# Risky Entrepreneurship and Wealth

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**Abstract**

This paper quantifies the importance of two financial frictions, a missing market for entrepreneurial risk and borrowing constraints, for reducing entrepreneurial activity in the US economy. I use the Kauffman Firm Survey to document facts about the risks new entrepreneurs take and the sources of their initial financing. Based on these facts, I build a model of occupational choice, where agents choose to become workers or entrepreneurs, and endogenous risk choice, where entrepreneurs choose from a menu of risky projects. On average, riskier projects deliver higher returns. The model features two financial frictions. First, a missing market for entrepreneurial risk prevents entrepreneurs from insuring themselves against entrepreneurial income risk and the risk of business failure. Second, borrowing constraints limit the size of a business that an entrepreneur can start based on their personal wealth. Agents select into entrepreneurship based on both their entrepreneurial ability and their personal wealth. I calibrate the model to match firm dynamics in the Kauffman Firm Survey and US wealth inequality as measured by the Survey of Consumer Finances. I use the calibrated model to measure the quantitative importance of these two frictions in reducing entrepreneurial risk-taking. Completing the missing market for entrepreneurial risk increases output by 17%, while fully relaxing borrowing constraints increases output by 9%. There is a significant interaction between the two frictions so that removing both frictions leads to a 43% increase in output. The endogenous choice of risk generates heterogeneity in rates of return, allowing wealthier entrepreneurs to earn higher returns on their wealth. Given fixed occupational choices, this endogenous choice explains 40% of the wealth shares of the wealthiest 1%. Finally, I study a simple policy experiment where I provide unemployment insurance benefits to unsuccessful entrepreneurs. Despite the presence of adverse selection, the partial insurance scheme increases output by 4% as entrepreneurs choose to take on higher risk projects.

**Keywords:** Entrepreneurship, Risk-Taking, Financial Frictions, Aggregate Productivity, Wealth Inequality

**JEL Codes:** D31, G32, J24, L26

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# 1 Introduction

A large literature studies how entrepreneurship is affected by borrowing constraints. These constraints are important determinants of aggregate productivity and output, and are an important explanation for the high degree of wealth inequality observed in the US[1]. However, these borrowing constraints are not the only obstacle entrepreneurs face. It is well known that entrepreneurship entails substantial uninsurable idiosyncratic risk[2]. Is the inability to insure this idiosyncratic risk quantitatively important for reducing entrepreneurial risk taking? And what implications does this missing insurance market have for the distribution of wealth? Answering these questions is vital to the design of public policy, as governments around the world seek to balance redistribution with the promotion of entrepreneurship in order to boost job creation and productivity growth.

To answer these questions, I develop a quantitative theory of entrepreneurial risk taking where entrepreneurs face two distinct financial frictions. First, borrowing constraints limit the amount of capital an entrepreneur can invest in. Second, a missing market for entrepreneurial risk prevents entrepreneurs from insuring the substantial income risk and capital investment risk they face. I use micro data on new US firms and the distribution of US wealth to quantify the strength of both of these frictions in the model. To measure their impact on aggregate productivity, output, and the distribution of wealth I then run counterfactuals where individuals do not face these frictions.

I build a model of occupational choice and endogenous risk choice. Agents are endowed with two types of ability, entrepreneurial and worker. Each period, they choose to be an entrepreneur or a worker. Conditional on being an entrepreneur they choose the risk type of their business from a menu of risky options. Higher risk businesses earn higher expected rewards. There are two assets in the model, capital and a risk-free financial asset. Borrowing constraints limit borrowing to a certain fraction of the invested capital stock, forcing entrepreneurs to invest their own personal wealth in the business. A key timing assumption is that individuals must choose their occupation and entrepreneurs must choose their business's risk type before shocks to their ability and their businesses' productivity are realized. The missing market for entrepreneurial risk means that individuals cannot trade resources between different states of the world where they receive different shocks. Wealthier agents are able to self-finance a larger capital stock and are able to self-insure themselves against fluctuations in entrepreneurial income. As a consequence, selection into entrepreneurship is based on both ability and wealth.

The distribution of wealth influences aggregate productivity in this model. As wealthier agents are better able to overcome both financial frictions, they are more likely to become entrepreneurs and conditional on being entrepreneurs are more likely to choose high-risk projects. The distribution of wealth will therefore affect the distribution of active entrepreneur's abilities and the distribution of project types in the economy. Both of these distributions will then influence aggregate productivity, as the

[1]For example, see Buera (2009) for how borrowing constraints impact aggregate productivity and output. See Cagetti and De Nardi (2006) for how they can explain high top-tail wealth inequality.

[2]For example, see Fairlie et al. (2018) who document the exit rates of entrepreneurs and DeBacker et al. (2018) who document that entrepreneurial income is much more volatile than employment income conditional on entrepreneurial survival.

higher-risk projects have higher expected productivity.

At the same time, the distribution of wealth is going to be shaped by these two financial frictions. Entrepreneurs will face strong savings motives to save as a consequence of both financial frictions. Borrowing constraints encourage entrepreneurs to save in order to invest in more capital and expand the size of their business. The missing market for entrepreneurial risk encourages entrepreneurs to engage in precautionary savings against future business failure. As a consequence of both of these savings motives, entrepreneurs accumulate wealth at higher rates than workers. Within entrepreneurs, wealthier entrepreneurs choose higher-risk higher-reward projects and earn higher expected rates of return on their wealth leading to higher top-tail wealth concentration.

I use the Kauffman Firm Survey (KFS), a panel of 4,928 new US firms, to motivate key model features, to quantify the strength of both financial frictions, and to validate the predictions of the model. To motivate the timing structure in the model, I show that many entrepreneurs in the KFS start businesses, earn cumulatively negative profits, and then exit. To motivate the model's exogenous collateral constraint, I also show that entrepreneurs who invest more of their own money in their businesses are more likely to raise funds from other sources and to raise more money from external sources conditional on raising some.

To quantify the importance of both frictions, I calibrate the model to match model generated moments to their empirical counterparts. In the model, I simulate an 8 year panel of new firms that corresponds to the Kauffman Firm Survey, an 8 year panel of 4,928 new US firms. To discipline the strength of the borrowing constraints, I match the ratio of average debt to average equity from the Kauffman Firm Survey. To discipline the nature of idiosyncratic entrepreneurial risk, I match the survival rates, the auto-correlation of employment, and the relative employment of different groups of firms. To ensure the model can also generate realistic wealth inequality, I match moments from the model's simulated cross-sectional distribution of wealth to micro data on wealth from the Survey of Consumer Finances.

The model predicts that wealthier individuals should invest more in their businesses and start riskier projects. To test this prediction with the Kauffman Firm Survey Data, I use the cumulative profits over the first eight years of a firm's operation as a measure of success. I show that KFS entrepreneurs who invest more of their own money in their business earn more dispersed cumulative profits, even after controlling for firm size. I interpret this greater dispersion to indicated that entrepreneurs who invest more are starting riskier businesses. I then show that the model can generate this same pattern.

In order to understand the relative importance of the two financial frictions in the model for output and productivity. I remove each friction and compare the resulting equilibria. I first complete the missing market for entrepreneurial risk by introducing a full set of contingent claims. Individuals are able to buy or sell securities that pay off based on the realizations of their personal shocks at actuarially fair prices. The ability to fully trade resources across different future states of the world leads entrepreneurs to switch to higher risk projects that have higher expected productivity. Completing the market also encourages unconstrained entrepreneurs to invest more and improves selection into entrepreneurship. Together these effects lead to a 17% increase in output. Second, I relax the borrowing constraint by allowing entrepreneurs to invest in any amount of capital regardless of their personal net-worth. This leads some entrepreneurs to switch to higher risk projects, but mainly increases capital investment as formerly constrained entrepreneurs increase the size of their business. I find that relaxing the borrowing constraints increases aggregate output by 9%. Removing both of these friction leads to all entrepreneurs choosing higher risk projects and greatly increases capital investment. As a result output increases by 43%, demonstrating a substantial interaction between these two financial frictions.

The model's endogenous choice of risk had implications for explaining the cross-sectional distribution of wealth. Since entrepreneurs endogenously choose their business type, higher wealth agents choose higher risk projects. Wealthy entrepreneurs therefore earn higher average returns than wealth-poor entrepreneurs. These higher returns increase the concentration of wealth at the top right tail. To quantify the contribution of this endogenous risk choice to wealth inequality, I compare an economy with this endogenous choice of risk to an exogenous risk economy, where the same projects are randomly allocated to new entrepreneurs. In order to keep the proportion of entrepreneurs fixed, I fix the occupational choices in both of these economies. I find that the share of wealth held by the top 1% wealthiest individuals declines by 40% when comparing the endogenous risk economy to the exogenous risk economy.

Finally, given the quantitative importance of the missing market for entrepreneurial risk, I study a simple policy designed to provide partial insurance to entrepreneurs. I provide an unemployment benefit to unsuccessful entrepreneurs that is modelled as an income floor. This benefit is paid for with lump-sum taxes on all entrepreneurs. Low levels of this benefit encourage some entrepreneurs to switch to riskier projects, resulting in a 4% increase to output. At higher levels of this benefit, low ability entrepreneurs enter because the benefit is higher than their income as workers. This adverse selection means that as the benefit increases output falls below its level in the benchmark economy without the benefit.

## Related Literature

The first contribution of this paper is to build a model where two distinct financial frictions distort the decision to become an entrepreneur. This model combines insights from two distinct strands of the literature. First, papers such as Kihlstrom and Laffont (1979) and Cressy (2000) argue that uninsurable idiosyncratic entrepreneurial risk is a major determinant for the decision to become an entrepreneur, either because of heterogeneity in risk-preferences or because wealth makes individuals more willing to take risks. Second, models of the choice to become an entrepreneur like Cagetti and De Nardi (2006) or Buera (2009) show how borrowing constraints discourage wealth-poor individuals from starting businesses. While individuals in these borrowing constraint models face some shocks to their productivity, their occupational choice takes place after the realization of the current period's shocks. As such individual's occupational choice is not distorted by a missing market for entrepreneurial risk. By contrast, the occupational choice in this paper is risky. Agents must choose their current period's occupation before knowing the current realization of their earnings in both occupations. Since entrepreneurial income is riskier than labour income, the occupational choice involves not just the choice between the level of income but also the variance. Both of these mechanisms can speak to a wider empirical literature showing that entrepreneurs are disproportionately wealthy and that the wealthy are disproportionately entrepreneurial. See in particular Evans and Jovanovic (1989), Quadrini (1999), Gentry and Hubbard (2004), and Hurst and Lusardi (2004). The focus on the missing market for entrepreneurial risk is also motivated by work showing that entrepreneurs face a high degree of uninsurable idiosyncratic risk. See for example, Castro et al. (2015), DeBacker et al. (2018), and Fairlie et al. (2018).

The second contribution of this paper is to quantify the importance of both financial frictions in reducing aggregate TFP and output in the US economy. This builds on Buera (2009), who quantifies the welfare costs of borrowing constraints in the US due borrowing constraints. He finds that welfare costs are substantial due to misallocation of talent and under-capitalized entrepreneurs operating businesses that are much smaller than their optimal size. This paper measure the productivity and output losses due to an separate financial friction, the missing market for entrepreneurial risk. In addition it provides an additional channel through which borrowing constraints can reduce productivity and output, by distorting entrepreneur's choice of risky projects.

This contribution relates to an emerging body of work showing that providing additional insurance to individuals can increase rates of self-employment. In particular, this paper's policy analysis provides a theoretical framework to understand the empirical results in Hombert et al. (2014), who show that providing unemployment insurance to self-employed individuals in France lead to more new business starts without any deterioration in quality. In addition, papers studying developing country contexts such as Bianchi and Bobba (2013) or Karlan et al. (2014) demonstrate that providing people with better insurance mechanisms can increase self-employment or investment more effectively than relaxing borrowing constraints. In developed country contexts, Olds (2016), and Gottlieb et al. (2018), both show how policy changes that provide additional insurance to individuals can increase self-employment. In particular, the

The quantitative results in this paper, showing that both borrowing constraints and the missing market for entrepreneurial risk are quantitatively important frictions that reduces aggregate output and productivity, have important policy implications. Papers such as Meh (2005), Bruggemann (2017), and Guvenen et al. (2018) study the effects of different government taxes on entrepreneurs given the existence of binding borrowing constraints. Work by Panousi and Reis (2016) considers optimal taxation with uninsurable capital income risk. This paper suggests that entrepreneurial tax policies should be designed with both of these financial frictions in mind. In addition, these results have implications for the literature that uses the differences in financial frictions to explain cross-country differences in TFP. Papers such as Buera et al. (2011), Midrigan and Xu (2014), and Castro and Ševčík (2017) all focus on how differences in borrowing constraints across countries can explain TFP differentials. This paper suggests that looking at cross-country difference in risk-sharing or insurance mechanisms may be a fruitful area of future research.

The third contribution of this paper is to study how wealth influences the endogenous choice of entrepreneurial risk. A closely related paper is Choi (2017). He uses US Census Bureau data to show that individuals who had higher paying jobs prior to starting a business take larger risks, as measured by higher exit rates, more dispersion in growth, and faster average growth conditional on survival. By contrast this paper considers how wealth, rather than labour market opportunities, encourages more risk taking by entrepreneurs, and studies the implications of the endogenous choice of risk for the distribution of wealth. Vereshchagina and Hopenhayn (2009) study how wealth impacts both the choice to become an entrepreneur and the endogenous choice of risk. In their three period model, wealth-poor entrepreneurs take larger risks in the hopes of being able to be entrepreneurs in the future. I find the opposite result, that it is wealthier individuals who take larger risks.

Panousi and Papanikolaou (2012) study risk taking in publicly traded firms. Using Compustat data, they document that firm investment falls as firm-specific risk rises and that this effect is stronger when managers own more of the firm. Their result provides micro evidence that concerns about firm-specific risk shifts investment decisions. By contrast, this paper considers how entrepreneurs choose what type of business to start, when given a choice between different risk-reward types. This channel is likely to be more important for private firms, which are the focus of this paper. More broadly, Gabler and Poschke (2013) also considers how less risk-taking leads to aggregate productivity losses. The mechanisms however differ substantially, as they consider a framework with risk-neutral firms facing distortions to allocative efficiency, while I consider risk-averse entrepreneurs facing a missing market for entrepreneurial risk.

Finally, I contribute to the literature on explaining the causes of the distribution of wealth by considering the impact of the endogenous choice of risk on wealth accumulation. This mechanism is consistent with recent empirical evidence from Scandinavian countries. Bach et al. (2018) and Fagereng et al. (2018) show that wealthier individuals earn persistently higher rates of return on their assets. I build on quantitative work by Cagetti and De Nardi (2006) who show that a model of entrepreneurship with both endogenous borrowing constraints and a strong bequest motive can generate the high concentration of wealth observed in US data. This paper uses a similar calibration strategy to match the wealth distribution, but does so incorporating both borrowing constraints and the missing market for entrepreneurial risk, which allows entrepreneurs to endogenously choose the level of risk they wish to undertake.

# 2 Static Model

In this section, I build a simple static model to illustrate the two key frictions explored in this paper.

## 2.1 Environment

Agents live for a single period and have preferences over consumption $u(c)$, where $u(\cdot)$ exhibits decreasing absolute risk aversion[3].

Agents are initially endowed with an amount of wealth given by $e \in [0, \bar{e}]$ and a known entrepreneurial ability $h^E \in [\underline{h}^E, \bar{h}^E]$. At the beginning of the period, agents

[3]Decreasing absolute risk aversion is defined by:

$$\frac{\partial}{\partial c}\left(-\frac{u''(c)}{u'(c)}\right) = -\frac{u'(c)u'''(c) - [u''(c)]^2}{[u'(c)]^2} < 0$$

Note that many common utility functions such as log and CRRA exhibit decreasing absolute risk aversion.

must choose to be an entrepreneur or a worker based on their state $(e, h^E)$.

All agents have identical productivity as a worker. If they choose to be a worker, they will earn a wage of $w$ with certainty.

If an agent chooses to be an entrepreneur, they must choose the level of business risk $x \in [0, h^E]$. Choosing a higher level of risk will deliver higher expected productivity at the cost of more dispersed productivity. With probability $p$ the entrepreneur's risk is successful, and their productivity is boosted by $\psi x$. With the complementary probability $(1-p)$ the risk is unsuccessful and their productivity is reduced by $x$. The entrepreneur's realized productivity is therefore given by $z$:

$$z = \begin{cases} h^E + \psi x & \text{with probability} \quad p \\ h^E - x & \text{with probability} \quad 1-p \end{cases} \tag{1}$$

where $\psi > \frac{1-p}{p}$, so that the expected value of $z$ increases with $x$.

There are two assets, capital $(k)$ and a risk-free financial asset $(a)$. At the beginning of the period, agents choose how much of their initial endowment $(e)$ to invest in each asset:

$$e = a + k \tag{2}$$

Agents can only hold a positive amount of capital, and potentially face a borrowing constraint in the financial asset. The borrowing constraint is given by:

$$a \geq -\phi k \tag{3}$$

where $\phi \in [0, 1]$. Note that this effectively limits the maximum amount of capital that an agent can invest in to $\frac{e}{1-\phi}$. When $\phi = 0$, entrepreneurs are not able to borrow in the financial asset. When $\phi = 1$ entrepreneurs are able to invest in any amount of capital $k$ regardless of their initial endowment of wealth.

The financial asset $a$ pays off $(1 + r^a)$ units in all states of the world.

An entrepreneur with a realized productivity $z$ and a capital stock $k$ will produce output according to:

$$y = z^{1-\gamma} k^{\alpha} \tag{4}$$

For values of $\phi < 1$, this model thus exhibits borrowing constraints, which prevent entrepreneurs from investing in a level of capital stock that exceeds a multiple $(\frac{1}{1-\phi})$ of their endowed wealth $e$. It also exhibits a missing market for entrepreneurial risk, in that there is no way for agents to insure themselves against the risks to their realized productivity $z$.

## 2.2 Agent's Problem

Depending on their initial endowments of wealth $(e)$ and entrepreneurial productivity $(z)$, agents make an occupational choice decision between being workers and entrepreneurs according to:

$$V(h^E, e) = \max\{V^E(h^E, e), V^W(e)\} \tag{5}$$

Worker's Value Function:

$$V^W(e) = u(w + (1 + r^a)e) \tag{6}$$

Entrepreneur's Value Function:

$$V^E(h^E, e) = \max_{x,k,a} pu\left[(h^E + \psi x)^{1-\gamma}k^\alpha + (1 + r^a)a\right] + (1-p)u\left[(h^E - x)^{1-\gamma}k^\alpha + (1 + r^a)a\right] \tag{7}$$

subject to:

$$\begin{aligned} e &= a + k \\ a &\geq -\phi k \end{aligned}$$

where $p\psi > (1 - p)$ and $\phi \in [0, 1]$.

