Session 3

Singularities

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3.1 Rapporteur talk: Singularities, by Gary W. Gibbons

3.1.1 *Introduction*

Einstein's General Relativity is incomplete because

- It predicts that **gravitational collapse** , both at the Big Bang and inside black holes, brings about spacetime singularities as at which the theory breaks down
- It gives no account of 'matter 'as opposed to geometry , and in particular the nature of classical 'particles '
- It is incompatible with quantum mechanics

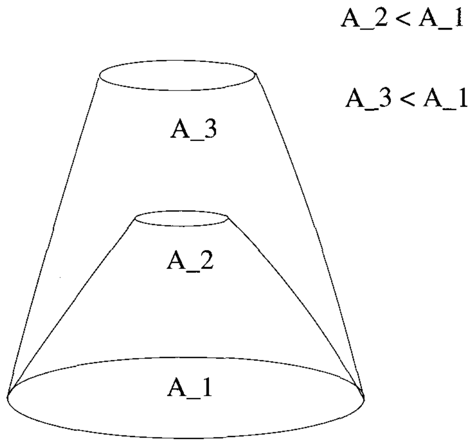
I have been asked to review the first problem.
I will cover the following topics.

- Singularity Theorems
- Cosmic Censorship
- Classical Boundary conditions and stabiity
- Higher dimensional resolutions
- Singularities at the end of Hawking Evaporation
- Maldacena's conjecture

3.1.2 *Singularity Theorems*

First discovered in Friedmann-Lemaitre models, it was shown by Roger Penrose [19] that these arise if closed trapped surfaces occur during gravitational collapse and work by Geroch , Hawking and Penrose [20] showed that as long as matter satisfies various **positive energy conditions**, then spacetime singularities are inevitable in the future of certain types of Cauchy data.

Thus unlike classical Yang-Mills theory [1] and scalar fields theories with renormalisable potentials, **Leibniz-Laplace Determinism** breaks down for General Relativity. It can at best be an effective theory.



The area of a closed trapped 2-surface decreases in both the inward *and* the outward directions if pushed to the future along its two lightlike normals.

The singularity theorem give very little information about the nature of the singularities, in effect they deny the existence of timelike or null geodesically complete spacetimes. The reason for the incompleteness is not predicted.

It is widely believed that incompleteness is due to divergences of curvature invariants [2], or the components of the curvature in certain privileged, example parallelly propagated frames.

The singularity theorems also fail to predict the scale at which singularities arise. Indeed classical general relativity has no in built scale,

$$g_{\mu\nu} \longrightarrow \lambda^2 g_{\mu\nu} \tag{1}$$

with λ constant is a symmetry of the theory.

Many people believe that the resolution of the problem of singularities will come from modifications of the Einstein equations due to Quantum Gravity at the Planck scale, but this is by no means obvious. The necessary modification could, in principle, have nothing to do with quantum mechanics.

It might for example entail the introduction of higher curvature terms [3] . However examples tend to show that some singularities still remain, e.g. those in singular pp-waves which are in effect solutions of almost all theories of gravity. In addition many, but not all admit ghosts.

Well motivated modifications of Einstein's theory include supergravity theories and the low energy limits of String theories.

[1] or possibly Born-Infeld theory
[2] but these may all vanish, e.g. for singular pp waves
[3] For example Born-Infeld Gravity [26]

However, the singularity theorems also apply to classical supergravity theories in all relevant dimensions since the matter fields satisfy the energy conditions. If supermatter is added, then only if potentials for scalars are positive (which cannot happen for pure supergravity) could singularities conceivably be avoided. However one may also truncate to the pure gravity sector and we are back to the same problem.

The same problem arises in String Theory in the zero slope limit. Only higher curvature terms could could conceivably evade the problem.

3.1.2.1 *The Strong Energy Condition*

$$\boxed{T_{\hat{0}\hat{0}} + \sum_i T_{\hat{i}\hat{i}} \geq 0} \tag{2}$$

is the most important energy condition ('Gravity is attractive').

It can only fail if potentials for scalars are positive.

As an aside , one should note that unless it fails, cosmic acceleration (e.g. a positive cosmological constant, $\Lambda > 0$, is impossible [27]).

Thus there can be no inflation in pure supergravity theories, or the zero slope limit of String theory.[4]

If one takes the view that the problem of singularities will be resolved quantum mechanically, one might be tempted to argue that no particular classical spacetime is of particular significance, and that classical or semi-classical studies of singularities are misguided.

At a fundamental level that is probably correct, and is a certainly a valid criticism of much current speculation on the final outcome of Hawking evaporation for example, but as a practical matter almost all of the large scale universe appears to be essentially classical. Astrophysicists should not need quantum gravity to understand X-ray sources or the black hole at the centre of our galaxy.

Thus we need understand better classical singularities.

3.1.3 *Cosmic Censorship*

The singularities that arise from localised gravitational collapse are associated with black holes. Intuitively, Penrose's **Cosmic Censorship Hypothesis** [21] postulates that all singularities are hidden inside the event horizon, i.e. inside $\dot{I}^-(\mathcal{I}^+)$, the boundary of the past of future null infinity, $\mathcal{I}^+$, the latter is usually assume to be complete.

Investigating this problem is extremely challenging mathematically. At present one is limited to looking at spherically symmetric spacetimes coupled to matter, e.g. massless scalar field, or, in the non-spherical case, to numerical simulations.

[4]except in models with time-dependent extra dimensions which have other problems

The work of Christodoulou on spherically symmetric massless scalar collapse [40][41][42][43] shows that Cosmic Censorship in the strict sense fails, because of transient singularities associated with a very special choice of initial data.

