

Risky Entrepreneurship and Wealth

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Abstract

This paper studies how the uninsurable nature of entrepreneurial risk reduces entrepreneurial activity and affects aggregate output, productivity, and the distribution of wealth. I model the occupational choice of individuals who can choose to become workers or entrepreneurs. Individuals that choose to be entrepreneurs also choose how risky a business to start, with higher-risk businesses leading to higher expected productivity. The model features two distinct financial frictions. First, a missing market for entrepreneurial risk prevents entrepreneurs from insuring themselves against entrepreneurial income risk and the risk of business failure. Second, the size of an entrepreneur's business is limited by borrowing constraints, which have been extensively studied in the previous literature. I calibrate the strength of these two financial frictions using micro data on new U.S. firms from the Kauffman Firm Survey. I find that completing the missing market for entrepreneurial risk improves aggregate productivity by 9%, which is more than twice as much as relaxing borrowing constraints would. I also find that completing the missing market for entrepreneurial risk reduces the share of wealth held by the wealthiest 1% by two thirds. In a policy experiment, I show that a partial insurance scheme for unsuccessful entrepreneurs increases aggregate productivity as it encourages entrepreneurs to start riskier businesses.

Keywords: Entrepreneurship, Risk-Taking, Financial Frictions, Aggregate Productivity, Wealth Inequality

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1 Introduction

Entrepreneurs play a crucial role in determining aggregate output, productivity, and the distribution of wealth by actively managing and owning private businesses. Decisions that entrepreneurs make directly affect their businesses' output and productivity (Smith et al., 2019). These decisions have aggregate implications, as Asker et al. (2015) estimate that privately held firms account for 69% of private-sector employment and 59% of aggregate sales. The ownership of these private businesses help make entrepreneurs disproportionately wealthy. In the Survey of Consumer Finances only 11% of households contain at least one entrepreneur, but these households own 40% of U.S. wealth and constitute 65% of the wealthiest 1% of households. Understanding how entrepreneurs decide to start, run, and growth their businesses is therefore vital for understanding these aggregate economic outcomes.

In this paper, I study the quantitative importance of two distinct financial frictions, a missing market for entrepreneurial risk and borrowing constraints, for aggregate output, productivity, and the distribution of wealth. I examine how these financial frictions distort individuals' decisions to start new businesses, their decisions to pursue riskier or safer business ideas, and their investment decisions. I then ask how these distorted decisions impact aggregate economic outcomes.

This paper makes two main contributions. First, I measure the losses to aggregate productivity and output from the missing market for risk in the U.S. economy. A large literature has studied how financial frictions distort entrepreneurial decisions and the impacts on aggregate productivity and output. While previous work has focused on borrowing constraints, little attention has been paid to how a missing market for risk can distort entrepreneurial decisions. My paper fills this gap and explores the interaction between this missing market and borrowing constraints. I find that the missing market for entrepreneurial risk is twice as important as borrowing constraints for aggregate productivity and that the interaction effect between these two financial frictions is 40% of the sum of their individual impacts on aggregate output.

Second, I study how the missing market for entrepreneurial risk contributes to wealth inequality. I build on quantitative work by Cagetti and De Nardi (2006) who show that a model of entrepreneurship with both borrowing constraints and a luxury bequest motive can generate the high concentration of wealth observed in U.S. data. My paper uses a similar calibration strategy to match the wealth distribution, but does so incorporating both borrowing constraints and a missing market for entrepreneurial risk. I find that removing the two financial frictions have opposing effects on wealth inequality. Completing the missing market for entrepreneurial risk reduces wealth inequality, while relaxing the borrowing constraints increases wealth inequality.

To study the aggregate effects of these two financial frictions, I build a dynamic equi-

librium model of entrepreneurship. Individuals have two types of ability, entrepreneurial and worker. Each period, they choose to be an entrepreneur or a worker. Individuals that choose to become an entrepreneur also decide how risky a business to start, with higher risk businesses resulting in higher expected productivity. A missing market for risk prevents individuals from insuring against shocks to their abilities or their businesses' productivity. Entrepreneurs also face borrowing constraints that limit the amount of capital they can invest in. Wealth helps individuals overcome both of these financial frictions, as it allows them to self-insure against the risks they face and self-finance a large capital stock. As a consequence, wealth plays an important role in an individual's decision to become an entrepreneur and in their decision of how risky a business to start.

I use micro data from the Kauffman Firm Survey, a panel of 4,928 new U.S. firms, to quantify the importance of the two financial frictions. The model predicts that wealthier individuals should both invest more in their businesses and start riskier businesses. I therefore split the sample of firms into a low-investment and high-investment group in both the data and in a simulated panel of new firms. To discipline the nature of entrepreneurial risk in the model, I separately match the survival rates and employment dynamics of these two groups of firms from the model to the data. To discipline the strength of borrowing constraints I match the ratio of average debt to average equity across all firms. Since wealth allows individuals to partially overcome the financial frictions they face, it's vital for the quantitative analysis that the model also matches the distribution of wealth that we observe in the data. Therefore, I also match moments from the model's simulated cross-sectional distribution of wealth to micro data on wealth from the Survey of Consumer Finances.

In the calibrated model, both financial frictions play an important role distorting individual choices. The missing market for risk discourages entrepreneurs from choosing high risk businesses and discourages individuals from becoming entrepreneurs. If entrepreneurs had access to complete insurance markets they would all choose to start a business with the highest expected productivity and then fully insure themselves. Faced with the missing market, some entrepreneurs will choose lower expected productivity businesses because these businesses have a lower probability of failure and generate more certain income. Similarly, because entrepreneurial income is more volatile than worker income, some individuals choose to be workers, even though their expected income as an entrepreneur is higher.

Borrowing constraints limit the size of many entrepreneur's businesses and discourages individuals from becoming entrepreneurs. In the absence of borrowing constraints, a wealth-poor individual with high entrepreneurial ability might rent a large stock of capital in order to operate a large scale business. Borrowing constraints prevent wealth-poor entrepreneurs from renting as much capital as they would like, forcing them to operate inefficiently small businesses. Smaller businesses reduce the total income an individual

can earn as an entrepreneur, and may reduce it below what they can earn as a worker. As a consequence, some wealth-poor individuals choose to be workers though they would prefer to be entrepreneurs if they could operate a larger business.

The main quantitative analysis I perform is to remove each financial friction and compare the resulting steady state equilibria. I first complete the missing market for risk by introducing a full set of state-contingent assets. Each individual is able to buy or sell securities at actuarially fair prices that pay off based on their individual abilities and their businesses' productivity. I then relax the borrowing constraints, allowing entrepreneurs to invest in any amount of capital regardless of their personal net-worth. Finally, I simultaneously complete the missing market for risk and relax the borrowing constraints to study the interaction between the two financial frictions.

I find that completing the missing market for entrepreneurial risk improves aggregate productivity by 9% and aggregate output by 8%. The state-contingent assets allow individuals to transfer resources from the state of the world where a business is successful to the state of the world where their business is unsuccessful. As a direct consequence, all entrepreneurs choose to run the business with the highest expected productivity and use the state-contingent assets to insure themselves against the risks associated with this.

Relaxing borrowing constraints increases aggregate productivity by 4% and aggregate output by 7%. Entrepreneurs that were previously constrained to operate inefficiently small businesses are now able to invest in much more capital. Some wealth-poor individuals with high entrepreneurial ability switch from being workers to being entrepreneurs. Without the constraint, they can now generate more income as entrepreneurs because they can run larger businesses. As a consequence of the increased investment, the aggregate capital stock increases by 10% and so the response of aggregate output is larger than the response of aggregate productivity.

When I simultaneously complete the missing market for risk and relax the borrowing constraint, aggregate productivity increases by 13% and aggregate output increases by 21%. As before, completing the missing market for risk means that all entrepreneurs will start businesses with the highest expected productivity. Now that these entrepreneurs are unconstrained they can also invest in far more capital. Relative to the benchmark economy with both financial frictions, the aggregate capital stock increases by 33%. The interaction between the financial frictions means that aggregate output increases by 6 percentage points more than the sum of the increases from removing either friction separately ($8\% + 7\% = 15\% < 21\%$).

The financial frictions also impact the distribution of wealth in the economy. Completing the missing market for entrepreneurial risk reduces wealth inequality. The wealth Gini falls from 0.83 to 0.77 and the share of aggregate wealth held by the wealthiest 1% decreases from 28% to 9%. This decrease results because in the benchmark economy, wealthier entrepreneurs choose to start higher risk businesses. These high-risk businesses

earn higher expected returns on average, leading to faster wealth accumulation by the wealthy entrepreneurs. When all entrepreneurs are able to insure against business risk, they all choose to start the highest expected productivity businesses, removing this difference in the rates of return.

By contrast, relaxing borrowing constraints increases wealth inequality, except at the very top. The wealth Gini rises from 0.83 to 0.87, even though the share of aggregate wealth held by the wealthiest 1% decreases from 28% down to 25%. The lack of borrowing constraints allow entrepreneurs to increase their leverage. More leveraged entrepreneurs earn higher profits when they receive good productivity shocks and lose more when they receive negative productivity shocks. The overall increase in wealth inequality is driven by increases in wealth inequality between entrepreneurs.

Completing the missing market for risk and relaxing borrowing constraints at the same time reduces wealth inequality. The wealth Gini falls from 0.83 to 0.68 and the share of aggregate wealth held by the wealthiest 1% decreases from 28% to 6%. Wealth inequality between entrepreneurs falls as all entrepreneurs choose to start the highest expected productivity businesses possible. In addition, the difference between average entrepreneurial wealth and average worker wealth declines by almost 50%. This occurs both because wages rise substantially, and because the lack of both financial frictions substantially reduces entrepreneur's incentives to save.

Finally, I demonstrate that these quantitative results have implications for public policy. Despite the presence of adverse selection, I show that a government can increase aggregate productivity if they provide a simple unemployment benefit to unsuccessful entrepreneurs. This benefit is successful because it encourages entrepreneurs to start riskier businesses, by partially insuring them against business failure. Adverse selection is a potential problem in the economy if low entrepreneurial-ability individuals choose to become entrepreneurs, not because they expect to make substantial income as entrepreneurs, but because the benefits are larger than the wages they would earn in the labour market. I find that at moderate levels of the unemployment benefit many entrepreneurs start higher risk businesses boosting aggregate productivity without inducing many low-ability individuals to become entrepreneurs. At higher levels of the benefit, adverse selection overwhelms the positive benefits of higher risk taking and aggregate output decreases.

In section 2, I illustrate the two distinct financial frictions studied in this paper within a static model. Section 3 provides evidence on entrepreneurial risk taking and funding sources from the Kauffman Firm Survey that motivates the modelling choices in this paper. I extend the static model into a dynamic model in section 4. Section 5 explains how I quantify the strength of the two financial frictions using the micro data. I report the results of the quantitative analysis where I remove the two financial frictions in section 7. Section 8 studies the effects of a simple government policy that provides partial insurance to entrepreneurs. Section 9 concludes.

Related Literature

My paper contributes to a large literature studying how financial frictions distort entrepreneurial decisions and the impacts on aggregate productivity and output. My paper studies how the missing market for risk, a previously understudied financial friction distorts entrepreneurial decisions. Previous work in this area has focused almost exclusively on borrowing constraints. For example, papers such as [Buera \(2009\)](#), [Buera et al. \(2011\)](#), [Moll \(2014\)](#), [Midrigan and Xu \(2014\)](#), and [Castro and Ševčík \(2017\)](#), study how borrowing constraints impact entrepreneurial decisions, including whether to start a business, which sector to start businesses in, whether to adopt more capital-intensive technology, or how much to invest in human capital.

My paper also contributes to a literature studying the determinants of the distribution of wealth. Two recent surveys of this literature, [De Nardi and Fella \(2017\)](#) and [Benhabib and Bisin \(2018\)](#), both consider differences in the earned rates of return to be key drivers of the high concentration of wealth among the wealthiest. In my model, entrepreneur's choice of business risk generates a previously unstudied channel for explaining the high concentration of wealth. Wealthier entrepreneurs choose higher risk businesses that on average earn them higher returns. Empirical evidence from Scandinavian countries, [Bach et al. \(2018\)](#) and [Fagereng et al. \(2018\)](#), shows that wealthier individuals do in fact earn persistently higher rates of return on their assets.

This paper is closely related to three papers that study entrepreneur's choice of risk. First, [Choi \(2017\)](#) uses U.S. Census Bureau data to provide evidence that individuals who had higher paying jobs prior to starting a business take larger risks, as measured by higher exit rates, more dispersion in growth, and faster average growth conditional on survival. In a quantitative model, he demonstrates the importance of entrepreneur's labour market options for encouraging entrepreneurial risk taking. By contrast, my paper considers how wealth, rather than an entrepreneur's labour market opportunities, encourages risk taking and measures how aggregate productivity and output would change with complete insurance markets. My paper is also distinct from [Choi \(2017\)](#) in that I study the implications of entrepreneurial risk choice for the distribution of wealth.

Second, [Vereshchagina and Hopenhayn \(2009\)](#) study how wealth impacts both the choice to become an entrepreneur and the choice of business risk. In their model, wealth-poor entrepreneurs start riskier businesses due to a non-concavity in the value function created by the choice to be a worker. I find the opposite result with wealth-poor individuals starting less risky businesses. The key difference between these models is that in my model higher risk businesses deliver higher expected productivity while in the [Vereshchagina and Hopenhayn \(2009\)](#) model all businesses have the same expected productivity.

Third, [Gabler and Poschke \(2013\)](#) studies how risk-taking impacts aggregate productivity. They consider a framework where distortions to allocative efficiency discourage

risk-neutral firms from investing in risky experimentation that may lead to productivity growth. My framework differs by considering how risk-averse entrepreneurs facing a missing market for risk choose how risky a business to start, and how much completing that missing market would improve aggregate productivity and output.

More broadly, my paper is related to a large literature on the decision to become an entrepreneur. In my model individuals decide to become an entrepreneur based on their abilities and two financial frictions. These two financial frictions correspond to two different strands of literature studying the decision to start a business. First, papers such as [Kihlstrom and Laffont \(1979\)](#) and [Cressy \(2000\)](#) argue that uninsurable entrepreneurial risk is a major determinant of the decision to become an entrepreneur, either because of heterogeneity in risk-preferences or because wealth makes individuals more willing to take risks. Second, a literature starting with [Evans and Jovanovic \(1989\)](#) considers how borrowing constraints will influence the decision to become an entrepreneur. See also [Quadrini \(1999\)](#), [Gentry and Hubbard \(2004\)](#), and [Hurst and Lusardi \(2004\)](#). My paper incorporates both of these mechanisms of occupational selection and explores how they interact.

Empirical work documenting that entrepreneurs face a high degree of uninsurable idiosyncratic risk motivate the focus in my paper on the missing market for entrepreneurial risk. Entrepreneurship is risky both because of the risk of business failure and the volatile nature of entrepreneurial income. [Fairlie et al. \(2018\)](#) documents that only 20 percent of new U.S. businesses survive for five years. [DeBacker et al. \(2018\)](#) shows that entrepreneurial earnings are far more volatile than employment earnings, even when conditioning on survival. More generally, [Castro et al. \(2015\)](#) measures that plant-level idiosyncratic shocks are far more important for firms than aggregate shocks for U.S. manufacturing plants. [Panousi and Papanikolaou \(2012\)](#) provides evidence that in publicly traded firms, some measure of the firm’s idiosyncratic risk is chosen by the managers.

Finally, this paper is related to a large literature on government policy for entrepreneurs. In particular, this paper’s policy analysis complements the empirical results in [Hombert et al. \(2014\)](#), who study a reform in France that extended unemployment insurance to self-employed individuals. They find that the reform led to more new businesses that had higher productivity than incumbents. Both [Olds \(2016\)](#) and [Gottlieb et al. \(2018\)](#) provide empirical evidence that additional insurance increases self-employment. Two papers, [Bianchi and Bobba \(2013\)](#) in a Mexican context and [Karlan et al. \(2014\)](#) in Ghanaian context, both argue that providing people with better insurance mechanisms can increase self-employment or investment more effectively than relaxing borrowing constraints.

Quantitative work on government policy for entrepreneurs tends to focus on a single financial friction. [Meh \(2005\)](#), [Bruggemann \(2017\)](#), and [Guvenen et al. \(2019\)](#) all study the effects of different government taxes on entrepreneurs given the existence of binding borrowing constraints. By contrast, [Panousi and Reis \(2016\)](#) considers optimal taxation

with uninsurable capital income risk. The quantitative results in my paper suggests that entrepreneurial policies should not ignore either of these frictions, as the missing market for risk, borrowing constraints, and the interaction between the two are all quantitatively important.

2 Static Model

In this section, I build a simple static model to illustrate the two financial frictions explored in this paper and analytically characterize how wealth allows individuals to overcome these frictions.

2.1 Environment

Agents live for a single period and have preferences over consumption $u(c)$, where $u(\cdot)$ exhibits decreasing absolute risk aversion¹.

Agents are initially endowed with wealth $e \in [0, \bar{e}]$ and a known entrepreneurial ability $h^E \in [\underline{h}^E, \bar{h}^E]$. At the beginning of the period, agents must choose to be an entrepreneur or a worker based on their wealth and ability (e, h^E) .

All agents have identical productivity as a worker. If they choose to be a worker, they will earn a wage of w with certainty.

If an agent chooses to be an entrepreneur, they must choose the riskiness of their business $x \in [0, h^E]$. Choosing a riskier business will result in higher expected productivity at the cost of more dispersed productivity. With probability p the entrepreneur's risk is successful, and their productivity is boosted by ψx . With the complementary probability $(1 - p)$ the risk is unsuccessful and their productivity is reduced by x . The entrepreneur's realized productivity is therefore given by z :

$$z = \begin{cases} h^E + \psi x & \text{with probability } p \\ h^E - x & \text{with probability } 1 - p \end{cases} \quad (1)$$

where $\psi > \frac{1-p}{p}$, so that the expected value of z increases with x .

There are two assets, capital (k) and a risk-free financial asset (a). At the beginning of the period, agents choose how much of their initial endowment (e) to invest in each asset:

¹Decreasing absolute risk aversion is defined by:

$$\frac{\partial}{\partial c} \left(-\frac{u''(c)}{u'(c)} \right) = -\frac{u'(c)u'''(c) - [u''(c)]^2}{[u'(c)]^2} < 0$$

Note that many common utility functions such as log and CRRA exhibit decreasing absolute risk aversion.

$$e = a + k \quad (2)$$

Agents can only hold a positive amount of capital, and potentially face a borrowing constraint in the financial asset. The borrowing constraint is given by:

$$a \geq -\phi k \quad (3)$$

where $\phi \in [0, 1]$. When $\phi = 0$, entrepreneurs are not able to borrow in the financial asset. When $\phi = 1$ entrepreneurs are able to invest in any amount of capital k regardless of their initial endowment of wealth. For values of $\phi < 1$, this model exhibits borrowing constraints, which prevent entrepreneurs from investing in a level of capital stock that exceeds a multiple $\left(\frac{1}{1-\phi}\right)$ of their endowed wealth e .

The financial asset a pays off $(1 + r^a)$ units in all states of the world. Without any access to state-contingent assets, this model exhibits a missing market for entrepreneurial risk as entrepreneurs are unable to insure themselves against the risks to their realized productivity z .

An entrepreneur with a realized productivity z and a capital stock k will produce output according to:

$$y = z^{1-\gamma} k^\alpha \quad (4)$$

2.2 Agent's Problem

Depending on their initial endowments of wealth (e) and entrepreneurial productivity (z), agents make an occupational choice decision between being workers and entrepreneurs according to:

$$V(h^E, e) = \max\{V^E(h^E, e), V^W(e)\} \quad (5)$$

Worker's Value Function:

$$V^W(e) = u(w + (1 + r^a)e) \quad (6)$$

Entrepreneur's Value Function:

$$V^E(h^E, e) = \max_{x, k, a} p u \left[(h^E + \psi x)^{1-\gamma} k^\alpha + (1 + r^a)a \right] + (1-p) u \left[(h^E - x)^{1-\gamma} k^\alpha + (1 + r^a)a \right] \quad (7)$$

subject to:

$$e = a + k$$

$$a \geq -\phi k$$

where $\psi p > (1 - p)$ and $\phi \in [0, 1]$.

