endogenous choice of risk. This is due to the presence of the unsecured borrowing limit, but the role of wealth is greatly diminished due to the removal of the two main financial frictions.

Figure 14 shows the investment patterns for the highest ability entrepreneur in the benchmark economy and in the economy with neither financial friction. Entrepreneurs of all wealth

Figure 13: Comparing Occupational Choice and Endogenous Risk Choice

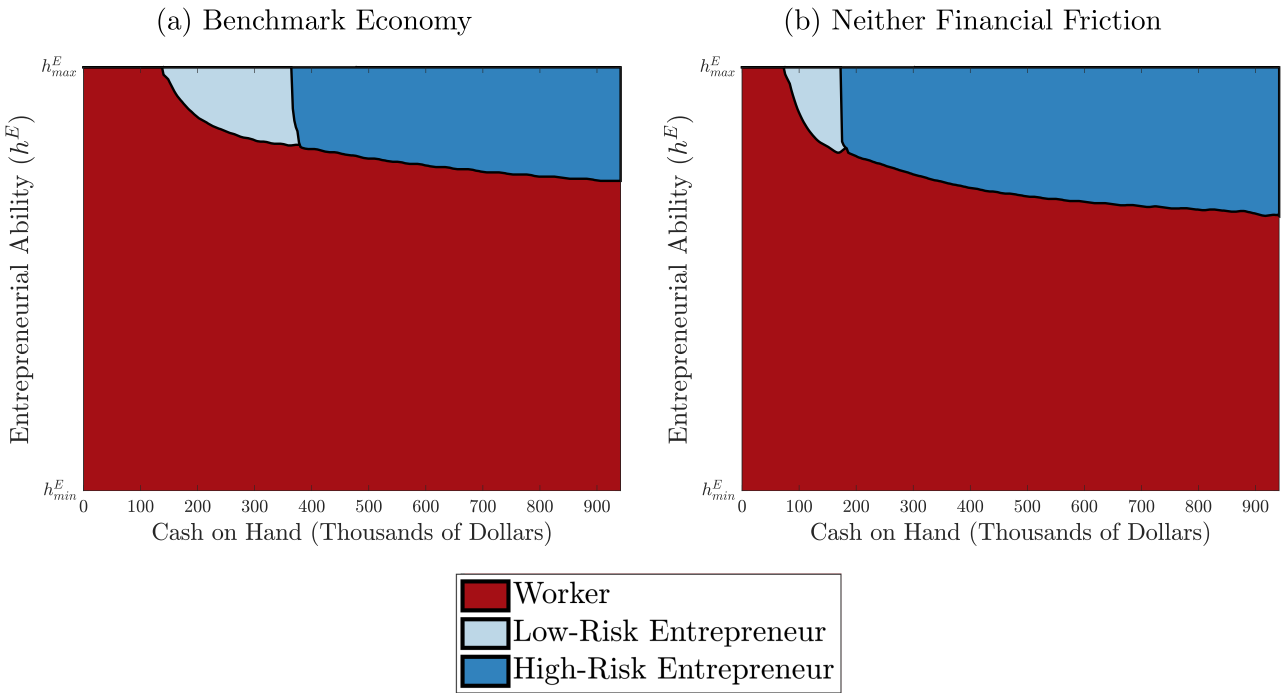
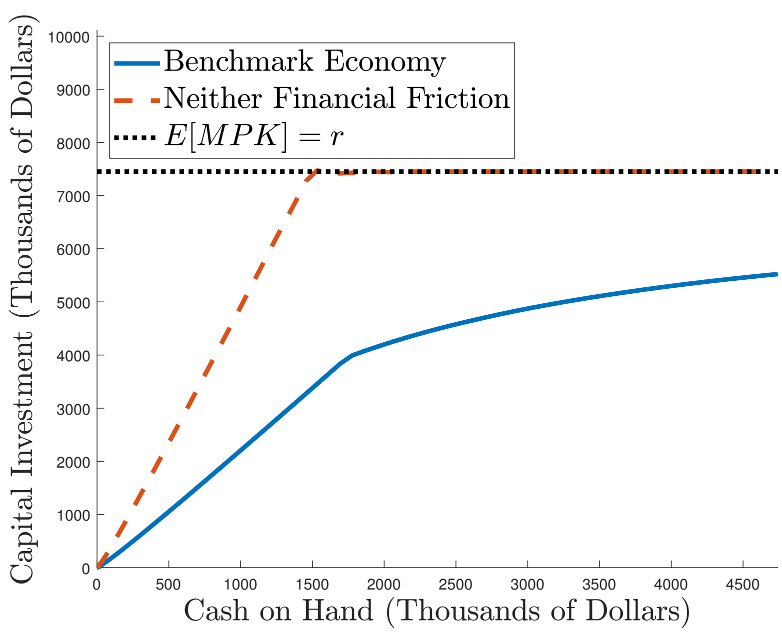


Figure 14: Capital Investment for the Highest Ability Entrepreneur



Capital investment for the highest ability entrepreneur in the benchmark economy with both frictions (solid blue line) and in the economy with neither friction (dashed orange).

levels invest substantially more than when they face the two financial frictions. Even with $\phi = 1$, the combination of the unsecured borrowing limit, the capital liquidation costs, and the inability to default discourages wealth-poor entrepreneurs from investing up to the point where the expected marginal product of capital is equal to the interest rate, as entrepreneurs ensure they have enough resources to repay their debts while still enjoying non-negative consumption in future periods.

6 Policy Analysis

The quantitative results in section 5 highlight how important the missing market for entrepreneurial risk is for discouraging entrepreneurial activity. Providing additional insurance to entrepreneurs may therefore encourage risk-taking. However, any government policy that seeks to provide partial insurance to entrepreneurs is likely to run into the same adverse selection and moral hazard problems that prevent a private sector insurance market for entrepreneurial risk from existing.

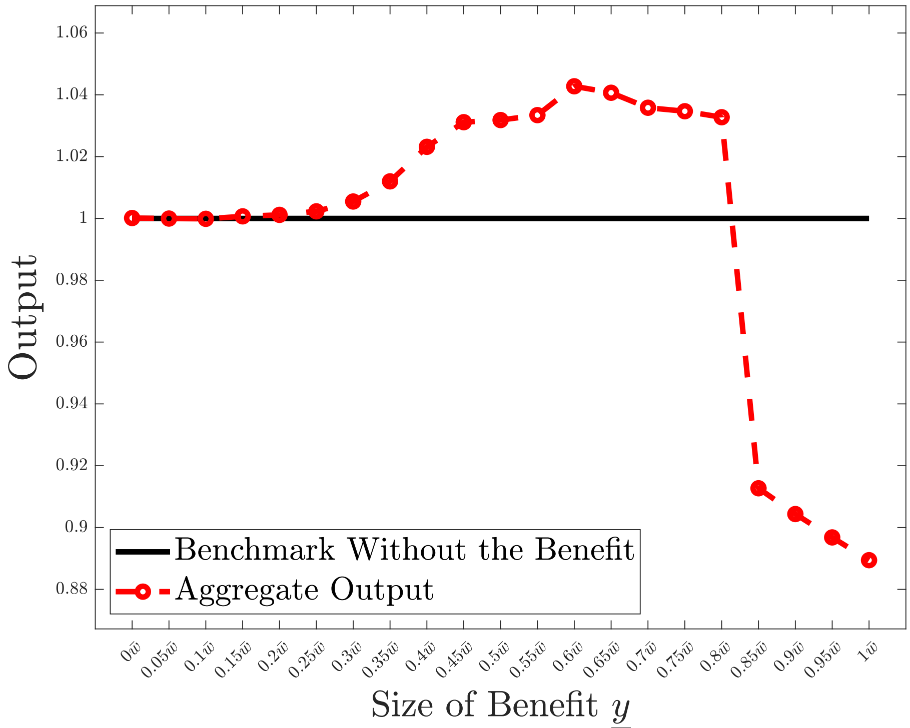
I therefore study a government policy designed to encourage risk taking, without assuming the government can observe the abilities or expected productivities of their projects. Specifically, I consider an unemployment-like benefit that tops up the incomes of unsuccessful entrepreneurs. I model this as a transfer that tops up entrepreneurial income to an income floor given by $\underline{y}$. This benefit is paid for with a proportional tax τ on entrepreneurial income that clears the government's budget constraint. An entrepreneur's income is therefore given by:

$$y = \max\left\{(1-\tau)[(zh^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn] + r^a a, \underline{y}\right\} \tag{10}$$

