Risky Entrepreneurship and Wealth

Baxter Robinson*

University of Toronto

This Version: August 15, 2019

Latest Version

Abstract

This paper studies how entrepreneurial risk-taking is affected by entrepreneur's wealth. I use the Kauffman Firm Survey to document facts about the risks new entrepreneurs take and the sources of their initial financing. Based on these facts, I build a model of occupational choice, where agents choose to become workers or entrepreneurs, and endogenous risk choice, where entrepreneurs choose from a menu of risky projects. On average, riskier projects deliver higher returns. The model features two frictions. First, a missing market for entrepreneurial risk prevents entrepreneurs from insuring themselves against entrepreneurial income risk and the risk of business failure. Second, borrowing constraints limit the size of a business that a entrepreneur can start based on their personal wealth. As a consequence of both of these frictions, agents select into entrepreneurship based on both their entrepreneurial ability and their personal wealth. I calibrate the model to match both the firm dynamics of new privately held US firms and US wealth inequality. I use the calibrated model to measure the quantitative importance of these two frictions in reducing entrepreneurial risk-taking. Removing either friction increases output, productivity, and wages in the economy. I find that completing the missing market for risk increases aggregate productivity by much more than reducing borrowing constraints, as entrepreneurs choose to start higher risk businesses, which are more productive on average. The endogenous choice of risk also generates heterogeneity in rates of return, which can help explain the high concentration of top tail US wealth.

Keywords: Entrepreneurship, Risk-Taking, Financial Frictions, Aggregate Productivity, Wealth Inequality

JEL Codes: D31, G32, J24, L26

*baxter.robinson@mail.utoronto.ca

I thank Burhan Kuruscu, Gueorgui Kambourov, and Sebastian Dyrda for invaluable guidance on this project. This paper has also benefited from discussions with Stephen Ayerst, Bulent Guler, Bariş Kaymak, Marcus Poschke, Benjamin Pugsley, Marc-Antoine Schmidt, as well as seminar participants at the University of Toronto Macro Brown Bag Seminar Series, the 2018 CEA Meetings, the 2019 CIREQ PhD Student Conference, the University of Toronto Public Discussion Group, the 2019 CEA Meetings, and the 2019 North American Summer Meeting of the Econometric Society. I thank the Kauffman Foundation for sponsoring my access to the Kauffman Firm Survey Data.

1 Introduction

Entrepreneurs are disproportionately important for the distributions of both income and wealth. In the 2004 Survey of Consumer Finances, 11% of households contain at least one entrepreneur, defined as someone who both owns and actively manages a business. These households earn 25% of aggregate income and own 40% of aggregate wealth. Despite their high levels of income and wealth, entrepreneurs in the US receive favourable tax treatment and direct government support in the form of loans and grants. For example, in 2017 the US Small Business Administration, a US Federal Government agency, had an outstanding loan portfolio to small and medium sized businesses of more than $131 billion dollars (US Small Business Administration, 2017). Why does the US government give favourable tax treatment and direct government support to those who are already wealthy? Presumably, these benefits are conferred on entrepreneurs in order to both encourage individuals to become entrepreneurs and to encourage existing entrepreneurs to increase their investments. Understanding precisely why individuals become entrepreneurs and their incentives to invest is therefore vital for the design of public policy.

In this paper, I study how entrepreneurial risk impacts the relationship between entrepreneurship and wealth. It is well known that entrepreneurship is risky. Many new businesses fail (Fairlie et al., 2018) and entrepreneurial income is far more volatile than labour income (DeBacker et al., 2018). Given the risks involved, wealthier people may be more likely to start a business if they are better able to self-insure their consumption when their business fails or when their entrepreneurial income is particularly low. By contrast, poor entrepreneurs may have to substantially reduce their consumption when their business fails or if they earn low income. Therefore if two equally-able individuals are considering whether to start a riskier business, the wealthier individual should be more likely to choose to become an entrepreneur. Of course, at the same time, financial constraints on the amount entrepreneurs can borrow may also make wealthier individuals more likely to start a business. If wealth poor individuals cannot afford to pay initial fixed costs or cannot rent a sufficient amount of capital, wealthier individuals should again be more likely to become entrepreneurs, even conditional on ability.

Using the confidential version of the Kauffman Firm Survey (KFS), a panel of 4,928 new US firms, I document facts about the risks new entrepreneurs face and how they fund their businesses. Motivated by these facts, I then build a quantitative model of entrepreneurship. Each period agents choose whether to become entrepreneurs or workers, and conditional on being an entrepreneur, choose from a menu of risky project types. These types differ in the degree of risk and reward they offer, with higher risk products delivering higher expected rewards. The model has two key frictions, a missing market for entrepreneurial risk and borrowing constraints. These frictions generate selection into entrepreneurship based on both entrepreneurial ability and wealth, and generate strong savings motives for entrepreneurs. I use this model to quantify the relative importance of these two frictions in reducing entrepreneurial activity in the economy. Removing either friction increases the economy's output, wages, and productivity. Completing the missing insurance market raising aggregate productivity by 11%, while removing the borrowing constraints on capital investment increases aggregate productivity by 2%. Removing both frictions leads to a gain of 15%, showing that there is some interaction between the two frictions. I evaluate the extent to which the endogenous choice of risk can help explain the high concentration of wealth in the

US by comparing a economy with the endogenous choice of risk, where entrepreneurs choose a project type, to an exogenous allocation of risk, where entrepreneurs are randomly allocated a project type. I find that the endogenous choice of risk can help explain a substantial share of the top 1% wealth share.

In the Kauffman Firm Survey data, I show that many entrepreneurs start businesses, earn cumulatively negative profits and then exit, suggesting that idiosyncratic risk is an essential feature of entrepreneurship. The existence of these unsuccessful entrepreneurial attempts motivate the timing structure in the model. Agents must make an occupational choice decision before realizing shocks to their entrepreneurial ability or to their businesses' productivity. This creates the potential for real losses to be experienced each period by individuals who chose to be entrepreneurs and then receive bad shocks. Even if the expected return to entrepreneurship is higher than the expected return to being a worker, risk averse agents missing the ability to insure themselves against this risk may choose to be workers, or choose lower-risk lower-return entrepreneurial projects.

I also show that KFS entrepreneurs who invest more of their own money in their businesses, earn more dispersed profits, even after controlling for firm size. I interpret this result as an indication that entrepreneurs who invest more in their businesses, who are likely to be wealthy, on average choose to start businesses that entail a greater degree of risk. Finally, I show how entrepreneurs who invest more of their own money in their businesses, are more likely to raise funds from other sources and to raise more money from external sources conditional on raising some. These patterns are consistent with the idea that entrepreneurs face borrowing constraints that self-finance can help alleviate.

Motivated by these facts, I build a model of occupational selection and endogenous risk choice. Agents are endowed with two types of ability, entrepreneurial and worker. Each period, they choose to be an entrepreneur or a worker, and conditional on being an entrepreneur choose how risky a business to operate. Higher risk businesses earn higher rewards in expectation. Two frictions prevent the efficient selection of agents, based solely on relative ability, into entrepreneurship. First, a missing market for entrepreneurial risk prevents entrepreneurs from insuring the substantial income risk and capital investment risk they face. Second, financial frictions, in the form of a collateral constraint, limit the amount of capital an entrepreneur can invest in. Wealth helps agents overcome both frictions, because wealthier agents are better able to self-insure themselves against fluctuations in entrepreneurial income, and at the same time are able to invest in a larger capital stock. As a consequence, selection into entrepreneurship is based on both ability and wealth.

Using the model, I measure the quantitative importance of these two frictions in reducing entrepreneurial risk taking in the economy. I calibrate this model to match moments about new firms in the Kauffman Firm Survey and moments about wealth inequality in the US form the Survey of Consumer Finances. Within the calibrated model, I first complete the missing market for entrepreneurial risk by introducing a full set of contingent claims. Agents are able to purchase securities that pay off based on the realizations of their own shocks at actuarially fair prices. Second, I relax the borrowing constraint by allowing entrepreneurs to invest in any amount of capital regardless of their personal net-worth. Finally, I shut down both frictions and compare the resulting equilibria. Removing either friction raises output, wages, and productivity in the economy.

The endogenous choice of risk within the model also has implications for the cross-sectional distribution of wealth. Since entrepreneurs endogenously choose their business type, higher wealth agents choose higher risk projects. As a consequence, wealthy entrepreneurs earn higher returns on average than wealth-poor entrepreneurs. These higher returns increase the concentration of wealth at the top right tail. I quantify the contribution of endogenous risk choice to generating dispersion in wealth by contrasting two economies, one with an endogenous choice of project type, to an exogenous risk economy, where projects are randomly allocated to entrepreneurs.

Related Literature

There is a large literature studying the decision to become an entrepreneur. Since at least Evans and Jovanovic (1989), much of the literature has focused on the importance of personal wealth in overcoming borrowing constraints. If financial frictions prevent wealth-poor individuals from borrowing, they will be unable to pay fixed set up costs or invest in a large capital stock. This will discourage or prevent them from entering entrepreneurship. Empirically, it is well known that entrepreneurship is highly correlated with wealth. For example, Gentry and Hubbard (2004) show that in the Survey of Consumer Finances, entrepreneurial households hold a disproportionate amount of wealth and that their wealth-to-income ratios are significantly higher than non-entrepreneurial households. Hurst and Lusardi (2004) present a more mixed picture. They document that in the Panel Study of Income Dynamics (PSID), the probability of selecting into entrepreneurship is only increasing in wealth for the top 5% of the wealth distribution. They find that both prior and future bequests predict individuals becoming entrepreneurs, suggesting this relationship between wealth and entrepreneurship is not solely driven by liquidity constraints.

An alternative mechanism that determines entrepreneurial selection, originally proposed by Kihlstrom and Laffont (1979), is a missing market for entrepreneurial risk. In their model, heterogeneity in risk-aversion drives more risk-averse individuals to be workers while more risk-tolerant individuals become entrepreneurs. Cressy (2000) argues that this uninsurable entrepreneurial risk can also generate a relationship between wealth and entrepreneurship. As long as agents' preferences exhibit decreasing absolute relative risk aversion, wealthier individuals will be more likely to take on a risky entrepreneurial venture. When entrepreneurial income is low, wealth-poor individuals may have to substantially curtail consumption, leading to large losses in utility, while wealthy entrepreneurs will be better able to self-insure their consumption using wealth held outside their business.

There is certainty substantial evidence that entrepreneurs face a high degree of idiosyncratic risk. Decker et al. (2016) calculate that US firm exit rates are consistently 8%-10% per year. Even conditional on survival, DeBacker et al. (2018) document that entrepreneurial income volatility is substantially higher than employment income using IRS data. While both exit and entrepreneurial income volatility may be correlated with the business cycle, much of this risk is idiosyncratic. Studying US manufacturing firms, Castro et al. (2015) shows that 80% of unexplained revenue TFP volatility is due to idiosyncratic shocks rather than aggregate shocks. More recent work in developing country contexts such as Bianchi and Bobba (2013) or Karlan et al. (2014) demonstrate that providing people with better insurance mechanisms can increase self-employment or investment more effectively than relaxing borrowing constraints. In a developed country context, Hombert et al. (2014), Olds (2016), and Gottlieb et al. (2018), all show how policy changes that provide additional insurance to individuals can increase self-employment.

The first contribution of this paper is to measure how important these two frictions, borrowing constraints, and a missing market for entrepreneurial risk induce selection into entrepreneurship based on wealthy. To do so, I build a model of entrepreneurship based upon Cagetti and De Nardi (2006). Like in their model, agents have two types of ability, worker and entrepreneurial, and make an occupational choice decision in each period. Unlike in their model, the occupational choice in this paper is risky. Agents must choose their current period's occupation before knowing the current realization of their earnings in both occupations. Since entrepreneurial income is riskier than labour income, the occupational choice involves not just the choice between the level of income but also the variance. In addition, capital liquidation costs induce additional losses when an entrepreneur decides to shut down their business.

One natural consequence of borrowing constraints is the misallocation of talent. Buera (2009) shows that financial frictions induces low-ability wealthy individuals to become entrepreneurs at the expense of high-ability poor individuals. This misallocation in talent can substantially reduce aggregate productivity. While people may be able to save out of these constraints over time, Moll (2014) finds that the short run transition dynamics can still be important. Papers such as Buera et al. (2011), Midrigan and Xu (2014), and Castro and Ševčík (2017) provide evidence that financial frictions can have large impacts on aggregate productivity and help explain large cross-country differences in TFP.

The second contribution of this paper is to document how the missing market for entrepreneurial risk can also reduce aggregate productivity. It does so through three channels. First, like borrowing constraints, the missing market for entrepreneurial risk distorts occupational choice decisions, discouraging high-ability individuals from becoming entrepreneurs. Second, since higher-risk higher-reward projects have higher expected productivity, there are aggregate productivity losses when entrepreneurs choose lower-risk lower-reward projects. Finally, the missing market for entrepreneurial risk encourages risk-averse entrepreneurs to invest less in their businesses than risk-neutral individuals would. I quantify the productivity gains from each of these channels.

By studying this endogenous choice of entrepreneurial risk, this paper is also related to a number of papers on how entrepreneurs choose risk. A closely related paper is Choi (2017). He uses US Census Bureau data to show that individuals who had higher paying jobs prior to starting a business take larger risks, as measured by higher exit rates, more dispersion in growth, and faster average growth conditional on survival. Relative to his contribution this paper considers how wealth, rather than labour market opportunities, encourages more risk taking by entrepreneurs. Vereshchagina and Hopenhayn (2009) study a similar question to this paper, looking at how wealth impacts both the choice to become an entrepreneur and the endogenous choice of risk. In their three period model, wealth-poor entrepreneurs take larger risks in the hopes of being able to be entrepreneurs in the future. I find the opposite result, disproportionately, it is wealthier individuals who take larger risks.

Panousi and Papanikolaou (2012) study risk taking in publicly traded firms. Using Compustat data, they document that firm investment falls as firm-specific risk rises and that this effect is stronger when managers own more of the firm. Their result provides micro evidence that aversion to firm-specific risk shifts investment decisions. By contrast, this paper considers how entrepreneurs choose what type of business to start, when given a choice between different risk-reward types. This channel is likely to be more important for private firms, which are the focus of this paper. More broadly, Gabler and Poschke (2013) also considers how less risk-taking leads to aggregate productivity losses. The mechanisms however differ, as they consider a framework with risk-neutral firms facing distortions to allocative efficiency, while I consider risk-averse entrepreneurs facing a missing market for entrepreneurial risk.

Finally, this paper is also related to a literature that aims to explain the cross-sectional distribution of wealth observed in the data. Quadrini (1999) argues that financial frictions give rise to strong savings motives for entrepreneurs. Cagetti and De Nardi (2006) show that a model of entrepreneurship with both endogenous borrowing constraints and a strong bequest motive can well approximate the observed distribution of wealth. More recently, empirical evidence from Scandinavian countries, Bach et al. (2018) and Fagereng et al. (2018), show that wealthier individuals earn persistently higher rates of return on their assets. This paper contributes to this literature by considering a model with both borrowing constraints and rate of return heterogeneity. Specifically, the endogenous choice of risk means that wealthier individuals choose the higher-risk higher-reward project and so on average may earn higher returns than wealth-poor entrepreneurs. The third contribution of this paper is to examine how much additional wealth concentration this mechanism can generate.

In section 2, I illustrate the two key mechanisms, a missing market for entrepreneurial risk and borrowing constraints in a simple two-period model. Section 3 provides some descriptive evidence on the importance of these mechanisms using data from the Kauffman Firm Survey. Motivated by these facts, I extend the two-period model into a perpetual youth model in section 4. I explain the calibration strategy for this model in section 5 and report the results of the quantitative analysis in section 6. Section 7 concludes.

2 Two Period Model

In this section, I construct a simple two-period model to illustrate two key frictions explored in this paper.

2.1 Environment

Agents live for two periods and have Epstein-Zin preferences over consumption in the two periods, given by:

$$U = \frac{c_1^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \beta \frac{\left(\mathbb{E}\left[c_2^{1-\sigma}\right]\right)^{\frac{1-\frac{1}{\theta}}{1-\sigma}}}{1-\frac{1}{\theta}} \tag{1}$$

where $\theta > 0$ is the elasticity of intertemporal substitution and $\sigma > 0$ is the coefficient of relative risk aversion.

At the start of the first period, agents have an endowment of initial wealth $e \in [0, \infty)$ and a known entrepreneurial ability $z \in [\underline{z}, \bar{z}]$. Based on these initial states, each agent must choose to be an entrepreneur or a worker and then make a consumption-savings decision. In the second period, entrepreneurs hire labour, produce, pay worker's wages and consume the remaining output. Workers supply labour and consume.

All agents have identical productivity as a worker. If an agent chooses to be a worker, they will earn a wage of w with certainty in the second period.

By contrast, entrepreneurship is risky and a missing market for entrepreneurial risk prevents entrepreneurs from insuring away this risk. Agents that choose to be an entrepreneur choose from a menu of risky project types $x \in \{x_1, x_2, ..., x_{n_x}\}$. Higher project types deliver higher expected productivity, but at the cost of more disperse productivity. With probability p, the entrepreneur's project is successful, and their productivity is boosted by ψx. With the complementary probability the risk fails and their productivity is reduced by x. Therefore in the second period, the entrepreneur's realized productivity is given by $\tilde{z}$:

$$\tilde{z} = \begin{cases} z + \psi x & \text{with probability} \quad p \\ z - x & \text{with probability} \quad 1-p \end{cases} \tag{2}$$

where $\psi > \frac{1-p}{p}$, so that the expected value of $\tilde{z}$ increases with x.

There are two assets, capital (k) and a risk-free financial asset (a). Agents can only hold a positive amount of capital, and face a borrowing constraint in the financial asset. The borrowing constraint is given by:

$$a \geq \underline{a} - \phi k \tag{3}$$

Since workers gain no return on investing in capital, they will optimally choose not the invest in any. As a consequence, they can borrow up to a maximum of $\underline{a} \leq 0$. Entrepreneurs who invest in capital can pledge that capital as collateral and increase their borrowing by $\phi \in [0, 1]$ units of a for each additional unit of k. Not that this means the maximum amount of capital an entrepreneur can invest in is therefore limited by their personal savings, unless $\phi = 1$.