2.3 Model Predictions

Depending on the tightness of the borrowing constraints ϕ , and each individual's wealth and entrepreneurial ability (e, h^E) , entrepreneurs may be unconstrained ($a > -\phi k$) or constrained ($a = -\phi k$) in the amount of capital they are investing in.

2.3.1 Case 1: Unconstrained

If the entrepreneur is unconstrained then optimal x and k are given by:

$$x^* = h^E \frac{1 - \left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}}{1 + \psi \left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}} \quad (8)$$

$$k^* = \left(\frac{\alpha}{[1 + r^a]} \frac{\left[p(h^E + \psi x)^{1-\gamma} + (1-p)(h^E - x)^{1-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right) \right]}{\left[p + (1-p) \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right) \right]} \right)^{\frac{1}{1-\alpha}} \quad (9)$$

where

$$\bar{c} = (h^E + \psi x)^{1-\gamma} k^{*\alpha} + (1 + r^a) a^*$$

$$\underline{c} = (h^E - x)^{1-\gamma} k^{*\alpha} + (1 + r^a) a^*$$

$$a^* = e - k^*$$

Note that if $x = 0$, the entrepreneurial project would be risk-free. In that case, $\bar{c} = \underline{c}$, and so the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$ will be equal to one. Since $\psi p > 1 - p$, the numerator of the right hand side of equation (8) is strictly positive, and all entrepreneurs choose a strictly positive level of risk $x > 0$.

Proposition 1: When unconstrained, risk taking is increasing in wealth

$$\frac{\partial x^*}{\partial e} > 0 \quad (10)$$

Proof See appendix [A.1](#).

Intuition

As an individual's wealth increases, more of their final consumption is derived from their endowed wealth and less from their income. Wealthier entrepreneurs are more willing to take a higher level of risk for a higher expected income because when their business is unsuccessful they still have abundant consumption from the wealth outside their business.

Implications

Wealthier entrepreneurs will choose higher-risk higher-productivity projects. The distribution of wealth will have a direct impact on the distribution of riskiness of businesses that get started, which will determine aggregate productivity in the economy. Starting higher expected productivity businesses means that wealthier individuals will also earn higher expected returns on their wealth.

Proposition 2: When unconstrained, individuals are more likely to become entrepreneurs as their wealth e increases

$$\frac{\partial V^E}{\partial e} > \frac{\partial V^W}{\partial e} \Big|_{V^E=V^W} \quad (11)$$

Proof See appendix [A.2](#).

Intuition

As an individual's wealth increases, more of their final consumption is derived from their endowed wealth and less from their income. Wealthier individuals are more willing to select a riskier occupation because if their income is low they still have abundant consumption from the wealth outside their business.

Implications

Even in the absence of borrowing constraints on capital ($\phi = 1$), the missing market for entrepreneurial risk will generate selection into entrepreneurship based on wealth, rather than solely on ability. This financial friction can therefore misallocate talent across occupations if low-ability wealthy individuals are becoming entrepreneurs at the expense of high-ability wealth-poor individuals.

2.3.2 Case 2: Constraint Binds

When the entrepreneur is constrained optimal k is given by:

$$k^c = \frac{e}{1 - \phi} \quad (12)$$

The expression for optimal x when the borrowing constraint binds is identical to the unconstrained case (8).

Proposition 3: When constrained, borrowing constraints affect the choice of risk

$$\frac{\partial x^*}{\partial \phi} < 0 \quad (13)$$

Proof See appendix A.3.

Intuition

As constrained entrepreneurs are able to invest in more capital they reduce risk taking. They do so because risk and size are substitutes in this static model. Taking more risk x or investing in more capital k both increase income when the business risk is successful and reduce it when the business risk is unsuccessful.

Implications

This proposition demonstrates that borrowing constraints can interact with the missing market for entrepreneurial risk that generates heterogeneity in the riskiness of businesses. The potential for this interaction necessitates studying borrowing constraints and the missing market for entrepreneurial risk together in order to quantify the impact of either of these frictions.

In this static model, tighter borrowing constraints unambiguously increase the risk taking of entrepreneurs. While this substitution of scale for risk is also a force in the dynamic model of section 4, an additional dynamic effect will quantitatively dominate this one and so tighter borrowing constraints will lead to less risk taking.

Proposition 4: Tighter borrowing constraints reduce the value of entrepreneurship for wealth-poor individuals

$$\frac{\partial V^E}{\partial \phi} > 0 \quad (14)$$

Proof See appendix A.4.

Intuition Constrained entrepreneurs invest in less capital than they would like to. As

a consequence their entrepreneurial income is lower than if they were unconstrained and this reduces the value of being an entrepreneur for them.

Implications

Borrowing constraints can also generate misallocation of talent as low-ability wealthy individuals become entrepreneurs at the expense of high-ability wealth poor individuals.

3 Stylized Facts

In this section I document key stylized facts about entrepreneurship from the confidential version of the Kauffman Firm Survey. These facts motivate the modelling choices in the static model in section 2 and the dynamic model in section 4.

3.1 Kauffman Firm Survey Data

The Kauffman Firm Survey is a single cohort panel of 4,928 new U.S. firms. All firms are founded in the year 2004, and the survey follows them until they exit or until 2011. It was designed to provide a representative sample of all new businesses started in 2004². The firms include businesses that were independently founded, purchased from an existing business or purchased as a franchise, and exclude any inherited businesses, any non-profits, and businesses that were started as a branch or subsidiary of an existing business. For each firm, information for up to 10 owners is provided.

These firms are highly heterogeneous. As table 1 shows, the majority of firms are non-employers in the first year, though many go on to hire at least one worker later. Most firms are owned and operated by a single entrepreneur, though a small proportion have multiple owners. The distribution of total investment is highly skewed with the mean investment almost twice as large as the median investment.

Table 1: Kauffman Firm Survey Summary Statistics

	Mean	p10	p50	p90	Number of Firms
Employment in Year 1	2.0	0	0	5	4,823
Employment in Year 8	5.2	0	1	10	2,000
Entrepreneurs	1.4	1	1	2	4,928
Total Cumulative Investment Over Years 1-8	480.5	67	277	762	3,488

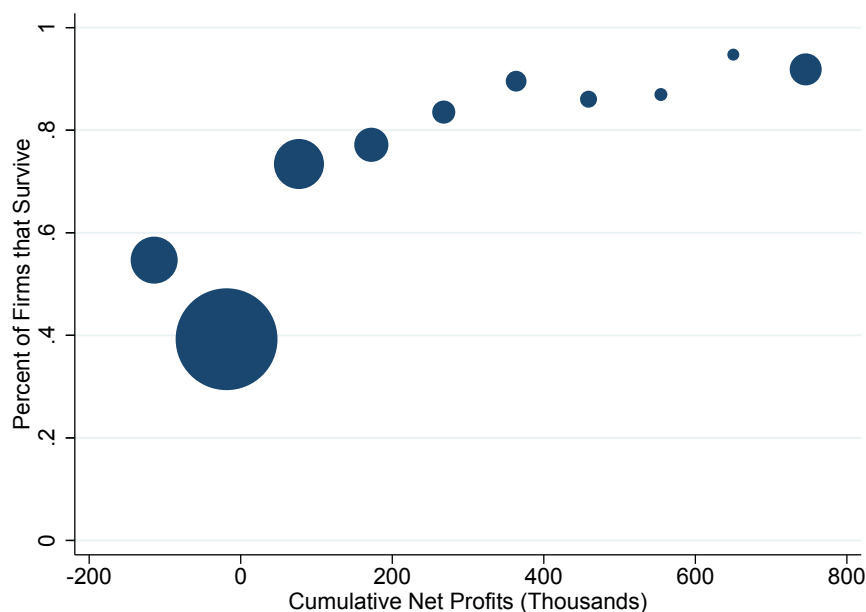
Summary statistics for the firms in the Kauffman Firm Survey. The number of firms in each row varies depending on data availability. Total investment is in thousands of dollars and included all sources of equity and debt.

²The sampling frame is taken from the Dun and Bradstreet U.S. Business database.

3.2 Idiosyncratic Risk

The model in section 2 shows how a missing market for entrepreneurial risk can generate selection into entrepreneurship based on wealth. If this missing market is quantitatively important for who selects into entrepreneurship, two things must be true. First, entrepreneurs must face a significant degree of uncertainty about the potential success of the business³. Second, if the business is unsuccessful entrepreneurs must bear real losses that make them worse off than if they did not start a business.

Figure 1: Firm Survival to Year 8 by Cumulative Profits



The 8-year survival rates of firms in ten bins of cumulative net profits. Cumulative profits have been winsorized between the 5th and 95th percentiles and then firms are sorted into the ten equally-sized bins between these percentiles. The size of each circle is proportional to the number of observations in that bin.

For evidence that these entrepreneurs face idiosyncratic risk, I examine the survival rates and cumulative profits over the eight year panel. I calculate the survival rate as the number of firms either operating in the final sample year or that have been merged or sold in a previous year, divided by the total number of firms with a known status in the final year. Without information about the final sale price, it is difficult to assess whether firms that are merged or sold constitute a successful or unsuccessful outcome for entrepreneurs. I include these firms in survival in order to provide a conservative estimate

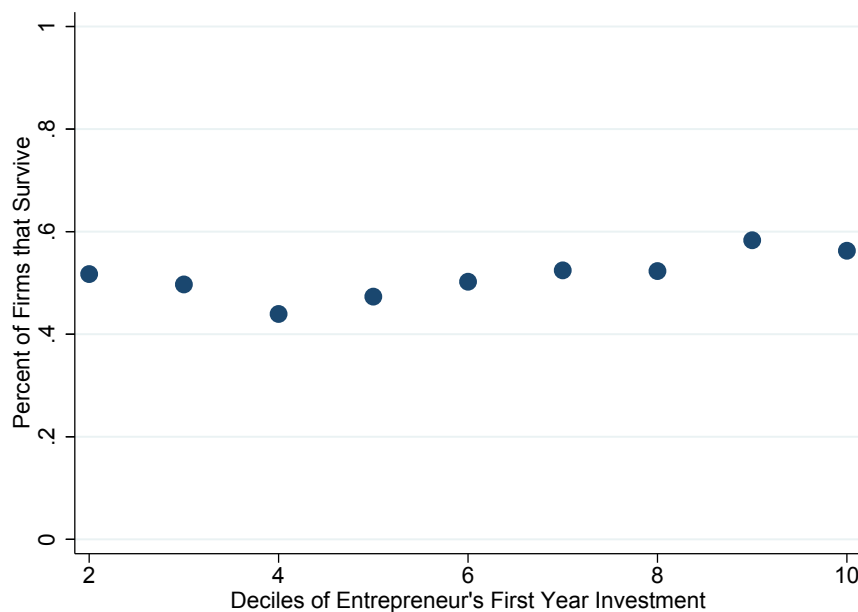
³Note that both uncertainty about entrepreneurial ability as in [Jovanovic \(1982\)](#), which decreases over time as entrepreneurs learn about their ability, and idiosyncratic risk to the business model as in [Hopenhayn \(1992\)](#), can provide the necessary uncertainty about the success of the business.

of the likelihood of undesirable outcomes. See appendix B.1 for additional details on the calculation of survival.

45% of the firms in the Kauffman Firm Survey shut down all operations by the end of their eighth year. Figure 1 shows that the firms that earn low or negative profits are much less likely to survive. In the absence of idiosyncratic risk, it is difficult to imagine why so many of these entrepreneurs would choose to enter, lose money, and then exit.

These firm exits are not confined to low-investment firms. Figure 2 shows that exit is common across the distribution of investment. It is not just low-investment firms, which may have been started to provide temporary self-employment⁴ that exit. While firms in the top decile of initial entrepreneurial investments have slightly higher survival rates than lower-investment firms, 42% of these firms have exited by the end of the eighth year.

Figure 2: Firm Survival to Year 8 by Initial Investment



Survival rates of firms grouped into deciles based on the total equity invested by the entrepreneurs in the first year. This measure excludes equity invested by individuals who do not have an active management role in the firm.

Firm exits are also not confined to a specific set of industries. Table 2 shows the proportion of firms that survive to the end of the sample according to their 2-digit NAICs code. Dropping industries with fewer than 50 observations, survival rates range from 43% of firms in the case of Retail Trade (44), to 64% in Manufacturing (33).

⁴See Galindo da Fonseca (2017), who documents different patterns of entrepreneurial activity based on employment status before starting a business.

Table 2: Firm Survival to Year 8 By Industry

Industry (2 digit NAICs Code)	Percent Surviving in Year 8	Number of Firms
Retail Trade (44)	42.9	238
Retail Trade (45)	43.7	229
Transportation and Warehousing (48)	45.3	86
Construction (23)	46.4	306
Finance and Insurance (52)	51.2	164
Other Services (except Public Administration) (81)	51.4	389
Health Care and Social Assistance (62)	52.1	94
Administrative, Support, Waste Management, and Remediation Services (56)	52.6	310
Manufacturing (32)	54.5	123
Accommodation and Food Services (72)	54.5	88
Information (51)	55.6	144
Real Estate and Rental and Leasing (53)	56.9	160
Wholesale Trade (42)	57.4	188
Arts, Entertainment, and Recreation (71)	57.6	92
Professional, Scientific, and Technical Services (54)	58.4	1,058
Manufacturing (33)	63.5	425

3.3 Choice of Risk

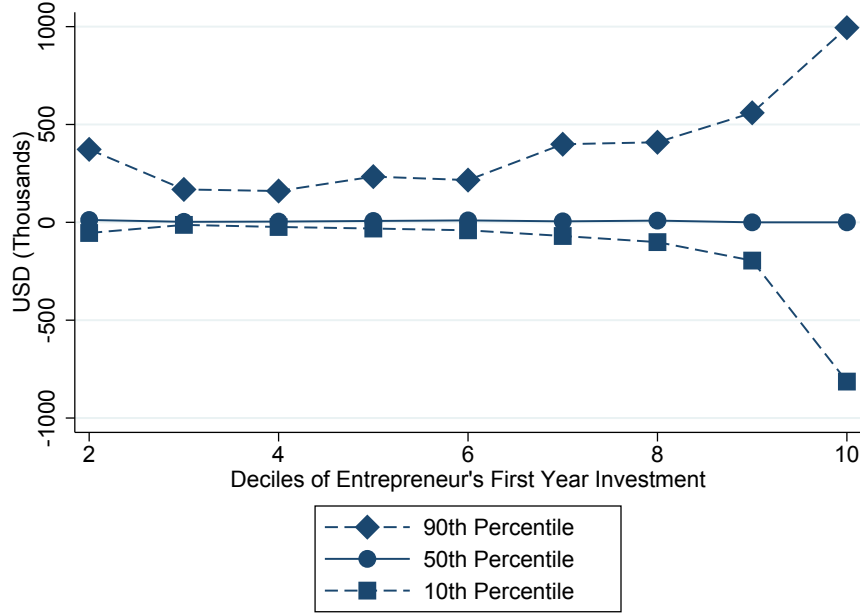
The model in section 2 predicts that wealthier entrepreneurs will both invest more in their business and choose businesses with a higher level of risk. While the ex-ante risk of a business is not directly observable, the ex-post outcomes of a group of firms is. In order to compare the risk taken by different entrepreneurs, I examine the dispersion in outcomes within different groups of firms. If wealthier entrepreneurs are taking more risk, there should be more dispersion in their outcomes than among a group of poorer entrepreneurs.

The Kauffman Firm Survey does not provide information about entrepreneur’s wealth when they start their business. However, the amount of money that they have been able to directly invest in the business is informative about their wealth. The survey asks detailed information about the sources of firm’s funding. In particular, it distinguishes between the money an entrepreneur has invested directly and money they have personally borrowed. I therefore use the amount of money an entrepreneurs is directly investing in their own business as a proxy for their net worth.

Therefore, in order to test the prediction of the model in section 2, I compare the dispersion in cumulative profits across firms in the different deciles of entrepreneur’s own investment, excluding external sources of equity. If entrepreneurs who invest more in

their businesses earn more dispersed cumulative profits, this may indicate that these entrepreneurs are in fact starting businesses with more idiosyncratic risk.

Figure 3: Firm's Cumulative Profits Over First 8 Years



The 10th, 50th and 90th percentiles of the distribution of cumulative profits for firms in each decile of entrepreneur's initial investment. Note that almost 20% of firms invest nothing in the first year of operation, and so the bottom two deciles are represented by a single point labelled "2".

Figure 3 clearly shows that firms with larger initial investments by their entrepreneurs have much more dispersed cumulative profits over the eight years of operations. However, since initial investment is highly correlated with firm size, it is not clear whether this greater dispersion is a mechanical consequence of these firms operating at a larger scale or if these entrepreneurs are starting fundamentally riskier businesses.

To test whether entrepreneurs who invest more also earn more dispersed cumulative profits while controlling for firm size, I run two regressions. First, I regress a firm's cumulative profits on the initial investment of their entrepreneurs, controlling for firm size with average employment over the years of operation. A positive coefficient on initial entrepreneur's own investment ($\hat{\alpha}_1 > 0$) means that entrepreneurs who invest more are on average earning higher cumulative profits.

$$\text{Cumulative Profits}_i = \alpha_0 + \alpha_1 \text{Initial Entrepreneur's Own Investment}_i + \alpha X_i + \epsilon_i \quad (15)$$

Second, I construct an auxiliary regression, where the square of the predicted residuals from the primary regression are regressed on the initial investment of their entrepreneurs

and the same controls. A positive coefficient on initial entrepreneur’s own investment ($\hat{\beta}_1 > 0$) suggests that entrepreneurs who invest more are earning more dispersed cumulative profits, after controlling for firm size. If entrepreneurs who are investing more in their businesses are also starting riskier businesses, then their cumulative profits should be more dispersed, even after controlling for the mechanical effects of size.

$$\hat{\epsilon}_i^2 = \beta_0 + \beta_1 \text{Initial Entrepreneur's Own Investment}_i + \beta X_i + \eta_i \quad (16)$$

Note that this is a slight variation on the common [Breusch and Pagan \(1979\)](#) method for testing for heteroscedasticity. Their method computes a single test statistic based on the explanatory power of the auxiliary regression, to test whether the residuals from the primary regression are homoscedastic. In this context, I am not interested in whether the predicted residuals are generally heteroscedastic. Instead, I want to know whether one regressor, the entrepreneur’s initial investments, can predict their absolute magnitude. If so, entrepreneurs with larger initial investments are earning more dispersed cumulative profits, even after controlling for size.

Table 3 shows that entrepreneurs who invest more of their own money in their businesses on average earn higher profits, and on average, earn more dispersed profits. Both of these results are true after controlling for both the number of employees and the level of debt. I take this to be evidence of greater risk taking on the part of the high-investment entrepreneurs. In appendix B.4 I show that this same result holds for cumulative sales. In appendix B.5, I discuss why the lack of labour market information for the entrepreneurs prevents me from calculating useful rates of return for these firms.