Figure 15 plots aggregate output as a function of the size of the unemployment-like benefit for entrepreneurs ($\underline{y}$), expressed as a fraction of the average wage in the benchmark economy ($\bar{w}$). At low levels, the benefit raises output in the economy. Even small insurance benefits are enough to encourage more entrepreneurs to choose the higher risk project, as illustrated in figure 16. As the higher risk project has higher expected productivity output increases. At its maximum efficacy, the policy can generate aggregate output 4.3% higher than in the benchmark economy. This is despite the fact that the proportional tax discourages both selection into entrepreneurship and investment in entrepreneurial capital.

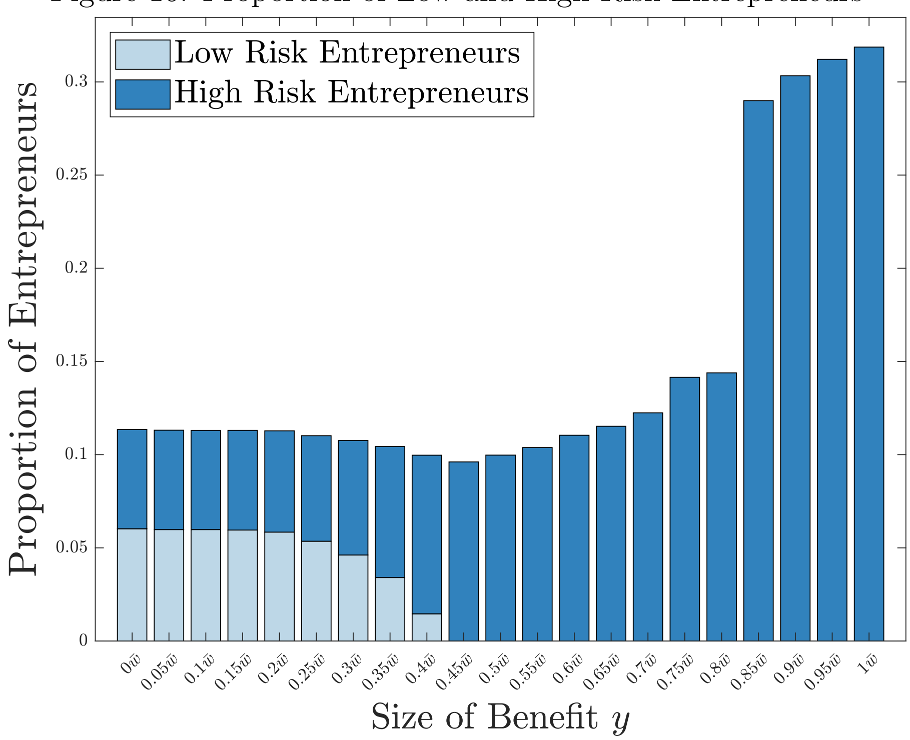
Note that this simple policy actually helps alleviate both financial frictions. In the benchmark economy, some high ability entrepreneurs start high risk businesses, receive bad productivity shocks, and then exit. As they do so, they receive low income for at least one period

Figure 15: Aggregate Output Depending on the Size of Benefit



$\underline{y}$ is the income floor given to all unsuccessful entrepreneurs. $\bar{w}$ is the average wage in the benchmark economy without the benefit.

Figure 16: Proportion of Low and High Risk Entrepreneurs



$\underline{y}$ is the income floor given to all unsuccessful entrepreneurs. $\bar{w}$ is the average wage in the benchmark economy without the benefit.

and also lose a portion of their capital stock as they liquidate their businesses to exit. They exit with lower wealth than they started with and so will be able to invest in less capital if they choose to start another new business. In the economy with the benefit, not only does the benefit provide partial insurance against bad shocks, but it also helps re-capitalize unsuccessful entrepreneurs. In this way, the unemployment-like benefit for entrepreneurs partially alleviates both the missing market for risk and the borrowing constraints.

Of course at higher levels of benefit, adverse selection becomes a significant problem with this policy. Individuals who have low entrepreneurial ability become entrepreneurs simply because the income floor is higher than what they can earn in the labour market, even if they have no expectation of being able to generate entrepreneurial income. As a consequence, aggregate output begins to fall when the benefit rises too high. As can be seen in figure 15, aggregate output decreases sharply when the benefit exceeds 80% of the benchmark wage, as more and more low-ability individuals select into entrepreneurship.

Moral hazard is also present in this policy experiment. Entrepreneurs have weaker incentives to invest in capital if they know that there is some probability that their income will fall below the threshold ($\underline{y}$). Below this level of income, the benefit will increase their income to $\underline{y}$ regardless of how much capital they have invested in. In practice, this source of moral hazard has only a small impact on investment behaviour given that entrepreneurs' incomes are low only when they receive bad productivity shocks, which is also when the marginal product of capital is low.

This simple insurance scheme is not an optimal policy, but it illustrates that partial insurance schemes can help governments encourage entrepreneurial risk taking, even in a context with both adverse selection and moral hazard. Clearly, the practical design of this type of partial insurance scheme would require careful analysis that is beyond the scope of this paper, but the quantitative results in section 5 suggest that the potential returns to this type of policy are large.

7 Conclusion

This paper studies the quantitative importance of two distinct financial frictions for output, productivity, and wealth inequality. It contributes to a large literature studying how financial frictions distort the decisions of entrepreneurs and how those distortions reduce aggregate output. The key contribution is to study an understudied financial friction, the missing market for entrepreneurial risk, and compares it to the well studied effects of borrowing constraints. I find that the missing market for risk causes larger losses to aggregate productivity than borrowing constraints and is also a key determinant of top tail wealth inequality.

I present descriptive evidence from the Kauffman Firm Survey that new entrepreneurs face a high degree of idiosyncratic risk and that entrepreneurs who invest more of their own money are more likely to raise external funds. Motivated by these facts, I build a model of occupational choice and business risk choice. I calibrate the strength of the two financial frictions in my model to micro data from the Kauffman Firm Survey. In the calibrated model both financial frictions play an important role distorting individuals decisions. Removing each of the financial frictions, I study changes in the patterns of occupational selection, the choice of the riskiness of businesses, and investment. Finally, I study a policy that provides partial insurance to entrepreneurs that encourages them to take more risk, showing that it is successful at increasing aggregate output despite the presence of adverse selection and moral hazard.

Governments around the world seek to balance redistribution with the promotion of entrepreneurship. This paper's results suggest that a promising area is to study the design of partial insurance schemes that encourage more individuals to take the risk of becoming an entrepreneur and encourage entrepreneurs to pursue more innovative business ideas. Not only does this type of policy have the potential to increase economic efficiency, but it also has the potential to reduce wealth inequality.

