Session 5

Emergent Spacetime

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5.1 Rapporteur talk: Emergent Spacetime, by Nathan Seiberg

5.1.1 *Introduction*

The purpose of this talk is to review the case for the idea that space and time will end up being emergent concepts; i.e. they will not be present in the fundamental formulation of the theory and will appear as approximate semiclassical notions in the macroscopic world. This point of view is widely held in the string community and many of the points which we will stress are well known.

Before we motivate the idea that spacetime should be emergent, we should discuss the nature of space in string theory. We do that in section 2, where we review some of the ambiguities in the underlying geometry and topology. These follow from the dualities of string theory. T-duality leads to ambiguities at the string length l_s and the quantum dualities lead to ambiguities at the Planck length $l_p \ll l_s$. All these ambiguities in the geometry are associated with the fact that as we try to probe the space with increasing resolution, the probes we use become big and prevent us from achieving the desired accuracy.

The discussion about ambiguities in space will lead us to make some comments about locality. In particular, we will ask whether to expect locality in a space or in one of its duals.

In section 3 we will briefly mention some of the peculiar non-gravitational theories which are found as certain limits of string theory. Some of them are expected to be standard field theories, albeit without a Lagrangian. Others, like theories on a noncommutative space or little string theory, are not local quantum field theory. They exhibit interesting nonlocal behavior.

In section 4 we will make the case that general covariance is likely to be a derived concept.

Section 5 will present several examples of emergent space. First we will discuss the simplest examples which do not involve gravity. Then we will turn to four classes of examples of emergent space: the emergent two-dimensional (worldsheet) gravity from the matrix model, the celebrated gauge/gravity duality, linear dilaton backgrounds, and the BFSS matrix model. We will discuss some of their properties and will stress the similarities and the differences between them. In particular, we will discuss their finite temperature behavior as a diagnostic of the system in extreme conditions.

Section 6 will be devoted to emergent time. Here we do not have concrete examples. Instead, we will present some of the challenges and confusions that this idea poses. We will also mention that understanding how time emerges will undoubtedly shed new light on some of the most important questions in theoretical physics including the origin of the Universe.

We will summarize the talk in section 7 where we will also present some general speculations.

Before we start we should mention some important disclaimers. As we said, most of the points which will be discussed here are elementary and are well known in the string community. We apologize for boring you with them. Other points will be inconclusive because they reflect our confusions. Also, not all issues and all points of view will be presented. Instead, the presentation will be biased by my prejudice and my own work. For example, the discussion will focus on string theory (for textbooks, see [1], [2]), and other approaches to quantum gravity will not be reviewed. Since this talk is expected to lead to a discussion, we will present certain provocative and perhaps outrageous ideas. Finally, there will be very few references, mostly to reviews of the subject, rather than to original papers.

5.1.2 *Ambiguous space*

5.1.2.1 *Ambiguous space in classical string theory*

We start this section by discussing the ambiguities in the geometry and the topology which exist already at string tree level. These are usually referred to as *T-duality* (for reviews, see e.g. [3], [4]).

Consider strings propagating in some background fields (e.g. metric). Clearly, these background fields should satisfy the equations of motion. Then, it turns out that different backgrounds can lead to the same physics without any observable difference between them. Therefore, there is no unique answer to the question: *"What is the background metric?"* and the background geometry is ambiguous.

Intuitively, these ambiguities arise from the extended nature of the string. Features in the geometry which are smaller than the string length $l_s = \sqrt{\alpha'}$ cannot be detected using a string probe whose characteristic size is l_s.[1]

[1] D-branes [2] which are smaller than l_s can sometime lead to a more precise metric, but different kinds of D-branes lead to different answers and therefore the ambiguity is not resolved.

The simplest and most widely known example of this ambiguity is the equivalence between a circle with radius R and a circle with radius α'/R. A slightly more peculiar example is the equivalence between a circle with radius $R = 2\sqrt{\alpha'}$ and a $\mathcal{Z}_2$ quotient of a circle (a line segment) with $R = \sqrt{\alpha'}$. This example demonstrates that even the topology is ambiguous. Furthermore, we can start with a circle of radius R, smoothly change it to $R = 2\sqrt{\alpha'}$, then use the duality with the line segment and then change the length of the line segment. This way we start with a circle which is not dual to a line segment and we continuously change its topology to a line segment which is not dual to a circle.

A characteristic feature of these dualities is the role played by momentum and winding symmetries. In the example of the two circles with radii R and α'/R momentum conservation in one system is mapped to winding conservation in the other. Momentum conservation arises from a geometric symmetry (an isometry) of the circle. It is mapped to winding conservation which is a *stringy symmetry*. This is a manifestation of the stringy nature of T-duality and it makes it clear that it is associated with the extended nature of the string.

In some situations there exists a description of the system in terms of a *macroscopic background*; i.e. the space and all its features are larger than l_s. This is the most natural description among all possible dual descriptions. However, two points should be stressed about this case. First, even though this description is the most natural one, there is nothing wrong with all other T-dual descriptions and they are equally valid. Second, it is never the case that there is more than one such macroscopic and natural description.

More elaborate and richer examples of this fundamental phenomenon arise in the study of Calabi-Yau spaces. Here two different Calabi-Yau spaces which are a "mirror pair" (for a review, see e.g. [5]) lead to the same physics. Furthermore, it is often the case that one can continuously interpolate between different Calabi-Yau spaces with different topology. These developments had dramatic impact on mathematics (see e.g. [5], [6]).

Another kind of T-duality is the cigar/Sine-Liouville duality [7]. One side of the duality involves the cigar geometry: a semi-infinite cylinder which is capped at one side. It has a varying dilaton, such that the string coupling at the open end of the cigar vanishes. This description makes it clear that the shift symmetry around the cigar leads to conserved momentum. However, the string winding number is not conserved, because wound strings can slip through the capped end of the cigar. The other side of this duality involves an infinite cylinder. Here the winding conservation is broken by a condensate of wound strings. The cigar geometry is described by a two-dimensional field theory with a nontrivial metric but no potential, while its dual, the Sine-Liouville theory, is a theory with a flat metric but a nontrivial potential. This example again highlights the importance of the winding modes. It also demonstrates that the T-duality ambiguity is not limited to compact dimensions.

Here the ambiguity is between two different non-compact systems (an infinite and a half infinite cylinder).

From the worldsheet point of view T-duality represents an exact equivalence between different two-dimensional conformal field theories. Therefore, the phenomenon of T-duality persists beyond classical string theory, and extends to all orders in perturbation theory. Furthermore, in some situations one can argue that T-duality is a gauge symmetry. This observation means that T-duality is exact and it cannot be violated non-perturbatively.

The phenomenon of T-duality leads us to ask two interesting questions. First, is l_s a minimum length; i.e. is the notion of distance ill defined below l_s? Second, is the theory local in one space, or in its T-dual space, or in neither? We will return to these questions below.

Before we leave the topic of ambiguities in classical string theory we would like to mention another important stringy phenomenon which is associated with the extended nature of the string. The high energy density of string states is such that the canonical ensemble of free strings does not exist above a certain temperature $T_H \sim \frac{1}{l_s}$, which is known as the *Hagedorn temperature* [1], [2]. The relevant modes which lead to this phenomenon are long strings. They have large entropy and hence the partition function diverges at T_H. Equivalently, when Euclidean time is compactified on a circle of radius $R = \frac{1}{2\pi T}$ (with thermal boundary conditions) an instability appears when $R \leq \frac{1}{2\pi T_H}$. This instability is associated with strings which are wound around the Euclidean time circle. T_H could be a limiting temperature, beyond which the theory does not exist. Alternatively, this phenomenon could mean that the system undergoes a first order phase transition to another phase. That phase could exhibit the fundamental degrees of freedom more clearly. Again we see that the theory tries to hide its short distance behavior.

5.1.2.2 *Ambiguous space in quantum string theory*

Quantum mechanics introduces new ambiguities in space which are related to new dualities (for reviews, see e.g. [2], [4]). These ambiguities go beyond the obvious ambiguities due to the quantum fluctuations. Here the characteristic length scale is the Planck length $l_p \ll l_s$.

An intuitive argument explaining the origin of these ambiguities is the following. If we want to explore space with resolution of order r, the uncertainly principle tells us that we need to use energy $E > \frac{1}{r}$. This energy has to be concentrated in a region of size r. But in the presence of gravitational interactions, this concentration of energy creates a black hole unless $r > l_p$. Therefore, *we cannot explore distances smaller than the Planck length.*

It is important to stress that although the ambiguities in the quantum theory are often described as of different nature than the ambiguities in the classical theory, fundamentally they are quite similar. Both of them are associated with the breakdown of the standard small distance/high energy connection – as we try to increase the energy of a probe it becomes bigger and does not allow us to explore short distances.

The *quantum dualities*, which are also known as S-duality or U-duality, extend the classical T-duality and lead to a beautiful and coherent picture of stringy dualities. These exchange highly quantum situations with semiclassical backgrounds, exchange different branes, etc. As in the classical dualities, among all dual descriptions there is at most one description which is natural because it is semiclassical. All other dual descriptions are very quantum mechanical.

5.1.2.3 *Comments about locality*

We now turn to some comments about locality in string theory.

Quantum field theory is local. This locality guarantees that the theory is causal. We would like string theory also to be causal or at least macroscopically causal. Furthermore, we know that at long distances string theory behaves like quantum field theory and therefore it is macroscopically local. But is string theory local also over short distances?

One piece of evidence in favor of locality is the analyticity of the perturbative string S-matrix. Normally, causality and locality lead to analyticity. Since the string S-matrix is analytic, it is likely that string theory is local. However, it is logically possible that a slightly weaker condition than locality and therefore of causality can also guarantee the analyticity of the S-matrix.

One reason string theory might not be local in a standard way is the extended nature of the interacting objects, the strings. At the most naive and intuitive level locality of string interactions is not obvious. Even though two strings interact at a point to form a third string, this interaction is nonlocal when viewed from the point of view of the center of masses of the interacting strings. It is known that this nonlocality is harmless and is consistent with the analyticity of the S-matrix.[2]

We would like to comment about locality and the cosmological constant. The old fashioned point of view of the cosmological constant problem suggested that its value is related to some kind of a UV/IR mixing and to violation of naive locality – the short distance theory somehow reacts to long distance fluctuations and thus sets the value of the cosmological constant. A more modern point of view on the subject is that the cosmological constant is set anthropically (see, e.g. [8]). It remains to be seen whether the cosmological constant is a hint about some intrinsic nonlocality in the theory.

The ambiguities we discussed above might hint at some form of nonlocality. We have stressed that increasing the energy of a probe does not lead to increased resolution. Instead, the probe becomes bigger and the resolution is reduced. This point is at the heart of the various dualities and ambiguities in the background. We have already asked whether we expect locality in a space, or in its dual space.

[2]In open string field theory a basis based on the string midpoint replaces the basis based on the center of mass and then the interaction appears to be local.

It is hard to imagine that the theory can be simultaneously local in both of them. Then, perhaps it is local in neither. Of course, when a macroscopic weakly coupled natural description exists, we expect the theory to be at least approximately local in that description.

It is important to stress that although intuitively the notion of locality is obvious, this is not the case in string theory or in any generally covariant theory. The theory has no local observables. Most of the observables are related to the S-matrix or other objects at infinity. These do not probe the detailed structure of the theory in the interior. Therefore, without local observables it is not clear how to precisely define locality.

We will argue below that space and time should be emergent concepts. So if they are not fundamental, the concept of locality cannot be fundamental as well. It is possible that locality will end up being ill defined, and there will be only an approximate notion of locality when there is an approximate notion of spacetime.

5.1.3 *Non-standard theories without gravity*

Next, let us digress slightly to review some of the non-standard theories without gravity that were found by studying various limits of string theory. These theories exhibit interesting and surprising new phenomena. We expect that these theories and their peculiar phenomena will be clues to the structure of the underlying string theory. Since they are significantly simpler than string theory, they could be used as efficient laboratories or toy models.

The first kind of surprising theories are new local field theories which cannot be given a standard Lagrangian description. These are superconformal field theories in five or six dimensions with various amount of supersymmetry. The most symmetric examples are the six-dimensional (2,0) theories (for a review, see e.g. [9]). They are found by taking an appropriate scaling limit of string theory in various singularities or on coincident 5-branes. The existence of these theories calls for a new formulation of local quantum field theory without basing it on a Lagrangian.

Another class of interesting non-gravitational theories are *field theories on non-commutative spaces* (for a review, see e.g. [10]). These theories do not satisfy the standard rules of local quantum field theory. For example, they exhibit a UV/IR mixing which is similar to the UV/IR mixing in string theory – as the energy of an object is increased its size becomes bigger.

The most enigmatic theories which are derived from string theory are the *little string theories* (for a review, see e.g. [11]). These non-gravitational theories exhibit puzzling stringy behavior. The stringy nature of these theories arises from the fact that they appear by taking a certain scaling limit of string theory (in the presence of NS5-branes or some singularities) while keeping α' fixed. One stringy phenomenon they exhibit is T-duality. This suggests that despite the lack of gravity, these theories do not have a local energy momentum tensor. Otherwise, there should have been several different energy momentum tensors which are related by T-duality. It was also argued that because of their high energy behavior these theories cannot have local observables. Finally, these theories exhibit Hagedorn spectrum with a Hagedorn temperature which is below T_H of the underlying string theory. It was suggested that this Hagedorn temperature is a limiting temperature; i.e. the canonical ensemble does not exist beyond that temperature.