These singularities are associated with a discrete self-similar behaviour, referred to as critical behaviour, first uncovered numerically by Choptuik [44].

It is rather doubtful that this behaviour will survive quantum corrections.

The strategy of Christodoulou is essentially to reduce the problem to a 1+1 dimensional non-linear wave problem.

This technique has been exploited recently by Dafermos [35][36][34] [33][32] who is able to treat charged gravitational collapse and establish a form of cosmic censorship in the presence of Cauchy horizons assuming the existence of closed trapped surfaces and also to justify the assumption that $\mathcal{I}^+$ is complete.

Dafermos's methods also extend to higher dimensions [46] where, following numerical numerical work by Bizon, Chmaj and Schmidt in 4+1 [30] and 8+1 [31] Dafermos and Holzegel [29] were recently able to extend some of these results vacuum gravitational collapse in 4+ 1 .

There has been a great deal of work on homogeneous solutions of Einstein's equations, particularly near singularities. This can be partially extended to inhomogeneous solutions provided one assumes spatial derivatives are small (velocity dominated approximation).

On this basis, Belinsky, Lifshitz and Khalatnikov proposed that generically, singularities are of chaotic, oscillatory, Mixmaster type, first seen in Bianchi IX models.

Recent mathematically rigorous work tends to confirm that this may happen, at least for an open set of Cauchy data.

The story in higher dimensions will be the subject of a report by T Damour.

3.1.4 *Classical Boundary Conditions and Stability*

The basic problem raised by spacetime by singularities is what boundary conditions are to be posed in their presence? Cosmic Censorship is an attempt to evade that problem as long as one is outside the event horizon. Even if it is true, what happens inside the horizon?.

The choice of boundary conditions may affect questions of genericity or stability.

In some cases, despite singularities the choice of boundary condition may be unique.[5]

Typically in gravitational situations however, this is not the case and choices must be made.

For example, in a recent paper, Gibbons, Hartnoll and Ishibashi [5] showed that there is a choice of boundary conditions such that even negative mass Schwarzschild

[5] for example the unique self-adjoint extension for the Hydrogen Atom

is stable against linear perturbations[6].

Another case of non-uniqueness concerns quantum fields near cosmic strings and orbifolds singularities.

3.1.5 *Boundary Conditions in Cosmology*

The issue of boundary conditions is particularly important in cosmology. For example one typically thinks of Minkowski spacetime as being stable.However this is manifestly not the case if one considers perturbations which do not die off at large distances.

3.1.5.1 *Instability of Flat space*

Consider Kasner solutions of the vacuum Einstein equations,

$$ds^2 = -dt^2 + t^{2p_1} dx^2 + t^{2p_2} dy^2 + t^{2p_3} dz^2 \,, \tag{3}$$

where p_1, p_2, p_3 are constants such that

$$p_1 + p_2 + p_3 = 1 = p_1^2 + p_2^2 + p_3^2 \,. \tag{4}$$

Unless one of the p_i is equal to 1, these metrics have a singularity at $t = 0$. If we set $t = 1 - t'$, then the metric near $t = 1$ starts out looking like a small deformation of the flat metric, with a small homogeneous mode growing linearly with t'. Ultimately, however, non-linear effects take over and the universe ends in a Big Crunch at $t' = 1$, *i.e.* $t = 0$.

This instability is universal in gravity theories, and is closely related to the modulus problem in theories with extra dimensions.

Consider, for example, the exact ten-dimensional Ricci-flat metric

$$ds^2 = t^{1/2}(-dt^2 + d\mathbf{x}^2) + t^{1/2} g_{mn}(y) dy^m dy^n, \tag{5}$$

where $g_{mn}(y)$ is a six-dimensional metric on a Calabi-Yau space K. This starts off at $t = 1$ looking like $E^{3,1} \times K$ with a small perturbation growing linearly in $t' = 1 - t$. However by the time it reaches $t' = 1$, $t = 0$, the solution has evolved to give a spacetime singularity. From the point of view of the four-dimensional reduced theory, the logarithm of the volume of the Calabi-Yau behaves like a massless scalar field – the modulus field which is sometimes thought of as a kind of Goldstone mode for a spontaneously-broken global scaling symmetry. This causes an isotropic expansion or contraction of the three spatial dimensions, with the scale factor $a(\tau)$ going like $\tau^{\frac{1}{3}}$, which is what one expects for a fluid whose energy density equals its pressure.

In recent work, Chenm Gibbons, Hu and Pope[9] [10] have shown that a wide variety of BPS brane configurations, including the Horawa-Witten model are cosmologically unstable. .

[6] non-linear stability remains problematic

3.1.5.2 *Hořava-Witten solution from Heterotic M-Theory*

The equations of motion for the metric and ϕ may be consistently obtained from the Lagrangian

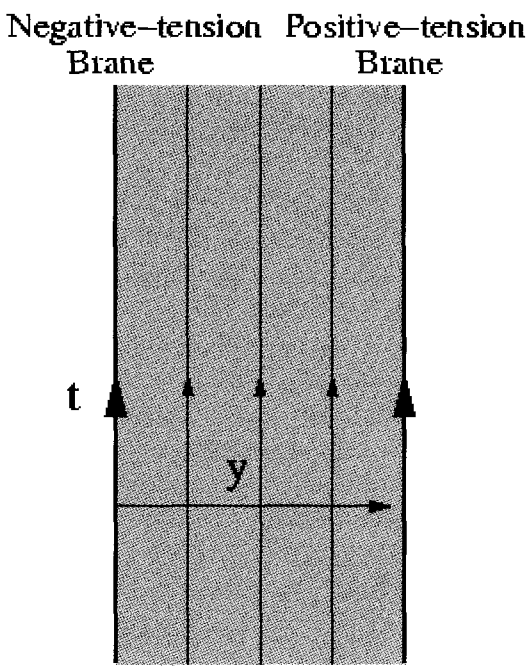
$$\mathcal{L}_5 = \sqrt{-g}(R - \frac{1}{2}(\partial\phi)^2 - m^2\, e^{2\phi})\,. \tag{6}$$

The scalar field ϕ characterises the size of the internal Calabi-Yau space.