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A Empirical Facts Appendix

The Kauffman Firm survey is a panel data set of 4,928 new firms that start in 2004. Given that the data is a survey, sample attrition and non-response is an issue. In order to make best use of the full sample, if respondents did not provide an exact amount of the variable in dollars, respondents were asked to provide a range of the amount instead. Throughout this paper, I use the midpoints of these ranges for these observations.

The Kauffman Firm survey is a stratified sample based on technology level and gender of owners. All tables and figures in this paper therefore use the Kauffman Firm survey's provided weights. Several figures in this paper are organized by the amount of the entrepreneur's own equity in the first three years of operation. These tables and figures use the third year's longitudinal weights[10], to ensure the measure of entrepreneur's own equity is not affected by any non-response. Unless otherwise mentioned, other tables and figure are weighted by the full longitudinal weights from the eighth year of operation[11]. The choice of weights does not substantially affect any of these results.

A.1 Measuring Survival

In table 7, I report the current operational status of all firms in the Kauffman Firm Survey over the 8 years of the survey. Firm exit is common. While non-response and sample attrition mean that the status of 701 firms are not available in the final year, 1,901 firms or 45% of the firms with known status have permanently shut down by the end of the sample. An additional 30 were temporarily shut down.

Without observing sale prices or the terms of mergers, it is impossible to asses whether firms that are merged or sold constitute successful outcomes for entrepreneurs. Throughout this paper, I therefore calculate the survival rate as the number of firms either operating or

[10] wgt_3_long within the dataset.
[11] wgt_7_long within the dataset.

Table 7: Business Status Over Time

Year	Operating	Shut Down	Merged or Sold	Temp Shut Down	Unknown
2004	4,928	0	0	0	0
2005	3,998	260	43	66	561
2006	3,390	581	90	124	743
2007	2,915	880	135	98	900
2008	2,606	1,224	175	58	865
2009	2,408	1,474	211	41	794
2010	2,126	1,692	249	45	816
2011	2,007	1,901	289	30	701

Tabulation of the operational status of firms in the Kauffman Firm Survey over the first eight years of operation. Note that when a firm is merged or sold, it exits the sample and so no more information about its operational status is available.

that have been merged or sold in a previous year, divided by the total number of firms with a known status. It is possible that some of the firms that were sold or merged were sold for low prices or merged under unfavorable terms for the entrepreneurs, so this measure of survival is likely an underestimate of desirable outcomes. If surveyors have a more difficult time contacting the entrepreneurs of firms who have ceased operations, then this measure of survival is further biased upwards.

A.2 Sources of Funding Details

New firms in the Kauffman Firm Survey raise their initial funds from a variety of sources. Table 8 updates a similar table in Robb and Robinson (2014) with the full sample of KFS data. By far the most common source of funding is the entrepreneurs' own investment. For 89% of firms, entrepreneurs are putting their own money into the business, these investments are often modest, with the median amount being only $25,000.

Note that while 53% of firms are able to borrow, most of these loans are personal debt. If the business is unsuccessful and entrepreneurs choose to exit, these debts cannot be discharged without filing for personal bankruptcy. Raising external equity is much less common, with only 15% of firms raising equity beyond the actively managing owners, though these firms typically invest substantially more than those without any external equity.

Figures 17 and 18 show the amounts of debt and external equity raised by firms for each of the ten deciles of initial investment, defined as the amount of their own money that the entrepreneurs invest in the business within the first three years. Note that these graphs have a

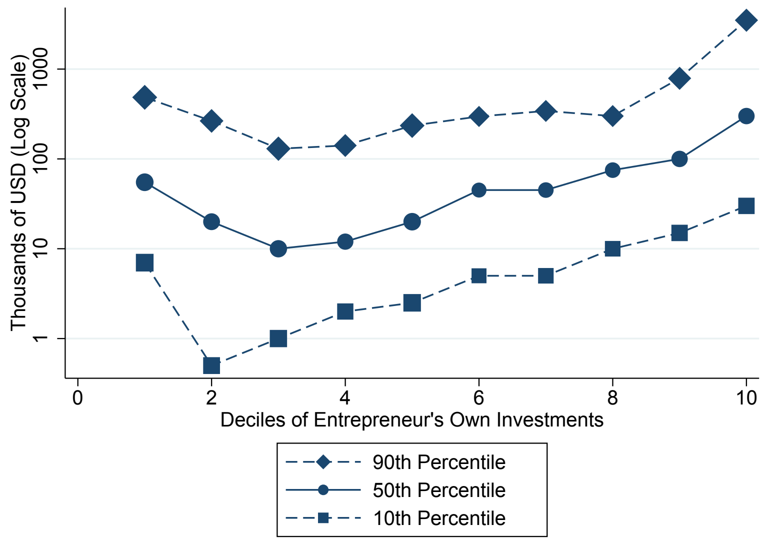
Table 8: Sources of Funding for Firms in the Kauffman Firm Survey

Source	Percentage with > 0	Mean given > 0	Percentiles 25th	50th	75th	Number of Firms
Own Equity	89	166	8	25	80	3,655
External Equity	15	1,151	10	50	203	640
Outside Investors	6	758	10	90	290	247
Parents	5	83	10	25	73	242
Other Companies	4	1,565	20	100	600	154
Spouses	3	82	5	15	30	137
Venture capital	1	8,669	85	450	6,125	60
Government Agencies	1	698	53	250	850	44
Other	1	995	10	25	135	23
Any Debt	53	714	15	55	210	2,302
Personal Debt	46	405	10	40	128	2,011
Bank	32	308	15	50	140	1,413
Family	20	51	5	15	46	938
Other Individuals	6	434	4	15	50	252
Any Other Sources	3	1,489	10	33	110	160
Business Debt	28	701	15	50	211	1,264
Bank	17	873	26	84	269	747
Family	8	84	5	15	45	356
Non-Bank Financial	6	357	12	48	200	262
Owners	4	331	15	48	200	158
Government Agencies	2	1,052	30	125	330	89
Other Individuals	2	153	5	24	100	74
Employees	1	56	5	14	40	38
All Funding Sources	94	520	10	45	181	3,687

The sources of funding for firms in the Kauffman Firm Survey. All values are in thousands of US dollars and are cumulative over the first eight years of operation. The first column reports the percentage of firms that received any money from each funding source, the second column gives the mean amount raised conditional on raising some money from that source, the third through fifth columns give the 25th, 50th and 75th percentiles of the amount raised conditional on raising some money from that source. The final column gives the number of firms that raise any money from each funding sources. "Personal Debt" is debt owed by an entrepreneur, while "Business Debt" is debt owed by the business.

log-scale. Across the first three deciles, there appears to be somewhat of a decline, suggesting that there is a small fraction of firms who are able to substitute owners investment for external funds, but over the rest of the distribution, larger entrepreneur's investments are correlated with larger amounts of external funds.

Figure 17: The Amount of Debt Raised by Firms



The 10$^{\text{th}}$, 50$^{\text{th}}$ and 90$^{\text{th}}$ percentiles of the distribution of total debt raised by firms across the different deciles of the total of entrepreneur's own investments, conditional on raising some debt. Note the y-axis is a log-scale.