In the second period, entrepreneurs who have a realized productivity of $\tilde{z}$ and have invested in capital k will hire labour n and produce output according to:

$$y = \tilde{z}^{1-\gamma}(k^{\alpha} n^{1-\alpha})^{\gamma} \tag{4}$$

2.2 Agent's Problems

Depending on their initial endowments of wealth (e) and entrepreneurial productivity (z), agents make an occupational choice decision between being workers and entrepreneurs according to:

$$V_1(z, e) = \max\{V_1^E(z, e), V_1^W(e)\} \tag{5}$$

Worker's Problem

Based on their initial endowment of wealth (e), workers make a simple consumption-savings decision between two periods. They face no uncertainty, as they will earn w with certainty in the second period.

$$V_1^W(e) = \max_{c_1,a} \frac{c_1^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \beta \left[\mathbb{E} \left(V_2^W(a)^{\frac{1-\sigma}{1-\frac{1}{\theta}}} \right) \right]^{\frac{1-\frac{1}{\theta}}{1-\sigma}} \tag{6}$$

s.t.

$$c_1 + a = e$$
$$a \geq \underline{a}$$

In the second period, workers consume all of their resources.

$$V_2^W(a) = \frac{c_2^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} \tag{7}$$

where

$$c_2 = a(1+r) + w$$

Entrepreneur's Problem

Based on their initial endowment of wealth (e, z), entrepreneurs choose how much to consume today (c_1), how risky a business to start $x \in \{x_1, x_2, ..., x_{n_x}\}$, the amount of capital to invest in (k), and how much to borrow or save in the financial asset (a).

$$V_1^E(e) = \max_{c_1,x,k,a} \frac{c_1^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \beta \left[\mathbb{E} \left(V_2^E(\tilde{z},k,a)^{\frac{1-\sigma}{1-\frac{1}{\theta}}} \right) \right]^{\frac{1-\frac{1}{\theta}}{1-\sigma}}$$

s.t.

$$c_1 + a + k = e$$
$$a \geq \underline{a} - \phi k$$
$$\tilde{z} = \begin{cases} z + \psi x & \text{with probability} \quad p \\ z - x & \text{with probability} \quad 1-p \end{cases}$$

where $\psi > \frac{1-p}{p}$ and $\phi \in [0,1]$.

In the second period, the entrepreneur chooses how much labour to hire after the realization of the shock to their productivity:

$$V_2^E(\tilde{z},k,a) = \max_n \frac{c_2^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}}$$

s.t.

$$c_2 = \tilde{z}^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma - wn + (1-\delta)k + a(1+r)$$

2.3 Model Predictions

Prediction 1: The missing market for entrepreneurial risk induces agents to select into entrepreneurship based on both ability and wealth

When there are no borrowing constraints ($\phi = 1$), both entrepreneurial ability and wealth increase the value of becoming an entrepreneur relative to a worker. Higher entrepreneurial ability (z) strictly increases realized productivity ($\tilde{z}$) and so increases the income agents will receive if they are entrepreneurs. Wealth increases the value of entrepreneurship because of the risky nature of entrepreneurship. If the entrepreneur receives the bad shock, they earn less than when they receive the good shock. For wealthy individuals, this loss of earnings may not make a big difference, as their consumption is primarily determined by their wealth. By contrast, wealth-poor individuals consume mainly out of their income. Since agents are risk averse, even if the expected entrepreneurial income is higher than income as a worker, wealth-poor agents may still choose to be workers because the loss of utility in the unsuccessful low-consumption state is enough to outweigh the higher utility in the successful high-consumption state.

Figure 1: Occupational Selection without Borrowing Constraints

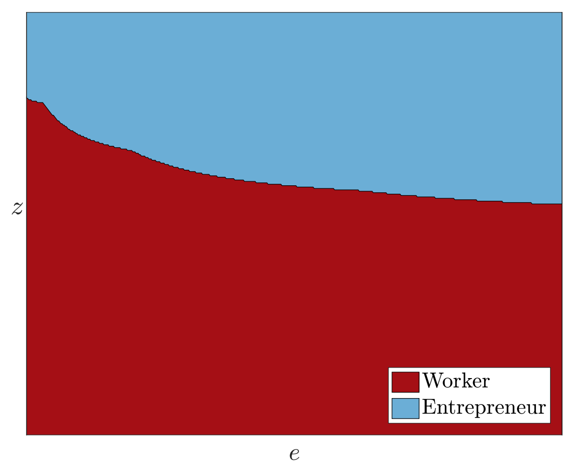
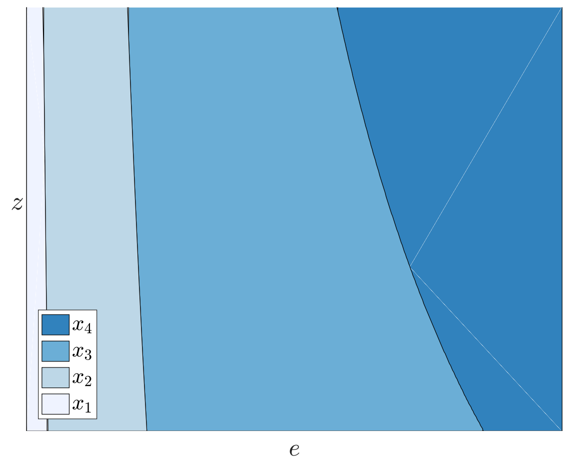


Figure 1 shows selection in the model with no borrowing constraints ($\phi = 1$). This pattern of selection misallocates some entrepreneurial talent. Output would strictly increase if some of the high-wealth low-ability entrepreneurs would switch occupations with the low-wealth high-entrepreneurial-ability workers.

Prediction 2: Wealthier agents choose higher risk projects

Wealth-poor agents choose less-risky projects. Even though their expected productivity is higher, the riskier projects result in much lower income when entrepreneurs receive the bad shocks. For wealth-poor agents, these bad shocks result in a severe cut to their consumption, which they seek to avoid. By contrast, wealthier agents choose to

Figure 2: Endogenous risk choice by entrepreneurial ability z and initial endowed wealth e



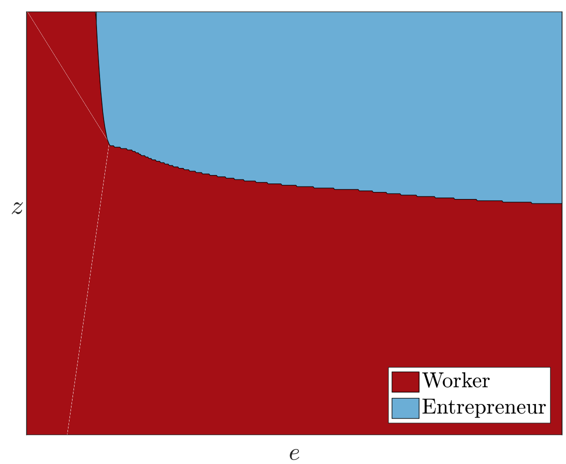
start the riskier projects. Much of their consumption is determined by their endowed wealth, so the income cut as a result of the bad shock is not as costly in utility terms. This result requires preferences that exhibit decreasing absolute risk aversion[1].

[1] Note that most common preferences using in economics, including log, CRRA, and Epstein-Zin preferences, all exhibit decreasing absolute risk aversion.

Prediction 3: Borrowing constraints increase the selection into entrepreneurship based on wealth

Imposing a borrowing constraint ($\phi < 1$) limits the amount of capital entrepreneurs can invest in based on their initial endowment of wealth. If entrepreneurs cannot invest in sufficient capital to operate at their optimal scale, this will lower their entrepreneurial income regardless of the success of the business. As a consequence, agents with low levels of wealth choose to become workers because their entrepreneurial income is so curtailed, even if they have very high entrepreneurial ability.

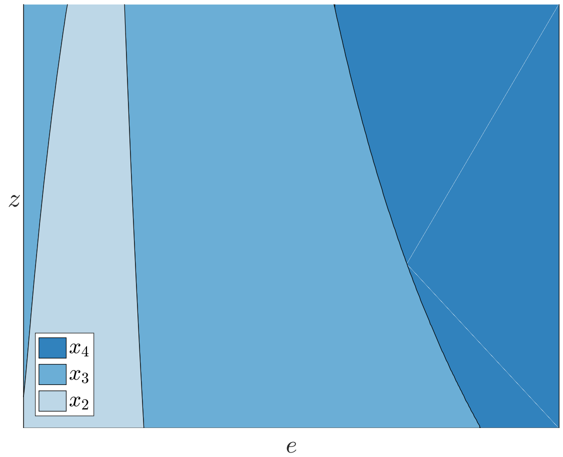
Figure 3: Occupational Selection with Borrowing Constraints



Prediction 4: Borrowing constraints distort the endogenous choice of risk

Borrowing constraints ($\phi < 1$) alter entrepreneur's incentive for taking risks. As borrowing constraints limit their income, entrepreneurs respond by choosing a higher level of endogenous risk x than they would if they were unconstrained. This happens because the risks of a higher x and a larger scale are substitutes. Higher x increases the entrepreneur's income in the good state and reduces it in the bad state. Conditional on the amount saved, the amount invested in k (and therefore the amount borrowed in a) increases the entrepreneur's income in the good state and reduces net income (profits minus debt repayment) in the bad state. In the two period model, this leads to an increase in endogenous risk for the constrained individuals relative to the case where they were unconstrained. In the full perpetual youth model in 4, the impact of borrowing constraints on the endogenous choice of risk becomes ambiguous due to dynamic considerations.

Figure 4: Endogenous risk choice by entrepreneurial ability z and endowed wealth e.



3 Stylized Facts

In this section I document facts about entrepreneurship from the confidential version of the Kauffman Firm Survey. These facts motivate the modelling choices in the static model from section 2 and the dynamic model from section 4.

3.1 Kauffman Firm Survey Data

The Kauffman Firm Survey is a single cohort panel of 4,928 new US firms. All firms are founded in the year 2004, and the survey follows them until they exit or until 2011. It was designed to provide a representative sample of all new businesses started in 2004[2]. The firms include businesses that were independently founded, purchased from an existing business or purchased as a franchise and exclude any inherited businesses, any non-profits, and businesses that were started as a branch or subsidiary of an existing business. For each firm, information about up to 10 owners is provided.

These firms are highly heterogeneous. As table 1 shows, the majority of firms are non-employers in the first year, though many go on to hire at least one worker later. More firms are owned and operated by a single entrepreneur, though a small proportion have multiple owners. The distribution of total investment is highly skewed with the mean investment almost twice as large as the median investment.

[2]The sampling frame is taken from the Dun and Bradstreet US Business database.

Table 1: Summary Statistics

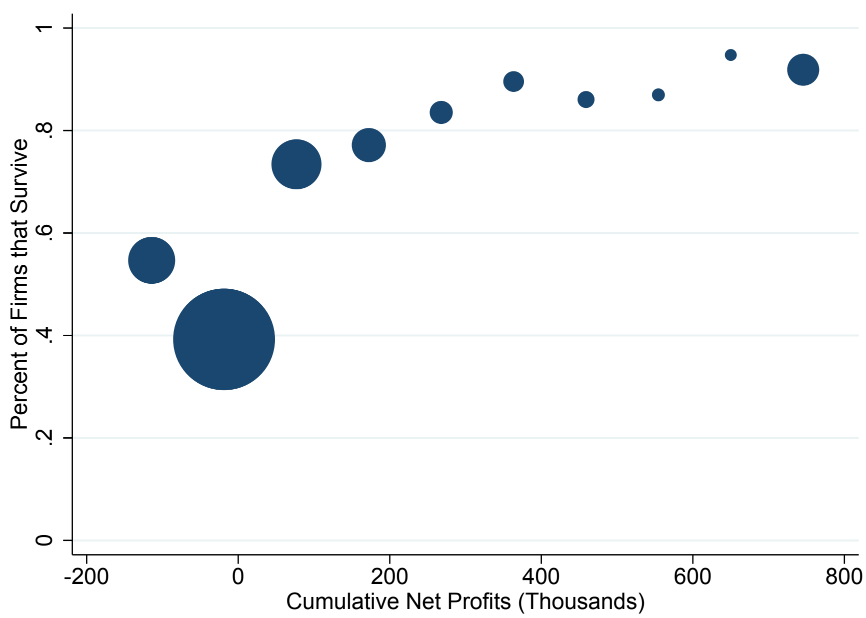
	Mean	p10	p50	p90	n Firms
Employment (Year 1)	2.0	0	0	5	4,823
Employment (Year 8)	5.2	0	1	10	2,000
Entrepreneurs	1.4	1	1	2	4,928
Total Investment (Years 1-8)	480.5	67	277	762	3,488

Total investment is in thousands of US dollars and included funding from both equity and debt. The number of firms in each row varies depending on data availability.

3.2 Business Losses

The model in section 2 shows how a missing market for entrepreneurial risk can generate selection into entrepreneurship based on wealth. If this missing market is quantitatively important for who selects into entrepreneurship, two things must be true. First, entrepreneurs must face a significant degree of uncertainty about the potential success of the business[3]. Second, if the business is unsuccessful entrepreneurs must bear real losses that make them worse off than if they did not start a business.

Figure 5: Survival by Cumulative Profits



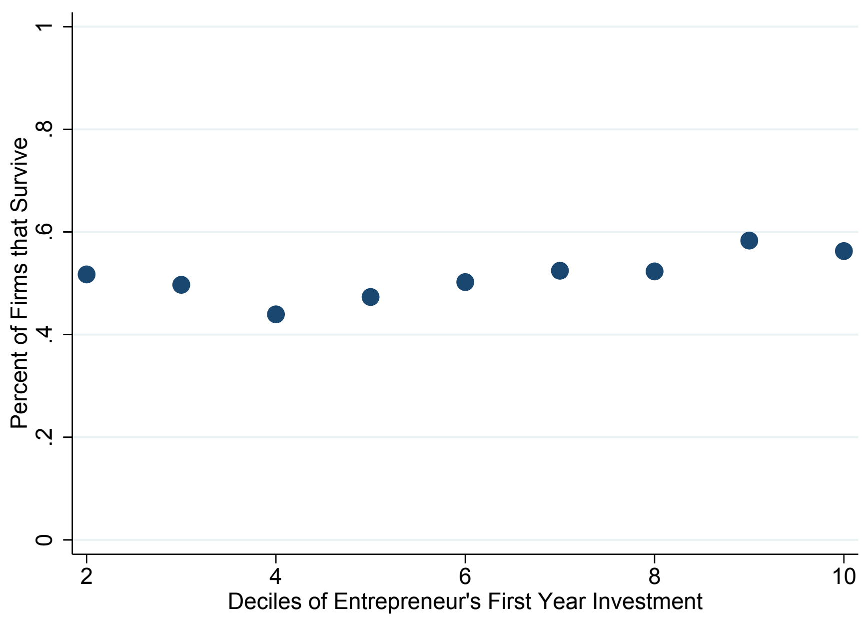
This figure shows the survival frequency of firms in ten bins of cumulative net profits. Cumulative profits have been winsorized between the 5th and 95th percentiles. The size of each circle is proportional to the number of observations in that bin.

[3]Note that both uncertainty about entrepreneurial ability as in Jovanovic (1982), which decreases over time as entrepreneurs learn about their ability, and idiosyncratic risk to the business model as in Hopenhayn (1992), can provide the necessary uncertainty about the success of the business.

Figure 5 shows firm survival rates are much lower for firms that have earned negative cumulative profits since founding. This suggests that as entrepreneurs earn negative profits, they disproportionately choose to exit. In the absence of idiosyncratic risk, it is difficult to imagine why these entrepreneurs would enter in order to first lose money and then exit.

Firm exits are not confined to low-investment firms, which could have been intended as temporary self-employment in response to an unemployment spell[4]. Firm exits are common across both low investment and high investment firms. Figure 6 shows the survival rates of firms based on the investment made in each firm in the first year. While survival is slightly higher among businesses with larger investments, among the top 10% of firms by initial investment, only 58% of these businesses survive the full 8 years of the sample.

Figure 6: Survival by Initial Investment



This figure shows the survival rates of firms grouped into deciles based on the total equity invested in the first year.

Firm exits are also not confined to a specific set of industries. Table 2 shows the proportion of firms that survive to the end of the sample according to their 2-digit NAICs code. I calculate the survival rate as the number of firms either operating in the final sample year or that have been merged or sold in a previous year, divided by the total number of firms with a known status in the final year. Without information about the final sale price, it is difficult to assess whether firms that are merged or sold constitute a successful or unsuccessful outcome for entrepreneurs. I include these firms

[4]See Galindo da Fonsec (2017), who documents different patterns of entrepreneurial activity based on employment status before starting a business.

in survival in order to provide a conservative estimate of the likelihood of undesirable outcomes. See appendix A.1 for additional details. Dropping industries with fewer than 50 observations, survival rates range from 43% of firms in the case of Retail Trade (44), to 64% in Manufacturing (33).

Table 2: Survival By Industry

Industry (2 digit NAICs Code)	Survival Rate	n Firms
Retail Trade (44)	42.9 %	238
Retail Trade (45)	43.7 %	229
Transportation and Warehousing (48)	45.3 %	86
Construction (23)	46.4 %	306
Finance and Insurance (52)	51.2 %	164
Other Services (except Public Administration) (81)	51.4 %	389
Health Care and Social Assistance (62)	52.1 %	94
Administrative, Support, Waste Management, and Remediation Services (56)	52.6 %	310
Manufacturing (32)	54.5 %	123
Accommodation and Food Services (72)	54.5 %	88
Information (51)	55.6 %	144
Real Estate and Rental and Leasing (53)	56.9 %	160
Wholesale Trade (42)	57.4 %	188
Arts, Entertainment, and Recreation (71)	57.6 %	92
Professional, Scientific, and Technical Services (54)	58.4 %	1,058
Manufacturing (33)	63.5 %	425

The existence of entrepreneurs that start businesses, earn negative cumulative profits, and then exit, demonstrates that entrepreneurs face significant idiosyncratic risk. Entrepreneurs who start unsuccessful businesses are likely to face two sources of potential losses to their personal net worth, relative to a counter-factual where they did not start a business. First, they may lose some of all of the money they invested in the business. Table 8 shows that in the median exiting firm, the value of a firm's assets are worth 55% of the initial investment. Second, unsuccessful entrepreneurs devote a substantial amount of their time to operating their businesses. This time has an opportunity cost, namely the wages or salaries the entrepreneurs could make in the labour market. Table 9 shows that around half of entrepreneurs pay themselves a salary out of the business, thus exiting entrepreneurs may have accumulated several years of forgone earnings before they decide to exit.

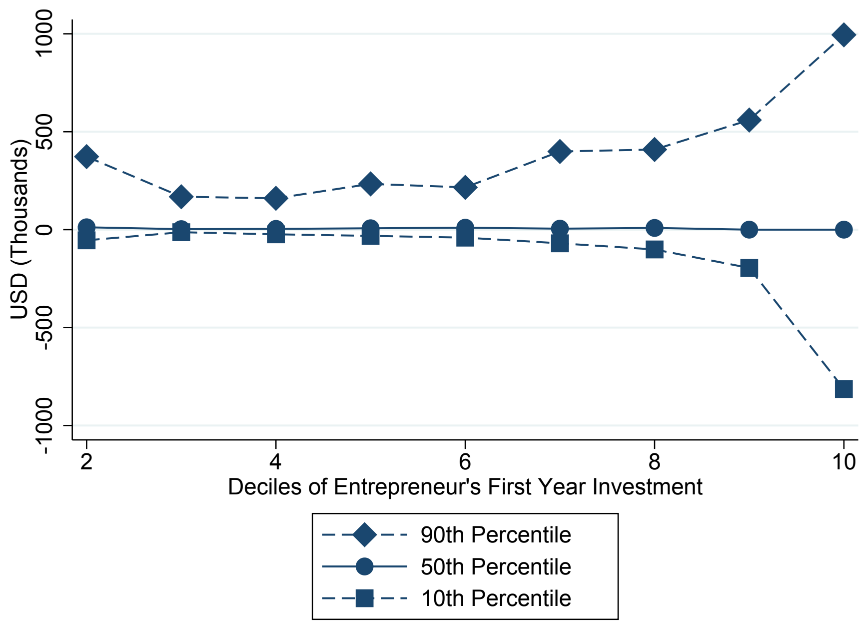
3.3 Choice of Risk

The model in section 2 predicts that wealthier entrepreneurs will both invest more in their business and choose businesses with a higher level of risk. While the ex-ante risk of a business is not directly observable, the ex-post outcomes of a group of firms is. In order to compare the risk taken by different entrepreneurs, I examine the dispersion in outcomes within different groups of firms. If wealthier entrepreneurs are taking more risk, there should be more dispersion in their outcomes than among a group of poorer entrepreneurs.