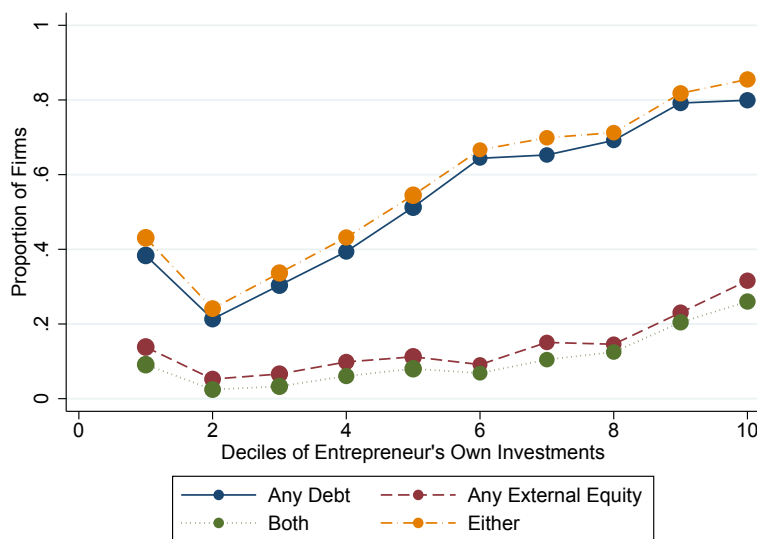
Table 3: Dispersion of Cumulative Profits

Regression (15):	Dependent Variable: Cumulative Profits				
	(1)	(2)	(3)	(4)	(5)
Own Investment in First Year	1.098*** (0.0832)	1.112*** (0.0832)	1.110*** (0.0835)	0.874*** (0.128)	0.712*** (0.190)
Average Employment		-62.64*** (17.32)	-62.91*** (18.07)	-58.98** (18.33)	-60.71*** (18.39)
Average Employment ²		0.210*** (0.0388)	0.211*** (0.0399)	0.152** (0.0473)	0.167*** (0.0490)
Employer			436.8 (312.6)	405.0 (314.1)	411.2 (314.1)
Total Investment in First Year				0.241* (0.0992)	0.409* (0.176)
Total Debt in First Year					-0.318 (0.275)
2 Digit NAICs Codes	No	No	Yes	Yes	Yes
Breush-Pagan	9766.4	25372.9	44478.4	46826.8	51476.7
Regression (16):	Dependent Variable: Squared Predicted Residuals ($\hat{\epsilon}_i^2$)				
Own Investment in First Year	0.10*** (0.03)	0.09** (0.03)	0.09** (0.03)	0.15** (0.05)	0.27*** (0.07)
Average Employment		27.43*** (6.52)	27.74*** (6.73)	27.07*** (6.82)	28.38*** (6.83)
Average Employment ²		-0.05*** (0.01)	-0.05*** (0.01)	-0.04* (0.02)	-0.05** (0.02)
Employer			8.06 (116.42)	13.68 (116.82)	8.98 (116.64)
Total Investment in First Year				-0.06 (0.04)	-0.19** (0.07)
Total Debt in First Year					0.24* (0.10)
2 Digit NAICs Codes	No	No	Yes	Yes	Yes
Observations	4513	4507	4507	4487	4487

3.4 Sources of Funding

How able are new entrepreneurs to raise external funds? While entrepreneurs invest their own money in 89% of these firms, only 15% of firms raise any equity from external sources beyond the actively managing entrepreneurs. 53% of the firms are funded with some debt, much of which is personal debt is taken out in the name of the entrepreneur rather than in the business's name. Figure 4 relates the proportion of firms that raise any debt or any external equity to the amount of their own money that the entrepreneurs are investing. Entrepreneurs who invest more of their own money in a firm are more likely to raise external funds, whether from debt or external equity.

Figure 4: The Proportion of Firms with External Sources of Funding

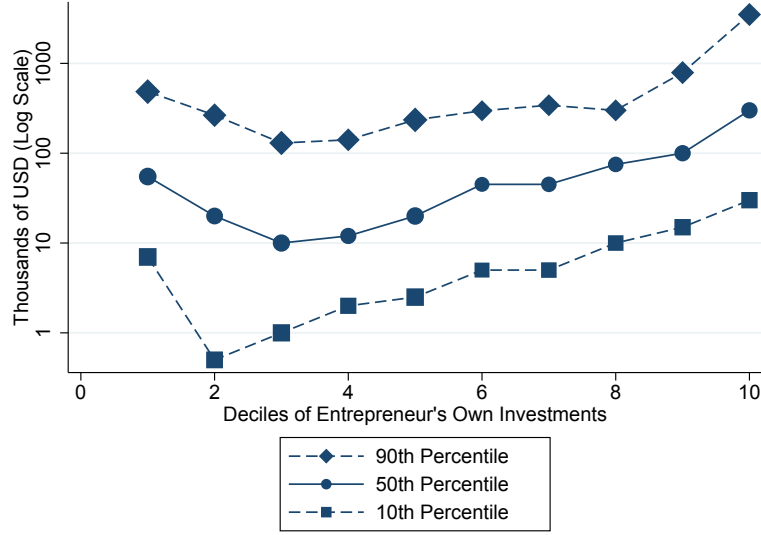


The proportion of firms that have raised some debt, some external equity, both, or neither across the distribution of entrepreneur's initial investment. Firms are sorted into deciles based on the total amount of their entrepreneur's own investment in the firm in the first year of operations.

Figures 5 and 6 relate how much debt and how much external equity to the amount of their own money that entrepreneurs are investing. Note that these graphs have a log-scale. Across the first three deciles, there appears to be somewhat of a decline, suggesting that there is a small fraction of firms who are able to substitute owners investment for external funds, but over the rest of the distribution, larger entrepreneur's investments are correlated with larger amounts of external funds.

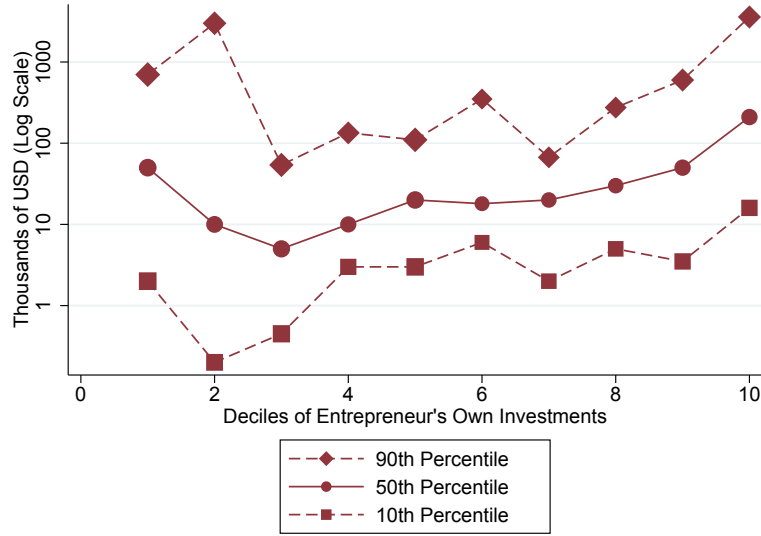
These figures suggest that both on the extensive and on the intensive margin, entrepreneurs who invest more of their own money raise more external funds. These patterns are consistent with the idea that wealth-poor entrepreneurs are unable to borrow to

Figure 5: The Amount of Debt Raised by Firms



The 10th, 50th and 90th percentiles of the distribution of total debt raised by firms across the different deciles of the total of entrepreneur's own investments, conditional on raising some debt. Note the y-axis is a log-scale.

Figure 6: The Amount of External Equity Raised by Firms



The 10th, 50th and 90th percentiles of the distribution of total external equity raised by firms across the different deciles of the total of entrepreneur's own investments, conditional on raising some external equity. Note the y-axis is a log-scale.

finance their business. They motivate the modelling of borrowing constraints as a collateral constraint, which allows entrepreneurs to borrow more only as they invest more. See table 13 for a detailed breakdown of the sources of debt and external equity.

4 Dynamic Model

In order to quantify the relative importance of the missing market for entrepreneurial risk and borrowing constraints, I build a dynamic general equilibrium model where individuals choose whether to be workers or entrepreneurs and entrepreneurs choose the riskiness of the businesses they start.

4.1 Environment

There are a unit measure of agents. Each agent faces a constant probability $(1 - \psi)$ of dying every period and has preferences given by:

$$U = \sum_{t=0}^{\infty} (\psi\beta)^t \frac{c^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} \quad (17)$$

Where β is the discount factor.

Each agent has two types of ability, their ability as a worker (h^W) and their ability as an entrepreneur (h^E). At the beginning of each period, an agent will choose whether to operate as a worker or as an entrepreneur for that period.

If the agent chooses to be a worker, they will supply labour inelastically and earn wh^W , where w is the common wage.

The first period an agent decides to be an entrepreneur, they start a business by choosing how risky a business to start from a menu of risky options $x \in \{x_1, x_2, \dots, x_{n_x}\}$, and investing in a capital stock k . Once chosen, the business's riskiness x is fixed. In a future period, if an entrepreneur wants to change the riskiness of their business they must shut down their business and liquidate their capital stock before they are able to select a new level of business risk.

After all agents make an occupational choice decisions and all new entrepreneurs decide on the riskiness of their business, all agents receive shocks to both their ability as a worker h^W and their ability as an entrepreneur h^E . While both types of ability are partially persistent, neither are perfectly so, and so agents face idiosyncratic income risk from choosing either occupation.

In addition, entrepreneurs also receive a project productivity shock (z) for their business. z is drawn from a distribution that depends on the riskiness of their business x . Higher x businesses have higher expected z but also more dispersed z . The productivity

of a business depends on both the firm-specific productivity shock as well as the entrepreneur's entrepreneurial ability h^E . Once they received their shocks, entrepreneurs hire an amount of labour n at wage rate w and produce according to:

$$y = (zH^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma \quad (18)$$

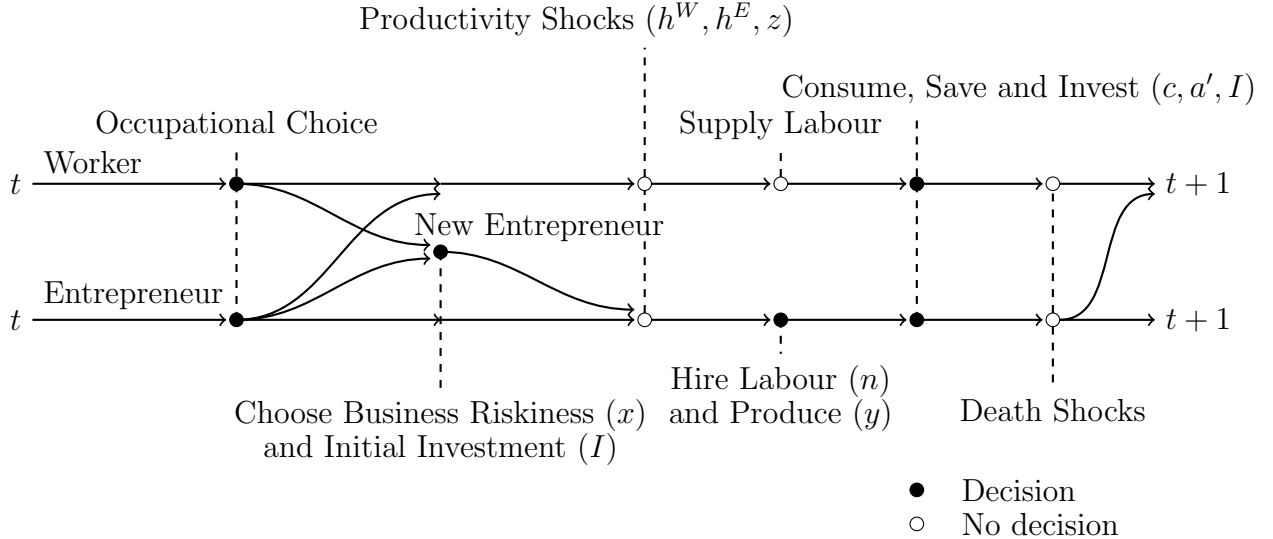
After entrepreneurs produce and pay their employees, all agents make a consumption, savings, and investment decision. The model has two assets. Only entrepreneurs can invest in capital k , which depreciates at rate δ . Capital is also illiquid, so that liquidating one unit of capital produces only $\frac{1}{\chi} < 1$ units of consumption. Given an investment of I , an entrepreneur's capital stock k evolves according to:

$$k' = \begin{cases} k(1 - \delta) + I & \text{if } I \geq 0 \\ k(1 - \delta) + \chi I & \text{if } I < 0 \end{cases} \quad (19)$$

Agents can also save and borrow in a liquid financial asset a , that pays a constant $1 + r^A$ in all states of the world. All agents can borrow up to an exogenous unsecured borrowing limit $\underline{a} \leq 0$. In addition, for each unit of capital that an entrepreneur invests in, they can borrow an addition $\phi \in [0, 1]$ units of a . Thus the borrowing constraint is given by:

$$a \geq \underline{a} - \phi k \quad (20)$$

Figure 7: Timing in the Dynamic Model



After their consumption, savings, and investment decisions, agents will die with probability $(1 - \psi)$. If they were an entrepreneur, their invested capital stock is liquidated.

All agents that die are immediately replaced by a descendent who inherits the full value of their liquidated assets. Figure 7 summarizes the timing in the model.

4.2 Agent's Problems

Worker's Problem

A worker makes a consumption-savings decision, and at the beginning of the next period will choose between being a worker (V^W) and becoming a new entrepreneur (V^{NE}):

$$V^W(a, h^W, h^E) = \max_{a', c} \frac{c^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta \max \{ \mathbb{E} [V^W(a', h^{W'}, h^{E'})], V^{NE}(a', h^{W'}, h^{E'}) \} \quad (21)$$

s.t.

$$a' + c = wh^W + (1 + r^a)a$$

$$a' \geq \underline{a}$$

New Entrepreneur's Problem

An agent that has decided to start a new business chooses the riskiness of their business x . Given their current financial assets a , they also choose how much to invest in the business (I) and how much to borrow or save in the financial assets \tilde{a} . They will then operate as an entrepreneur later this period with a capital stock \tilde{k} .

$$V^{NE}(a, h_{-1}^W, h_{-1}^E) = \max_{\tilde{a}, I, x} \mathbb{E} [V^E(\tilde{a}, \tilde{k}, h^W, h^E, z, x)] \quad (22)$$

s.t.

$$\tilde{a} = a - I$$

$$\tilde{a} \geq \underline{a} - \phi \tilde{k}$$

$$\tilde{k} = I$$

Entrepreneur's Problem

An entrepreneur that has a business will choose an amount of labour n to hire, consumption c , savings $(I + a')$, and investment I . At the beginning of the next period, they will choose between shutting down their business to become a worker (V^W), shutting down their business to start a new business (V^{NE}) and continuing to operate the same business (V^E).

$$V^E(a, k, h^W, h^E, z, x) = \max_{n, c, a', I} \frac{c^{1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} + \psi \beta \max \left\{ \begin{array}{c} \mathbb{E} [V^W(a', h^{W'}, h^{E'})], \\ V^{NE}(a', h^W, h^E), \\ \mathbb{E} [V^E(a', k', h^{W'}, h^{E'}, z', x)] \end{array} \right\} \quad (23)$$

s.t.

$$c + a' + I = (zh^E)^{1-\gamma} (k^\alpha n^{1-\alpha})^\gamma - wn + (1 + r^a)a$$

$$a' \geq \underline{a} - \phi k'$$

$$k' = \begin{cases} k(1 - \delta) + I & \text{if } I \geq 0 \\ k(1 - \delta) + \chi I & \text{if } I < 0 \end{cases}$$

Note that if the entrepreneur decides to start a new business, they cannot use the capital from their current business. They must first fully liquidate it and then invest in a new capital stock for the new business.

4.3 Equilibrium

An equilibrium is a set of value functions $\{V^W, V^{NE}, V^E\}$, occupational choices, a set of policy functions $\{c^W, a'^W, \tilde{a}^{NE}, I^{NE}, x^{NE}, c^E, a'^E, I^E, n^E\}$, a distribution of agents $\{\Gamma^E(a, k, h^W, h^E, z, x), \Gamma^W(a, h^W, h^E)\}$, and a price w such that

1. The policy functions solve the individual's problems given by (21), (22) and (23).
2. All markets clear:
 - Labour

$$\int h d\Gamma^W(a, h^W, h^E) = \int n^E d\Gamma^E(a, k, h^W, h^E, z, x)$$

- Final Goods

$$\begin{aligned} \int (c + a') d\Gamma^W(a, h^W, h^E) + \int (c + a' + I) d\Gamma^E(a, k, h^W, h^E, z, x) = \\ \int ((zh^E)^{1-\gamma} (k^\alpha n^{1-\alpha})^\gamma + (1 + r^A)a) d\Gamma^E(a, k, h^W, h^E, z, x) \\ + \int ((1 + r^A)a) d\Gamma^W(a, h^W, h^E) \end{aligned}$$

3. The distribution of agents is stationary

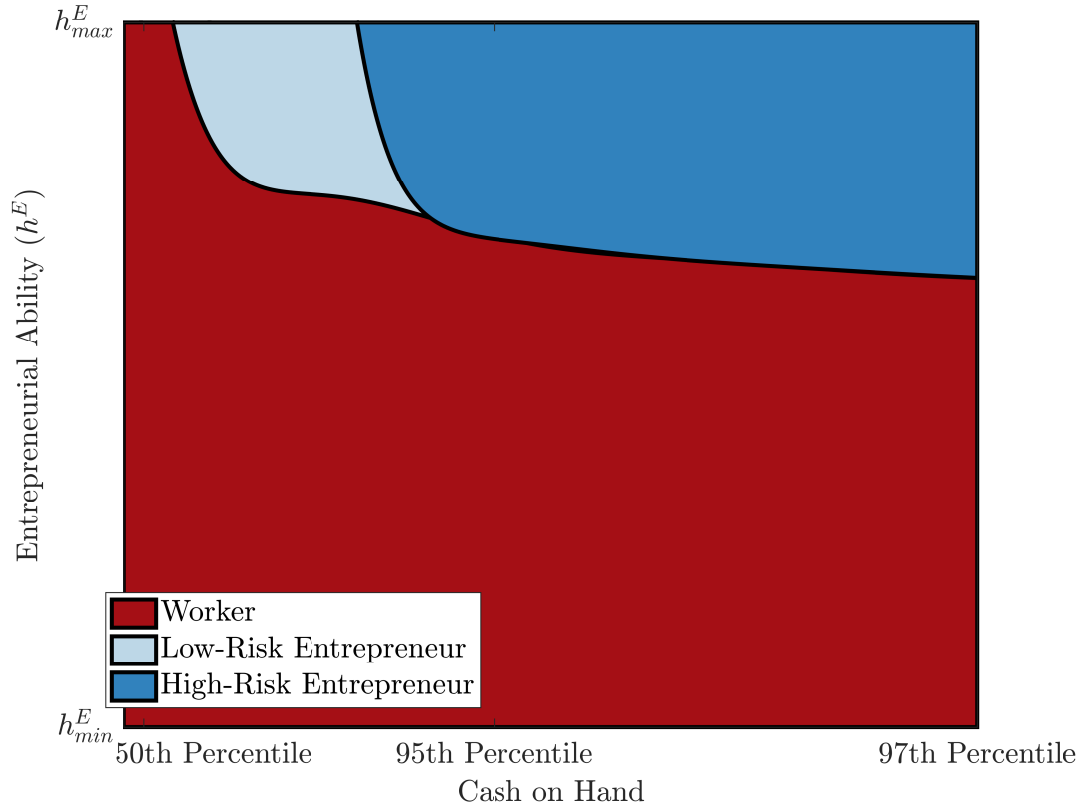
$$\Gamma^E(a, k, h^W, h^E, z, x) = \Gamma^{E'}(a, k, h^W, h^E, z, x)$$

$$\Gamma^W(a, h^W, h^E) = \Gamma^{W'}(a, h^W, h^E)$$

Note that I assume the economy is a small open economy, and so the financial asset can be in net positive or negative supply.

4.4 Model Predictions

Figure 8: Patterns of Occupational Selection and Risk Choice in the Model



Patterns of occupational choice and entrepreneurial risk choice in the calibrated model. Given a worker with a particular worker ability (h^W), the graph shows how the worker would select their occupation depending on their current level of entrepreneurial productivity (h^E) and their current cash on hand ($a(1+r^A)+wh^W$). The x-axis labels the level of wealth needed to make it into the top 50%, the top 5% and the top 3% of the wealth distribution.

An individual's two abilities and their wealth all play a role in their occupational choices and if they become an entrepreneur in their choice of business risk. Figure 8

illustrates these choices in the model. For a worker with the lowest worker ability (h^W), the graph compares the decisions for a worker depending on their current level of entrepreneurial productivity (h^E) and their current level of cash on hand ($a(1+r^A)+wh^W$). The level of cash on hand captures the total resources available to consume or save by an individual. Note that for computational reasons, entrepreneurs choose between two different business risk levels, a relatively safe business and a relatively risky business.