A.3 Calculating Returns on Equity

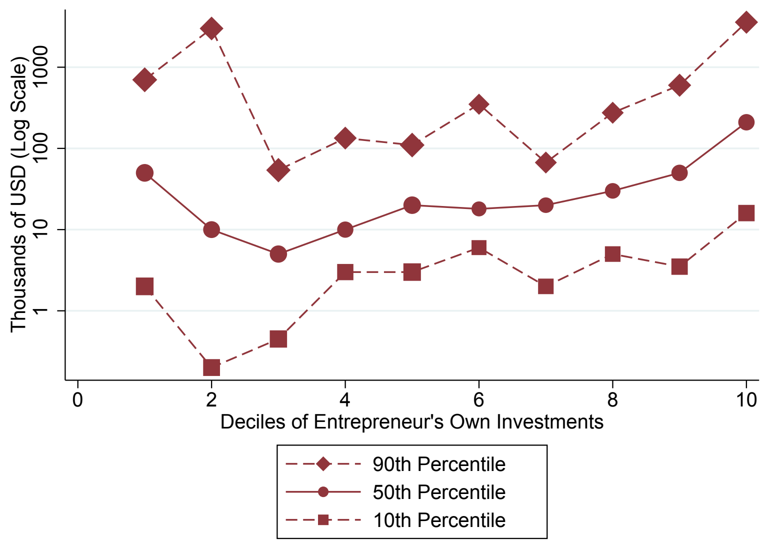
For each year t, I calculate a firm i's return on equity as:

$$\text{Measured Return on Equity}_{i,t} = \frac{\pi_{i,t}}{\text{Cumulative Equity}_{i,t}} \tag{11}$$

where $\pi_{i,t}$ is the profits or losses of firm i in year t, and Cumulative Equity$_{i,t}$ is the total amount invested in the firm up to and including in year t.

Figure 8 shows dispersion in average returns on equity across different deciles of entrepreneur's own initial investment. In order to construct this measure, I figure compute a firm level arithmetic average return on equity as $\bar{\pi}_i = \sum_{t=1}^{T_i} \text{Measured Return on Equity}_{i,t}$, where T_i is the last year the firm appears in the sample. This calculated average return on equity is then used to calculate the coefficient of variation by decile in figure 8.

Figure 18: The Amount of External Equity Raised by Firms



The 10^{th}, 50^{th} and 90^{th} percentiles of the distribution of total external equity raised by firms across the different deciles of the total of entrepreneur's own investments, conditional on raising some external equity. Note the y-axis is a log-scale.

Table 9: Survival by Cumulative Profits and Average Return on Equity

	Cumulative Profits		Return on Equity	
Decile	Percent of Firms that Survive to Year 8	Number of Firms	Percent of Firms that Survive to Year 8	Firms
1	62	339	14	306
2	44	349	35	309
3	37	353	50	315
4	32	364	58	317
5	27	346	58	309
6	49	349	66	301
7	61	340	66	302
8	75	343	62	311
9	77	355	64	309
10	89	352	66	309

How well does the average return on equity capture the success or failure of these firm? One way to test this is to compare survival rates of firms with different levels of return on equity. If the calculated return on equity is a good measure of success, we should expect to see that firms with high calculated average returns on equity are much more likely to survive than firms with low calculated average returns on equity.

Table 9 shows that firms that have a low calculate return on equity are unlikely to still be operating at the end of the eight year sample. Indeed having a low calculated return on equity is a much better predictor of firm exit than have low cumulative profits. However, high calculated returns on equity are much less correlated with survival than high levels of cumulative profits.

Why are these calculated returns on equity not a better predictor of success than cumulative profits? Given that many of these firms are relatively small scale, the ideal calculation of a firm's rate of return would include the entrepreneur's salaries and forgone earnings. Unfortunately, the Kauffman Firm Survey does not contain information about the amount of salary paid to entrepreneurs or about their forgone earnings. Omitting entrepreneurial salaries may substantially bias the measured return on equity if entrepreneurs choose to receive some of the returns to their business in the form of a salary rather than in the form of dividends.

Similarly, omitting forgone labour earnings may substantially bias the estimated returns if the value of the entrepreneur's time input is a substantial input into the business. This is likely to be the case for many small-scale firms, where the value of the entrepreneur's time may be larger than the financial investment. Ignoring this input may greatly inflate the rates of return earned by small-scale firms. With more data, it would be desirable to calculate the following cumulative rate of return:

$$\text{Return on Equity}_{i,t} = \frac{\text{Dividends}_{i,t} + \text{Entrepreneur's Salary}_{i,t} + \Delta\text{Firm Value}_{i,t}}{\text{Cumulative Equity}_{i,t} + \text{Cumulative Forgone Salary}_{i,t}} \tag{12}$$

Unfortunately, the Kauffman Firm Survey does not contain information about salaries paid to entrepreneurs ($\text{Salary}_{i,t}$), the change in the firm's value in year t ($\Delta\text{Firm Value}_{i,t}$), or the forgone salaries of entrepreneurs ($\text{Cumulative Forgone Salary}_{i,t}$). As a result it is difficult to calculate a more informative measure of the returns to running these businesses.

A.4 Wealth Inequality Moments

In section 4, I calibrate the model to several wealth inequality moments taken from the 2004 Survey of Consumer Finances. I define entrepreneurs as households that both own a business and actively manage that business. As this paper does not study passive business ownership, I drop households from the sample if they are passive business owners, unless they own multiple businesses and actively manage at least one of them. As there is no retirement state in the model of section 3, I drop households of retirement age. For single person households, I drop the household if the individual is older than 65. For two person households, I drop the household if the average age of the two individuals is greater than 65.

Table 10 shows that the distribution of wealth does not change substantially if either retired households or passive business owners are dropped. The first row shows the top wealth shares for the whole SCF sample, designed to be representative of the US population. The second drops households of retirement age. The third row drops all passive business owners, who are not also active business owners. The fourth drops both households of retirement age and the passive business owners.

Table 10: The Distribution of Wealth

	Wealth Share of Top					Wealth Gini	Entrepreneur Wealth Gini	Proportion with Negative Net Worth
	1%	5%	10%	20%	50%			
Full Sample	33	57	69	83	97	0.79	0.75	0.07
Drop Retirement Age	34	57	69	82	97	0.80	0.75	0.08
Drop Passive Business Owners	31	56	68	82	97	0.78	0.75	0.07
Drop Retirement Age and Passive Business Owners	32	56	67	81	97	0.79	0.75	0.08
Drop All Business Owners	15	37	52	71	95	0.73	.	0.08

Top wealth shares in the US economy across several samples.

In table 11, I show that two of the key targeted model moments do depend on the subsample chosen. Since both retired households and passive business owners own a substantial amount of wealth, their inclusion makes the wealth differences between entrepreneurs and non-entrepreneurs look less stark. I use the statistics from the fourth row for the calibration of my model.

Table 11: Model Moments by Subsample

	Ratio of Wealth of Entrepreneurs to All Others	Proportion of Entrepreneurs	Proportion of Entrepreneurs in Wealthiest 1%
Full Sample	5.70	0.11	0.60
Drop Retirement Age	6.29	0.12	0.65
Drop Passive Business Owners	6.42	0.11	0.69
Drop Retirement Age and Passive Business Owners	7.15	0.12	0.74

B Static Model

In this section, I build a simple static model to illustrate the two financial frictions explored in this paper and analytically characterize how wealth allows individuals to overcome these frictions.

B.1 Environment

Agents live for a single period and have preferences over consumption $u(c)$, where $u(\cdot)$ exhibits decreasing absolute risk aversion[12].

Agents are initially endowed with wealth $e \in [0, \bar{e}]$ and a known entrepreneurial ability $h^E \in [\underline{h}^E, \bar{h}^E]$. At the beginning of the period, agents must choose to be an entrepreneur or a worker based on their wealth and ability (e, h^E).

All agents have identical productivity as a worker. If they choose to be a worker, they will earn a wage of w with certainty.

If an agent chooses to be an entrepreneur, they must choose the riskiness of their business $x \in [0, h^E]$. Choosing a riskier business will result in higher expected productivity at the cost of more dispersed productivity. With probability p the entrepreneur's risk is successful, and their productivity is boosted by ψx. With the complementary probability $(1-p)$ the risk is unsuccessful and their productivity is reduced by x. The entrepreneur's realized productivity is therefore given by z:

$$z = \begin{cases} h^E + \psi x & \text{with probability} \quad p \\ h^E - x & \text{with probability} \quad 1-p \end{cases} \tag{13}$$

where $\psi > \frac{1-p}{p}$, so that the expected value of z increases with x.

