CHAPTER 11

The philosophy of left and right

Some boys and girls find it very difficult to learn which is their right and which their left hand. Likewise, when they begin writing, some confuse their p's and q's, or their b's and d's. Kant had the brilliant insight that children are right to be confused. Something deeply puzzling is involved. In a famous argument published in a four-page essay in 1768, Kant diagnosed the children's problem and found in it a beautiful justification for Newton's absolute space.

Kant's startling argument

If two objects have the same size and shape, they are called "congruent". If an object is removed from a place exactly its size, a congruent object can be put in the same place. But your hands are "incongruent": they have different shapes. A left hand cannot be inserted into an empty right-hand glove. This gave Kant pause. Hands have similar parts and yet are incongruent. He called two incongruent objects that have the same parts arranged in the same way, *incongruent counterparts*. They are a pair of counterparts because they are so similar, and yet they are incongruent.

What makes left and right hands different? They each have the same number of fingers, and each of the fingers is attached to a palm. What accounts for their difference? We want to say that the fingers point "in different directions", but surely a direction outside the hand cannot determine the hand's shape.

Hands are fleshy, complicated creatures living in three-dimensional space. Consider a simpler example. The letter "b" consists of a small circle and a vertical line, and looks like a the palm of a left hand with everything but the thumb amputated. Likewise, the letter "d" looks like the palm of a right hand that has lost everything but its thumb. Suppose that the "serifs" have been removed, so that these two letters are exact incongruent counterparts: small circles with a line on the left and right (b and d). Suppose further that these letters can be slid around on the surface of this page, but not lifted up and reattached to it. That is, suppose the letters live only within the two-dimensional plane surface of the page.

Oddly, no matter how the b is twirled around or shuffled back and forth, it can never fill the exact place of a d as long as it remains flat on the page. Kant was deeply troubled by this. Here b and d have the same parts, and each line is attached to a circle. What makes them incongruent? We want to say the little lines "point in different directions", but can the shape of an object depend on which way it is pointing? We tend to think of each object in the world as "independent" in some sense. For example, we think an object's shape surely belongs to it, and is independent of what surrounds it. Kant considered this carefully, and was forced to conclude that the letters were incongruent because of *something* outside.

What makes things congruent or incongruent? Kant made the general assumption that at most three factors are involved. An object has parts. An object also has relations inside itself, that is, between or among its parts. These "inner" relations may hold the parts together and make the object into a whole. Finally, an object is involved in "outer" relations to whatever is outside or surrounds it. Kant began with the simple idea that the letters are incongruent because of one of these three factors:

Incongruency due to outer relations

A. The letters b and d are incongruent. (P)
B. If they are incongruent, then this is caused by some difference in their parts, inner relations or outer relations. (P)
C. So this is caused by some difference in their parts, inner relations or outer relations. (from A,B)
D. But the parts of b and d are the same. (P)
E. And the inner relations of b and d are the same. (P)
F. So the incongruency is caused by some difference between the outer relations of b and d. (C,D,E)
G. If something is caused (directly) by some difference between things, it is caused (indirectly) by the things. (P)

H. So, the incongruency is caused by the outer relations of b and d. (F,G)

This conclusion is already surprising. Our simple idea that the letters are incongruent because their little lines point "in different directions" has led to a much more profound fact. We might say: the shape of an object depends on something outside it. This is so peculiar we should check the premises. Surely the parts of b and d are the same (D): they each consist of a circle and a line. Some criticize the next premise (E), and say that the inner relations are different, that is, that the circle and line are attached in different ways. But look carefully; this criticism is mistaken. In each case, the line is attached at one of its ends, and intersects without penetrating the circle (i.e. is tangential to the circle). The inner relations are the same. This is what confuses children. No amount of staring at just the b or the d themselves will reveal any difference: same parts, same angles, same kind of connections. The difference must be outside.

Kant has already discovered something surprising, but he has much greater ambitions. He claims that all this leads to a new proof for the existence of space. Aristotle was wrong about the plenum; the atomists and Newton were right to assert the existence of space. He continues the argument as follows:

From outer relations to space

I. Outer relations are outer relations to other objects or to space. (P)
J. So the incongruency is caused by the outer relations of b and d to other objects or to space. (H,I)
K. But the incongruency is not caused by outer relations to other objects. (P!)
L. So the incongruency is caused by outer relations to space. (J,K)
M. If something is a cause (or a relatum of a cause), then it exists. (P)
N. So space exists. (L,M)

This is a stunning conclusion. Kant's insight into a child's confusion has led to a powerful new proof for the existence of space.

Mathematicians call the outer relation of an object to the environment its orientation. They say that b and d are incongruent because they have different orientations.

But should we believe Kant? The premise with the exclamation mark (K) is what philosophers call a "strong assumption". This means that a premise contains a powerful new idea, and helps push the argument much nearer to the conclusion. But strong premises are dangerous. They try to do so much that they are often untrustworthy. Compare this to a rock climber scaling a cliff. Every inch the climber moves upwards creates a new choice between safe and incremental moves from one crevice to a nearby one, or long, more dangerous stretches. The difficult "strong moves" may be more challenging and open up new routes, but by making them the climber risks a dangerous fall. Strong premises are similar. They often represent surprising new inspirations, but just as often are a stretch too far and can send an argument crashing down. Will Kant's assumption bear so much weight?

Kant believes his assumption K is obvious. Some of his remarks indicate why. First, he says that the incongruency is due to the letter's shape itself, and shape does not depend on *which* other objects are around. The b on a page would have the same shape even if there were no d's on the page or anywhere else. Secondly, he emphasizes this point with another short argument. This might be called the "empty universe argument". Kant simply assumes that a letter or hand would still have the same shape even if the rest of the entire universe were empty. From this, he reasons that the outer relations on which the shape depends cannot be relations to other objects – because there are no others in such a universe.

Both of these remarks simply restate the idea that shape does not depend on other objects. They do not make Kant's strong assumption seem obvious. Although the structure of Kant's argument is clear, the argument is finally not persuasive. One assumption (K) seems necessary for the conclusion, but is not obvious. In itself, Kant's argument is suggestive and interesting, but not finally a proof for the existence of absolute space.

According to Kant the shape of your hands depends on the universe.

Kant is rescued

Fortunately, later research provided very interesting support for Kant, and showed that incongruency is very probably caused by outer relations to space. With this help, Kant's argument has been rehabilitated and now stands as one of the most provocative arguments for absolute space. The central point is simple. The letters b and d are incongruent only when confined to the surface of this page. No horizontal sliding will allow one into the place occupied by the other. But if we are allowed to lift the letters off the page and rotate them around, it is easy to show they are congruent because they exactly fit into the same place, and therefore have the same size and shape. Thus two objects that are incongruent in two dimensions are in fact congruent in three dimensions. Whether or not objects are congruent *depends on the dimensions of the space they inhabit*.

The same is true of left and right hands. In a strange space that had four spatial dimensions in addition to time, a left hand could be "rotated" into a right hand. A left hand could thus be fitted into a right-hand glove simply by flipping it around.

Another intriguing example is provided by a long, thin, rectangular plastic strip whose end is twisted halfway around (by 180 degrees) and smoothly glued to its other end. This loop with a twist is called a *Möbius strip* after its inventor, the mathematician A. F. Möbius. Surprisingly, a letter b that slides around this strip and makes a complete journey around the loop will return as a letter d (when viewed from the same direction, as if the strip were transparent and the letter were in the surface). That is, the letters b and d are incongruent in an ordinary, flat two-dimensional space, but not in a two-dimensional space with a twist! The incongruency of the letters depends on the overall shape of the space!

Suppose, likewise, that the entire universe had some sort of peculiar twist in it like the Möbius strip. Astronauts travelling in one direction would then find themselves back at their starting point. In

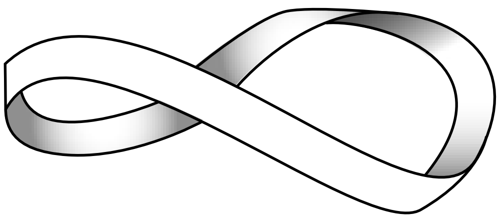

Figure 11.1 *A Möbius strip has one side and one edge. It is two-dimensional like a flat piece of paper, but has a different topology.*

this case, a left-hand glove could be converted into a right-hand glove just by sending it along with the astronauts through the twist. In fact, the astronauts too would return mirror-reversed: their hearts would be on their right sides!

Mathematicians say that a space with a twist in it has a different *topology* from ordinary, flat space. The word "topology" just means the study of place, and is the name of an important branch of mathematics today. The topology of a space is the way its points are connected to each other, and this stays the same if the distances between points are shrunk or expanded. Analogously, a balloon's shape and size change as it is blown up, but its topology doesn't change: the bonds between molecules stretch but do not break.

The fact that incongruence depends on dimensionality and topology very strongly suggests that Kant's strong assumption (K) was correct. The letters b and d are incongruent because of outer relations, but not outer relations to other objects. Since altering the space affects whether or not the letters are incongruent, their shape must depend on the surrounding space, and not on the objects contained in it. Research continues on this subject, but many philosophers think Kant's argument is good evidence for some form of spatial structure over and above the bodies they contain.

After Kant's investigations of the peculiarities of incongruent counterparts, they played an extraordinary role in chemistry and physics. Two molecules that are incongruent counterparts of each other are called "isomers" in chemistry (from the Greek: "iso" is "same" and "mer" is "parts"). Many medicines and industrial chemicals depend on the remarkably different properties of isomeric molecules.

There was tremendous surprise in 1956 when two physicists, Tsung-Dao Lee and Chen Ning Yang, showed that incongruent counterparts play a role in fundamental physics. They studied very fragile subatomic particles, which can be produced by physicists but quickly decay and fall apart. Some of these particles come in pairs of incongruent counterparts; that is, pairs of particles that have the same properties except that their shapes are mirror images of each other (like hands). In a series of dramatic experiments, Madam Wu (Chien-Shiung Wu), a physicist in New York, showed that the lifetime of certain particles depends on whether they were left-handed or right-handed! That is, even the most fundamental physical laws are sensitive to handedness. The excitement about this discovery was so great that Lee and Yang were given the Nobel prize in record time.