Agents with low entrepreneurial ability (h^E) will become workers regardless of their wealth. For agents with high entrepreneurial ability, their choices will depend on their level of wealth. If the agent has little wealth, they will choose to be a worker. The borrowing constraint limits the scale of the business a poor entrepreneur can operate, which limits their entrepreneurial income. In addition, if the business fails, the poor agent will earn very low income that period and consume very little.

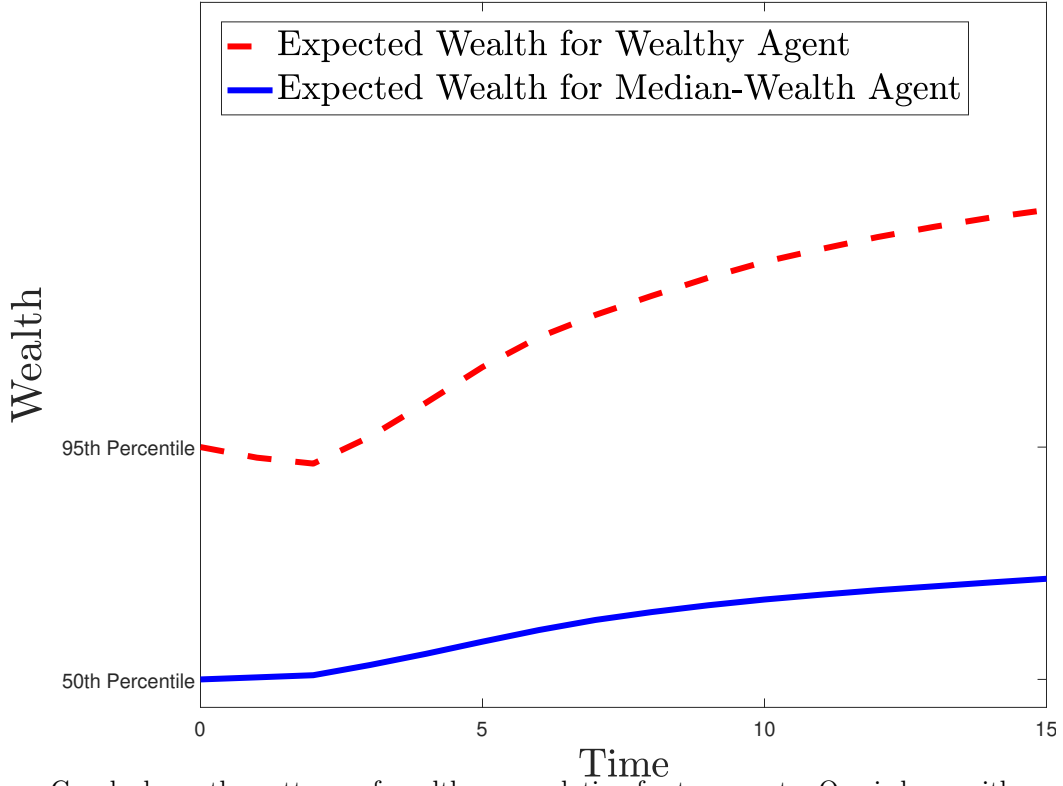
Agents with high entrepreneurial ability and moderate wealth will choose to be entrepreneurs operating the low risk project. While the expected productivity of the high-risk project is greater, the higher risk makes it unattractive. A bad productivity shock would leave the entrepreneur with low income for at least one period, and if the shock was low enough that they decided to exit, the entrepreneur would have to liquidate their capital stock, leading to a loss of wealth.

Agents with high entrepreneurial ability and high wealth will choose to be entrepreneurs operating the high risk project. Since they have sufficient wealth to self-insure any bad productivity shocks, they choose the higher risk project with higher expected productivity.

Given these choices, wealthier agents accumulate wealth faster than poor agents with the same abilities. Figure 9 illustrates this dynamic for two agents born with identical entrepreneurial and worker abilities. Both agents are born at time 1 as workers with the maximum entrepreneurial and worker ability. However, they are endowed with different initial levels of wealth, one born with an inheritance equal to the median level of wealth in the economy and one born with an inheritance equal to the 95th percentile of wealth in the economy.

The wealthier agent immediately starts a high risk business. They have some probability of receiving initial productivity shocks low enough that they will shut down their business, liquidate their capital and start a new high risk business next period. This process is costly, and so on average the wealth of the wealthy agent declines briefly. As soon as they start a business with a good initial shock though, they tend to stay in business, receiving high productivity and earning high returns on their wealth. The wealthy agent then saves a high proportion of their entrepreneurial income for two reasons. First, because the project is still risky, the agent wants to engage in precautionary savings in order to self-insure against the project's failure. Second, because of the borrowing constraint, the entrepreneur wants to save more in order to operate a larger business next period and earn an even higher income. As a consequence the wealth of the wealthy agent

Figure 9: Wealth Accumulation for Individuals with Different Initial Wealth



Graph shows the patterns of wealth accumulation for two agents. One is born with the median level of wealth in the economy and one born with an inheritance equal to the 95th percentile of wealth in the economy. The two lines correspond to the average level of wealth given all the possible shocks to their abilities and business productivities that they could receive.

accumulates wealth quickly.

The agent born with median wealth initially chooses to be a worker, and save up to become an entrepreneur in the future. Depending on their shocks, they will likely start a business after a few periods of saving. However, when they choose to do so they will start the low-risk business. Starting the high-risk business is simply too risky, as low productivity shocks would induce them to liquidate their capital and exit after having earned little, or even lost money, in that period. As time progresses, these entrepreneurs continue to save. Some of the luckier individual eventually go on to accumulate enough wealth to start the high risk business, but many remain with the low risk business.

5 Calibration

I calibrate this model to match micro data on new firm dynamics and wealth inequality by using the Kauffman Firm Survey and the 2004 Survey of Consumer Finances. While more recent data for the Survey of Consumer Finances is available, I use the 2004 wave as it represents the distribution of wealth in the year that all of the Kauffman Firm Survey firms were founded. For a set of parameters that are difficult to identify using my data, I rely on commonly used values in the literature. I then jointly match twelve model moments to twelve moments in the data by varying twelve parameters.

Parameterization of the Ability and Productivity Processes

I parameterize both the worker and entrepreneurial ability processes with AR(1) processes:

$$\log(h^{i'}) = \rho_{h^i} \log(h^i) + \epsilon_{h^i}$$

where $\epsilon_{h^i} \sim N\left(\frac{\mu_{h^i}}{1-\rho_{h^i}}, \sigma_{h^i}^2\right)$ for $i \in \{E, W\}$

In order to keep the problem computationally tractable, I restrict myself to the case where entrepreneurs choose between two project types, a relatively safe project (x_1) and a relatively risky project (x_2). Each project type receives productivity shocks z the follows an AR(1) process:

$$\log(z') = \rho_{z|x_i} \log(z) + \epsilon_{z|x_i}$$

$$\epsilon_{z|x_i} \sim N\left(\frac{\mu_{z|x_i}}{1-\rho_{z|x_i}}, \sigma_{\epsilon_{z|x_i}}^2\right)$$

for $i \in \{1, 2\}$.

I use the Rowenhorst method to discretize these four AR(1) processes $\{h^W, h^E, z_{x_1}, z_{x_2}\}$.

Externally Chosen Parameters

A period in the model is a year. I fix ten parameters to commonly used values in the literature. I set the capital share α to $\frac{1}{3}$ and the interest rate r^A to 4% per year. I set the decreasing returns to scale parameter γ to 0.85 as in [Midrigan and Xu \(2014\)](#) and the depreciation rate to 6% per year. The time discount factor is set such that $\beta = \frac{1}{\psi} \frac{1}{1+r^A}$. I set the coefficient of relative risk aversion to 1.50. I set the probability of death ($1 - \psi$) to $\frac{1}{40}$ so that the expected working lifetime is 40 years.

I normalize the average level of worker ability (μ_{h^W}) to 1. I set the persistence of the labour income process ρ_{ϵ_h} to 0.9, which is in the range of empirical estimates according to [Guvenen \(2007\)](#). The standard deviation of the innovation σ_{ϵ_h} is set to 0.2.

Internally Calibrated Parameters

This paper is about quantifying the strength of financial frictions on the decision to start a business and the decision to start businesses with different levels of risk. In order to quantify the importance of these financial frictions, I must match the firm dynamics of new firms. In addition, I must match the distribution of wealth because of the importance of wealth for overcoming these financial frictions. To match both the firm dynamics and the distribution of wealth, I jointly match twelve moments from the model to the data by varying the remaining twelve parameters.

In the model, I simulate an eight year panel of new firms, corresponding to the Kauffman Firm Survey's eight year panel. In the data I observe the ratio of debt and equity that these new firms start with, and so to pin down the tightness of the borrowing constraint (ϕ), I match the ratio of average debt to average equity in the panel to the ratio of debt to equity in the data.

The degree of idiosyncratic entrepreneurial risk is vital to this model. Obviously, it is not possible to directly observe the distribution of risk each entrepreneur is drawing from. However, the distribution of firm outcomes is informative about the nature of entrepreneurial risk that all entrepreneurs are facing. The model predicts that wealthier individuals will select higher risk projects and invest more in them. Therefore I separate firm both in the model simulation and in the data at the 90th percentile of owner's own investment over the first three years. I compute moments separately for the firms in the top 10% of first-three-year investment and those in the bottom 90% of first-three-year investment.

To discipline the size and persistence of productivity shocks, I match the survival rates and the auto-correlation of employment for both these high-investment and low-investment firms. In order to pin down the differences in average productivity between the low-risk and high risk project, I match the ratio of average employment in these high-investment firms to the average employment in the low-investment firms at the eighth year of the panel, conditional on survival. Of course these moments on firm dynamics, (survival, auto-correlation of employment, and average productivity) are all jointly generated by shocks to the business's productivity and shocks to the entrepreneur's ability. Both types of shocks likewise influence savings motives and so the accumulation of wealth in the model.

Since this model is primarily about the differences in savings behaviour of the entrepreneurs and the workers, I match the ratio of average wealth of entrepreneurs to workers, the proportion of entrepreneurs in the economy, and the proportion of entrepreneurs in the wealthiest 1%. Following [Cagetti and De Nardi \(2006\)](#), I match the wealth Gini, and since this model is a model of heterogeneous entrepreneurs, I also match the wealth Gini of the group of entrepreneurs to ensure that the degree of wealth inequality within

entrepreneurs matches the data. Finally, to replicate the bottom of the wealth distribution I match the level of unsecured borrowing (\underline{a}) to ensure the correct proportion of agents have negative net worth.

Table 4: Benchmark Calibration

Target	Data	Model	Parameter	Value
Exogenously Chosen				
40 Year Working Lifespan			$1 - \psi$	0.03
Capital Share			α	0.33
Interest Rate			r^A	0.04
Entrepreneurial Share			$1 - \gamma$	0.12
Depreciation			δ	0.06
Discount Factor			β	0.96
Coefficient of Relative Risk Aversion			σ	1.50
Average Labour Ability			μ_{h^W}	1.00
Dispersion of Labour Ability			σ_{h^W}	0.20
Persistence of Labour Ability			ρ_{h^W}	0.90
Endogenously Calibrated				
Ratio of Average Debt to Average Equity	1.35	1.13	ϕ	0.64
Proportion of Entrepreneurs	0.11	0.14	χ	1.02
Proportion with Negative Net Worth	0.08	0.09	\underline{a}	-1.07
Wealth Ratio of Entrepreneurs to Workers	7.14	7.04	μ_{h^E}	-1.09
Wealth Gini of Entrepreneurs	0.75	0.59	σ_{h^E}	1.70
Wealth Gini	0.79	0.83	ρ_{h^E}	0.98
Proportion of Entrepreneurs in Wealthiest 1%	0.74	0.67	$\mu_{z x_1}$	0.53
Autocorrelation of Employment (Low Investment)	0.03	0.13	$\sigma_{z x_1}$	0.06
Survival Rate (Low Investment)	0.50	0.50	$\rho_{z x_1}$	0.72
Relative Employment (High-to-Low Investment)	4.23	3.86	$\mu_{z x_2}$	-4.16
Autocorrelation of Employment (High Investment)	0.12	0.04	$\sigma_{z x_2}$	5.34
Survival Rate (High Investment)	0.60	0.23	$\rho_{z x_2}$	0.72

6 Validation

Within the model, I simulate an eight year panel of new firms that correspond to the Kauffman Firm Survey's eight year panel. Within this simulated panel, I am able to compare the behaviour of new entrepreneurs in the model to the data along several dimensions that are not directly targeted in the calibration.

Table 5: Dispersion in Cumulative Profits in the Model

	Kauffman Firm Survey	Model Simulation
Own Investment	0.27*** (0.07)	0.37*** (0.03)
Average Employment	28.38*** (6.83)	1.79*** (0.19)
Average Employment ²	-0.05** (0.02)	-0.00*** (0.00)
Employer	8.98 (116.64)	
Total Investment	-0.19** (0.07)	
Total Debt	0.24* (0.10)	0.26*** (0.05)
Constant	Yes	Yes
2 Digit NAICs Codes	Yes	No
Observations	4487	5000

6.1 Dispersion in Cumulative Profits

Table 5 compares the results of regression (16) from the data to the model’s simulation. Just as in the data, entrepreneurs in the model who invest more in their own business earn significantly more dispersed cumulative profits even after controlling for the size of the business.s

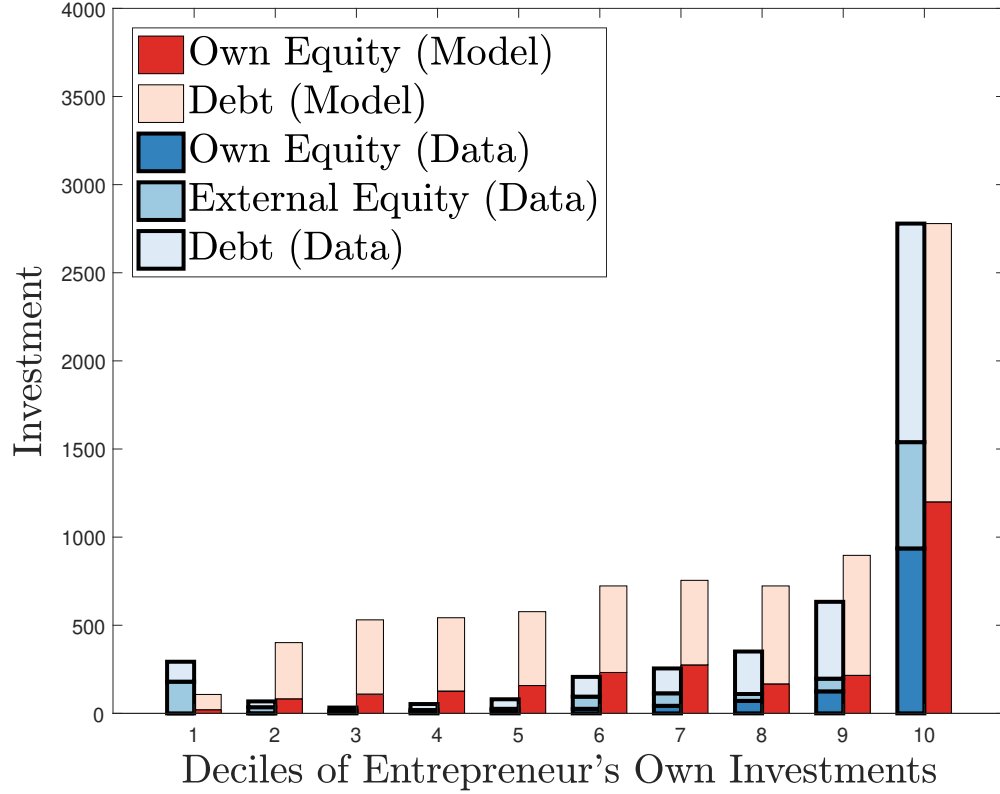
$$\text{Cumulative Profits}_i = \alpha_0 + \alpha_1 \text{Initial Entrepreneur's Own Investment}_i + \alpha X_i + \epsilon_i \quad (15)$$

$$\epsilon_i^2 = \beta_0 + \beta_1 \text{Initial Entrepreneur's Own Investment}_i + \beta X_i + \eta_i \quad (16)$$

6.2 Distribution of Investment

Figure (10) compares the composition investment in the data and in the model. The model can replicate both the skewness in the amount of invested in the firms and the fact that high-investment firms rely heavily on debt.

Figure 10: Composition of Investment for Firms in the Model and the Data



7 Quantitative Analysis

A central question in this paper is how important is the missing market for entrepreneurial risk relative to borrowing constraints for reducing entrepreneurial activity in the economy. In order to assess the quantitative significance of both of these frictions, I remove first one, then the other, and finally both from the model in order to compare the resulting equilibria.

First, I complete this missing market by introducing state-contingent assets. In the benchmark economy, there is a single risk-free asset. In the complete markets economy, each agent can purchase assets that pay off in future states of the world based on their individual realizations of h^W, h^E and z . An asset a_i pays off $1 + r^A$ in the state of the world $i \in H^W \times H^E \times Z$. Each of these assets are sold at actuarially fair price q_i by competitive risk-neutral financial intermediaries.

$$q(h^{W'}, h^{E'}, z') = \text{Prob}(h^{W'}, h^{E'}, z' | h^W, h^E, z, x) \quad (24)$$

In order to separate the impact of the missing market for entrepreneurial risk from

the impact of the borrowing constraint, I initially keep the borrowing constraint in the economy unchanged. In the benchmark economy the borrowing constraint requires that $a \geq \underline{a} + \phi k$. In the complete markets economy, the borrowing constraint requires that:

$$\sum_i q_i a_i \geq \underline{a} + \phi k \quad (25)$$

Thus an entrepreneur with the same net worth in both economies can invest in the same maximum amount of capital.

Obviously, considering the perfect completion of the missing market for entrepreneurial risk is not directly policy-relevant. In the real world, information frictions will always create moral hazard and adverse selection problems that make it difficult to provide insurance to entrepreneurs at actuarially fair prices. In this complete markets economy I assume that financial intermediaries have perfect information over the exact abilities of potential entrepreneurs and the expected productivity of these projects. Access to this degree of information is impossible in the real world, however this exercise is still a useful counterfactual to help us understand what the potential gains are. Understanding the size of these potential gains helps us understand which financial frictions public policy should aim to mitigate. In section 8, I consider policy implications of these frictions without requiring that a government have perfect information about private individuals.

To remove borrowing constraints, I simply set $\phi = 1$, so that entrepreneurs can invest in any level of capital stock unrestricted by their personal net worth. Note that I keep the unsecured level of debt \underline{a} constant. I do this because I'm interested in entrepreneurial borrowing constraints that prevent entrepreneurs from raising funds to put into their business, as has been the focus of a large literature, rather than general borrowing constraints that apply to individuals for consumption. The policy implications of these two counterfactuals are very different. Eliminating the unsecured level of debt would allow many individuals to self-insure themselves by accumulating large debts, though this would require adding default into the model.

I then compare four economies. The benchmark economy with both financial frictions, an economy where state-contingent assets complete the missing market for risk but entrepreneurs still face borrowing constraints ($\phi < 1$), the economy where entrepreneurs can borrow the full value of their capital ($\phi = 1$) but the insurance market for risk is missing, and finally an economy with both a complete market for risk and unrestricted capital investment ($\phi = 1$). Table 6 reports aggregate output, wages, capital stock, and productivity in these four economies.