There are two assets, capital (k) and a risk-free financial asset (a). At the beginning of the period, agents choose how much of their initial endowment (e) to invest in each asset:

$$e = a + k \tag{14}$$

Agents can only hold a positive amount of capital, and potentially face a borrowing con-

[12] Decreasing absolute risk aversion is defined by:

$$\frac{\partial}{\partial c}\left(-\frac{u''(c)}{u'(c)}\right) = -\frac{u'(c)u'''(c) - [u''(c)]^2}{[u'(c)]^2} < 0$$

Note that many common utility functions such as log and CRRA exhibit decreasing absolute risk aversion.

straint in the financial asset. The borrowing constraint is given by:

$$a \geq -\phi k \tag{15}$$

where $\phi \in [0,1]$. When $\phi = 0$, entrepreneurs are not able to borrow in the financial asset. When $\phi = 1$ entrepreneurs are able to invest in any amount of capital k regardless of their initial endowment of wealth. For values of $\phi < 1$, this model exhibits borrowing constraints, which prevent entrepreneurs from investing in a level of capital stock that exceeds a multiple $\left(\frac{1}{1-\phi}\right)$ of their endowed wealth e.

The financial asset a pays off $(1+r^a)$ units in all states of the world. Without any access to state-contingent assets, this model exhibits a missing market for entrepreneurial risk as entrepreneurs are unable to insure themselves against the risks to their realized productivity z.

An entrepreneur with a realized productivity z and a capital stock k will produce output according to:

$$y = z^{1-\gamma} k^{\alpha} \tag{16}$$

B.2 Agent's Problem

Depending on their initial endowments of wealth (e) and entrepreneurial productivity (z), agents make an occupational choice decision between being workers and entrepreneurs according to:

$$V(h^E, e) = \max\{V^E(h^E, e), V^W(e)\} \tag{17}$$

Worker's Value Function:

$$V^W(e) = u(w + (1+r^a)e) \tag{18}$$

Entrepreneur's Value Function:

$$V^E(h^E, e) = \max_{x,k,a} pu\left[(h^E + \psi x)^{1-\gamma} k^{\alpha} + (1+r^a)a\right] + (1-p)u\left[(h^E - x)^{1-\gamma} k^{\alpha} + (1+r^a)a\right] \tag{19}$$

subject to:

$$e = a + k$$
$$a \geq -\phi k$$

where $p\psi > (1-p)$ and $\phi \in [0,1]$.

B.3 Model Predictions

Depending on the tightness of the borrowing constraints ϕ, and each individual's wealth and entrepreneurial ability (e, h^E), entrepreneurs may be unconstrained $(a > -\phi k)$ or constrained $(a = -\phi k)$ in the amount of capital they are investing in.

B.3.1 Case 1: Unconstrained

If the entrepreneur is unconstrained then optimal x and k are given by:

$$x^* = h^E \frac{1 - \left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}}{1 + \psi \left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}} \tag{20}$$

$$k^* = \left(\frac{\alpha}{[1 + r^a]} \frac{\left[p(h^E + \psi x)^{1-\gamma} + (1-p)(h^E - x)^{1-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]}{\left[p + (1-p)\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]} \right)^{\frac{1}{1-\alpha}} \tag{21}$$

where

$$\begin{aligned} \bar{c} =& (h^E + \psi x)^{1-\gamma} k^{*\alpha} + (1 + r^a) a^* \\ \underline{c} =& (h^E - x)^{1-\gamma} k^{*\alpha} + (1 + r^a) a^* \\ a^* =& e - k^* \end{aligned}$$

Note that if $x = 0$, the entrepreneurial project would be risk-free. In that case, $\bar{c} = \underline{c}$, and so the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$ will be equal to one. Since $\psi p > 1 - p$, the numerator of the right hand side of equation (20) is strictly positive, and all entrepreneurs choose a strictly positive level of risk $x > 0$.

Proposition 1: When unconstrained, risk taking is increasing in wealth

$$\frac{\partial x^*}{\partial e} > 0 \tag{22}$$

Proof See section B.4.1.

Intuition

As an individual's wealth increases, more of their final consumption is derived from their endowed wealth and less from their income. Wealthier entrepreneurs are more willing to take a higher level of risk for a higher expected income because when their business is unsuccessful they still have abundant consumption from the wealth outside their business.

Implications

Wealthier entrepreneurs will choose higher-risk higher-productivity projects. The distribution of wealth will have a direct impact on the distribution of riskiness of businesses that get started, which will determine aggregate productivity in the economy. Starting higher expected productivity businesses means that wealthier individuals will also earn higher expected returns on their wealth.

Proposition 2: When unconstrained, individuals are more likely to become entrepreneurs as their wealth e increases

$$\frac{\partial V^E}{\partial e} > \frac{\partial V^W}{\partial e}\bigg|_{V^E=V^W} \tag{23}$$

Proof See section B.4.2.

Intuition

As an individual's wealth increases, more of their final consumption is derived from their endowed wealth and less from their income. Wealthier individuals are more willing to select a riskier occupation because if their income is low they still have abundant consumption from the wealth outside their business.

Implications

Even in the absence of borrowing constraints on capital ($\phi = 1$), the missing market for entrepreneurial risk will generate selection into entrepreneurship based on wealth, rather than solely on ability. This financial friction can therefore misallocate talent across occupations if low-ability wealthy individuals are becoming entrepreneurs at the expense of high-ability wealth-poor individuals.

B.3.2 Case 2: Constraint Binds

When the entrepreneur is constrained optimal k is given by:

$$k^c = \frac{e}{1-\phi} \tag{24}$$

The expression for optimal x when the borrowing constraint binds is identical to the unconstrained case (20).

Proposition 3: When constrained, borrowing constraints affect the choice of risk

$$\frac{\partial x^*}{\partial \phi} < 0 \tag{25}$$

Proof See section B.4.3.

Intuition

As constrained entrepreneurs are able to invest in more capital they reduce risk taking. They do so because risk and size are substitutes in this static model. Taking more risk x or investing in more capital k both increase income when the business risk is successful and reduce it when the business risk is unsuccessful.

Implications

This proposition demonstrates that borrowing constraints can interact with the missing market for entrepreneurial risk that generates heterogeneity in the riskiness of businesses. The potential for this interaction necessitates studying borrowing constraints and the missing market for entrepreneurial risk together in order to quantify the impact of either of these frictions.

In this static model, tighter borrowing constraints unambiguously increase the risk taking of entrepreneurs.

Proposition 4: Tighter borrowing constraints reduce the value of entrepreneurship for wealth-poor individuals

$$\frac{\partial V^E}{\partial \phi} > 0 \tag{26}$$

Proof See section B.4.4.

Intuition Constrained entrepreneurs invest in less capital than they would like to. As a consequence their entrepreneurial income is lower than if they were unconstrained and this reduces the value of being an entrepreneur for them.

Implications

Borrowing constraints can also generate misallocation of talent as low-ability wealthy individuals become entrepreneurs at the expense of high-ability wealth poor individuals.

B.4 Static Model Proofs

B.4.1 Proposition 1

Using the chain rule, we have

$$\frac{\partial x^*}{\partial e} = \left[\frac{\partial x^*}{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}\right]\left[\frac{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial e}\right] \tag{27}$$

The first term of the RHS of (27)

$$\frac{\partial x^*}{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)} = -\frac{h^E(1+\psi)\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}-1}}{\gamma\left[1+\psi\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}\right]^2} < 0 \tag{28}$$

i.e. the optimal risk taking x^* is decreasing in the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$.