Even if we live in a three-dimensional space without twists and thus left and right hands must remain incongruent counterparts, Kant's argument is strengthened by the possibility that more dimensions would render them congruent.

CHAPTER 12

The unreality of time

British philosophy is sometimes celebrated and sometimes satirized as sturdy common sense. It tends to be grounded in facts and logic, and prefers science to mysticism. But for a generation or two during the late 1800s and early 1900s, a loose movement called British Idealism came to dominate philosophy in the universities. The major figures – Bradley, McTaggart, Green and Alexander – often disagreed among themselves, but they typically denied the reality of space and time, claiming that the world of science and appearance was contradictory. They believed instead in some sort of higher, spiritual reality. They were rational mystics.

James McTaggart supported his metaphysical idealism by advancing a famous argument against the existence of time. His attack has survived the rest of his philosophy, and continues to be widely discussed. When the argument was first published in 1908, three years after Einstein's papers on special relativity, debates over the nature of time were quite fashionable. Questions about whether time was another "dimension" and whether the world was really a four-dimensional block universe began to emerge. Independently of Einstein's theories, McTaggart saw clearly that such views might be incompatible with real change. If time is like a spatial dimension, he argued, then there is no such process, no "change", whereby one thing becomes another.

The A-series and the B-series

The truth-maker principle insists that if a sentence is true of the world, then something in the world makes it true. This seems to be common sense and is treated by many philosophers as bedrock, as obviously correct in some sense. But there is a well-known challenge to the principle that was already raised in one form by Aristotle. Suppose that the sentence "Princess Diana died in a car crash" is true. What makes it true? What is the truth-maker for this sentence?

We want to say that the sentence is true because the crash really happened; metal did smash into concrete. But this event occurred in the past. If the past is gone completely, and does not now exist in any way, what makes it true that the crash did occur? Is it our present memories, or the traces of paint still left on the underpass in Paris? This cannot be correct, because Princess Diana would still be dead even if she was forgotten and the underpass scoured clean.

The general problem of statements about the past and the future, and what makes them true or false, is deeply puzzling. There is no consensus among philosophers about their truth-makers; some even doubt whether such statements could be true or false. Perhaps it is not true now that Princess Diana died in the past? For some philosophers, however, this problem leads them to suppose that the past (and the future?) has some form of "existence". The event of Princess Diana's death does "exist" in some sense, and this past event is the missing truth-maker. These philosophers debate the kind of existence that past events might have, but generally suppose it to be a paler, ghostly existence: more "abstract" and less robust than present events.

In his attack on time, McTaggart seemed to take it for granted that past and future events have some sort of existence, and perhaps the truth of statements about the past persuaded him that this was obvious. Thus he speaks as if all events "have positions in time" even when they are far in the past or future. To have a position or any property requires that events exist in some sense. Given this fundamental assumption, he sets to work abolishing time.

Since all events have positions in time, what makes some events earlier or past, and others later or future? McTaggart says there are only two ways of accounting for the order of events in time. Perhaps events are linked by relations into long chains, and these relations make some events earlier and some events later. Thus the marriage of Diana is earlier than her death because there is a relation between the two events that fixes their order. Since the marriage is always earlier than her death, the relation between them is permanent. McTaggart calls the long chain of events linked together by permanent relations a *B-series*. Although he does not use the term, this B-series is very similar to the block universe discussed above (this is easy to remember

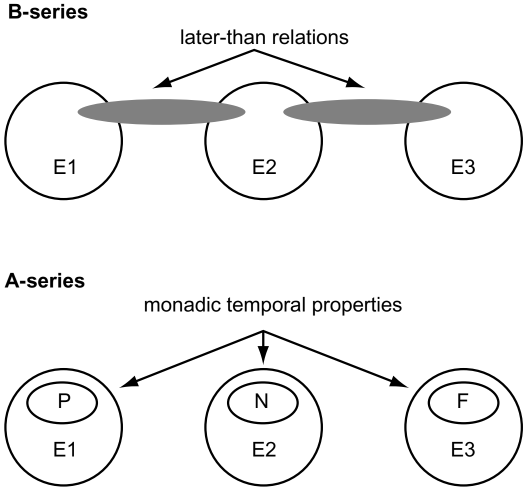

Figure 12.1 *McTaggart's two series. In the B-series, event E2 is made later than E1 by a later-than relation, and event E3 is made later than E2 by a similar relation. In the A-series, the three events E1, E2 and E3 have properties that make them past, now or future.*

since both begin with B). In both, events are static points locked into their positions in space and time, and there is no real change or becoming.

On the other hand, McTaggart says, perhaps there are no such relations, and events simply have the special properties of "being past", "being now" or "being future" (i.e. P, N, and F). Diana's death is earlier than now simply because it *is past*, or it *has past-ness*. The passage of time just is the shifting and changing of these properties. An event that begins with the property of being in the future, fleetingly becomes present, and then has the property of being past. The event of Diana's death never changes – the car always crashes – but its temporal property shifts and changes. McTaggart calls such a collection of events with P, N, and F properties an *A-series*.

In sum, McTaggart starts by saying that time must be one of the following:

- *A-series*: individual events have the property of being past, now or future, and these properties determine their position in time. These properties change.

- *B-series*: individual events are linked into a chain by earlier-than and later-than relations, and this linked chain is the order of time. These relations are permanent.

This terminology is so well-known in the philosophy of space and time that it is worth memorizing. Assuming that all events have positions in time, and therefore have an order in time, McTaggart's distinction seems right. The idea is that any order must be due either to something "between" the events or to something "in" the events. The relations of the B-series are somehow between events and link them together, and each property of the A-series is within an individual event.

> For McTaggart, since all events in the past, present and future exist in some sense, their being is not temporal; their temporality must then be in their properties or relations.

The B-series is contradictory

The above distinction now turns into a dilemma. McTaggart shows that no matter which we choose, we encounter contradictions. His first argument is straightforward:

The B-series implies that time is unreal

A.	Time is a B-series.	(P)
B.	The B-series is events in earlier-than/later-than relations.	(P)
C.	Events in earlier-than/later-than relations are unchanging.	(P)
D.	So time is unchanging.	(from A,B,C)
E.	But time is change.	(P)
F.	So time is contradictory.	(D,E)
G.	There are no contradictions.	(P)
H.	So there is no time.	(F,G)

There are two strong premises here that should be examined closely. The idea that events in earlier-than/later-than relations are unchanging, premise C, is the idea that Diana's marriage is *always* before her death. This order between the events is permanent. The claim that time is change, premise E, shows that McTaggart is aiming to show that there is no becoming, no process in which an individual event loses some properties and gains others. Thus the argument is really aimed at time as a process of becoming. McTaggart is, therefore, correct that in a "block universe" there is no room for true becoming or movement of any kind.

In short, if time is relations, and these relations are unchanging, then there is no true change.

The A-series is contradictory

To save the reality of time, we must therefore place our hopes in the A-series. McTaggart's argument here is deeper and more original, but also more involved. There are two main ideas. The first stems from the idea that the properties P, N and F are contrary properties, and therefore cannot all belong to the same event without giving rise to a contradiction. But, McTaggart emphasizes, they do all belong to the same event: each event has the property of being future, of being now and of being past. Why then is there no contradiction?

When faced with a contradiction, draw a distinction. Clearly, an event has the properties P, N and F *in different respects*. But what are these respects? We want to say that an event has the properties *at different times*. It is future first, and then at a later time exists now, and at an even later time becomes past. But this simple response leads to trouble. The properties P, N and F were meant to endow an event with a position in time, but now to avoid contradiction we must introduce a "meta-time". We can label moments of this new meta-time P′, N′ and F′ (pronounced "P prime", etc.).

McTaggart is clearly correct here. If we think of P, N and F as ordinary properties, then they must belong to events in different respects. But now he pounces. If the meta-times P′, N′ and F′ are also ordinary properties, and also contraries, then we have an infinite regress. That is, a further set of moments of "meta-meta-time" will be needed to prevent P′, N′ and F′ from belonging to an event all at once. Thus P′, N′ and F′ lead to P″, N″ and F″, which lead to P‴, N‴ and F‴, and so on to infinity. Thus McTaggart's first idea is that the properties of the A-series lead to an infinite regress. His second is that this infinity should be rejected.

At each level of the properties, the next level *must* exist to avoid a contradiction. But this means that *all* the levels to infinity must exist to avoid contradiction. That is, the whole infinity of levels must be actual and complete in some sense. Philosophers say that such infinities are *vicious*: to avoid a contradiction in some ordinary property we must assume a towering, actual infinity. This is unacceptable.

The argument is that the A-series leads to an infinite regress, this regress is vicious and thus we must reject the A-series:

The A-series implies that time is unreal

A. Time is an A-series. (P)
B. The A-series is events with contrary properties: P, N and F. (P)
C. Thus, time is events with contrary properties. (from A,B)
D. If something has contrary properties, it has them in different respects (to avoid contradiction). (P)
E. Thus time is events with different respects: P′, N′ and F′. (C,D)
F. But these different respects are also contrary properties. (P)
G. Therefore, time is events with an infinite regress of properties. (E,F, and induction)
H. Contradiction is avoided only if the infinity is actual (i.e. if *every* set of properties is accompanied by a further set of properties). (P)
I. But no infinity is actual. (P)
J. Therefore, time is contradictory events. (G,H,I)
K. There are no contradictions. (P)
L. Therefore, there is no time. (J,K)

McTaggart's attack on the A-series is a deep argument that sharpens our thinking about time. By combining this with his earlier attack, McTaggart will conclude that, since time must be either an A-series or a B-series but both lead to contradictions, there is no time at all. Time is not real.