5.1.4 *Derived general covariance*

The purpose of this section is to argue that general covariance which is the starting point of General Relativity might not be fundamental. It could emerge as a useful concept at long distances without being present in the underlying formulation of the theory.

General covariance is a *gauge symmetry*. As with other gauge symmetries, the term "symmetry" is a misnomer. Gauge symmetries are not symmetries of the Hilbert space; the Hilbert space is invariant under the entire gauge group. Instead, gauge symmetries represent a redundancy in our description of the theory. (It is important to stress, though, that this is an extremely useful redundancy which allows us to describe the theory in simple local and Lorentz invariant terms.)

Indeed, experience from duality in field theory shows that gauge symmetries are not fundamental. It is often the case that a theory with a gauge symmetry is dual to a theory with a different gauge symmetry, or no gauge symmetry at all. A very simple example is Maxwell theory in 2+1 dimensions. This theory has a $U(1)$ gauge symmetry, and it has a dual description in terms of a free massless scalar without a local gauge symmetry. More subtle examples in higher dimensions were found in supersymmetric theories (for reviews, see e.g. [12], [13]).

If ordinary gauge symmetries are not fundamental, it is reasonable that general covariance is also not fundamental. This suggests that the basic formulation of the theory will not have general covariance. General covariance will appear as a derived (and useful) concept at long distances.

An important constraint on the emergence of gauge symmetries follows from the Weinberg-Witten theorem [14]. It states that if the theory has massless spin one or spin two particles, these particles are gauge particles. Therefore, the currents that they couple to are not observable operators. If these gauge symmetries are not present in some formulation of the theory, these currents should not exist there. In particular, it means that if an ordinary gauge symmetry emerges, the fundamental theory should not have this symmetry as a global symmetry. In the context of emergent general covariance, this means that the fundamental theory cannot have an energy momentum tensor.

If we are looking for a fundamental theory without general covariance, it is likely that this theory should not have an underlying spacetime. This point is further motivated by the fact that General Relativity has no local observables and perhaps no local gauge invariant degrees of freedom. Therefore, there is really no need for an underlying spacetime. Spacetime and general covariance should appear as approximate concepts which are valid only macroscopically.

5.1.5 *Examples of emergent space*

5.1.5.1 *Emergent space without gravity*

The simplest examples of emergent space are those which do not involve gravity. Here the starting point is a theory without a fundamental space, but the resulting answers look approximately like a theory on some space. The first examples of this kind were the *Eguchi-Kawai* model and its various variants (for a review, see e.g. [15]). Here a d dimensional $SU(N)$ gauge theory is formulated at one point. The *large* N answers look like a gauge theory on a macroscopic space.

Certain extensions of the (twisted) Eguchi-Kawai model are theories on a *non-commutative space* (for a review, see e.g. [10]). Here the coordinates of the space do not commute and are well defined only when they are macroscopic.

A physical realization of these ideas is the *Myers effect* [16]. Here we start with a collection of N branes in some background flux. These branes expand and become a single brane of higher dimension. The new dimensions of this brane are not standard dimensions. They form a so-called "fuzzy space." In the *large N limit* the resulting space becomes macroscopic and its fuzzyness disappears.

5.1.5.2 *Emergent space with gravity: matrix model of 2d gravity*

The first examples of emergent space with gravity and general covariance arose from the *matrix model of random surfaces* (for a review, see e.g. [17]). Here we start with a certain matrix integral or matrix quantum mechanics and study it in perturbation theory. Large Feynman diagrams of this perturbation expansion can be viewed as discretized two-dimensional surfaces.

This system is particularly interesting when the size of the matrices N is taken to infinity together with a certain limit of the parameters of the matrix integral. In this double scaling limit the two-dimensional surfaces become large and smooth and the system has an effective description in terms of random surfaces. The degrees of freedom on these surfaces are local quantum fields including a dynamical metric and therefore this description is generally covariant.

The formulation of these theories as matrix models does not have a two-dimensional space nor does it have general covariance. These concepts emerge in the effective description.

In addition to being interesting and calculable models of two-dimensional gravity, these are concrete examples of how space and its general covariance can be emergent concepts.

5.1.5.3 *Emergent space with gravity: Gauge/Gravity duality*

The most widely studied examples of emergent space with gravity are based on the AdS/CFT correspondence [18], [19], [20], [21]. This celebrated correspondence is the duality between string theory in AdS space and a conformal field theory at its boundary. Since other speakers in this conference will also talk about it, we will only review it briefly and will make a few general comments about it.

The bulk theory is a theory of gravity and as such it does not have an energy momentum tensor. The dual field theory on the boundary has an energy momentum tensor. This is consistent with the discussion above about emergent gravity (section 4), because the energy momentum tensor of the field theory is in lower dimensions than the bulk theory and reflects only its boundary behavior.

The operators of the boundary theory are mapped to string states in the bulk. A particularly important example is the energy momentum tensor of the boundary theory which is mapped to the bulk graviton. The correlation functions of the conformal field theory are related through the correspondence to string amplitudes in the AdS space. (Because of the asymptotic structure of AdS, these are not S-matrix elements.) When the field theory is deformed by relevant operators, the background geometry is slightly deformed near the boundary but the deformation in the interior becomes large. This way massive field theories are mapped to nearly AdS spaces.

The radial direction in AdS emerges without being a space dimension in the field theory. It can be interpreted as the renormalization group scale, or the energy scale used to probe the theory. The asymptotic region corresponds to the UV region of the field theory. This is where the theory is formulated, and this is where the operators are defined. The interior of the space corresponds to the IR region of the field theory. It is determined from the definition of the theory in the UV.

A crucial fact which underlies the correspondence, is the infinite warp factor at the boundary of the AdS space. Because of this warp factor, finite distances in the field theory correspond to infinite distances in the bulk. Therefore, a field theory correlation function of finitely separated operators is mapped to a gravity problem which infinitely separated sources.

An important consequence of this infinite warp factor is the effect of finite temperature. The boundary field theory can be put at finite temperature T by compactifying its Euclidean time direction on a finite circle of radius $R = \frac{1}{2\pi T}$. At low temperature, the only change in the dual asymptotically AdS background it to compactify its Euclidean time. Because of the infinite warp factor, the radius of the Euclidean time circle in the AdS space is large near the boundary, and it is small only in a region of the size of the AdS radius R_{AdS}. Therefore, most of the bulk of the space is cold. Only a finite region in the interior is hot. As the system is heated up, the boundary theory undergoes a thermal deconfinement phase transition. In the bulk it is mapped to the appearance of a Schwarzschild horizon at small radius and the topology is such that the Euclidean time circle becomes contractible. For a CFT on a 3-sphere, this phase transition is the Hawking-Page transition, and the dual high temperature background is AdS-Schwarzschild. Both above and below the transition the bulk asymptotes to (nearly) AdS. Most of it remains cold and it is not sensitive to the short distance behavior of string theory.

While the boundary field theory is manifestly local, locality in the bulk is subtle. Because of the infinite warp factor, possible violation of locality in the bulk over distances of order l_s could be consistent with locality at the boundary. In fact, it is quite difficult to find operators in the field theory which represent events in the bulk which are localized on scales of order R_{AdS} or smaller. This underscores the fact that it is not clear what we mean by locality, if all we can measure are observables at infinity.

These developments have led to many new insights about the two sides of the duality and the relation between them (for a review, see [21]). In particular, many new results about gauge theories, including their strong coupling phenomena like thermal phase transitions, confinement and chiral symmetry breaking were elucidated. The main new insight about gravity is its *holographic nature* – the boundary theory contains all the information about the bulk gravity theory which is higher dimensional. Therefore, the number of degrees of freedom of a gravity theory is not extensive. This is consistent with the lack of local observables in gravity.

5.1.5.4 *Emergent space with gravity: linear dilaton backgrounds*

Generalities Another class of examples of an emergent space dimension involves backgrounds with a linear dilaton direction. The string coupling constant depends on the position in the emergent direction, parameterized by the spatial coordinate ϕ, through $g_s(\phi) = e^{\frac{Q\phi}{2}}$ with an appropriate constant Q. Therefore, the string coupling constant vanishes at the boundary $\phi \to -\infty$. The other end of the space at $\phi \to +\infty$ is effectively compact.

Like the AdS examples, here the bulk string theory is also dual to a theory without gravity at the boundary. In that sense, this is another example of holography. However, there are a few important differences between this duality and the AdS/CFT duality.

In most of the linear dilaton examples the holographic theory is not a standard local quantum field theory. For example, the near horizon geometry of a stack of NS5-branes is a linear dilaton background which is holographic to the little string theory (for a review, see e.g. [11]). The stringy, non-field theoretic nature of the holographic theory follows from the fact that it has nonzero α', and therefore it exhibits T-duality.

Because of the vanishing interactions at the boundary of the space, the interactions take place in an effectively compact region (the strong coupling end). Therefore, we can study the S-matrix elements of the bulk theory. These are the observables of the boundary theory.

Unlike the AdS examples, the string metric does not have an infinite warp factor.

Here finite distances in the boundary theory correspond to finite distances (in string units) in the bulk. Therefore, it is difficult to define *local* observables in the boundary theory and as a result, the holographic theory is not a local quantum field theory.

This lack of the infinite warp factor affects also the finite temperature behavior of the system. Finite temperature in the boundary theory is dual to finite temperature in the entire bulk. Hence, the holographic theory can exhibit Hagedorn behavior and have maximal temperature.

Matrix model duals of linear dilaton backgrounds Even though the generic linear dilaton theory is dual to a complicated boundary theory, there are a few simple cases where the holographic theories are very simple and are given by the large N limit of certain matrix models.

The simplest cases involve strings in one dimension ϕ with a linear dilaton. The string worldsheet theory includes a Liouville field ϕ and a $c < 1$ minimal model (or in the type 0 theory a $\hat{c} < 1$ superminimal model). The holographic description of these *minimal string theories* is in terms of the large N limit of matrix integrals (for a review, see e.g. [22]).

Richer theories involve strings in two dimensions: a linear dilaton direction ϕ and time x (for a review, see e.g. [23]). Here the holographic theory is the large N limit of matrix quantum mechanics.

These two-dimensional string theories have a finite number of particle species. The bosonic string and the supersymmetric 0A theory have one massless boson, and the 0B theory has two massless bosons. Therefore, these theories do not have the familiar Hagedorn density of states of higher dimensional string theories, and correspondingly, their finite temperature behavior is smooth.

One can view the finite temperature system as a system with compact Euclidean time x. Then, the system has $R \to \alpha'/R$ T-duality which relates high and low temperature. As a check, the smooth answers for the thermodynamical quantities respect this T-duality.

It is important to distinguish the two different ways matrix models lead to emergent space. Above (section 5.2) we discussed the emergence of the two-dimensional string worldsheet with its worldsheet general covariance. Here, we discuss the target space of this string theory with the emergent holographic dimension ϕ.

Since the emergence of the holographic direction in these systems is very explicit, we can use them to address various questions about this direction. In particular, it seems that there are a number of inequivalent ways to describe this dimension. The most obvious description is in terms of the Liouville field ϕ. A second possibility is to use a free worldsheet field which is related to ϕ through a nonlocal transformation (similar to T-duality transformation). This is the Backlund field of Liouville theory. A third possibility, which is also related to ϕ in a nonlocal way arises more naturally out of the matrices as their eigenvalue direction. These different descriptions of the emergent direction demonstrate again that the ambiguity in the description of space which we reviewed above (section 2) is not limited to compact dimensions. It also highlights the question of locality in the space. In which of these descriptions do we expect the theory to be local? Do we expect locality in one of them, or in all of them, or perhaps in none of them?

2d heterotic strings We would like to end this subsection with a short discussion of the heterotic two-dimensional linear dilaton system. Even though there is no known holographic matrix model dual of this system, some of its peculiar properties can be analyzed.

As with the two-dimensional linear dilaton bosonic and type 0 theories, this theory also has a finite number of massless particles. But here the thermodynamics is more subtle. We again compactify Euclidean time on a circle of radius R. The worldsheet analysis shows that the system has $R \to \alpha'/2R$ T-duality. Indeed, the string amplitudes respect this symmetry. However, unlike the simpler bosonic system, here the answers are not smooth at the selfdual point $R = \sqrt{\alpha'/2}$. This lack of smoothness is related to long macroscopic strings excitations [24].

What is puzzling about these results is that they cannot be interpreted as standard thermodynamics. If we try to interpret the Euclidean time circle as a thermal ensemble with temperature $T = \frac{1}{2\pi R}$, then the transition at $R = \sqrt{\alpha'/2}$ has negative latent heat. This violates standard thermodynamical inequalities which follow from the fact that the partition function can be written as a trace over a Hilbert space Tr $e^{-H/T}$ for some Hamiltonian H. Therefore, we seem to have a contradiction between compactified Euclidean time and finite temperature. The familiar relation between them follows from the existence of a Hamiltonian which generates *local time evolution.* Perhaps this contradiction means that we cannot simultaneously have locality in the circle and in its T-dual circle. For large R the Euclidean circle answers agree with the thermal answers with low temperature. But while these large R answers can be extended to smaller R, the finite temperature interpretation ceases to make sense at the selfdual point. Instead, for smaller R we can use the T-dual circle, which is large, and describe the T-dual system as having low temperature.

5.1.5.5 *Emergent space in the BFSS matrix model*

As a final example of emergent space we consider the BFSS matrix model (for a review, see e.g. [25]). Its starting point is a large collection of D0-branes in the lightcone frame. The lightcone coordinate x^+ is fundamental and the theory is an ordinary quantum mechanical system with x^+ being the time.