3.1.5.3 *Exact static supersymmetric solution*

$$\begin{aligned} ds_5^2 &= \widetilde{H}\,(-dt^2 + d\mathbf{x}^2) + \widetilde{H}^4\, d\tilde{y}^2\,, \\ \widetilde{H} &= 1 + \tilde{k}\,|\tilde{y}|\,, \\ \phi &= -3\log \widetilde{H}\,, \end{aligned} \tag{7}$$

$\tilde{k}$ is a constant In the Hořava-Witten picture a second domain wall is introduce, at $y = L$, by taking y to be periodic with period $2L$, such that $y = L$ is identified with $y = -L$. Furthermore, one makes the Z_2 identification $y \leftrightarrow -y$.



3.1.5.4 *Time-dependent solutions*

$$ds_5^2 = H^{1/2}\,(-dt^2 + d\mathbf{x}^2) + H\,dy^2\,,$$
$$H = h\,t + k\,|y|\,, \tag{8}$$
$$\phi = -\frac{3}{2}\log H\,,$$

$k^2 = 8m^2/3$, and h is an arbitrary constant.

If we turn off the time dependence (by setting $h = 0$), the relation to the previous static solution is seen by making a coordinate transformation of the form $y = \tilde{y}^2$. If we set the parameter m in the Lagrangian to zero, the solution describes a **Kasner universe**.

When we lift the solution back to $D = 11$, the metric becomes

$$ds_{11}^2 = H^{-1/2}(-dt^2 + d\mathbf{x}^2) + H^{1/2}\,ds_{CY_6}^2 + dy^2\,. \tag{9}$$

The static solution, in the orbifold limit, can be viewed as an intersection of three equal-charge M5-branes. Turning off the brane charge, the time-dependent metric describes a direct product of a ten-dimensional Kasner universe and a line segment.

3.1.5.5 *Local Static Form: Chamblin-Reall picture*

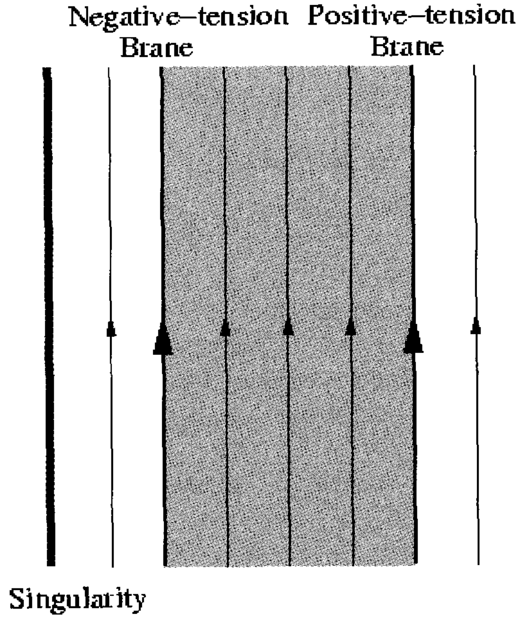
If we temporarily drop the modulus sign around y in (8), then the coordinate transformation from t and y to $\tilde{t}$ and r given by

$$dt = d\tilde{t} - \frac{hr^{1/2}}{k^2\,f}\,dr\,,$$
$$H = r\,, \tag{10}$$
$$f = 1 - \frac{h^2\,r^{1/2}}{k^2}$$

transforms the solution into the static form [25]

$$ds_5^2 = r^{1/2}\left(-f\,d\tilde{t}^2 + d\mathbf{x}^2 + r^{1/2}\,\frac{dr^2}{k^2\,f}\right),$$
$$\phi = -\frac{3}{2}\log r\,. \tag{11}$$