## 2.3 Model Predictions

### 2.3.1 Case 1: Unconstrained

Optimal $x$ and $k$ are given by:

$$x^* = h^E \frac{1 - \left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}}{1 + \psi \left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}} \left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}} \tag{8}$$

$$k^* = \left(\frac{\alpha}{[1 + r^a]} \frac{\left[p(h^E + \psi x)^{1-\gamma} + (1 - p)(h^E - x)^{1-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]}{\left[p + (1 - p)\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]}\right)^{\frac{1}{1-\alpha}} \tag{9}$$

where

$$\begin{aligned} \bar{c} =& (h^E + \psi x)^{1-\gamma}k^{*\alpha} + (1 + r^a)a^* \\ \underline{c} =& (h^E - x)^{1-\gamma}k^{*\alpha} + (1 + r^a)a^* \\ a^* =& e - k^* \end{aligned}$$

If $x = 0$, the entrepreneurial project would be risk-free. In that case, $\bar{c} = \underline{c}$, and so the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$ will be equal to one. Since $\psi p > 1-p$, the numerator of the right hand side of equation (8) is strictly positive, and all entrepreneurs choose a strictly positive level of risk $x > 0$.

**Proposition 1: Under case 1, risk taking $(x)$ is increasing in wealth $(e)$**

$$\frac{\partial x^*}{\partial e} > 0 \tag{10}$$

***Proof*** See appendix A.1.

***Intuition***

As an individual's wealth increases, more of their final consumption is derived from their endowed wealth and less from their income. Wealthier entrepreneurs are more willing to take a higher level of risk for a higher expected income because when their business is unsuccessful they still have abundant consumption from their wealth.

***Implications***

Wealthier entrepreneurs will choose higher-risk higher-reward projects. If these higher rewards are the consequence of higher productivity businesses, then the distribution of wealth is going to have a direct impact on the distribution of business types that get started which will determine aggregate productivity in the economy. These higher expected rewards may also have implications for the distribution of wealth, as wealthier individuals will earn higher expected returns on their wealth.

**Proposition 2: Under case 1, individuals are more likely to become entrepreneurs as their wealth $e$ increases**

$$\frac{\partial V^E}{\partial e} > \frac{\partial V^W}{\partial e}|_{V^E=V^W} \tag{11}$$

***Proof*** See appendix A.2.

***Intuition***

As an individual's wealth increases, more of their final consumption is derived from their endowed wealth and less from their income. Wealthier individuals are more willing to select a riskier occupation because if their income is low they still have abundant consumption from their wealth.

***Implications***

Even in the absence of borrowing constraints on capital ($\phi = 1$), the missing market for entrepreneurial risk will generate selection into entrepreneurship based on both wealth and ability. Potential this friction can misallocate talent across occupations if low-ability wealthy individuals are becoming entrepreneurs at the expense of high-ability wealth-poor individuals.

### 2.3.2 Case 2: Constraint Binds

Optimal $k$ is given by:

$$k^c = \frac{e}{1-\phi} \tag{12}$$

The expression for optimal $x$ when the borrowing constraint binds is identical to the unconstrained case (8).

**Proposition 3: Under case 2, borrowing constraints affect the choice of risk**

$$\frac{\partial x^*}{\partial \phi} < 0 \tag{13}$$

***Proof*** See appendix A.3.

***Intuition***
As constrained entrepreneurs are able to invest in more capital they reduce risk taking. They do so because risk and size are substitutes in this static model. Taking more risk $x$ or investing in more capital $k$ both increase income when the business risk is successful and reduce it when the business risk is unsuccessful.
***Implications***
This proposition demonstrates that borrowing constraints can interact with the missing market for entrepreneurial risk that generates the dispersion in risk taking. The potential for this interaction necessitates studying borrowing constraints and the missing market for entrepreneurial risk together in order to quantify the impact of either of these frictions.

In this static model, tighter borrowing constraints unambiguously increase the risk taking of entrepreneurs. While this substitution of scale for risk is a force in the dynamic model of section 4, an additional dynamic effect will quantitatively dominate this one, and so tighter borrowing constraints will lead to less risk taking.

**Proposition 4: Tighter borrowing constraints reduce the value of entrepreneurship for wealth-poor individuals**

$$\frac{\partial V^E}{\partial \phi} > 0 \tag{14}$$

***Proof*** See appendix A.4.

***Intuition*** Constrained entrepreneurs invest in less capital than they would like to. As a consequence their entrepreneurial income is lower than if they were unconstrained and so their value of being an entrepreneur is reduced.
***Implications***
Borrowing constraints can generate misallocation of talent as low-ability wealthy individuals become entrepreneurs at the expense of high-ability wealth poor individuals.

# 3 Stylized Facts

In this section I document key stylized facts about entrepreneurship from the confidential version of the Kauffman Firm Survey. These facts motivate the modelling choices in the static model in section 2 and the dynamic model in section 4.

## 3.1 Kauffman Firm Survey Data

The Kauffman Firm Survey is a single cohort panel of 4,928 new US firms. All firms are founded in the year 2004, and the survey follows them until they exit or until 2011. It was designed to provide a representative sample of all new businesses started in 2004[4]. The firms include businesses that were independently founded, purchased from an existing business or purchased as a franchise, and exclude any inherited businesses, any non-profits, and businesses that were started as a branch or subsidiary of an existing business. For each firm, information for up to 10 owners is provided.

These firms are highly heterogeneous. As table 1 shows, the majority of firms are non-employers in the first year, though many go on to hire at least one worker later. Most firms are owned and operated by a single entrepreneur, though a small proportion have multiple owners. The distribution of total investment is highly skewed with the mean investment almost twice as large as the median investment.

Table 1: Summary Statistics

| | Mean | p10 | p50 | p90 | n Firms |
|---|---|---|---|---|---|
| Employment (Year 1) | 2.0 | 0 | 0 | 5 | 4,823 |
| Employment (Year 8) | 5.2 | 0 | 1 | 10 | 2,000 |
| Entrepreneurs | 1.4 | 1 | 1 | 2 | 4,928 |
| Total Investment (Years 1-8) | 480.5 | 67 | 277 | 762 | 3,488 |

Summary statistics for the firms in the Kauffman Firm Survey. The number of firms in each row varies depending on data availability. Total investment is in thousands of US dollars and included all sources of both equity and debt.

## 3.2 Idiosyncratic Risk

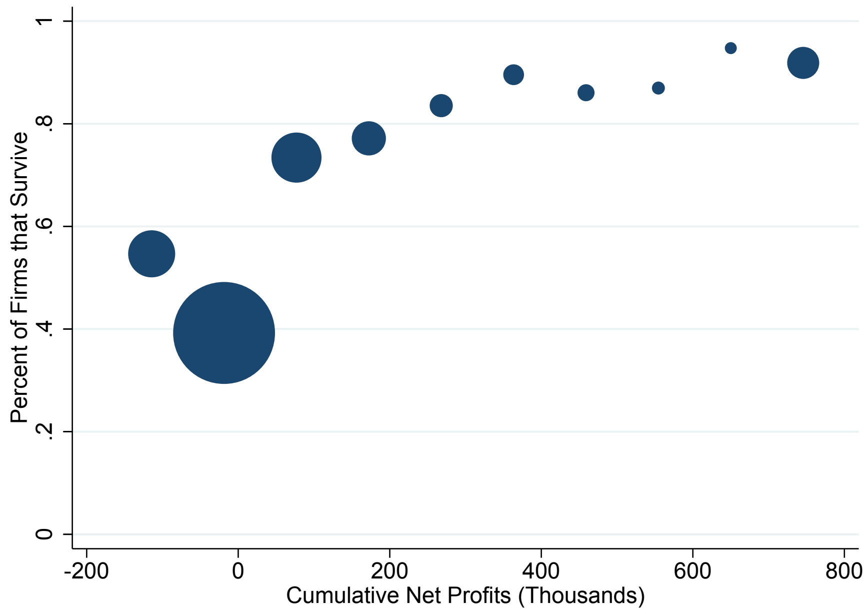
The model in section 2 shows how a missing market for entrepreneurial risk can generate selection into entrepreneurship based on wealth. If this missing market is quantitatively important for who selects into entrepreneurship, two things must be true. First, entrepreneurs must face a significant degree of uncertainty about the potential success of the business[5]. Second, if the business is unsuccessful entrepreneurs must bear real losses that make them worse off than if they did not start a business.

For evidence that these entrepreneurs face idiosyncratic risk, I examine the survival rates and cumulative profits over the 8 year panel. I calculate the survival rate as the number of firms either operating in the final sample year or that have been merged or sold in a previous year, divided by the total number of firms with a known status in the final year. Without information about the final sale price, it is difficult to assess

[4]The sampling frame is taken from the Dun and Bradstreet US Business database.

[5]Note that both uncertainty about entrepreneurial ability as in Jovanovic (1982), which decreases over time as entrepreneurs learn about their ability, and idiosyncratic risk to the business model as in Hopenhayn (1992), can potentially provide the necessary uncertainty about the success of the business.

Figure 1: Survival by Cumulative Profits




The 8-year survival rates of firms in ten bins of cumulative net profits. Cumulative profits have been winsorized between the 5th and 95th percentiles and then firms are sorted into the ten equally-sized bins between these percentiles. The size of each circle is proportional to the number of observations in that bin.

whether firms that are merged or sold constitute a successful or unsuccessful outcome for entrepreneurs. I include these firms in survival in order to provide a conservative estimate of the likelihood of undesirable outcomes. See appendix B.1 for additional details on the calculation of survival.

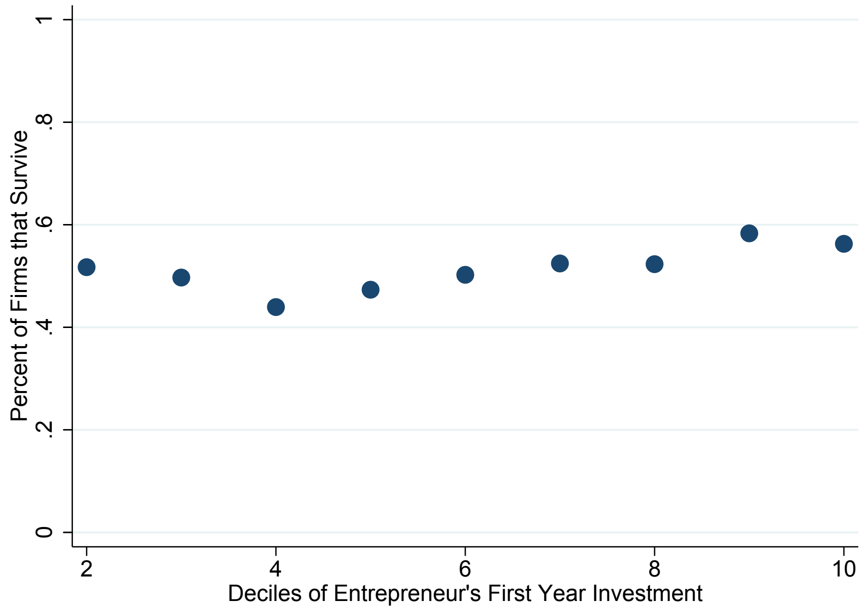
45% of the firms in the Kauffman Firm Survey shut down all operations by the end of their 8th year. Figure 1 shows that the firms that earn low or negative profits are much less likely to survive. In the absence of idiosyncratic risk, it is difficult to imagine why so many of these entrepreneurs would choose to enter, lose money, and then exit.

These firm exits are not confined to low-investment firms. Figure 2 shows that exit is common across the distribution of investment. It is not just low-investment firms, which may have been started to provide temporary self-employment[6] that exit. While firms in the top decile of initial entrepreneurial investments have slightly higher survival rates than lower-investment firms, 42% of these firms have exited by the end of the eighth year.

Firm exits are also not confined to a specific set of industries. Table 2 shows the proportion of firms that survive to the end of the sample according to their 2-digit NAICs code. Dropping industries with fewer than 50 observations, survival rates range from 43% of firms in the case of Retail Trade (44), to 64% in Manufacturing (33).

[6]See Galindo da Fonsec (2017), who documents different patterns of entrepreneurial activity based on employment status before starting a business.

Figure 2: Survival by Initial Investment




Survival rates of firms grouped into deciles based on the total equity invested by the entrepreneurs in the first year. This measure excludes equity invested by individuals who do not have an active management role in the firm.

Table 2: Survival By Industry

| Industry (2 digit NAICs Code) | Survival Rate | n Firms |
|---|---|---|
| Retail Trade (44) | 42.9 % | 238 |
| Retail Trade (45) | 43.7 % | 229 |
| Transportation and Warehousing (48) | 45.3 % | 86 |
| Construction (23) | 46.4 % | 306 |
| Finance and Insurance (52) | 51.2 % | 164 |
| Other Services (except Public Administration) (81) | 51.4 % | 389 |
| Health Care and Social Assistance (62) | 52.1 % | 94 |
| Administrative, Support, Waste Management, and Remediation Services (56) | 52.6 % | 310 |
| Manufacturing (32) | 54.5 % | 123 |
| Accommodation and Food Services (72) | 54.5 % | 88 |
| Information (51) | 55.6 % | 144 |
| Real Estate and Rental and Leasing (53) | 56.9 % | 160 |
| Wholesale Trade (42) | 57.4 % | 188 |
| Arts, Entertainment, and Recreation (71) | 57.6 % | 92 |
| Professional, Scientific, and Technical Services (54) | 58.4 % | 1,058 |
| Manufacturing (33) | 63.5 % | 425 |

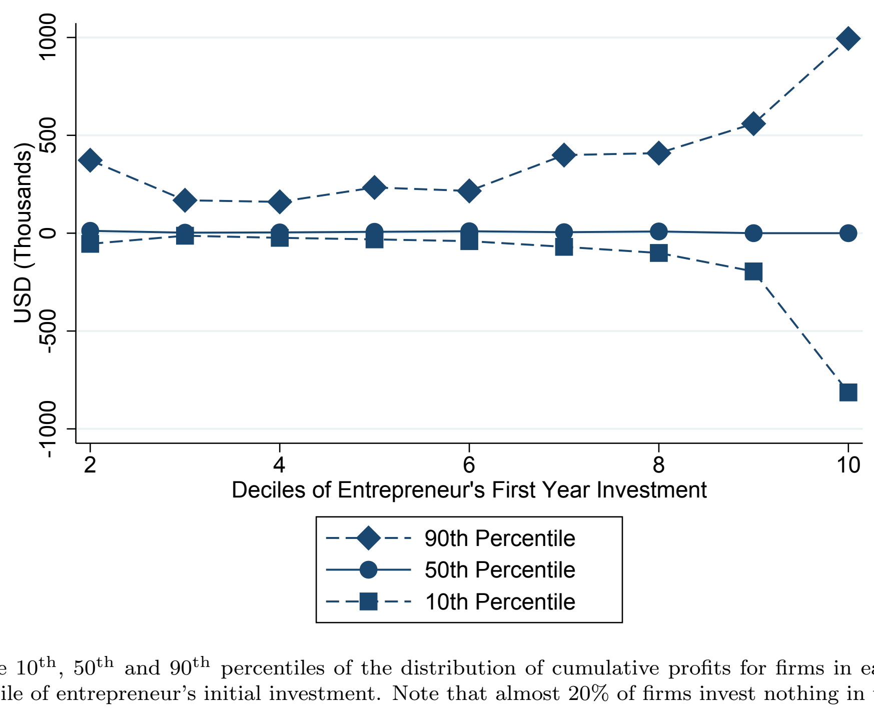
### 3.3 Choice of Risk

The model in section 2 predicts that wealthier entrepreneurs will both invest more in their business and choose businesses with a higher level of risk. While the ex-ante risk of a business is not directly observable, the ex-post outcomes of a group of firms is. In order to compare the risk taken by different entrepreneurs, I examine the dispersion in outcomes within different groups of firms. If wealthier entrepreneurs are taking more risk, there should be more dispersion in their outcomes than among a group of poorer entrepreneurs.

The Kauffman Firm Survey does not provide information about entrepreneur's wealth when they start their business. However, the amount of money that they have been able to directly invest in the business is informative about their wealth. The survey asks detailed information about the sources of firm's funding. In particular, it distinguishes between the money an entrepreneur has invested directly and money they have personally borrowed. I therefore use the amount of money an entrepreneurs is directly investing in their own business as a proxy for their net worth.

Therefore, in order to test the prediction of the model in section 2, I compare the dispersion in cumulative profits across firms in the different deciles of entrepreneur's own investment, excluding external sources of equity. If entrepreneurs who invest more in their businesses earn more dispersed cumulative profits, this may indicate that these entrepreneurs are in fact starting businesses with more idiosyncratic risk.

Figure 3: Dispersion in Cumulative Profits




The $10^{\text{th}}$, $50^{\text{th}}$ and $90^{\text{th}}$ percentiles of the distribution of cumulative profits for firms in each decile of entrepreneur's initial investment. Note that almost 20% of firms invest nothing in the first year of operation, and so the bottom two deciles are represented by a single point labelled "2".

Figure 3 clearly shows that firms with larger initial investments by their entrepreneurs have much more dispersed cumulative profits over the 8 years of operations. However, since initial investment is highly correlated with firm size, it is not clear whether this greater dispersion is a mechanical consequence of these firms operating at a larger scale or if these entrepreneurs are starting fundamentally riskier businesses.

To test whether entrepreneurs who invest more also earn more dispersed cumulative profits while controlling for firm size, I run two regressions. First, I regress a firm's cumulative profits on the initial investment of their entrepreneurs, controlling for firm size with average employment over the years of operation. A positive coefficient on initial entrepreneur's own investment ($\hat{\alpha}_1 > 0$) means that entrepreneurs who invest more are on average earning higher cumulative profits. I then construct an auxiliary regression, where the square of the predicted residuals from the primary regression are regressed on the initial investment of their entrepreneurs and the same controls. A positive coefficient on initial entrepreneur's own investment ($\hat{\beta}_1 > 0$) suggests that entrepreneurs who invest more are earning more dispersed cumulative profits, after controlling for firm size. If entrepreneurs who are investing more in their businesses are also starting riskier businesses, then their cumulative profits should be more dispersed, even after controlling for the mechanical effects of size.