The transverse coordinates of the branes x^i are the variables in the quantum mechanical system. They are not numbers. They are N dimensional matrices. The standard interpretation as positions of the branes arises only when the branes are far apart. Then the matrices are approximately diagonal and their eigenvalues are the positions of the branes. In that sense the transverse dimensions emerge from the simple quantum mechanical system.

The remaining spacetime direction, x^-, emerges holographically. It is related to the size of the matrices $N \sim p_-$ where p_- is the momentum conjugate to x^-.

5.1.6 *Emergent time*

After motivating the emergence of space it is natural to ask whether time can also emerge. One reason to expect it is that this will put space and time on equal footing – if space emerges, so should time. This suggests that time is also not fundamental. The theory will be formulated without reference to time and an approximate (classical) notion of macroscopic time, which is our familiar "time", will emerge. Microscopically, the notion of time will be ill defined and time will be fuzzy.

There are several obvious arguments that time should not be emergent:

(1) Even though we have several examples of emergent space, we do not have a single example of emergent time.
(2) We have mentioned some of the issues associated with locality in emergent space. If time is also emergent we are in danger of violating locality in time and that might lead to violation of causality.
(3) It is particularly confusing what it means to have a theory without fundamental time. Physics is about predicting the outcome of an experiment *before* the experiment is performed. How can this happen without fundamental time and without notions of "before and after"? Equivalently, physics is about describing the evolution of a system. How can systems evolve without an underlying time? Perhaps these questions can be avoided, if some order of events is well defined without an underlying time.
(4) More technically, we can ask how much of the standard setup of quantum mechanics should be preserved. In particular, is there a wave function? What is its probabilistic interpretation? Is there a Hilbert space of all possible wave functions, or is the wave function unique? What do we mean by unitarity (we cannot have unitary evolution, because without time there is no evolution)? Some of these questions are discussed in [26].

My personal prejudice is that these objections and questions are not obstacles to emergent time. Instead, they should be viewed as challenges and perhaps even clues to the answers.

Such an understanding of time (or lack thereof) will have, among other things, immediate implications for the physics of space-like and null singularities (for a review, see e.g. [27]) like the black hole singularity and the cosmological singularity. We can speculate that understanding how time emerges and what one means by a wave function will explain the meaning of *the wave-function of the Universe.* Understanding this wave function, or equivalently understanding the proper initial conditions for the Universe, might help resolving some of the perplexing questions of vacuum selection in string theory. For a review of some aspects of these questions see [8].

5.1.7 *Conclusions and speculations*

We have argued that spacetime is likely to be an emergent concept. The fundamental formulation of the theory will not have spacetime and it will emerge as an approximate, classical concept which is valid only macroscopically.

One challenge is to have emergent spacetime, while preserving some locality – at least macroscopic locality, causality, analyticity, etc. Particularly challenging are the obstacles to formulating physics without time. It is clear that in order to resolve them many of our standard ideas about physics will have to be revolutionized. This will undoubtedly shed new light on the fundamental structure of the theory.

Understanding how time emerges will also have other implications. It will address deep issues like the cosmological singularity and the origin of the Universe.

We would like to end this talk with two general speculative comments.

Examining the known examples of a complete formulation of string theory, like the various matrix models, AdS/CFT, etc., a disturbing fact becomes clear. It seems that many different definitions lead to a consistent string theory in some background. In particular, perhaps every local quantum field theory can be used as a boundary theory to define string theory in (nearly) AdS space. Perhaps every quantum mechanical system can be the holographic description of string theory in 1+1 dimensions. And perhaps even every ordinary integral defines string theory in one Euclidean dimension. With so many different definitions we are tempted to conclude that we should not ask the question: *"What is string theory?"* Instead, we should ask: *"Which string theories have macroscopic dimensions?"* Although we do not have an answer to this question, it seems that *large* N will play an important role in the answer.

Our second general comment is about reductionism – the idea that science at one length scale is derived (at least in principle) from science at shorter scales. This idea has always been a theme in all branches of science. However, if there is a basic length scale, below which the notion of space (and time) does not make sense, we cannot derive the principles there from deeper principles at shorter distances. Therefore, once we understand how spacetime emerges, we could still look for more basic fundamental laws, but these laws will not operate at shorter distances. This follows from the simple fact that the notion of "shorter distances" will no longer make sense. This might mean the *end of standard reductionism.*

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5.2 Discussion

S. Shenker I would like to speak in defense of redundancy. It is certainly true as you say that gauge symmetry really does reflect just a redundancy in the description. But sometimes, as we know, this redundancy is very useful. Formulating quantum electrodynamics with four gauge fields is good, because you can choose different ways of resolving the redundancy to make things like unitarity, or Lorentz covariance, clear in different gauges. And so, well, some of us have been worrying for a long time, and Gerard 't Hooft made these ideas quite explicit, perhaps what we should be thinking about is a description where we vastly enlarge the number of degrees of freedom in our description of whatever the thing we're trying to describe is, quantum gravity. And so we would then be able to resolve this redundancy in a way in which locality, to the extent it exists, is manifest, or holography is manifest in another gauge, or some other property is manifest. And then it would be too much to ask for a description in which all the desirable properties of the theory are clear, in one presentation.

N. Seiberg I would like to respond to that. I sympathize with your point of view, but recall that you wrote a paper about the matrix model, where the local reparametrization on the world sheet was absent. You had a description of two dimensional gravity, in the sense of the world sheet, without reparametrization freedom. So this is an example where gravity and general covariance emerge.

S. Shenker What is this statement: to do as I say, not as I do.

J. Harvey There is also saying that consistency is the hobgoblin of small minds.

E. Silverstein I just have a small comment on non-locality appearing in string theories. So another example occurs in AdS/CFT. You said we don't have any argument against non-locality at the string scale, but there is evidence for non-locality at a much larger scale, if you consider multi-trace deformations of the field theory. So if you include the internal space, the simplest versions being spheres or other Einstein spaces, the boundary conditions are grossly non-local on those dimensions, at a scale of order the AdS radius scale. So this is another indication that we might need to incorporate non-locality in a serious way.

S. Weinberg I have a comment and a question. The comment is that for a long time, I thought that general covariance was a red herring, because any particle with mass zero and spin two would have the properties that we derive from general covariance, as shown also by Feynman, and the great thing done by string theory, in this area, is to show that there has to be a particle of mass zero and spin two, while before string theory we didn't know why there had to be one. My question had to do with your remark that the S matrix in string theory is more analytic than in quantum field theory. I thought that in quantum field theory the S matrix was as analytic as it possibly could be, given the constraints of unitarity – I do not see how anything...

N. Seiberg What I had in mind is the good high energy behaviour. And I think the person who should really answer this sits next to you [G. Veneziano].

S. Weinberg I see what you mean. So you include the point at infinity in the analyticity.

N. Seiberg Yes.

S. Weinberg Oh well OK, thank you.

N. Seiberg The sign of my comment was not in this direction, it was actually in the opposite. This usually signals locality and causality. But I am not aware, maybe you can enlighten me on that, of an argument that this must mean that there is underlying locality. There might be some weaker statement than locality and causality which leads to the same kind of analyticity.

S. Weinberg Well, I have never been able to elevate it into a theorem; in my courses, I taught quantum field theory in such a way that locality emerges out of the requirements of Lorentz invariance plus the cluster decomposition principle, and then the cluster decomposition principle is more fundamental than locality. In fact, I think it would be an interesting challenge, to understand how cluster decomposition emerges from something like string theory.

N. Seiberg That is a very interesting point, because in particular it means that there could be something fuzzy and non-local at short distance, as long as when you separate things, things are well-behaved.

S. Weinberg That is the real test. The locality, we can live without, but I do not see how science is possible without the cluster decomposition principle.

M. Douglas As a candidate to a model with emergent time, how about 2D quantum gravity coupled to c greater than twenty-five matter, where the Liouville becomes time-like.

J. Harvey How about it?...

W. Fischler What is the requirement of analyticity, if the emergent space-time and its asymptopia do not allow for an S matrix?

N. Seiberg Well, if you have this asymptotic region, you can scatter particles...

W. Fischler No, I am saying, what if, there is no asymptopia that allows for an S matrix. Or do we say that maybe the requirement of analyticity forbids such space-times to be solutions of quantum gravity?

N. Seiberg I do not see anything wrong with compact space. It is very confusing, but I do not see anything wrong with that.

W. Fischler Let me just give you an example: de Sitter space-time, there is no S matrix. Asymptopia does not allow you to define such an object. So what requirement do we have about analyticity in this case? Or do we say de Sitter space-time is not a viable quantum mechanical solution to whatever theory of quantum gravity?

N. Seiberg I am not an expert on the subject. Some people believe that this class of questions and confusions perharps are trying to tell us that de Sitter space is not a stable solution. Other people, including some people in this room, strongly disagree with that.

W. Fischler It was a question, I just do not know the answer.

G. Horowitz I wanted to mention a couple of other examples of emergent time. I think one is $2+1$ gravity, which as you know can be written as a Chern-Simons theory, there is a state of zero metric, no space, no time, anything, and that's perfectly, you know, allowed and understood, and yet we see how we can get classical space-times out of that theory. And in string theory there is sort of at least a formal generalization of all of that in the purely cubic action in string field theory.

N. Seiberg Well, Chern-Simons theory has time, right? You write the integral $\int d^2x\, dt$ of something. So t appears there. I think we would like something which is a little bit more dramatic than this.

J. Harvey Yes, I would have thought space-time was there, it was just rewriting the action in a different form.

G. Horowitz There is a state of zero metric, so I am not sure what time or space would mean if the metric is zero.

A. Ashtekar In terms of this emergent time, there is a lot of literature, in the relativity circles, quantum gravity circles, about emergent time. Basically there is a relation to dynamics, and how one of the variables, under circumstances when there is actually a semiclassical space-time, can be taken to be time. But in general there would be no such variable, and then we would not have a good notion of time. In a particular example that I sketched, which had to do with scalar fields coupled with gravity in cosmology, in that example a scalar field is a good notion of time, in quantum theory I am talking about. But near the Big-Bang singularity, it becomes very very fuzzy, we do not have the standard notion, and again, it reemerges as the standard notion on the other side. So it seems to me that there are definite examples, certainly not a completely general theorem or anything like that, but lots and lots of such examples, that exist in the literature.

J. Harvey I think that this might be a good point to stop, have a coffee break, and I would like to continue this discussion, so if you have things to say, please make a note of that and we will continue in half an hour and have time for discussion after the next four talks.

5.3 Prepared Comments

5.3.1 *Tom Banks: The Holographic Approach to Quantum Gravity*

String Theory provides us with many consistent models of quantum gravity in space-times which are asymptotically flat or AdS. These models are explicitly holographic: the observables are gauge invariant boundary correlation functions. Typical cosmological situations do not have well understood asymptotic boundaries. They begin with a Big Bang, and can end with *e.g.* asymptotic de Sitter (dS) space. In order to formulate a string theory of cosmology, we have to find a more general formulation of the theory.

Einstein gravity has the flexibility to deal with a wide variety of asymptotic behaviors for space-time. It describes space-time in terms of a local, gauge variant, variable, the metric tensor, $g_{\mu\nu}(x)$. The corresponding object in the quantum theory is a preferred algebra of operators for a causal diamond in space-time. An *observer* is a large quantum system with a wealth of semi-classical observables. Our mathematical model of observers is a cut-off quantum field theory, with volume large in cutoff units. The semi-classical observables are averages of local fields over large volumes. Tunneling transitions between different values of these observables are suppressed by exponentials of the volume.

Experiment teaches us that there are many such observers in the real world, and that they travel on time-like trajectories. A theory of quantum gravity should reproduce this fact as a mathematical theorem, but it is permissible to use the idea of an observer in the basic formulation of the theory. A pair of points $P > Q$ on the trajectory of an observer defines a causal diamond: the intersection of the interior of the backward light cone of P with that of the forward light cone of Q. Conversely, a dense sequence of nested causal diamonds completely defines the trajectory.

The covariant entropy bound[5][6][7] associates an entropy with each causal diamond. For sufficiently small proper time between P and Q the entropy is always finite. Fischler and the present author have argued that the only general ansatz one can make about the density matrix corresponding to this entropy is that it is proportional to the unit matrix. This hypothesis provides us with a dictionary for translating concepts of Lorentzian geometry into quantum mechanics. A nested sequence of causal diamonds, describing the trajectory of an observer, is replaced by a sequence of finite dimensional Hilbert spaces, $\mathcal{H}_N$, with $\mathcal{H}_{n-1}$ a tensor factor in $\mathcal{H}_n$. The precise mapping of this sequence into space-time is partly a gauge choice. We will concentrate on Big Bang space-times, where it is convenient to choose the initial point of every causal diamond to lie on the Big Bang hypersurface. Each Hilbert space $\mathcal{H}_n$ is equipped with a sequence of time evolution transformations $U(k,n)$ with $1 \leq k \leq n$. A basic consistency condition is that $U(k,n) = U(k,m) \otimes V(k,m)$ if $k \leq m < n$. The unitary $V(k,m)$ operates on the tensor complement of $\mathcal{H}_m$ in $\mathcal{H}_n$. This condition guarantees that the notion of *particle horizon* usually derived from local field theory, is incorporated into holographic cosmology.