While the Kauffman Firm Survey does not ask entrepreneurs about their wealth when they first start their business, the amount of money they have been able to directly invest in the business is informative about their wealth. The survey distinguishes between money an entrepreneur is able to directly invest in a business and the amount of money they invest after taking out a personal loan, the amount they have been able to directly invest is a reasonable proxy for an entrepreneur's net worth.

Therefore, in order to test the prediction of the model in section 2, I compare the dispersion in cumulative profits across firms in the different deciles of entrepreneur's own investment, excluding external sources of equity. If entrepreneurs who invest more in their businesses earn more dispersed cumulative profits, this may indicate that these entrepreneurs are in fact starting businesses with more idiosyncratic risk.

Figure 7: Dispersion in Cumulative Profits



This figure shows 10^{th}, 50^{th} and 90^{th} percentiles of the distribution of cumulative profits separately for each decile of entrepreneur's initial investment. Note that almost 20% of firms invest nothing in the first year of operation, and so the bottom two deciles are represented by a single point, labelled "2".

Figure 7 clearly shows that firms with larger initial investments by their entrepreneurs have much more dispersed cumulative profits. However, since initial investment is highly correlated with firm size, it is not clear whether the greater dispersion is a mechanical consequence of these firms operating at a larger scale or a consequence of these entrepreneurs choosing higher scale.

To test whether entrepreneurs who invest more also earn more dispersed cumulative profits while controlling for size, I run two key regressions. First, I regress firm's cumulative profits on the initial investment of their entrepreneurs, controlling for size through employment, as well as other sources of funding and industry. A positive coefficient on initial entrepreneur's own investment ($\hat{\alpha}_1 > 0$) means that entrepreneurs who invest more are on average earning higher cumulative profits. In the second regression, I take the absolute value of the residuals from the first regression, and regress them on the initial investment of their entrepreneurs and the same controls. A positive coefficient on initial entrepreneur's own investment ($\hat{\beta}_1 > 0$) suggests that entrepreneurs who invest more are earning more dispersed cumulative profits (i.e. the cumulative profits are heteroskedastic). I take this to be evidence of greater risk taking on the part of high-investment entrepreneurs.

$$\text{Cumulative Profits}_i = \alpha_0 + \alpha_1 \text{Initial Entrepreneur's Own Investment}_i + \alpha X_i + \epsilon_i \quad (8)$$

$$|\hat{\epsilon}_i| = \beta_0 + \beta_1 \text{Initial Entrepreneur's Own Investment}_i + \beta X_i + \eta_i \quad (9)$$

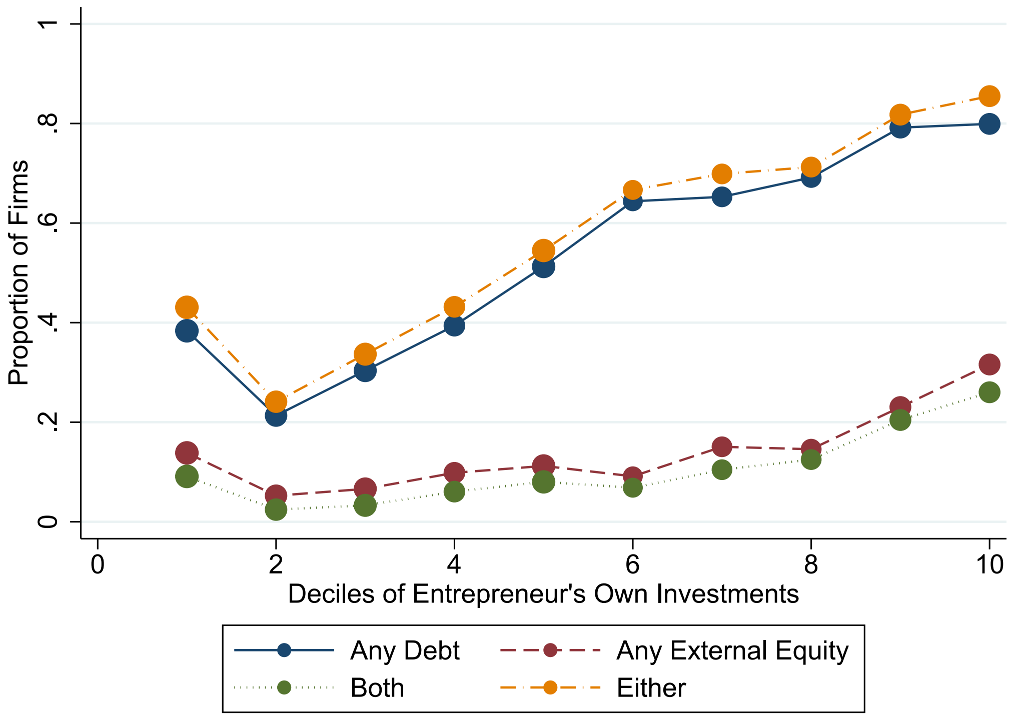
Table 3 shows that entrepreneurs who invest more of their own money in their businesses on average earn higher profits, and on average, earn more dispersed profits. Both of these results are true after controlling for both the number of employees and the level of debt. In appendix A.5 I show that this same result holds for cumulative sales. In appendix A.6 I discuss why the absence of labour market information for these entrepreneurs prevents me from calculating reasonable rates of return for these firms.

Table 3: Dispersion of Cumulative Profits Regressions

First Regression:	Cumulative Profits		
	(1)	(2)	(3)
Entrepreneur's Initial Own Equity	1.312***	1.313***	1.314***
	(0.0847)	(0.0859)	(0.0860)
Initial External Equity			-0.387*
			(0.181)
Initial Debt		0.528***	0.670***
		(0.109)	(0.128)
Employer		15.61	32.90
		(293.0)	(293.4)
Number of Employees		-33.21	-31.61
		(23.77)	(23.80)
Constant	70.95	-308.7	-271.5
	(131.3)	(1512.0)	(1512.3)
2 Digit NAICs Codes	No	Yes	Yes
Second Regression:	Absolute Value of Residuals		
	(1)	(2)	(3)
Entrepreneur's Initial Own Equity	0.468***	0.452***	0.449***
	(0.0842)	(0.0844)	(0.0844)
Initial External Equity			0.412*
			(0.177)
Initial Debt		0.116	-0.0110
		(0.107)	(0.125)
Employer		404.7	403.1
		(287.8)	(288.1)
Number of Employees		50.98*	44.90
		(23.35)	(23.36)
Constant	572.5***	482.8	359.2
	(130.6)	(1485.2)	(1484.9)
2 Digit NAICs Codes	No	Yes	Yes
N	4528	4419	4414

3.4 Sources of Funding

Figure 8: External Funds

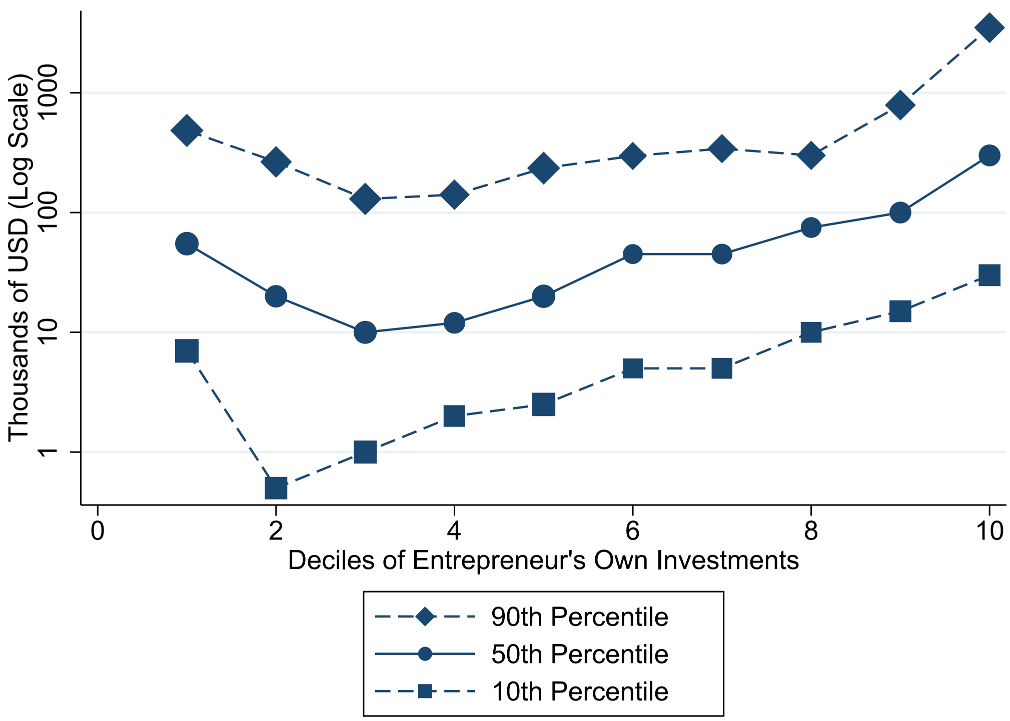


This graph shows the proportion of firms that have raised some debt, some external equity, both or neither across the distribution of initial investment. Firms are sorted into deciles based on the total amount their entrepreneur's own investment in the firm in the first year of operations. See table 12 for a detailed break-down of the sources of debt and external equity.

How able are new entrepreneurs to raise external funds? Figure 8 shows the proportion of firms that are able to raise any debt or any external equity in order to fund their firm. Clearly entrepreneurs who invest more of their own money in the firm are more likely to raise external funds, whether from debt or external equity. This is consistent with the idea that wealth-poor individuals are unable to access external funds.

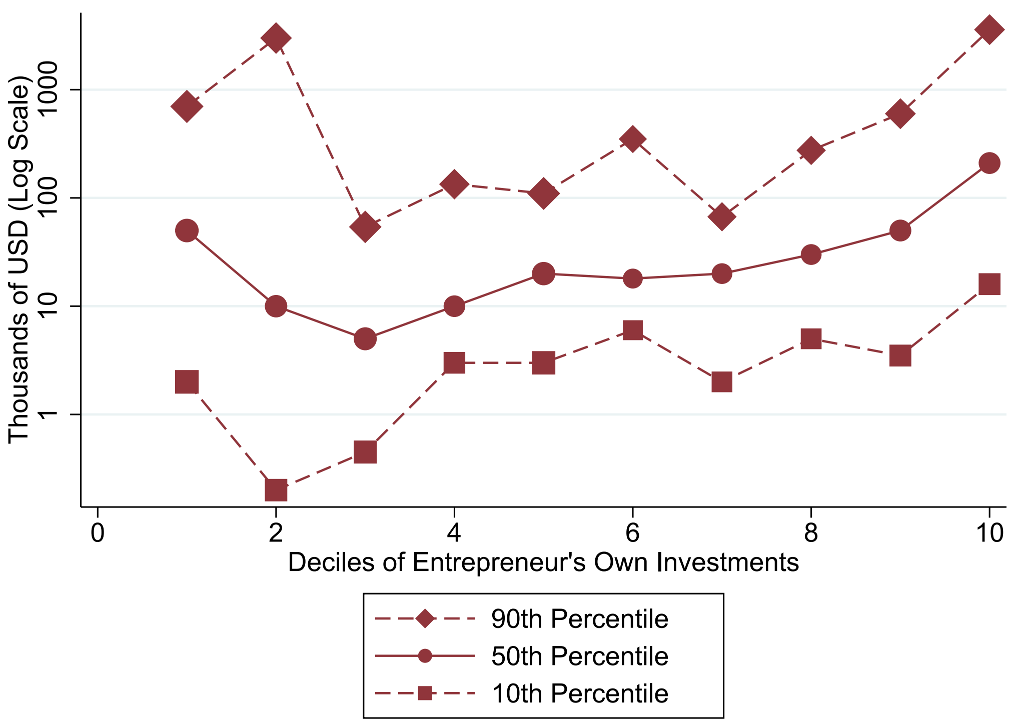
Figures 9 and 10 show how much debt and external equity that entrepreneurs raise broken down by the deciles of entrepreneurs' total own investments. Note that these graphs have a log-scale. Across the first three deciles, there appears to be somewhat of a decline, suggesting that there is a small fraction of firms who are able to substitute owners investment for external funds, but over the rest of the distribution, larger entrepreneur's investments are correlated with larger amounts of external funds. This is consistent with the presence of financial frictions, and motivate the modelling of these frictions as a collateral constraint.

Figure 9: Amount of Debt



This figure shows the 10^{th}, 50^{th} and 90^{th} percentiles of the distribution of total debt raised within the deciles of the total of entrepreneur's own investments, conditional on raising some debt. Note the y-axis is a log-scale.

Figure 10: Amount of External Equity



This figure shows the 10^{th}, 50^{th} and 90^{th} percentiles of the distribution of total external equity raised within the deciles of the total of entrepreneur's own investments, conditional on raising some external equity. Note the y-axis is a log-scale.

4 Full Model

In order to quantify the relative importance of the missing market for entrepreneurial risk and borrowing constraints, I extend the two period model of section 2 to a dynamic setting.

4.1 Environment

There are a unit measure of agents. Each agent faces a constant probability $(1-\psi)$ of dying every period and has CRRA preferences given by:

$$U = \sum_{t=0}^{\infty} (\psi\beta)^t \frac{c^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} \tag{10}$$

Each agent has two types of ability, their ability as a worker (h^W) and their ability as an entrepreneur (h^E). At the beginning of each period, they choose whether to work as a worker or as an entrepreneur for the period.

Workers supply labour inelastically and earn wh^W, where w is the common wage.

The first period an agent decides to be an entrepreneur, they start a business by choosing a business type x, and investing in a capital stock k. Once chosen, the business type x is fixed. In a future period, if an entrepreneur wants to change their business type they must shut down their business and liquidate their capital stock, then they are able to select a new business type and invest in a new capital stock.

After all agents make an occupational choice decisions and entrepreneurs decide on a business type, all agents receive shocks to both their ability as a worker h^W and their ability as an entrepreneur h^E. While both types of ability are partially persistent, neither are perfectly so, and so agents face idiosyncratic income risk from choosing either occupation.

In addition, entrepreneurs receive a project productivity shock (z) to the specific business they are currently operating. z is drawn from a distribution that depends on the chosen x. Higher x businesses have higher expected z but also more dispersed z. The productivity of a business depends on both the firm-specific productivity shock as well as the entrepreneur's entrepreneurial ability h^E. Once they received their shocks, entrepreneurs hire an amount of labour n at wage rate w and produce according to:

$$y = (zH^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma$$

$$\log(z') = \log(z) + \epsilon$$

$$\epsilon \sim N(0, \sigma_{\epsilon,x})$$

After production and factors are paid, all agents make a consumption, savings, and investment decision. The model has two assets. Only entrepreneurs can invest in capital k, which depreciates at rate δ. Capital is illiquid, so that liquidating a unit of capital produces only $\frac{1}{\chi} < 1$ units of consumption. Given some investment I, an entrepreneur's capital stock therefore evolved according to:

$$k' = \begin{cases} k(1-\delta) + I & \text{if } I \geq 0 \\ k(1-\delta) - \chi I & \text{if } I < 0 \end{cases} \tag{11}$$

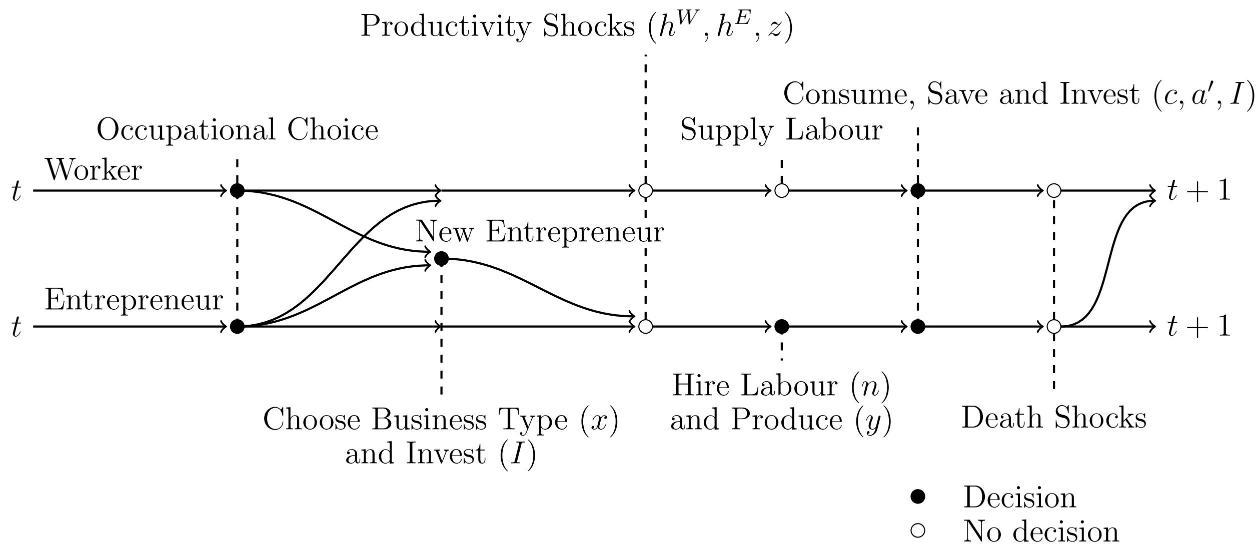
where $\chi > 1$.

Agents can also save and borrow in a liquid financial asset a, that pays a constant r^A each period. All agents can borrow up to an exogenous borrowing limit $\underline{a} \leq 0$. In addition, for each unit of capital that an entrepreneur invests in, they can use this capital as collateral and borrow an addition $\phi \in [0, 1]$ units of a. Thus the borrowing constraint is given by:

$$a \geq \underline{a} - \phi k$$

After their consumption, savings, and investment decisions, agents face a probability of dying of $(1 - \psi)$. When an agent dies, if they were an entrepreneur, their capital stock is immediately liquidated, and then the agent is immediately replaced by a descendent who inherits the full value of their assets. Figure 11 summarizes the timing of the model.