Note that this is a slight variation on the common Breusch and Pagan (1979) method for testing for heteroscedasticity. Their method computes a single test statistic based on the explanatory power of the auxiliary regression, to test whether the residuals from the primary regression are homoscedastic. In this context, I am not interested in whether the predicted residuals are generally heteroscedastic. I want to know whether one regressor, the entrepreneur's initial investments, can predict their absolute magnitude. If so, entrepreneurs with larger initial investments are earning more dispersed cumulative profits, even after controlling for size.

$$\text{Cumulative Profits}_i = \alpha_0 + \alpha_1 \text{Initial Entrepreneur's Own Investment}_i + \alpha X_i + \epsilon_i \quad (15)$$

$$\hat{\epsilon}_i^2 = \beta_0 + \beta_1 \text{Initial Entrepreneur's Own Investment}_i + \beta X_i + \eta_i \quad (16)$$

Table 3 shows that entrepreneurs who invest more of their own money in their businesses on average earn higher profits, and on average, earn more dispersed profits. Both of these results are true after controlling for both the number of employees and the level of debt. I take this to be evidence that of greater risk taking on the part of the high-investment entrepreneurs. In appendix B.4 I show that this same result holds for cumulative sales. In appendix B.5, I discuss why the lack of labour market information for the entrepreneurs prevents me from calculating useful rates of return for these firms.

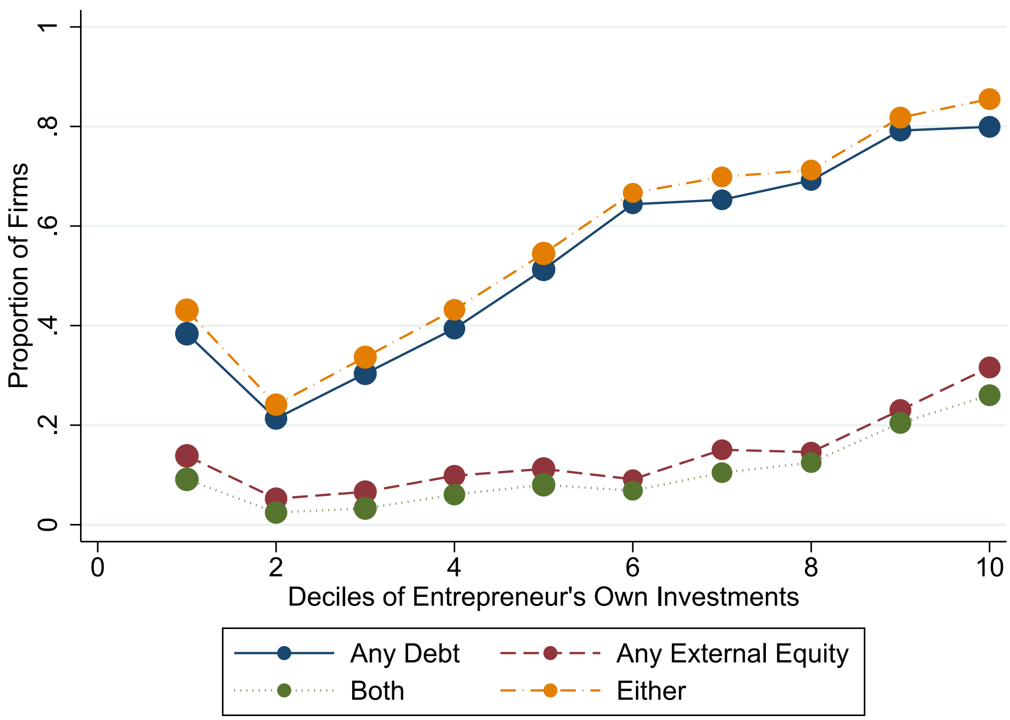
Table 3: Dispersion of Cumulative Profits

| Regression (15): | Kauffman Firm Survey: Cumulative Profits | | | | |
|---|---|---|---|---|---|
| Own Equity (Year 1) | 1.098*** | 1.112*** | 1.110*** | 0.874*** | 0.712*** |
| | (0.0832) | (0.0832) | (0.0835) | (0.128) | (0.190) |
| Average Employment | | -62.64*** | -62.91*** | -58.98** | -60.71*** |
| | | (17.32) | (18.07) | (18.33) | (18.39) |
| Average Employment$^2$ | | 0.210*** | 0.211*** | 0.152** | 0.167*** |
| | | (0.0388) | (0.0399) | (0.0473) | (0.0490) |
| Employer | | | 436.8 | 405.0 | 411.2 |
| | | | (312.6) | (314.1) | (314.1) |
| Total Investment (Year 1) | | | | 0.241* | 0.409* |
| | | | | (0.0992) | (0.176) |
| Total Debt (Year 1) | | | | | -0.318 |
| | | | | | (0.275) |
| Constant | Yes | Yes | Yes | Yes | Yes |
| 2 Digit NAICs Codes | No | No | Yes | Yes | Yes |
| N | 4513 | 4507 | 4507 | 4487 | 4487 |
| Breush-Pagan | 9766.4 | 25372.9 | 44478.4 | 46826.8 | 51476.7 |

| Regression (16): | Kauffman Firm Survey: Absolute Value of Residuals | | | | |
|---|---|---|---|---|---|
| Own Equity (Year 1) | 103438.3*** | 91834.9** | 91392.5** | 149071.3** | 270053.7*** |
| | (31297.5) | (31278.1) | (31081.2) | (47672.5) | (70590.7) |
| Average Employment | | 27431939.3*** | 27735636.5*** | 27071817.6*** | 28384184.7*** |
| | | (6515923.5) | (6731794.3) | (6818830.2) | (6830044.4) |
| Average Employment$^2$ | | -50803.8*** | -51148.6*** | -37135.7* | -48799.7** |
| | | (14589.2) | (14869.6) | (17590.0) | (18210.0) |
| Employer | | | 8061673.4 | 13676598.8 | 8979985.6 |
| | | | (116426650.6) | (116821952.5) | (116644344.0) |
| Total Investment (Year 1) | | | | -59827.2 | -185929.9** |
| | | | | (36912.0) | (65337.5) |
| Total Debt (Year 1) | | | | | 241370.4* |
| | | | | | (102115.9) |
| Constant | Yes | Yes | Yes | Yes | Yes |
| 2 Digit NAICs Codes | No | No | Yes | Yes | Yes |
| N | 4513 | 4507 | 4507 | 4487 | 4487 |

### 3.4 Sources of Funding

Figure 4: External Funds



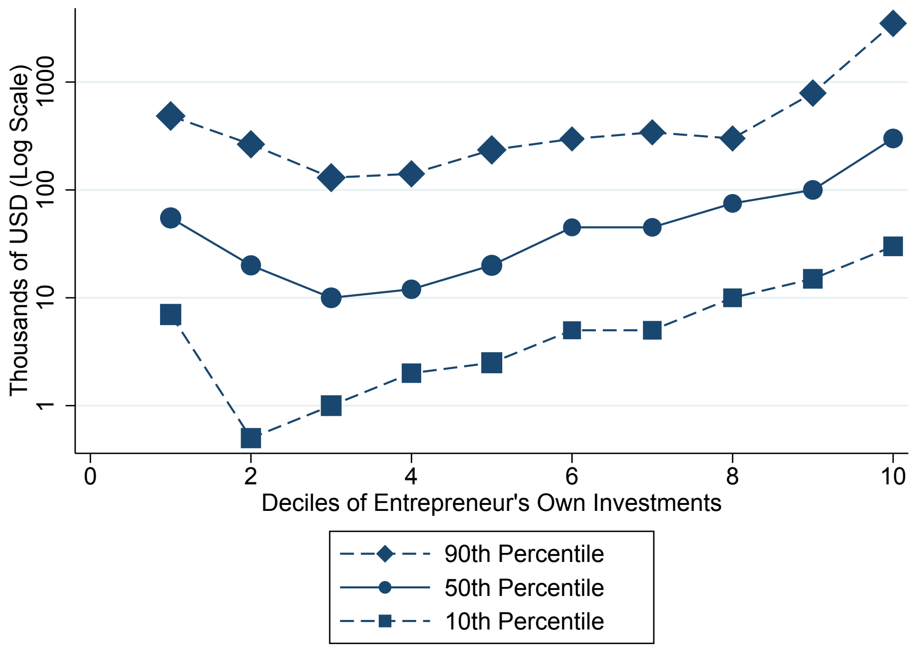

The proportion of firms that have raised some debt, some external equity, both, or neither across the distribution of entrepreneur's initial investment. Firms are sorted into deciles based on the total amount of their entrepreneur's own investment in the firm in the first year of operations.

How able are new entrepreneurs to raise external funds? Figure 4 shows the proportion of firms that are able to raise any debt or any external equity in order to fund their firm. Clearly entrepreneurs who invest more of their own money in the firm are more likely to raise external funds, whether from debt or external equity.

Figures 5 and 6 show how much debt and how much external equity entrepreneurs raise broken down by the deciles of entrepreneurs' total own investments. Note that these graphs have a log-scale. Across the first three deciles, there appears to be somewhat of a decline, suggesting that there is a small fraction of firms who are able to substitute owners investment for external funds, but over the rest of the distribution, larger entrepreneur's investments are correlated with larger amounts of external funds. See table 13 for a detailed breakdown of the sources of debt and external equity.

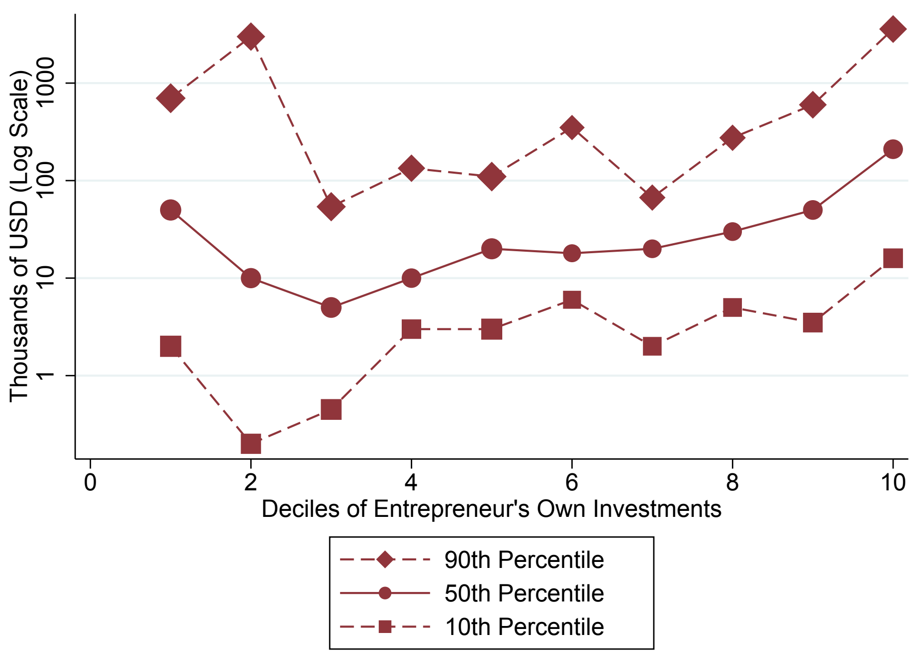
These figures suggest that both on the extensive and on the intensive margin, entrepreneurs who invest more of their own money raise more external funds. These patterns are consistent with the idea that wealth-poor entrepreneurs are unable to

Figure 5: Amount of Debt




The $10^{th}$, $50^{th}$ and $90^{th}$ percentiles of the distribution of total debt raised by firms across the different deciles of the total of entrepreneur's own investments, conditional on raising some debt. Note the y-axis is a log-scale.

Figure 6: Amount of External Equity




The $10^{th}$, $50^{th}$ and $90^{th}$ percentiles of the distribution of total external equity raised by firms across the different deciles of the total of entrepreneur's own investments, conditional on raising some external equity. Note the y-axis is a log-scale.

borrow to finance their business. They motivate the modelling of borrowing constraints as a collateral constraint, which allows entrepreneurs to borrow more only as they invest more.

# 4 Dynamic Model

In order to quantify the relative importance of the missing market for entrepreneurial risk and borrowing constraints, I build a dynamic general equilibrium model with occupational choice and endogenous risk choice.

## 4.1 Environment

There are a unit measure of agents. Each agent faces a constant probability $(1-\psi)$ of dying every period and has CRRA preferences given by:

$$U = \sum_{t=0}^{\infty} (\psi\beta)^t \frac{c^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} \tag{17}$$

Where $\beta$ is the discount factor.

Each agent has two types of ability, their ability as a worker $(h^W)$ and their ability as an entrepreneur $(h^E)$. At the beginning of each period, an agent will choose whether to work as a worker or as an entrepreneur for that period.

If the agent chooses to be a worker, they will supply labour inelastically and earn $wh^W$, where $w$ is the common wage.

The first period an agent decides to be an entrepreneur, they start a business by choosing a business type $x \in \{x_1, x_2, ..., x_{n_x}\}$, and investing in a capital stock $k$. Once chosen, the business type $x$ is fixed. In a future period, if an entrepreneur wants to change their business type they must shut down their business and liquidate their capital stock before they are able to select a new business type.

After all agents make an occupational choice decisions and all new entrepreneurs decide on a business type, all agents receive shocks to both their ability as a worker $h^W$ and their ability as an entrepreneur $h^E$. While both types of ability are partially persistent, neither are perfectly so, and so agents face idiosyncratic income risk from choosing either occupation.

In addition, entrepreneurs also receive a project productivity shock $(z)$ for their business. $z$ is drawn from a distribution that depends on the business type $x$. Higher $x$ businesses have higher expected $z$ but also more dispersed $z$. The productivity of a business depends on both the firm-specific productivity shock as well as the entrepreneur's entrepreneurial ability $h^E$. Once they received their shocks, entrepreneurs hire an amount of labour $n$ at wage rate $w$ and produce according to:

$$y = (zH^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma \tag{18}$$

After entrepreneurs produce and pay their employee workers, all agents make a consumption, savings, and investment decision. The model has two assets. Only entrepreneurs can invest in capital $k$, which depreciates at rate $\delta$. Capital is also illiquid, so that liquidating one unit of capital produces only $\frac{1}{\chi} < 1$ units of consumption. Given some investment $I$, an entrepreneur's capital stock therefore evolves according to:

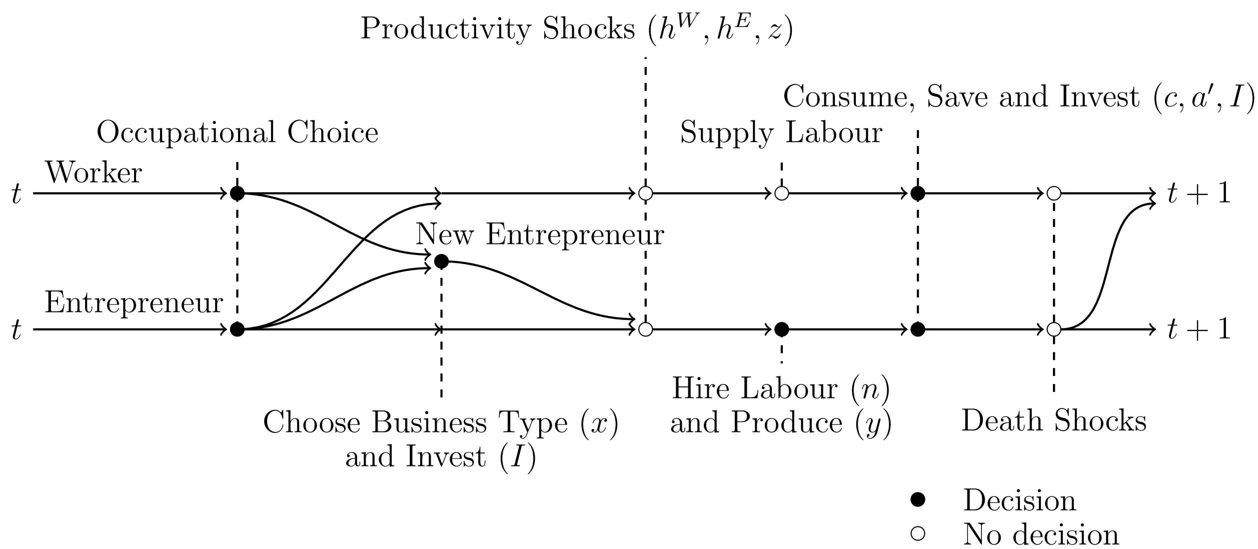
$$k' = \begin{cases} k(1-\delta) + I & \text{if } I \geq 0 \\ k(1-\delta) + \chi I & \text{if } I < 0 \end{cases} \tag{19}$$

Agents can also save and borrow in a liquid financial asset $a$, that pays a constant $r^A$ each period. All agents can borrow up to an exogenous unsecured borrowing limit $\underline{a} \leq 0$. In addition, for each unit of capital that an entrepreneur invests in, they can use this capital as collateral and borrow an addition $\phi \in [0,1]$ units of $a$. Thus the borrowing constraint is given by:

$$a \geq \underline{a} - \phi k \tag{20}$$

After their consumption, savings, and investment decisions, agents face a death shock. With probability $(1-\psi)$ an agent will die. If they were an entrepreneur, their invested capital stock is liquidated. All agents that die are immediately replaced by a descendent who inherits the full value of their liquidated assets. Figure 7 summarizes the timing of the model.

Figure 7: Timing in the Dynamic Model




## 4.2 Agent's Problems

### Worker's Problem

A worker makes a consumption-savings decision, and at the beginning of the next period will choose between being a worker $(V^W)$ and becoming a new entrepreneur $(V^{NE})$:

$$V^W(a, h^W, h^E) = \max_{a',c} \frac{c^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta \max\left\{\mathbb{E}\left[V^W(a', h^{W\prime}, h^{E\prime})\right], V^{NE}(a', h^{W\prime}, h^{E\prime})\right\} \tag{21}$$

s.t.

$$a' + c = wh^W + (1 + r^a)a$$

$$a' \geq \underline{a}$$

### New Entrepreneur's Problem

An agent that has decided to start a new business chooses a business type $x$. Given their current financial assets $a$, they also choose how much to invest in the business ($I$) and how much to borrow or save in the financial assets $\tilde{a}$. They will then operate as an entrepreneur this period with a capital stock $\tilde{k}$.

$$V^{NE}(a, h^W_{-1}, h^E_{-1}) = \max_{\tilde{a},I,x} \mathbb{E}\left[V^E(\tilde{a}, \tilde{k}, h^W, h^E, z, x)\right] \tag{22}$$

s.t.

$$\tilde{a} = a - I$$

$$\tilde{a} \geq \underline{a} - \phi\tilde{k}$$

$$\tilde{k} = I$$

### Entrepreneur's Problem

An entrepreneur that has a business will choose an amount of labour $n$ to hire, consumption $c$, savings $(I + a')$, and investment $I$. At the beginning of the next period, they will choose between shutting down their business to become a worker ($V^W$), shutting down their business to start a new business ($V^{NE}$) and continuing to operate the same business ($V^E$).

$$V^E(a, k, h^W, h^E, z, x) = \max_{n,c,a',I} \frac{c^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta \max\left\{\begin{array}{c} \mathbb{E}\left[V^W(a', h^{W\prime}, h^{E\prime})\right], \\ V^{NE}(a', h^W, h^E), \\ \mathbb{E}\left[V^E(a', k', h^{W\prime}, h^{E\prime}, z', x)\right] \end{array}\right\} \tag{23}$$

s.t.

$$c + a' + I = (zh^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn + (1 + r^a)a$$

$$a' \geq \underline{a} - \phi k'$$

$$k' = \begin{cases} k(1-\delta)+I & \text{if } I \geq 0 \\ k(1-\delta)+\chi I & \text{if } I < 0 \end{cases}$$

Note that if the entrepreneur decides to start a new business, they cannot use the capital from their current business. They must fully liquidate it this period.