McTaggart is assuming that all the properties of an event except P, N, F and so on, are permanent. Diana will always have died; only the time of the event can change. Thus only P, N, F and so on can be used to fend off contradictions.

CHAPTER 13

General relativity: is space curved?

> Errors of thought cost me two years of excessively hard work, until I finally recognised them as such at the end of 1915 and, after having ruefully returned to Riemann's idea of curved space, succeeded in linking the theory with the facts of astronomical experience. Now the happy achievement seems almost a matter of course, and any intelligent student can grasp it without too much trouble. But the years of anxious searching in the dark, with their intense longing, their alterations of confidence and exhaustion and the final emergence into the light – only those who have experienced it can understand that. (Einstein, 1934)

The 1905 special theory of relativity was limited to measurements made by equipment moving without acceleration. The general theory of relativity eliminates this special restriction. To achieve this, however, Einstein had to even more deeply revolutionize our concepts of space and time. Building on the insights of the earlier theory, he now argued that space can be bent by matter.

Thus we have reached a third stage in the philosophy of space and time. The atomists first proclaimed that space or "nothing" existed in order to solve the problem of change. Newton revived this doctrine when, despite the criticisms of Aristotle and Descartes, he needed absolute space for his broader theory. Einstein now completes his revolution with the momentous claim that *space changes*. That is, space has evolved from a static structure proposed to make sense of motion to a structure that itself changes.

Red rubber sheets

In Newton's classical physics, space tells bodies how to move. A body given an initial shove will follow a straight line in space unless deflected by some other force. Einstein thought that this "one-way" influence was peculiar. If space acted on bodies, should not bodies act on space? The germ of Einstein's later theories of space and time, his general theory of relativity, is that this circle is completed.

> *Central idea of the theory of general relativity:* Spacetime tells bodies how to move, and bodies tell spacetime how to curve.

But what is a curved space or curved spacetime? If space is emptiness, or even a kind of nothingness, how could that curve? What would space bend into?

Physicists begin to answer these questions by insisting that only observable entities may play a role in physics. Since space is invisible and we see no lines of latitude and longitude etched across the night sky, the straight lines through space can only be discovered by following bodies around and observing their paths. Thus physicists define a *straight line* through space as the *path traversed by a body moving without interference of any kind*, that is, *moving inertially*.

Suppose, however, that two bodies are shoved along two adjacent, parallel paths in the same direction, but later bump into each other. Newton would say that some force must have pushed them together. If they were moving inertially without disturbance they would have followed two parallel paths, and since parallel lines never intersect, the bodies would never collide. Einstein disagreed, and suggested another possibility. Suppose that the bodies bump into each other because space itself is curved. Suppose that space is crisscrossed in all directions by lines that inertial bodies follow, but that the fabric of space can be bent, twisted and curved. Then bodies coasting along "parallel" paths may indeed bump into each other if the paths converge.

It does sound strange to say that "straight lines curve" or that "parallel lines intersect", but this is just a superficial oddity of physicist's language. They adhere to the definition above even when parallel, straight lines intersect, and prefer simply to say that they have enlarged our notion of what "straight" and "parallel" mean.

Einstein's concept of curved space is often illustrated by the *rubber sheet analogy*. Strictly speaking, it is spacetime that is curved, but this analogy helps us picture a curved space. Imagine a large sheet of supple rubber stretched out and secured along its edges, perhaps like the surface of a large drum in an orchestra. A heavy lead weight placed in the centre of the sheet will sink down into a smooth, circular well. This gives us a picture of what it means to say that "mass causes space to curve". More interestingly, suppose that children begin shooting marbles back and forth along the rubber sheet. Suppose a marble is shot straight but not directly at the lead weight. Instead of following a "straight line" its path will bend towards the lead weight. Either it continues on away from the weight after this deflection, or it will spiral down into the well. There is no significant attraction between the lead weight and the marble. The lead weight affects the marble by distorting the spatial surface that it travels along.

This nicely shows how Newton's gravitational forces are replaced in Einstein's theories by curved spaces. For Newton, bodies move towards each other when they exert attractive forces. For Einstein, they influence each other indirectly by affecting the space between them. Each body curves the space in its environment; when other bodies coast along straight lines they veer toward the centre of curvature. Some say that, in general relativity, *gravitational forces are geometrized away*.

Suppose further that the lead weight on the rubber sheet was moved rapidly up and down. If the sheet were large enough and flexible enough, ripples of upward and downward motion would spread out across the sheet. Perhaps they would look like the circular waves in the surface of a pond caused by a falling stone. Similarly, Einstein's theory predicts that moving masses will cause travelling distortions in the fabric of space: *gravity waves*.

Thus the rubber sheet analogy illustrates the way mass curves space, the curvature of "straight" inertial paths and gravity waves.

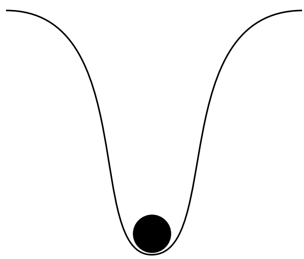

Figure 13.1 *The rubber sheet analogy. The heavy lead weight pulls the sheet down and creates a circular well.*

The rubber sheet analogy is cheating. It assumes that the weight and marbles are held down by ordinary gravity here on Earth. In a real curved space, inertia alone would keep them following along the curved lines of the space.

Eddington's eclipse expedition

The world was convinced of the truth of Einstein's outlandish conception of curved space in 1919. In the aftermath of Germany's defeat at the end of the First World War, the English astronomer Arthur Eddington organized an expedition to observe a total eclipse in South Africa. British confirmation of the theories of Einstein, a Swiss national working in Berlin, would at once advance physics and the cause of world peace.

How might observations made when the Sun was eclipsed by the Moon confirm the curvature of space? Light follows straight lines but, according to Einstein, these lines should bend around and towards heavy masses. Thus starlight travelling closely by and almost grazing the Sun should be slightly deflected towards it (like the marbles above). Ordinarily, the Sun's brilliance would drown out the starlight and make it impossible to observe this deflection. But during a total eclipse, the Moon would just block out the Sun's own light. The images of stars around the edge of the darkened Moon should be slightly out of place because their light was bent on its way to earthbound telescopes.

During the summer and early autumn of 1919, Eddington measured his blurry photographs and calculated his results. Word began to leak out that his team had confirmed Einstein's predictions about the degree of deflection within experimental error. Back in his office, according to a well-known story, when Einstein was first handed the historic telegram with a preliminary announcement of the results, he glanced at it, set it aside and continued talking to a doctoral student. When she asked about its contents, Einstein's calm report caused great jubilation. Einstein, however, remained calm. Somewhat amused, he said simply "But I knew all the time that the theory was correct". When pressed to admit that the experiment might have given negative evidence, he went on "In that case I'd have to feel sorry for God, because the theory is correct".

Eddington announced his results publicly in London at a meeting of the Royal Society, whose president some 200 years earlier had been Isaac Newton. The next day, 7 November 1919, the London *Times* carried the now famous headlines on its front page: "Revolution in Science – New Theory of the Universe – Newton's Ideas Overthrown". From that day, Einstein was a celebrity and space was curved.

Although historians have since cast some doubt on Eddington's first crude results, the bending of starlight is now routinely observed by astronomers. The Hubble Telescope orbiting Earth has produced beautiful images of what is now called *gravitational lensing* (see Appendix D for websites). This occurs, for example, when one distant galaxy sits far behind another nearer galaxy. As light streams around the nearer galaxy, the rays may be bent towards us from several different angles. The telescope will see multiple copies of the distant galaxy arrayed around the nearer one.

Physicists now accept that a substantial number of experiments confirm Einstein's ideas of a curved spacetime. Gravity waves, however, have been a significant exception. Even movements of very large bodies produce extremely weak waves, and they have so far proved impossible to detect here on Earth. In 1974, however, Taylor, Hulse and their colleagues produced the first, serious evidence for the existence of waves rippling through the fabric of space. They studied the motion of pairs of stars orbiting rapidly around each other, and showed that they lose energy at just the rate that would be expected if gravity waves were emanating outwards. In 1993, they won the Nobel prize for this discovery. Several large, ground-based projects are now underway to detect gravity waves originating in outer space.

> After Eddington published a famous mathematical treatise on general relativity, someone asked whether it was true that the theory was so difficult that only three people in the world understood it. Eddington looked startled and asked "Who's the third?"

The path to discovery

The idea that space is curved seems bizarre. Einstein emphasized that the train of thought that led to this conception depended heavily on certain *thought experiments*. In these imaginary situations, Einstein used his reason and intuition to investigate physics without mathematics or experiment, and at times this method made Einstein appear more like a modern philosopher than a hard-nosed physicist. Two of these now famous thought experiments illustrate one route Einstein took to the discovery of general relativity, and also are good examples of the breathtaking beauty of Einstein's arguments.

Suppose that some surveyors live on the surface of a large *spinning wheel* and make a series of length measurements. Since *acceleration* means a change in speed or a change in direction away from straight line motion, every part of the wheel except its very centre is accelerating as it rotates. Such surveyors might, for example, measure the radius of the wheel from its centre to its rim, and also its circumference. For an ordinary stationary wheel, the circumference is about three times longer than the diameter. That is, the length of the circumference divided by the length of the diameter is the number π (pi), which is about $3\frac{1}{7}$. But, Einstein claimed, this would not be true for the spinning wheel.

Remember, he said, the lesson of special relativity: faster speeds mean shorter lengths. That is, there is *length contraction in the direction of travel*. Special relativity applies only to measurements made with rulers and clocks moving at steady speeds in the same direction but, Einstein argued, it also *applies approximately to accelerating bodies*. During very short moments of time, even an accelerating body moves at about the same speed. A sports car may accelerate to high speeds within 20 seconds, but during each of those seconds its speed is more or less the same. Thus during short moments we can apply special relativity to measurements made by accelerating rulers and clocks, and predict that they too will find length contraction and time dilation.