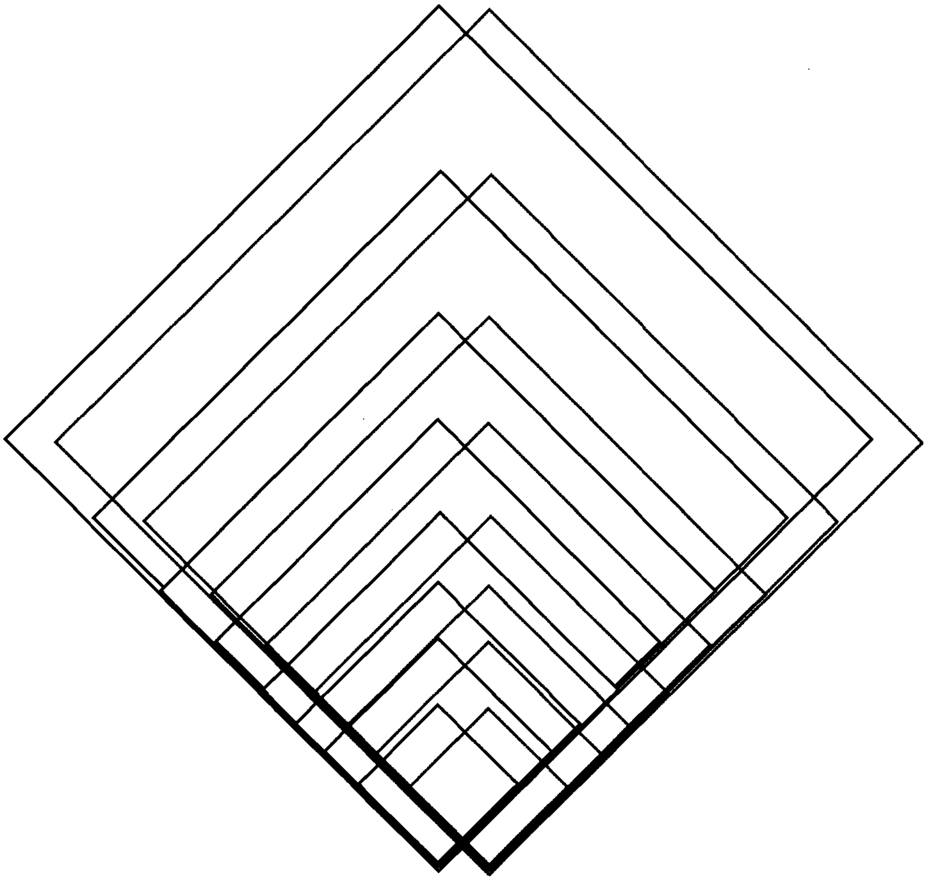

Fig. 5.1 A Sequence of Causal Diamonds in a Big Bang Space-time Defines an Observer. A Lattice of Overlapping Diamonds Defines Space-Time.

The particle horizon condition guarantees that a certain set of degrees of freedom will interact only among themselves before a given fixed time. It makes the apparent increase of spatial volume with cosmological time, compatible with quantum unitarity and a Planck scale cutoff. In fact, the discretization of time implicit in this formalism is not a simple cut-off. Subsequent Hilbert spaces in the sequence have dimension increasing by a fixed factor, which we will specify below. This implies a fixed area cutoff in space-time, which generically corresponds to a time cut-off which goes to zero with the size of causal diamonds.

To model an entire space-time we need a collection of time-like observers, with overlapping causal diamonds. We introduce a spatial lattice, which defines the topology of the non-compact dimensions of space on the initial time-slice[3]. This topology is conserved in time, and for the moment we will take it to be that of flat

[3]The compact dimensions will be dealt with in a completely different manner below.

$d-1$ dimensional space. The geometry of the lattice should be thought of as another gauge choice. The formalism must be built in such a way that true semi-classical measurements do not depend on it. It is not clear whether any real measurement will depend on the geometric structure of the spatial lattice. Asymptotically infinite space-times are defined as limits, and the boundary observables will be independent of the structure of the lattice. Finite space-times will have a built in restriction on the accuracy of measurements, stemming from the inability of a finite quantum system to make arbitrarily precise measurements on itself. It may be that this *a priori* lack of precision will make the micro-structure of the spatial lattice truly unmeasurable.

To model a family of time-like observers, we introduce a sequence of Hilbert spaces $\mathcal{H}_n(\mathbf{x})$ (an observer) for each point $\mathbf{x}$ on the lattice. It is convenient to choose an *equal area time slicing* in which the dimension of $\mathcal{H}_n(\mathbf{x})$ depends only on n. For each pair of space-time points we introduce an overlap Hilbert space $\mathcal{O}(m, \mathbf{x}, n, \mathbf{y})$ which encodes the physics of the maximal causal diamond in the overlap of the causal diamonds described by $\mathcal{H}_m(\mathbf{x})$ and $\mathcal{H}_n(\mathbf{y})$. Each Hilbert space is equipped with a sequence of unitaries and there are an infinite number of complicated consistency conditions relating the time evolution in different Hilbert spaces. The basic claim is that every solution of these consistency conditions is a quantum space-time[4] In the limit where all causal diamonds are large it should determine a solution of Einstein's equations, coupled to sensible matter. *Note that there is no sense in which the quantum system itself can be thought of as a sum over space-time histories. Metrics appear only as a semi-classical artifact, and are truly emergent.*

5.3.1.1 *The variables of quantum gravity*

The holographic principle suggests that the fundamental geometrical object in Lorentzian space-time is the holographic screen of a causal diamond. This is a spatial $d-2$ surface on which all the information in the diamond is projected. Consider a little area or *pixel* on the holographic screen. It determines a null direction, which penetrates the pixel, and the screen element transverse to this null ray. The Cartan-Penrose equation

$$\bar{\psi}\gamma_\mu\psi\gamma^\mu\psi = 0,$$

encodes all of this information in a pure spinor ψ, determined up to a real or complex constant, depending on the dimension.

It is natural to quantize the real independent components, S_a of the pure spinor associated with a pixel by writing

$$[S_a, S_b]_+ = \delta_{ab}$$

[4]A quantum Big Bang cosmology for the choice we have made of the relation between Hilbert spaces for a single observer.

which is the most general rule giving a finite number of states per pixel and covariant under the transverse $SO(d-2)$ little group of the null vector $\bar{\psi}\gamma^{\mu}\psi$. It breaks the projective invariance of the CP equation to a Z_2 (for each pixel), which we treat as a gauge symmetry and identify with fermion parity $(-1)^F$, enforcing the usual connection between spin and statistics. This quantization rule implements the Bekenstein Hawking relation between quantum entropy, and area. The logarithm of the dimension of the Hilbert space of the irreducible representation of this algebra is the area of a fundamental pixel, in d dimensional Planck units. Compact dimensions lead to an enlarged pixel algebra, incorporating charges for Kaluza-Klein Killing symmetries, their magnetic duals, and wrapped brane configurations. Note that it is precisely these quantum numbers which remain unchanged under the topology changing duality transformations of string/M-theory. We will ignore the complications of compactification in the brief description which follows.

After performing a Klein transformation using the $(-1)^F$ gauge symmetry, the operator algebra of an entire causal diamond takes the form,

$$[S_a(n), S_b(m)]_+ = \delta_{ab}\delta_{mn}.$$

The middle alphabet labels stand for individual pixels. More generally we say that the geometry of the holographic screen is pixelated by replacing its algebra of functions by a finite dimensional algebra, and these labels stand for a general basis in that algebra. If we use finite dimensional non-abelian function algebras, we can have finite causal diamonds with exact rotational invariance, which would be appropriate for describing the local physics in asymptotically symmetric spacetimes.

The $S_a(n)$ operators should be thought of as transforming in the spinor bundle over the holographic screen. Informally, we can say that the algebra of operators of a pixel on the holoscreen, is described by the degrees of freedom of a massless superparticle which exits the holoscreen via that pixel. In a forthcoming paper[1], I will describe how an infinite dimensional limit of such a construction can reproduce the Fock space of eleven dimensional supergravity. The basic idea is to find a sequence of algebras which converges to

$$\mathcal{A}_{11} \equiv R_{[0,1]} \otimes M(S^9),$$

where $R_{[0,1]}$ is the unique, hyperfinite Type II_{∞} von Neumann factor, and $M(S^9)$ the algebra of measurable functions on the sphere. We take our quantum operator algebra to be operator valued linear functionals $S(q)$, $q \in \mathcal{S}[\mathcal{A}_{11}]$, which are invariant under inner automorphisms of $\mathcal{A}_{11}$. $\mathcal{S}$ is the spinor bundle over the algebra. The projectors in $R_{[0,1]}$ are characterized, up to inner automorphism, by their trace, which is a real number between 0 and ∞. A general invariant linear functional is determined by its value on a finite sum of projectors. Thus, the quantum algebra consists of finite collections of operators of the form $S_a(p_i)q^a(\mathbf{\Omega}_i)$ where p_i is a positive number and $\mathbf{\Omega}_i$ a direction on S^9. These parametrize a null momentum

$p_i(1, \mathbf{\Omega}_i)$. It can be shown that the resulting Hilbert space on which the quantum algebra is represented is the SUGRA Fock space. Massless particles and the overall scale of null momenta appear in a manner reminiscent of Matrix Theory[4], while S^9 parametrizes the direction of null momenta. The holographic formalism can thus reproduce the kinematics of M-theory. One would like to show that the dynamical consistency conditions lead to equations which determine the scattering matrix uniquely.

5.3.1.2 *Holographic cosmology*

Only one solution of the dynamical consistency conditions of holographic cosmology has been found[9]. In this model, the dynamics of a given observer is described, at each time t by a Hamiltonian which is a random *irrelevant* perturbation of a random bilinear Hamiltonian

$$H = \sum S_a(m) h_{mn} S_a(n).$$

The term irrelevant is used because, for large $t \times t$ matrices h, the bilinear dynamics approaches that of free massless $1+1$ dimensional fermions.

A simple prescription for the overlap Hilbert spaces, combined with this ansatz for the Hamiltonian, satisfies all the consistency conditions. One then observes the emergence, at large times, of a flat FRW geometry, with equation of state $p = \rho$. That is, one can define a distance function on the lattice in terms of which lattice points are causally disconnected at a given time. This geometry satisfies the scaling laws of the FRW universe. Moreover, in the large time limit, the exact quantum dynamics of the system is invariant under the conformal Killing symmetry of this geometry.

The $p = \rho$ cosmology was previous introduced by Fischler and the author[8] in terms of a heuristic picture of a *dense black hole fluid*. This is a system in which black holes continuously coalesce to form a single horizon filling black hole. The random fermion model above is a precise mathematical realization of this idea. Based on the heuristic picture, one can develop an ansatz for a more normal universe, with normal regions originally arising as small defects in the black hole fluid. This leads to a cosmology which can solve all of the standard cosmological puzzles, with only a small amount of inflation just before nucleosynthesis. Inflation is necessary only to stretch the scale of fluctuations generated during the $p = \rho$ era, to the size of our current horizon. The resulting cosmological model depends on only a few parameters, and in one parameter range the fluctuations are entirely generated in the $p = \rho$ era. They are *exactly* scale invariant, between sharp infrared and ultraviolet cutoffs. This fluctuation spectrum is, in principle, distinguishable from that of inflationary models. It might explain the apparent disagreement between inflationary models and the data at low L. We do not yet have predictions for the gravitational wave spectrum in this parameter range.

In other parameter regimes, inflation must generate the observed fluctuations, and this implies some kind of hybrid model, since the scale of inflation must be quite low. In this regime there are no observable tensor fluctuations. Even in this regime, holographic cosmology is an advance over standard inflation, because the primordial $p = \rho$ regime sets up the right initial conditions for inflation, starting from a fairly generic primordial state. Indeed, Penrose[10] and others have argued that conventional inflation models *do not* resolve the question of why the universe began in a low entropy state. In holographic cosmology, this might be resolved by the following line of argument: the most generic initial condition is the uniform $p = \rho$ fluid. The more normal universe, initially consists of defects in this fluid: regions where not all of the degrees of freedom in a horizon volume are excited. This has lower entropy, but can evolve into a stable normal universe if the following two conditions are met:

- The initial matter density in the normal regions is a dilute fluid of black holes. This fluid must be sufficiently homogeneous that black hole collisions do not result in a recollapse to the $p = \rho$ phase.
- The initial normal region either is a finite volume fraction of the infinite $p = \rho$ system, or contains only a finite number of degrees of freedom in total. The latter case, which is entropically favored, evolves to a de Sitter universe. Thus, holographic cosmology predicts a de Sitter universe with the largest cosmological constant compatible with the existence of observers. If the gross features of the theory of a small Λ universe is uniquely determined by Λ, then this may predict a universe with physics like our own.

5.3.1.3 *de Sitter space*

Holographic cosmology predicts that the asymptotic future is a de Sitter space, so it behooves us to construct a quantum theory for that symmetric space-time. We will restrict attention to 4 dimensions, which may be the only case where the quantum theory of de Sitter space is defined. In four dimensions, the holographic screen of the maximal causal diamond of any observer following a time-like geodesic, is a two sphere of radius R. Our general formalism tells us that we must pixelate the surface of this sphere in order to have a finite number of quantum states[3][11]. The most elegant pixelation is given by the fuzzy sphere.