## 4.3 Equilibrium

An equilibrium is a set of value functions $\{V^W, V^{NE}, V^E\}$, occupational choices, a set of policy functions $\{c^W, a'^W, \tilde{a}^{NE}, I^{NE}, x^{NE}, c^E, a'^E, I^E, n^E\}$, a distribution of agents $\{\Gamma^E(a,k,h^W,h^E,z,x), \Gamma^W(a,h^W,h^E)\}$, and a price $w$ such that

1. The policy functions solve the problems given by 21, 22 and 23.
2. All markets clear:
   - Labour

$$\int h d\Gamma^W(a,h^W,h^E) = \int n^E d\Gamma^E(a,k,h^W,h^E,z,x)$$

   - Final Goods

$$\int (c+a')d\Gamma^W(a,h^W,h^E) + \int (c+a'+I)d\Gamma^E(a,k,h^W,h^E,z,x) =$$
$$\int \left((zh^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma + (1+r^A)a\right) d\Gamma^E(a,k,h^W,h^E,z,x)$$
$$+ \int ((1+r^A)a)d\Gamma^W(a,h^W,h^E)$$

3. The distribution of agents is stationary

$$\Gamma^E(a,k,h^W,h^E,z,x) = \Gamma^{E\prime}(a,k,h^W,h^E,z,x)$$

$$\Gamma^W(a,h^W,h^E) = \Gamma^{W\prime}(a,h^W,h^E)$$

Note that I assume the economy is a small open economy, and so that financial asset can be in net positive or negative supply.

## 4.4 Model Predictions

### Patterns of Selection

Figure 8 illustrates the occupational choice and endogenous risk choice in the model. For a worker with the lowest worker ability ($h^W$), the graph shows how the worker would select their occupation depending on their current level of entrepreneurial productivity ($h^E$) and their current level of cash on hand ($a(1+r^A)+wh^W$), which captures the wealth of the agent.

Agents with low entrepreneurial ability ($h^E$) will become workers regardless of their wealth. For agents with high entrepreneurial ability, their choices will depend on their level of wealth. If the agent has little wealth, they will choose to be a worker. The borrowing constraint limits the scale of the business a poor entrepreneur can operate, which limits their entrepreneurial income. In addition, if the business fails, the poor agent will earn very low income that period and consume very little.

Agents with high entrepreneurial ability and moderate wealth will choose to be entrepreneurs operating the low risk project. While the expected productivity of the high-risk project is greater, the higher risk makes it unattractive. A bad productivity shock would leave the entrepreneur with low income for at least one period, and if the shock was low enough that they decided to exit, the entrepreneur would have to liquidate their capital stock. would curtail the entrepreneur's consumption substantially and make exit more likely.

Agents with high entrepreneurial ability and high wealth will choose to be entrepreneurs operating the high risk project. Since they have sufficient wealth to self-insure any bad productivity shocks, they choose the higher risk project with higher returns.

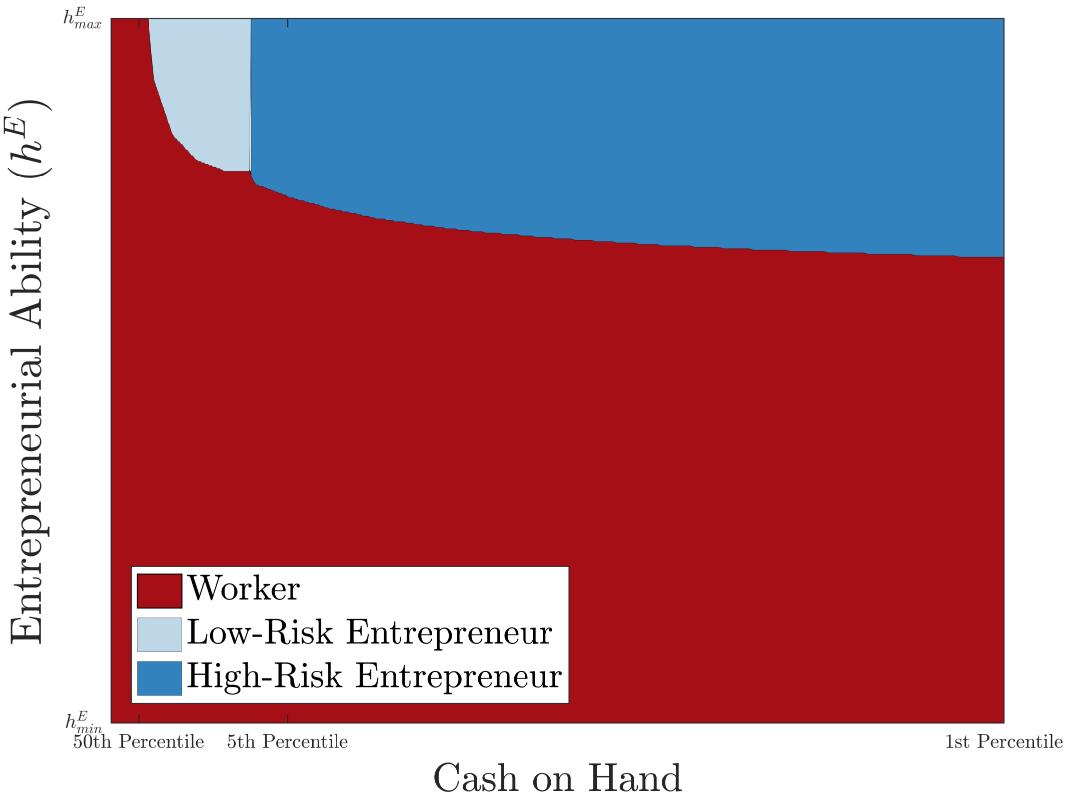
### Wealth Accumulation

In this model, wealthier agents accumulate wealth faster than poor agents given the same abilities. Figure 9 illustrates this dynamic for two agents with identical entrepreneurial and worker abilities. Both agents are born at time 1 as workers with the highest entrepreneurial and worker ability. However, they are endowed with different initial levels of wealth.

The wealthier agent immediately starts a high risk business. In this illustrative example, they receive the highest project productivity and earn high returns on their wealth. The wealthy agent saves a high proportion of their entrepreneurial income for two reasons. First, because the project is risky, the agent wants to engage in precautionary savings in order to self-insure against the project's failure. Second, because of the borrowing constraint, the entrepreneur wants to save more in order to operate a larger business next period and earn an even higher income.

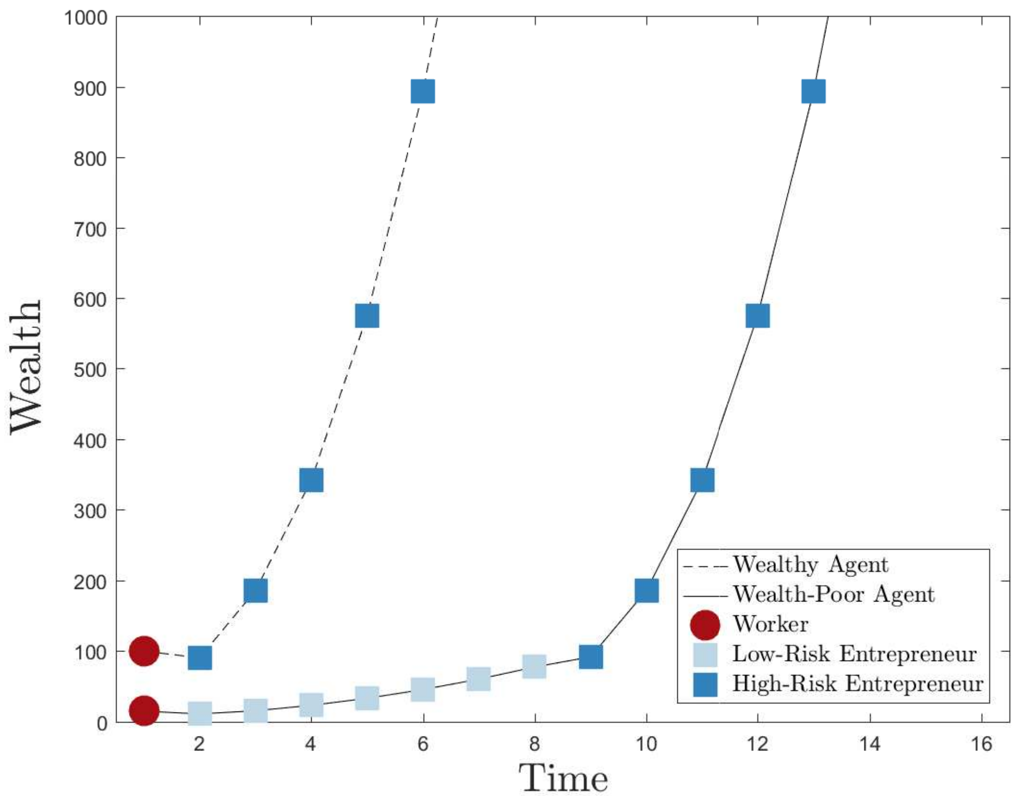
Lacking the wealth to self insure against large fluctuations in income, the wealth poor-agent starts a low risk business. In this illustrative example, they receive the highest project productivity, but earn substantially lower entrepreneurial income. They save at high rates and slowly accumulate enough wealth until they are willing to shut down their safe business and start a new risky business. Of course, if they receive any negative productivity shocks before then it will take them even longer to accumulate the necessary net worth to be willing to start a risky business.

Figure 8: Selection Patterns




Patterns of occupational choice and endogenous risk choice in the calibrated model. Given a worker with a particular worker ability ($h^W$), the graph shows how the worker would select their occupation depending on their current level of entrepreneurial productivity ($h^E$) and their current cash on hand ($a(1+r^A)+wh^W$). The x-axis labels the level

Figure 9: Wealth Accumulation Patterns



# 5 Calibration

I calibrate this model to match micro data on new firm dynamics and wealth inequality by using the Kauffman Firm Survey and the 2004 Survey of Consumer Finances. For a set of parameters that are difficult to identify using my data, I rely on commonly used values in the literature. I then jointly match twelve model moments to twelve moments in the data by varying twelve parameters.

### Parameterization of the Ability and Productivity Processes

I parameterize both the worker and entrepreneurial ability processes with AR(1) processes:

$$\log(h^{i\prime}) = \rho_{h^i}\log(h^i) + \epsilon_{h^i}$$

where $\epsilon_{h^i} \sim N\left(\frac{\mu_{h^i}}{1-\rho_{h^i}}, \sigma^2_{h^i}\right)$ for $i \in \{E, W\}$

In order to keep the problem computationally tractable, I restrict myself to the case where entrepreneurs choose between two project types, a relatively safe project ($x_1$) and a relatively risky project ($x_2$). Each project type receives productivity shocks $z$ the follows an AR(1) process:

$$\log(z') = \rho_{z|x_i}\log(z) + \epsilon_{z|x_i}$$

$$\epsilon_{z|x_i} \sim N\left(\frac{\mu_{z|x_i}}{1-\rho_{z|x_i}}, \sigma^2_{\epsilon_z|x_i}\right)$$

for $i \in \{1, 2\}$.

I use the Rowenhorst method to discretize these four AR(1) processes $\{h^W, h^E, z_{x_1}, z_{x_2}\}$.

### Externally Chosen Parameters

A period in the model is a year. I fix ten parameters to commonly used values in the literature. I set the capital share $\alpha$ to $\frac{1}{3}$ and the interest rate $r^A$ to 4% per year. I set the decreasing returns to scale parameter $\gamma$ to 0.85 as in Midrigan and Xu (2014) and the depreciation rate to 6% per year. The time discount factor is set such that $\beta = \frac{1}{\psi}\frac{1}{1+r^A}$. I set the coefficient of relative risk aversion to 1.50. I set the probability of death $(1-\psi)$ to $\frac{1}{40}$ so that the expected working lifetime is 40 years.

I normalize the average level of worker ability ($\mu_{h^W}$) to 1. I set the persistence of the labour income process $\rho_{\epsilon_h}$ to 0.9, which is in the range of empirical estimates according to Guvenen (2007). The standard deviation of the innovation $\sigma_{\epsilon_h}$ is set to 0.2.

### Internally Calibrated Parameters

I jointly calibrate twelve parameters to match twelve model generated moments to the corresponding moments in the data. Since the main mechanism it proposes has to do with patterns of selection and wealth accumulation for entrepreneurs, it must match the firm dynamics of new firms. Since this model is designed to generate the empirically observed wealth inequality, it must also be consistent with micro economic data on household wealth holdings.

In order to match the model's dynamics to observable firm dynamics, I simulate in the model an 8 year panel of new firms, corresponding to the Kauffman Firm Survey's 8 year panel. In the data I observe the ratio of debt and equity that these new firms start with, and so to pin down the tightness of the borrowing constraint ($\phi$), I match the ratio of average debt to average equity in the panel to the ratio of debt to equity in the data.

The degree of idiosyncratic entrepreneurial risk is vital to this model. Obviously, it is not possible to directly observe the distribution of risk each entrepreneur is drawing from. However, the distribution of firm outcomes is informative about the nature of entrepreneurial risk that all entrepreneurs are facing. The model predicts that wealthier individuals will select higher risk projects and invest more in them. Therefore I separate firm both in the model simulation and in the data at the 90th percentile of owner's own investment over the first three years. I compute moments separately for the firms in the top 10% of first-three-year investment and those in the bottom 90% of first-three-year investment.

To discipline the size and persistence of productivity shocks, I match the survival rates and the auto-correlation of employment for both these high-investment and low-investment firms. In order to pin down the differences in average productivity between the low-risk and high risk project, I match the ratio of average employment in these high-investment firms to the average employment in the low-investment firms at the eighth year of the panel, conditional on survival.

Of course these moments on firm dynamics, (survival, auto-correlation of employment, and average productivity) are all jointly generated by shocks to the business's productivity and shocks to the entrepreneur's ability. Both types of shocks likewise influence savings motives and so the accumulation of wealth in the model. Thus it is vital that the model also match micro evidence on the wealth inequality in the US economy. To match up with the distribution of firms when the Kauffman Firm Survey firms are initially founded, I use the 2004 wave of the Survey of Consumer Finances.

Since this model is primarily about the differences in savings behaviour of the entrepreneurs and the workers, I match the ratio of average wealth of entrepreneurs to workers, the proportion of entrepreneurs in the economy, and the proportion of entrepreneurs in the wealthiest 1%. Following Cagetti and De Nardi (2006), I match the wealth Gini, and since this model is a model of heterogeneous entrepreneurs, I also match the wealth Gini of the entrepreneurs to ensure that the degree of wealth inequality within entrepreneurs matches the data.

Finally, to replicate the bottom of the wealth distribution I match the level of unsecured borrowing ($\underline{a}$) to ensure the correct proportion of agents have negative net worth.

Table 4: Benchmark Calibration

| Target | Data | Model | Parameter | Value |
|---|---|---|---|---|
| Exogenously Chosen | | | | |
| Capital Share | | | $\alpha$ | 0.33 |
| Interest Rate | | | $r^A$ | 0.04 |
| Entrepreneurial Share | | | $1-\gamma$ | 0.12 |
| Depreciation | | | $\delta$ | 0.06 |
| Discount Factor | | | $\beta$ | 0.96 |
| Coefficient of Relative Risk Aversion | | | $\sigma$ | 1.50 |
| Elasticity of Intertemporal Substitution | | | $\theta$ | 0.67 |
| Average Labour Ability | | | $\mu_{h^W}$ | 1.00 |
| Dispersion of Labour Ability | | | $\sigma_{h^W}$ | 0.20 |
| Persistence of Labour Ability | | | $\rho_{h^W}$ | 0.90 |
| Endogenously Calibrated | | | | |
| Ratio of Average Debt to Average Equity | 1.35 | 1.31 | $\phi$ | 0.57 |
| Proportion of Entrepreneurs | 0.11 | 0.14 | $\chi$ | 1.07 |
| Proportion with Negative Net Worth | 0.08 | 0.07 | $\underline{a}$ | -3.63 |
| Wealth Ratio of Entrepreneurs to Workers | 7.14 | 6.91 | $\mu_{h^E}$ | -4.18 |
| Wealth Gini of Entrepreneurs | 0.75 | 0.60 | $\sigma_{h^E}$ | 1.01 |
| Wealth Gini | 0.79 | 0.66 | $\rho_{h^E}$ | 0.94 |
| Proportion of Entrepreneurs in Wealthiest 1% | 0.74 | 0.80 | $\mu_{z\|x_1}$ | 2.17 |
| Autocorrelation of Employment | 0.03 | 0.06 | $\sigma_{z\|x_1}$ | 0.08 |
| Survival Rate | 0.50 | 0.45 | $\rho_{z\|x_1}$ | 0.73 |
| Relative Employment | 4.23 | 3.80 | $\mu_{z\|x_2}$ | -2.28 |
| Autocorrelation of Employment | 0.12 | 0.06 | $\sigma_{z\|x_2}$ | 4.89 |
| Survival Rate | 0.60 | 0.36 | $\rho_{z\|x_2}$ | 0.79 |

# 6 Validation

In the model, I simulate an 8 year panel of new firms corresponding to the Kauffman Firm Survey's 8 year panel. Within this simulated panel, I am able to compare the behaviour of new entrepreneurs to the untargeted stylized facts from Kauffman Firm Survey.