This can be recognised as a black 3-brane, with an horizon at $f = 0$.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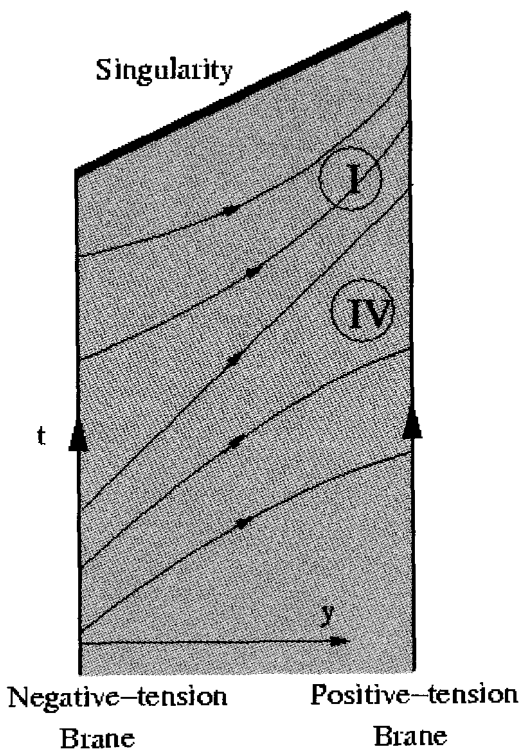
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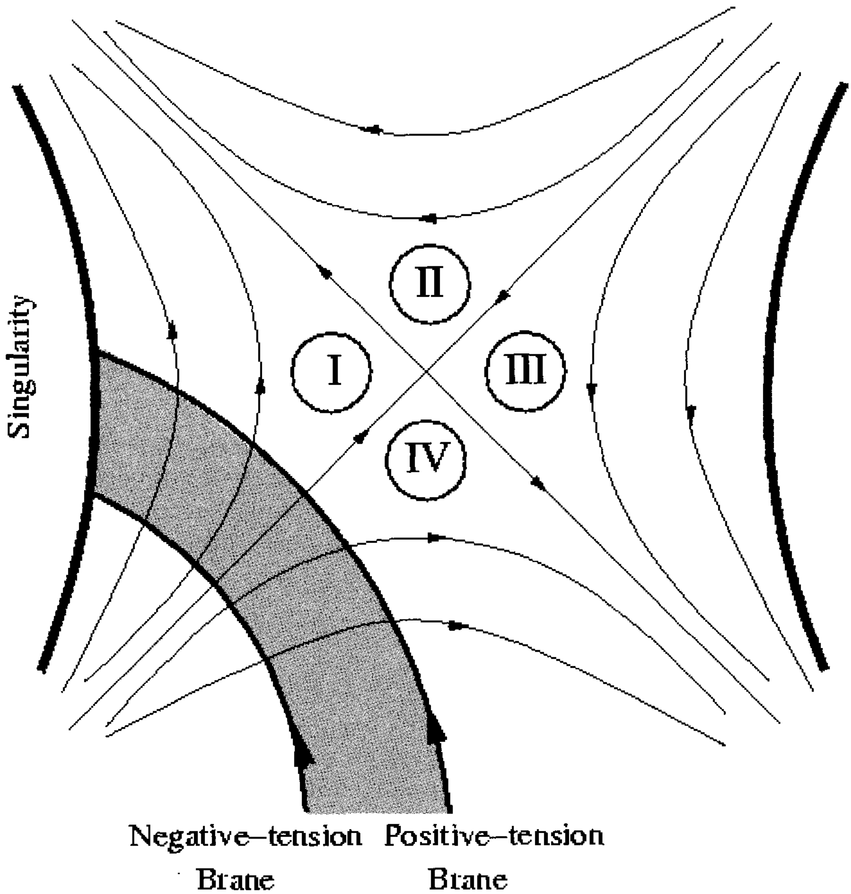

3.1.6 *Higher dimensional resolutions*

Serini, Einstein, Pauli and Lichnerowicz were able to show that there are no static or stationary soliton like solutions of the vacuum Einstein equations without horizons (see [45] for a review of this type of result). The presence of an horizon implies a singularity. These results extend straightforwardly [45] to include the sort of matter encountered in ungauged supergravity theories and Klauza-Klein theory[7] or the zero slope limits of String Theory. They follow essentially because these theories do not admit a length scale: rigid dilation

$$g_{\mu\nu} \to \lambda^2 g_{\mu\nu} \qquad \lambda \text{ constant}, \tag{12}$$

is a symmetry of the equations of motion.

[7] with the proviso that the fields in four dimensions are regular, see later

The situation improves if one considers the Kaluza-Klein monopoles of Gross Perry [3] and Sorkin [4].

$$ds^2 = dt^2 + V^{-1}(dx^5 + \omega_i dx^i)^2 + V d\mathbf{x}^2\,, \tag{13}$$

$$\operatorname{grad} V = \operatorname{curl} \omega \tag{14}$$

$$V = 1 + \sum_a \frac{M_a}{|\mathbf{x} - \mathbf{x}_a|} \tag{15}$$

Periodicity of x^5 imposes a quantisation condition on the Klauza-Klein charges and on the magnetic monopole moment.

In addition, the singularities of the four-dimensional metric receive a **Higher Dimensional Resolution**: they are mere coordinate artifacts in five dimensions. In this way, they evade the Pauli-Einstein theorem.

Gibbons, Horowitz and Townsend [2] have shown that higher dimensional resolutions are quite common. However the problem of singularities and the ultimate outcome of gravitational collapse and Hawking evaporation cannot be solved in this this way. Moreover, the solution is unstable in the sense that

$$ds^2 = dt^2 + \tilde{V}^{-1}(dx^5 + \omega_i dx^i)^2 + \tilde{V} d\mathbf{x}^2\,, \tag{16}$$

$$\operatorname{grad} V = \operatorname{curl} \omega \tag{17}$$

$$\tilde{V} = -ht + \sum_a \frac{M_a}{|\mathbf{x} - \mathbf{x}_a|} \tag{18}$$

is an exact, time-dependent solution.

This is a particular example of the general tendency of higher dimensions to undergo gravitational collapse [17], which is fatal to the dimensional reduction programme unless some means can be found to stabilize the various 'moduli 'fields.

These examples also underscore the need for **a theory of initial conditions** in order to understand cosmology and the initial singularity or big bang. As emphasised by Penrose among others, elementary thermodynamic arguments indicate that the Universe began in a very special state and even proponents of eternal inflation have had to concede, following Borde, Guth and Vilenkin [28] , that eternity is past incomplete. In other words if inflation is past eternal then spacetime is geodesically incomplete. **Penrose's Weyl Curvature Hypothesis** postulates a connection between gravitational entropy and Weyl curvature[8] and hence demands of the universe that the initial singularity has vanishing, or possibly finite, Weyl curvature, as in F-L-R-W. models.In general such singularities are called isotropic and Tod and co-workers [22] [23] [24] have proven existence and uniqueness results for the associated Cauchy problem. However there is as yet no derivation of this condition from something deeper and it remains a purely classical viewpoint.