Figure 11: Timing in the model



4.2 Agent's Problems

Worker's Problem

A worker makes a simple consumption-savings decision, and at the beginning of the next period will choose between being a worker (V^W) and becoming a new entrepreneur (V^{NE}):

$$V^W(a, h^W, h^E) = \max_{a',c} \frac{c^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta \max\left\{\mathbb{E}\left(V^W(a', h^{W\prime}, h^{E\prime})\right), V^{NE}(a', h^{W\prime}, h^{E\prime})\right\} \tag{12}$$

s.t.

$$a' + c = wh^W + (1 + r^a)a$$

$$a' \geq \underline{a}$$

New Entrepreneur's Problem

An agent that has decided to start a new business chooses a business type x. Given their current financial assets a, they also choose how much to invest in the business (I) and how much to borrow or save in the financial assets $\tilde{a}$.

$$V^{NE}(a, h_{-1}^{W}, h_{-1}^{E}) = \max_{\tilde{a}, I, x} \mathbb{E}\left(V^{E}(\tilde{a}, \tilde{k}, h^{W}, h^{E}, z, x)\right) \tag{13}$$

s.t.

$$\tilde{a} = a - I$$
$$\tilde{a} \geq \underline{a} - \phi \tilde{k}$$
$$\tilde{k} = I$$

Entrepreneur's Problem

An entrepreneur that has a business will choose an amount of labour n to hire, consumption c, savings $(I + a')$, and investment I. At the beginning of the next period, they will choose between shutting down their business to become a worker (V^{W}), shutting down their business to start a new business (V^{NE}) and continuing to operate the same business (V^{E}).

$$V^{E}(a, k, h^{W}, h^{E}, z, x) = \max_{n, c, a', I} \frac{c^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta \max \left\{ \begin{array}{c} \mathbb{E}\left(V^{W}(a', h^{W\prime}, h^{E\prime})\right), \\ V^{NE}(a', h^{W}, h^{E}), \\ \mathbb{E}\left(V^{E}(a', k', h^{W\prime}, h^{E\prime}, z', x)\right) \end{array} \right\} \tag{14}$$

s.t.

$$c + a' + I = (zh^{E})^{1-\gamma}(k^{\alpha} n^{1-\alpha})^{\gamma} - wn + (1 + r^{a})a$$
$$a' \geq \underline{a} - \phi k'$$

$$k' = \begin{cases} k(1-\delta) + I & \text{if } I \geq 0 \\ k(1-\delta) - \chi I & \text{if } I < 0 \end{cases}$$

Note that if the entrepreneur decides to start a new business, they cannot use the capital from their current business. They must fully liquidate it this period.

4.3 Equilibrium

An equilibrium is a set of value functions $\{V^{W}, V^{NE}, V^{E}\}$, occupational choices, a set of policy functions $\{c^{W}, a'^{W}, \tilde{a}^{NE}, I^{NE}, x^{NE}, c^{E}, a'^{E}, I^{E}, n^{E}\}$, a distribution of agents $\{\Gamma^{E}(a, k, h^{W}, h^{E}, z, x), \Gamma^{W}(a, h^{W}, h^{E})\}$, and a price w such that

1. The policy functions solve the problems given by 12, 13 and 14.
2. All markets clear:

- Labour

$$\int h d\Gamma^W(a, h^W, h^E) = \int n^E d\Gamma^E(a, k, h^W, h^E, z, x)$$

- Final Goods

$$\int (c + a') d\Gamma^W(a, h^W, h^E) + \int (c + a' + I) d\Gamma^E(a, k, h^W, h^E, z, x) =$$
$$\int \left((zh^E)^{1-\gamma}(k^\alpha n^{1-\alpha})^\gamma + (1 + r^A)a\right) d\Gamma^E(a, k, h^W, h^E, z, x)$$
$$+ \int ((1 + r^A)a) d\Gamma^W(a, h^W, h^E)$$

3. The distribution of agents is stationary

$$\Gamma^E(a, k, h^W, h^E, z, x) = \Gamma^{E\prime}(a, k, h^W, h^E, z, x)$$
$$\Gamma^W(a, h^W, h^E) = \Gamma^{W\prime}(a, h^W, h^E)$$

4.4 Model Predictions

Patterns of Selection

Figure 12 illustrates the occupational choice and endogenous risk choice in the model. For a worker with the lowest worker ability (h^W), the graph shows how the worker would select their occupation depending on their current level of entrepreneurial productivity (h^E) and their current level of cash on hand ($a(1 + r^A) + wh^W$).

Regardless of how high their entrepreneurial productivity is, if the agent has little wealth, they will choose to be a worker. The borrowing constraint prevents poor entrepreneurs from operating at a large scale, limiting their income as an entrepreneur. As a consequence, a worker with little wealth and the highest entrepreneurial productivity will still choose to be a worker.

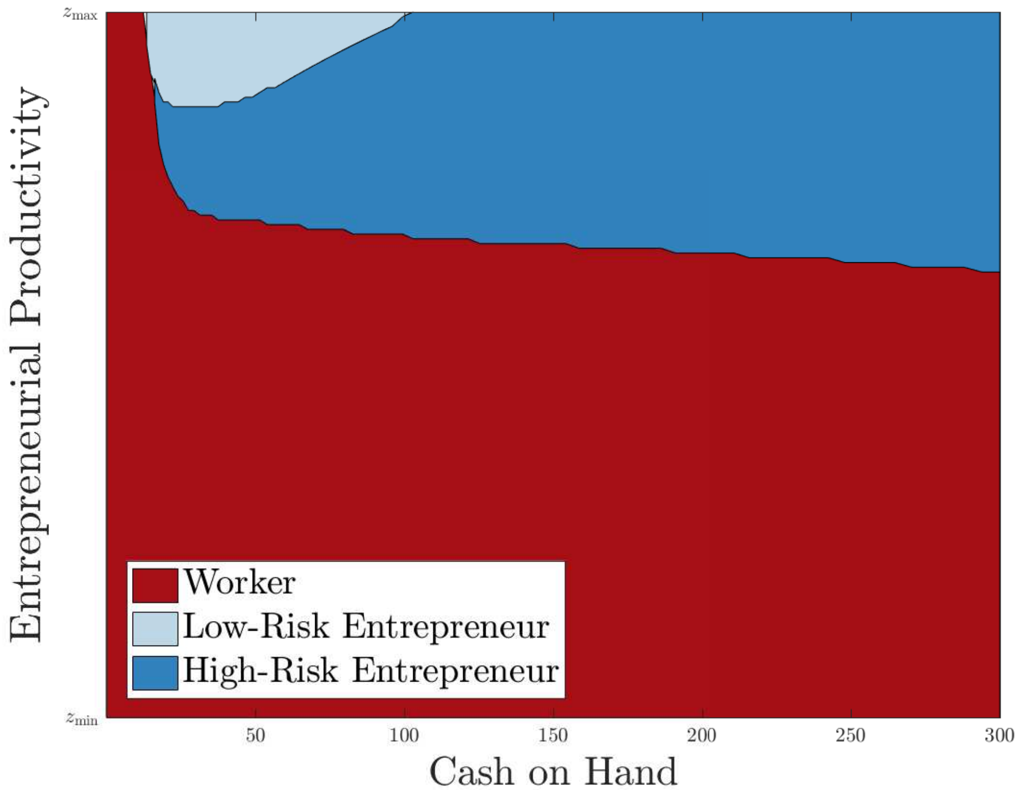
Agents with high entrepreneurial ability and moderate wealth will choose to be entrepreneurs operating the low risk project. While the expected productivity of the high-risk project is greater, the higher risk makes it unattractive. Bad productivity shocks would curtail the entrepreneur's consumption substantially and make exit more likely.

Agents with high entrepreneurial ability and high wealth will choose to be entrepreneurs operating the high risk project. Since they have sufficient wealth to self-insure any bad productivity shocks, they choose the higher risk project with higher returns.

Agents with moderate entrepreneurial ability and moderate wealth switch directly from being workers to starting a high risk project. Since they are less productive, they are more likely to exit entrepreneurship for any given bad productivity shock. The relatively high likelihood of exit means that these entrepreneurs care little about their productivity if they receive a bad productivity shock. As a consequence they pick the high risk project because it results in much higher productivity in good states of the world, and much lower productivity in bad states of the world, when they will choose to be workers[5].

[5] This effect is closely related to the mechanism explored in Vereshchagina and Hopenhayn (2009).

Figure 12: Selection Patterns



This figure shows the patterns of selection in the calibrated model. Given a worker with a particular worker ability (h^W), the graph shows how the worker would select their occupation depending on their current level of entrepreneurial productivity (h^E) and their current cash on hand.

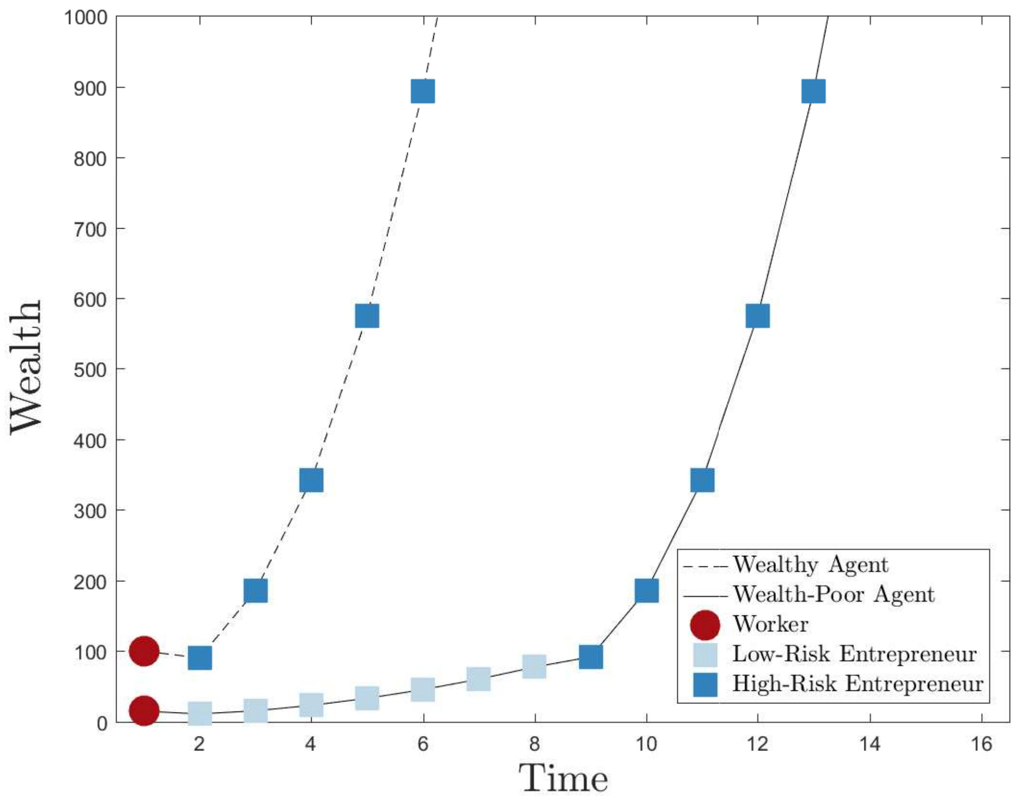
Wealth Accumulation

In this model, wealthier agents accumulate wealth faster than poor agents given the same abilities. Figure 13 illustrates this dynamic for two agents with identical entrepreneurial and worker abilities. Both agents are born at time 1 as workers with the highest entrepreneurial and worker ability. However, they are endowed with different initial levels of wealth.

The wealthier agent immediately starts a high risk business. In this illustrative example, they receive the highest project productivity and earn high returns on their wealth. The wealthy agent saves a high proportion of their entrepreneurial income for two reasons. First, because the project is risky, the agent wants to engage in precautionary savings in order to self-insure against the project's failure. Second, because of the borrowing constraint, the entrepreneur wants to save more in order to operate a larger business next period and earn an even higher income.

Lacking the wealth to self insure against large fluctuations in income, the wealth poor-agent starts a low risk business. In this illustrative example, they receive the highest project productivity, but earn substantially lower entrepreneurial income. They save at high rates and slowly accumulate enough wealth until they are willing to shut down their safe business and start a new risky business. Of course, if they receive any negative productivity shocks before then it will take them even longer to accumulate the necessary net worth to be willing to start a risky business.

Figure 13: Wealth Accumulation Patterns



5 Calibration

I calibrate this model to match micro data on wealth inequality and new firm dynamics by using the 2004 Survey of Consumer Finances and the Kauffman Firm Survey. For a set of parameters that are difficult to identify using my data, I rely on commonly used values in the literature. I then jointly match twelve model moments to twelve moments in the data by varying twelve parameters.

Parameterization of the Ability and Productivity Processes

I parameterize both the worker and entrepreneurial ability processes with AR(1) processes:

$$\log(h^{i\prime}) = \rho_{h^i} \log(h^i) + \epsilon_{h^i}$$

where $\epsilon_{h^i} \sim N\left(\frac{\mu_{h^i}}{1-\rho_{h^i}}, \sigma^2_{h^i}\right)$ for $i \in \{E, W\}$

In order to keep the problem computationally tractable, I restrict myself to the case where entrepreneurs choose between two project types, a relatively safe project (x_1) and a relatively risky project (x_2). Each project type receives productivity shocks z the follows an AR(1) process:

$$\log(z') = \rho_{z|x_i} \log(z) + \epsilon_{z|x_i}$$

$$\epsilon_{z|x_i} \sim N\left(\frac{\mu_{z|x_i}}{1-\rho_{z|x_i}}, \sigma^2_{\epsilon_z|x_i}\right)$$

for $i \in \{1, 2\}$.

I use the Rowenhorst method to discretize these four AR(1) processes $\{h^W, h^E, z_{x_1}, z_{x_2}\}$.

Externally Chosen Parameters

A period in the model is a year. I fix ten parameters to commonly used values in the literature. I set the capital share α to $\frac{1}{3}$ and the interest rate r^A to 4% per year. I set the decreasing returns to scale parameter γ to 0.85 as in Midrigan and Xu (2014) and the depreciation rate to 6% per year. The time discount factor is set such that $\beta = \frac{1}{1+r^A}$. I set the coefficient of relative risk aversion to 1.50. I set the probability of death $(1-\psi)$ to $\frac{1}{40}$ so that the expected working lifetime is 40 years.

I normalize the average level of worker ability (μ_{h^W}) to 1. I set the persistence of the labour income process ρ_{ϵ_h} to 0.9, which is in the range of empirical estimates according to Guvenen (2007). The standard deviation of the innovation σ_{ϵ_h} is set to 0.2.

Internally Calibrated Parameters

I jointly calibrate twelve parameters to match twelve model generated moments to the corresponding moments in the data. Since this model is designed to generate the empirically observed wealth inequality, it must be consistent with micro economic data on household wealth holdings. Since the main mechanism it proposes has to do with patterns of selection and wealth accumulation for entrepreneurs, it must match the firm dynamics of new firms.

In order to match the model's dynamics to observable firm dynamics, I simulate in the model an 8 year panel of new firms, corresponding to the Kauffman Firm Survey's 8 year panel. In the data I observe the ratio of debt and equity that these new firms start with, and so to pin down the tightness of the borrowing constraint ϕ, I match the ratio of debt to equity in the panel to the ratio of debt to equity in the data.

The nature of entrepreneurial risk is of vital importance to this model. Obviously, it is not possible to directly observe the distribution of risk each entrepreneur is drawing from. However, the distribution of firm outcomes is informative about the nature of entrepreneurial risk that all entrepreneurs are facing. The model predicts that wealthier individuals will select higher risk projects and invest more in them. Therefore I separate firm both in the model simulation and in the data at the 90th percentile of investment over the first three years. I compute moments separately for the firms in the top 10% of first-three-year investment and those in the bottom 90% of first-three-year investment.

To discipline the size and persistence of productivity shocks, I match the survival rates and the auto-correlation of employment for both these high-investment and low-investment firms. In order to pin down the differences in average productivity between the low-risk and high risk project, I match the ratio of average employment in these high-investment firms to the average employment in the low-investment firms at the eighth, and last, year of the panel, conditional on survival.

Of course these moments on firm dynamics, (survival, auto-correlation of employment, and average productivity) are all jointly generated by shocks to the business's productivity and shocks to the entrepreneur's ability. Both types of shocks likewise influence savings motives and so the accumulation of wealth in the model. Thus it is vital that the model also match micro evidence on the wealth inequality in the US economy. To match up with the distribution of firms when the Kauffman Firm Survey firms are initially founded, I use the 2004 wave of the Survey of Consumer Finances.

Since this model is primarily about the differences in savings behaviour of the entrepreneurs and the workers, I match the ratio of average wealth of entrepreneurs to workers, the proportion of entrepreneurs in the economy, and the proportion of entrepreneurs in the wealthiest 1%. Following Cagetti and De Nardi (2006), I match the wealth gini. Since this model is a model of heterogeneous entrepreneurs, I also match the wealth gini of the entrepreneurs to ensure that the degree of wealth inequality within entrepreneurs matches the data.

Finally, to replicate the bottom of the wealth distribution I match the level of unsecured borrowing ($\underline{a}$) to ensure the correct proportion of agents have negative net worth.

Table 4: Benchmark Calibration

Target	Data	Model	Parameter	Value
Exogenously Chosen				
Capital Share			α	0.33
Interest Rate			r^A	0.04
Entrepreneurial Share			$1-\gamma$	0.12
Depreciation			δ	0.06
Discount Factor			β	0.96
Coefficient of Relative Risk Aversion			σ	1.50
Elasticity of Intertemporal Substitution			θ	0.67
Average Labour Ability			μ_{h^W}	1.00
Dispersion of Labour Ability			σ_{h^W}	0.20
Persistence of Labour Ability			ρ_{h^W}	0.90
Endogenously Calibrated				
Ratio of Average Debt to Average Equity	1.35	1.32	ϕ	0.57
Proportion of Entrepreneurs	0.11	0.14	χ	1.09
Proportion with Negative Net Worth	0.08	0.12	$\underline{a}$	-3.83
Wealth Ratio of Entrepreneurs to Workers	7.14	7.28	μ_{h^E}	-4.23
Wealth Gini of Entrepreneurs	0.75	0.59	σ_{h^E}	1.05
Wealth Gini	0.79	0.70	ρ_{h^E}	0.95
Proportion of Entrepreneurs in Wealthiest 1%	0.74	0.79	$\mu_{z\|x_1}$	2.10
Autocorrelation of Employment	0.03	0.07	$\sigma_{z\|x_1}$	0.13
Survival Rate	0.50	0.48	$\rho_{z\|x_1}$	0.73
Relative Employment	4.23	4.13	$\mu_{z\|x_2}$	-2.31
Autocorrelation of Employment	0.12	0.05	$\sigma_{z\|x_2}$	4.80
Survival Rate	0.60	0.33	$\rho_{z\|x_2}$	0.79

6 Quantitative Analysis

6.1 The Relative Importance of Two Frictions

A central question in this paper is how important is the missing market for entrepreneurial risk relative to borrowing constraints on generating selection into entrepreneurship on the basis of wealth. In order to assess the quantitative significance of both of these frictions, I remove first one, then the other, and finally both from the model in order to compare the resulting equilibria.