The second term of the RHS of (27)

$$\frac{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial e} = \frac{u''(\underline{c})u'(\bar{c})\frac{\partial \underline{c}}{\partial e} - u''(\bar{c})u'(\underline{c})\frac{\partial \bar{c}}{\partial e}}{\left[u'(\bar{c})\right]^2} \tag{29}$$

with

$$\frac{\partial \underline{c}}{\partial e} = 1 + r^a + \left[\alpha k^{*\alpha-1}(h^E - x^*)^{1-\gamma} - 1 - r^a\right]\frac{\partial k^*}{\partial e} - (1-\gamma)(h^E - x^*)^{-\gamma}k^{*\alpha}\frac{\partial x^*}{\partial e} \tag{30}$$

$$\frac{\partial \bar{c}}{\partial e} = 1 + r^a + \left[\alpha k^{*\alpha-1}(h^E + \psi x^*)^{1-\gamma} - 1 - r^a\right]\frac{\partial k^*}{\partial e} + (1-\gamma)\psi(h^E + \psi x^*)^{-\gamma}k^{*\alpha}\frac{\partial x^*}{\partial e} \tag{31}$$

Using (30) and (31) in (29), we obtain

$$\begin{aligned}\frac{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial e} = \frac{u'(\underline{c})}{u'(\bar{c})}\Bigg\{&(1+r^a)\left(\frac{u''(\underline{c})}{u'(\underline{c})} - \frac{u''(\bar{c})}{u'(\bar{c})}\right)\\ &+ \frac{\partial k^*}{\partial e}\left[\left(\alpha k^{*\alpha-1}(h^E - x^*)^{1-\gamma} - 1 - r^a\right)\frac{u''(\underline{c})}{u'(\underline{c})} - \left(\alpha k^{*\alpha-1}(h^E + \psi x^*)^{1-\gamma} - 1 - r^a\right)\frac{u''(\bar{c})}{u'(\bar{c})}\right]\\ &+ \frac{\partial x^*}{\partial e}(1-\gamma)k^{*\alpha}\left[-(h^E - x^*)^{-\gamma}\frac{u''(\underline{c})}{u'(\underline{c})} - \psi(h^E + \psi x^*)^{-\gamma}\frac{u''(\bar{c})}{u'(\bar{c})}\right]\Bigg\}\end{aligned} \tag{32}$$

Since $\bar{c} > \underline{c}$, decreasing absolute risk aversion $\frac{\partial}{\partial c}\left(-\frac{u''(c)}{u'(c)}\right) < 0$ implies that:

$$\frac{u''(\underline{c})}{u'(\underline{c})} < \frac{u''(\bar{c})}{u'(\bar{c})} \tag{33}$$

As a consequence, the first term in the brace of (32) is strictly negative.
Diminishing marginal utility ($u''(c) < 0$) and (33) imply that the term multiplying $\frac{\partial k^*}{\partial e}$ on the second line is positive because it can be rewritten as:

$$(1+r^a)\left(\frac{u''(\bar{c})}{u'(\bar{c})} - \frac{u''(\underline{c})}{u'(\underline{c})}\right) + \left(\frac{u''(\underline{c})}{u'(\underline{c})}(h^E - x^*)^{1-\gamma} - \frac{u''(\bar{c})}{u'(\bar{c})}(h^E + \psi x^*)^{1-\gamma}\right)\alpha k^{*\alpha-1} > 0.$$

The term multiplying $\frac{\partial x^*}{\partial e}$ on the third line is also positive using the fact that $u''(c) < 0$.

On the other hand,

$$\begin{aligned}
\frac{\partial k^*}{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)} = {} & \frac{\alpha k^{*\alpha}}{(1-\alpha)(1+r^a)\left[p + (1-p)\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]^2}\Bigg\{(1-\gamma)\frac{\partial x^*}{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}\times \\
& \times\left[\psi p(h^E + \psi x^*)^{-\gamma} - (1-p)(h^E - x^*)^{-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]\left[p + (1-p)\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right] \\
& - p(1-p)\Big((h^E + \psi x^*)^{1-\gamma} - (h^E - x^*)^{1-\gamma}\Big)\Bigg\}
\end{aligned} \tag{34}$$

The first term on the second line of (34) is zero.

$$\psi p(h^E + \psi x^*)^{-\gamma} - (1-p)(h^E - x^*)^{-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right) = 0 \tag{35}$$

To show this, we have:

$$\begin{aligned}
(20) \iff & x^*\left(1 + \psi\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}\right) = h^E\left(1 - \left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}\right) \\
\iff & x^* + \psi x^*\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}} = h^E - h^E\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}} \\
\iff & (h^E + \psi x^*)\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}} = h^E - x^*
\end{aligned}$$

$$\iff (h^E + \psi x^*)(\psi p)^{-\frac{1}{\gamma}} = (h^E - x^*)\,(1-p)^{-\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{-\frac{1}{\gamma}}$$
$$\iff \psi p(h^E + \psi x^*)^{-\gamma} = (1-p)(h^E - x^*)^{-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)$$
$$\iff \psi p(h^E + \psi x^*)^{-\gamma} - (1-p)(h^E - x^*)^{-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right) = 0$$

Hence,

$$\frac{\partial k^*}{\partial \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)} = -\frac{\alpha p(1-p)\Big((h^E + \psi x^*)^{1-\gamma} - (h^E - x^*)^{1-\gamma}\Big) k^{*\alpha}}{(1-\alpha)(1+r^a)\left[p + (1-p)\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]^2} < 0 \tag{36}$$

By contradiction, suppose that

$$\frac{\partial \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial e} \geq 0$$

This implies, using (28) and (36), that

$$\frac{\partial x^*}{\partial e} = \left[\frac{\partial x^*}{\partial \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}\right] \left[\frac{\partial \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial e}\right] \leq 0 \tag{37}$$

and

$$\frac{\partial k^*}{\partial e} = \left[\frac{\partial k^*}{\partial \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}\right] \left[\frac{\partial \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial e}\right] \leq 0 \tag{38}$$

This implies from (32) that all three terms are weakly negative with the first term strictly negative. This is a contradiction, so it must be that:

$$\frac{\partial \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial e} < 0$$

Hence,

$$\frac{\partial x^*}{\partial e} = \left[\frac{\partial x^*}{\partial \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}\right] \left[\frac{\partial \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial e}\right] > 0 \tag{39}$$

B.4.2 Proposition 2

From (18), we have

$$\frac{\partial V^W(e)}{\partial e} = (1+r^a)u'(w + (1+r^a)e) > 0$$

and

$$\frac{\partial^2 V^W(e)}{\partial e^2} = (1+r^a)^2 u''(w + (1+r^a)e) < 0$$

Applying the envelope theorem to (19), we have

$$\frac{\partial V^E(h^E, e)}{\partial e} = (1+r^a)\big[pu'(\bar{c}) + (1-p)u'(\underline{c})\big] > 0$$

and

$$\frac{\partial^2 V^E(h^E, e)}{\partial e} = (1+r^a)^2\big[pu''(\bar{c}) + (1-p)u''(\underline{c})\big] < 0$$

Thus, both V^W and V^E are strictly increasing and strictly concave in wealth e.