## 6.1 Dispersion in Cumulative Profits

Table 5 replicates regression (16) on the simulated data. Just as in the KFS (see table 3), entrepreneurs who invest more in their own business earn more dispersed cumulative profits even after controlling for the size of the business.

$$\text{Cumulative Profits}_i = \alpha_0 + \alpha_1 \text{Initial Entrepreneur's Own Investment}_i + \alpha X_i + \epsilon_i \quad (15)$$

Table 5: Dispersion in Cumulative Profits in the Model

| Second Regression: | Model Simulation: Absolute Value of Residuals | | |
|---|---|---|---|
| | (1) | (2) | (3) |
| Initial Own Equity | 83.05*** | 3.013*** | 2.371*** |
| | (1.700) | (0.133) | (0.139) |
| Average Employment | | 5.448*** | 0.789 |
| | | (0.654) | (0.741) |
| Average Employment$^2$ | | -0.0498*** | -0.0148 |
| | | (0.0109) | (0.0109) |
| Initial Debt | | | -0.901*** |
| | | | (0.168) |
| Constant | Yes | Yes | Yes |
| N | 10000 | 10000 | 10000 |

$$\hat{\epsilon}_i^2 = \beta_0 + \beta_1 \text{Initial Entrepreneur's Own Investment}_i + \beta X_i + \eta_i \tag{16}$$

## 6.2 Distribution of Investment

Figures (10) and (11) compares the distribution of cumulative investment in the data and in the model. As can be seen from figure (10) the distribution of investment is fairly skewed and high-investment firms still rely heavily on debt. Figure (11) shows that the model can replicate both of these features.

Figure 10: KFS Data: Investment Composition by Decile of Initial Own Investment

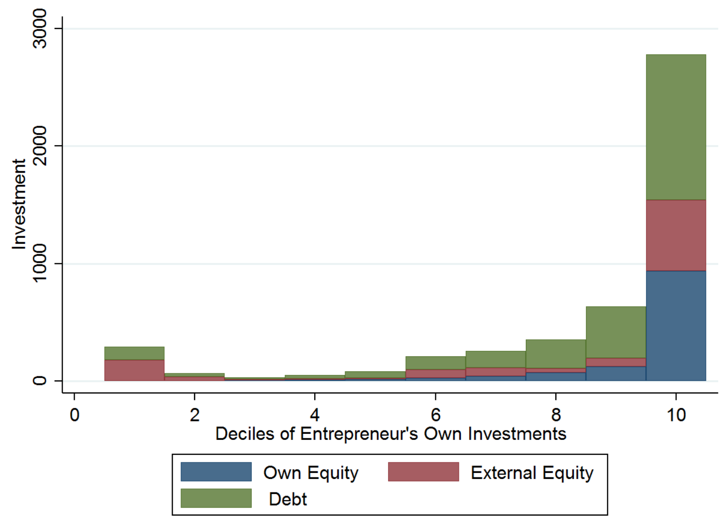
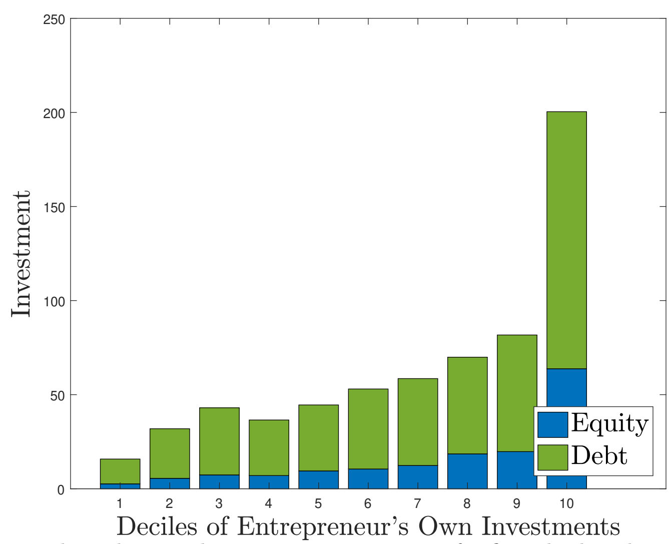



Figure 11: Model Simulation: Investment by Decile of Initial Own Investment




These two figures show the cumulative investment sources for firms broken down by the decile of entrepreneur's own initial investment. The top figure is the Kauffman Firm Survey Data and the bottom figure is the model's simulation.

# 7 Quantitative Analysis

## 7.1 The Relative Importance of Two Frictions

A central question in this paper is how important is the missing market for entrepreneurial risk relative to borrowing constraints for reducing entrepreneurial activity in the economy. In order to assess the quantitative significance of both of these frictions, I remove first one, then the other, and finally both from the model in order to compare the resulting equilibria.

In order to assess the importance of the missing market for entrepreneurial risk, I complete this missing market by introducing contingent claims. In the benchmark economy, there is a single risk free asset. In the contingent claims economy, each agent can purchase assets that pay off in future states of the world based on their individual realizations of $h^W, h^E$ and $z$. An asset $a_i$ pays off $1 + r^A$ in the state of the world $i \in H^W \times H^E \times Z$. Each of these assets are sold at actuarially fair price $q_i$ by competitive risk-neutral financial intermediaries.

$$q(h^{W\prime}, h^{E\prime}, z') = \text{Prob}(h^{W\prime}, h^{E\prime}, z' | h^W, h^E, z, x) \tag{24}$$

In order to separate the impact of the missing market for entrepreneurial risk from the impact of the borrowing constraint, I keep the borrowing constraint in the economy unchanged. In the benchmark economy the borrowing constraint requires that $a \geq \underline{a} + \phi k$. In the contingent claims economy, the borrowing constraint requires that:

$$-\sum_i q_i a_i \geq \underline{a} + \phi k \tag{25}$$

Thus an entrepreneur with the same amount of net worth in both economies can invest in the same maximum amount of capital.

Obviously, considering the perfect completion of the missing market for entrepreneurial risk is not directly policy-relevant. In the real world, information frictions will always create moral hazard and adverse selection problems that make it difficult to provide insurance to entrepreneurs. This complete market assumes financial intermediaries will have perfect information over the exact abilities of potential entrepreneurs and expected productivity of projects. This level of information is infeasible in the real world, however this exercise is still a useful counter-factual to help us understand what the potential gains are so that we can think about what frictions public policy should aim to mitigate. In section 8, I consider the policy implications of this friction.

To remove borrowing constraints, I simply set $\phi = 1$, so that entrepreneurs can invest in any level of capital stock, unrestricted by their personal net worth. I then compare four economies. The benchmark economy with both frictions, an economy where contingent claims complete the missing insurance market but entrepreneurs still face borrowing constraints ($\phi < 1$), the economy where entrepreneurs can borrow the full value of their capital ($\phi = 1$) with the missing market for entrepreneurial risk, and finally an economy with both contingent claims and unrestricted capital investment ($\phi = 1$). Table 6 reports aggregate output, wages, capital stock, and TFP in these four economies.

Table 6: Productivity Losses from the Two Friction

| | $Y$ | $w$ | $K$ | TFP | Percentage of Workers | Percentage of Entrepreneurs | Percentage of Low Risk | Percentage of High Risk |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 1.00 | 1.00 | 1.00 | 1.00 | 84.0 | 16.0 | 13.7 | 2.2 |
| Contingent Claims | 1.17 | 1.13 | 1.09 | 1.11 | 86.3 | 13.7 | 0.0 | 13.7 |
| No Borrowing Constraint | 1.09 | 1.07 | 1.22 | 1.02 | 84.9 | 15.1 | 12.3 | 2.7 |
| Both | 1.43 | 1.36 | 1.92 | 1.15 | 87.3 | 12.7 | 0.0 | 12.7 |

Aggregate moments from comparing four economies. The first four columns list output ($Y$), wage ($w$), capital stock ($K$), and total factor productivity (TFP) relative to the benchmark economy. The latter four columns report the percentage of agents in the economy working as workers or entrepreneurs and the percentage of agents in the economy that are entrepreneurs operating the low risk and the high risk project. "Contingent Claims" refers to the economy with individual contingent claims. "No Borrowing Constraints" refers to the economy where $\phi = 1$, so entrepreneurs can invest in any amount of capital. "Both" refers to the economy with both contingent claims and $\phi = 1$.

### 7.1.1 Conditional Claims

Introducing conditional claims increases output in the economy in four distinct ways. First, with the ability to insure against entrepreneurial risk, entrepreneurs exclusively choose the high risk project, rather than the low risk project. As the higher-risk project has a higher expected productivity, this boosts productivity and aggregate output. Secondly, individuals with higher entrepreneurial ability are more likely to select into entrepreneurship. Third, conditional on wealth, entrepreneurs invest more in their businesses. Finally, because entrepreneurs earn higher incomes as a result of all three previous reasons, they accumulate more wealth and thus are able to run larger businesses, despite the continued presence of borrowing constraints.

Figure 12 shows the amount of capital invested by the most productive individuals in the risky project based on the total cash on hand available to them at the end of a period. In the benchmark economy, because of the substantial capital investment risks entrepreneurs face, they invest less than the expected marginal product of capital. With the introduction of contingent claims, entrepreneurs insure themselves against bad shocks, and so choose to invest more in their businesses. In both economies, relatively wealth-poor individuals are constrained by their personal wealth and borrow up to the exogenous borrowing limit.

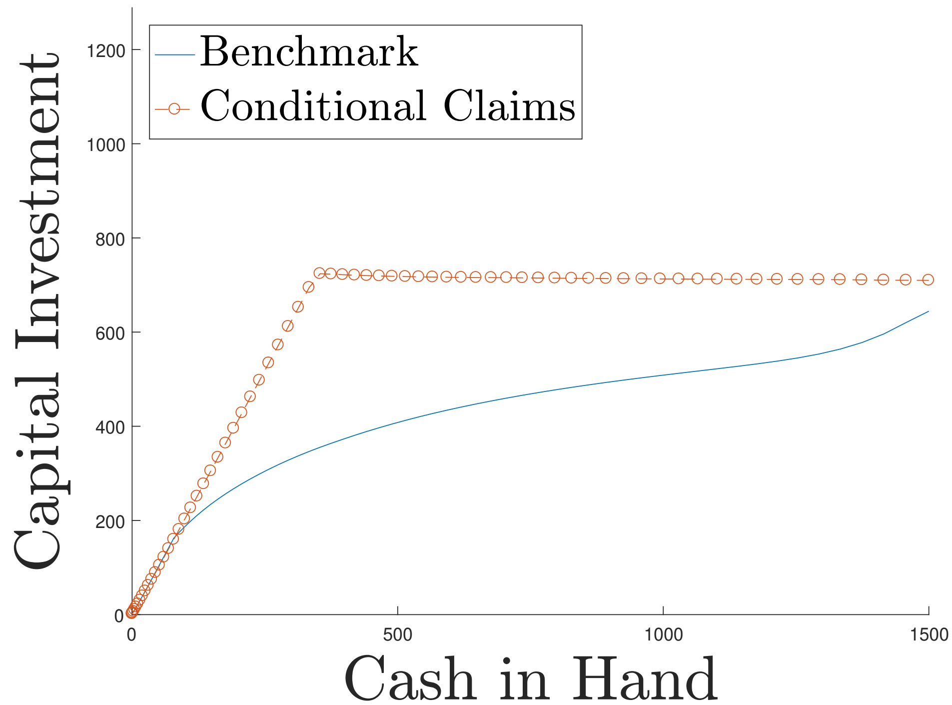


Figure 12: Capital Investment

#### 7.1.2 Unrestricted Capital Investment

Removing borrowing constraints likewise increases output in the same four way as the introduction of contingent claims. First, the removal of borrowing constraints encourage more, but not all, entrepreneurs to start the high risk project. Secondly, individuals with higher entrepreneurial ability are more likely to select into entrepreneurship. Third, conditional on wealth, entrepreneurs invest more in their businesses. Finally, because entrepreneurs earn higher incomes as a result of all three previous reasons, they accumulate more wealth and are thus able to run larger businesses, despite the missing market for entrepreneurial risk.

Figure 13 shows the investment patterns for the highest productivity entrepreneur in the benchmark and unrestricted investment economies. Wealth poor individuals in the unrestricted investment economy are able to invest more than those in the benchmark economy because of the lack of borrowing constraint. However, because of the substantial capital investment risk, they still invest less than the expected marginal product of capital.

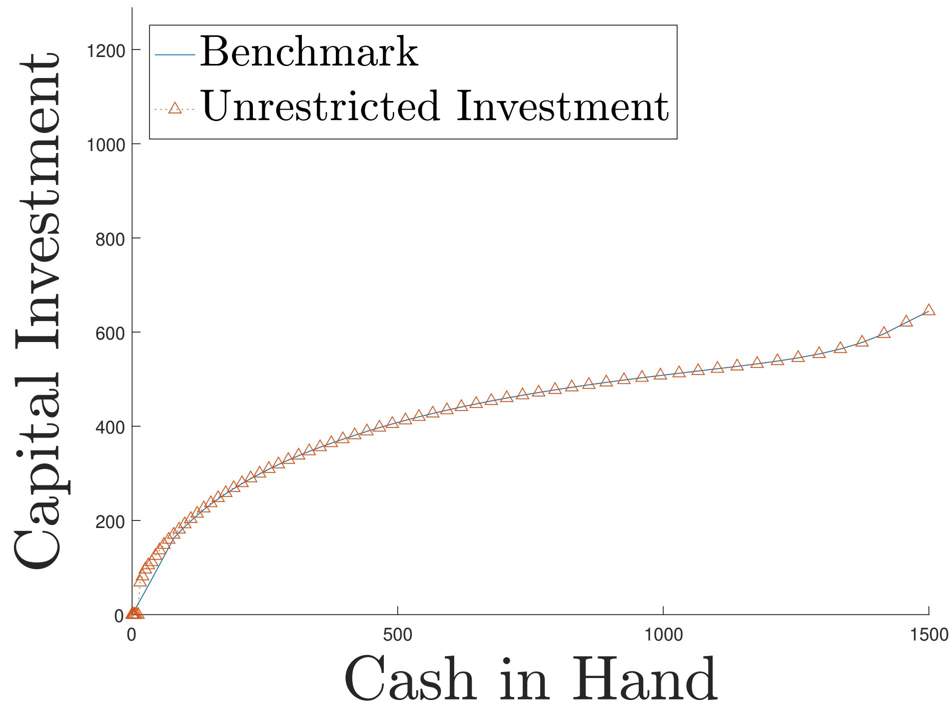



Figure 13: Capital Investment

## 7.2 Endogenous Risk and Wealth Inequality

In the benchmark economy, risk is endogenous in the sense that entrepreneurs can choose between projects with levels of risk and reward. Wealthier entrepreneurs optimally choose higher-risk higher-reward projects and so in expectation earn higher average returns than poorer entrepreneurs. These higher rates of return constitute a form of rate of return heterogeneity that can potentially have an important role shaping the aggregate distribution of wealth. A key question is how much additional dispersion in wealth this endogenous choice of risk results in. To measure the quantitative importance of this channel, I compare the benchmark endogenous risk economy with an exogenous risk economy, where new entrepreneurs are not able to choose their project type and instead are randomly allocated a project.

However, in the benchmark economy, entrepreneurs strictly prefer one of the project types, so the removal of project choice will strictly reduces the value of entrepreneurship for all agents. Naively comparing two economies, one with endogenous project choice and one with exogenous project allocation, will then conflate the effects of the endogenous choice of risk with changes in occupational selection patterns. In order to provide an isolated assessment of the contribution of the endogenous risk choice to wealth inequality, I first fix occupations. I consider a variant of the model where agents are born as either workers or entrepreneurs, and can never change occupations. This also allows me to compare distributions of wealth that have a consistent percentage of entrepreneurs.

I proceed in several steps. Starting from the benchmark economy, I first consider an economy where entrepreneurs can make an endogenous risk choice but all agents have a fixed occupation. I then compare this economy to another economy with fixed

Table 7: Occupational Choice, Endogenous Risk Choice and Wealth Inequality

| | | Wealth Share of Top | | | | | Percentage of | Entrepreneurs operating | |
|---|---|---|---|---|---|---|---|---|---|
| Occupations | Risk | 1% | 5% | 10% | 20% | 50% | Entrepreneurs | Low Risk | High Risk |
| SCF Data | | 33 | 57 | 69 | 83 | 94 | 12.2 | | |
| Chosen | Endogenous | 29 | 64 | 75 | 85 | 95 | 13.7 | 11.8 | 1.9 |
| Fixed | Endogenous | 50 | 68 | 75 | 86 | 102 | 13.7 | 10.8 | 2.9 |
| Fixed | Exogenous | 30 | 66 | 74 | 83 | 97 | 13.7 | 10.8 | 2.9 |
| Chosen | Exogenous | 24 | 52 | 67 | 80 | 94 | 12.1 | 11.1 | 1.0 |

Top wealth shares across four economies. The first five columns list the percentage of aggregate wealth held by selected top percentiles. Note that one number is greater than 100 because individuals in the model can hold negative wealth. The latter three columns report the percentage of entrepreneurs in the economy and the percentage of agents in the economy that are entrepreneurs operating the low risk and the high risk projects.

occupations where entrepreneurs are randomly allocated a project and cannot switch projects. Finally, I report the results from an economy where agents can choose their occupations, but that conditional on becoming an entrepreneur their project type is randomly allocated. In this last economy, the effects of endogenous risk choice are conflated with the changing occupational selection decisions. Table 7 compares the top wealth shares across these four economies.

Comparing rows 1 and 2, shows how important the labour market option is for entrepreneurs. In the benchmark economy, when entrepreneurs receive bad shocks they always have the option to return to the labour market. As a consequence, precautionary savings is much lower than in the Fixed Occupations Endogenous Risk economy. Without the option to go into the labour market, entrepreneurs save at much higher rates. They do so both because they anticipate that at some point they may face a period of low productivity, and the fact that the borrowing constraints will limit future income unless they can self-finance a large stock of capital. As a consequence of these strong savings motives, entrepreneurs save much more than in the benchmark economy. Workers, without the possibility of ever becoming an entrepreneur, save very little. As a consequence, the average difference in wealth between entrepreneurs and workers increases sharply.

Keeping the occupational choice fixed, comparing rows 2 and 3 shows how different top-tail wealth inequality looks in a model with endogenous risk compared to an exogenous allocation of projects. In the endogenous risk economy, wealthy entrepreneurs choose the high risk project, earn high returns on average, and so accumulate wealth much faster than relatively poor entrepreneurs. In the exogenous risk economy, projects are randomly allocated, and so wealthy and wealth-poor entrepreneurs have the same chance at earning these high returns. The difference between the two distributions is mostly confined to the top 1%, as in the Fixed Occupation Endogenous Risk Economy, the extremely high top wealth share of 50% is generated because the wealthy on average earn very high rates of return on large quantities of wealth. Switching to exogenous project allocation means these entrepreneurs are no longer able to earn these higher returns, leading to much lower wealth accumulation.

Note that I also keep the wage rate constant in these comparison economies, rather than clearing the labour market. This is important as changes in the wage rate can have large impacts on the optimal scale of operations for an entrepreneur with a given productivity and so drastically change their savings behaviour.