In order to assess the importance of the missing market for entrepreneurial risk, I complete this missing market by introducing contingent claims. In the benchmark economy, there is a single risk free asset. In the contingent claims economy, each agent can purchase assets that pay off in future states of the world based on the agent's realizations of h^W, h^E and z. An asset a_i pays off 1 in the state of the world $i \in H^W \times H^E \times Z$. Each of these assets are sold at price q_i by competitive risk-neutral financial intermediaries at actuarially fair prices.

In order to separate the impact of the missing market for entrepreneurial risk from

Table 5: Productivity Losses from the Two Friction

	Y	w	K	TFP	Percentage of Workers	Percentage of Entrepreneurs	Percentage of Low Risk	Percentage of High Risk
Benchmark	1.00	1.00	1.00	1.00	84.0	16.0	13.7	2.2
Contingent Claims	1.17	1.13	1.09	1.11	86.3	13.7	0.0	13.7
No Borrowing Constraint	1.09	1.07	1.22	1.02	84.9	15.1	12.3	2.7
Frictionless	1.43	1.36	1.92	1.15	87.3	12.7	0.0	12.7

the impact of the borrowing constraint, I keep the borrowing constraint in the economy unchanged. In the benchmark economy the borrowing constraint requires that $-a \leq \phi k$. In the contingent claims economy, the borrowing constraint requires that:

$$-\sum_i q_i a_i \leq \phi k$$

Thus an entrepreneur with the same amount of net worth across the two economies can invest in the same amount of capital.

Obviously, considering the perfect completion of the missing market for entrepreneurial risk is not a policy-relevant exercise. In the real world, information frictions will always make it difficult to provide insurance to entrepreneurs. The perfect ability of the financial intermediaries to identify the exact abilities of potential entrepreneurs and expected productivity of projects in this exercise is infeasible in the real world. This complete market exercise is however a useful counter-factual. Understanding the relative impact of the missing market for entrepreneurial risk and the borrowing constraint, as well as any interaction between the two, is key for the design of public policy to encourage entrepreneurship.

To remove borrowing constraints, I simply set $\phi = 1$, so that entrepreneurs can invest in any level of capital stock, unrestricted by their personal net worth. Note however, that for reasons of tractability, I do not allow entrepreneurs to default.

I then compare four economies. The benchmark economy with both frictions, a complete markets economy with borrowing constraints ($\phi < 1$), the un-constrained economy with incomplete markets but where entrepreneurs are unrestricted in the amount of capital ($\phi = 1$) they can invest in, and finally a friction-less economy with complete markets and unrestricted capital investment ($\phi = 1$). Table 5 reports aggregate output, wages, capital stock, and TFP in these four economies.

6.1.1 Conditional Claims

Introducing conditional claims increases output in the economy in four distinct ways. First, with the ability to insure against entrepreneurial risk, entrepreneurs exclusively choose the high risk project, rather than the low risk project. As the higher-risk project has a higher expected productivity, this boosts aggregate output. Secondly, individuals with higher entrepreneurial ability are more likely to select into entrepreneurship. Third, conditional on wealth, entrepreneurs invest more in their businesses. Finally, because entrepreneurs earn higher incomes as a result of all three previous reasons, they accumulate more wealth and thus are able to run larger businesses, despite the continued presence of borrowing constraints.

Figure 14 shows the amount of capital invested by the most productive individuals in the risky project based on the total cash on hand available to them at the end of a period. In the benchmark economy, because of the substantial capital investment risks entrepreneurs face, they invest less than the expected marginal product of capital. With the introduction of contingent claims, entrepreneurs insure themselves against bad shocks, and so choose to invest more in their businesses. In both economies, relatively wealth-poor individuals are constrained by their personal wealth and borrow up to the exogenous borrowing limit.

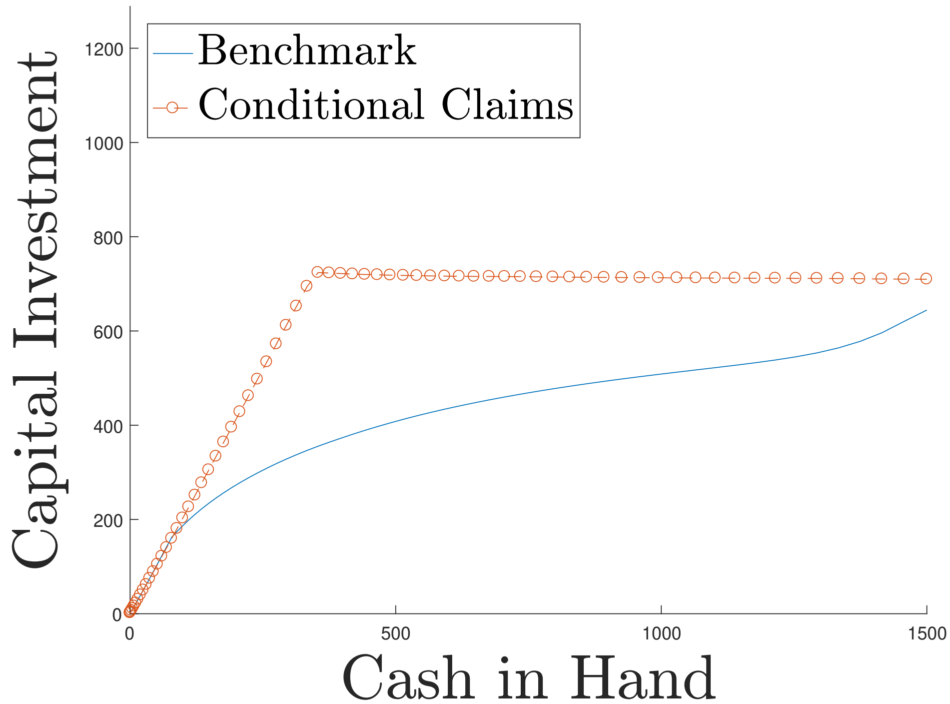
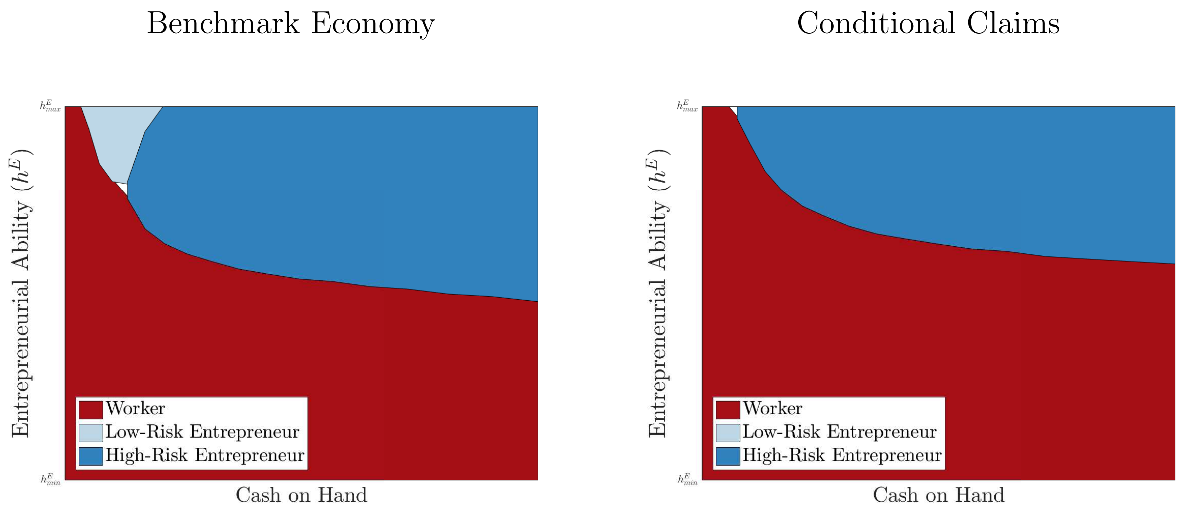


Figure 14: Capital Investment

Figure 15 compares the patterns of occupational choice and endogenous risk choice between the benchmark economy and the economy with conditional claims. In the economy with conditional claims, entrepreneurs all choose the high risk project, because they can insure away the risk of bad outcomes and enjoy the higher expected income in every state of the world.

Figure 15: Comparing Occupational Choice and Endogenous Risk Choice



6.1.2 Unrestricted Capital Investment

Removing borrowing constraints likewise increases output in the same four way as the introduction of contingent claims. First, the removal of borrowing constraints encourage more, but not all, entrepreneurs to start the high risk project. Secondly, individuals with higher entrepreneurial ability are more likely to select into entrepreneurship. Third, conditional on wealth, entrepreneurs invest more in their businesses. Finally, because entrepreneurs earn higher incomes as a result of all three previous reasons, they accumulate more wealth and are thus able to run larger businesses, despite the continued presence of borrowing constraints.

Figure 16 shows the investment patterns for the highest productivity entrepreneur in the benchmark and unrestricted investment economies. Wealth poor individuals in the unrestricted investment economy are able to invest more than those in the benchmark economy because of the lack of borrowing constraint. However, because of the substantial capital investment risk, they still invest less than the expected marginal product of capital.

Figure 17 shows the pattern of selection in the two economies. Note from table 5 that the proportion of entrepreneurs running the high risk project increases after the borrowing constraints are removed. With borrowing constraints, if a wealth-poor entrepreneur starts the high risk project and receives a good shock, they will not be able to operate a large business and earn as much income as a wealthier entrepreneur operating the same project. In this way, the existence of borrowing constraints distort the endogenous risk choice of the entrepreneurs. Without the constraints, low-wealth entrepreneurs are more likely to start the high risk project because they are able to expand faster when they receive good shocks and earn more income. However, given the incomplete market, relatively wealth-poor entrepreneurs still choose the low-risk project.

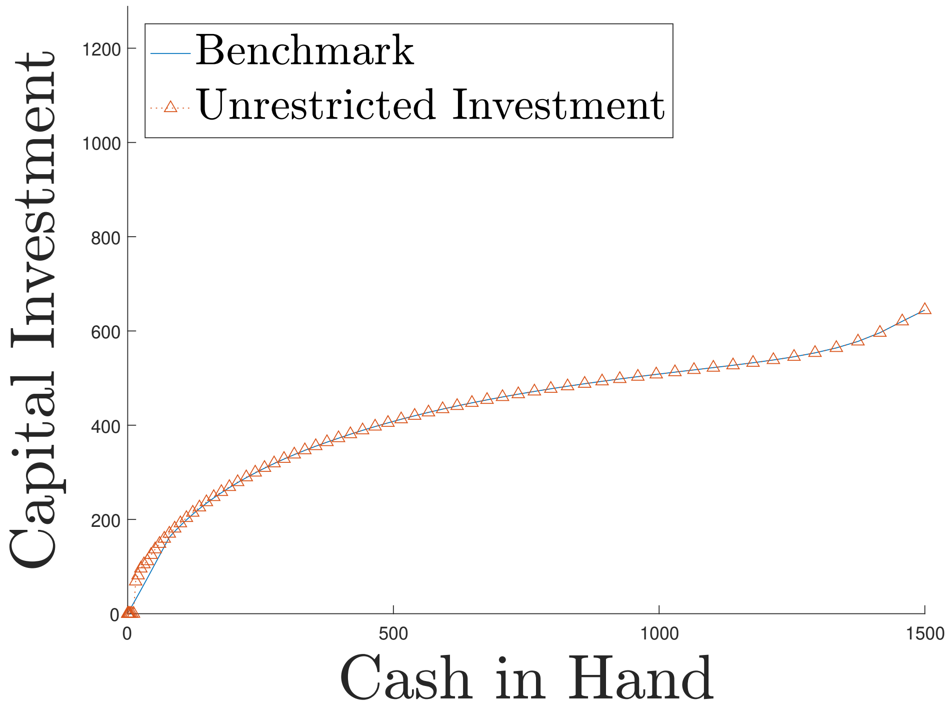
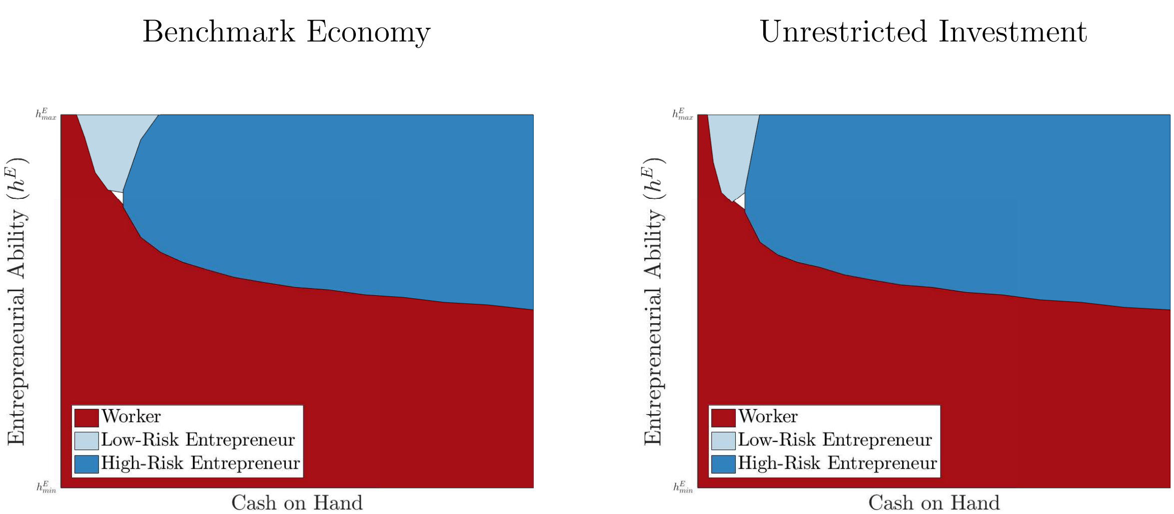


Figure 16: Capital Investment

Figure 17: Comparing Occupational Choice and Endogenous Risk Choice



6.1.3 Both

When both the contingent claims are introduced and capital investment is unrestricted, output again increases through all four channels. Figure 18 shows the investment patterns for the highest productivity entrepreneur in the benchmark economy and the economy with both frictions removed. The availability of contingent claims means that entrepreneurs invest fully up to the expected marginal product of capital, and the lack of borrowing constraints on capital investment, mean that they do so with even less wealth than in the contingent claims economy.

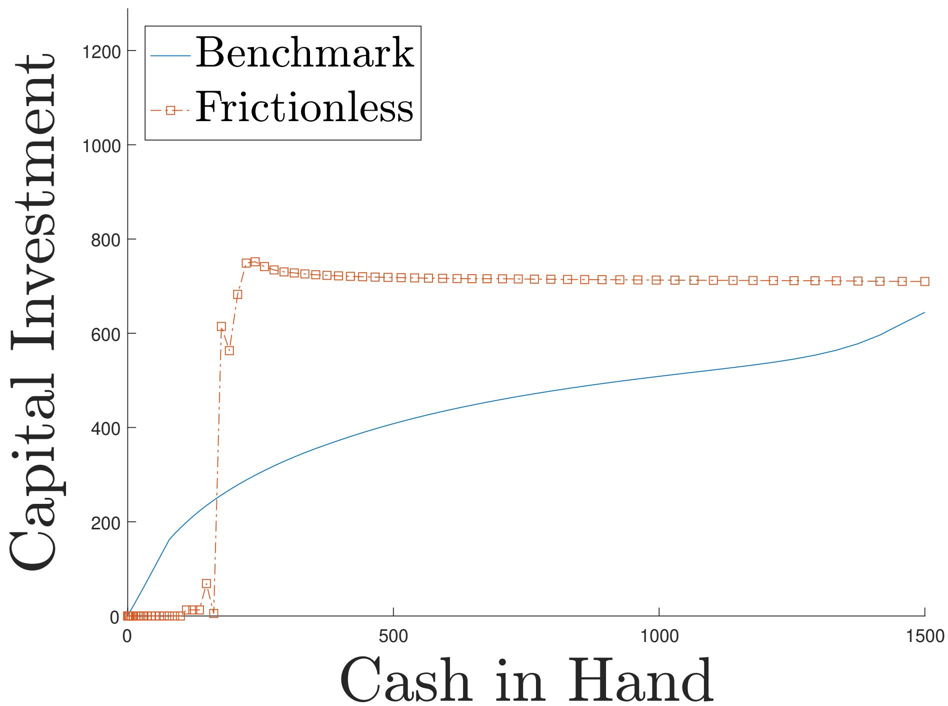
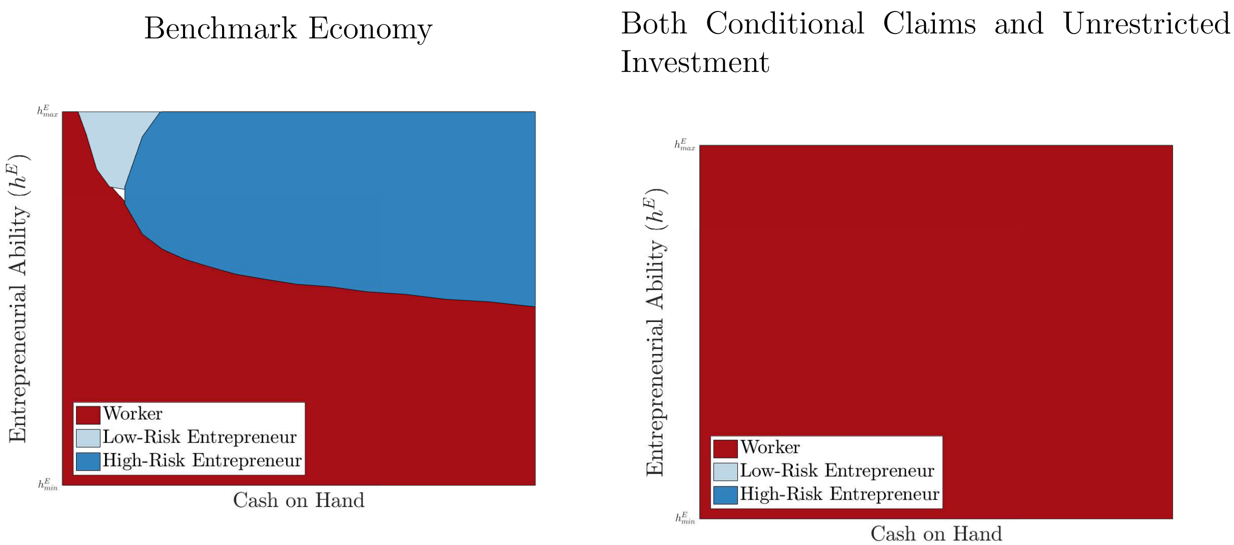


Figure 18: Capital Investment

Figure 19: Comparing Occupational Choice and Endogenous Risk Choice



6.2 Endogenous Risk and Wealth Inequality

In the benchmark economy, risk is endogenous in the sense that entrepreneurs choose between different projects with different risks and rewards. Wealthier entrepreneurs optimally choose higher risk higher reward projects and so in expectation earn higher average returns than poorer entrepreneurs. These higher rates of return constitute a form of rate of return heterogeneity, that can potentially help explain the cross-sectional dispersion in wealth we observe in the data. A key question is how much additional dispersion in wealth this endogenous choice of risk results in. To measure the quantitative importance of this channel, I compare the benchmark endogenous risk economy with an exogenous risk economy, where new entrepreneurs are are not able to choose their project type and instead are randomly allocated a project.