The spinor bundle over the fuzzy sphere consists of complex $N \times N + 1$ matrices ψ_i^A, transforming in the $[N] \otimes [N+1]$ dimensional representation of $SU(2)$. If these are quantized as fermions:

$$[\psi_i^A, (\psi^\dagger)_B^j]_+ = \delta_i^j \delta_B^A,$$

then the Hilbert space of the system has entropy $N(N+1)\ln 2$, which will agree with the area formula for large N if $N \propto R$.

The natural Hamiltonian, H, for a geodesic observer in dS space would seem to be the one which generates motion along the observer's time-like Killing vector.

However, once quantum mechanics is taken into account, there are no stable localized states. Everything decays back into the dS vacuum. Classically, the vacuum has zero eigenvalue of H, but the semiclassical results of Gibbons and Hawking can be explained if we assume instead that the spectrum of H is spread more or less randomly between 0 and something of order the dS temperature, $T_{dS} = \frac{1}{2\pi R}$, with level density $e^{-\pi(RM_P)^2}$. The vacuum state corresponding to empty dS space is the thermal density matrix $\rho = e^{-\frac{H}{T_{dS}}}$. This ansatz explains the thermal nature of dS space, as well as its entropy, which for large R is very close to the log of the dimension of the Hilbert space.

However, the spectrum of H has no relation to our familiar notions of energy. In [12] the latter concept was argued to emerge only in the large R limit. It is an operator P_0 which converges to the Poincaré Hamiltonian of the limiting asymptotically flat space-time, in the reference frame of the static observer. The conventional argument that $H \to P_0$ is wrong. This is true for the mathematical action of Killing vectors on finite points in dS space. However, physical generators in GR are defined on boundaries of space-time. The cosmological horizon of dS space converges to null infinity in asymptotically flat space, and the two generators act differently on the boundary. The boundary action motivates the approximate commutation relation:

$$[H, P_0] \approx \frac{1}{R} P_0,$$

which says that eigenvalues of P_0 much smaller than the maximal black hole mass, are approximately conserved quantum numbers, which resolve the degeneracy of the spectrum of H. The corresponding eigenstates of P_0 are localized in the observer's horizon volume. Semiclassical physics indicates that

$$\mathrm{tr}\ e^{-H/T_{dS}} \delta(P_0 - E) \approx e^{-\frac{E}{T_{dS}}} \mathrm{tr}\ e^{-H/T_{dS}}.$$

This relation can be explained if, for small P_0 eigenvalue,E, the entropy deficit of the corresponding eigenspace, relative to the dS vacuum is equal to $(2\pi R)E$. This formula can be explicitly verified for black hole states, if we identify the mass parameter in the Kerr-Newman- de Sitter black hole with the Poincare eigenvalue.

Based on this picture it is relatively easy to identify black hole states in terms of the fermionic pixel operators. We work in the approximation in which all states of the vacuum ensemble are exactly degenerate, as well as all black hole eigenstates corresponding to the same classical solution. The vacuum is, in this approximation, just the unit density matrix. A black hole of radius K is the density matrix of all states satisfying

$$\psi_i^A |BH>= 0,$$

for $0 \leq i \leq K$ and $0 \leq A \leq K + 1$. This satisfies the geometrical relation between radius and entropy. For $K \ll N$ we can define the Hamiltonian P_0 (in T_{dS} units) to be the entropy deficit, as explained above. The choice of which K and $K + 1$

indices are used in the above equation is a gauge choice, equivalent to the choice of a particular horizon volume.

A similar analysis leads to a guess about the description of particle states in dS space. If, using ideas from quantum field theory, we ask for the maximal entropy states in dS space which contain no black holes whose radius goes to infinity with R, then we find that they are made from massless particles (or other conformal field theory degrees of freedom) with a typical momentum of order $(\frac{M_P}{R})^{-\frac{1}{2}}$. The entropy of such states scales like $(RM_P)^{3/2}$. These are the only states of the dS theory which are kept in the Poincare invariant $R \rightarrow \infty$ limit. We can view the full entropy of dS space as built up from $(RM_P)^{1/2}$ independent horizon volumes[2], each filled with such maximal entropy particle states.

In terms of the matrix ψ_i^A, we model these degrees of freedom as follows. Write the block decomposition

$$\psi = \begin{pmatrix} 1 & 2 & 3 & \ldots & M \\ M & 1 & 2 & \ldots & M-1 \\ \ldots & \ldots & \ldots & \ldots & \ldots \\ 2 & 3 & 4 & \ldots & 1 \end{pmatrix},$$

where each block is an independent $M \times M$ matrix, with $M \sim N^{1/2}$. The integer labels on the blocks refer to a given horizon volume, of which there are of order M. In a future publication [13] I hope to show, using Matrix Theory ideas, similar to those described in [1], that the degrees of freedom in a single block correspond, in the large M limit to those of a single 4 dimensional supergraviton. The integer M will be the longitudinal momentum of the supergraviton in units of the cutoff.

If this idea works, it is clear that corrections to the commutator $[P_0, Q_\alpha]$ for the super-Poincare algebra will be of order $N^{-\frac{1}{2}}$, which implies the scaling law for the gravitino mass postulated in [3].

The holographic approach to quantum gravity is thus a promising generalization of string theory which is applicable to cosmological backgrounds. Much work remains to be done to refine its principles, make more explicit contact with string theory, find non-perturbative equations for S-matrices in asymptotically flat space, and solve the quantum theory of de Sitter space.

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5.3.2 *Igor Klebanov: Confinement, Chiral Symmetry Breaking and String Theory*

The AdS/CFT duality [1–3] provides well-tested examples of emergent spacetimes. The best studied example is the emergence of an $AdS_5 \times \mathbf{S}^5$ background of type IIB string theory, supported by N units of quantized Ramond-Ramond flux, from the $\mathcal{N}=4$ supersymmetric $SU(N)$ gauge theory. The emergent spacetime has radii of curvature $L = (g_{\rm YM}^2 N)^{1/4}\sqrt{\alpha'}$. In the 't Hooft large N limit, $g_{\rm YM}^2 N$ is held fixed [4]. This corresponds to the classical limit of the string theory on $AdS_5 \times \mathbf{S}^5$ (the string loop corrections proceed in powers of $1/N^2$). The traditional Feynman graph perturbative expansion is in powers of the 't Hooft coupling $g_{\rm YM}^2 N$. The AdS/CFT duality allows us to develop a completely different perturbation theory that works for large 't Hooft coupling where the emergent spacetime is weakly curved. The string scale corrections to the supergravity limit proceed in powers of $\frac{\alpha'}{L^2} = \left(g_{\rm YM}^2 N\right)^{-1/2}$.

The metric of the Poincaré wedge of AdS_d is

$$ds^2 = \frac{L^2}{z^2}\left(dz^2 - (dx^0)^2 + \sum_{i=1}^{d-2}(dx^i)^2\right) . \tag{1}$$

Here $z \in [0,\infty)$ is an emergent dimension related to the energy scale in the gauge theory. In the AdS/CFT duality, fields in AdS space are dual to the gauge invariant local operators [2, 3]. The fundamental strings are dual to the chromo-electric flux lines in the gauge theory, providing a string theoretic set-up for calculating the quark anti-quark potential [5]. The quark and anti-quark are placed near the boundary of Anti-de Sitter space ($z = 0$), and the fundamental string connecting them is required to obey the equations of motion following from the Nambu action. The string bends into the interior ($z > 0$), and the maximum value of the z-coordinate is proportional to the separation r between quarks. An explicit calculation of the string action gives an attractive Coulombic $q\bar{q}$ potential [5]. Historically, a dual string description was expected mainly for confining gauge theories where long confining flux tubes have string-like properties. In a pleasant surprise, we have seen that a string description can apply to non-confining theories as well, due to the presence of extra dimensions in the string theory.

It is also possible to generalize the AdS/CFT correspondence in such a way that the $q\bar{q}$ potential is linear at large distances. In an effective 5-dimensional approach [6] the necessary metric is

$$ds^2 = \frac{dz^2}{z^2} + a^2(z)\big(- (dx^0)^2 + (dx^i)^2\big) \tag{2}$$

and the space must end at a maximum value of z where the "warp factor" $a^2(z_{\rm max})$ is finite. Placing widely separated probe quark and anti-quark near $z = 0$, we find that the string connecting them bends toward larger z until it stabilizes at $z_{\rm max}$ where its tension is minimized at the value $\frac{a^2(z_{\rm max})}{2\pi\alpha'}$. Thus, the confining flux tube is described by a fundamental string placed at $z = z_{\rm max}$ parallel to one of the x^i-directions. This establishes a duality between "emergent" chromo-electric flux tubes and fundamental strings in certain curved string theory backgrounds.

Several 10-dimensional supergravity backgrounds dual to confining gauge theories are now known, but they are somewhat more complicated than (2) in that the compact directions are "mixed" with the 5-d (x^μ, z) space. Witten [7] constructed a background in the universality class of non-supersymmetric pure glue gauge theory. While in this background there is no asymptotic freedom in the UV, hence no dimensional transmutation, the background has served as a simple model of confinement where many infrared observables have been calculated using the classical supergravity. For example, the lightest glueballs correspond to normalizable fluctuations around the supergravity solution.

Introduction of $\mathcal{N} = 1$ supersymmetry facilitates construction of confining gauge/string dualities. A useful method to generate $\mathcal{N} = 1$ dualities (for reviews, see [8, 9]) is to place a stack of N D3-branes at the tip of a Calabi-Yau cone, whose base is Y_5. In the near-horizon limit, one finds the background $AdS_5 \times Y_5$, which is conjectured to be dual to the superconformal gauge theory on the D3-branes. Furthermore, for spaces Y_5 whose topology is $\mathbf{S}^2 \times \mathbf{S}^3$, the conformal invariance may be broken by adding M D5-branes wrapped over the $\mathbf{S}^2$ at the tip of the cone. The gauge theory on such a combined stack is no longer conformal; it exhibits a novel pattern of quasi-periodic renormalization group flow, called a duality cascade [10, 9].

To date, the most extensive study of a cascading gauge theory has been carried out for a 6-d cone called the conifold. Here one finds a $\mathcal{N} = 1$ supersymmetric $SU(N) \times SU(N+M)$ theory coupled to chiral superfields A_1, A_2 in the $(\mathbf{N}, \overline{\mathbf{N}+\mathbf{M}})$ representation, B_1, B_2 in $(\overline{\mathbf{N}}, \mathbf{N}+\mathbf{M})$, with a quartic superpotential [11]. The M wrapped D5-branes create M units of R-R flux through the 3-cycle in the conifold. This flux creates a "geometric transition" to the deformed conifold $\sum_{a=1}^4 w_a^2 = \epsilon^2$, where the 3-cycle is blown up. An exact non-singular supergravity solution dual to the cascading gauge theory, incorporating the 3-form and the 5-form R-R field strengths and their back-reaction on the geometry, is the *warped deformed conifold* [10]

$$ds^2 = h^{-1/2}(\tau)\left(-(dx^0)^2 + (dx^i)^2\right) + h^{1/2}(\tau) d\tilde{s}_6^2 \ , \tag{3}$$

where $d\tilde{s}_6^2$ is the Calabi-Yau metric of the deformed conifold with radial coordinate τ. The 5-from R-R field, which is dual to N, decreases as the theory flows to the infrared (towards smaller τ).

What is the field theoretic interpretation of this effect? After a finite amount of RG flow, the $SU(N + M)$ group undergoes a Seiberg duality transformation [12]. After this transformation, and an interchange of the two gauge groups, the new gauge theory is $SU(\tilde{N}) \times SU(\tilde{N} + M)$ with the same matter and superpotential, and with $\tilde{N} = N - M$. The self-similar structure of the gauge theory under the Seiberg duality is the crucial fact that allows this pattern to repeat many times.

If $N = (k+1)M$, where k is an integer, then the duality cascade stops after k steps, and we find a $SU(M) \times SU(2M)$ gauge theory. This IR gauge theory exhibits a multitude of interesting effects visible in the dual supergravity background. One of them is confinement, which follows from the fact that the warp factor h is finite and non-vanishing at the smallest radial coordinate, $\tau = 0$, which roughly corresponds to $z = z_{\max}$ in an effective 5-d approach (2). This implies that the $q\bar{q}$ potential grows linearly at large distances. The confinement scale is proportional to $\epsilon^{2/3}$. The geometric transition that generates ϵ is dual in the gauge theory to a non-perturbative quantum deformation of the moduli space of vacua, which originates from *dimensional transmutation.* It breaks the Z_{2M} chiral R-symmetry, which rotates the complex conifold coordinates w_a, down to Z_2.

The string dual also incorporates the Goldstone mechanism due to a spontaneous breaking of the $U(1)$ baryon number symmetry [13]. Because of the $\mathcal{N} = 1$ SUSY, this produces a moduli space of confining vacua. In the $SU(M) \times SU(2M)$ gauge theory there exist baryonic operators $\mathcal{A} = A_1^M A_2^M$, $\mathcal{B} = B_1^M B_2^M$, which satisfy the "baryonic branch" relation $\mathcal{AB} =$ const. If the gauge theory were treated classically, this constant would vanish and the baryon symmetry would be unbroken. In the full quantum theory the constant arises non-perturbatively and deforms the moduli space [14]. The warped deformed conifold of [10] is dual to the locus $|\mathcal{A}| = |\mathcal{B}|$ in the gauge theory. Remarkably, the more general "throat" backgrounds, the *resolved warped deformed conifolds* corresponding to the entire baryonic branch, were constructed in [15]. These backgrounds were further studied in [16] where various observables were calculated along the baryonic branch. It was shown that a D3-brane moving on a resolved warped deformed conifold has a monotonically rising potential that asymptotes to a constant value at large radius. Therefore, when such a construction is embedded into a string compactification, it may serve as a model of inflationary universe, with the position of the 3-brane on the throat playing the role of the inflaton field, as in [17].