Define $y^{CE}(e)$ as the certainty-equivalent income of the entrepreneur, which is implicitly given by:

$$\begin{aligned} u(y^{CE}(e) + (1+r^a)e) = pu\left[(h^E + \psi x^*)^{1-\gamma} k^{*\alpha} - (1+r^a)k^* + (1+r^a)e\right] \\ + (1-p)u\left[(h^E - x^*)^{1-\gamma} k^{*\alpha} - (1+r^a)k^* + (1+r^a)e\right] \end{aligned}$$

For any agent on the margin between choosing to be a worker or an entrepreneur, we have:

$$V^W(e) = V^E(h^E, e)$$

Therefore for the marginal agent, the wage is equal to the certainty-equivalent amount of income ($w = y^{CE}(e)$) because

$$\begin{aligned} u(w + (1+r^a)e) = u(y^{CE}(e) + (1+r^a)e) = pu\left[(h^E + \psi x^*)^{1-\gamma} k^{*\alpha} - (1+r^a)k^* + (1+r^a)e\right] \\ + (1-p)u\left[(h^E - x^*)^{1-\gamma} k^{*\alpha} - (1+r^a)k^* + (1+r^a)e\right] \end{aligned}$$

Then, for any wage rate less than the certainty equivalent income $w < y^{CE}(e)$, the individual will choose to be an entrepreneur, and for any wage rate greater than the certainty equivalent income $w < y^{CE}(e)$, the individual will choose to be a worker. Given decreasing absolute risk aversion, certainty equivalent income strictly decreases with endowed wealth (e).
Since V^W and V^E are strictly increasing and strictly concave in e, then there must be a single crossing point between V^W and V^E.
Thus, for a marginal agent, we have:

$$\frac{\partial V^E}{\partial e} > \frac{\partial V^W}{\partial e}\bigg|_{V^E = V^W}$$

B.4.3 Proposition 3

We have from (28) that

$$\frac{\partial x^*}{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)} < 0$$

Also,

$$\frac{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial\phi} = \frac{u''(\underline{c})u'(\bar{c})\frac{\partial\underline{c}}{\partial\phi} - u''(\bar{c})u'(\underline{c})\frac{\partial\bar{c}}{\partial\phi}}{[u'(\bar{c})]^2} \tag{40}$$

with

$$\frac{\partial\underline{c}}{\partial\phi} = \left[\alpha k^{*\alpha-1}(h^E - x^*)^{1-\gamma} - 1 - r^a\right]\frac{\partial k^*}{\partial\phi} - (1-\gamma)(h^E - x^*)^{-\gamma}k^{*\alpha}\frac{\partial x^*}{\partial\phi} \tag{41}$$

$$\frac{\partial\bar{c}}{\partial\phi} = \left[\alpha k^{*\alpha-1}(h^E + \psi x^*)^{1-\gamma} - 1 - r^a\right]\frac{\partial k^*}{\partial\phi} + (1-\gamma)\psi(h^E + \psi x^*)^{-\gamma}k^{*\alpha}\frac{\partial x^*}{\partial\phi} \tag{42}$$

Hence,

$$\begin{aligned}\frac{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial\phi} = \frac{u'(\underline{c})}{u'(\bar{c})}\Bigg\{&\frac{\partial k^*}{\partial\phi}\left[\left(\alpha k^{*\alpha-1}(h^E - x^*)^{1-\gamma} - 1 - r^a\right)\frac{u''(\underline{c})}{u'(\underline{c})} - \left(\alpha k^{*\alpha-1}(h^E + \psi x^*)^{1-\gamma} - 1 - r^a\right)\frac{u''(\bar{c})}{u'(\bar{c})}\right]\\ &+ \frac{\partial x^*}{\partial\phi}(1-\gamma)k^{*\alpha}\left[-(h^E - x^*)^{-\gamma}\frac{u''(\underline{c})}{u'(\underline{c})} - \psi(h^E + \psi x^*)^{-\gamma}\frac{u''(\bar{c})}{u'(\bar{c})}\right]\Bigg\}\end{aligned} \tag{43}$$

But,

$$\frac{\partial k^*}{\partial\phi} = \frac{e}{(1-\phi)^2} > 0 \tag{44}$$

Diminishing marginal utility ($u''(c) < 0$) and (33) imply that the term multiplying $\frac{\partial k^*}{\partial e}$ is positive because it can be rewritten as:

$$(1 + r^a)\left(\frac{u''(\bar{c})}{u'(\bar{c})} - \frac{u''(\underline{c})}{u'(\underline{c})}\right) + \left(\frac{u''(\underline{c})}{u'(\underline{c})}(h^E - x^*)^{1-\gamma} - \frac{u''(\bar{c})}{u'(\bar{c})}(h^E + \psi x^*)^{1-\gamma}\right)\alpha k^{*\alpha-1} > 0.$$

The second term of (43) is also positive since $u''(c) < 0$.
Therefore,

$$\frac{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial\phi} > 0$$

Hence,

$$\frac{\partial x^*}{\partial\phi} = \left[\frac{\partial x^*}{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}\right]\left[\frac{\partial\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)}{\partial\phi}\right] < 0 \tag{45}$$

B.4.4 Proposition 4

$$\begin{aligned}
\frac{\partial V^E}{\partial \phi} &= pu'(\bar{c})\frac{\partial \bar{c}}{\partial \phi} + (1-p)u'(\underline{c})\frac{\partial \underline{c}}{\partial \phi} \\
&= pu'(\bar{c})\left\{\left[\alpha k^{*\alpha-1}(h^E+\psi x^*)^{1-\gamma} - 1 - r^a\right]\frac{\partial k^*}{\partial \phi} + (1-\gamma)\psi(h^E+\psi x^*)^{-\gamma}k^{*\alpha}\frac{\partial x^*}{\partial \phi}\right\} \\
&\quad + (1-p)u'(\underline{c})\left\{\left[\alpha k^{*\alpha-1}(h^E-x^*)^{1-\gamma} - 1 - r^a\right]\frac{\partial k^*}{\partial \phi} - (1-\gamma)(h^E-x^*)^{-\gamma}k^{*\alpha}\frac{\partial x^*}{\partial \phi}\right\} \\
&= \frac{\partial k^*}{\partial \phi}\Big\{pu'(\bar{c})\left[\alpha k^{*\alpha-1}(h^E+\psi x^*)^{1-\gamma} - 1 - r^a\right] + (1-p)u'(\underline{c})\left[\alpha k^{*\alpha-1}(h^E-x^*)^{1-\gamma} - 1 - r^a\right]\Big\} \\
&\quad + (1-\gamma)u'(\bar{c})k^{*\alpha}\frac{\partial x^*}{\partial \phi}\left\{p\psi(h^E+\psi x^*)^{-\gamma} - (1-p)(h^E-x^*)^{-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right\} \qquad (46)
\end{aligned}$$

The last term of the last equality is equal to zero (see (35) for more details).

Hence,

$$\begin{aligned}
\frac{\partial V^E}{\partial \phi} = \frac{\partial k^*}{\partial \phi}\Big\{&pu'(\bar{c})\left[\alpha k^{*\alpha-1}(h^E+\psi x^*)^{1-\gamma} - 1 - r^a\right] \\
&+ (1-p)u'(\underline{c})\left[\alpha k^{*\alpha-1}(h^E-x^*)^{1-\gamma} - 1 - r^a\right]\Big\} > 0 \qquad (47)
\end{aligned}$$

because $\frac{\partial k^*}{\partial \phi} = \frac{e}{(1-\phi)^2} > 0$ and the term in the braces is positive.

C Computational Details

I solve the value functions $\{V^W, V^{NE}, V^E\}$, occupational choices, and the set of policy functions $\{c^W, a'^W, \tilde{a}^{NE}, \tilde{k}^{NE}, x^{NE}, c^E, a'^E, k'^E, n^E\}$ to maximize the problems given by 5, 6 and 7. To do so, I adapt an algorithm from Dyrda and Pugsley (2018). To aid the computational tractability of the problem, I solve for the decision rules on a grid of resources r, rather than on both a and k. I use 125 grid points for r in order to solve the decision rules. For the entrepreneurial ability, worker ability, and business productive, I use three grid points each. With two different risky business types, there are then $54 = 3 \times 3 \times 3 \times 2$ different relative abilities between entrepreneurship and being a worker.