In the benchmark, entrepreneurs strictly prefer one of the project types, so the removal of project choice will strictly reduces the value of entrepreneurship for all agents. Naively comparing two economies one with endogenous project choice and one with exogenous project allocation will then conflate the effects of the endogenous choice of risk with changes in occupational selection. In order to provide a fair assessment of the contribution of the endogenous risk choice to wealth inequality, I compare several different economies. Starting from the benchmark economy, I first consider an economy where entrepreneurs can make an endogenous risk choice but all agents have a fixed occupation, in that they are born as either workers or entrepreneurs and cannot change occupation throughout their lifetime. I then compare this economy to an economy where entrepreneurs are randomly allocated a project with fixed occupations. Table 6 compares the top wealth shares across these economies.

Comparing rows 1 and 2, shows how important the labour market option is for entrepreneurs. In the benchmark economy, when entrepreneurs receive bad shocks they always have the option to return to the labour market. As a consequence, precaution-

Table 6: Occupational Choice, Endogenous Risk Choice and Wealth Inequality

	Wealth Shares of the Top				
	1%	5%	10%	20%	50%
SCF Data	33	57	69	83	94
Occupational Choice Endogenous Risk (Benchmark)	29	64	75	85	95
Fixed Occupations Endogenous Risk	50	68	75	86	102
Fixed Occupations Exogenous Risk	30	66	74	83	97
Occupational Choice Exogenous Risk	24	52	67	80	94

ary savings is much lower than in the Fixed Occupations Endogenous Risk economy. Without the option to go into the labour market, entrepreneurs save at much higher rates. They do so both because they anticipate that at some point they may face a period of low productivity, and the fact that the borrowing constraints will limit future income unless they can self-finance a large stock of capital. As a consequence of these strong savings motives, entrepreneurs save much more than in the benchmark economy. Workers, without the possibility of ever becoming an entrepreneur, save very little. As a consequence, wealth inequality increases sharply along entrepreneur-worker lines.

Keeping the occupational choice fixed, comparing rows 2 and 3 shows how different top-tail wealth inequality looks in a model with endogenous risk vs. exogenous risk. In the endogenous risk economy, wealthy entrepreneurs choose the high risk project, earn high returns on average, and so accumulate wealth much faster than relatively poor entrepreneurs. In the exogenous risk economy, projects are randomly allocated, and so both wealthy and wealth-poor entrepreneurs. Note how there is little difference in the top wealth shares throughout the rest of the distribution. This mechanism mostly has an impact at the top.

Note that I also keep the wage rate constant in these comparison economies, rather than clearing the labour market. This is important as changes in the wage rate can have large impacts on the optimal scale of operations for an entrepreneur with a given productivity and so drastically change their savings behaviour.

7 Conclusion

This paper studies the relationship between wealth and entrepreneurial risk. I present descriptive evidence from the Kauffman Firm Survey that new entrepreneurs face a high degree of idiosyncratic risk, but that many are able to borrow substantial amounts of debt. Motivated by these facts, I build a quantitative model of occupational choice and endogenous risk choice. The model features two frictions, a missing market for entrepreneurial risk and borrowing constraints. I use this model to measure the relative impact of these two frictions on productivity in the US economy. While both frictions reduce aggregate output and productivity, I find that the missing market for entrepreneurial risk is far more important than borrowing constraints. I also use the model to study wealth inequality, showing that the missing market for entrepreneurial risk can also generate substantial wealth inequality in a quantitative model.

Overall, the results in this paper clearly suggest that government policies designed to encourage entrepreneurship should take into account the high degree of entrepreneurial risk that new entrepreneurs face. While providing greater access to loans may help encourage entrepreneurship, designing policy that does this and also promotes better risk-sharing mechanisms is likely to be far more effective. Given the obvious moral hazard and adverse selection problems inherent in entrepreneurial risk sharing, I leave the design of these risk-sharing government policy to future work.

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A Kauffman Firm Survey Facts

A.1 Measuring Survival

In table 7, I report the current operational status of all firms in the Kauffman Firm Survey over the 8 years of the survey. Firm exit is common. While non-response and sample attrition mean that the status of 701 firms are not available in the final year, 1,901 firms or 45% of the firms with known status have permanently shut down by the end of the sample. An additional 30 were temporarily shut down.

Table 7: Business Status

Year	Operating	Shut Down	Merged or Sold	Temp Shut Down	Unknown
0	4,928	0	0	0	0
1	3,998	260	43	66	561
2	3,390	581	90	124	743
3	2,915	880	135	98	900
4	2,606	1,224	175	58	865
5	2,408	1,474	211	41	794
6	2,126	1,692	249	45	816
7	2,007	1,901	289	30	701

A.2 Remaining Assets

Initial financial investments in a business may be used to purchase capital, to cover initial operating losses or to pay fixed set-up costs. Upon business exit, only the funds used to purchase tangible capital are potentially recoverable, and even then liquidation costs are likely to impose losses on the investment. To investigate what fraction of initial investments are potentially recoverable, I examine the ratio of net assets, measured in the last year of operation for exiting firms or in the eight year of operation for surviving firms, to owner's total cumulative investment in the firm. Table 8 displays percentiles of this ratio for both surviving and exiting firms. The table is further broken down by the quintile of initial investment.

The median exiting firm had net assets worth 55% of the total cumulative investment in the firm. Assuming zero liquidation costs, the owners of this firm would have lost 45% of their initial investment plus the return they could have earned on a risk-free asset. Of course, liquidated values, particularly of illiquid assets such as inventories and accounts receivable, would likely be lower than these values. They key insight here is that many exiting entrepreneurs will experience a substantial loss to their financial investment.

A.3 Entrepreneur's Working Hours

Even for those entrepreneurs who are paying themselves a salary, entrepreneurs may not be paying themselves the full opportunity costs of their time. Hall and Woodward (2010) document that venture-capital backed entrepreneurs are typically paid less than

Table 8: Ratio of Net Assets to Cumulative Investment

Quintile	p10	p25	p50	p75	p90	n Firms
Survivors						
1	0.00	0.09	0.49	2.00	7.42	229
2	0.00	0.12	0.73	3.37	16.94	425
3	0.01	0.18	0.68	1.81	5.09	271
4	0.00	0.14	0.60	1.75	4.20	269
5	0.01	0.11	0.45	1.37	3.00	328
All Survivors	0.00	0.12	0.60	1.96	6.17	1,522
Exiters						
1	-0.02	0.00	0.36	1.99	5.90	133
2	0.00	0.10	0.64	2.31	6.80	458
3	0.00	0.11	0.68	2.00	6.63	282
4	0.00	0.15	0.60	1.50	3.12	273
5	0.00	0.10	0.44	1.21	4.39	239
All Exiters	0.00	0.10	0.55	1.79	5.80	1,385

This table shows percentiles on the ratio of net assets to cumulative investment. Net assets are the sum of all self-reported firm assets less the self-reported firm liabilities. The top half of the table shows statistics for the distribution of surviving firms separately based on the five quintiles of first year equity invested. Likewise the bottom half of the table shows statistics for the distribution of exiting firms separately based on the five quintiles of first year equity invested.

their outside option in the labour market in order to encourage effort. More generally, if borrowing constraints are binding, entrepreneurs could pay themselves less in order to save within the firm and accumulate additional capital. In both cases, lower compensation during the start up period is compensated for by higher returns later. Of course, if the firm exits before those returns materialize, the entrepreneur suffers a real economic loss. Note here the potential interaction of borrowing constraints and the missing market for entrepreneurial risk, which will be explored in section 6.

Tables 10 and 11 compare the proportion of entrepreneurs who report paying themselves a salary based on their weekly hours of work in the first and final years. Only a third of the entrepreneurs working less than 25 hours a week are paying themselves a salary. For entrepreneurs working more than 35 hours a week, 57% are paying themselves a salary in the first year, while 68% are paying themselves a salary in the eighth year. Hours worked are self-reported usual hours worked. Note that both of these graphs include only the survey-respondent entrepreneur, rather than all of the entrepreneurs working on a business.

A.4 Detailed Breakdown of Funding Sources

New firms in the Kauffman Firm Survey raise their initial funds from a variety of sources. Table 12 updates a similar table in Robb and Robinson (2014) with the full sample of KFS data. By far the most common source of funding is the entrepreneurs' own investment. For 89% of firms, entrepreneurs are putting their own money into

Table 9: Entrepreneurs Paying Themselves a Salary by Year

Year	Prop Salaried	n Entrepreneurs
2004	0.47	6,916
2005	0.53	5,673
2006	0.55	4,776
2007	0.55	4,057
2008	0.53	3,617
2009	0.54	3,304
2010	0.53	2,859
2011	0.51	2,715

Table 10: Entrepreneurs Paying Themselves a Salary by Hours Worked, First Year

Hours	Prop Salaried	n Entrepreneurs
<25	0.33	1,288
25-35	0.43	442
35-44	0.54	655
45-54	0.56	758
55-65	0.58	887
65<	0.58	780
Total	0.49	4,900

the business, these investments are often modest, with the median amount being only $25,000.

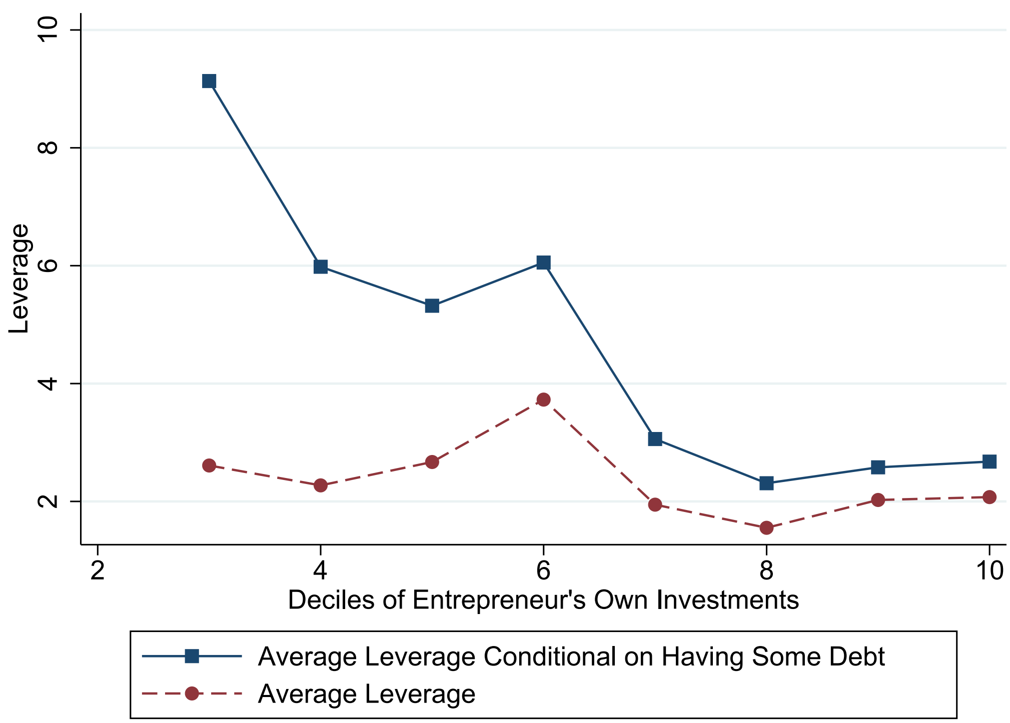
Note that while 53% of firms are able to borrow from a bank, most of these loans are personal debt. If the business is unsuccessful and entrepreneurs choose to exit, these debts cannot be discharged without filing for personal bankruptcy. Raising external equity is much less common, with only 15% of firms raising equity beyond the actively managing owners, though these firms typically invest substantially more than those without any external equity.

Figure 20 shows the average leverage ratio of firms. I exclude the bottom two deciles of initial investment for legibility. Conditional on having some debt, the leverage ratio decreases sharply over the distribution of initial investment. Given the the proportion of firms who take out some debt increases sharply over the distribution of initial investment (see figure 8), the unconditional mean leverage ratio is relatively flat, around 2.

Table 11: Entrepreneurs Paying Themselves a Salary by Hours Worked, Eighth Year

Hours	Prop Salaried	n Entrepreneurs
<25	0.33	569
25-35	0.52	174
35-44	0.62	303
45-54	0.68	344
55-65	0.73	313
65<	0.70	182
Total	0.56	1,892

Figure 20: Leverage



This graph shows the debt to equity ratio across the distribution of initial investment. Firms are sorted into deciles based on the total amount of equity invested in the firm in the first year of operations.

Table 12: Funding Sources

Source	%	Mean >0	p25	p50	p75	n Firms
Own Equity	89	166	8	25	80	3,655
External Equity	15	1,151	10	50	203	640
Outside Investors	6	758	10	90	290	247
Parents	5	83	10	25	73	242
Other Companies	4	1,565	20	100	600	154
Spouses	3	82	5	15	30	137
Venture capital	1	8,669	85	450	6,125	60
Government Agencies	1	698	53	250	850	44
Other	1	995	10	25	135	23
Any Debt	53	714	15	55	210	2,302
Personal Debt	46	405	10	40	128	2,011
Bank	32	308	15	50	140	1,413
Family	20	51	5	15	46	938
Other Individuals	6	434	4	15	50	252
Any Other Sources	3	1,489	10	33	110	160
Business Debt	28	701	15	50	211	1,264
Bank	17	873	26	84	269	747
Family	8	84	5	15	45	356
Non-Bank Financial	6	357	12	48	200	262
Owners	4	331	15	48	200	158
Government Agencies	2	1,052	30	125	330	89
Other Individuals	2	153	5	24	100	74
Employees	1	56	5	14	40	38
All Funding Sources	94	520	10	45	181	3,687

This table shows the funding sources of firms. The first column reports the percentage of firms that received some money from each possible funding source, the second column gives the mean amount raised conditional on raising some money from that source, the third through fifth columns give the 25^{th}, 50^{th} and 75^{th} percentiles of the amount raised conditional on raising some money from that source. The final column gives the number of firms this corresponds to. All values are in thousands of US dollars and include investment across the first 8 years of operation.

A.5 Dispersion in Cumulative Sales

In subsection 3.3, I showed that the level and dispersion of cumulative profits are significantly higher for high initial own investment firms than for low initial own investment firms. In this subsection, I test for the level and dispersion of cumulative sales, using regressions 15 and 16.

$$\text{Cumulative Sales}_i = \alpha_0 + \alpha_1 \text{Initial Owner Investment}_t + \alpha X_i + \epsilon_i \tag{15}$$

$$|\hat{\epsilon}_i| = \beta_0 + \beta_1 \text{Initial Owner Investment}_t + \beta X_i + \eta_i \tag{16}$$

Table 13 shows that firms with higher initial own investments have significantly higher cumulative sales, even after controlling for initial employment, external sources of funding and industry. In the second regression, the point estimate is positive, indicating that firms with higher initial own investments have more dispersed cumulative sales, though it is only significant at the 10% level.

Table 13: Dispersion of Cumulative Sales Regressions

First Regression: Cumulative Sales			
	(1)	(2)	(3)
Entrepreneur's Initial Own Equity	3.729***	3.647***	3.647***
	(0.251)	(0.251)	(0.251)
Initial External Equity			-0.0197
			(0.588)
Initial Debt		1.617	1.601
		(1.293)	(1.300)
Employer		-197.8	-185.7
		(898.2)	(899.9)
Number of Employees		647.1***	649.4***
		(76.17)	(76.32)
Constant	3109.2***	377.7	371.5
	(407.9)	(4616.5)	(4620.2)
2 Digit NAICs Code	No	Yes	Yes
Second Regression: Absolute Value of Cumulative Sales			
	(1)	(2)	(3)
Entrepreneur's Initial Own Equity	0.668**	0.562*	0.561*
	(0.246)	(0.241)	(0.241)
Initial External Equity			0.235
			(0.564)
Initial Debt		0.708	0.648
		(1.239)	(1.246)
Employer		-53.34	-44.55
		(860.6)	(862.2)
Number of Employees		644.7***	646.0***
		(72.98)	(73.12)
Constant	4943.5***	809.3	774.4
	(400.1)	(4423.2)	(4426.4)
2 Digit NAICs Code	No	Yes	Yes
N	4112	4017	4012

A.6 Dispersion in Rates of Return

In subsection 3.3 and A.5, I showed that the level and dispersion of cumulative profits and cumulative sales are significantly higher for high initial own investment firms than for low initial own investment firms. In this subsection, I attempt to calculate firm level rates of return in order to provide an additional test of the dispersion in firm outcomes.

I construct cumulative rates of return for each firm using the following equation:

$$\text{Cumulative Rate of Return} = \frac{\sum_{t=0}^{7} \beta^t \left(\text{Dividends}_t + \text{Salary}_t\right) + \beta^7 \text{Firm Value}_7}{\sum_{t=0}^{7} \beta^t (\text{Equity Invested}_t + \text{Forgone Salary}_t)} \tag{17}$$

It is vital to include salaries in this calculation as they represent part of the return to starting a business. The split between dividends and salaries is more likely to depend on tax code provisions or incentive compatible contracts with passive owners, rather than the relative economic value of the entrepreneur's labour and financial capital to the business. In addition, including forgone labour earnings is essential in this calculation. The entrepreneurs in this sample are highly heterogeneous. Some operate small-scale firms with minimal assets and require little financial investment. Without including the opportunity cost of their time, the estimate of their rate of return would be inflated by the missing factor. I measure or proxy for each element of this equation in the following way:

- Dividends_t: Dividends are directly measured in the data by a question that asks "Thinking of calendar year 2004, how much money, if any, did you and other owners withdraw from the business for personal use? This includes any dividends paid." it should therefore include all dividend payments as well as any other cash withdrawals from the business.
- Salary_t: Information about the exact salaries paid to entrepreneurs is not available, though whether a salary was paid to each entrepreneur is recorded. In order to proxy for salaries paid, I use the total wage bill of the firm, divided by the number of employees including the salaried entrepreneurs.
- Firm Value_7: For a termination value of the firm, I use the total assets of the firm minus the total liabilities in the final year.
- Equity Invested_t: Equity invested is directly measured in the data by a series of questions that ask how much money was received from active owners, angel investors, other companies, governments, parents of owners, spouses of owners, venture capital firms and an other category.
- Forgone Salary_t: No information about the previous labour market activities of these entrepreneurs is available. In order to proxy for their forgone labour earnings, I run a Mincerian regression in the Survey of Consumer Finances estimating total annual labour market earnings on demographics. I then use the coefficients from this regression to predict annual labour market earnings for the entrepreneurs in the Kauffman Firm Survey. I then multiply these predicted annual labour market earnings by entrepreneur's reported weekly hours of work.
- I discount all values using $\beta = \frac{1}{1.02}$.

To validate these rates of return, I compare survival frequencies of firms in different deciles of cumulative profits to survival frequencies of firms in different deciles of cumulative rate of return. If the rates of return are accurately capturing entrepreneurial success, then the survival frequencies should be much more closely aligned with the rates of return than the profits.