Throughout its history, string theory has been intertwined with the theory of strong interactions. The AdS/CFT correspondence [1–3] succeeded in making precise connections between conformal 4-dimensional gauge theories and superstring theories in 10 dimensions. This duality leads to many dynamical predictions about strongly coupled gauge theories. Extensions of the AdS/CFT correspondence to confining gauge theories provide new geometrical viewpoints on such important phenomena as chiral symmetry breaking, dimensional transmutation, and quantum deformations of moduli spaces of supersymmetric vacua. They allow for studying glueball spectra, string tensions, and other observables. The throat backgrounds that arise in this context may have applications also to physics beyond the standard model, and to cosmological modeling.

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5.3.3 *Juan Maldacena: Comments on emergent space-time*

Einstein looked at his equation

$$G_{\mu\nu} = T_{\mu\nu} \tag{1}$$

and he noticed that the left hand side is very beautiful and geometrical. On the other hand, the right hand side is related to the precise dynamics of matter and it depends on all the details of particle physics. Why isn't the right hand side as nice, beautiful and geometric as the left hand side?.

String theory partially solves this problem since in string theory there is no sharp distinction between matter and geometry. All excitations are described by different modes of a string. However, giving a stringy spacetime involves more than fixing the metric, it involves setting the values of all *massive string modes*. The classical string equations are given by the β functions of the two dimensional conformal field theory [1]

$$\beta_{g_i} = 0 \tag{2}$$

These equations unify gravity and matter dynamics. However, these are just the classical equations and one would like to find the full quantum equations that describe spacetime.

In order to understand the full structure of spacetime we need to go beyond perturbation theory. There are several ways of doing this depending on the asymptotic boundary conditions. The earliest and simplest examples are the "old matrix models" which describe strings in two or less dimensions [2]. We also have the BFSS matrix model which describes 11 dimensional flat space [3]. Another example is the gauge/gravity duality (AdS/CFT)[4, 5]. In all these examples we have a relation which says that an ordinary quantum mechanical system with no gravity is dual to a theory with gravity. Some of the dimensions of space are an emergent phenomenon, they are not present in the original theory but they appear in the semiclassical analysis of the dynamics.

In the gauge theory/gravity duality we have a relation of the form [5]

$$\Psi[ag] \sim e^{(a^D + \cdots)} Z_{Field\ theory}[g] \tag{3}$$

which relates the large a limit of the wavefunction of the universe to the field theory partition function, where a is the scale factor for the metric on a slice of the geometry.

Note that in this relation, the full stringy geometry near the boundary determines the field theory. It determines the lagrangian of the field theory. The full partition function is then equivalent to performing the full sum over interior stringy geometries. In the ordinary ADM parametrization we can think of the dynamical variables of 3+1 dimensional general relativity as given by 3-geometries. The analogous role in string theory is then played by the space of couplings in the field theory, since these are the quantities that the wavefunction depends on. By deforming the examples we know, it seems that it might be possible, in principle, to obtain any field theory we could imagine. In this way we see that the configuration space for a quantum spacetime seems to be related to the space of all possible field theories. This is a space which seems dauntingly large and hard to manage. So, in some sense, the wavefunction of the universe is the answer to all questions. At least all questions we can map to a field theory problem.

After many years of work on the subject there are some things that are not completely well understood. For example, it is not completely clear how locality emerges in the bulk. An important question is the following. What are the field theories that give rise to a macroscopic spacetime?. In other words, we want theories where there is a big separation of scales between the size of the geometry and the scale where the geometric description breaks down. Let us consider an AdS_4 space whose radius of curvature is much larger than the planck scale. Then the corresponding $2+1$ dimensional field theory has to have a number of degrees of freedom which goes as

$$c \sim \frac{R^2_{AdS}}{l^2_{Planck}} \tag{4}$$

In addition we need to require that all single particle or "single trace" operators with large spin should have a relatively large anomalous dimension. In other words, if we denote by Δ_{lowest} the lowest scaling dimension of operators with spin larger than two. Then we expect that the gravity description should fail at a distance scale given by

$$L \sim \frac{R_{AdS}}{\Delta_{lowest}} \tag{5}$$

It is natural to think that the converse might also be true. Namely, if we have a theory where all single trace higher spin operators have a large scaling dimension, then the gravity description would be good.

By the way, this implies that the dual of bosonic Yang Mills would have a radius of curvature comparable to the string scale since, experimentally, the gap between the mesons of spin one and spin larger than one is not very large.

One of the most interesting questions is how to describe the interior of black holes. The results in this area are suggesting that the interior geometry arises from an analytic continuation from the outside. Of course, we know that this is how we obtained the classical geometry in the first place. But the idea is that, even in a more precise description, perhaps the interior exists only as an analytic continuation [6]. A simple analogy that one could make here is the following. One can consider a simple gaussian matrix integral over $N \times N$ matrices [2]. By diagonalizing the matrix we can think in terms of eigenvalues. We can consider observables which are defined in the complex plane, the plane where the eigenvalues live. It turns out that in the large N limit the eigenvalues produce a cut on the plane and now these observables can be analytically continued to a second sheet. In the exact description the observables are defined on the plane, but in the large N approximation they can be defined on both sheets.

Faced with this situation the first reaction would be to say that the interior does not make sense. On the other hand we could ask the question: What is wrong with existence only as an approximate analytic continuation?. This might be good enough for the observers living in the interior, since they cannot make exact measurements anyway.

It seems that in order to make progress on this problem we might need to give up the requirement of a precise description and we might be forced to think about a framework, where even in principle, quantities are approximate.

One of the main puzzles in the emergence of space-time is the emergence of time. By a simple analogy with AdS/CFT people, have proposed a dS/CFT [7]. The idea is to replace the formula (3) by a similar looking formula except that the left hand side is the wavefunction of the universe in a lorentzian region, in a regime where it is peaked on a de-Sitter universe. Note that a given field theory is useful to compute a specific amplitude, but in order to compute probabilities we need to consider different field theories at once. For example, we should be able to vary the parameters defining the field theory. In AdS/CFT the way we fill the interior depends on the values of the parameters of the field theory. In this case this dependence translates into a dependence on the question we ask. So, for example, let us suppose that the de-Sitter ground state corresponds to a conformal field theory. If we are interested in filling this de-Sitter space with some density of particles, then we will need to add some operators in the field theory and these operators might modify the field theory in the IR. So they modify the most likely geometry in the past. So it is clear that in this framework, our existence will be part of the input. On the other hand, it is hard to see how constraining this is. In particular, empty de-Sitter space is favored by an exponentially large factor $e^{1/\Lambda}$. On the other hand, it is unclear that requiring our existence alone would beat this factor and produce the much less enthropic early universe that seems to have existed in our past.

Of course, dS/CFT suffers from the problem that we do not know a single example of the duality. Moreover, de-Sitter constructions based on string theory produce it only as a metastable state. In any case, some of the above remarks would also apply if we were to end up with a $\Lambda = 0$ supersymmetric universe in the far future. In that case, we might be able to have a dual description of the physics in such a cosmological $\Lambda = 0$ universe. It seems reasonable to think that these hypothetical dual descriptions would give us the amplitudes to end in particular configurations. In order to compute probabilities about the present we would have to sum over many different future outcomes.

In summary, precise dual descriptions are expected to exist only when the space-time has well defined stable asymptotics. In all other situations, we expect that the description of physics might be fundamentally imprecise. Let us hope that we will soon have a clear example of a description of a cosmological singularity.

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5.3.4 *Alexander Polyakov: Beyond space-time*

In what follows I shall briefly describe various mechanisms operating in and around string theory. This theory provides a novel view of space-time. I would compare it with the view of heat provided by statistical mechanics. At the first stage the word "heat" describes our feelings. At the second we try to quantify it by using equations of thermodynamics. And finally comes an astonishing hypothesis that heat is a reflection of molecular disorder. This is encoded in one of the most fascinating relations ever, the Boltzmann relation between entropy and probability.

Similar stages can be discerned in string theory. The first is of course the perception of space-time. The second is its description using the Einstein equations. The third is perhaps a possibility to describe quantum space-time by the boundary gauge theory. Let us discuss in more details our limited but important knowledge of the gauge/string correspondence.

5.3.4.1 *Gauge /String correspondence*

It consists of several steps. First we try to describe the dynamics of a non-abelian flux line by some string theory. That means, among other things that the Wilson loop $W(C)$ must be represented as a sum over 2d random surfaces immersed in the flat 4d space-time and bounded by the contour C. Surprisingly, strings in 4d behave as if they are living in the 5d space, the fifth (Liouville) dimension being a result of quantum fluctuations[1] . More detailed analyses shows that while the 4d space is flat, the 5d must be warped with the metric

$$ds^2 = d\varphi^2 + a^2(\varphi)d\overrightarrow{x}^2 \tag{1}$$

where the scale factor $a(\varphi)$ must be determined from the condition of conformal symmetry on the world sheet [2]. This is the right habitat for the gauge theory strings. If the gauge theory is conformally invariant (having a zero beta function) the isometries of the metric must form a conformal group. This happens for the space of constant negative curvature, $a(\varphi) \sim \exp c\varphi$ [3]. The precise meaning of the gauge/strings correspondence [4],[5] is that there is an isomorphism between the single trace operators of a gauge theory, e.g. $Tr(\nabla^k F_{\mu\nu}\nabla^l F_{\lambda\rho}...)$ and the on-shell vertex operators of the string, propagating in the above background. In other words, the $S-$ matrix of a string in the 5d warped space is equal to a correlator of a gauge theory in the flat 4d space. The Yang -Mills equations of motion imply that the single trace operators containing $\nabla_\mu F_{\mu\nu}$ are equal to zero. On the string theory side it corresponds to the null vectors of the Virasoro algebra, leading to the linear relations between the vertex operators. If we pass to the generating functional of the various Yang- Mills operators, we can encode the above relation in the formula

$$\Psi_{WOE}[h_{\mu\nu}(x), ...] = \langle \exp \int dx h_{\mu\nu}(x) T_{\mu\nu}(x) + ...\rangle_{Y.-M} \tag{2}$$

Here at the left hand side we have the "wave function of everything" (WOE). It is obtained as a functional integral over 5d geometries with the metric $g_{mn}(x, y)$, where $y = \exp -c\varphi$, satisfying asymptotic condition at infinity ($y \to 0$) $g_{\mu\nu} \to \frac{1}{y^2}(\delta_{\mu\nu} + h_{\mu\nu}(x))$. It differs from the "wave function of the universe" by Hartle and Hawking only by the y^{-2} factor. On the right side we have an expression defined in terms of the Yang- Mills only, $T_{\mu\nu}$ being its energy- momentum tensor. The dots stand for the various string fields which are not shown explicitly. An interesting unsolved problem is to find the wave equation satisfied by Ψ. It is not the Wheeler -de Witt equation. The experience with the loop equations of QCD tells us that the general structure of the wave equation must be as following

$$\mathcal{H}\Psi = \Psi * \Psi \tag{3}$$

where $\mathcal{H}$ is some analogue of the loop Laplacian and the star product is yet to be defined. This conjectured non-linearity may lead to the existence of soliton-like WOE-s.

The formula (2) , like the Boltzmann formula, is relating objects of very different nature. This formula has been confirmed in various limiting cases in which either LHS or RHS or both can be calculated. I suspect that, like with the Boltzmann formula, its true meaning will still be discussed a hundred years from now.

5.3.4.2 *de Sitter Space and Dyson's instability*

Above we discussed the gauge/ strings duality for the geometries which asymptotically have negative curvature. What happens in the de Sitter case ? It is not very clear. There have been a number of attempts to understand it [6]. We will try here a different approach. It doesn't solve the problem, but perhaps gives a sense of the right direction.

Let us begin with the 2d model, the Liouville theory. Its partition function is given by

$$Z(\mu) = \int D\varphi \exp\{-\frac{c}{48\pi}\int d^2x(\frac{1}{2}(\partial\varphi)^2 + \mu e^{\varphi}) \tag{4}$$

For large c (the Liouville central charge) one can use the classical approximation. The classical solution with positive μ describes the AdS space with the scalar curvature -μ.By the use of various methods [7] one can find an exact answer for the partition function, $Z \sim \mu^{\alpha}$ where $\alpha = \frac{1}{12}[c - 1 + \sqrt{(c-1)(c-25)}]$. In order to go to the de Sitter space we have to change $\mu \Rightarrow -\mu$. Then the partition function acquires an imaginary part, $\operatorname{Im} Z \sim \sin \pi\alpha|\mu|^{\alpha}$. It seems natural to assume that the imaginary part of the Euclidean partition function means that the de Sitter space is intrinsically unstable. This instability perhaps means that due to the Gibbons -Hawking temperature of this space it "evaporates" like a simple black hole. In the latter its mass decreases with time, in the de-Sitter space it is the cosmological constant. If we define the Gibbons- Hawking entropy S in the usual way, $S = (1 - \beta\frac{\partial}{\partial\beta})\log Z$, we find another tantalizing relation, $\operatorname{Im} Z \sim e^S$,which holds in the classical limit, $c \to \infty$. Its natural interpretation is that the decay rate of the dS space is proportional to the number of states, but it is still a speculation, since the precise meaning of the entropy is not clear. For further progress the euclidean field theory, used above, is inadequate and must be replaced with the Schwinger-Keldysh methods.