In the complete markets economy aggregate productivity is 9% higher than in the benchmark economy. Much of this increase is due to the fact that all entrepreneurs operate the high risk project, which has higher expected productivity. There are also some wealth-poor high-ability individuals who were previously workers who choose to

Table 6: Aggregate Economic Outcomes With and Without the Two Financial Frictions

	Output	Wages	Capital	Productivity	Percentage of		Entrepreneurs operating	
					Workers	Entrepreneurs	Low Risk	High Risk
Benchmark Economy	1.00	1.00	1.00	1.00	86.1	13.9	9.9	3.9
Complete Market for Risk	1.08	1.08	0.99	1.09	85.9	14.1	0.0	14.1
Relaxed Borrowing Constraints	1.07	1.06	1.10	1.04	85.2	14.8	8.6	6.3
Neither Friction	1.21	1.25	1.33	1.13	82.1	17.9	0.0	17.9

The benchmark economy has both a missing market for risk and borrowing constraints. The complete market for risk economy has borrowing constraints ($\phi < 1$). The relax borrowing constraints economy, entrepreneurs face no borrowing constraints ($\phi = 1$) but cannot insure themselves against risk. In the Neither Friction economy, entrepreneurs face no borrowing constraints ($\phi = 1$) and state-contingent assets complete the market for risk. The first four columns list output, wages, capital stock and productivity relative to the benchmark economy.

become entrepreneurs. As existing entrepreneurs are more productivity and with new entrepreneurs entering, aggregate labour demand increases substantially. Wages rise 8% to clear the market. Due to the rise in the wage there is a small decline of less than 1% in the aggregate capital stock. Overall, aggregate output increases by 8%.

In the relaxed borrowing constraints economy aggregate productivity increases by 4% as some entrepreneur switch to the higher risk project and more high-ability low-wealth individuals decide to start businesses. Formerly constrained entrepreneurs invest in far more capital, leading to an increase in the aggregate capital stock of 10%. Wages rise by 6% to clear the labour market. Overall, aggregate output increases by 7%.

In the economy with neither friction, aggregate productivity increases by 13% as all entrepreneurs switch to the higher risk project and many more high-ability individuals decide to start businesses regardless of their wealth. Unconstrained by their personal net worth entrepreneurs invest in far more, leading to an aggregate capital stock 33% higher than the benchmark economy. Output increases by 21%. Note that 6 percentage points of this increase is due to the interaction of the two financial frictions. In subsections 7.1, 7.2, and 7.3 I illustrate how individuals decisions change across these three economies in partial equilibrium.

Table 7 reports selected statistics about the distribution of wealth in the four economies. The benchmark economy does a reasonable job of matching the high concentration of wealth at the top, with 28% of aggregate wealth held by the wealthiest 1% of households, compared to 32% in the data.

Completing the missing market substantially reduces wealth inequality in the economy. The wealth Gini declines from 0.83 to 0.77 and the share of wealth owned by the wealthiest 1% declines from 28% down to 9%. This substantial decline in inequality occurs mainly between entrepreneurs as all entrepreneurs switch to running the high-risk project. The wealth Gini of entrepreneurs declines from 0.59 down to 0.35.

By contrast, relaxing the borrowing constraints increases wealth inequality in the econ-

Table 7: Wealth Inequality With and Without the Two Financial Frictions

	Wealth Share of Top					Wealth Gini	Entrepreneur Wealth Gini	Proportion with Negative Net Worth	Entrepreneur to Worker Wealth Ratio
	1%	5%	10%	20%	50%				
SCF Data	32	56	67	81	97	0.79	0.75	0.08	7.14
Benchmark	28	63	74	87	98	0.83	0.59	0.09	7.04
Complete Market for Risk	9	35	59	85	98	0.77	0.35	0.03	6.99
Relaxed Borrowing Constraints	25	69	86	91	98	0.87	0.76	0.12	5.95
Neither Friction	6	28	50	72	93	0.68	0.53	0.08	3.60

The benchmark economy has both a missing market for risk and borrowing constraints. The complete market for risk economy has borrowing constraints ($\phi < 1$). The relax borrowing constraints economy, entrepreneurs face no borrowing constraints ($\phi = 1$) but cannot insure themselves against risk. In the Neither Friction economy, entrepreneurs face no borrowing constraints ($\phi = 1$) and state-contingent assets complete the market for risk. Entrepreneur Wealth Gini computes the Gini coefficient for wealth on the group of entrepreneurs.

omy, except at the very top. While the share of wealth owned by the wealthiest 1% declines from 28% to 25%, the wealth Gini rises from 0.83 to 0.87. The lack of borrowing constraints encourages entrepreneurs to increase their leverage. As a consequence, entrepreneurs that receive good productivity shocks on highly leveraged businesses accumulate wealth much faster, those that receive bad shocks lose wealth faster due to their higher leverage. The wealth Gini of entrepreneurs increases from 0.59 to 0.76.

Moving from the benchmark economy to the economy with neither friction reduces wealth inequality. The wealth Gini declines from 0.83 to 0.68 and the share of wealth owned by the wealthiest 1% declines from 28% down to 6%.

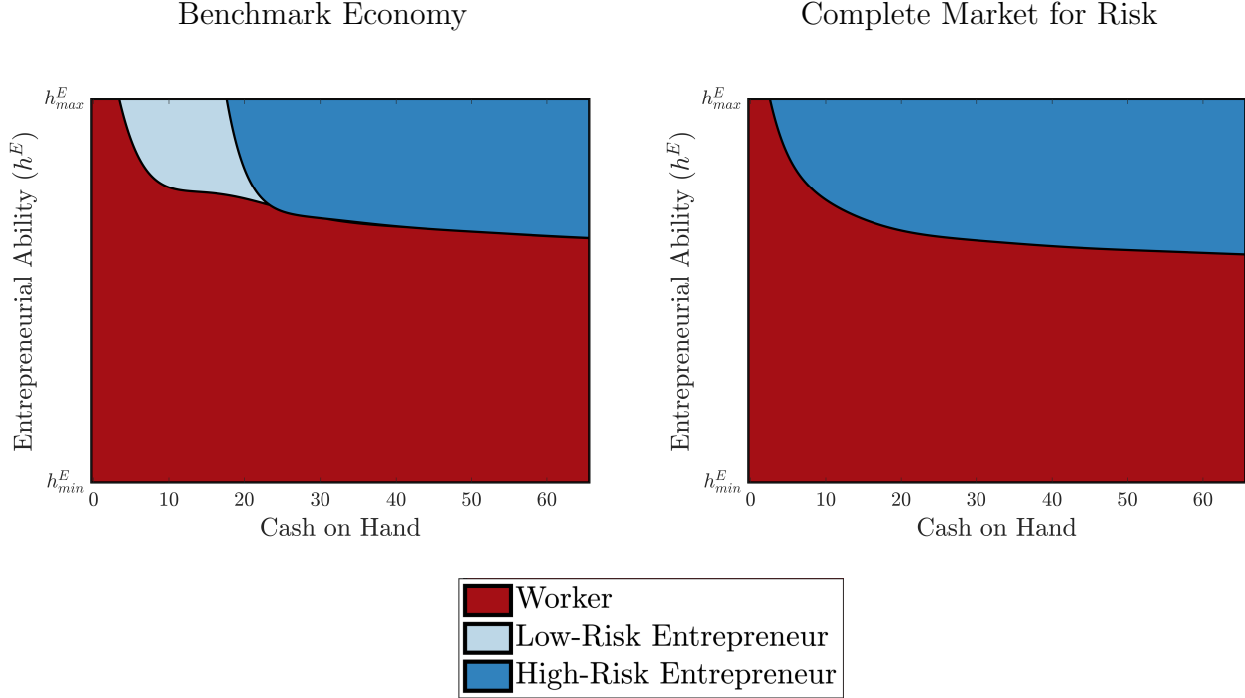
7.1 Completing the Missing Market for Risk

Introducing conditional claims increases output in the economy in four distinct ways. First, with the ability to insure against entrepreneurial risk, entrepreneurs exclusively choose the high risk project, rather than the low risk project. As the higher-risk project has a higher expected productivity, this boosts productivity and aggregate output. Secondly, individuals with higher entrepreneurial ability are more likely to select into entrepreneurship. Third, conditional on wealth, unconstrained entrepreneurs invest more in their businesses. Finally, because entrepreneurs earn higher incomes as a result of these three previous effects, they accumulate more wealth and thus are able to run larger businesses, despite the continued presence of borrowing constraints.

Figure 11 compares the patterns of occupational choice and endogenous risk choice between the benchmark economy and the economy with a complete market for risk. In the economy with state-contingent assets, entrepreneurs all choose the riskier project with higher expected productivity. They then insure themselves against the substantial risks associated with this using the state-contingent assets.

There is a small decrease in the wealth that makes individuals indifferent between

Figure 11: Comparing Occupational Choice and Endogenous Risk Choice

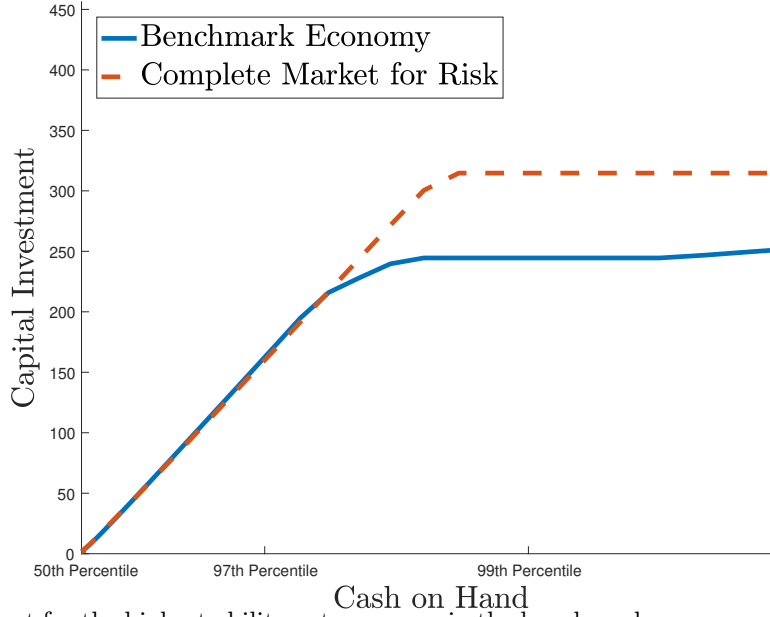


Patterns of occupational selection and endogenous risk choice in the benchmark economy with both frictions (left) and in the economy with state-contingent assets (right).

being workers or entrepreneurs. As a consequence, a few more high-ability low-wealth individuals become entrepreneurs. However the relatively small change in occupational selection demonstrates that it is primarily borrowing constraints that generate these patterns, while the choice of project risk is clearly dependent on the missing market for risk.

Figure 12 shows the amount of capital invested by the most productive entrepreneur in the economy in the risky project. In the benchmark economy, even when the entrepreneur is unconstrained they invest less than the expected marginal product of capital because of the substantial capital investment risks they face. With the introduction of state-contingent assets, entrepreneurs insure themselves against bad shocks, and so choose to invest more in their businesses. In both economies, relatively wealth-poor individuals are constrained by their personal wealth and borrow up to the exogenous borrowing limit.

Figure 12: Capital Investment for the Highest Ability Entrepreneur



Capital investment for the highest ability entrepreneur in the benchmark economy with both frictions (solid blue line) and in the economy with contingent claims (dashed orange). The x-axis labels the cash on hand required to be in the top 50%, top 5% and top 1% of the wealth distribution in the benchmark economy.

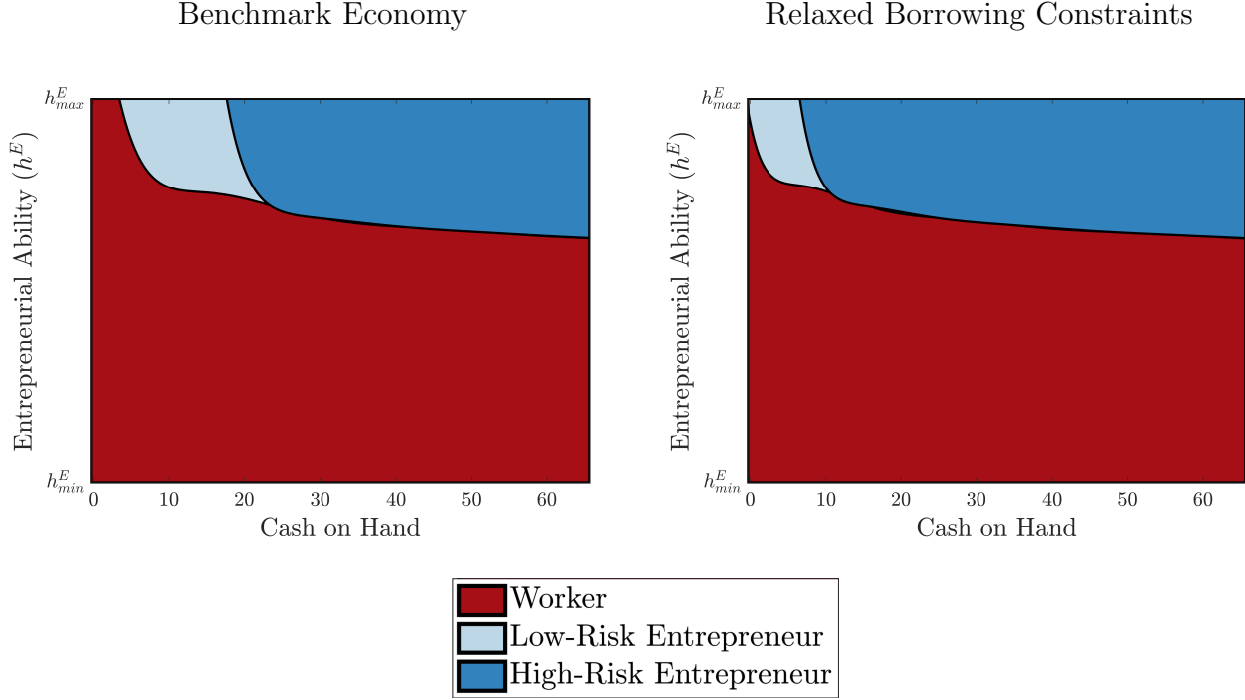
7.2 Relaxing The Borrowing Constraints

Relaxing entrepreneurs' borrowing constraints likewise increases output through the same four channels as the introduction of state-contingent assets, though the strength of these channels differ between the two counterfactuals. First, the removal of borrowing constraints encourage more, but not all, entrepreneurs to start the high risk project. Secondly, individuals with higher entrepreneurial ability are more likely to select into entrepreneurship. Third, conditional on wealth, entrepreneurs invest more in their businesses. Finally, because entrepreneurs earn higher incomes as a result of all three previous reasons, they accumulate more wealth and are thus able to run larger businesses, despite the missing market for entrepreneurial risk.

Figure 13 shows the pattern of occupational choice and risk choice in economies with and without borrowing constraints. Without borrowing constraints, many more wealth-poor individuals with high entrepreneurial ability switch from being workers to being entrepreneurs. They do so because they are now able to operate much larger scale businesses and so generate far more entrepreneurial income.

Relaxing the borrowing constraints also induced relatively wealth-poor entrepreneurs to switch from the low-risk project to the high risk project. With borrowing constraints, if a wealth-poor entrepreneur starts a high risk project, receives a bad productivity shock and loses wealth either through consuming out of their savings or because they must

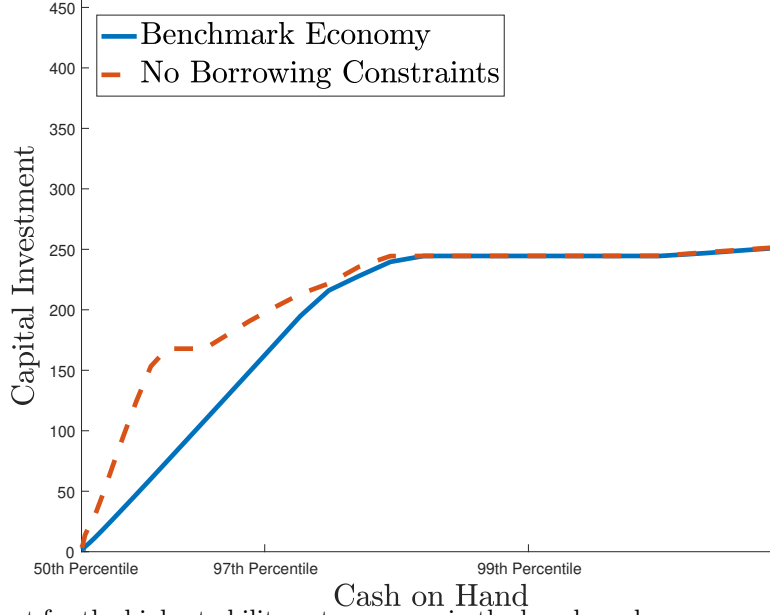
Figure 13: Comparing Occupational Choice and Endogenous Risk Choice



liquidate their capital stock, the next business they start must be operated at an even smaller scale because of their limited wealth. Without borrowing constraints, this negative effect on serial entrepreneurship doesn't take place, as entrepreneurs are able to operate subsequent businesses at any scale. As a consequence removing the borrowing constraints increases the proportion of entrepreneurs who operate the high-risk project.

Figure 14 shows the investment patterns for the highest productivity entrepreneur in the benchmark and unrestricted investment economies. Wealth poor individuals in the unrestricted investment economy are able to invest more than those in the benchmark economy because of the lack of borrowing constraint. However, because of the substantial capital investment risk, they still invest less than the expected marginal product of capital.

Figure 14: Capital Investment for the Highest Ability Entrepreneur



Capital investment for the highest ability entrepreneur in the benchmark economy with both frictions (solid blue line) and in the economy with no borrowing constraints ($\phi = 1$) (dashed orange). The x-axis labels the cash on hand required to be in the top 50%, top 5% and top 1% of the wealth distribution in the benchmark economy.

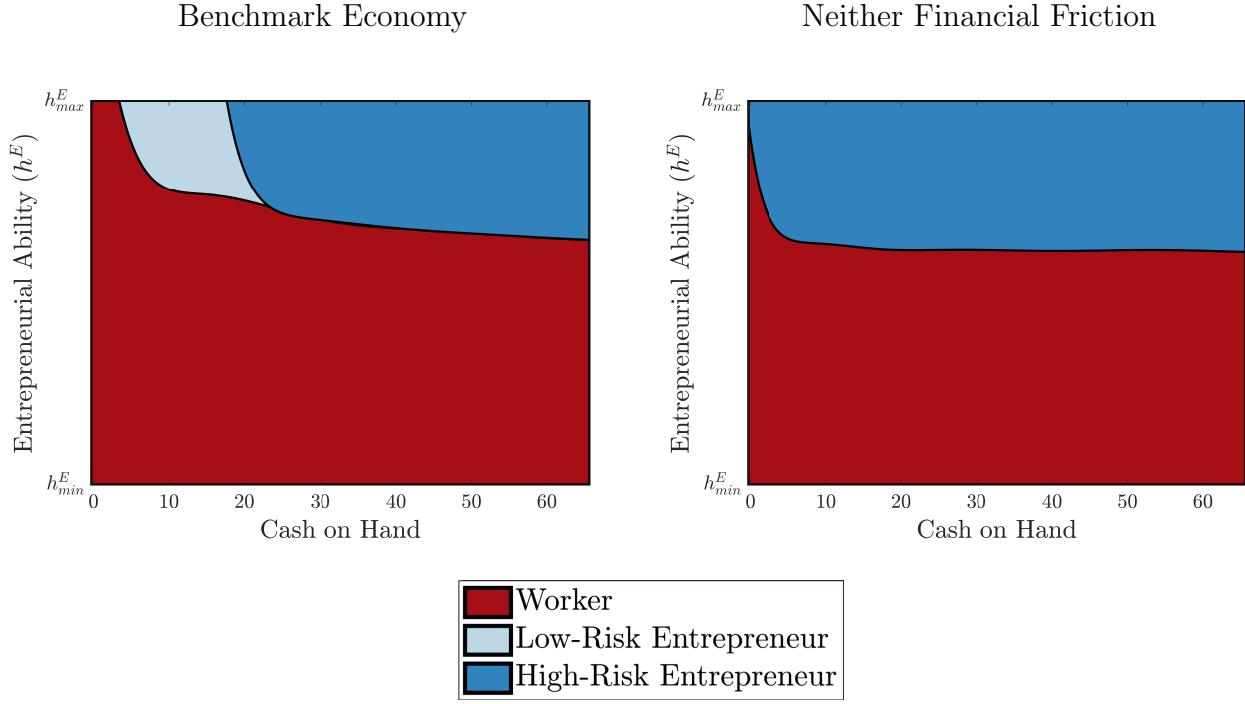
7.3 Neither Financial Friction

An economy without either of the financial frictions has higher output through the same four channels, though the effects are amplified by the interaction between the two financial frictions. First, the state-contingent assets allow every entrepreneur to start the high-risk project. Secondly, individuals with higher entrepreneurial ability are much more likely to select into entrepreneurship regardless of their wealth. Third, conditional on wealth, entrepreneurs invest much more in their businesses. Finally, because entrepreneurs earn higher incomes as a result of all three previous reasons, they accumulate more wealth though this has a greatly diminished role in their decision making.