Hartle and Hawking's No Boundary Proposal achieves a similar purpose but at the expense of leaving the realm of Lorentzian metrics. In principle this is

[8] despite the example of de-Sitter spacetime

a complete quantum mechanical answer to the problem of singularities but so far has only been explored at the semi-classical level. Probably all the resources of String/M-theory will be required for a full treatment.

3.1.7 *Singularities at the end of Hawking Evaporation*

The ultimate fate of the singularities inside black holes is inextricably mixed up with the ultimate end of Hawking Evaporation. It may be shown (e.g. Kodama [1]) that the well known classical spacetime model incoporating back re-action must contain a (transient) naked singularity.

However it is by no means clear that the semi-classical approximation applies. One must surely have to take into account the **quantum interference of spacetimes**.

But how to do this?

3.1.8 *Maldacena's Conjecture*

One way in which this might be achieved is to consider Hawking Evaporation in AdS. String theory in the bulk is supposedly dual to conformal field theory on the conformal boundary. The latter is believed to be unitary and non-singular, hence so must the former.

Much work has been done relating black holes in AdS_5 and $\mathcal{N} = 4$ SU(N) SUSYM. A more tractable case is AdS_3. Using it, Maldacena has suggested [6] a deep connection between unitarity and the **ergodic properties of quantum fields** and this has recently been taken up by Barbon and Rabinovici [14] [15], [16] and by Hawking himself [7][8].

However, presumably the case of greatest physical interest AdS_4 which has received much less attention. Little is known about the CFT.

Klebanov and Polyakov [13] have proposed a correspondence valid at weak coupling but this invovles a bulk theory containing **infinitely many spins**. Apart from some work of Hartnoll and Kumar [11] and Warnick[12], little detailed work has been done on this case.

Hertog, Horowitz and Maeda have argued [39][38] that cosmic scensorship is easier to violate in AdS backgrounds, but the remain uncertainties about the details [35][37].

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3.2 Discussion

A. Linde Eternal inflation indeed requires initial conditions and is not future geodesically complete. However, one should recall the difference between being past eternal and past complete. For any point, any geodesic starting in it has finite distance, however, for any geodesic, there is a longer one. So for a particular observer, the universe begins and ends, but not the universe as a whole. This rectifies a common misconception.

G. Gibbons I agree, there is no big bang in that model.

A. Strominger Cosmic censorship proposed a solution to the singularity problems, namely that they always lie behind horizons. However, Hawking radiation shows that black holes become smaller over time, and that presumably Planckian effects take over the dynamics at some point in time. So, cosmic censorship would not anymore seem to make general relativity complete in any way. Why is it then so close to being true?

G. Gibbons Recall that the cosmic censorship idea came from a practical question in X-ray cosmology. Did general relativity need to be changed at the scale these experimenters were interested in? The conclusion they came to was that there was no obvious reason why general relativity should break down at macroscopic scales. Still, cosmic censorship is an important property for classical general relativity, and it is still important to know whether it is satisfied.

A. Strominger If it was not, we could see naked singularities in the sky.

G. Gibbons Which was always Penrose's viewpoint. The counter was put forward by more conservative opponents.

S. Weinberg If only on aesthetic grounds, it seems obvious that at short distance scales additional terms in the gravitational action will become important. General relativity is an effective theory as is the theory of soft pions. Obviously, it is not the whole answer. Original singularity theorems are important in showing that higher order terms will become relevant at some point.

G. Gibbons Yes, however, we distinguish two possibilities: they can become important already classically, or at the quantum level. The latter is more plausible.

S. Weinberg There is no problem with ghosts and the higher derivative terms. They arise only when misusing perturbation theory.

D. Gross Indeed, there is no problem. But only if you have a sensible theory to do perturbation theory in, which is not a priori given.

3.3 Prepared Comments

3.3.1 *Gary Horowitz: Singularities in String Theory*

I would like to give a brief overview of singularities in string theory. Many different types of singularities have been shown to be harmless in this theory. They are resolved by a variety of different mechanisms using different aspects of the theory. Some rely on the existence of other extended objects called branes, while others are resolved just in perturbative string theory. However it is known that not all singularities are resolved, and as we will review below, this necessary for the theory to have a stable ground state. There are very few general results in this subject. In particular, there is nothing like the singularity theorems of general relativity which give general conditions under which singularities form. So far, singularities have been studied on a case by case basis.

The starting point for our discussion is the fact that strings sense spacetime differently than point particles. Perturbative strings feel the metric through a two dimensional field theory called a sigma model. This means that two spacetimes which give rise to equivalent sigma models are indistinguishable in string theory. Apparently trivial changes to the sigma model can result in dramatically different spacetimes. Let me give two examples:

1) T-duality: If the spacetime metric is independent of a periodic coordinate x, then a change of variables in the sigma model describes strings on a new spacetime with $g_{xx} \to 1/g_{xx}$ [1, 2].

2) Mirror symmetry: In string theory, we often consider spacetimes of the form $M_4 \times K$ where M_4 is four dimensional Minkowski spacetime and K is a Calabi-Yau space [3], i.e. a compact six dimensional Ricci flat space. One can show that changing a sign in the (supersymmetric) sigma model changes the spacetime from $M_4 \times K$ to $M_4 \times K'$ where K' is topologically different Calabi-Yau space [4].

Using these facts it is easy to show that spacetimes which are singular in general relativity can be nonsingular in string theory. A simple example is the quotient of Euclidean space by a discrete subgroup of the rotation group. The resulting space, called an orbifold, has a conical singularity at the origin. Even though this leads to geodesic incompleteness in general relativity, it is completely harmless in string theory [5]. This is essentially because strings are extended objects.

The orbifold has a very mild singularity, but even curvature singularities can be harmless in string theory. A simple example follows from applying T-duality to rotations in the plane. This results in the metric $ds^2 = dr^2 + (1/r^2)d\phi^2$ which has a curvature singularity at the origin. However strings on this space are completely equivalent to strings in flat space.