Consider how this affects the surveyors on the spinning wheel, which is, by assumption, a well-formed circle. They measure the radius by laying their rulers along one of the spokes. Since these rulers will lie at right angles to the direction of motion, they are not contracted at all. However, rulers laid along the circumference will experience length contraction as the wheel spins. More copies of the ruler would fit into the circumference. This means that the measured length of the circumference divided by the measured length of the radius will give an answer different to that for the stationary wheel. Instead of a number about three, the result will be much larger. The rulers along the circumference shrink while those along the radius stay the same, and so the ratio increases. Nonetheless, the wheel is a circle: it is the same distance from its hub to its circumference in every direction. This is remarkable. According to Einstein, laws of geometry known since ancient times "do not apply" to rapidly accelerating bodies. In sum, Einstein's thought experiment with the spinning wheel taught him that *acceleration distorts geometry*.

Building on this insight, a second thought experiment led Einstein deeper, toward the core ideas of general relativity. Imagine that an experimenter is in a small room like the inside of a *lift*, and that the room is located in outer space far from any planet or any other source of gravity. Suppose further that a hook and long rope is attached to the outside of the room at its top, and that God or some other being steadily pulls the room upwards. This will cause the room to move up faster and faster with constant acceleration. Anything floating or unsupported in the room will settle to the floor and remain there as long as the acceleration continues.

Einstein realized that, for the experimenter enclosed in the room, the upward acceleration would feel the same as a gravitational force pulling down. In fact, *no experiment performed inside the room could distinguish between an upward acceleration and downward gravitational pull*. Perhaps objects fell to the floor because God was pulling the room up, or perhaps some unknown planet had swum into the neighbourhood and its gravity was pulling everything downwards. No experiment could decide. In sum, Einstein's thought experiment taught him that *acceleration is equivalent to gravitation*. If no experiment can detect a difference, he concluded, they are the same as far as physics is concerned.

The reason for this equivalence is that gravity affects all bodies in the same way: it is a universal force. A gravitational force will pull *all* the bodies in the room down at the same rate, and thus is indistinguishable from an upward acceleration, which tries to pull all the bodies upwards. In contrast, electric forces affect only bodies that carry electric charges. Thus an electric attraction pulling down on the room would affect only some of the objects in the room. The experimenter inside would quickly see that only charged bodies were affected and conclude that an electric force was present. It is the universality of gravity that makes it equivalent to an upward acceleration.

Einstein's line of reasoning reached a stunning climax when he combined the lessons of these two thought experiments. Put somewhat crudely, he triumphantly argued:

Core argument for general relativity

A. Acceleration distorts geometry. (P: from spinning wheel)
B. Acceleration is equivalent to gravitation. (P: from lift)
C. Therefore, gravitation distorts geometry. (from A,B)

This is the line of reasoning that Einstein used to argue that "matter tells spacetime how to bend": matter is the source of gravitation and gravitation bends spacetime. Thus spacetime is curved and behaves like a flexible rubber sheet.

This is a bewildering, fantastic series of ideas. No one would take them seriously if Einstein had not used them to make some very surprising predictions that experiments later confirmed. We have met one example above. If we assume that light follows straight lines, then the paths taken by light should be distorted by any nearby matter that distorts the geometry in its neighbourhood. This is the effect confirmed by the eclipse expedition and by gravitational lensing. Light does follow straight lines, but the lines are bent by large masses!

Many other predictions followed. Returning to the spinning wheel, Einstein argued that a clock on the rim would run more slowly than a similar clock left at the centre of the wheel. This is just another application of the idea that faster speeds mean longer hours. Since the rim is accelerating, Einstein argued that acceleration also causes time dilation. But since "acceleration is equivalent to gravitation", he concluded that *gravitation causes time dilation*. Clocks that feel stronger gravity pulling them down run more slowly.

In 1960, Pound and Rebka confirmed this claim directly by placing two similar clocks at the top and bottom of a tower on the campus of Harvard University. Since the clock at the bottom was nearer to the earth and therefore felt its gravitational pull more strongly, Einstein's theory predicted that this lower clock would run more slowly. Although the difference was very, very slight because the clocks were so near each other, delicate measurements found just the effect Einstein predicted!

In fact, Einstein took his ideas one step further. Rather than saying that mass causes gravity, which distorts the geometry of space, he asserted more simply that mass is directly associated with distorted geometry. In other words, rather than picturing gravity as something contained within and distorting space, he said they were one and the same thing: he identified gravity with the curvature of space. The rubber sheet analogy helps make this clear. The marbles rolling past the lead weight are deflected by the curvature of the rubber sheet:

there is no need to assume some gravitational force or field that exists over and above the sheet and pulls on the passing marbles. Thus, for Einstein, gravity just is the curvature of space. As before, the forces have been geometrized away.

> Einstein's thought experiments are not a priori reasoning. They mix reasoning and experience.

Equivalence and the bucket

This insight that gravitation was equivalent to an acceleration was key in Einstein's struggle to generalize his earlier theory of special relativity. Remember that this was limited to special cases; namely, to measurements made with unaccelerated or inertial rulers and clocks. In the early theory, Einstein simply did not know how to handle acceleration. Newton thought that acceleration was so strange that it provided the best evidence for absolute space. Einstein rejected absolute space, but simply ignored the problem of acceleration in the early theory. Now his thought experiment suggested a new way to think about acceleration. Instead of pointing to the existence of absolute space, acceleration was the equivalent of a gravitational field. Thus instead of accepting Newton's talk of a mysterious, invisible space containing all bodies, Einstein could say that all effects of acceleration in the room were due to gravitation.

Thus Einstein's thought experiment with the small room was also his answer to Newton's bucket argument. Einstein agreed that the surface of the water was concave, but denied that this was due to accelerated motion relative to absolute space. The concavity was evidence for the presence of some gravitational field pulling the water out to the sides of the bucket. This claim is very much alive and at the centre of contemporary debate, as discussed in Chapter 15.

The shape of the universe

The universe is very, very large. Although light is fast, it takes it about three years to travel from the nearest star to Earth. A typical galaxy like our Milky Way galaxy contains about 100,000,000,000 stars; light crosses from one end to the other in about 50,000 years. In human terms, galaxies are incredibly huge. But they are only tiny specks compared to the whole universe. There may be 50,000,000 galaxies in the universe. Some cosmologists speculate that a beam of starlight would take a hundred billion years to travel through the entire universe. If that is true, then even the Hubble Telescope orbiting around Earth can see only a small portion of the whole universe.

In ancient times, the Roman poet Lucretius argued vividly that the universe must be infinite. Suppose, he said, that there was an end to space. Then we could stand at the brink and hurl our spear across the edge. If the spear rebounded, we would know that there was something beyond the end. If the spear did not rebound, then it carried on into more space. In either case, there must be something beyond the brink, something beyond the supposed end of space. Thus, he concluded, it is not really an end, and space must continue on infinitely. Newton too thought that his absolute space was infinite. But, once Einstein had persuaded us that space can curve and bend, questions about the shape and size of the universe were opened to experimental tests. Interestingly, Einstein raised the possibility that the universe might have a limited finite size but not have any boundary. That is, the universe may be "finite and unbounded". This does not make sense at first, but can be explained by a simple analogy.

Consider the surface of a child's balloon. The surface itself is a two-dimensional curved space. Although the surface has a limited size, say about 50 square centimetres, the surface has no boundary. An ant crawling over the balloon could move forwards endlessly without meeting a boundary. Thus a curved space that turns back on to itself can be finite even though it has no boundaries.

We live in a four-dimensional spacetime, and Einstein considered the possibility that our universe is curved in such a way that it is finite and unbounded like the balloon. This would have a surprising consequence. Just as an ant on the balloon crawling in a straight line would return to its starting point, astronauts travelling in a straight line for a very long time would find themselves back on Earth! This seems crazy, but there are experiments that might provide evidence that this is the case. Some astronomers are pointing their powerful telescopes in diametrically opposite directions, and hope that they might see the same, distant galaxy in both directions. Most think there is little chance of success, but we might be surprised.

There is now a wide variety of experiments being performed to explore the shape of the universe, and more are planned. We may have hard evidence soon.

CHAPTER 14

The fall of geometry: is mathematics certain?

> We reverence ancient Greece as the cradle of western science. Here for the first time the world witnessed the miracle of a logical system which proceeded from step to step with such precision that every single one of its propositions was absolutely indubitable – I refer to Euclid's geometry. This admirable triumph of reasoning gave the human intellect the necessary confidence in itself for its subsequent achievements. (Einstein, 1933)

Some seek truth and some doubt it. Some are dedicated to seeking progress in our knowledge of reality, and some find it all too absurd. In the European tradition, the battles between these two warring tribes took place in the shadow of a great fortress. Defenders of truth could raise their fingers over the heads of the sceptics and point upwards to mathematics: a shining crystal palace of certainty surrounded by thick walls of deductions and demonstrations. But the revolution in theories of space and time during the twentieth century finally levelled this fortress. Sceptics have overrun even mathematics. Much of the fashionable relativism that has flourished during the past 30 years took heart from this downfall of mathematics, and with good reason.

Euclid and his *Elements*

Geometry was the glory of ancient Greece. Building on its beginnings in Egypt and Babylon, the ancient Greeks pursued geometry with extraordinary passion and precision for a thousand years, and first built up the fortress of mathematics on firm foundations. For them, geometry was the study of the real properties of real things: it was a branch of metaphysics. Like their other theories, it was object-oriented. They did not, in the first place, think of triangles and spheres as inhabiting a "geometric space". Instead, they typically drew concrete figures in the sand or on their slates, and investigated their properties and inner relations.

The Greeks did more than reveal the beauty of geometry in a thousand different theorems. They organized this knowledge, and invented a new method that grounded it securely in the simplest truths. This "axiomatic method" is one of our most precious inheritances. Even in ancient times, it was applied broadly to physical science (optics) and philosophy (by Proclus). This method is essentially the same as that used in the step-by-step arguments here. It begins by identifying the basic premises, and then carefully illuminates the reasoning that leads from them to some new truth in the conclusion.