Table 14: Survival by Cumulative Profits and Cumulative Rate of Return

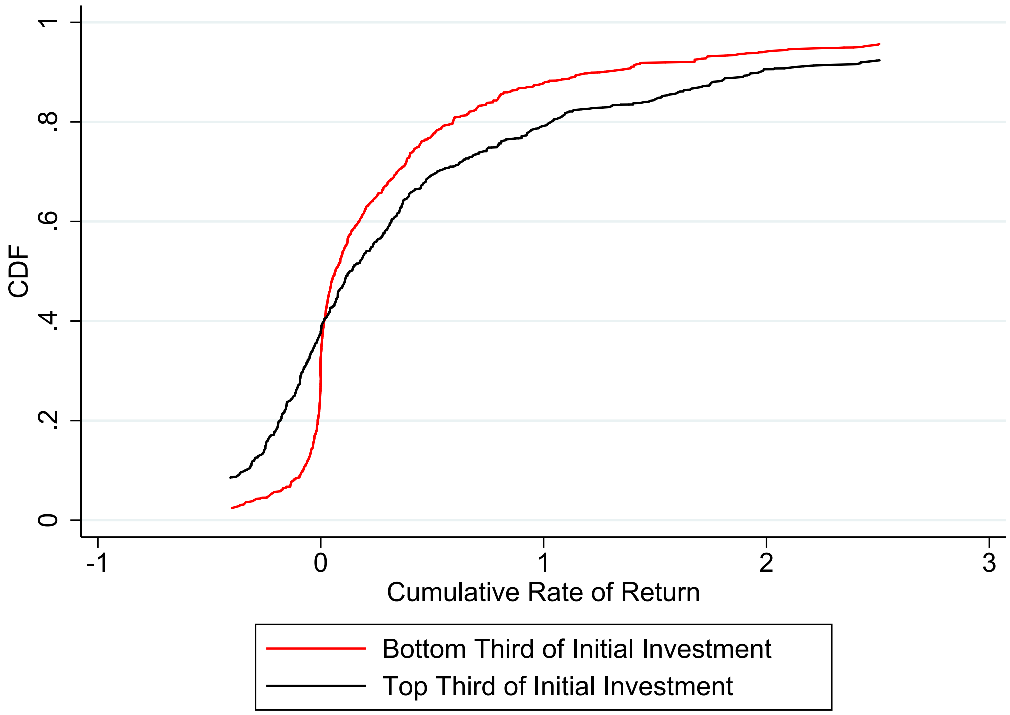
Decile	Survival by Profit decile	n Firms	Survival by RoR decile	n Firms
1	0.57	321	0.27	234
2	0.43	343	0.42	247
3	0.34	344	0.42	241
4	0.28	432	0.49	242
5	0.29	230	0.49	238
6	0.48	330	0.49	237
7	0.59	328	0.53	240
8	0.74	323	0.46	224
9	0.78	342	0.60	219
10	0.90	335	0.55	205

In table 14, it is clear that the calculated rates of return is a much better measure of business success at the bottom of the distribution than profits. While the majority of firms with negative profits are exiting, some of those that remain have relatively high rates of return, generally because they are accumulating substantial assets while earning negative profits. At the top, the rate of return measure is much less successful than cumulative profits at predicting survival.

If these entrepreneurs who are making larger initial investments are really taking on more risk, their rates of return should be more dispersed. Successful entrepreneurs should earn higher rates of return while unsuccessful entrepreneurs should earn lower rates of return. Comparing the distribution of returns should be informative about the relative levels of risk. I compare the distribution of rates of return for the top third of entrepreneur's own initial investments to the bottom third of this measure.

Figure 21 plots the cumulative density of the returns of the firms in the bottom third of initial investment against those in the top third of initial investment. Higher initial investment firms are earning more dispersed rates of return. On the far left, there is a greater mass of high initial investment firms that are earning negative rates of return, while on the far right, there are more high initial investment firms that are earning high rates of return. I interpret this as suggestive evidence that entrepreneurs who are investing more in their businesses are also taking on more risk.

Figure 21: Dispersion in Rates of Return



This figure shows the cumulative distribution of cumulative returns for firms. It plots the distribution of returns for firms in both the bottom third and top third of the distribution of owner's initial investment. The figure is truncated at the top and bottom 5% of returns.

A.7 Entrepreneur's Biggest Challenge

In the 5th through 8th years of operation, entrepreneurs were asked "What was the most challenging problem your business faced in calendar year X?". The responses are provided in table 15. The responses suggests that most entrepreneurs are far more concerned about the risks they face than any lack of credit. Unfortunately, as this question was only asked for the years from 2008-2011, it is not clear how much the responses to this question are driven by the US financial crisis. Yet as economic conditions recover after 2008, there is no increase the number of entrepreneurs who consider the lack of credit to be their primary challenge, which suggests that for many firms binding borrowing constraints are not a major concern for entrepreneurs only 8 years after starting a business.

A.8 Firm Growth

Table 15: Entrepreneur's Self-Reported Most Challenging Problem

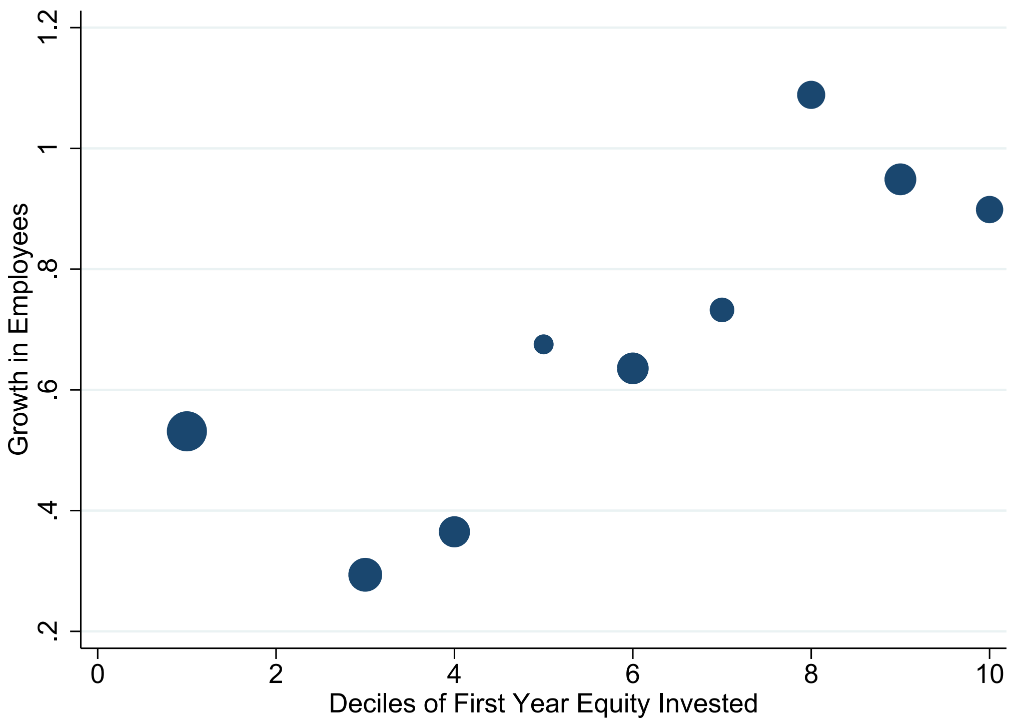
Year	2008	2009	2010	2011
An inability to obtain credit	4 %	5 %	4 %	4 %
Slow or lost sales	53 %	45 %	42 %	35 %
Falling real estate values	5 %	5 %	4 %	4 %
The cost and/or terms of credit	2 %	2 %	1 %	1 %
The unpredictability of business conditions	24 %	23 %	26 %	31 %
Some other problem	11 %	13 %	15 %	16 %
Customers or clients not making payments or paying late	2 %	8 %	8 %	8 %
Number of Entrepreneurs	2,566	2,369	2,094	1,971

Figure 22 shows that firms with larger initial investments in the first year experience more growth in employment conditional on surviving to the eighth year of operation. This simple fact stands in stark contrast with standard models of firm dynamics and borrowing constraints based on Cobb-Douglas production functions. Because the marginal product of capital is high at low levels of capital, entrepreneurs operating sub-optimally small firms have strong incentives to save out of their borrowing constraints and so rapidly increase size.

Figure 23 shows the average growth rate of net assets across the distribution of initial equity investment. As in the previous graph, I sort firms into 10 deciles based on the total equity invested in the firm in the first year. I then compare the net assets of all surviving firms in the eighth year of operation to their net assets in the first year of operations.

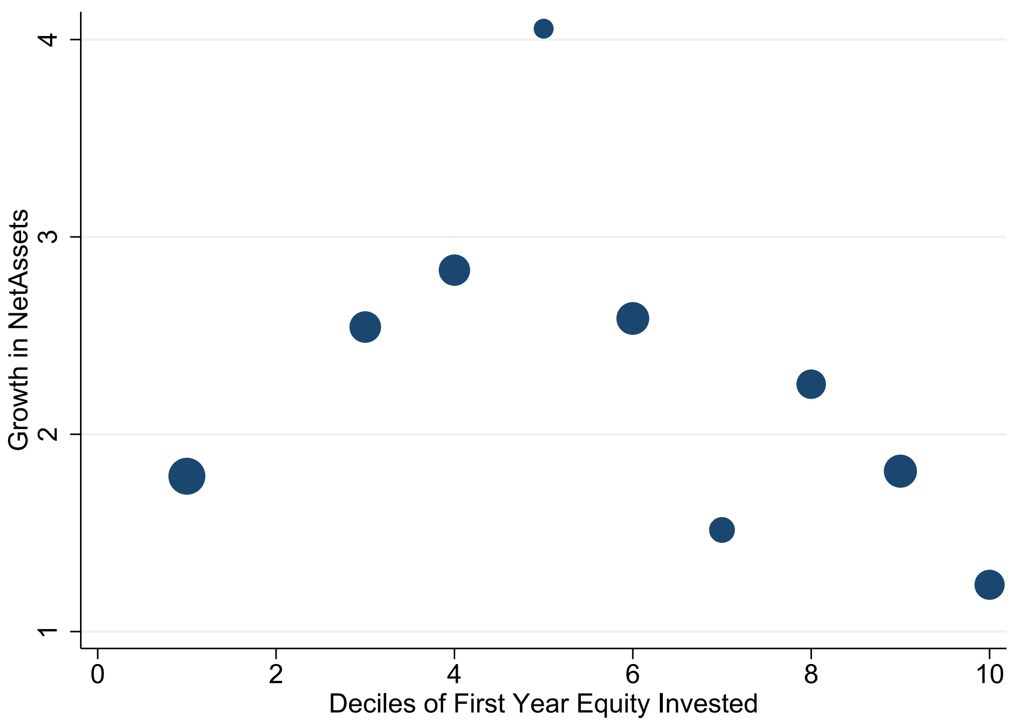
Classic models of entrepreneurship and borrowing constraints, such as Cagetti and De Nardi (2006) and Buera (2009), suggest that entrepreneurs who are financially constrained will operate firms at sub-optimal scale. As they operate, they should save out of retained earnings to accumulate firm assets. Thus a key measure of which entrepreneurs are most borrowing-constrained are the firms that ex-post have accumulated assets at the fastest rate.

Figure 22: Employment Growth by Initial Investment



This table shows the average growth of the employment of firms across the distribution of initial investment. Firms are sorted into deciles based on the total amount of equity invested in the firm in the first year of operations. Note that the growth rates have been Winsorized to remove the effect of outliers.

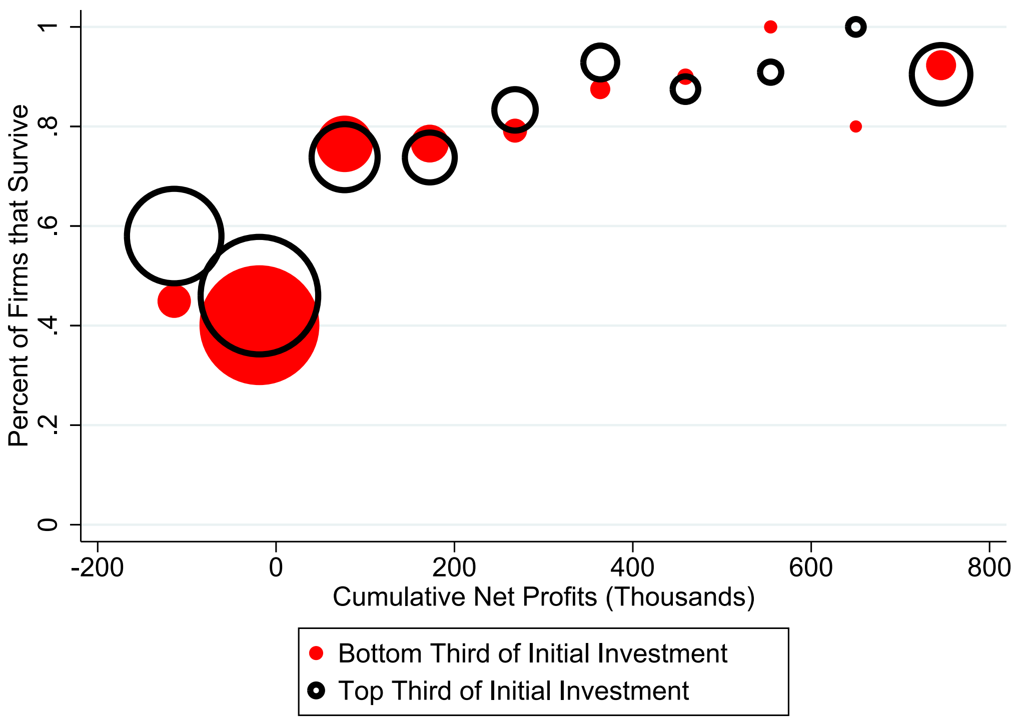
Figure 23: Net Asset Growth by Initial Investment



This graph shows the average growth of firm net assets across the distribution of initial investment. Firms are sorted into deciles based on the total amount of equity invested in the firm in the first year of operations. Note that the growth rates have been winsorized to remove the effect of outliers.

A.9 Survival by Cumulative Profits

Figure 24: Survival by Cumulative Profits



This figure shows the survival frequency of firms in ten bins of cumulative net profits for two groups of firms in thousands of 2004 US dollars. The size of each circle is proportional to the number of observations in that bin. The firms are the top and bottom third of the distribution of initial investment, defined as the total amount of the entrepreneur's own money they invest in the business. This initial investment does not include debt or other sources of investment, such as angel investors, investments from businesses, entrepreneur's family investments, or venture capital funding. Both distributions have been winsorized at the top and bottom 5% of the total distribution of cumulative profits.

B Survey of Consumer Finances Facts

In this section I document several facts from the Survey of Consumer Finances in regards to the relationship between entrepreneurship and wealth. I focus on the year 2004, as that is the year in which all of the businesses in the Kauffman Firm Survey are started. As such, it is the relevant wealth distribution to study selection into entrepreneurship based on wealth.

B.1 Wealth Moments

In section 5, I calibrate the model to several wealth inequality moments taken from the 2004 Survey of Consumer Finances.

In the SCF data, I define entrepreneurs by active business ownership. Active business owners must own or shares ownership of a business that they also actively manage. As this paper does not study passive business ownership, I drop households from the sample if they are passive business owners, unless they they own multiple businesses and actively manage at least one of them. As there is no retirement savings motive in the model of section 4, I drop households of retirement age. For single person households, I drop the household if the individual is older than 65. For two person households, I drop the household if the average age of the two individuals is greater than 65.

In table 16, I show that the top wealth shares in the US economy do not change substantially as a consequence of dropping either retired households or passive business owners. The first column shows the top wealth shares for the whole SCF sample, designed to be representative of the US population. The second drops households of retirement age. The third column drops all passive business owners, who are not also active business owners. While the fourth drops both households of retirement age and the passive business owners. I use this fourth column as the relevant comparison for my model's wealth distribution.

Table 16: Top Wealth Shares

	Full Sample	Drop > 65	Drop PBO	Drop Both
Top 1%	33.2%	33.5%	31.1%	32.1%
Top 5%	57.5%	57.4%	55.6%	56.0%
Top 10%	69.5%	68.6%	67.8%	67.1%
Top 20%	82.9%	82.4%	81.9%	81.4%
Top 50%	97.5%	97.1%	97.3%	97.0%
Wealth Gini	0.79	0.80	0.78	0.79

This table documents top wealth shares in the US economy. "Drop > 65" refers to the sub-sample after dropping all households aged over 65. "Drop PBO" refers to the sub-sample after dropping all households that own a business they do not actively manage without also owning a business they actively manage. "Drop Both" refers to the sub-sample after imposing both the age and passive business ownership restrictions.

In table 17, I show that two of the key targeted model moments do depend on the sub-sample chosen. Since both retired households and passive business owners own a substantial amount of wealth, their inclusion makes the wealth differences between entrepreneurs and non-entrepreneurs look less stark. Again, I use the statistics from the fourth column for the calibration of my model.

Table 17: Model Moments by Subsample

	Everyone	Drop > 65	Drop PBO	Drop Both
Wealth Ratio of Entrepreneurs to Everyone Else	5.70	6.29	6.42	7.14
Proportion of Entrepreneurs in Wealthiest 1%	0.60	0.65	0.69	0.74

B.2 Wealth To Income Ratios

One key empirical fact that has motivated the study of borrowing constraints in entrepreneurship, is the fact that entrepreneurs have higher wealth-to-income ratios than workers. Quadrini (1999) table 1 regresses the wealth to income ratios of households in the PSID and the SCF on a binary variable for entrepreneurship, the level of income, and a polynomial in age. He finds that the coefficient on the entrepreneurship dummy to be positive and significant, suggesting that entrepreneurial households have on average higher wealth-to-income ratios than other households. The first column of table 18 replicates this result with data from the Survey of Consumer Finances.

Table 18: Wealth To Income Ratios of Entrepreneurs

	(1) $\frac{\text{Wealth}}{\text{Income}}$	(2) $\frac{\text{Non-Bus Wealth}}{\text{Income}}$	(3) $\frac{\text{Fin Wealth}}{\text{Income}}$
Entrepreneur	13.2912***	2.1269	0.4986
	(1.4984)	(1.2048)	(0.4788)
Income	Yes	Yes	Yes
Age FEs	Yes	Yes	Yes
Educ FEs	Yes	Yes	Yes
Observations	4498	4498	4498

Standard errors in parentheses

* $p < 0.05$, ** $p < 0.01$, *** $p < 0.001$

However, the quantitative significance of this result is somewhat questionable. As documented in DeBacker et al. (2018), entrepreneurial earnings are much more volatile than labour earnings. The following simple numerical exercise illustrates how more volatile incomes can also generate this result.