As mentioned above, string theory has exact solutions which are the product of four dimensional Minkowski space and a compact Calabi-Yau space. A given Calabi-Yau manifold usually admits a whole family of Ricci flat metrics. So one can construct a solution in which the four large dimensions stay approximately flat and the geometry of the Calabi-Yau manifold changes slowly from one Ricci flat metric to another. In this process the Calabi-Yau space can develop a curvature singularity. In many cases, this is the result of a topologically nontrivial sphere S^2 or S^3 being shrunk down to zero area. It has been shown that when this happens, string theory remains completely well defined. The evolution continues through the geometrical singularity to a nonsingular Calabi-Yau space on the other side.

There are two qualitatively different ways in which this can happen. In one case, an S^2 collapses to zero size and then re-expands as a topologically different S^2. This is known as a flop transition. It was shown in [6] that the mirror description of this is completely nonsingular. Under mirror symmetry, this transition corresponds to evolution through nonsingular metrics. In the second case, an S^3 collapses down to zero size and re-expands as an S^2. This is called a *conifold singularity*. This transition is nonsingular if you include branes wrapped around the S^3 [7]. As long as the area of the surface is nonzero, these degrees of freedom are massive, and it is consistent to ignore them. However when the surface shrinks to zero volume these degrees of freedom become massless, and one must include them in the analysis. When this is done, the theory is nonsingular. These examples show that topology can change in a nonsingular way in string theory.

I will divide the remaining examples of singularity resolution into three classes depending on whether the singularities are timelike, null, or spacelike. Some spacetimes with timelike singularities can be replaced by entirely smooth solutions. In some cases this involves replacing the singularity with a source consisting of a smooth distribution of branes as in the "enhancon" [8]. Other cases can be done purely geometrically and do not need a source [9]. In this case, the smooth solution has less symmetry than the singular one. Although there is no argument here that strings in the singular space are equivalent to strings in the nonsingular space, there are arguments that the nonsingular description is the correct description of the physical situation.

Branes carry charges which source higher rank generalizations of a Maxwell field called RR fields. The gravitational field produced by a collection of branes wrapped around cycles often contain null singularities. In some cases, one can find nonsingular geometries with the same charge but no brane sources. This is possible since they contain nontrivial topology which supports nonzero RR flux. Many examples of this have been found for solutions involving two charges [10]. This phenomena of branes being replaced by fluxes is generally called geometric transitions.

Under certain conditions, string theory has tachyons, i.e. states with $m^2 < 0$. In the past, these tachyons were mysterious, but recently they have been understood as just indicating an instability of the space. In fact, tachyons can be very useful in avoiding black hole and cosmological singularities. There are situations in which a tachyon arises in the evolution toward a spacelike singularity. The evolution past this point is then governed by the dynamics of the tachyon and no longer agrees with general relativity [11].

Despite all these examples, it is simply not true that all singularities are removed in string theory. Nonlinear gravitational plane waves are not only vacuum solutions to general relativity, but also exact solutions to string theory [12]. These solutions contain arbitrary functions describing the amplitude of each polarization of the wave. If one of the amplitudes diverge at a finite point, then the plane wave is singular. One can study string propagation in this background and show that in some cases, the string does not have well behaved propagation through this curvature singularity [13]. The divergent tidal forces cause the string to become infinitely excited.

It is a good thing that string theory does not resolve all singularities. Consider the Schwarzschild solution with $M < 0$. This describes a negative mass solution with a naked singularity at the origin. If this singularity was resolved, there would be states with arbitrarily negative energy. String theory would not have a ground state. This argument, of course, is not restricted to string theory but applies to any candidate quantum theory of gravity.

One of the main goals of quantum gravity is to provide a better understanding of the big bang or big crunch singularities of cosmology. Perhaps the most fundamental question is whether they provide a true beginning or end of time, or whether there is a bounce. Hertog and I have recently studied this question using the AdS/CFT correspondence [14], which states that string theory in spacetimes which are asymptotically anti de Sitter (AdS) is equivalent to a conformal field theory (CFT). We found supergravity solutions in which asymptotically AdS initial data evolve to big crunch singularities [15]. The dual description involves a CFT with a potential unbounded from below. In the large N limit, the expectation value of some CFT operators diverge in finite time. A minisuperspace approximation leads to a bounce, but there are arguments that this is not possible in the full CFT. Although more work is still needed to completely understand the dual description, this suggests that a big crunch is not a big bounce [15].

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3.4 Discussion

Prepared comment by S. Shenker.

D. Gross In the context of resolving the information paradox, it has been suggested by J. Maldacena and S. Hawking that puzzles disappear when we take into account different semi-classical space-times. When resolving singularities, might it not be the case that taking seriously superpositions of geometries would similarly resolve puzzles?

S. Shenker We have defined in pedestrian fashion an observable with a pole (on the second sheet) and we can ask what happens to that pole. What happens to the observable might be related to many geometries contributing . A first signal of that would be large $g_s \propto 1/N$ corrections of the sort $g_s^2(\frac{1}{t-t_c})^x$ for some power x. The closer the geodesic gets to the singularity at $t = t_c$ the larger the quantum effects are. That could be interpretable as many geometries becoming important. Or as branes becoming light. Imagine further that we have a power series $g_s^n(\frac{1}{t-t_c})^{xn}$ that we resum, and then taking a double scaling limit. That might lead to an understanding of non-perturbative physics at the singularity.

A. Ashtekar In how far do your conclusions depend on analyticity?

S. Shenker They heavily do. Analyticity is a crucial property of quantum field theory and it would be bad to lose it.

G. Gibbons You have these two disjoint components to the boundary and a field theory on each. How, within field theory, do they talk to one another?