Figure 15 compares the pattern of occupational choice and risk choice in an economy with both financial frictions to an economy with neither. In the economy with neither friction, the availability of state-contingent assets mean that no entrepreneurs start the low-risk business. With neither financial friction, wealth's role in determining whether someone chooses to be an entrepreneur or a worker is greatly diminished. There is a still a small effect due to the unsecured borrowing limit, but this is relatively minor compared to the previous economies.

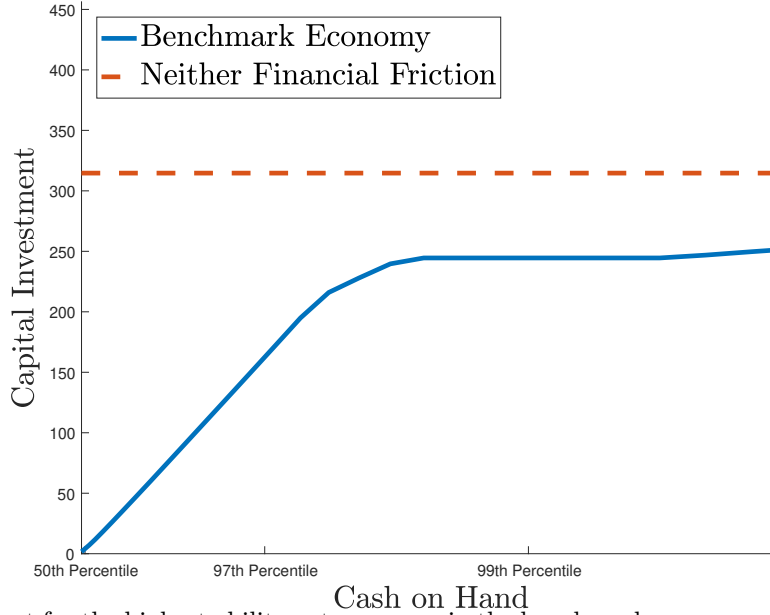
Figure 16 shows the investment patterns for the highest productivity entrepreneur in the benchmark economy and in the economy with neither financial friction. In the absence of both financial frictions, entrepreneurs all invest up to the point where the expected

Figure 15: Comparing Occupational Choice and Endogenous Risk Choice



marginal product of capital is equal to the interest rate, regardless of their personal net worth.

Figure 16: Capital Investment for the Highest Ability Entrepreneur



Capital investment for the highest ability entrepreneur in the benchmark economy with both frictions (solid blue line) and in the economy neither financial frictions (dashed orange) so agents have access to contingent claims and can borrow any amount of capital ($\phi = 1$). The x-axis labels the cash on hand required to be in the top 50%, top 5% and top 1% of the wealth distribution in the benchmark economy.

8 Policy Analysis

The quantitative results in section 7 highlight how important the missing market for entrepreneurial risk is for discouraging entrepreneurial activity. Providing additional insurance to entrepreneurs may therefore encourage risk-taking. However, any government policy that seeks to provide partial insurance to entrepreneurs is likely to run into the same adverse selection and moral hazard problems that prevent a private sector insurance market for entrepreneurial risk from existing.

I therefore study a policy in the presence of adverse selection designed to encourage risk taking by entrepreneurs. The policy is a unemployment-style benefit that tops up the incomes of unsuccessful entrepreneurs. Adverse selection is a problem with this scheme if many workers with low entrepreneurial ability become entrepreneurs. They may do so if the benefit is larger than their labour market opportunities, even if they have no expectation of being able to generate any income as entrepreneurs.

I model a simple unemployment insurance benefit for entrepreneurs as a transfer that tops up entrepreneurial income to an income floor given by \underline{y} . This benefit is paid for with a lump-sum tax T on all other entrepreneurs that exactly clears the government's budget constraint. An entrepreneur's income is therefore given by:

$$y = \max((zh^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn + r^a a - T, \underline{y}) \quad (26)$$

Table 8: Aggregate Moments With and Without the Unemployment Benefit for Entrepreneurs

	Output	Wages	Capital	Productivity	Workers	Entrepreneurs	Low Risk	High Risk
Benchmark Economy	1.00	1.00	1.00	1.00	86.1	13.9	9.9	3.9
With the Benefit	1.08	1.06	1.00	1.07	86.7	13.3	0.0	13.3

Aggregate moments across economies with different levels of entrepreneurial unemployment benefits. All benefits are expressed in terms of the equilibrium wage in the benchmark economy (\bar{w}) without any unemployment benefits. The first four columns list output (Y), wage (w), capital stock (K), and total factor productivity (TFP) relative to the benchmark economy. The latter four columns report the percentage of agents in the economy working as workers or entrepreneurs and the percentage of agents in the economy that are entrepreneurs operating the low risk and the high risk project.

Table 8 compares the aggregate moments of the benchmark economy with the economy with an entrepreneurial income floor (y) worth half the average wage in the benchmark. The benefit is successful at encouraging all of the entrepreneurs to switch from the low-risk project to the high-risk project. This substantially increases aggregate productivity. Aggregate output increases by 8%.

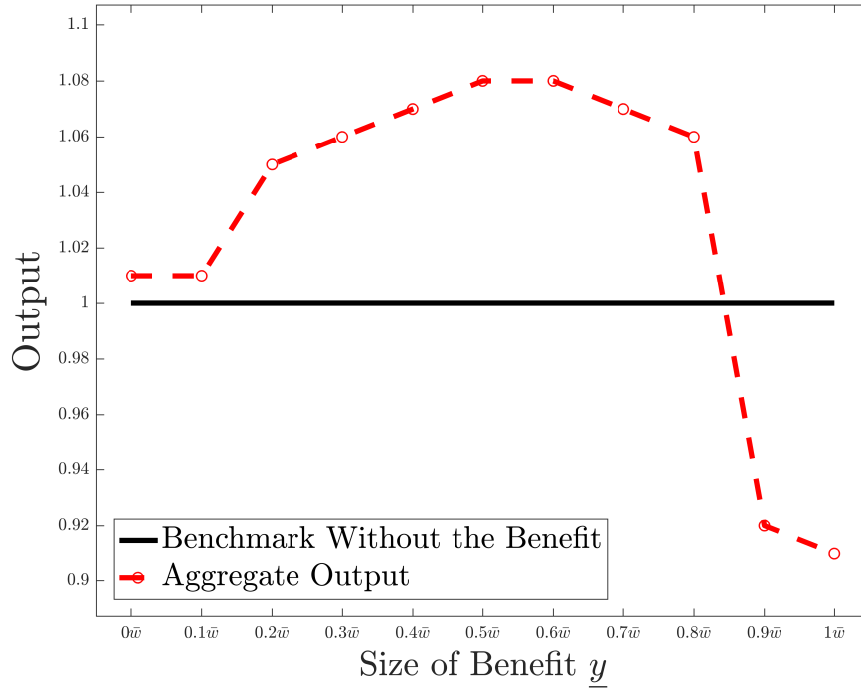
How is this partial insurance scheme able to generate such a large impact on aggregate output by only partially completing the market? The answer is that the benefit actually helps alleviate both financial frictions. In the benchmark economy, some high ability entrepreneurs start businesses receive bad productivity shocks and exit. As they do so, they receive low income for at least one period and also lose a portion of their capital stock as they liquidate their businesses to exit. They exit with lower wealth and so if they choose to start a new business it will be of a smaller scale than they were previously able to operate. In the economy with the benefit, not only does the benefit provide partial insurance against bad shocks, but it also helps re-capitalize unsuccessful entrepreneurs.

Figure 18 shows aggregate output as a function of the size of the unemployment benefit for entrepreneurs (y). At low levels, the benefit raises output in the economy. Even small insurance benefits are enough to encourage some entrepreneurs to choose the higher risk project. As the higher risk project has higher expected productivity output increases. Labour demand also increases raising the wage, and as a consequence the proportion of entrepreneurs initially falls.

At much higher levels of benefit, adverse selection becomes an increasing problem. Individuals who have low entrepreneurial ability enter simply because the income floor is higher than what they can earn in the labour market. As a consequence output falls as worse entrepreneurs require the benefit.

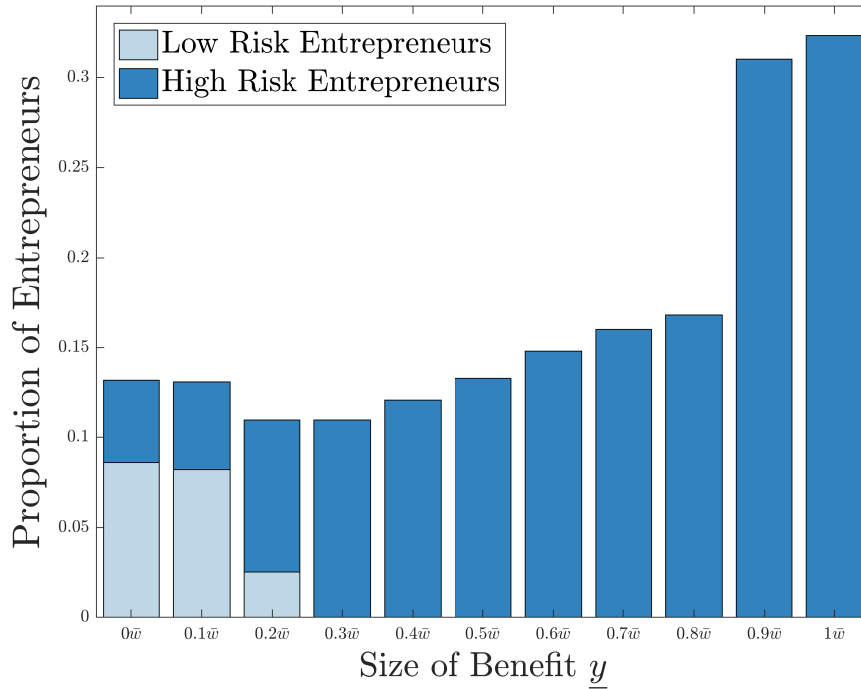
This simple insurance scheme is not an optimal policy and it ignores possible moral hazard problems that may arise in the real world. However, it does illustrate that well-

Figure 17: Aggregate Output Depending on the Size of Benefit



\underline{y} is the income floor given to all unsuccessful entrepreneurs. \bar{w} is the average wage in the benchmark economy without the benefit.

Figure 18: Proportion of Entrepreneurs Depending on the Size of Benefit



\underline{y} is the income floor given to all unsuccessful entrepreneurs. \bar{w} is the average wage in the benchmark economy without the benefit.

designed partial insurance schemes can help governments encourage entrepreneurial risk taking, even in a context with adverse selection. The quantitative results in section 7 suggest that seeking to provide partial insurance schemes to entrepreneurs has much greater potential to raise output and productivity than alleviating entrepreneur’s borrowing constraints.

9 Conclusion

My paper studies the quantitative importance of two financial frictions for output, productivity, and wealth inequality. My paper contributes to a large literature studying how financial frictions distort the decisions of entrepreneurs and how those distortions reduce aggregate output. I study an understudied financial friction, the missing market for entrepreneurial risk, and compares it to the well studied effects of borrowing constraints. I find that the missing market for risk causes larger aggregate productivity losses than borrowing constraints and that its interaction effect with these constraints is important.

I present descriptive evidence from the Kauffman Firm Survey that new entrepreneurs face a high degree of idiosyncratic risk and that entrepreneurs who invest more of their own money are more likely to raise external funds. Motivated by these facts, I build a model of occupational choice and business risk choice. I calibrate the strength of the two financial frictions in my model to micro data on new U.S. firms. I then validate the model by showing that the model can generate a number of untargeted patterns in the data. In the calibrated model both financial frictions play an important role distorting individuals decisions. Removing each of the financial frictions, I study changes in the patterns of occupational selection, the choice of the riskiness of businesses, and investment. Finally, I study a policy that provides partial insurance to entrepreneurs that encourages them to take more risk.

Governments around the world seek to balance redistribution with the promotion of entrepreneurship. This paper’s results suggest that a promising area of future work would be to study the design of partial insurance schemes that encourage more individuals to take the risk of becoming an entrepreneur and encourage entrepreneurs to pursue more innovative business ideas. The results of this paper suggest that this type of policy can both increase economic efficiency and reduce inequality.

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A Static Model Proofs

A.1 Proposition 1

Optimal risk taking x^* is decreasing in the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$

$$\frac{\partial x^*}{\partial \frac{u'(\underline{c})}{u'(\bar{c})}} = - \frac{z \left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \frac{1}{\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right)^{\frac{1}{\gamma}-1}}{1 + \psi \left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right)^{\frac{1}{\gamma}}} - z \psi \left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \frac{1}{\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right)^{\frac{1}{\gamma}-1} \frac{1 - \left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right)^{\frac{1}{\gamma}}}{\left[1 + \psi \left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right)^{\frac{1}{\gamma}} \right]^2} < 0 \quad (27)$$

Optimal capital investment k^* is also decreasing in the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$

$$\begin{aligned} \frac{\partial k^*}{\partial \frac{u'(\underline{c})}{u'(\bar{c})}} &= \frac{1}{1-\alpha} \left(\frac{\alpha}{[1+r^a]} \frac{\left[p(h^E + \psi x)^{1-\gamma} + (1-p)(h^E - x)^{1-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) \right]}{\left[p + (1-p) \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) \right]} \right)^{\frac{1}{1-\alpha}-1} \frac{\alpha}{1+r^a} \\ &\times \left[\frac{(1-p)(h^E - x)^{1-\gamma}}{p + (1-p) \frac{u'(\underline{c})}{u'(\bar{c})}} - (1-p) \frac{\left[p(h^E + \psi x)^{1-\gamma} + (1-p)(h^E - x)^{1-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) \right]}{\left[p + (1-p) \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) \right]^2} \right] < 0 \end{aligned} \quad (28)$$

Cancelling positive terms:

$$\left[\frac{(1-p)(h^E - x)^{1-\gamma}}{p + (1-p) \frac{u'(\underline{c})}{u'(\bar{c})}} - (1-p) \frac{\left[p(h^E + \psi x)^{1-\gamma} + (1-p)(h^E - x)^{1-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) \right]}{\left[p + (1-p) \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) \right]^2} \right] < 0 \quad (29)$$

Rearranging:

$$(1-p)(h^E - x)^{1-\gamma} \left[p + (1-p) \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) \right] < (1-p) \left[p(h^E + \psi x)^{1-\gamma} + (1-p)(h^E - x)^{1-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) \right] \quad (30)$$

$$p(h^E - x)^{1-\gamma} + (1-p)(h^E - x)^{1-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) < p(h^E + \psi x)^{1-\gamma} + (1-p)(h^E - x)^{1-\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right) \quad (31)$$

$$p(h^E - x)^{1-\gamma} < p(h^E + \psi x)^{1-\gamma} \quad (32)$$

$$-x < \psi x \quad (33)$$

Taking the derivative of the ratio of marginal utilities w.r.t. endowed wealth:

$$\begin{aligned} \frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} &= \frac{1}{u'(\bar{c})} u''(\underline{c}) \left(-(h^E - x)^{-\gamma} (1 - \gamma) k^\alpha \frac{\partial x}{\partial e} + [(h^E - x)^{1-\gamma} \alpha k^{\alpha-1} - (1 + r^a)] \frac{\partial k}{\partial e} + [1 + r^a] \right) \\ &- \frac{u'(\underline{c})}{[u'(\bar{c})]^2} u''(\bar{c}) \left(\psi(h^E + \psi x)^{-\gamma} (1 - \gamma) k^\alpha \frac{\partial x}{\partial e} + [(h^E + \psi x)^{1-\gamma} \alpha k^{\alpha-1} - (1 + r^a)] \frac{\partial k}{\partial e} + [1 + r^a] \right) \end{aligned} \quad (34)$$

Rearranging:

$$\begin{aligned} \frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} &= (1 + r^a) \left(\frac{u''(\underline{c})}{u'(\underline{c})} - \frac{u''(\bar{c})}{u'(\bar{c})} \right) \\ &+ \frac{\partial x}{\partial e} (1 - \gamma) k^\alpha \left(-(h^E - x)^{-\gamma} \frac{u''(\underline{c})}{u'(\underline{c})} - \psi(h^E + \psi x)^{-\gamma} \frac{u''(\bar{c})}{u'(\bar{c})} \right) \\ &+ \frac{\partial k}{\partial e} \left([(h^E - x)^{1-\gamma} \alpha k^{\alpha-1} - (1 + r^a)] \frac{u''(\underline{c})}{u'(\underline{c})} - [(h^E + \psi x)^{1-\gamma} \alpha k^{\alpha-1} - (1 + r^a)] \frac{u''(\bar{c})}{u'(\bar{c})} \right) \end{aligned} \quad (35)$$

Since $\bar{c} > \underline{c}$, decreasing absolute risk aversion $\frac{\partial}{\partial c} \left(-\frac{u''(c)}{u'(c)} \right) < 0$ implies that:

$$-\frac{u''(\underline{c})}{u'(\underline{c})} > -\frac{u''(\bar{c})}{u'(\bar{c})} \implies \frac{u''(\underline{c})}{u'(\underline{c})} < \frac{u''(\bar{c})}{u'(\bar{c})} \quad (36)$$

As a consequence the first line of 35 is negative. Diminishing marginal utility ($u''(c) < 0$) implies that the term multiplying $\frac{\partial x}{\partial e}$ on the second line is positive. Likewise, diminishing marginal utility and the fact that $(h^E - x)^{1-\gamma} \alpha k^{\alpha-1} < (1 + r^a)$ implies that the term multiplying $\frac{\partial k}{\partial e}$ on the third line is positive.