# 8 Policy Implications

The quantitative results in section 7.1 highlight how important the missing market for entrepreneurial risk is for discouraging entrepreneurial activity. Governments that seek to promote entrepreneurship should therefore not restrict themselves to trying to alleviate borrowing constraints through loan programs. In this section I describe a simple policy experiment that demonstrates how extending unemployment benefits to entrepreneurs can raise output in the economy.

A missing market for entrepreneurial risk discourages individuals from starting businesses or starting high-risk businesses because of their low utility when their business is unsuccessful. Providing any degree of insurance to unsuccessful entrepreneurs may help encourage them to take more risk ex-ante.

I model a simple unemployment insurance benefit for entrepreneurs as a transfer that tops up entrepreneurial income to an income floor given by $\underline{y}$. This benefit is paid for with a lump-sum tax $T$ on all other entrepreneurs. An entrepreneur's income is therefore given by:

$$y = \max((zh^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn + (1 + r^a)a - T, \underline{y}) \tag{26}$$

Table 8 shows the steady state of the economy with different levels of entrepreneurial income floor ($\underline{y}$). I express this benefit relative to the equilibrium wage in the benchmark economy ($\bar{w}$). At low levels, the benefit raises output and TFP in the economy. Even small insurance benefits are enough to encourage some entrepreneurs to choose the higher risk project. As the higher risk project has an expected higher productivity output increases. Labour demand also increases raising the wage, and as a consequence the proportion of entrepreneurs initially falls.

At much higher levels of benefit, adverse selection becomes an increasing problem. Individuals who have low entrepreneurial ability enter simply because the income floor is higher than what they can earn in the labour market. As a consequence output falls.

Obviously, this simple insurance scheme is not an optimal policy, nor would it necessarily deal with moral hazard problems that may arise in the real world. However, it does illustrate the usefulness of partial insurance schemes that can help government encourage entrepreneurial risk taking. The quantitative results in section 7.1 suggest that seeking to provide partial insurance schemes to entrepreneurs has much greater potential to raise output and productivity than alleviating entrepreneur's borrowing constraints.

Table 8: Aggregate Moments with Different Levels of Unemployment Benefit

| Size of $\underline{y}$ | $Y$ | $w$ | $K$ | TFP | % Workers | % Entrepreneurs | % Low Risk | % High Risk |
|---|---|---|---|---|---|---|---|---|
| Benchmark | 1.00 | 1.00 | 1.00 | 1.00 | 83.4 | 16.6 | 9.2 | 7.4 |
| $0.00\bar{w}$ | 1.00 | 1.00 | 1.00 | 1.00 | 83.4 | 16.6 | 9.2 | 7.4 |
| $0.10\bar{w}$ | 1.02 | 1.01 | 1.00 | 1.02 | 84.4 | 15.6 | 3.6 | 12.0 |
| $0.20\bar{w}$ | 1.04 | 1.03 | 1.01 | 1.03 | 84.2 | 15.8 | 0.0 | 15.8 |
| $0.30\bar{w}$ | 1.04 | 1.04 | 1.01 | 1.04 | 83.1 | 16.9 | 0.0 | 16.9 |
| $0.40\bar{w}$ | 1.04 | 1.05 | 1.00 | 1.05 | 81.9 | 18.1 | 0.0 | 18.1 |
| $0.50\bar{w}$ | 1.03 | 1.07 | 0.99 | 1.06 | 80.5 | 19.5 | 0.0 | 19.5 |
| $0.60\bar{w}$ | 1.03 | 1.08 | 0.99 | 1.06 | 79.3 | 20.7 | 0.0 | 20.7 |
| $0.70\bar{w}$ | 1.03 | 1.09 | 0.98 | 1.07 | 78.4 | 21.6 | 0.0 | 21.6 |
| $0.80\bar{w}$ | 1.02 | 1.09 | 0.97 | 1.07 | 77.1 | 22.9 | 0.0 | 22.9 |
| $0.90\bar{w}$ | 0.86 | 1.10 | 0.80 | 1.06 | 60.5 | 39.5 | 0.0 | 39.5 |
| $1.00\bar{w}$ | 0.84 | 1.10 | 0.78 | 1.06 | 59.4 | 40.6 | 0.0 | 40.6 |

Aggregate moments across economies with different levels of entrepreneurial unemployment benefits. All benefits are expressed in terms of the equilibrium wage in the benchmark economy ($\bar{w}$) without any unemployment benefits. The first four columns list output ($Y$), wage ($w$), capital stock ($K$), and total factor productivity (TFP) relative to the benchmark economy. The latter four columns report the percentage of agents in the economy working as workers or entrepreneurs and the percentage of agents in the economy that are entrepreneurs operating the low risk and the high risk project.

# 9 Conclusion

This paper studies the quantitative importance of two financial frictions for output, productivity, and wealth inequality. I present descriptive evidence from the Kauffman Firm Survey that new entrepreneurs face a high degree of idiosyncratic risk and that entrepreneurs who invest more of their own money are more likely to raise external funds. Motivated by these facts, I build a quantitative model of occupational choice and endogenous risk choice. The model features two frictions, a missing market for entrepreneurial risk and borrowing constraints. I calibrate the model to US data on new firms and wealth inequality. I then validate the model by showing that the model can generate a number of untargeted patterns in the data.

The paper makes three key contributions. First, I build a model in which both a missing market for entrepreneurial risk and borrowing constraints distort occupational choice. The model predicts that wealthier individuals will select higher-risk projects leading to higher returns. Second, I quantify the relative importance of the missing market for entrepreneurial risk and borrowing constraints in the US economy. I find that completing the missing market for entrepreneurial risk raises output by almost twice as much as relaxing borrowing constraints do, and that these two frictions interact in an important way. I illustrate the importance of these quantitative results with a simple policy experiment. This policy experiment demonstrates that even though realizing the full gains are likely infeasible, carefully designing public policy with these frictions in mind can lead to higher output. Finally, I show that entrepreneur's endogenous choice of risk is a powerful mechanism that can help explain the high top wealth shares we observe in the data.

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# A Static Model Proofs

## A.1 Proposition 1

Optimal risk taking $x^*$ is decreasing in the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$

$$\frac{\partial x^*}{\partial \frac{u'(\underline{c})}{u'(\bar{c})}} = -\frac{z\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\frac{1}{\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}-1}}{1+\psi\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}} - z\psi\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\frac{1}{\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}-1}\frac{1-\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}}{\left[1+\psi\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}\right]^2} < 0 \tag{27}$$

Optimal capital investment $k^*$ is also decreasing in the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$

$$\frac{\partial k^*}{\partial \frac{u'(\underline{c})}{u'(\bar{c})}} = \frac{1}{1-\alpha}\left(\frac{\alpha}{[1+r^a]}\frac{\left[p(h^E+\psi x)^{1-\gamma}+(1-p)(h^E-x)^{1-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]}{\left[p+(1-p)\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]}\right)^{\frac{1}{1-\alpha}-1}\frac{\alpha}{1+r^a}$$
$$\times\left[\frac{(1-p)(h^E-x)^{1-\gamma}}{p+(1-p)\frac{u'(\underline{c})}{u'(\bar{c})}} - (1-p)\frac{\left[p(h^E+\psi x)^{1-\gamma}+(1-p)(h^E-x)^{1-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]}{\left[p+(1-p)\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]^2}\right] < 0 \tag{28}$$

Cancelling positive terms:

$$\left[\frac{(1-p)(h^E-x)^{1-\gamma}}{p+(1-p)\frac{u'(\underline{c})}{u'(\bar{c})}} - (1-p)\frac{\left[p(h^E+\psi x)^{1-\gamma}+(1-p)(h^E-x)^{1-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]}{\left[p+(1-p)\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right]^2}\right] < 0 \tag{29}$$

Rearranging:

$$(1-p)(h^E-x)^{1-\gamma}\left[p+(1-p)\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right] < (1-p)\left[p(h^E+\psi x)^{1-\gamma}+(1-p)(h^E-x)^{1-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)\right] \tag{30}$$

$$p(h^E-x)^{1-\gamma}+(1-p)(h^E-x)^{1-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right) < p(h^E+\psi x)^{1-\gamma}+(1-p)(h^E-x)^{1-\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right) \tag{31}$$

$$p(h^E-x)^{1-\gamma} < p(h^E+\psi x)^{1-\gamma} \tag{32}$$

$$-x < \psi x \tag{33}$$

Taking the derivative of the ratio of marginal utilities w.r.t. endowed wealth:

$$\frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} = \frac{1}{u'(\bar{c})} u''(\underline{c}) \left( -(h^E - x)^{-\gamma}(1-\gamma)k^{\alpha}\frac{\partial x}{\partial e} + \left[(h^E - x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)\right]\frac{\partial k}{\partial e} + [1+r^a] \right)$$
$$- \frac{u'(\underline{c})}{[u'(\bar{c})]^2} u''(\bar{c}) \left( \psi(h^E + \psi x)^{-\gamma}(1-\gamma)k^{\alpha}\frac{\partial x}{\partial e} + \left[(h^E + \psi x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)\right]\frac{\partial k}{\partial e} + [1+r^a] \right) \tag{34}$$

Rearranging:

$$\begin{aligned} \frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} &= (1+r^a)\left(\frac{u''(\underline{c})}{u'(\underline{c})} - \frac{u''(\bar{c})}{u'(\bar{c})}\right) \\ &\quad + \frac{\partial x}{\partial e}(1-\gamma)k^{\alpha}\left(-(h^E - x)^{-\gamma}\frac{u''(\underline{c})}{u'(\underline{c})} - \psi(h^E + \psi x)^{-\gamma}\frac{u''(\bar{c})}{u'(\bar{c})}\right) \\ + \frac{\partial k}{\partial e}&\left([(h^E - x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)]\frac{u''(\underline{c})}{\underline{c}} - [(h^E + \psi x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)]\frac{u''(\bar{c})}{u'(\bar{c})}\right] \end{aligned} \tag{35}$$

Since $\bar{c} > \underline{c}$, decreasing absolute risk aversion $\frac{\partial}{\partial c}\left(-\frac{u''(c)}{u'(c)}\right) < 0$ implies that:

$$-\frac{u''(\underline{c})}{u'(\underline{c})} > -\frac{u''(\bar{c})}{u'(\bar{c})} \implies \frac{u''(\underline{c})}{u'(\underline{c})} < \frac{u''(\bar{c})}{u'(\bar{c})} \tag{36}$$

As a consequence the first line of 35 is negative. Diminishing marginal utility ($u''(c) < 0$) implies that the term multiplying $\frac{\partial x}{\partial e}$ on the second line is positive. Likewise, diminishing marginal utility and the fact that $(h^E - x)^{1-\gamma}\alpha k^{\alpha-1} < (1+r^a)$ implies that the term multiplying $\frac{\partial k}{\partial e}$ on the third line is positive.

Suppose now that the ratio of marginal utilities $\frac{u'(\underline{c})}{u'(\bar{c})}$ weakly increasing in endowed wealth $e$:

$$\frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} \geq 0 \tag{37}$$

This implies that:

$$\frac{\partial x^*}{\partial e} = \frac{\partial x^*}{\partial \frac{u'(\underline{c})}{u'(\bar{c})}} \frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} \leq 0 \tag{38}$$

and that:

$$\frac{\partial k^*}{\partial e} = \frac{\partial k^*}{\partial \frac{u'(\underline{c})}{u'(\bar{c})}} \frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} \leq 0 \tag{39}$$

But if this is true, then all three terms in 35 are weakly negative with the first term strictly negative. This is a contradiction, so it must be that:

$$\frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial e} < 0 \tag{40}$$

From 27 and 40 it must be that $\frac{\partial x}{\partial e} > 0$ $\square$

## A.2 Proposition 2

This argument is originally from Cressy (2000). For any agent on the margin between choosing to be a worker or an entrepreneur:

$$\begin{aligned} V^W(e) &= V^E(e,z) \\ u(w+(1+r^a)e) &= pu\left[(h^E+\psi x^*)^{1-\gamma}k^{*\alpha}+(1+r^a)a^*\right]+(1-p)u\left[(h^E-x^*)^{1-\gamma}k^{*\alpha}+(1+r^a)a^*\right] \end{aligned}$$

Increases in endowed wealth will raise the value of entrepreneurship relative to being a worker iff:

$$\frac{\partial V^W}{\partial e} < \frac{\partial V^E}{\partial e} \tag{41}$$

Applying the envelope theorem:

$$u'(w+(1+r^a)e) < pu'(\bar{c}) + (1-p)u'(\underline{c}) \tag{42}$$

For the marginal agent, it must be that $\underline{c} < w + (1+r^a)e < \bar{c}$. Applying Jensen's inequality to the marginal utility function, 42 is true if the marginal utility function is convex. Preferences that exhibit decreasing absolute risk aversion will have:

$$\frac{\partial}{\partial e}\left(-\frac{u''}{u'}\right) = -\left(\frac{u'''u' - (u')^2}{(u')^2}\right) < 0 \text{ only if } u''' > 0 \tag{43}$$

## A.3 Proposition 3

$$\frac{\partial x}{\partial \phi} = h^E\left(-\frac{\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\frac{1}{\gamma}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}-1}}{1+\psi\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}} - \psi\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\frac{1-\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}}{\left[1+\psi\left(\frac{1-p}{\psi p}\right)^{\frac{1}{\gamma}}\left(\frac{u'(\underline{c})}{u'(\bar{c})}\right)^{\frac{1}{\gamma}}\right]^2}\right)\frac{\partial\frac{u'(\underline{c})}{u'(\bar{c})}}{\partial\phi} < 0 \tag{44}$$

$$\frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial \phi} = \frac{u''(\underline{c})}{u'(\bar{c})}\left((z-x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)\right)\frac{e}{(1-\phi)^2} - \frac{u'(\underline{c})}{[u'(\bar{c})]^2}u''(\bar{c})\left((z+\psi x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)\right)\frac{e}{(1-\phi)^2} > 0 \quad (45)$$

Rearranging:

$$\left[\frac{u''(\underline{c})}{u'(\underline{c})}\left((z-x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)\right) - \frac{u''(\bar{c})}{u'(\bar{c})}\left((z+\psi x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)\right)\right]\frac{e}{(1-\phi)^2} > 0 \quad (46)$$

$$(1+r^a)\left(\frac{u''(\bar{c})}{u'(\bar{c})} - \frac{u''(\underline{c})}{u'(\underline{c})}\right) + \left(\frac{u''(\underline{c})}{u'(\underline{c})}(z-x)^{1-\gamma} - \frac{u''(\bar{c})}{u'(\bar{c})}(z+\psi x)^{1-\gamma}\right)\alpha k^{\alpha-1} > 0 \quad (47)$$

Since $\bar{c} > \underline{c}$, decreasing absolute risk aversion $\frac{\partial}{\partial c}\left(-\frac{u''(c)}{u'(c)}\right) < 0$ implies that:

$$-\frac{u''(\underline{c})}{u'(\underline{c})} > -\frac{u''(\bar{c})}{u'(\bar{c})} \implies \frac{u''(\underline{c})}{u'(\underline{c})} < \frac{u''(\bar{c})}{u'(\bar{c})} \quad (48)$$

As a consequence the first term of 47 is positive, and the second term is positive for any $x \geq 0$. Therefore $\frac{\partial \frac{u'(\underline{c})}{u'(\bar{c})}}{\partial \phi} > 0$.

Since the term in brackets in 45 is negative, this implies:

$$\frac{\partial x}{\partial \phi} < 0 \quad (49)$$

## A.4 Proposition 4

$$\frac{\partial V^E}{\partial \phi} = pu'(\bar{c})\left[(z+\psi x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)\right]\frac{e}{(1-\phi)^2} + (1-p)u'(\underline{c})\left[(z-x)^{1-\gamma}\alpha k^{\alpha-1} - (1+r^a)\right]\frac{e}{(1-\phi)^2} > 0 \quad (50)$$

Rearranging:

$$\alpha k^{\alpha-1}\left(pu'(\bar{c})(z+\psi x)^{1-\gamma} + (1-p)u'(\underline{c})(z-x)^{1-\gamma}\right) > (1+r^a)\left(pu'(\bar{c}) + (1-p)u'(\underline{c})\right) \quad (51)$$

$$\left[\frac{\alpha}{(1+r^a)}\frac{\left(pu'(\bar{c})(z+\psi x)^{1-\gamma}+(1-p)u'(\underline{c})(z-x)^{1-\gamma}\right)}{(1+r^a)\left(pu'(\bar{c})+(1-p)u'(\underline{c})\right)}\right]^{\frac{1}{1-\alpha}} > k \tag{52}$$

Note that the left hand side of this express is the expression for $k^*$ from 9, and all constrained entrepreneurs will have $k < k^*$.

# B Kauffman Firm Survey Facts

## B.1 Measuring Survival

In table 9, I report the current operational status of all firms in the Kauffman Firm Survey over the 8 years of the survey. Firm exit is common. While non-response and sample attrition mean that the status of 701 firms are not available in the final year, 1,901 firms or 45% of the firms with known status have permanently shut down by the end of the sample. An additional 30 were temporarily shut down.

Table 9: Business Status

| Year | Operating | Shut Down | Merged or Sold | Temp Shut Down | Unknown |
|---|---|---|---|---|---|
| 2004 | 4,928 | 0 | 0 | 0 | 0 |
| 2005 | 3,998 | 260 | 43 | 66 | 561 |
| 2006 | 3,390 | 581 | 90 | 124 | 743 |
| 2007 | 2,915 | 880 | 135 | 98 | 900 |
| 2008 | 2,606 | 1,224 | 175 | 58 | 865 |
| 2009 | 2,408 | 1,474 | 211 | 41 | 794 |
| 2010 | 2,126 | 1,692 | 249 | 45 | 816 |
| 2011 | 2,007 | 1,901 | 289 | 30 | 701 |

Tabulation of the operational status of firms in the Kauffman Firm Survey over the first eight years of operation. Note that when a firm is merged or sold, it exits the sample and so no more information about its operational status is available.

## B.2 Entrepreneur's Working Hours

Table 10 shows that over the first eight years of operation, about half of the entrepreneurs pay themselves a salary.

Even for those entrepreneurs who are paying themselves a salary, entrepreneurs may not be paying themselves the full opportunity costs of their time. Hall and Woodward (2010) document that venture-capital backed entrepreneurs are typically paid less than their outside option in the labour market in order to encourage effort. More generally, if borrowing constraints are binding, entrepreneurs could pay themselves less in order to save within the firm and accumulate additional capital. In both cases, lower compensation during the start up period is compensated for by higher returns later. Of course, if the firm exits before those returns materialize, the entrepreneur suffers a real

Table 10: Entrepreneurs Paying Themselves a Salary by Year

| Year | Prop Salaried | n Entrepreneurs |
|---|---|---|
| 2004 | 0.47 | 6,916 |
| 2005 | 0.53 | 5,673 |
| 2006 | 0.55 | 4,776 |
| 2007 | 0.55 | 4,057 |
| 2008 | 0.53 | 3,617 |
| 2009 | 0.54 | 3,304 |
| 2010 | 0.53 | 2,859 |
| 2011 | 0.51 | 2,715 |

economic loss. Note here the potential interaction of borrowing constraints and the missing market for entrepreneurial risk.