S. Shenker As J. Maldacena observed, the state in which you evaluate the correlation functions is a correlated state, the Hartle-Hawking state.

G. Gibbons So, you are not thinking of the quantum field theories as disjoint?

S. Shenker Well, you cannot build these states from non-singular data (if that is your question).

G. Gibbons I am more concerned with the quantum mechanics on the boundary. I thought the idea was to reduce the problem to standard quantum mechanics?

S. Shenker There is a corollary to that which is that if you have two Hilbert spaces in this case, one for each boundary.

G. Gibbons Is this a true generalization of quantum mechanics?

S. Shenker I am not going to agree with that. Perhaps somebody else would like to.

3.5 Prepared Comments

3.5.1 *Eva Silverstein: Singularities: Closed String Tachyons and Singularities*

3.5.1.1 *Singularities and Winding Modes*

A basic problem in gravitational physics is the resolution of spacetime singularities where general relativity breaks down. The simplest such singularities are conical singularities arising from orbifold identifications of flat space, and the most challenging are spacelike singularities inside black holes (and in cosmology). Topology changing processes also require evolution through classically singular spacetimes. In this contribution I will briefly review how a phase of closed string tachyon condensate replaces, and helps to resolve, basic singularities of each of these types. Finally I will discuss some interesting features of singularities arising in the small volume limit of compact negatively curved spaces.

In the framework of string theory, several types of general relativistic singularities are replaced by a phase of closed string tachyon condensate. The simplest class of examples involves spacetimes containing 1-cycles with antiperiodic Fermion boundary conditions. This class includes spacetimes which are globally stable, such as backgrounds with late-time long-distance supersymmetry and/or AdS boundary conditions.

In the presence of such a circle, the spectrum of strings includes winding modes around the circle. The Casimir energy on the worldsheet of the string contributes a negative contribution to the mass squared, which is of the form

$$M^2 = -\frac{1}{l_s}^2 + \frac{L^2}{l_s^4} \tag{1}$$

where L is the circle radius and $1/l_s^2$ is the string tension scale. For small L, the winding state develops a negative mass squared and condenses, deforming the system away from the $L < l_s$ extrapolation of general relativity. This statement is under control as long as L is static or shrinking very slowly as it crosses the string scale.

Examples include the following. Generic orbifold singularities have twisted sector tachyons, *i.e.* tachyons from strings wound around the angular direction of the cone. The result of their condensation is that the cone smooths out [1], as seen in calculations of D-brane probes, worldsheet RG, and time dependent GR in their regimes of applicability (see [2] for reviews). Topology changing transitions in which a Riemann surface target space loses a handle or factorizes into separate surfaces are also mediated by winding tachyon condensation [3]. Tachyon condensation replaces certain spacelike singularities of a cosmological type in which some number of circles shrinks homogeneously in the far past (or future) [4].

Finally, tachyons condensing *quasilocally* over a spacelike surface appear in black hole problems and in a new set of examples sharing some of their features [5][6].

One interesting new example is an AdS/CFT dual pair in which an infrared mass gap (confinement) arises at late times in a system which starts out in an unconfined phase out on its approximate Coulomb branch. As an example, consider the $\mathcal{N} = 4$ SYM theory on a (time dependent) Scherk-Schwarz circle, with scalar VEVs turned on putting it out on its Coulomb branch. As the circle shrinks to a finite size and the scalars roll back toward the origin, the infrared physics of the gauge theory becomes dominated by a three dimensional confining theory. The gravity-side description of this is via a shell of D3-branes which enclose a finite region with a shrinking Scherk-Schwarz cylinder. When the cylinder's radius shrinks below the string scale, a winding tachyon turns on. At the level of bulk spacetime gravity, a candidate dual for the confining theory exists [7]; it is a type of "bubble of nothing" in which the geometry smoothly caps off in the region corresponding to the infrared limit of the gauge theory. This arises in the time dependent problem via the tachyon condensate phase replacing the region of the geometry corresponding to the deep IR limit of the field theory.

For all these reasons, it is important to understand the physics of the tachyon condensate phase. The tachyon condensation process renders the background time-dependent; the linearized solution to the tachyon equation of motion yields an exponentially growing solution $T \propto \mu e^{\kappa X^0}$. As such there is no a priori preferred vacuum state. The simplest state to control is a state $|out>$ obtained by a Euclidean continuation in the target space, and describes a state in which nothing is excited in the far future when the tachyon dominates. This is a perturbative analogue of the Hartle-Hawking choice of state. At the worldsheet level (whose self-consistency we must check in each background to which we apply it), the tachyon condensation shifts the semiclassical action appearing in the path integrand. String amplitudes are given by

$$< \prod \int V > \sim \int DX^0 D\vec{X} e^{-S_E} \prod \int V \tag{2}$$

where I work in conformal gauge and suppress the fermions and ghosts. Here X^μ are the embedding coordinates of the string in the target space and $\int V$ are the integrated vertex operators corresponding to the bulk asymptotic string states appearing in the amplitude. The semiclassical action in the Euclidean theory is

$$S_E = S_0 + \int d^2\sigma \mu^2 e^{2\kappa X^0} \hat{T}(\vec{X}) \tag{3}$$

with S_0 the action without tachyon condensation and $\hat{T}(\vec{X})$ a winding (sine-Gordon) operator on the worldsheet. These amplitudes compute the components of the state $|out>$ in a basis of multiple free string states arising in the far past bulk spacetime when the tachyon is absent. The tachyon term behaves like a worldsheet potential energy term, suppressing contributions from otherwise singular regions of the path integration.