Suppose now that the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$ weakly increasing in endowed wealth e :

$$\frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} \geq 0 \quad (37)$$

This implies that:

$$\frac{\partial x^*}{\partial e} = \frac{\partial x^*}{\partial \frac{u'(\underline{c})}{u'(\bar{c})}} \frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} \leq 0 \quad (38)$$

and that:

$$\frac{\partial k^*}{\partial e} = \frac{\partial k^*}{\partial \frac{u'(\underline{c})}{u'(\bar{c})}} \frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} \leq 0 \quad (39)$$

But if this is true, then all three terms in 35 are weakly negative with the first term

strictly negative. This is a contradiction, so it must be that:

$$\frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} < 0 \quad (40)$$

From 27 and 40 it must be that $\frac{\partial x}{\partial e} > 0$ \square

A.2 Proposition 2

This argument is originally from Cressy (2000). For any agent on the margin between choosing to be a worker or an entrepreneur:

$$V^W(e) = V^E(e, z)$$

$$u(w + (1 + r^a)e) = pu \left[(h^E + \psi x^*)^{1-\gamma} k^{*\alpha} + (1 + r^a)a^* \right] + (1 - p)u \left[(h^E - x^*)^{1-\gamma} k^{*\alpha} + (1 + r^a)a^* \right]$$

Increases in endowed wealth will raise the value of entrepreneurship relative to being a worker iff:

$$\frac{\partial V^W}{\partial e} < \frac{\partial V^E}{\partial e} \quad (41)$$

Applying the envelope theorem:

$$u'(w + (1 + r^a)e) < pu'(\bar{c}) + (1 - p)u'(\underline{c}) \quad (42)$$

For the marginal agent, it must be that $\underline{c} < w + (1 + r^a)e < \bar{c}$. Applying Jensen's inequality to the marginal utility function, 42 is true if the marginal utility function is convex. Preferences that exhibit decreasing absolute risk aversion will have:

$$\frac{\partial}{\partial e} \left(-\frac{u''}{u'} \right) = - \left(\frac{u'''u' - (u')^2}{(u')^2} \right) < 0 \text{ only if } u''' > 0 \quad (43)$$

A.3 Proposition 3

$$\frac{\partial x}{\partial \phi} = h^E \left(-\frac{\left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \frac{1}{\gamma} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right)^{\frac{1}{\gamma}-1}}{1 + \psi \left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right)^{\frac{1}{\gamma}}} - \psi \left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \frac{1 - \left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right)^{\frac{1}{\gamma}}}{\left[1 + \psi \left(\frac{1-p}{\psi p} \right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})} \right)^{\frac{1}{\gamma}} \right]^2} \right) \frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial \phi} < 0 \quad (44)$$

$$\begin{aligned} \frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial \phi} &= \frac{u''(\underline{c})}{u'(\bar{c})} \left((z-x)^{1-\gamma} \alpha k^{\alpha-1} - (1+r^a) \right) \frac{e}{(1-\phi)^2} \\ &\quad - \frac{u'(\underline{c})}{[u'(\bar{c})]^2} u''(\bar{c}) \left((z+\psi x)^{1-\gamma} \alpha k^{\alpha-1} - (1+r^a) \right) \frac{e}{(1-\phi)^2} > 0 \end{aligned} \quad (45)$$

Rearranging:

$$\left[\frac{u''(\underline{c})}{u'(\underline{c})} \left((z-x)^{1-\gamma} \alpha k^{\alpha-1} - (1+r^a) \right) - \frac{u''(\bar{c})}{u'(\bar{c})} \left((z+\psi x)^{1-\gamma} \alpha k^{\alpha-1} - (1+r^a) \right) \right] \frac{e}{(1-\phi)^2} > 0 \quad (46)$$

$$(1+r^a) \left(\frac{u''(\bar{c})}{u'(\bar{c})} - \frac{u''(\underline{c})}{u'(\underline{c})} \right) + \left(\frac{u''(\underline{c})}{u'(\underline{c})} (z-x)^{1-\gamma} - \frac{u''(\bar{c})}{u'(\bar{c})} (z+\psi x)^{1-\gamma} \right) \alpha k^{\alpha-1} > 0 \quad (47)$$

Since $\bar{c} > \underline{c}$, decreasing absolute risk aversion $\frac{\partial}{\partial c} \left(-\frac{u''(c)}{u'(c)} \right) < 0$ implies that:

$$-\frac{u''(\underline{c})}{u'(\underline{c})} > -\frac{u''(\bar{c})}{u'(\bar{c})} \implies \frac{u''(\underline{c})}{u'(\underline{c})} < \frac{u''(\bar{c})}{u'(\bar{c})} \quad (48)$$

As a consequence the first term of 47 is positive, and the second term is positive for any $x \geq 0$. Therefore $\frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial \phi} > 0$.

Since the term in brackets in 45 is negative, this implies:

$$\frac{\partial x}{\partial \phi} < 0 \quad (49)$$

A.4 Proposition 4

$$\begin{aligned} \frac{\partial V^E}{\partial \phi} &= pu'(\bar{c}) \left[(z+\psi x)^{1-\gamma} \alpha k^{\alpha-1} - (1+r^a) \right] \frac{e}{(1-\phi)^2} \\ &\quad + (1-p)u'(\underline{c}) \left[(z-x)^{1-\gamma} \alpha k^{\alpha-1} - (1+r^a) \right] \frac{e}{(1-\phi)^2} > 0 \end{aligned} \quad (50)$$

Rearranging:

$$\alpha k^{\alpha-1} \left(pu'(\bar{c})(z+\psi x)^{1-\gamma} + (1-p)u'(\underline{c})(z-x)^{1-\gamma} \right) > (1+r^a) \left(pu'(\bar{c}) + (1-p)u'(\underline{c}) \right) \quad (51)$$

$$\left[\frac{\alpha}{(1+r^a)} \frac{(pu'(\bar{c})(z+\psi x)^{1-\gamma} + (1-p)u'(\underline{c})(z-x)^{1-\gamma})}{(1+r^a)(pu'(\bar{c}) + (1-p)u'(\underline{c}))} \right]^{\frac{1}{1-\alpha}} > k \quad (52)$$

Note that the left hand side of this express is the expression for k^* from 9, and all constrained entrepreneurs will have $k < k^*$.

B Kauffman Firm Survey Facts

B.1 Measuring Survival

In table 9, I report the current operational status of all firms in the Kauffman Firm Survey over the 8 years of the survey. Firm exit is common. While non-response and sample attrition mean that the status of 701 firms are not available in the final year, 1,901 firms or 45% of the firms with known status have permanently shut down by the end of the sample. An additional 30 were temporarily shut down.

Table 9: Business Status Over Time

Year	Operating	Shut Down	Merged or Sold	Temp Shut Down	Unknown
2004	4,928	0	0	0	0
2005	3,998	260	43	66	561
2006	3,390	581	90	124	743
2007	2,915	880	135	98	900
2008	2,606	1,224	175	58	865
2009	2,408	1,474	211	41	794
2010	2,126	1,692	249	45	816
2011	2,007	1,901	289	30	701

Tabulation of the operational status of firms in the Kauffman Firm Survey over the first eight years of operation. Note that when a firm is merged or sold, it exits the sample and so no more information about its operational status is available.

B.2 Entrepreneur's Working Hours

Table 10 shows that over the first eight years of operation, about half of the entrepreneurs pay themselves a salary.

Even for those entrepreneurs who are paying themselves a salary, entrepreneurs may not be paying themselves the full opportunity costs of their time. [Hall and Woodward \(2010\)](#) document that venture-capital backed entrepreneurs are typically paid less than their outside option in the labour market in order to encourage effort. More generally, if borrowing constraints are binding, entrepreneurs could pay themselves less in order to

Table 10: Entrepreneurs Paying Themselves a Salary by Year

Year	Proportion of Entrepreneurs Receiving a Salary	Number of Entrepreneurs
2004	0.47	6,916
2005	0.53	5,673
2006	0.55	4,776
2007	0.55	4,057
2008	0.53	3,617
2009	0.54	3,304
2010	0.53	2,859
2011	0.51	2,715

save within the firm and accumulate additional capital. In both cases, lower compensation during the start up period is compensated for by higher returns later. Of course, if the firm exits before those returns materialize, the entrepreneur suffers a real economic loss. Note here the potential interaction of borrowing constraints and the missing market for entrepreneurial risk.

Tables 11 and 12 compare the proportion of entrepreneurs who report paying themselves a salary based on their weekly hours of work in the first and final years. Only a third of the entrepreneurs working less than 25 hours a week are paying themselves a salary. For entrepreneurs working more than 35 hours a week, 57% are paying themselves a salary in the first year, while 68% are paying themselves a salary in the eighth year. Hours worked are self-reported usual hours worked. Note that both of these graphs include only the survey-respondent entrepreneur, rather than all of the entrepreneurs working on a business.

Table 11: Entrepreneurs Paying Themselves a Salary by Hours Worked in the First Year

Year	Proportion of Entrepreneurs Receiving a Salary	Number of Entrepreneurs
<25	0.33	1,288
25-35	0.43	442
35-44	0.54	655
45-54	0.56	758
55-65	0.58	887
65<	0.58	780
Total	0.49	4,900

Table 12: Entrepreneurs Paying Themselves a Salary by Hours Worked in the Eighth Year

Year	Proportion of Entrepreneurs Receiving a Salary	Number of Entrepreneurs
<25	0.33	569
25-35	0.52	174
35-44	0.62	303
45-54	0.68	344
55-65	0.73	313
65<	0.70	182
Total	0.56	1,892

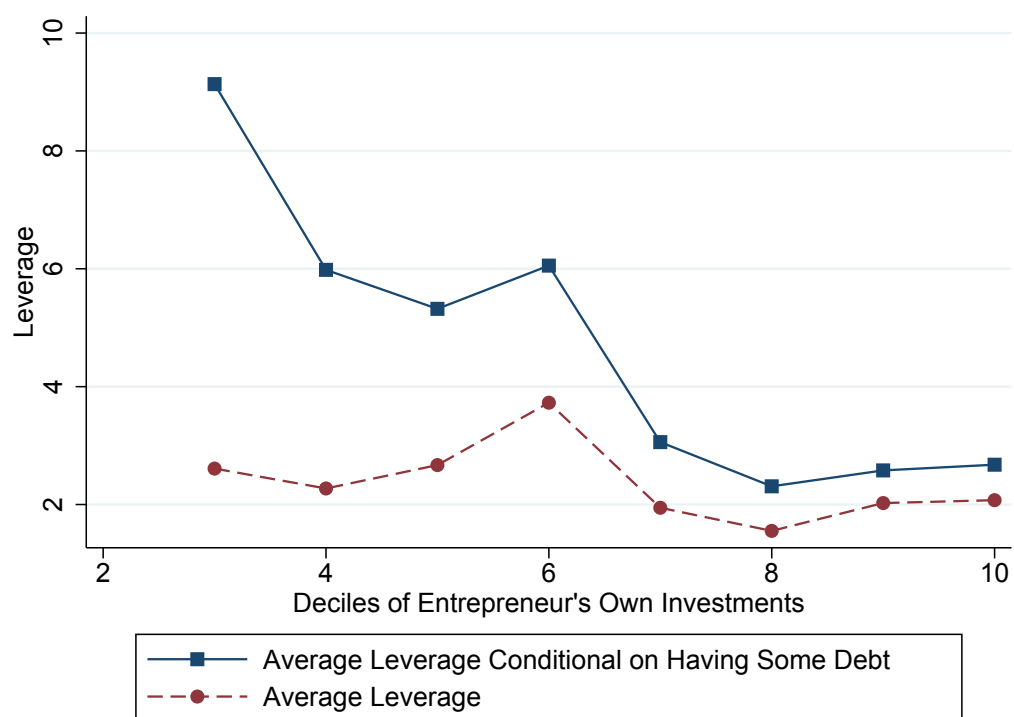
B.3 Sources of Funding Details

New firms in the Kauffman Firm Survey raise their initial funds from a variety of sources. Table 13 updates a similar table in [Robb and Robinson \(2014\)](#) with the full sample of KFS data. By far the most common source of funding is the entrepreneurs' own investment. For 89% of firms, entrepreneurs are putting their own money into the business, these investments are often modest, with the median amount being only \$25,000.

Note that while 53% of firms are able to borrow, most of these loans are personal debt. If the business is unsuccessful and entrepreneurs choose to exit, these debts cannot be discharged without filing for personal bankruptcy. Raising external equity is much less common, with only 15% of firms raising equity beyond the actively managing owners, though these firms typically invest substantially more than those without any external equity.

Figure 19 shows the average leverage ratio of firms. I exclude the bottom two deciles of initial investment for legibility. Conditional on having some debt, the leverage ratio decreases sharply over the distribution of initial investment. Given the the proportion of firms who take out some debt increases sharply over the distribution of initial investment (see figure 4), the unconditional mean leverage ratio is relatively flat.

Figure 19: Leverage



The debt to equity ratio across the distribution of initial investment. Firms are sorted into deciles based on the total amount of equity invested in the firm in the first year of operations. The bottom two deciles are excluded for legibility.

Table 13: Sources of Funding for Firms in the Kauffman Firm Survey

Source	Percentage with > 0	Mean given > 0	Percentiles			Number of Firms
			25 th	50 th	75 th	
Own Equity	89	166	8	25	80	3,655
External Equity	15	1,151	10	50	203	640
Outside Investors	6	758	10	90	290	247
Parents	5	83	10	25	73	242
Other Companies	4	1,565	20	100	600	154
Spouses	3	82	5	15	30	137
Venture capital	1	8,669	85	450	6,125	60
Government Agencies	1	698	53	250	850	44
Other	1	995	10	25	135	23
Any Debt	53	714	15	55	210	2,302
Personal Debt	46	405	10	40	128	2,011
Bank	32	308	15	50	140	1,413
Family	20	51	5	15	46	938
Other Individuals	6	434	4	15	50	252
Any Other Sources	3	1,489	10	33	110	160
Business Debt	28	701	15	50	211	1,264
Bank	17	873	26	84	269	747
Family	8	84	5	15	45	356
Non-Bank Financial	6	357	12	48	200	262
Owners	4	331	15	48	200	158
Government Agencies	2	1,052	30	125	330	89
Other Individuals	2	153	5	24	100	74
Employees	1	56	5	14	40	38
All Funding Sources	94	520	10	45	181	3,687

The sources of funding for firms in the Kauffman Firm Survey. All values are in thousands of US dollars and are cumulative over the first eight years of operation. The first column reports the percentage of firms that received any money from each funding source, the second column gives the mean amount raised conditional on raising some money from that source, the third through fifth columns give the 25th, 50th and 75th percentiles of the amount raised conditional on raising some money from that source. The final column gives the number of firms that raise any money from each funding sources.

“Personal Debt” is debt owed by an entrepreneur, while “Business Debt” is debt owned by the business.

B.4 Dispersion in Cumulative Sales

In subsection 3.3, I showed that the level and dispersion of cumulative profits are significantly higher for high initial own investment firms than for low initial own investment firms. In this subsection, I test whether the same holds for the level and dispersion of cumulative sales.

$$\text{Cumulative Sales}_i = \alpha_0 + \alpha_1 \text{Initial Owner Investment}_t + \alpha X_i + \epsilon_i \quad (53)$$

$$\hat{\epsilon}_i^2 = \beta_0 + \beta_1 \text{Initial Owner Investment}_t + \beta X_i + \eta_i \quad (54)$$

Table 14 shows that firms with higher initial own investments have significantly higher cumulative sales, even after controlling for initial employment, external sources of funding and industry. In the second regression, firms with higher initial own investments have more significantly more dispersed cumulative sales.

Table 14: Dispersion of Cumulative Sales

Regression (53):	Dependent Variable: Cumulative Profits				
	(1)	(2)	(3)	(4)	(5)
Own Investment in First Year	3.988*** (0.245)	3.692*** (0.235)	3.692*** (0.235)	4.823*** (0.523)	5.156*** (0.582)
Average Employment		1022.0*** (55.83)	1044.3*** (58.37)	1057.5*** (58.77)	1040.5*** (60.21)
Average Employment ²		-1.883*** (0.187)	-1.973*** (0.192)	-1.965*** (0.192)	-1.929*** (0.194)
Employer			-781.0 (914.9)	-760.7 (917.6)	-744.5 (917.6)
Total Investment in First Year				-1.154* (0.476)	-1.499** (0.545)
Total Debt in First Year					1.731 (1.331)
2 Digit NAICs Codes	No	No	Yes	Yes	Yes
Breush-Pagan	3279.7	23612.6	27030.8	31440.3	31438.7
Regression (54):	Dependent Variable: Squared Predicted Residuals ($\hat{\epsilon}_i^2$)				
	(1)	(2)	(3)	(4)	(5)
Own Investment in First Year	0.52*** (0.14)	0.39** (0.13)	0.39** (0.13)	1.50*** (0.28)	1.50*** (0.31)
Average Employment		277.58*** (30.58)	285.09*** (31.71)	296.49*** (31.66)	296.26*** (32.44)
Average Employment ²		-0.66*** (0.10)	-0.69*** (0.10)	-0.68*** (0.10)	-0.68*** (0.10)
Employer			-639.92 (497.11)	-613.72 (494.34)	-613.91 (494.44)
Total Investment in First Year				-1.13*** (0.26)	-1.13*** (0.29)
Total Debt in First Year					0.02 (0.72)
2 Digit NAICs Codes	No	No	Yes	Yes	Yes
Observations	4098	4092	4092	4076	4076

B.5 Calculating Rates of Return

In subsection 3.2 and B.4, I showed that the level and dispersion of cumulative profits and cumulative sales are significantly higher for high initial own investment firms than for low initial own investment firms. I would also like to test whether the firm's rates of return follows the same pattern.

Given that many of these firms are relatively small scale, it is vital to include both salaries and forgone earnings in this calculation. Omitting entrepreneurial salaries may substantially bias the measure, as the split between dividends and salaries is more likely to depend on tax code provisions or contracting frictions with passive owners than on the relative economy value of an entrepreneur's hours vs. financial contribution to the business. Omitting forgone labour earnings is likewise essential, as for many small-scale firms the value of the entrepreneur's time is a major input, if not bigger than the financial investment, in the business. Ignoring these inputs would greatly inflate the rates of return earned by small scale firms. Therefore, I construct cumulative rates of return for each firm using the following equation:

$$\text{Cumulative Rate of Return} = \frac{\sum_{t=0}^7 \beta^t (\text{Dividends}_t + \text{Salary}_t) + \beta^7 \text{Firm Value}_7}{\sum_{t=0}^7 \beta^t (\text{Equity Invested}_t + \text{Forgone Salary}_t)} \quad (55)$$

- Dividends_t : Dividends are directly measured in the data by a question that asks "Thinking of calendar year 2004, how much money, if any, did you and other owners withdraw from the business for personal use? This includes any dividends paid." it should therefore include all dividend payments as well as any other cash withdrawals from the business.
- Salary_t : Information about the exact salaries paid to entrepreneurs is not available, though whether a salary was paid to each entrepreneur is recorded. In order to proxy for salaries paid, I use the total wage bill of the firm, divided by the number of employees including the salaried entrepreneurs.
- Firm Value_7 : For a termination value of the firm, I use the total assets of the firm minus the total liabilities in the final year.
- Equity Invested_t : Equity invested is directly measured in the data by a series of questions that ask how much money was received from active owners, angel investors, other companies, governments, parents of owners, spouses of owners, venture capital firms and an other category.
- Forgone Salary_t : No information about the previous labour market activities of these entrepreneurs is available. In order to proxy for their forgone labour earnings, I run a Mincerian regression in the Survey of Consumer Finances estimating total annual labour market earnings on demographics. I then use the coefficients from this regression to predict annual labour market earnings for the entrepreneurs in

the Kauffman Firm Survey. I then multiply these predicted annual labour market earnings by entrepreneur’s reported weekly hours of work.

- I discount all values using $\beta = \frac{1}{1.02}$.

To validate these rates of return, I compare the survival rates of firms in different deciles of cumulative profits to survival frequencies of firms in different deciles of cumulative rate of return. If the rates of return are accurately capturing entrepreneurial success, then the survival rates should be much more closely aligned with the rates of return than the profits.

Table 15: Survival by Cumulative Profits and Cumulative Rate of Return

Decile	Survival by Profit Decile	Number of Firms	Survival by Return Decile	Number of Firms
1	0.57	321	0.27	234
2	0.43	343	0.42	247
3	0.34	344	0.42	241
4	0.28	432	0.49	242
5	0.29	230	0.49	238
6	0.48	330	0.49	237
7	0.59	328	0.53	240
8	0.74	323	0.46	224
9	0.78	342	0.60	219
10	0.90	335	0.55	205

Table 15 shows that the calculated rates of return are not a good predictor of survival. At the bottom decile, the low calculated rates of return seem to correctly identify a group of firms that are much less likely to survive. However, throughout the higher deciles there is little increase across the distribution of calculated rates of return. I conclude that these rates of return are not a good measure of the financial success of these businesses.

B.6 Entrepreneur’s Biggest Challenge

In the 5th through 8th years of operation, entrepreneurs were asked “What was the most challenging problem your business faced in calendar year X?”. The responses are provided in table 16. In every year, less than 10% of entrepreneurs consider credit issues, either “an inability to obtain credit” or “the cost and/or terms of credit” to be their largest challenge. By contrast the majority of entrepreneurs say that their biggest challenge is either “the unpredictability of business conditions” and “slow or lost sales”, answers that may reflect the risks the firm faces. These patterns suggests that most entrepreneurs are far more concerned about the risks they face than any lack of credit. Unfortunately,

as this question was only asked for the years from 2008-2011, it is not clear how much the responses to this question are driven by aggregate risks from the US financial crisis, rather than idiosyncratic risk to their individual business. Yet as economic conditions recover after 2008, there is no increase the number of entrepreneurs who consider the lack of credit to be their primary challenge, which suggests that for many firms binding borrowing constraints may not be a major concern for entrepreneurs 8 years after starting a business.