Tables 11 and 12 compare the proportion of entrepreneurs who report paying themselves a salary based on their weekly hours of work in the first and final years. Only a third of the entrepreneurs working less than 25 hours a week are paying themselves a salary. For entrepreneurs working more than 35 hours a week, 57% are paying themselves a salary in the first year, while 68% are paying themselves a salary in the eighth year. Hours worked are self-reported usual hours worked. Note that both of these graphs include only the survey-respondent entrepreneur, rather than all of the entrepreneurs working on a business.

Table 11: Entrepreneurs Paying Themselves a Salary by Hours Worked, First Year

| Hours | Prop Salaried | n Entrepreneurs |
|---|---|---|
| <25 | 0.33 | 1,288 |
| 25-35 | 0.43 | 442 |
| 35-44 | 0.54 | 655 |
| 45-54 | 0.56 | 758 |
| 55-65 | 0.58 | 887 |
| 65< | 0.58 | 780 |
| Total | 0.49 | 4,900 |

## B.3 Sources of Funding Details

New firms in the Kauffman Firm Survey raise their initial funds from a variety of sources. Table 13 updates a similar table in Robb and Robinson (2014) with the full sample of KFS data. By far the most common source of funding is the entrepreneurs' own investment. For 89% of firms, entrepreneurs are putting their own money into the business, these investments are often modest, with the median amount being only $25,000.

Note that while 53% of firms are able to borrow, most of these loans are personal

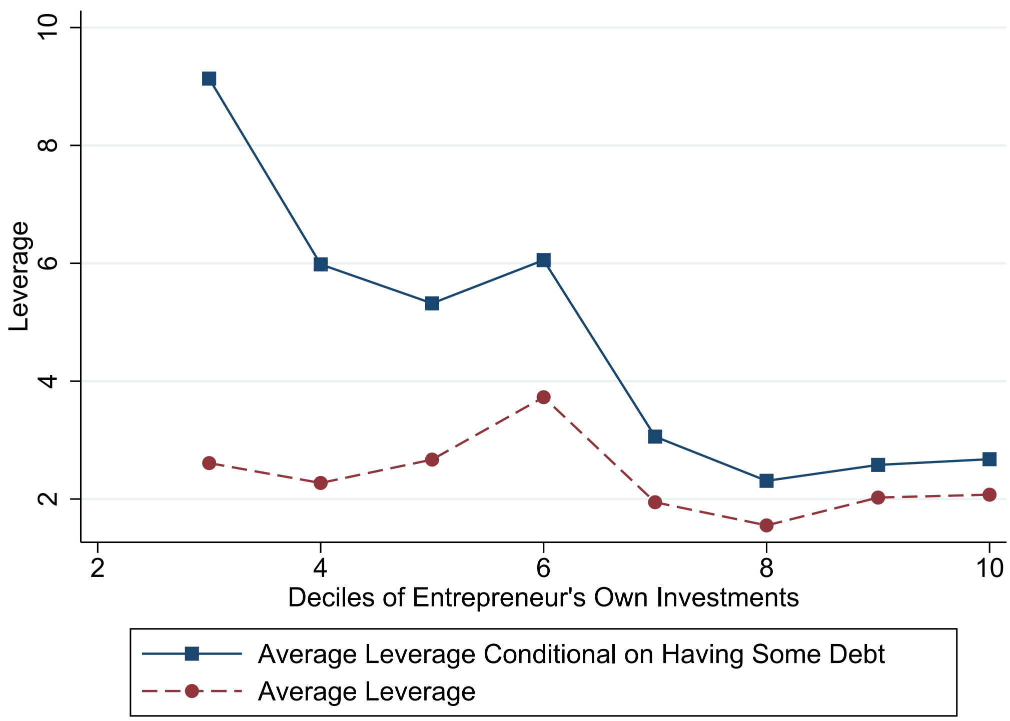
Table 12: Entrepreneurs Paying Themselves a Salary by Hours Worked, Eighth Year

| Hours | Prop Salaried | n Entrepreneurs |
|---|---|---|
| <25 | 0.33 | 569 |
| 25-35 | 0.52 | 174 |
| 35-44 | 0.62 | 303 |
| 45-54 | 0.68 | 344 |
| 55-65 | 0.73 | 313 |
| 65< | 0.70 | 182 |
| Total | 0.56 | 1,892 |

debt. If the business is unsuccessful and entrepreneurs choose to exit, these debts cannot be discharged without filing for personal bankruptcy. Raising external equity is much less common, with only 15% of firms raising equity beyond the actively managing owners, though these firms typically invest substantially more than those without any external equity.

Figure 14 shows the average leverage ratio of firms. I exclude the bottom two deciles of initial investment for legibility. Conditional on having some debt, the leverage ratio decreases sharply over the distribution of initial investment. Given the the proportion of firms who take out some debt increases sharply over the distribution of initial investment (see figure 4), the unconditional mean leverage ratio is relatively flat.

Figure 14: Leverage




The debt to equity ratio across the distribution of initial investment. Firms are sorted into deciles based on the total amount of equity invested in the firm in the first year of operations. The bottom two deciles are excluded for legibility.

Table 13: Funding Sources

| Source | % | Mean | p25 | p50 | p75 | n Firms |
|---|---|---|---|---|---|---|
| Own Equity | 89 | 166 | 8 | 25 | 80 | 3,655 |
| External Equity | 15 | 1,151 | 10 | 50 | 203 | 640 |
| Outside Investors | 6 | 758 | 10 | 90 | 290 | 247 |
| Parents | 5 | 83 | 10 | 25 | 73 | 242 |
| Other Companies | 4 | 1,565 | 20 | 100 | 600 | 154 |
| Spouses | 3 | 82 | 5 | 15 | 30 | 137 |
| Venture capital | 1 | 8,669 | 85 | 450 | 6,125 | 60 |
| Government Agencies | 1 | 698 | 53 | 250 | 850 | 44 |
| Other | 1 | 995 | 10 | 25 | 135 | 23 |
| Any Debt | 53 | 714 | 15 | 55 | 210 | 2,302 |
| Personal Debt | 46 | 405 | 10 | 40 | 128 | 2,011 |
| Bank | 32 | 308 | 15 | 50 | 140 | 1,413 |
| Family | 20 | 51 | 5 | 15 | 46 | 938 |
| Other Individuals | 6 | 434 | 4 | 15 | 50 | 252 |
| Any Other Sources | 3 | 1,489 | 10 | 33 | 110 | 160 |
| Business Debt | 28 | 701 | 15 | 50 | 211 | 1,264 |
| Bank | 17 | 873 | 26 | 84 | 269 | 747 |
| Family | 8 | 84 | 5 | 15 | 45 | 356 |
| Non-Bank Financial | 6 | 357 | 12 | 48 | 200 | 262 |
| Owners | 4 | 331 | 15 | 48 | 200 | 158 |
| Government Agencies | 2 | 1,052 | 30 | 125 | 330 | 89 |
| Other Individuals | 2 | 153 | 5 | 24 | 100 | 74 |
| Employees | 1 | 56 | 5 | 14 | 40 | 38 |
| All Funding Sources | 94 | 520 | 10 | 45 | 181 | 3,687 |

The sources of funding for firms in the KFS. All values are in thousands of US dollars and are cumulative over the first eight years of operation. The first column reports the percentage of firms that received any money from each funding source, the second column gives the mean amount raised conditional on raising some money from that source, the third through fifth columns give the $25^{\text{th}}$, $50^{\text{th}}$ and $75^{\text{th}}$ percentiles of the amount raised conditional on raising some money from that source. The final column gives the number of firms that raise any money from each funding sources.

"Personal Debt" is debt owed by an entrepreneur, while"Business Debt" is debt owned by the business.

## B.4 Dispersion in Cumulative Sales

In subsection 3.3, I showed that the level and dispersion of cumulative profits are significantly higher for high initial own investment firms than for low initial own investment firms. In this subsection, I test whether the same holds for the level and dispersion of cumulative sales.

$$\text{Cumulative Sales}_i = \alpha_0 + \alpha_1 \text{Initial Owner Investment}_t + \alpha X_i + \epsilon_i \tag{53}$$

$$\hat{\epsilon}_i^2 = \beta_0 + \beta_1 \text{Initial Owner Investment}_t + \beta X_i + \eta_i \tag{54}$$

Table 14 shows that firms with higher initial own investments have significantly higher cumulative sales, even after controlling for initial employment, external sources of funding and industry. In the second regression, firms with higher initial own investments have more significantly more dispersed cumulative sales.

Table 14: Dispersion of Cumulative Profits

| Regression (53): | Kauffman Firm Survey: Cumulative Sales | | | | |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| Own Equity (Year 1) | 3.988*** | 3.692*** | 3.692*** | 4.823*** | 5.156*** |
| | (0.245) | (0.235) | (0.235) | (0.523) | (0.582) |
| Average Employment | | 1022.0*** | 1044.3*** | 1057.5*** | 1040.5*** |
| | | (55.83) | (58.37) | (58.77) | (60.21) |
| Average Employment$^2$ | | -1.883*** | -1.973*** | -1.965*** | -1.929*** |
| | | (0.187) | (0.192) | (0.192) | (0.194) |
| Employer | | | -781.0 | -760.7 | -744.5 |
| | | | (914.9) | (917.6) | (917.6) |
| Total Investment (Year 1) | | | | -1.154* | -1.499** |
| | | | | (0.476) | (0.545) |
| Total Debt (Year 1) | | | | | 1.731 |
| | | | | | (1.331) |
| Constant | Yes | Yes | Yes | Yes | Yes |
| 2 Digit NAICs Codes | No | No | Yes | Yes | Yes |
| N | 4098 | 4092 | 4092 | 4076 | 4076 |
| Breush-Pagan | 3279.7 | 23612.6 | 27030.8 | 31440.3 | 31438.7 |

| Regression (54): | Kauffman Firm Survey: Absolute Value of Residuals | | | | |
|---|---|---|---|---|---|
| | (1) | (2) | (3) | (4) | (5) |
| Own Equity (Year 1) | 516889.5*** | 390999.7** | 392691.8** | 1499572.9*** | 1498831.0*** |
| | (140365.5) | (128636.6) | (127884.3) | (281694.0) | (313618.2) |
| Average Employment | | 277576708.9*** | 285089333.6*** | 296491940.8*** | 296257271.0*** |
| | | (30582699.7) | (31714108.3) | (31661144.0) | (32439458.6) |
| Average Employment$^2$ | | -663012.0*** | -691960.1*** | -679306.0*** | -679404.1*** |
| | | (102336.8) | (104175.8) | (103614.2) | (104747.0) |
| Employer | | | -639918743.3 | -613722711.1 | -613913751.3 |
| | | | (497113683.9) | (494343476.6) | (494440868.0) |
| Total Investment (Year 1) | | | | -1130881.9*** | -1129389.1*** |
| | | | | (256276.6) | (293549.2) |
| Total Debt (Year 1) | | | | | 15817.2 |
| | | | | | (717108.4) |
| Constant | Yes | Yes | Yes | Yes | Yes |
| 2 Digit NAICs Codes | No | No | Yes | Yes | Yes |
| N | 4098 | 4092 | 4092 | 4076 | 4076 |

## B.5 Calculating Rates of Return

In subsection 3.2 and B.4, I showed that the level and dispersion of cumulative profits and cumulative sales are significantly higher for high initial own investment firms than for low initial own investment firms. I would also like to test whether the firm's rates of return follows the same pattern.

Given that many of these firms are relatively small scale, it is vital to include both salaries and forgone earnings in this calculation. Omitting entrepreneurial salaries may substantially bias the measure, as the split between dividends and salaries is more likely to depend on tax code provisions or contracting frictions with passive owners than on the relative economy value of an entrepreneur's hours vs. financial contribution to the business. Omitting forgone labour earnings is likewise essential, as for many small-scale firms the value of the entrepreneur's time is a major input, if not bigger than the financial investment, in the business. Ignoring these inputs would greatly inflate the rates of return earned by small scale firms. Therefore, I construct cumulative rates of return for each firm using the following equation:

$$\text{Cumulative Rate of Return} = \frac{\sum_{t=0}^{7} \beta^t \left(\text{Dividends}_t + \text{Salary}_t\right) + \beta^7 \text{Firm Value}_7}{\sum_{t=0}^{7} \beta^t (\text{Equity Invested}_t + \text{Forgone Salary}_t)} \tag{55}$$

- $\text{Dividends}_t$: Dividends are directly measured in the data by a question that asks "Thinking of calendar year 2004, how much money, if any, did you and other owners withdraw from the business for personal use? This includes any dividends paid." it should therefore include all dividend payments as well as any other cash withdrawals from the business.
- $\text{Salary}_t$: Information about the exact salaries paid to entrepreneurs is not available, though whether a salary was paid to each entrepreneur is recorded. In order to proxy for salaries paid, I use the total wage bill of the firm, divided by the number of employees including the salaried entrepreneurs.
- $\text{Firm Value}_7$: For a termination value of the firm, I use the total assets of the firm minus the total liabilities in the final year.
- $\text{Equity Invested}_t$: Equity invested is directly measured in the data by a series of questions that ask how much money was received from active owners, angel investors, other companies, governments, parents of owners, spouses of owners, venture capital firms and an other category.
- $\text{Forgone Salary}_t$: No information about the previous labour market activities of these entrepreneurs is available. In order to proxy for their forgone labour earnings, I run a Mincerian regression in the Survey of Consumer Finances estimating total annual labour market earnings on demographics. I then use the coefficients from this regression to predict annual labour market earnings for the entrepreneurs in the Kauffman Firm Survey. I then multiply these predicted annual labour market earnings by entrepreneur's reported weekly hours of work.
- I discount all values using $\beta = \frac{1}{1.02}$.

To validate these rates of return, I compare the survival rates of firms in different deciles of cumulative profits to survival frequencies of firms in different deciles of cumulative rate of return. If the rates of return are accurately capturing entrepreneurial success, then the survival rates should be much more closely aligned with the rates of return than the profits.

Table 15: Survival by Cumulative Profits and Cumulative Rate of Return

| Decile | Survival by Profit decile | n Firms | Survival by RoR decile | n Firms |
|---|---|---|---|---|
| 1 | 0.57 | 321 | 0.27 | 234 |
| 2 | 0.43 | 343 | 0.42 | 247 |
| 3 | 0.34 | 344 | 0.42 | 241 |
| 4 | 0.28 | 432 | 0.49 | 242 |
| 5 | 0.29 | 230 | 0.49 | 238 |
| 6 | 0.48 | 330 | 0.49 | 237 |
| 7 | 0.59 | 328 | 0.53 | 240 |
| 8 | 0.74 | 323 | 0.46 | 224 |
| 9 | 0.78 | 342 | 0.60 | 219 |
| 10 | 0.90 | 335 | 0.55 | 205 |

Table 15 shows that the calculated rates of return are not a good predictor of survival. At the bottom decile, the low calculated rates of return seem to correctly identify a group of firms that are much less likely to survive. However, throughout the higher deciles there is little increase across the distribution of calculated rates of return. I conclude that these rates of return are not a good measure of the financial success of these businesses.

## B.6 Entrepreneur's Biggest Challenge

In the 5$^{th}$ through 8$^{th}$ years of operation, entrepreneurs were asked "What was the most challenging problem your business faced in calendar year X?". The responses are provided in table 16. In every year, less than 10% of entrepreneurs consider credit issues, either "an inability to obtain credit" or "the cost and/or terms of credit" to be their largest challenge. By contrast the majority of entrepreneurs say that their biggest challenge is either "the unpredictability of business conditions" and "slow or lost sales", answers that may reflect the risks the firm faces. These patterns suggests that most entrepreneurs are far more concerned about the risks they face than any lack of credit. Unfortunately, as this question was only asked for the years from 2008-2011, it is not clear how much the responses to this question are driven by aggregate risks from the US financial crisis, rather than idiosyncratic risk to their individual business. Yet as economic conditions recover after 2008, there is no increase the number of entrepreneurs who consider the lack of credit to be their primary challenge, which suggests that for many firms binding borrowing constraints may not be a major concern for entrepreneurs 8 years after starting a business.

Table 16: Entrepreneur's Self-Reported Most Challenging Problem

| Year | 2008 | 2009 | 2010 | 2011 |
|---|---|---|---|---|
| An inability to obtain credit | 4 % | 5 % | 4 % | 4 % |
| Slow or lost sales | 53 % | 45 % | 42 % | 35 % |
| Falling real estate values | 5 % | 5 % | 4 % | 4 % |
| The cost and/or terms of credit | 2 % | 2 % | 1 % | 1 % |
| The unpredictability of business conditions | 24 % | 23 % | 26 % | 31 % |
| Some other problem | 11 % | 13 % | 15 % | 16 % |
| Customers or clients not making payments or paying late | 2 % | 8 % | 8 % | 8 % |
| Number of Entrepreneurs | 2,566 | 2,369 | 2,094 | 1,971 |

# C Survey of Consumer Finances Facts

In this section I document several facts from the Survey of Consumer Finances (SCF) about the relationship between entrepreneurship and wealth. I focus on the 2004 wave of the SCF, as that is the year in which all of the businesses in the Kauffman Firm Survey are started.

## C.1 Wealth Moments

In section 5, I calibrate the model to several wealth inequality moments taken from the 2004 Survey of Consumer Finances. I define entrepreneurs as households that both own a business and actively manage that business. As this paper does not study passive business ownership, I drop households from the sample if they are passive business owners, unless they own multiple businesses and actively manage at least one of them. As there is no retirement state in the model of section 4, I drop households of retirement age. For single person households, I drop the household if the individual is older than 65. For two person households, I drop the household if the average age of the two individuals is greater than 65.

Table 17 shows that the distribution of wealth does not change substantially if either retired households or passive business owners are dropped. The first column shows the top wealth shares for the whole SCF sample, designed to be representative of the US population. The second drops households of retirement age. The third column drops all passive business owners, who are not also active business owners. The fourth drops both households of retirement age and the passive business owners.

In table 18, I show that two of the key targeted model moments do depend on the sub-sample chosen. Since both retired households and passive business owners own a substantial amount of wealth, their inclusion makes the wealth differences between entrepreneurs and non-entrepreneurs look less stark. I use the statistics from the fourth column for the calibration of my model.