One historian has asserted that all the prominent mathematicians in later antiquity were students or scholars in Plato's academy, or students of these and so on. The most prominent of these lineal descendants was Euclid, who lived in Alexandria, Egypt around 300BCE: about 50 years after Plato's death. Euclid's famous book on geometry, the *Elements*, is one of the great books of European culture. Its beauty is twofold. By brilliantly creating and arranging his proofs, he was able to show that the mathematical discoveries of the preceding centuries could be traced back to *five* premises. Just five ideas formed the foundations of the entire crystal palace.

Secondly, Euclid concluded his treatise with the touch of a real virtuoso. Plato had written of the beauty and symmetry of "regular solids", which became known later as the "Platonic solids". These are three-dimensional geometric figures like the cube, whose sides and angles are all the same. Oddly, even though there are an infinity of differently shaped sides that might be fitted together in infinitely different ways, there are only five different regular solids. There are only five ways of building perfectly symmetrical solids with flat sides. At the end of his treatise, Euclid leapt from two-dimensional plane figures into the realm of solids. In the great climax of Greek geometric thought, he proved that no future mathematician would ever discover another perfect solid: that the five Platonic solids were perfect and complete.

Historians do find it difficult to judge how much of the *Elements* is original and how much Euclid merely collected and systematized the efforts of earlier geometers: after Euclid, scribes neglected copying the scrolls of earlier geometers and in time their works were lost altogether.

Euclid's basic premises contained a time bomb, which later was to topple the edifice he so carefully erected. His five premises were supposed to be the most basic and simple truths, grounded in the deepest simplicities of Being. But consider each in its turn:

Euclid's five postulates

I. There is a straight line between any two points.
II. A line can be continued in the same direction.
III. A circle can be constructed around any point with any radius.
IV. All right angles are equal to each other.
V. Lines which are not parallel will, if continued to infinity, intersect somewhere.

The first four are plausible basic truths, but the fifth suddenly extends common sense into the great unknown. How would we prove what happens at infinity? What experience or intuition could help us here?

According to legend, Plato had inscribed above the door of his Academy, "Let no one ignorant of geometry enter here".

The rise of non-Euclidean geometries

Already, in ancient times, there was discomfort with Euclid's fifth postulate. Beginning with the Renaissance in the fifteenth century and continuing in the Scientific Revolution in the seventeenth century, the prestige of mathematics rose again and new suspicions were levelled at his mysterious fifth postulate. Many ambitious mathematicians sought to outdo Euclid by cleaning up his axioms by finding fewer or simpler ones that would serve as a more secure foundation for the queen of sciences. Some 300 years of searching produced only mounting frustrations. Mathematicians could neither do without Euclid's fifth postulate or its equivalents, nor show that it was wrong or faulty. It was there, ugly and stubborn, like a wart on the face of mathematics.

In desperation, several innovators finally adopted an indirect strategy. Instead of seeking for a deeper truth behind or underneath the fifth postulate, they proposed to prove it using a *reductio ad* *absurdum*. They would *assume* that Euclid's postulate was false and, using this reversed axiom together with the other four, would derive contradictions. That is, they would perversely assume that all parallel lines intersect! Surely, they thought, such an abomination would lead to contradictions. By leading this false assumption into a contradiction, that is, by reducing it to an absurdity, they would reveal its falsity. Surely, this would prove that Euclid's postulate could not be false, and was a necessary foundation stone for a true and consistent mathematics.

This result was astonishing, and led to some very deep thinking on the part of Europe's leading mathematicians. They had combined the false premise with Euclid's four other, simple premises and begun proving things, hoping to squeeze out a contradiction. But one theorem led to another and soon they had a large number of new theorems but no contradiction. They continued on, hoping the bad apple they had started with would spoil the whole barrel of new theorems, but could find no contradictions at all. To their surprise, they had soon built up an entire alternate mathematics, as if God had created a psychotic alternative universe and they had stumbled on its strange system of geometry: a world where parallel lines met each other!

Mathematicians soon realized that they could build pictures or models of these strange worlds. The simplest was the two-dimensional surface of a sphere. Consider a globe plastered over with a map of Earth. The lines of longitude cut the equator at right angles and all intersect at the north and south poles. But one definition of parallel lines is that they cut another line at the same angle. Thus at the equator, the lines of longitude are *parallel*, but they nonetheless *intersect* at the poles. In fact, all the largest circles that can be drawn on the globe are parallel – according to this definition – and yet intersect. Unwittingly, the mathematicians had created a new axiom system to describe the geometry of circles drawn on the surface of a globe. The strange universe they had created was not Euclid's, but it was free of contradictions. *It was the geometry of a curved space.*

In the early-nineteenth century, Karl Friedrich Gauss (1777–1855), one of the greatest mathematicians of modern times, pondered these unexpected developments. He had a daring, tickling insight. A very small patch of the globe appears flat. This is why we used to think the earth was flat. In each small bit of the surface, parallel lines do not meet and Euclid's old geometry holds true. But suppose, Gauss said, that *our universe*, the physical world we live in, is just like that!

Suppose that we experience only a small bit of the universe, and thus believe that parallel lines do not intersect. Suppose that parallel lines extended across the universe far enough do intersect, like lines of longitude at the poles. Suppose we live in a universe that is not described by Euclid's geometry. Suppose the universe is curved. These thoughts must have made Gauss dance.

Although Gauss's thoughts seemed a mere fantasy about lines extending across the cosmos, he brilliantly saw that his ideas could be tested here on Earth. The key was that triangles on a curved space differ from those on a flat space. Both are constructed of three line segments meeting at three angles. In the flat space described by Euclid, the sum of those angles was always two right angles (180 degrees). But this is not true in curved space: *the sum of the three angles in a triangle in curved space can be more than two right angles*. To see this, consider the large triangle on the globe constructed from two lines of longitude that intersect at right angles at the North Pole. If the part of the equator that lies between them is taken as the third side, then a triangle is formed between the North Pole and the equator. Every one of its angles is a right angle; thus, the sum of its three angles is three right angles. The curved space permits lines meeting at large angles to fit together to form a triangle.

Gauss was so seized by this fact that he soon persuaded his friends to test his ideas. They knew that small triangles would appear to be flat, and have angles that added up to two right angles. But triangles that were large enough might begin to reflect any curvature in space, and have angles that added up to more than that. Gauss and his friends hauled telescopes and other apparatus to the tops of three

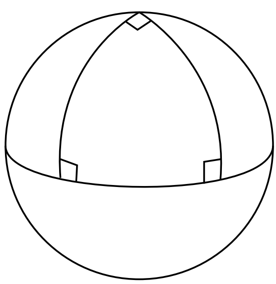

Figure 14.1 *The geometry of a curved space. The two lines drawn from the North Pole to the equator, with the equator itself, form a triangle with three right angles.*

mountains in Germany. With lights flashing on two of them, and the telescope on the third, they were able to measure the angles between the mountaintops. Repeating this, they found all three angles. Alas, after careful calibrations and calculations, they discovered that, within experimental error, the sum of the three angles was two right angles. Euclid was right! His geometry described the true structure of the world. Defeated, Gauss trudged back down the muddy slopes, not realizing that he had simply been two centuries too early. Although his challenge to Euclid was stillborn, the new geometries blossomed into an exciting sub-speciality of mathematics. Soon new paradises of exotic geometries were being invented and explored in the minds of mathematicians all over Europe. Euclid still reigned, but subversive energies lurked in the shadows.

As we have seen, Einstein worked to extend his special theory into the more general theory, which included gravity, for eight long years, from 1907 to 1915. This was a titanic struggle of late nights, frustrating mistakes and time-wasting failed ideas. At one point, Einstein began grappling with the idea that gravity might distort geometry. This was a breathtaking intuition, but Einstein was a poor mathematician and knew too little of advanced mathematics to express his ideas. Desperate, he finally appealed for help to a friend from his days at university. Now a professional mathematician, the friend surprised Einstein by saying that an entire sub-branch of mathematics had been developed to describe the geometry of curved spaces. The mathematical language Einstein had been groping for in the dark had already been worked out in detail, but Einstein had not known.

This new high-powered language for describing curved spaces had been worked out by mathematicians merely for its sheer beauty. A few of them (like Clifford and Riemann) speculated that Gauss was perhaps correct, and that further experiments should be done to investigate the geometry of the physical universe. But in the main, curved spaces were studied as pure abstractions. Once in Einstein's hands, however, the new language helped him make the final breakthroughs to his general theory. The experiments celebrated as confirmations of Einstein's theory also confirmed that Gauss was indeed correct. The geometry of curved spaces, with triangles whose angles did not add to two right angles, was the geometry of our world.

The lesson is a profound one. The axiomatic method was born out of a desire to isolate the essence of ideas and their relations. Euclid's masterpiece applied the method and surprisingly revealed that assumptions about parallels were a cornerstone of geometry. Two thousand years later, this paid off in a wholly unexpected way: it inspired the development of non-Euclidean geometries and brought Einstein's general relativity to fruition. The step-by-step analysis of arguments is a precious inheritance.

> Mathematicians used to think that they were studying the structure of space or the properties of figures; alternative geometries taught them they were merely studying possible models.

What does curved space curve into?

Anyone first encountering the idea of curved space often asks what it is curved into. Generally, things bend by moving into some unoccupied space. But if the whole universe consists of curved space, *what could it bend into*? Physicists usually reject this question, and this section explores one of the motivations for this neglect.

We usually think of ourselves as three-dimensional creatures living in a three-dimensional world. But suppose there was a two-dimensional universe called "Flatland" populated by dismally thin two-dimensional creatures, the "Flatlanders". They might slide around their world very contentedly and never feel that they had been deprived of a dimension, just as we feel cosy in our three dimensions. Flatlanders might develop a sophisticated geometry, and, indeed, might decide that their space was curved (as if they were, unbeknown to themselves, living on the surface of a balloon). They might notice, for example, that the angles in large triangles did not add up to two right angles. Their measurements would, of course, all take place *within* their world and thus would have to describe the curvature of their space in a language that referred only to their two-dimensional world. It would have to be an *intrinsic description*.