In higher dimensions we can try once again the method of analytic continuation from the AdS space. The AdS geometry is dual to a conformally invariant gauge field theory. In the strong coupling limit (which we consider for simplicity only) the scalar curvature of the AdS, $R \propto \frac{1}{\sqrt{\lambda}}$ ($\lambda = g_{YM}^2 N$). So, the analytic continuation we should be looking for is $\sqrt{\lambda} \Rightarrow -\sqrt{\lambda}$. In order to understand what it means in the gauge theory, let us notice that in the same limit the Coulomb interaction of two charges is proportional to $\sqrt{\lambda}$ [8]. Hence under the analytic continuation we get a theory in which the same charges attract each other. Fifty years ago Dyson has shown that the vacuum in such a system is unstable due to creation of the clouds of particles with the same charge. It is natural to conjecture that Dyson's instability of the gauge theory translates into the intrinsic instability of the de Sitter space. Once again the cosmological constant evaporates.

5.3.4.3 *Descent to four dimensions*

Critical dimension in string theory is ten . How it becomes four ? If we consider type two superstrings, the 10d vacuum is stable, at least perturbatively., and stays 10d. Let us take a look at the type zero strings, which correspond to a non-chiral GSO projections. These strings contain a tachyon, described by a relevant operator of the string sigma model. Relevant operators drive a system from one fixed point to another. According to Zamolodchikov's theorem, the central charge must decrease in the process. That means that the string becomes non-critical and the Liouville field must appear. The Liouville dimension provides us with the emergent "time" in which the system evolves and changes its effective dimensionality (the central charge). As the "time" goes by, the effective dimensionality of the system goes down. If nothing stops it, we should end up with the $c = 0$ system which has only the Liouville field. It is possible,however, that non-perturbative effects would stop this slide to nothingness [10]. In four dimensions we have the B -field instantons, described by the formula (at large distances) $(dB)_{\mu\nu\lambda} = q\epsilon_{\mu\nu\lambda\rho}\frac{x_\rho}{x^4}$. In the modern language they correspond to the NS5 branes. These instantons form a Coulomb plasma with the action $S \sim \sum \frac{q_i q_j}{(x_i - x_j)^2}$. As was explained in [10] , the Debye screening in this plasma causes "string confinement", turning the string into a membrane. Formally this is described by the relation

$$\langle \exp i \int B_{\mu\nu} d\sigma_{\mu\nu} \rangle \sim e^{-aV} \tag{5}$$

where we integrate over the string world sheet and V is the volume enclosed by it. There is an obvious analogy with Wilson's confinement criterion. While the gravitons remain unaffected, the sigma model description stops being applicable and hopefully the sliding stops at 4d.

5.3.4.4 *Screening of the cosmological constant*

Classical limits in quantum field theories are often not straightforward. For example, classical solutions of the Yang - Mills theory describing interaction of two charges have little to do with the actual interaction. The reason is that because of the strong infrared effects the effective action of the theory has no resemblance to the classical action. In the Einstein gravity without a cosmological constant the IR effects are absent and the classical equations make sense. This is because the interaction of gravitons contain derivatives and is irrelevant in the infrared.

The situation with the cosmological term is quite different, since it doesn't contain derivatives. Here we can expect strong infrared effects [11] , see also [9] for the recent discussion.

Let us begin with the 2d model (3) . The value of μ in this lagrangian is subject to renormalization. Perturbation theory generates logarithmic corrections to this quantity. It is easy to sum up all these logs and get the result $\mu_{ph} = \mu(\frac{\Lambda}{\mu_{ph}})^{\beta}$, with $\beta = \frac{1}{12}[c - 13 - \sqrt{(c-1)(c-25)}]$.Here Λ is an UV cut-off while the physical (negative) cosmological constant μ_{ph} provides a self-consistent IR cut-off. We see that in this case the negative cosmological constant is anti-screened.

In four dimensions the problem is unsolved. For a crude model one can look at the IR effect of the conformally flat metrics. If the metric $g_{\mu\nu} = \varphi^2\delta_{\mu\nu}$ is substituted in the Einstein action S with the cosmological constant Λ,the result is $S = \int d^4x[-\frac{1}{2}(\partial\varphi)^2 + \Lambda\varphi^4]$. There is the well known non-positivity of this action. This is an interesting topic by itself, but here we will not discuss it and simply follow the prescription of Gibbons and Hawking and change $\varphi \Rightarrow i\varphi$. After that we obtain a well defined φ^4 theory with the coupling constant equal to Λ. This theory has an infrared fixed point at zero coupling, meaning that the cosmological constant screens to zero.

There exists a well known argument against the importance of the infrared effects. It states that in the limit of very large wave length the perturbations can be viewed as a change of the coordinate system and thus are simply gauge artefacts.This argument is perfectly reasonable when we discuss small fluctuations at the fixed background (see [12] for a different point of view). However in the case above the effect is non-perturbative- it is caused by the fluctuation of the metric near zero, not near some background. In this circumstances the argument fails. Indeed, if we look at the scalar curvature, it has the form $R \sim \varphi^{-3}\partial^2\varphi$. We see that while for the perturbative fluctuations it is always small because of the second derivatives, when φ is allowed to be near zero this smallness can be compensated. In the above primitive model the physical cosmological constant is determined from the equation $\Lambda_{ph} = \frac{const}{\log\frac{1}{\Lambda_{ph}}}$ which always has a zero solution. One would expect that in the time-dependent formalism we would get a slow evaporation instead of this zero. The main challenge for these ideas is to go beyond the conformally flat fluctuations. Perhaps gauge/ strings correspondence will help.

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5.4 Discussion

G. 't Hooft I would like to make sort of a claim or statement and then a question. Actually it bears on Nati's talk, but also others have mentioned emergent space and emergent time, and I claim that any theory you have allows a rigorous definition of time, not a fuzzy one, and even a rigorous definition of space, and not a fuzzy definition of space. And the argument goes as follows. Assume you have some theory that is supposed to explain some phenomenon. A priori there was no space, no time in the question you've been asking. You just have a theory. Then the theory will contain variables and equations, and a lot of prescriptions how to solve these equations, if it is a good theory. And I claim that, as soon as you have indicated the order by which you have to solve the equations, that order defines causality in your theory, and that defines a notion of time. So time is basically the order by which you have to solve the equations. If you think a little bit, that's exactly for instance how a theory of the planetary system works. The time, the notion of time among the planets is the order by which you solve the equations. If you solve the equations in the wrong order, you might have forgotten that two planets might collide, and then you get impossible answers. So you have to know, exactly that time is the order by which you solve the equations. And, so that is a rigorous definition, there's no way to fool around with that, because if you solve them in the wrong order, you might get the wrong answer. Similarly however, you can also make a rigorous definition of space. And that is because, well, I must assume some form of reduction of a theory into simple equations. If you write down infinitely complicated equations, you don't really know what you're doing, you have to reduce them to simple equations. And then you can ask, two sets of variables, how many equations are they away from each other? And that defines a distance between variables, and that eventually defines space. If you think a little bit, that's the way our present space-time seems to work, that two systems are far away if you have to solve differential equations very very many times before you reach from one point to the other point. So I think that any theory should contain some notion of space as well as time, and in a discussion during the break, the question was asked: does this defines a continuous space-time? And I would say no, most time and space would be discrete in this sense, but two variables like that, connected with by an equation, that defines them to be nearest neighbours, that defines a distance one. And then, so any theory in some sense looks like a lattice.

J. Harvey I cannot resist comparing, that if this defines not only the order of time, but the rate at which time proceeds, then I am sure time proceeds much faster in your part of the world than in my part of the world.

G. 't Hooft But anyway, the question comes then, that if you would drop the notion of reduction, then what are you doing, and is that not a direct contradiction in terms, that we do want theories to be based on simple equations, therefore reduction, in that sense, seems to be absolutely necessary to me, and if you do not have that, should you then not do something else, go into music instead of theoretical physics? That is the question.

J. Polchinski So I would like to respond to Abhay's comments from earlier and ask a question. In canonical general relativity, you write the wave function in terms of geometries at one time, solve the hamiltonian constraint, and time emerges as correlations in that wave functions. But when you talk about emergent gauge symmetry and AdS/CFT, there is something more that happens. Because there's a set of variables which are actually completely neutral under the gauge symmetry, you don't have a hamiltonian constraint, you have variables that satisfy it trivially, and they're related in a complicated way to the other ones. But you can solve the theory, and write the observables entirely in terms of the gauge invariant observables, the ones on which the hamiltonian constraint is already solved. And the question then is: is there some analogue of this which is known, say, in other approaches to quantum gravity?

J. Harvey Does anyone want to answer that question or respond to it? Nati?

N. Seiberg I think the answer is exactly the comment I made to Steve, that in the matrix model description of two dimensional gravity this is exactly what happens.

E. Rabinovici In regard to emerging space-times I have a comment. I think we should attempt to use methods which we learned from statistical mechanics, appropriately modified for gravity, to study possible phases of gravity. And one phase which I think is necessary, we should study more, is that in which α' is infinity, or in other words where the string scale vanishes. And I think once we understand that, it could help also understand more the emergence of space-time.

G. Dvali I just wanted to comment that all these important questions about emerging nature of gravity and space-time at short distances, in the UV, probably should be also asked about large distances, in the infrared, because after all, we only understand, experimentally at least, we only understand the nature of space-time and gravity at intermediate distances, and we have no idea what is happening beyond the 10^{28} centimeters, and we know that something is going on there, the universe is accelerating. Normally we are attributing it to a cosmological constant, but it may very well be that string theory encodes new far infrared scale, and so the nature of space-time and gravity gets dramatically modified there. So space-time may emerge in the UV, and also it happens something in the IR. This question also should be studied.

J. Harvey I think that is sort of along the lines of what Nati called the old approach to the cosmological constant problem, that there's some confusion between UV and IR, which I think is perhaps old, but not completely forgotten.

J. Maldacena Yes, I was going just to mention that this question of very long distances outside our horizon—so even if we had a precise description, sort of reductionist description in the far future where the universe is infinitely large and so on, it might involve regions outside our horizon, and then when we ask a question about our universe, we need to sum over everything that's going on in the outside, so we have to sum over theories and so on. That would be probably an important part.

T. Damour I am confused about in what sense really dynamical gravity emerges in AdS/CFT, and the apparent contradiction with the theorem of Weinberg and Witten. So, in what we saw from Sasha and Juan, the $h_{\mu\nu}$ on the boundary does not satisfy any constraint, so it's not really dynamical; how will the Wheeler-de Witt equation come out? Or let me ask a more practical question, are there correlators, multi-correlators in super Yang-Mills, where I see the massless spin two pole of a dynamical graviton emerging, in super Yang-Mills?

J. Maldacena The answer is no, because the graviton is not a massless particle in four dimensions. So it is a massless particle in five dimensions. So we don't have a local stress tensor in five dimensions, we have a local stress tensor in four. And so the gravity that emerges is the five dimensional gravity, and this is an important point.

G. Gibbons I have a question really for Tom Banks, which is that the formalism he outlined seems to take causal structures primary and given once and for all. So do you envisage that the causal structure varies and fluctuates, or that we have always a fixed causal structure? And if the latter, one normally says that nine tenths of the metric, in dimension four, are given by the causal structure, so you also seem not to be allowing the gravitational field to fluctuate, if you took the view that the causal structure was fixed.

T. Banks In this formalism, the causal structure is put in, but the variables that are going to describe the geometry are quantum fluctuating, so it's a quantum mechanical causal structure. You get a geometry that comes out of it in the large area limit which is unique, and indeed that causal structure will be fixed, but it will only be fixed in the large area limit where things are approximately semiclassical.

G. Gibbons So for you locality is really not a problem, it is fixed. We always have a local theory if I understand you ... and it makes sense to say things commute at space-like separations...

T. Banks There are certainly no operators here that you can define at local points that commute in space-like separations in this formalism. Perhaps we should, I should talk to you privately about this.

J. Harvey David, did you have something you wanted to say?

D. Gross Yes. I think Gerard correctly pointed out some of the things one would like space and time to satisfy in an emergent scenario. The problem is that it might not be obvious how to do that when one looks for the principle that will lead to an ordering and one time. Perhaps some kind of reductionism, although given the ultraviolet-infrared connection we are not even sure of this. Since we don't know what that principle is then how do we know that, if space-time is emergent, one time will emerge? One question about space-time that has not come up here, which has always intrigued me, is why can we easily imagine alternate topologies, alternate dimensions of space, different than what we see around us, but it seems impossible to imagine more than one time. Is a single time anthropically selected? (This is a modern strategy for eliminating things you don't understand.) Or is it simply impossible to imagine physics with more than one time? So when you give up the foundational setting of physics, as people are struggling to do, you really would like to know what principles lead to a unique causal ordering.