Table 16: Entrepreneur’s Self-Reported Most Challenging Problem

Percent who say that ... is their most challenging problem	2008	2009	2010	2011
An inability to obtain credit	4	5	4	4
The cost and/or terms of credit	2	2	1	1
Slow or lost sales	53	45	42	35
The unpredictability of business conditions	24	23	26	31
Falling real estate values	5	5	4	4
Some other problem	11	13	15	16
Customers or clients not making payments or paying late	2	8	8	8
Number of Entrepreneurs	2,566	2,369	2,094	1,971

C Survey of Consumer Finances Facts

In this section I document several facts from the Survey of Consumer Finances (SCF) about the relationship between entrepreneurship and wealth. I focus on the 2004 wave of the SCF, as that is the year in which all of the businesses in the Kauffman Firm Survey are started.

C.1 Wealth Moments

In section 5, I calibrate the model to several wealth inequality moments taken from the 2004 Survey of Consumer Finances. I define entrepreneurs as households that both own a business and actively manage that business. As this paper does not study passive business ownership, I drop households from the sample if they are passive business owners, unless they own multiple businesses and actively manage at least one of them. As there is no retirement state in the model of section 4, I drop households of retirement age. For single person households, I drop the household if the individual is older than 65. For two person households, I drop the household if the average age of the two individuals is greater than 65.

Table 17 shows that the distribution of wealth does not change substantially if either retired households or passive business owners are dropped. The first column shows the top wealth shares for the whole SCF sample, designed to be representative of the US population. The second drops households of retirement age. The third column drops all passive business owners, who are not also active business owners. The fourth drops both households of retirement age and the passive business owners.

Table 17: The Distribution of Wealth

	Wealth Share of Top					Wealth Gini	Entrepreneur Wealth Gini	Proportion with Negative Net Worth
	1%	5%	10%	20%	50%			
Full Sample	33	57	69	83	97	0.79	0.75	0.07
Drop Retirement Age	34	57	69	82	97	0.80	0.75	0.08
Drop Passive Business Owners	31	56	68	82	97	0.78	0.75	0.07
Drop Retirement Age and Passive Business Owners	32	56	67	81	97	0.79	0.75	0.08
Drop All Business Owners	15	37	52	71	95	0.73	.	0.08

Top wealth shares in the US economy across several samples. “Full Sample” is the full survey. “Drop > 65” refers to the sub-sample after dropping all households of retirement age. “Drop PBO” refers to the sub-sample after dropping all households that own a business they do not actively manage without also owning a business they actively manage. “Drop Both” refers to the sub-sample after imposing both the age and passive business ownership restrictions.

In table 18, I show that two of the key targeted model moments do depend on the sub-sample chosen. Since both retired households and passive business owners own a substantial amount of wealth, their inclusion makes the wealth differences between entrepreneurs and non-entrepreneurs look less stark. I use the statistics from the fourth column for the calibration of my model.

Table 18: Model Moments by Subsample

	Ratio of Wealth of Entrepreneurs to All Others	Proportion of Entrepreneurs	Proportion of Entrepreneurs in Wealthiest 1%
Full Sample	5.70	0.11	0.60
Drop Retirement Age	6.29	0.12	0.65
Drop Passive Business Owners	6.42	0.11	0.69
Drop Retirement Age and Passive Business Owners	7.15	0.12	0.74

C.2 Wealth To Income Ratios

One key empirical fact that has motivated the study of borrowing constraints for entrepreneurs is the fact that entrepreneurs have higher wealth-to-income ratios than non-entrepreneurial households. [Quadrini \(1999\)](#) regresses the wealth to income ratios of

households in the PSID and the SCF on a binary variable for entrepreneurship, the level of income, and a polynomial in age. He finds that the coefficient on the entrepreneurship dummy to be positive and significant, suggesting that entrepreneurial households have higher average wealth-to-income ratios than other households. The first column of table 19 replicates this result with data from the 2004 Survey of Consumer Finances.

Table 19: Wealth To Income Ratios of Entrepreneurs

	(1) <u>Wealth</u> <u>Income</u>	(2) <u>Non-Bus Wealth</u> <u>Income</u>	(3) <u>Fin Wealth</u> <u>Income</u>
Entrepreneur	13.2912*** (1.4984)	2.1269 (1.2048)	0.4986 (0.4788)
Income	Yes	Yes	Yes
Age FEs	Yes	Yes	Yes
Educ FEs	Yes	Yes	Yes
Observations	4498	4498	4498

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

However, a higher average wealth-to-income ratio does not necessarily imply that entrepreneurs are subject to borrowing constraints. Higher volatility in entrepreneurial earnings can generate the same result, and DeBacker et al. (2018) shows that entrepreneurial earnings are in fact much more volatile than labour earnings. The following simple numerical example illustrates how more volatile incomes can also generate higher average wealth-to-income ratios.

Consider an economy populated by 2 workers and 2 entrepreneurs. All agents have the same level of wealth, equal to 3. All agents have the same expected income, equal to 1. Worker's income is certain and equal to one. However, entrepreneurs face some risk, and half the time receive 0.5 and half the time receive 2. Calculating the average of the wealth-to-income ratios for the two types of workers will give:

$$\overline{W/I}^{\text{Worker}} = \frac{1}{2} \left[\frac{3}{1} + \frac{3}{1} \right] = 3$$

$$\overline{W/I}^{\text{Entrepreneur}} = \frac{1}{2} \left[\frac{3}{0.5} + \frac{3}{1.5} \right] = 4$$

In this simple example the entrepreneurs average wealth-to-income ratio is much higher than the workers because of their more volatile income, despite the fact that both groups have the same average income and average wealth. To determine whether this arithmetic consideration drives the regression result, table 20 compares the average wealth-to-income

ratios ($\overline{W}/\overline{I}$) and the ratio of average wealth to average income (\bar{W}/\bar{I}) for entrepreneurial households and non-entrepreneurial households.

$$\overline{W}/\overline{I} = \frac{1}{n} \sum_{i=1}^n \frac{W_i}{I_i} \quad (56)$$

$$\bar{W}/\bar{I} = \frac{\frac{1}{n} \sum_{i=1}^n W_i}{\frac{1}{n} \sum_{i=1}^n I_i} \quad (57)$$

Table 20: Wealth to Income Comparison

	Average Wealth to Income Ratio (\bar{W}/\bar{I})		Ratio of Average Wealth to Average Income ($\overline{W}/\overline{I}$)	
	Entrepreneurs	Others	Entrepreneurs	Others
99.5% - 100%	57.9	39.0	16.9	15.4
99% - 99.5%	48.9	27.0	15.3	11.8
95% - 99%	19.7	25.9	11.1	11.8
90% - 95%	15.9	14.5	8.9	9.3
80% - 90%	9.9	12.0	6.6	6.8
60% - 80%	5.9	7.0	3.9	4.0
40% - 60%	3.6	3.3	2.2	2.0
20% - 40%	1.3	1.2	0.8	0.7
0% - 20%	-0.3	-0.2	-0.3	-0.2
Total	12.4	4.8	6.4	2.7

Table 20 shows that the differences between entrepreneurial households and non-entrepreneurial households are smaller when calculated with the ratio of average wealth to average income, rather than the average wealth-to-income ratio. This suggests that the more volatile nature of entrepreneurial income is at least one part of the explanation of this result that entrepreneurs typically have higher wealth-to-income ratios.

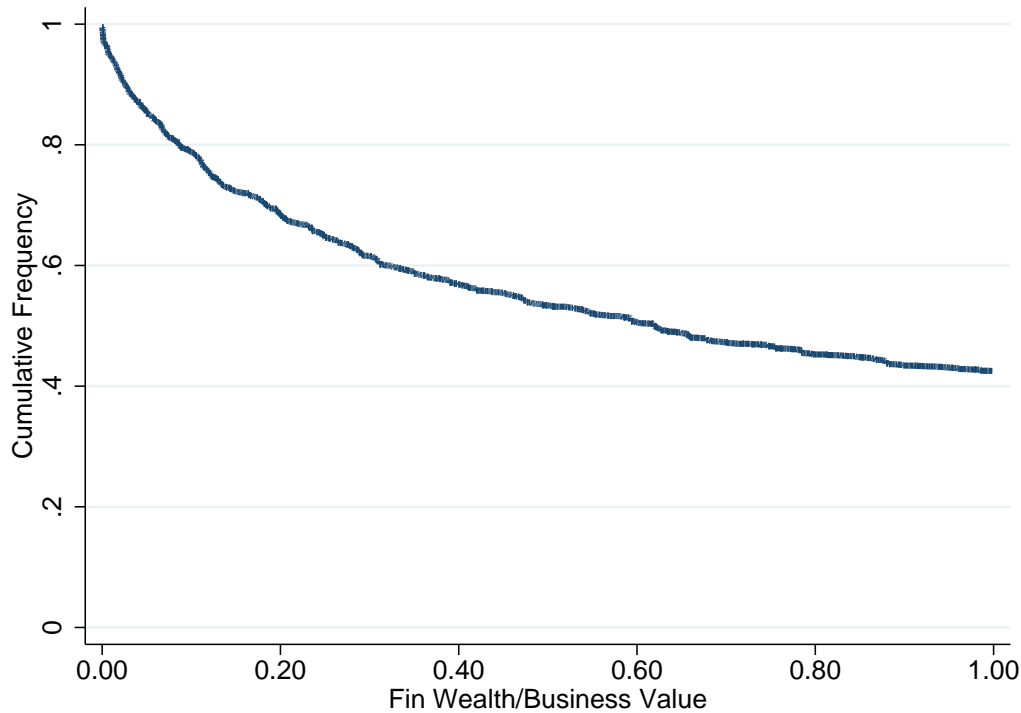
C.3 Entrepreneurial Wealth Composition

If an entrepreneur is borrowing constrained, they should have relatively few financial assets outside their business. If borrowing constraints raise the cost of external financing, entrepreneurs should liquidate most of their financial assets in order to fund their business internally. One measure of the strength of borrowing constraints that entrepreneurs face is then the proportion of their assets they keep outside the business.

To assess this, I calculate the ratio of financial wealth to business value for each entrepreneur. Entrepreneurs who are borrowing constrained, and wish to expand the scale of their business should have low values of this ratio, as they should liquidate financial assets in order to invest in their business. Figure 20 shows the cumulative distribution

of this ratio for all entrepreneurs. I plot the inverse CDF, so that the far left side of the graph shows that 100% of entrepreneurs could liquidate financial assets worth 1% of their business and the far right side of the graph shows that just over 40% of entrepreneurs could liquidate financial assets worth 100% of their business. While there are certainly some entrepreneurs who have few financial assets outside of their business, the vast majority have substantial financial assets outside of their business.

Figure 20: Inverse CDF of the Ratio of Financial Wealth to Business Value



The inverse cumulative density function of the ratio of financial wealth to business value. Intuitively, this graph gives the proportion of entrepreneurs would be able to liquidate their financial investments and make a $x\%$ investment in their firm.

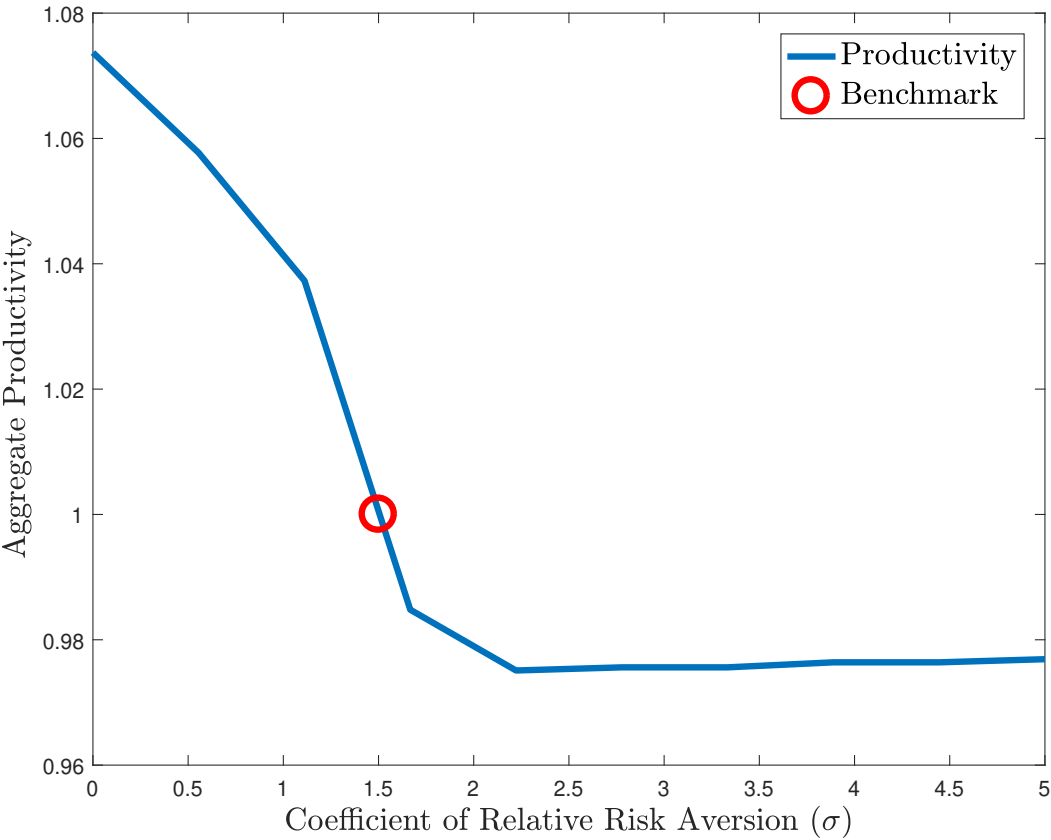
D Quantitative Robustness

D.1 Robustness: Coefficient of Relative Risk Aversion

Of course, the importance of the missing market for entrepreneurial risk depends critically on the value of risk-aversion used in the economy. In figure 21, I plot the TFP of the benchmark economy and how this varies over different values of σ , the coefficient of relative risk aversion. Aggregate productivity declines as the agents that populate the economy become more risk-averse. This operates primarily through the endogenous choice of risky

projects, as agent's risk aversion increases, fewer agents are willing to start a high-risk project with high expected productivity. As a consequence, more agents operate small-scale safe projects.

Figure 21: Aggregate Productivity and the Coefficient of Relative Risk Aversion



E Computational Algorithm

E.1 Decision Rules in the Benchmark Economy

The goal is to solve for the value functions $\{V^W, V^{NE}, V^E\}$, occupational choices, and the set of policy functions $\{c^W, a'^W, \tilde{a}^{NE}, \tilde{k}^{NE}, x^{NE}, c^E, a'^E, k'^E, n^E\}$ to maximize the problems given by 21, 22 and 23.

I proceed by adapting an algorithm from [Dyrda and Pugsley \(2018\)](#). To aid the computational tractability of the problem, I solve for the decision rules on a grid of resources r , rather than on both a and k . I solve first for the value functions, conditional on which occupation will be chosen at the beginning of the next period and then maximize over these occupational choices.

Given a wage w , I solve for the decision rules by:

- Initial Guess:
 - Set $i = 0$,
 - Guess $c_i(\cdot)$ is a constant fraction of total resources
 - Guess $r'_{i,0}(\cdot) = r$
 - Guess $V_i(\cdot) = \frac{c_i}{1-\frac{1}{\theta}} / (1 - \psi\beta)$
 - Guess all agents choose to stay in their current occupation in the next period
- Begin Value Function loop
 - $i = i + 1$
 - Calculate the derivatives of each value function $V_i^E(\cdot), V_i^{NE}(\cdot), V_i^W(\cdot)$ w.r.t. total resources r
 - Use the envelope condition to update current consumption:

$$c_i^E = \left(\frac{\partial V_i^E(\cdot)}{\partial r} \right)^\theta$$

$$c_i^{NE} = \left(\frac{\partial V_i^{NE}(\cdot)}{\partial r} \right)^\theta$$

$$c_i^W = \left(\frac{\partial V_i^W(\cdot)}{\partial r} \right)^\theta$$

- Set $j = 0$
- Begin Portfolio Allocation loop:
 - * $j = j + 1$
 - * For each future state of the world m , calculate $\lambda_m = \frac{\partial V'}{\partial r'}|_{r'_{i,j}(m)}$

- * Use the FOC w.r.t. k' to determine the optimal level of k'

$$k'_{i,j} = \frac{\left[\sum_m \text{Prob}_m \lambda_m (z_m h_m^E)^{\frac{1-\gamma}{\alpha+(1-\gamma)(1-\alpha)}} \right]^{\alpha+\frac{(1-\gamma)(1-\alpha)}{1-\gamma}} \left(\frac{1-\alpha}{w} \right)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}} \gamma^{\frac{1}{1-\gamma}} \alpha^{\alpha+\frac{(1-\gamma)(1-\alpha)}{1-\gamma}}}{\sum_m \text{Prob}_m \lambda_m (\delta + r^A)^{\alpha+\frac{(1-\gamma)(1-\alpha)}{1-\gamma}}}$$

- * If that level of k' exceeds the entrepreneur's ability to borrow, reduce it to the maximum amount consistent with the borrowing constraint and the current level of consumption:

$$k'_{i,j}{}^E = \min \left(k'_{i,j}, \frac{r - c_i^E - \underline{a}}{1 - \phi} \right)$$

$$k'_{i,j}{}^{NE} = \min \left(k'_{i,j}, \frac{r - f - c_i^{NE} - \underline{a}}{1 - \phi} \right)$$

- * Use the budget constraint to determine the resulting saving or borrowing constraint in a' :

$$a'^E = r - c_i^E - k'_{i,j}{}^E$$

$$a'^{NE} = r - f - c_i^E - k'_{i,j}{}^{NE}$$

- * Update next period resources for each future state of the world m :

$$r'_{i,j}(m) = (z_m h_m^E)^{1-\gamma} (k^\alpha n_m^{1-\alpha})^\gamma - w n_m + (1 - \delta) k'_{i,j} + a'(1 + r^A)$$

$$\text{where } n_m = (z_m h_m^E)^{1-\gamma} k^{\alpha\gamma} \gamma \left(\frac{1-\alpha}{w} \right)^{\frac{1}{\alpha+(1-\gamma)(1-\alpha)}}$$

- * Calculate distance between $r_{i,j-1}$ and $r_{i,j}$ for all m
 - * If distance is less than tolerance, end loop, else return to beginning of Portfolio Allocation loop
- As workers will not invest in capital, their savings can be directly backed out by the budget constraint

$$a'^W = r - c_i^W$$

- Update conditional value functions:

$$V_i^E = \frac{c_i^{E1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} + \psi\beta \sum_m \text{Prob}_m V_{i-1}^E(r'_{i,j}{}^E, m)$$

$$V_i^{NE} = \frac{c_i^{NE1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} + \psi\beta \sum_m \text{Prob}_m V_{i-1}^{NE}(r'_{i,j}{}^{NE}, m)$$

$$V_i^W = \frac{c_i^{W1-\frac{1}{\theta}}}{1 - \frac{1}{\theta}} + \psi\beta \sum_m \text{Prob}_m V_{i-1}^W(r'_{i,j}{}^W, m)$$

- Perform occupational choice to obtain unconditional value functions:

$$V_i^E(r, h^W, h^E, z, x) = \max(V_i^E(r, h^W, h^E, z, x), V_i^{NE}(r, h^W, h^E), V_i^W(r, h^W, h^E))$$

$$V_i^W(r, h^W, h^E) = \max(V_i^{NE}(r, h^W, h^E), V_i^W(r, h^W, h^E))$$

- Calculate distance between $V_i^E(\cdot)$ and $V_{i-1}^E(\cdot)$ and between $V_i^W(\cdot)$ and $V_{i-1}^W(\cdot)$
- If distance is less than tolerance, end loop, else return to beginning of Value Function loop

With the decision rules solve, I then simulate a fixed mass of agents on a discretion grid of $a \times k \times h \times z \times x$. I guess a uniform distribution over this state space and then iterate until the distribution converges. I use a bisection search to determine the wage that clears the labour market in this economy.