Gauss invented a way of describing a curved space that depended on features and measurements confined within that space. This language is complete, and does not require any reference to what may be outside the space. The Flatlanders, after all, may be correct that their universe is only two-dimensional, that there is not-even-nothing outside their horizontal world. Geometers say that Gauss's intrinsic descriptions need not refer to any outer, *embedding space*.

Physicists have adopted Gauss's viewpoint. Curved spaces are entirely autonomous. Since a curved space can be described internally, curvature does not presuppose any outside space for it to "curve into". In part, this is motivated by their empiricism. If physical theory does not need to mention a structure, if measurements and predictions do not depend on it, then the structure has no place in physics. Embedding spaces are superfluous.

> Physicists say our universe can be curved even though there is nothing outside it.

The loss of certainty

So far we have two intersecting stories. Einstein's monumental struggles to extend special relativity led him to believe that space was somehow curved. He did not know that investigations of Euclid's fifth postulate had led mathematicians to the development of an elaborate new language for curved spaces. When Einstein adopted this language in his new general theory, he was able to make precise and surprising predictions. Their confirmation persuaded us that the world was correctly described by these exotic geometries – that Euclid was overthrown. These facts tumbled philosophy into a new world, and must affect the way every philosopher thinks. They mark a new age in the history of reason.

Amid the busy hurly-burly of our ordinary lives, the philosopher's search for permanent truth, for deeper, lasting understanding, seems not only impractical but also hopelessly naive. From Plato onwards, however, philosophers could always point at mathematics as an ideal of certainty and knowledge. They could say that truth was attainable, that they had proud, noble, valuable examples here ready at hand. Under the flag of Euclid's geometry, philosophers could rally the troops and fend off the sceptics. For some 2,000 years, Euclid's axioms were thought true. They were precise insights into the nature of Being, into the structure of our world. The edifice of inferences and theorems he built on his axioms was a model for deriving truth from truth, an ideal for reason everywhere to struggle towards.

General relativity led to a profound re-evaluation of Euclid. Before Einstein, the exotic geometries of curved spaces were often thought to be mere fantasies, games that mathematicians might play during their idle hours. But Einstein showed that experiment favoured these non-Euclidean geometries. It was the world that was fantastic. Thus Euclid's axioms were not insights into Being. There were reinterpreted as mere historical accidents. They were arbitrary starting points chosen by a mathematician with a limited imagination. Just as people who do not travel much might think that the earth is flat, mathematicians studying only circles and lines on their little slates might think that Euclid's axioms were obvious. If they had first studied larger systems like triangles formed by mountains or stars, they might have observed that three angles in a triangle do not always add to two right angles. They might have chosen different axioms.

To add insult to injury, even Euclid's reasoning was found to be flawed. With the rise of more powerful logics in the twentieth century, the *Elements* was carefully scrutinized and found to contain a catalogue of minor errors and omissions. Although all of Euclid's theorems did follow from his axioms, his proofs were sometimes sloppy or incomplete by modern standards. Euclid was cut down to human size.

Mourning the loss of their hero, philosophers retreated. They had thought of axioms as truths, as the deepest, most secure foundations for their systems. Now axioms became mere assumptions, mere premises. They were provisional starting points, which another philosopher might or might not accept. Instead of pillars driven into bedrock, axioms were merely floating ideas selected for convenience on given occasions and disposed of tomorrow.

Non-Euclidean geometries, of course, did not replace Euclid with a new crystal certainty. Once Einstein has shown that experiment would decide which geometry described the physical world, he made all geometry uncertain. Future experiments and deeper theories may show that even these curved spaces are superficial, special cases. Deep down, our world may be neither Euclidean nor non-Euclidean. There is no longer any certainty.

Mathematicians pioneered this new view of axioms, as they were the first to be acutely aware that Euclid had competitors. Among philosophers, Russell loudly propagandized for the new viewpoint in his *Principia Mathematica*, where he emphasized that his assumptions were doubtful and provisional. He claimed only that they were sufficient to derive the conclusions he needed, and not that they were necessarily true in any deeper sense. After Einstein and early work by the philosopher Hans Reichenbach, Euclid's downfall was widely accepted: *axioms became assumptions.*

For many philosophers, these developments represent real progress, a welcome sign that philosophy has thrown off its dreams of achieving truths that escape from human fallibility. Complete certainty was always beyond our grasp, they say, and the myth that mathematicians had found it simply confused philosophers. We are not diminished by Euclid's downfall; we are liberated. Living with a sense that all knowledge is as fragile or as strong as we make it is a mark of maturity.

Einstein transformed not only our image of the physical world, but also our belief in reason and philosophy itself.

CHAPTER 15

The resurrection of absolutes

Philosophy does make progress. The achievement of philosophers of space and time over the past 30 years has been extraordinarily important and far-reaching. The dramatic claims made by Einstein and many other physicists about the death of Newton's absolute space have been rebutted. The nature of spacetime has been substantially clarified in ways that would have astonished the pioneers.

This success is all the more significant because it has taken place in the face of hostility from many physicists. Even today, many or most physicists cling to some of the naive early claims made about relativity theory, which survive as a kind of folklore in the physics community. There are exceptional physicists who contributed to recent developments. But philosophers deserve recognition for penetrating through the fog that surrounded the foundations of spacetime theories and for moving the debate ahead.

Ancient and medieval philosophers debated the existence of universals. Was there, they asked, a single, universal "form of red" present in each red thing and somehow making it red? Realists argued that such universals were needed to *explain* the similarities between colours and all the properties we see around us. Tough-minded nominalists insisted that universals were merely common names, and resisted entities that were not solid, respectable individuals. The controversies below provide a modern parallel to these debates. One side accepts invisible spatial structures to make sense of what we see; the other derides this as extravagant metaphysics and adheres closely to concrete observables. Unlike the ancient debates, which linger still, there is a strong if not unanimous sense that decisive progress has been made here.

The debate below is of sweeping importance not only for physics and philosophy but for intellectual culture as a whole. The existence of space as an entity in its own right seems to be an issue for metaphysicians. But the interpretation of relativity theory is at once the interpretation of our deepest scientific theory and of science itself. It cuts quickly to the questions of where we and the universe came from, and whether reason can discern what lies behind appearances. The future is fighting its way out of this debate.

Modern relationalism

Recall, from above, that Leibniz attacked absolute space. He asserted that, since all motion was relative, absolute place or absolute velocity could never be observed. Leibniz had, however, no answer to the bucket argument. In a flash of genius, Newton noticed that the sloshing water in the rotating bucket was independent of surrounding bodies. He argued that this acceleration was strong evidence for absolute space. Many philosophers and physicists found Newton's vast, invisible, unobservable absolute space irritating, and wished to banish it from physics. But the bucket argument proved a thorn in their side. They tended simply to ignore or overlook Newton's insight. They certainly never had a plausible answer to it.

Three hundred years after Newton, this changed with Ernst Mach, an Austrian physicist now remembered whenever we say that a jet plane has flown at "Mach Two". Unusually, Mach was also a path-breaking historian of physics and fully realized the importance of Newton's bucket. Like Leibniz, he detested absolute space; unlike Leibniz, he took up the challenge of the bucket argument. His reply inspired the young Einstein, and opened the way for Einstein's heretical rejection of absolute space. Thus Mach is now recognized as the originator of attempts in the twentieth century to rid physics of Newton's absolutes. We will see below how far they succeeded.

Mach's famous reply to Newton's bucket argument is driven by a strict *epistemological* assumption. Mach was a prominent advocate of *positivism*, an important philosophy in the late-nineteenth century and early-twentieth century, which insisted that all knowledge was based on experience. It was an extreme form of empiricism. Positivists denigrated any knowledge not based on experience as mere "metaphysics", and this term became a label for old-fashioned philosophy. For positivists, these were merely old superstitions and, like belief in witches, had to be left behind by the march of progress. Physics, in particular, had to expel everything not founded firmly on observation and measurement. The following summary of Mach's argument connects this viewpoint to the attack on absolute space:

Mach's reply to the bucket argument

A. The physical cause of the concavity in the bucket is motion. (P)
B. Motion is either relative or absolute (and nothing else). (P)
C. Therefore, the physical cause of the concavity is either relative or absolute motion. (from A,B)
D. But absolute motion is not observable. (P)
E. Physical causes are observable. (P)
F. Therefore, absolute motion is not a physical cause. (D,E)
G. Therefore, the physical cause of the concavity is relative motion. (C,F)

As we have seen, Newton had argued that the concavity *could not* be caused by relative motion. The spinning water was at rest relative to the bucket and surely the concavity couldn't be caused by rotation relative to the distant surroundings, that is, to the tree, stars and so on. With his missionary zeal for positivism, Mach is certain that causes must be observable. This leads him to conclude that the concavity *must* be caused by relative motion, since only relative motions are observable. Thus it must be the tree or stars that cause the concavity. For him, there is no alternative, no option of invoking unobservable absolute space. For Mach, the spinning water slides up against the sides of the bucket because the water is rotating relatively to the distant stars. Somehow their distant masses grab the water in the bucket.

Mach's general philosophy of positivism had led him to a specific view of space and time that philosophers call *Relationalism*. In this ontology, *the only things that exist are bodies and the relations between them*. Here, "bodies" may mean things with matter or energy, and includes fields of energy like those of electricity or magnetism. The key point is that there is no "container space" independent of bodies, no spatial or temporal structures *over and above* bodies and their relations. Thus there are no places that have an existence distinct from bodies. Things are not in places. The motivation for this relationalism is the rejection of unobservables: bodies and relations are observable, while spatial containers are not.