Consider an economy populated by 2 workers and 2 entrepreneurs. All agents have the same level of wealth, equal to 3. All agents have the same expected income, equal to 1. Worker's income is certain and equal to one. However, entrepreneurs face some risk, and half the time receive 0.5 and half the time receive 2. Calculating the average of the wealth-to-income ratios for the two types of workers will give:

$$\overline{W/I}^{\text{Worker}} = \frac{1}{2}\left[\frac{3}{1} + \frac{3}{1}\right] = 3$$

$$\overline{W/I}^{\text{Entrepreneur}} = \frac{1}{2}\left[\frac{3}{0.5} + \frac{3}{1.5}\right] = 4$$

Simply because the entrepreneurs have more volatile income, their average wealth-to-income ratio is much higher than the workers, despite the fact that both groups have the same average income and average wealth. To determine whether this arithmetic consideration drives the regression result, table 19 compares the average of individual level wealth-to-income to the group's ratio of average wealth to average income for selected percentiles of the wealth distribution.

$$\overline{W/I} = \frac{1}{n}\sum_{i=1}^{n}\frac{W_i}{I_i}$$

$$\bar{W}/\bar{I} = \frac{\frac{1}{n}\sum_{i=1}^{n} W_i}{\frac{1}{n}\sum_{i=1}^{n} I_i}$$

Table 19: Individual vs. Group Level Wealth-to-Income Ratios

	$\overline{W/I}$		$\bar{W}/\bar{I}$	
	Entrepreneurs	Others	Entrepreneurs	Others
	Indiv E	Indiv W	Group E	Group W
99.5% - 100%	57.9	39.0	16.9	15.4
99% - 99.5%	48.9	27.0	15.3	11.8
95% - 99%	19.7	25.9	11.1	11.8
90% - 95%	15.9	14.5	8.9	9.3
80% - 90%	9.9	12.0	6.6	6.8
60% - 80%	5.9	7.0	3.9	4.0
40% - 60%	3.6	3.3	2.2	2.0
20% - 40%	1.3	1.2	0.8	0.7
0% - 20%	-0.3	-0.2	-0.3	-0.2
Total	12.4	4.8	6.4	2.7

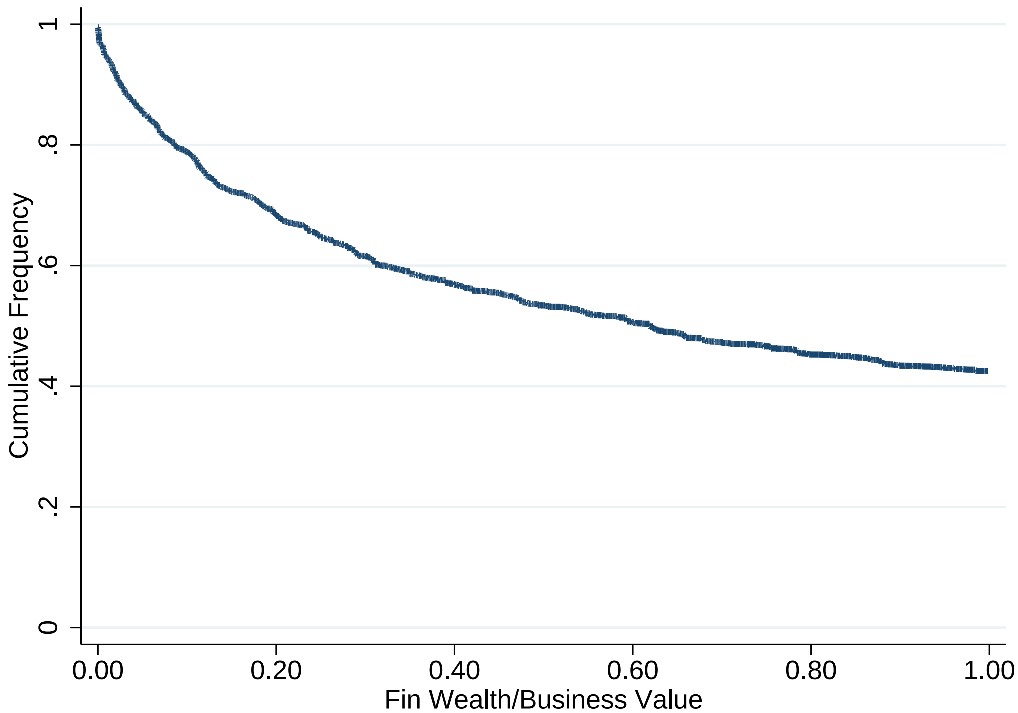
The wealth-to-income ratios of entrepreneurial households look much closer to other households when calculated by the group's ratio of average wealth to average income. This suggests that the more volatile nature of entrepreneurial income is at least one part of the explanation of this result that entrepreneurs typically have higher wealth-to-income ratios.

B.3 Entrepreneurial Wealth Composition

If an entrepreneur is borrowing constrained, they should have relatively few financial assets outside their business. If borrowing constraints raise the cost of external financing, entrepreneurs should liquidate most of their financial assets in order to fund their business internally. One measure of the strength of borrowing constraints that entrepreneurs face is then the proportion of their assets they keep outside the business.

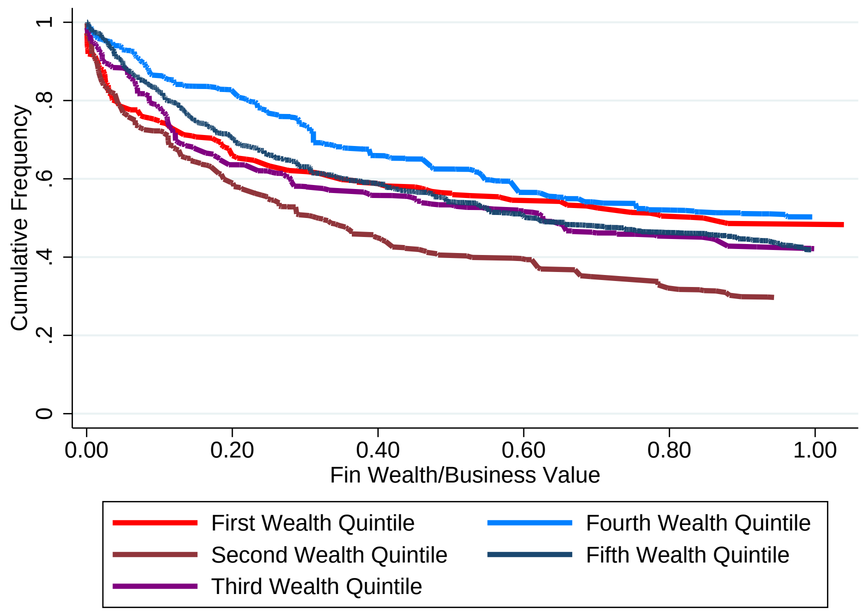
To assess this, I calculate the ratio of financial wealth to business value for each entrepreneur. Entrepreneurs who are borrowing constrained, and wish to expand the scale of their business should have low values of this ratio, as they should liquidate financial assets in order to invest in their business. Figure 25 shows the distribution of this ratio for all entrepreneurs. I plot the inverse CDF, so that the far left side of the graph shows that 100% of entrepreneurs could liquidate financial assets worth 1% of their business and the far right side of the graph shows that just over 40% of entrepreneurs could liquidate financial assets worth 100% of their business. While there are certainly some entrepreneurs who have few financial assets outside of their business, the vast majority have substantial financial assets outside of their business. Figures 27 and 26 break down this graph by quintiles of business value and quintiles of net worth respectively.

Figure 25: Cumulative Frequency of the Ratio of Financial Wealth to Business Value



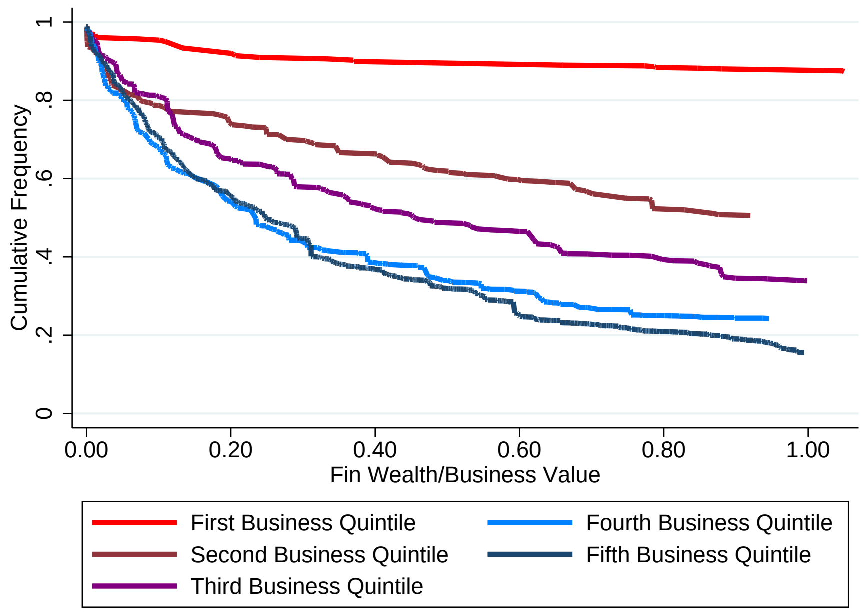
This figure shows what proportion of entrepreneurs would be able to liquidate their financial investments and make an x% investment in their firm. The horizontal axis plots the ratio of financial assets to business value and the vertical axis plots the inverse CDF.

Figure 26: Cumulative Frequency of the Ratio of Financial Wealth to Business Value, by Net Worth



This figure shows what proportion of entrepreneurs would be able to liquidate their financial investments and make an x% investment in their firm. The horizontal axis plots the ratio of financial assets to business value and the vertical axis plots the inverse CDF. Each of the five lines corresponds to a different quintile of networth.

Figure 27: Cumulative Frequency of the Ratio of Financial Wealth to Business Value, By Business Value



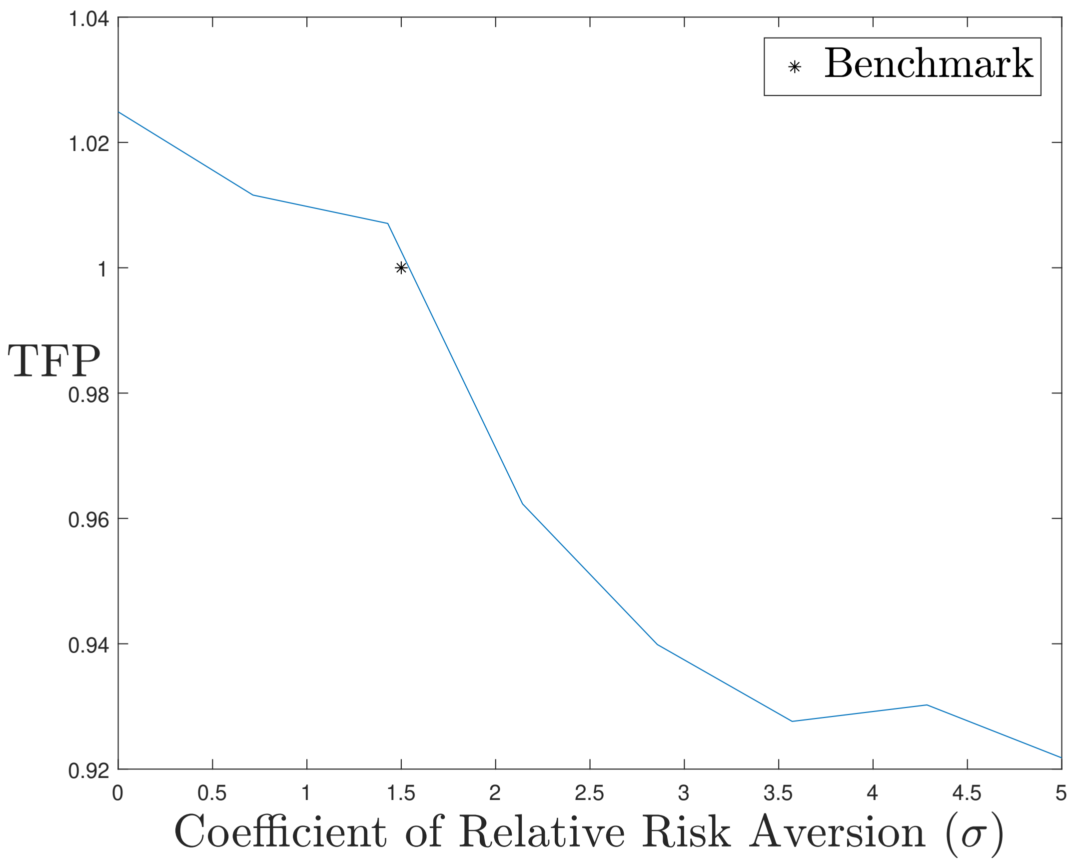
This figure shows what proportion of entrepreneurs would be able to liquidate their financial investments and make an x% investment in their firm. The horizontal axis plots the ratio of financial assets to business value and the vertical axis plots the inverse CDF. Each of the five lines corresponds to a different quintile of the value of businesses.

C Quantitative Robustness

C.1 Robustness: Coefficient of Relative Risk Aversion

Of course, the importance of the missing market for entrepreneurial risk depends critically on the value of risk-aversion used in the economy. In figure 28, I plot the TFP of the benchmark economy and how this varies over different values of σ, the coefficient of relative risk aversion. Aggregate productivity declines as the agents that populate the economy become more risk-averse. This operates primarily through the endogenous choice of risky projects, as agent's risk aversion increases, fewer agents are willing to start a high-risk project with high expected productivity. As a consequence, more agents operate small-scale safe projects.

Figure 28: Aggregate Productivity and the Coefficient of Relative Risk Aversion



D Computational Algorithm

D.1 Decision Rules in the Benchmark Economy

The goal is to solve for the value functions $\{V^W, V^{NE}, V^E\}$, occupational choices, and the set of policy functions $\{c^W, a'^W, \tilde{a}^{NE}, \tilde{k}^{NE}, x^{NE}, c^E, a'^E, k'^E, n^E\}$ to maximize the problems given by 12, 13 and 14.

I proceed by adapting an algorithm from Dyrda and Pugsley (2018). To aid the computational tractability of the problem, I solve for the decision rules on a grid of resources r, rather than on both a and k. I solve first for the value functions, conditional on which occupation will be chosen at the beginning of the next period and then maximize over these occupational choices.

Given a wage w, I solve for the decision rules by:

- Initial Guess:
  - Set $i = 0$,
  - Guess $c_i(\dot{})$ is a constant fraction of total resources
  - Guess $r'_{i,0}(\cdot) = r$
  - Guess $V_i(\cdot) = \frac{c_i^{1-\frac{1}{\theta}}}{1-\frac{1}{\theta}}/(1-\psi\beta)$
  - Guess all agents choose to stay in their current occupation in the next period
- Begin Value Function loop
  - $i = i + 1$
  - Calculate the derivatives of each value function $V_i^E(\cdot), V_i^{NE}(\cdot), V_i^W(\cdot)$ w.r.t. total resources r
  - Use the envelope condition to update current consumption:

$$c_i^E = \left(\frac{\partial V_i^E(\cdot)}{\partial r}\right)^\theta$$

$$c_i^{NE} = \left(\frac{\partial V_i^{NE}(\cdot)}{\partial r}\right)^\theta$$

$$c_i^W = \left(\frac{\partial V_i^W(\cdot)}{\partial r}\right)^\theta$$

  - Set $j = 0$
  - Begin Portfolio Allocation loop:
    * $j = j + 1$
    * For each future state of the world m, calculate $\lambda_m = \frac{\partial V'}{\partial r'}|_{r'_{i,j}(m)}$
    * Use the FOC w.r.t. k' to determine the optimal level of k'

$$k'_{i,j} = \frac{\left[\sum_m \text{Prob}_m \lambda_m (z_m h_m^E)^{\frac{1-\gamma}{\alpha+(1-\gamma)(1-\alpha)}}\right]^{\alpha+\frac{(1-\gamma)(1-\alpha)}{1-\gamma}} \left(\frac{1-\alpha}{w}\right)^{\frac{\gamma(1-\alpha)}{(1-\gamma)}} \gamma^{\frac{1}{1-\gamma}} \alpha^{\alpha+\frac{(1-\gamma)(1-\alpha)}{1-\gamma}}}{\sum_m \text{Prob}_m \lambda_m (\delta + r^A)^{\alpha+\frac{(1-\gamma)(1-\alpha)}{1-\gamma}}}$$

* If that level of k' exceeds the entrepreneur's ability to borrow, reduce it to the maximum amount consistent with the borrowing constraint and the current level of consumption:

$$k'^{E}_{i,j} = \min\left(k'_{i,j}, \frac{r - c_i^E - \underline{a}}{1-\phi}\right)$$

$$k'^{NE}_{i,j} = \min\left(k'_{i,j}, \frac{r - f - c_i^{NE} - \underline{a}}{1-\phi}\right)$$

* Use the budget constraint to determine the resulting saving or borrowing constraint in a':

$$a'^{E} = r - c_i^E - k'^{E}_{i,j}$$

$$a'^{NE} = r - f - c_i^E - k'^{NE}_{i,j}$$

* Update next period resources for each future state of the world m:

$$r'_{i,j}(m) = (z_m h_m^E)^{1-\gamma}(k^\alpha n_m^{1-\alpha})^\gamma - wn_m + (1-\delta)k'_{i,j} + a'(1+r^A)$$

where $n_m = (z_m h_m^E)^{1-\gamma} k^{\alpha\gamma}\gamma \left(\frac{1-\alpha}{w}\right)^{\frac{1}{\alpha+(1-\gamma)(1-\alpha)}}$

* Calculate distance between $r_{i,j-1}$ and $r_{i,j}$ for all m
* If distance is less than tolerance, end loop, else return to beginning of Portfolio Allocation loop

- As workers will not invest in capital, their savings can be directly backed out by the budget constraint

$$a'^{W} = r - c_i^W$$

- Update conditional value functions:

$$V_i^E = \frac{c_i^{E\,1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta\sum_m \text{Prob}_m V_{i-1}^E(r'^{E}_{i,j}, m)$$

$$V_i^{NE} = \frac{c_i^{NE\,1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta\sum_m \text{Prob}_m V_{i-1}^E(r'^{NE}_{i,j}, m)$$

$$V_i^W = \frac{c_i^{W\,1-\frac{1}{\theta}}}{1-\frac{1}{\theta}} + \psi\beta\sum_m \text{Prob}_m V_{i-1}^W(r'^{W}_{i,j}, m)$$

- Perform occupational choice to obtain unconditional value functions:

$$V_i^E(r, h^W, h^E, z, x) = \max(V_i^E(r, h^W, h^E, z, x), V_i^{NE}(r, h^W, h^E), V_i^W(r, h^W, h^E))$$

$$V_i^W(r, h^W, h^E) = \max(V_i^{NE}(r, h^W, h^E), V_i^W(r, h^W, h^E))$$

- Calculate distance between $V_i^E(\cdot)$ and $V_{i-1}^E(\cdot)$ and between $V_i^W(\cdot)$ and $V_{i-1}^W(\cdot)$
- If distance is less than tolerance, end loop, else return to beginning of Value Function loop

With the decision rules solve, I then simulate a fixed mass of agents on a discretion grid of $a \times k \times h \times z \times x$. I guess a uniform distribution over this state space and then iterate until the distribution converges. I use a bisection search to determine the wage that clears the labour market in this economy.