Table 17: Top Wealth Shares

| | Full Sample | Drop > 65 | Drop PBO | Drop Both |
|---|---|---|---|---|
| Top 1% | 33.2% | 33.5% | 31.1% | 32.1% |
| Top 5% | 57.5% | 57.4% | 55.6% | 56.0% |
| Top 10% | 69.5% | 68.6% | 67.8% | 67.1% |
| Top 20% | 82.9% | 82.4% | 81.9% | 81.4% |
| Top 50% | 97.5% | 97.1% | 97.3% | 97.0% |
| Wealth Gini | 0.79 | 0.80 | 0.78 | 0.79 |

Top wealth shares in the US economy across several samples. "Full Sample" is the full survey. "Drop > 65" refers to the sub-sample after dropping all households of retirement age. "Drop PBO" refers to the sub-sample after dropping all households that own a business they do not actively manage without also owning a business they actively manage. "Drop Both" refers to the sub-sample after imposing both the age and passive business ownership restrictions.

Table 18: Model Moments by Subsample

| | Full Sample | Drop > 65 | Drop PBO | Drop Both |
|---|---|---|---|---|
| Wealth Ratio of Entrepreneurs to Everyone Else | 5.70 | 6.29 | 6.42 | 7.14 |
| Proportion of Entrepreneurs in Wealthiest 1% | 0.60 | 0.65 | 0.69 | 0.74 |

## C.2 Wealth To Income Ratios

One key empirical fact that has motivated the study of borrowing constraints for entrepreneurs is the fact that entrepreneurs have higher wealth-to-income ratios than non-entrepreneurial households. Quadrini (1999) regresses the wealth to income ratios of households in the PSID and the SCF on a binary variable for entrepreneurship, the level of income, and a polynomial in age. He finds that the coefficient on the entrepreneurship dummy to be positive and significant, suggesting that entrepreneurial households have higher average wealth-to-income ratios than other households. The first column of table 19 replicates this result with data from the 2004 Survey of Consumer Finances.

However, a higher average wealth-to-income ratio does not necessarily imply that entrepreneurs are subject to borrowing constraints. Higher volatility in entrepreneurial earnings can generate the same result, and DeBacker et al. (2018) shows that entrepreneurial earnings are in fact much more volatile than labour earnings. The following simple numerical example illustrates how more volatile incomes can also generate higher average wealth-to-income ratios.

Consider an economy populated by 2 workers and 2 entrepreneurs. All agents have the same level of wealth, equal to 3. All agents have the same expected income, equal to 1. Worker's income is certain and equal to one. However, entrepreneurs face some risk, and half the time receive 0.5 and half the time receive 2. Calculating the average of the wealth-to-income ratios for the two types of workers will give:

$$\overline{W/I}^{\text{Worker}} = \frac{1}{2}\left[\frac{3}{1} + \frac{3}{1}\right] = 3$$

Table 19: Wealth To Income Ratios of Entrepreneurs

| | (1) $\frac{\text{Wealth}}{\text{Income}}$ | (2) $\frac{\text{Non-Bus Wealth}}{\text{Income}}$ | (3) $\frac{\text{Fin Wealth}}{\text{Income}}$ |
|---|---|---|---|
| Entrepreneur | 13.2912*** | 2.1269 | 0.4986 |
| | (1.4984) | (1.2048) | (0.4788) |
| Income | Yes | Yes | Yes |
| Age FEs | Yes | Yes | Yes |
| Educ FEs | Yes | Yes | Yes |
| Observations | 4498 | 4498 | 4498 |

Standard errors in parentheses

$^{*}$ $p < 0.05$, $^{**}$ $p < 0.01$, $^{***}$ $p < 0.001$

$$\overline{W/I}^{\text{Entrepreneur}} = \frac{1}{2}\left[\frac{3}{0.5} + \frac{3}{1.5}\right] = 4$$

In this simple example the entrepreneurs average wealth-to-income ratio is much higher than the workers because of their more volatile income, despite the fact that both groups have the same average income and average wealth. To determine whether this arithmetic consideration drives the regression result, table 20 compares the average wealth-to-income ratios ($\overline{W/I}$) and the ratio of average wealth to average income ($\bar{W}/\bar{I}$) for entrepreneurial households and non-entrepreneurial households.

$$\overline{W/I} = \frac{1}{n}\sum_{i=1}^{n}\frac{W_i}{I_i} \tag{56}$$

$$\bar{W}/\bar{I} = \frac{\frac{1}{n}\sum_{i=1}^{n} W_i}{\frac{1}{n}\sum_{i=1}^{n} I_i} \tag{57}$$

Table 20 shows that the differences between entrepreneurial households and non-entrepreneurial households are smaller when calculated with the ratio of average wealth to average income, rather than the average wealth-to-income ratio. This suggests that the more volatile nature of entrepreneurial income is at least one part of the explanation of this result that entrepreneurs typically have higher wealth-to-income ratios.

Table 20: Wealth to Income Comparison

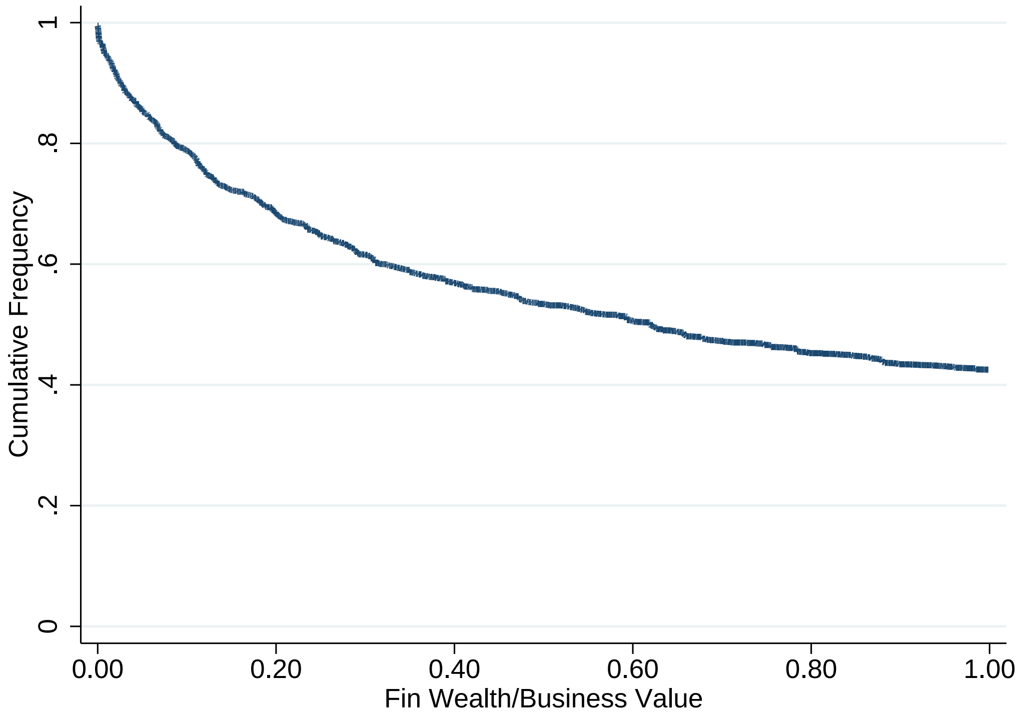
| | Average Wealth to Income Ratio ($\overline{W/I}$) | | Ratio of Average Wealth to Average Income ($\bar{W}/\bar{I}$) | |
|---|---|---|---|---|
| | Entrepreneurs | Others | Entrepreneurs | Others |
| 99.5% - 100% | 57.9 | 39.0 | 16.9 | 15.4 |
| 99% - 99.5% | 48.9 | 27.0 | 15.3 | 11.8 |
| 95% - 99% | 19.7 | 25.9 | 11.1 | 11.8 |
| 90% - 95% | 15.9 | 14.5 | 8.9 | 9.3 |
| 80% - 90% | 9.9 | 12.0 | 6.6 | 6.8 |
| 60% - 80% | 5.9 | 7.0 | 3.9 | 4.0 |
| 40% - 60% | 3.6 | 3.3 | 2.2 | 2.0 |
| 20% - 40% | 1.3 | 1.2 | 0.8 | 0.7 |
| 0% - 20% | -0.3 | -0.2 | -0.3 | -0.2 |
| Total | 12.4 | 4.8 | 6.4 | 2.7 |

## C.3 Entrepreneurial Wealth Composition

If an entrepreneur is borrowing constrained, they should have relatively few financial assets outside their business. If borrowing constraints raise the cost of external financing, entrepreneurs should liquidate most of their financial assets in order to fund their business internally. One measure of the strength of borrowing constraints that entrepreneurs face is then the proportion of their assets they keep outside the business.

To assess this, I calculate the ratio of financial wealth to business value for each entrepreneur. Entrepreneurs who are borrowing constrained, and wish to expand the scale of their business should have low values of this ratio, as they should liquidate financial assets in order to invest in their business. Figure 15 shows the cumulative distribution of this ratio for all entrepreneurs. I plot the inverse CDF, so that the far left side of the graph shows that 100% of entrepreneurs could liquidate financial assets worth 1% of their business and the far right side of the graph shows that just over 40% of entrepreneurs could liquidate financial assets worth 100% of their business. While there are certainly some entrepreneurs who have few financial assets outside of their business, the vast majority have substantial financial assets outside of their business.

Figure 15: Inverse CDF of the Ratio of Financial Wealth to Business Value



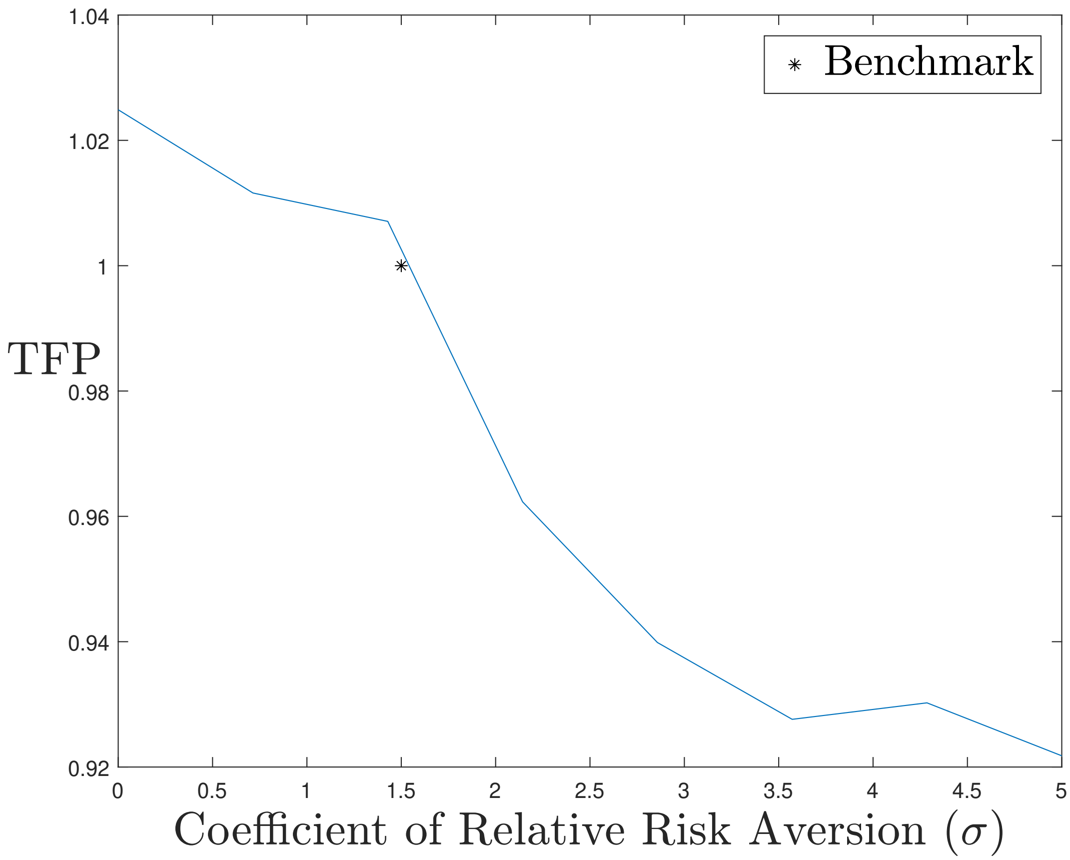

The inverse cumulative density function of the ratio of financial wealth to business value. Intuitively, this graph gives the proportion of entrepreneurs would be able to liquidate their financial investments and make a $x\%$ investment in their firm.

# D Quantitative Robustness

## D.1 Robustness: Coefficient of Relative Risk Aversion

Of course, the importance of the missing market for entrepreneurial risk depends critically on the value of risk-aversion used in the economy. In figure 16, I plot the TFP of the benchmark economy and how this varies over different values of $\sigma$, the coefficient of relative risk aversion. Aggregate productivity declines as the agents that populate the economy become more risk-averse. This operates primarily through the endogenous choice of risky projects, as agent's risk aversion increases, fewer agents are willing to start a high-risk project with high expected productivity. As a consequence, more agents operate small-scale safe projects.

Figure 16: Aggregate Productivity and the Coefficient of Relative Risk Aversion




# E Computational Algorithm

## E.1 Decision Rules in the Benchmark Economy

The goal is to solve for the value functions $\{V^W, V^{NE}, V^E\}$, occupational choices, and the set of policy functions $\{c^W, a'^W, \tilde{a}^{NE}, \tilde{k}^{NE}, x^{NE}, c^E, a'^E, k'^E, n^E\}$ to maximize the problems given by 21, 22 and 23.

I proceed by adapting an algorithm from Dyrda and Pugsley (2018). To aid the computational tractability of the problem, I solve for the decision rules on a grid of resources $r$, rather than on both $a$ and $k$. I solve first for the value functions, conditional on which occupation will be chosen at the beginning of the next period and then maximize over these occupational choices.

Given a wage $w$, I solve for the decision rules by:

- Initial Guess:
  - Set $i = 0$,
  - Guess $c_i(\dot{)}$ is a constant fraction of total resources
  - Guess $r'_{i,0}(\cdot) = r$
  - Guess $V_i(\cdot) = \frac{c_i^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} / (1 - \psi\beta)$

- Guess all agents choose to stay in their current occupation in the next period

- Begin Value Function loop
  - $i = i + 1$
  - Calculate the derivatives of each value function $V_i^E(\cdot), V_i^{NE}(\cdot), V_i^W(\cdot)$ w.r.t. total resources $r$
  - Use the envelope condition to update current consumption:

$$c_i^E = \left(\frac{\partial V_i^E(\cdot)}{\partial r}\right)^\theta$$

$$c_i^{NE} = \left(\frac{\partial V_i^{NE}(\cdot)}{\partial r}\right)^\theta$$

$$c_i^W = \left(\frac{\partial V_i^W(\cdot)}{\partial r}\right)^\theta$$

  - Set $j = 0$
  - Begin Portfolio Allocation loop:
    * $j = j + 1$
    * For each future state of the world $m$, calculate $\lambda_m = \frac{\partial V'}{\partial r'}|_{r'_{i,j}(m)}$
    * Use the FOC w.r.t. $k'$ to determine the optimal level of $k'$

$$k'_{i,j} = \frac{\left[\sum_m \text{Prob}_m \lambda_m (z_m h_m^E)^{\frac{1-\gamma}{\alpha+(1-\gamma)(1-\alpha)}}\right]^{\alpha+\frac{(1-\gamma)(1-\alpha)}{1-\gamma}} \left(\frac{1-\alpha}{w}\right)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}} \gamma^{\frac{1}{1-\gamma}} \alpha^{\alpha+\frac{(1-\gamma)(1-\alpha)}{1-\gamma}}}{\sum_m \text{Prob}_m \lambda_m (\delta + r^A)^{\alpha+\frac{(1-\gamma)(1-\alpha)}{1-\gamma}}}$$

    * If that level of $k'$ exceeds the entrepreneur's ability to borrow, reduce it to the maximum amount consistent with the borrowing constraint and the current level of consumption:

$$k'^E_{i,j} = \min\left(k'_{i,j}, \frac{r - c_i^E - \underline{a}}{1-\phi}\right)$$

$$k'^{NE}_{i,j} = \min\left(k'_{i,j}, \frac{r - f - c_i^{NE} - \underline{a}}{1-\phi}\right)$$

    * Use the budget constraint to determine the resulting saving or borrowing constraint in $a'$:

$$a'^E = r - c_i^E - k'^E_{i,j}$$

$$a'^{NE} = r - f - c_i^E - k'^{NE}_{i,j}$$

    * Update next period resources for each future state of the world $m$:

$$r'_{i,j}(m) = (z_m h_m^E)^{1-\gamma}(k^\alpha n_m^{1-\alpha})^\gamma - w n_m + (1-\delta)k'_{i,j} + a'(1 + r^A)$$

where $n_m = (z_m h_m^E)^{1-\gamma} k^{\alpha\gamma} \gamma \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha+(1-\gamma)(1-\alpha)}}$

  * Calculate distance between $r_{i,j-1}$ and $r_{i,j}$ for all $m$
  * If distance is less than tolerance, end loop, else return to beginning of Portfolio Allocation loop

- As workers will not invest in capital, their savings can be directly backed out by the budget constraint

$$a'^W = r - c_i^W$$

- Update conditional value functions:

$$V_i^E = \frac{{c_i^E}^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta \sum_m \mathrm{Prob}_m V_{i-1}^E(r'^E_{i,j}, m)$$

$$V_i^{NE} = \frac{{c_i^{NE}}^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta \sum_m \mathrm{Prob}_m V_{i-1}^E(r'^{NE}_{i,j}, m)$$

$$V_i^W = \frac{{c_i^W}^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta \sum_m \mathrm{Prob}_m V_{i-1}^W(r'^W_{i,j}, m)$$

- Perform occupational choice to obtain unconditional value functions:

$$V_i^E(r, h^W, h^E, z, x) = \max(V_i^E(r, h^W, h^E, z, x), V_i^{NE}(r, h^W, h^E), V_i^W(r, h^W, h^E))$$

$$V_i^W(r, h^W, h^E) = \max(V_i^{NE}(r, h^W, h^E), V_i^W(r, h^W, h^E))$$

- Calculate distance between $V_i^E(\cdot)$ and $V_{i-1}^E(\cdot)$ and between $V_i^W(\cdot)$ and $V_{i-1}^W(\cdot)$
- If distance is less than tolerance, end loop, else return to beginning of Value Function loop

With the decision rules solve, I then simulate a fixed mass of agents on a discretion grid of $a \times k \times h \times z \times x$. I guess a uniform distribution over this state space and then iterate until the distribution converges. I use a bisection search to determine the wage that clears the labour market in this economy.