The other comment I wanted to make was about locality in string theory and its connection with causality or the analyticity of the S-matrix. This is something we might understand. After all, we have the S-matrix in string theory, and we can ask why is it analytic? Why is string theory causal even though strings are non-local and do not interact locally, at least if you define space-time in terms of the center of mass of the string (i.e. choose a gauge in which the center of mass of the string is identified with space-time). In string field theory, for example, the fields depend not on points in space-time, but on loops, and in terms of the center of mass of the string, as Nati pointed out, the interations of such fields are then non-local. But this is not the way to define space-time in string field theory. In fact, locality becomes manifest and one can derive the analytic properties of the S-matrix if you either work in light-cone gauge, where strings interact at the same light cone time at a single point, or covariantly, if you define space-time to as the midpoint of the string, where the interaction takes place (at least in open string field theory). In both of these cases one can understand, in string perturbation theory, how locality and causality emerge from the interaction of non-local objects such as strings. On the other hand we do not know how to address what happens non-perturbatively. If we use the AdS/CFT duality to non-perturbatively define ten dimensional string theory we are hard pressed to recover locality in the bulk. In addition, in cases we now begin to understand, we see non-local structures emerge from string theory, such as non-commutative field theories, as a description of dual theory on the boundary. There is a whole program, which I would urge people to follow, to explore, in the context of field theory, what kinds of non-localities are allowable and controllable and sensible, expanding what we already know about the non-local field theories that are healthy boundary theories dual to string theories in the bulk.

J. Harvey One of the topics that Nati raised which I must say confuses me as well, is in situations where—string theory on a circle for example—where in different limits you would use one description or its T-dual. It's very tempting to think that you should have a formulation where you have both x-left and x-right, but it seems difficult to have locality in both and so which is one supposed to choose, or what principle determines that. I ... Do you feel like that's ... that there is a clear understanding of that, because I find that also very confusing?

D. Gross It is clear, because of this ambiguity, that locality or even what we mean by space-time is something like gauge invariance. It is a description that is inherently ambiguous and there are different descriptions which are useful for different purposes.

J. Harvey Right, but perhaps we are missing the additional gauge degrees of freedom that allow us to project in a clear way onto the different local descriptions right now. You could imagine that there's a formulation where—I guess people have attempted this—where both the variable and the T-dual are there at the same time but there is some additional redundancy.

M. Gell-Mann When I left this field and stopped following what was going on in detail, people had proposed a version of string theory in which there's another variable running along the string, and when you have that, then you can say that in a string vertex, the old string and the two new ones, or the two old ones and the one new one, are laid along one another, so that there is exact locality for every point on the string. It's not a question of the center of mass at all. And I assume that in the intervening years that hasn't disappeared. It's a much more satisfactory way of treating locality.

J. Harvey Alright. As a moderator I don't feel obliged to answer any questions, so if anybody else would like to answer that I'll... the author of a textbook, or...

G. Veneziano As far as I know, Murray is right, I mean, I thought that in — at least in some versions of string field theory — that is exactly what you do. You put a local coupling in terms of strings, namely when three strings overlap completely, then they interact. This is an invariant local concept, I think.

D. Gross That is absolutely wrong. It is nice to be able to make absolute statements. It is not true that strings interact when they overlap— were it the case that string theory interaction consisted of a vertex where strings totally overlapped, it would be infinitely more nonrenormalizable than ordinary quantum field theory, require an infinite number of constants, have no relation to two-dimensional geometry, and be totally different than the string theory that we know and love. Instead strings meet at one point (in light cone gauge) or overlap on half their length (in the covariant open string field theory approach). The problem is that, unless care is taken to define time carefully, this interaction need not be local in time.

G. Veneziano That is what I thought was the Witten open string field theory action, that it had really overlapping strings.

Note by the editors The discussion between Gross and Veneziano continued over lunch during which the misunderstandings were elucidated. Summarizing: while a local (say ϕ^3) interaction in field theory means that three fields interact when their coordinates all coincide ($x_1 = x_2 = x_3$), three strings interact when every point on one string also belongs to another string. Forcing all three strings to overlap completely by imposing $x_1(\sigma) = x_2(\sigma) = x_3(\sigma)$ would indeed lead to incurable UV divergences. Instead, the condition for having a string interaction is more like momentum conservation, but in coordinate space, i.e. $x_1+x_2+x_3 = 0$ (where string bits of opposite orientation are counted with a relative minus sign). This is how string theory provides its compromise between locality and non-locality.

G. Veneziano May I take advantage to... I am just wondering, in view also of what Sasha Polyakov has emphasized, namely that the AdS/CFT correspondence is between an on-shell theory in the bulk and an off-shell theory on the boundary, whether we should really see this as a correspondence, or, you know, just as a tool, OK? After all, suppose on the boundary we manage to have QCD, just QCD, no weak interactions, just QCD, then on the boundary itself, we just may be interested in the only observable, which is the S-matrix on the boundary. And that S-matrix on the boundary would not be in itself sufficient to determine what goes on in the bulk. In other words, it looks to me that this on- versus off-shell duality may mean that actually the boundary field theory is a tool to—the off-shell boundary field theory is just a tool rather than an equivalent thing.

A. Polyakov Well, you know, before you turned on gravity, you have a choice, then off-shell quantities are more or less well-defined. You don't have to consider necessarily the S-matrix. I think this – what we were taught in the days that field theory was despised, that the only thing which makes sense is the S-matrix, which is actually true in the theory of gravity, almost true. But in normal field theory, it is not true, so I think it's quite appropriate that since we make the contact between theory of gravity and the theory without gravity, on the gravity side we must have only on-shell amplitude, while on the theory without gravity side we may have all possible correlation functions, not necessarily on-shell.

J. Harvey It has always seemed to me that AdS/CFT should, you know, be perhaps a precise statement, but should allow for inexact statements. I mean, after all if we had discovered that we live in anti-de Sitter space with a very small cosmological constant rather than de Sitter space, and we went to the experimentalists in Fermilab and told them that what they observe and measure is not real, because it's not defined at infinity, I think they would regard us as rather useless. So it is clear that there has to be room for a description that is, you know, an exceedingly good approximation to observables defined at surfaces at infinity that are defined in a local way in the bulk. But how to actually do this within the machinery we currently have seems to me rather problematic.

D. Gross This indeed is a very deep problem, because we can imagine compact spaces in general relativity, and then we have no local gauge-invariant observables and no place for a holographic description in terms of something which doesn't include gravity, which is well-defined. I find the discussion of compact universes extremely puzzling. Nati said that he has no problem thinking about closed universes, but how do you think about it, Nati, or how does anyone think about what are physical observables in such situations?

N. Seiberg Maybe I should clarify what I have in mind. I have a lot of problems thinking about it, but I don't see any obstacle to their existence.

D. Gross Agreed, but given that they might exist, how do you think about them?

N. Seiberg Well, we have one example of this in two dimensions, which is the quantization of the world-sheet of the string. This is a very concrete example.

J. Maldacena Yes, it seems that in order to have a description of these closed universes, you have to allow yourself the possibility of not having an exact description, that the fundamental description will be fundamentally not precise, I think.

E. Silverstein In fact this can be borne out by holographically dualizing closed universes in the same way that we do for Randall-Sundrum, where indeed the dual description also has lower-dimensional gravity, and you can continue that down to two dimensional gravity plus a large amount of matter and obtain at least a simplification of the problem, but one which illustrates the limitations that you all are talking about.

A. Ashtekar David, what is wrong with gauge-fixing? So if you had closed universes, right, there is gauge-fixed description, and then of course there has to be consistency checks that different choices of gauges will give you the same...

D. Gross As Nati emphasized, gauge symmetries are redundancies in our description of nature, and presumably there are, as the AdS/CFT duality beautifully illustrates, formulations of generally covariant theories where there is no general covariance needed since there are only physical degrees of freedom used to describe the system, so...

A. Ashtekar Maybe I am saying the same.

D. Gross AdS/CFT is the one description of quantum gravity that is best defined in our toolbox, and using the gauge theory description of quantum gravity in 10 dimensions there is no such thing a gauge fixing, because there is no gauge to fix.

B. Greene So just back to the general question of emerging space-time, I wonder, and I am not sure about this, I wonder if it is worth trying to sharpen what one means by emerging space-time, and perhaps Gary's question highlights one instance. You can imagine a situation as you were describing, Chern-Simons theory with a zero metric, where you do have some background coordinate grid and then you can imagine the metric emerging as opposed to the coordinate system emerging, so one can ask: do we talk about emerging topology, emerging differential structure, do we talk about emerging complex structure, and then do we talk about emerging geometrical structure on top of those structures? I mean, I have always wondered: do exotic differential structures have any real role in anything that we're talking about? Could we see that if we spoke not just about emerging x and t, but emerging topological/differential structure, we'd see that maybe the exotic structures are there in some meaningful way and they need to be taken account of. And then one can talk about emerging geometrical structure on top of those. I do not know if that is a worthwhile framework to think about it, but it would help sharpen, I think, what we mean by emerging spacetime.

J. Harvey Well, it is certainly an interesting question whether things can emerge that we don't think describe reality and why they do not emerge.

T. Banks I want to emphasize a point that was made both by Juan and Nati about the question of trying to understand what we need to do to have a macroscopic spacetime, a spacetime with low curvature that's well-described by gravity. And I would claim that in the well-understood examples beyond $1 + 1$ dimensions, we always need to have supersymmetry that's either exact or restored asymptotically in the low-curvature region. The landscape proposal gives examples which claim that this is not a...

J. Harvey In order to have moduli spaces, you mean?

T. Banks No, well, in the BFSS matrix model, in order to have moduli spaces so that you can talk about large distance scattering, you need to have exact supersymmetry. In AdS/CFT, in all the examples that we really understand, in order to have a low-curvature AdS space, in the sense that Juan discussed, we have to have exact supersymmetry. There are claims in the literature about examples where that's not true, I think it's extremely important for us to try to find the conformal field theories that supposedly describe completely non-supersymmetric, very low curvature AdS space.

J. Harvey That is a good point.

S. Weinberg I would like to offer a remark that is so reactionary that I might be ejected from the room.

J. Harvey It is almost time for lunch, so do not worry.

S. Weinberg Listening to the discussion this morning, which I found very stimulating, I was nevertheless reminded of the kind of discussion that went on in the late 1930's and early 1940's about the problems of fundamental physics. There were internal problems of not knowing how to do calculations, and external problems of anomalies in cosmic rays, because although they didn't know it, they were confusing pions and muons, and most people at that time thought that fundamental new ideas were needed, that we had to go beyond quantum field theory as it had been constructed by Heisenberg and Pauli and others, and have something entirely new, something perhaps non-local, or a fundamental length. It turned out that the solution was to stick to quantum field theory, and that it worked. It occasionally occurs to me, well, maybe that is the solution now, that there is not an emergent space-time, that we just have three space and one time dimension, and that the solution is quantum field theory. Now why do you think that is not true? Well, obviously the reason, the most obvious reason is that gravity has problems that you get into, problems of ultraviolet divergences. That is really the wrong way to look at the problem, because if you allow all possible terms in the Lagrangian, with arbitrary powers of the curvature, you can cancel the divergences the same way you do in quantum electrodynamics. But then you say: oh, but the problem is that you have an infinite number of free constants, and the theory loses all predictive power when you go to sufficiently high energy. Well, that is not necessarily true, although it might be true. It might be that there is a fixed point in the theory, that is that there is a point in the infinite-dimensional space of all these coupling constants where the beta function vanishes. And furthermore, when that happens, you actually expect that the surface of trajectories which are attracted into that fixed point as you go to high energy is finite-dimensional. So the theory would in fact have precisely as much predictive power as an ordinary renormalizable field theory, although much more difficult to calculate since the fixed point would not be near the origin. Even so, you would then say: oh, but even so, this theory has a lot of free parameters, maybe a finite number, but where do they come from? What we were hoping for was that string theory would tell us how to calculate everything. And even there, that might not be true, it might be that the surface of trajectories that are attracted to the fixed point is one-dimensional. And we know that there are examples of extremely complicated field theories in which in fact there is a non-Gaussian fixed point with a one-dimensional attractive surface — just a line of trajectories that are ultravioletly attracted to the fixed point and therefore avoid problems when you go to large energy. That is shown by the existence of second order phase transitions. Basically, the condition is that the matrix of partial derivatives of all the various beta functions with respect to all the various coupling parameters should have a finite number of negative eigenvalues, and in the case of a second order phase transitions, in fact, there is just one negative eigenvalue: that is the condition. So it is possible that that is the answer. I suspect it is not. I suspect that these really revolutionary ideas are going to turn out to be necessary. But I think we should not altogether forget the possibility that there is no revolution that's needed, and that good old quantum field theory, although with a non-Gaussian fixed point, is the answer.

I. Klebanov I just had a comment on Brian's question. Of course in AdS/CFT, not all of spacetime is emergent: $3+1$, say, dimensions are put in from the beginning, but the other six emerge. And for example in the story I discussed, you can see the emergent Calabi-Yau with the complex structure epsilon emerging from the infrared effects of the field theory. And then I have a brief unrelated comment about locality. I think there are some speculations that you can formulate string theory just on a lattice, and string theory completely erases this lattice, as long as the lattice spacing is small enough. Namely, it's not just an approximation, it's the same theory. And one simple example is for example the $c = 1$ matrix model, where you discretize this one dimension, you can just show that for a lattice spacing small enough, you get exactly equivalent theory to a continuous dimension. So I think that's a relevant picture for locality.

J. Harvey I think we're probably a few minutes over and should wrap this up. So I think...

M. Douglas I had a very short postscript to Steve Weinberg—it's short. So, we used to be very confident that there would not be non-trivial quantum field theories in greater than four dimensions and now we believe there are non-trivial fixed point theories with lots of supersymmetry in six. So similarly, maybe the idea that gravity stops at two in quantum field theory will go up to four.

S. Weinberg Thank you, Mike.

J. Harvey Alright, on that—on that point we'll wrap things up. I guess lunch is in the usual place and the picture is at 1:15, is that correct?

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