The opposite of this view is *substantivalism*, which asserts that there is some container, some spatial and temporal structure over and above, and containing ordinary bodies. Newton was a substantivalist.

The terminology can be confusing. In ancient philosophy, space and substance were rival solutions to the problem of change. The atomists solved the problem by supposing that a vast, invisible space existed; Aristotle's common sense insisted that only ordinary bodies existed, and that these substances touched each other without gaps in a plenum. However, over time, the word "substance" began to mean anything solid and "substantial": anything existing on its own and independently of other things. Philosophers began to call Newton's space a "substance" when they wanted to emphasize that it existed independently of the bodies it contained.

John Earman usefully notes that relationalism insists that relations between bodies are *direct*, while substantivalism argues for *indirect* relations. According to the latter, the Sun is 150 million kilometres from Earth because the Sun is in one place, Earth in another, and the places are connected by a stretch of space. Thus the relation between bodies goes through non-bodies, that is, spatial structures. Such relations are indirect. A relationalist denies that the distance between the Sun and Earth is constituted by a spatial structure. For example, fields of gravity and electricity stretch between them and a relationalist may say that these somehow just are, or constitute, the separation between the Sun and Earth.

Thus relationalism and substantivalism disagree about whether space and time exist over and above ordinary bodies and their relations. More briefly, they disagree about whether space exists. For a generation now, this question has been at the core of contemporary philosophy of space and time.

> Relationalists are tough-minded empiricists who say only the concrete is real. Substantivalists are far-seeing metaphysicians who tolerate unobservables to "make sense" of things.

Mach motivates Einstein

Newton's prestige was so great that Mach's radical relationalism was very liberating for the young Einstein. It gave him the courage to consider whether absolute space and time were really needed in physics, and probably directly contributed to his great breakthroughs. As we have seen, the special theory of relativity of 1905 launched a direct attack on invariant distances and durations, and therefore seemed to dispose of both absolute space and absolute time. But Einstein knew that his revolution was incomplete. The theory was special because it was limited to the special case of rulers and clocks moving at steady speeds in straight lines. The theory applied only to measurements made by devices moving inertially, and not those undergoing acceleration.

The distinction between inertial movement and accelerated movement seems to depend on the existence of space. If there is a physical difference between them, there must be some standard that determines which is accelerated and which is not. But this standard must be space. It was for this reason that Mach was determined to show that acceleration was merely an ordinary kind of relative motion: not relative to absolute space, but relative to other ordinary bodies. Thus, for Mach, the coffee in an accelerating cup sloshes because the cup is accelerating relative to the distant stars.

Einstein was determined to vindicate this insight of Mach's. His work on the general theory of relativity was guided by this belief that he had to eliminate any fundamental distinction between inertial and accelerated motions. Both should be kinds of motion relative to other bodies, and should not be evidence for unseen spatial structures. In 1916, Einstein completed his general theory, and announced:

> In Newton's mechanics, and no less in the special theory of relativity, there is an inherent epistemological defect which was, perhaps for the first time, pointed out by Ernst Mach.
>
> In an epistemologically satisfactory law of causality, the reason given for any effect must be an observable fact of experience – ultimately only observable facts may appear as causes and effects. Newtonian mechanics does not give a satisfactory answer to this question [of why the spinning water in the bucket is concave], since it makes the unobservable cause of absolute space responsible for the observable effects.
>
> The general theory of relativity takes away from space and time the last remnant of physical objectivity. In this theory, both a body moving with uniform motion in a straight line and an accelerating body may with equal right be looked upon as "stationary".

In sum, Einstein thought he had vindicated Mach and vanquished Newton in two steps:

- Special relativity eliminated absolute distances and durations and velocities, but retained absolute acceleration (which was Newton's evidence for absolute space and time).
- General relativity eliminated absolute acceleration, and thus what was supposed to be the last physical evidence for absolute space and time.

The triumphant claim by Einstein that he had stripped space of its "last remnant of physical objectivity" has been extremely influential. Especially in the middle of the last century, and still to a large degree today, Einstein's pronouncement was regarded as gospel.

> Mach's positivism led him to deny the existence of atoms since they were too small to be observed. As he lay dying, legend has it that his students showed him a small "scintillation screen" where individual atoms threw off sparks, and he recanted.

Neo-Newtonian spacetime

Philosophers have fought back against Mach and Einstein and largely vindicated substantivalism and forms of absolute space. Although the battle still rages, and many points remain contentious, there is clearly a growing consensus against Einstein and his fellow relationalists. The next three sections briefly introduce this historic rehabilitation of absolute space. Here we discuss the question of whether absolutes can be justified within classical Newtonian physics, and then we return to relativity theory.

Newton's absolute space scandalized good empiricists and positivists. His ghostly, invisible space and unmeasurable absolute velocities were outrageous lapses for those crusading to rid physics of old-fashioned "metaphysics". The core of the relationalist attack on absolute space was the inference:

> Even before Einstein's relativity, we knew that absolute velocity is unmeasurable and absolute place is undetectable. This implies that there is no scientific justification for substantivalism: there is no spatial container over and above bodies and their relations.

This assertion that absolute space was merely "superfluous structure" without empirical justification was often used to browbeat backsliding defenders of Newton.

A milestone in philosophy of space and time was passed when a strong consensus emerged that the above inference was a mistake, thus robbing relationalists of a central weapon. One major reason is that philosophers now agree that substantivalism can be slimmed down and stripped of its superfluous structure. This lean and mean substantivalism can withstand empiricist arguments from the nature of motion. Thus *within classical physics* the arguments do not justify abandoning all structures over and above bodies and their relations.

This puts pressure on relationalism. If absolutes are to be abandoned altogether, it must be because of something new about relativity theory. Empiricism alone is not sufficient to banish absolutes.

The strategy for salvaging substantivalism involves three elements. First, concede to Mach that absolute velocities and absolute places are unobservable, and should be expunged from physics. Secondly, shift from considering three-dimensional spaces to four-dimensional spacetimes. Thirdly, strip down the structure of the four-dimensional spacetime so that it has just the properties and relations justified by observation and no more. The result is a theory or model of space and time known as *neo-Newtonian spacetime*. It is substantivalist and therefore "Newtonian" but, unlike Newton's space, it is four-dimensional and therefore a "spacetime".

Relative velocities and absolute accelerations are observable. Before neo-Newtonian spacetime was invented, no one knew how there could be absolute accelerations (as in the bucket) without absolute velocities and absolute space. Acceleration is, after all, just a change in velocity. The key feature of this theory is that it predicts just the needed combination: absolute accelerations but merely relative velocities.

The quickest way to understand the structure of this new spacetime theory is to compare it to a four-dimensional representation of Newton's original absolute space. Newton believed that the universe was only three-dimensional but a *four-dimensional model* of Newton's theory can be built as follows. Imagine there is a four-dimensional block universe that is neatly sliced, and that each slice is a three-dimensional world at an instant. Newton believed that "the same place existed at different times". In a four-dimensional model, this would mean that places on different slices are "the same" in some sense. They are not *one and the same* because they are on different slices, but have *some relation* that indicates that they share something in common. This relation can be symbolized by R(x,y). That is, any two places, say a and b, which are "the same place at different times" have the relation R(a,b). If these relations exist in our four-dimensional model, then it makes sense to say that places persist through time, and this is what Newton meant by absolute space. These cross-linking relations stretching from slice to slice thus provide a four-dimensional model of Newton's picture of space.

Using these relations to keep track of which places are which as time passes, we can define absolute velocities and absolute accelerations. Thus this four-dimensional model has the features that Newton wanted. But does it have too much? Can we modify the model in a way that abandons absolute place persisting through time, but keeps absolute acceleration? In other words, can we drop absolute place from our model but still pick out which movements are along straight lines at uniform speeds, that is, which movements are inertial and unaccelerated?

Suppose now that the relations R(x,y) do not exist in our four-dimensional block universe, and thus places on different slices are just different and unrelated: there is no sameness of place across time, and therefore no eternal absolute space. Instead, suppose there are three-place relations I(x,y,z) that stretch across the different slices. Three places, say a, b and c, have the relation I(a,b,c) just when a body moving inertially would pass through a on one slice, b on a later slice and c on an even later slice. If a body follows a path through spacetime such that any three places along its path are related by the new relations, then the body is moving inertially along the entire path. Any body deviating from an inertial path is accelerating absolutely.

This is the trick. Instead of defining persistent places, we directly define which movements are inertial. That is, instead of keeping track of places through different slices and using this structure to decide which bodies are moving inertially, we simply keep track of which trajectories are inertial.

If the real spacetime of our physical universe had the structure of this neo-Newtonian spacetime, then we expect to see absolute accelerations (which make the water in the bucket and coffee cups slosh) but only relative velocities. Since this is what we do see, we have an argument that this model correctly describes the real structure of our universe (at least in domains where only low velocities and thus no relativistic effects are involved). If this is so, however, then classical substantivalism is rescued. We have a theory that posits a container over and above bodies and their relations but has no undetectable superfluous structure. We have put Newton's absolutes on a diet, and saved them from Mach's empiricist scoldings.

Neo-Newtonian spacetime does not represent a complete victory for substantivalism. Other attacks can be made. For example, some of Leibniz's symmetry arguments apply equally to Newton's absolute space and neo-Newtonian spacetime. It is also true that the plausibility of this model rests on resolving questions about the status of the future and past in a four-dimensional block universe. Nonetheless, a prominent and seemingly persuasive attack has been rebutted. By shifting from Newton's three-dimensional absolutes to a four-dimensional spacetime, those who argue that space does exist in its own right have won an important battle.

> Model builders ask what properties and relations are needed to account for observations; they do not ask what accounts for those properties and relations.

Absolutes in general relativity

As we have seen, Newton believed that the effects of acceleration, like the sloshing of liquids, provided evidence for absolute space. Special relativity retained a peculiar role for acceleration by limiting its predictions to non-accelerating, inertially moving rulers and clocks, and this seemed to endorse Newton's insight. After ten years of labour, Einstein was understandably proud when he claimed to eliminate the "last remnant of objectivity" from spatial structure in his general theory. Many in the physics community still believe that Einstein was correct.