I solve first for the value functions, conditional on which occupation will be chosen at the beginning of the next period, and then maximize over these occupational choices. Given a wage w, I solve for the decision rules by:

- Initial Guess:

- Set $i = 0$,
- Guess $c_i(\cdot)$ is a constant fraction of total resources
- Guess $r_i'(\cdot) = r$
- Guess $V_i(\cdot) = \frac{c_i^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} / (1 - \psi\beta)$

• Begin Value Function loop

- $i = i + 1$
- Calculate the derivatives of each value function $V_i^E(\cdot), V_i^{NE}(\cdot), V_i^W(\cdot)$ w.r.t. total resources r
- Use the envelope condition to update current consumption:

$$c_i^E = \left(\frac{\partial V_i^E(\cdot)}{\partial r}\right)^\theta$$

$$c_i^{NE} = \left(\frac{\partial V_i^{NE}(\cdot)}{\partial r}\right)^\theta$$

$$c_i^W = \left(\frac{\partial V_i^W(\cdot)}{\partial r}\right)^\theta$$

- For entrepreneurs and new entrepreneurs, calculate the maximum possible amount of capital ($k'_{\max}$) the entrepreneur can invest in given their resources r, their consumption, and the borrowing constraint.[13]
- Evaluate the difference between the FOC w.r.t. k' and w.r.t a'
  * If $\frac{\partial V(\cdot)}{\partial k'} - \frac{\partial V(\cdot)}{\partial a'}|_{k'=k'_{\max}} > 0$ then set $k_i' = k'_{\max}$
  * Else, bisect over $k \in \{0, k'_{\max}\}$, using the differences between the FOC w.r.t. k' and the FOC w.r.t. a':
  $$\frac{\partial V(\cdot)}{\partial k'} - \frac{\partial V(\cdot)}{\partial a'}$$
- Note that using the difference between the first order conditions between the two assets effectively differences out the marginal utility of current consumption. As a consequence, the consumption-savings problem is separated from the portfolio allocation problem in a way that aids tractability.
- Use the budget constraint to determine the corresponding a'

$$a' = r - c_i - k'_{i,j}$$

[13] Note that in the relaxed borrowing constraints equilibrium, where $\phi = 1$, there is still a well-defined maximum amount of capital the entrepreneur can invest in. It is determined by a non-negativity constraint on future consumption, and is given by $a' \geq -\frac{1-\delta}{\chi(1+r^a)}k'$.

– For workers, their savings can be directly backed out by the budget constraint

$$a'^W = r - c_i^W$$

– Update conditional value functions

– Calculate the distances between $V_i^E(\cdot)$ and $V_{i-1}^E(\cdot)$, $V_i^{NE}(\cdot)$ and $V_{i-1}^{NE}(\cdot)$ and between $V_i^W(\cdot)$ and $V_{i-1}^W(\cdot)$

– If the maximum distance is less than tolerance, end loop, else return to beginning of Value Function loop

With the decision rules solved, I then simulate a unit mass of agents on a discrete grid of $r \times h^E \times h^W \times z \times x$. To approximate the distribution, I use 2,000 grid points for r. I guess a uniform distribution over this state space and then iterate until the distribution converges. I use Brent search to determine the wage that clears the labour market in this economy. I calculate moments using the method outlined in Ocampo and Robinson (2022).

D Quantitative Robustness

In this section, I study the impact of the missing market for entrepreneurial risk in the absence of the endogenous choice of risk. Here I assume that all entrepreneurs have access to only one type of risky entrepreneurial project ($n_x = 1$). I recalibrate the model to the same set of moments. However, given the single project choice, I do not distinguish between high and low investment projects, as in section 4. Instead, I compute the same statistics for the total population of firms in the Kauffman Firm Survey. Table 12 reports the calibrated parameter values alongside the targeted moments.

I then perform the same analysis as in section 5, but on the economy with a single entrepreneurial project type. Table 13 reports the steady-states of four economies. The benchmark 1-project economy with both financial frictions, a complete markets economy where state-contingent assets complete the missing market for risk but entrepreneurs still face borrowing constraints ($\phi < 1$), the relaxed borrowing constraints economy where entrepreneurs can borrow the full value of their capital ($\phi = 1$) but there are no state-contingent assets, and finally a neither friction economy with both a complete market for risk and unrestricted capital investment ($\phi = 1$).

The gains to aggregate productivity or output from removing either financial friction are smaller in the absence of the endogenous choice of risky projects. Completing the missing market for risk increases aggregate productivity by 5.3% with an endogenous choice of risk,

Table 12: 1-Project Calibration

Target	Data	Model	Parameter	Value
Exogenously Calibrated				
40 Year Working Lifespan			$1-\psi$	0.03
Capital Share			α	0.33
Interest Rate			r^A	0.04
Entrepreneurial Share			$1-\gamma$	0.12
Depreciation			δ	0.06
Discount Factor			β	0.96
Coefficient of Relative Risk Aversion			σ	1.50
Average Labour Ability			μ_{h^W}	1.00
Dispersion of Labour Ability			σ_{h^W}	0.20
Persistence of Labour Ability			ρ_{h^W}	0.90
Endogenously Calibrated				
Ratio of Average Debt to Average Equity	1.35	1.35	ϕ	0.79
Proportion of Entrepreneurs	0.11	0.13	χ	1.09
Wealth Ratio of Entrepreneurs to Workers	7.14	7.46	μ_{h^E}	-1.13
Wealth Gini of Entrepreneurs	0.75	0.52	σ_{h^E}	1.12
Wealth Gini	0.79	0.80	ρ_{h^E}	0.96
Proportion of Entrepreneurs in Wealthiest 1%	0.74	0.79	$\mu_{z\|x_1}$	0.97
Autocorrelation of Employment	0.61	0.69	$\sigma_{z\|x}$	1.45
Survival Rate	0.54	0.38	$\rho_{z\|x}$	0.72

and 2.4% without the endogenous choice of risk. Likewise, relaxing the borrowing constraints increases aggregate productivity by 2.9% with the endogenous choice of risk, or 0.5% without. Still in both cases, the aggregate productivity gains from completing the missing market for risk are higher than from relaxing the borrowing constraints.

Even in the absence of an endogenous choice of risky projects, the missing market for entrepreneurial risk significantly distorts entrepreneurial decisions. It does so primarily through discouraging capital investment by unconstrained entrepreneurs, but also through the extensive margin, where it discourages entry into entrepreneurship.

Note that the decrease in the capital stock that arises from relaxing the borrowing con-

Table 13: Change in Aggregate Outcomes

	Output	Wages	Capital	Productivity	Workers	Entrepreneurs
Benchmark Economy	0.0	0.0	0.0	0.0	87.2	12.8
Complete Markets	1.2	3.9	1.3	2.4	84.3	15.7
Relaxed Borrowing Constraints	0.4	0.5	-0.3	0.5	86.7	13.3
Neither Friction	2.1	4.5	4.8	2.1	84.8	15.2

straints in table 13 is due to general equilibrium effects. In partial equilibrium, as the borrowing constraints are relaxed, entrepreneurs are able to invest more and more wealth-poor high-ability individuals enter entrepreneurship. The capital invested rises substantially. Of course with more capital invested, labour demand is much higher, and so wages must rise to clear the labour market. As wages rise, less capital is invested.