A number of physicists and most philosophers of science now believe that Einstein was wrong. Although his theory has made many correct predictions, and remains the foundational theory for astronomy and cosmology, Einstein *misinterpreted* what his new mathematics implied. It is not true that general relativity eliminates absolutes. This reinterpretation of general relativity represents an important achievement of post-war philosophy of space and time.

There are two main lines of objections to Einstein's claims about general relativity, which can only be sketched here. The first concerns *boundary conditions* and the second *non-standard models*. Recall that, according to the rubber sheet analogy, we may imagine that spacetime is stretched and distorted by the presence of mass and energy. Large masses create curvature; regions that are mostly vacuum are typically flatter. At first it was thought that this made spacetime a kind of property of the distribution of mass and energy, that is, that mass and energy and their relations fully determined all aspects of spacetime.

To see that this is not the case, consider an ordinary drum. The sound made when a drum is struck depends on three things: the location and intensity of percussive whack, the elasticity and tautness of the material stretched across the drum *and* the size and shape of the drum. A small, child's drum and a large drum like those used in orchestras might be covered by the same taut material and struck in the same way, but they will produce very different tones. Thus the sound that emerges is an important combination of local and global factors. The reason is that the vibrations that produce the sound waves are extended across the whole drum, and will therefore depend on the "global" shape of the whole drum. The bang produced by the drumstick is a local occurrence, but the response is global.

In general relativity, mass produces curvature the way that the drumstick depresses the drumhead. The resulting curvature of spacetime, however, depends on the *shape of the whole universe*, just as the drum's tones depend on the shape of the whole drum. Physicists say that the curvature of spacetime depends on the "boundary conditions", just as the vibrations on the drumhead depend on the shape of the drumhead and the way it is tacked down at the edges.

This is important because it means that the distribution of mass and energy does not determine all of the properties of space and time. The overall curvature of the universe depends on mass and energy *and* the boundary conditions. Since these are independent factors, some aspects of space and time are independent of mass and energy. Mach's hopes that all of physics would depend only on the relations between observable mass and energy are thus dashed: as Newton proposed, spacetime seems to have some autonomous aspects, and exists over and above the bodies within it.

General relativity is a *local theory*: it tells how each little patch of spacetime is attached to its neighbours. This sort of theory leaves a great deal of *global freedom*. By analogy, in a chain of iron links, we know how each oval link is attached to its neighbours, but the overall shape of the chain is still very free. It may by stretched out in a straight line or be left in a pile on the floor.

This global freedom is important. It means that general relativity can make very precise predictions about what happens in small regions of space, say the size of a star or black hole, but leave great doubt about the shape of the universe as a whole. Unfortunately for relationalists, this freedom has also given scope for the construction of non-standard models. These are models of the universe that agree at each point with general relativity's local predictions but have bizarre global features. Some of these models prove that general relativity retains a role for absolutes. In particular, absolute acceleration seems to have reared its head again, and confirmed Newton's original claim that accelerations are evidence for spatial and temporal structures independent of their contents. These models are complicated but, when examined in detail, show Einstein's claims to have eliminated the "last remnants of objectivity" from spacetime were premature.

Philosophical victories are rarely clear-cut. No referee blows a whistle to stop play and declares a winner. Inventive minds will always push and probe more deeply. Nonetheless, the bulk of the arguments now favour sophisticated substantivalism over relationalism. The philosophers who led this revolt, Adolf Grunbaum, John Earman and Michael Friedman, fought against considerable resistance from physicists, and indeed against all those who too quickly acceded to Einstein's authority.

We admire the genius of Einstein's mathematics, and the success of its many predictions. But we also see that, as a pioneer, Einstein misread the trail of ideas that led him to his discoveries. We now understand the nature of Einstein's relativity revolution better than he did himself.

> Space or spacetime exists. It is not merely a creature of the matter and energy in its midst.

The bucket in orbit

NASA will soon launch a satellite built by a team based at Stanford University to conduct Newton's bucket experiment in outer space. The motivation for this experiment can be understood by returning to the rubber sheet analogy. Imagine that the heavy weight in the central well is grasped firmly, depressed downwards to grip the sheet, and twisted. The turning weight will form a whirlpool of spiral wrinkles radiating across the sheet and out to its edge. This effect can, however, be produced in the opposite way. Imagine that the weight is left alone in the centre of the sheet, and instead the entire outer edge of the sheet is given a sharp twist. Since the heavy weight will not immediately keep up with the rotating sheet, a similar whirlpool of wrinkles, this time converging on the centre, will form; soon the weight will feel the twist and begin rotating itself.

The twisting rubber sheet gives us a picture of Mach and Einstein's view of Newton's bucket. They claim that acceleration relative to absolute space does not produce the concavity on the water's surface. Instead, they say, it is the rotation relative to the surroundings that causes the concavity. In fact, they add, there is no way to distinguish whether the water is rotating or whether its surroundings are. Just as the rubber sheet suggests, if the universe was rotating around the bucket, the same concavity would arise. Thus the effect is due to the relation between the bucket and its surroundings, that is, the relative rotation. It does not depend on whether the bucket has some fictional absolute motion. There is, they say, no fact of the matter about which is rotating: the bucket or the starry heavens.

Very soon after Einstein published his general theory of relativity, it was shown that the theory did predict that a rotating universe would cause the concavity in the bucket. More precisely, it was shown that mass rotating around a central body would grab the body and produce a rotation. This effect is known as *frame dragging*. The lines followed by bodies moving inertially are known as their "reference frame", and these are dragged around and twisted like the rubber sheet above. Relationalists view these calculations as a vindication of their interpretation of the bucket. Newton's argument has always been a thorn in their side and this, they say, is the nail in the coffin of arguments for absolute space.

Regardless of these debates, the experimental confirmation of frame dragging would be an extraordinarily important test of general relativity. Most tests to date rely on astronomical observations, and therefore on distant objects we cannot manipulate directly. Physicists have proposed that a spherical weight sent into orbit would be influenced by the earth's rotation. On the earth's surface, the effect would be swamped by vibrations. Suspended in outer space, delicate measurements could distinguish the direct gravitational forces from the subtler influences produced by frame dragging. Accomplishing this, however, has required some of the most advanced physics and engineering ever poured into an experiment. The special weight in the satellite is said to be the most perfectly spherical object ever manufactured. It will be cooled to temperatures near absolute zero to eliminate the tiny vibrations of heat energy. (See Appendix D for websites to access for a further description.) The bucket has gone high-tech.

Substantivalists can hope for the success of this experiment without conceding that it counts against the existence of space in its own right. As Earman and others have argued, relationalists need to show not that the rotating stars would influence the water in the bucket, but that rotation would account for all of the observed concavity. This has not been done. Even in general relativity, rotation is absolute. Newton's deep insight into the puzzle of rotational acceleration has survived into the space age.

CHAPTER 16

The resilience of space

The concept of space was born in paradox and seemed to have the flimsiest claim to existence. Although nothing but mere empty extendedness, it helped make motion and change understandable. Aristotle's rugged common sense rejected "space" out of hand, and made do with his plenum of concrete objects. We have now seen that this ancient debate was preparation for the grander controversies over Newton and Einstein's concepts of space. Like the ancient atomists, Newton embraced space to make sense of motion. His law of inertia demanded a world of geometric lines, and the sloshing water in his bucket seemed to make absolute space almost visible. Like Aristotle, the tough-minded empiricists made war on this metaphysical extravagance. Mach, Einstein and contemporary relationalists all fought back against a space existing over and above its contents. Against Newton and Lorentz, they dispensed with "superfluous" structure and pushed physics back down towards concrete objects and their concrete relations.

But now the folklore that surrounds Einstein's relativity theory has been dispersed. Amid Einstein's many triumphs, he did transform and deepen the concept of space. He did not, however, reduce space to its contents. This flimsy nothing has proved resilient. In special relativity, we saw that the defence of absolute space in Lorentz's minority interpretation was not ruled out by experiment. Furthermore, it provided attractive explanations where the mainstream provided none. A philosopher would say that the case for eliminating absolute space there rested on a strict empiricist ideology, and was not compulsory. In general relativity, the case for some features of curved space remaining independent of matter was significantly strengthened. Boundary conditions and non-standard models proved that the anti-substantivalists had not clinched their case.

The theme connecting the chapters in Part II has been the centrality of real relations. Over and above the individuals in the universe, there must be some web of relations stretching between and uniting them. The philosophy of space and time turns crucially upon the nature of these relations. The ancient paradoxes exposed the knot of conflicting tensions underlying relations, and led to Aristotle's rejection of real relations. In Leibniz, we have the last great metaphysician who saw the depth and intractability of these problems, and likewise banished relations from his system.

In Part I we saw that the mainstream interpretation of special relativity turned the basic properties of distance and duration into relations. Here in Part II, the succeeding chapters have made a strong case for the robust reality of spatial relations. From Kant's incongruent counterparts, to the three-place relations of neo-Newtonian spacetime, and then to the role of absolutes in general relativity, relations between concrete individuals were constituted by something non-concrete. In short, spatial relations are as real as the things they relate.

The victory of substantivalism – of real spatial relations – is still in doubt. Physicists remain in thrall to Einstein's premature claims to have eliminated the objectivity of space. Among philosophers, the autonomy of space is still contested. But, from a long-term perspective, real spatial relations have been gaining credibility for some 400 years. Once mathematics had been rephrased in the language of equations, and once physics had adopted this new language, the ancient resistance to relations was rendered implausible. Modern critics of spatial relations retreated to the claim that there were somehow only concrete relations. But even this halfway house has failed to make sense of the physics of motion and change. Opinion is moving toward substantivalism.

If spatial relations are accepted as real, however, future generations will have to contend with the paradoxes they conceal.