Before moving to summarize the full calculation of basic amplitudes, let me note two heuristic indications that the tachyon condensation effectively masses up degrees of freedom of the system. First, the tachyon term in (3) behaves like a spacetime dependent mass squared term in the analogue of this action arising in the case of a first quantized worldline action for a relativistic particle [8]. Second, the dependence of the tachyon term on the spatial variables $\vec{X}$ is via a relevant operator, dressed by worldsheet gravity (which in conformal gauge is encapsulated in the fluctuations of the timelike embedding coordinate X^0). The worldsheet renormalization group evolution with scale is different from the time dependent evolution, since fluctuations of X^0 contribute. However in some cases, such as localized tachyon condensates and highly supercritical systems, the two processes yield similar endpoints. In any case, as a heuristic indicator of the effect of tachyon condensation, the worldsheet RG suggests a massing up of degrees of freedom at the level of the worldsheet theory as time evolves forward.

Fortunately we do not need to rely too heavily on these heuristics, as the methods of Liouville field theory enable us to calculate basic physical quantities in the problem. In the Euclidean state defined by the above path integral, regulating the bulk contribution by cutting off X^0 in the far past at $ln\mu_*$, one finds a partition function Z with real part

$$Re(Z) = -\frac{ln(\mu/\mu_*)}{\kappa}\hat{Z}_{free} \tag{4}$$

This is to be compared with the result from non-tachyonic flat space $Z_0 = \delta(0)\hat{Z}_{free}$ [4], where $\delta(0)$ is the infinite volume of time, and $\hat{Z}_{free}$ is the rest of the partition function. In the tachyonic background (4), the first factor is replaced by a truncated temporal volume which ends when the tachyon turns on. A similar calculation of the two point function yields the Bogoliubov coefficients corresponding to a pure state in the bulk with thermal occupation numbers of particles, with temperature proportional to κ. This technique was first suggested in [8], where it was applied to bulk tachyons for which $\kappa \sim 1/l_s$ and the resulting total energy density blows up. In the examples of interest for singularities, the tachyon arises from a winding mode for which $\kappa \ll 1/l_s$, and the method [8] yields a self-consistently small energy density [4]. In the case of an initial singularity, this gives a perturbative string mechanism for the Hartle/Hawking idea of starting time from nothing. This timelike Liouville theory provides a perturbative example of "emergent time", in the same sense that spatial Liouville theory provides a worldsheet notion of "emergent space".[9]

So far this analysis applied to a particular vacuum. It is important to understand the status of other states of the system. In particular, the worldsheet path integral has a saddle point describing a single free string sitting in the tachyon phase. Do putative states such as this with extra excitations above the tachyon condensate

[9]This was also noted by M. Douglas in the discussion period in the session on emergent spacetime, in which G. Horowitz also noted existing examples. As explained by the speakers in that session, no complete *non-perturbative* formulation involving emergent time exists, in contrast to the situation with spatial dimensions where matrix models and AdS/CFT provide examples (but see [13] for an interesting example of a null singularity with a proposed non-perturbative description in terms of matrix theory).

constitute consistent states? This question is important for the problem of unitarity in black hole physics and in more general backgrounds where a tachyon condenses quasilocally, excising regions of ordinary spacetime. If nontrivial states persist in the tachyon phase in such systems, this would be tantamount to the existence of hidden remnants destroying bulk spacetime unitarity.

In fact, we find significant indications that the state where a string sits in the tachyon phase does not survive as a consistent state in the interacting theory [9][6]. The saddle point solution has the property that the embedding coordinate X^0 goes to infinity in finite worldsheet time τ. This corresponds to a hole in the worldsheet, which is generically not BRST invariant by itself. If mapped unitarily to another hole in the worldsheet obtained from a correlated negative frequency particle impinging on the singularity, worldsheet unitarity may be restored. This prescription is a version of the Horowitz/Maldacena proposal of a black hole final state [10]; the tachyon condensate seems to provide a microphysical basis for this suggestion.

A more dynamical effect which evacuates the tachyon region also arises in this system. A particle in danger of getting stuck in the tachyon phase drags fields (for example the dilaton and graviton) along with it. The heuristic model of the tachyon condensate as an effective mass for these modes [8] suggests that the fields themselves are getting heavy. The resulting total energy of the configuration, computed in [6] for a particle of initial mass m_0 coupled with strength λ to a field whose mass also grows at late times like $M(x^0)$, is

$$E = m_0^2 \lambda^2 M(x^0) cos^2 \left(\int^{x^0} M(t')dt' \right) F(R) \tag{5}$$

This is proportional to a function $F(R)$ which increases with greater penetration distance R of the particle into the tachyon phase. Hence we expect a force on any configuration left in the tachyon phase which sources fields (including higher components of the string field). This does not mean every particle classically gets forced out of the tachyonic sector: for example in black hole physics, the partners of Hawking particles which fall inside the black hole provide negative frequency modes that correlate with the matter forming the black hole.

The analysis of this dynamical effect in generic states relies on the field-theoretic (worldsheet minisuperspace) model for tachyon dynamics. It is of interest to develop complete worldsheet techniques to analyze other putative vacua beyond the Euclidean vacuum. In the case of the Euclidean vacuum, the worldsheet analyses [8][4] reproduce the behavior expected from the heuristic model, so we have tentatively taken it as a reasonable guide to the physics in more general states as well.

The string-theoretic tachyon mode which drives the system away from the GR singularities is clearly accessible perturbatively. But it is important to understand whether the whole background has a self-consistent perturbative string description. In the Euclidean vacuum, this seems to be the case: the worldsheet amplitudes are shut off in the tachyon phase in a way similar to that obtained in spatial Liouville theory. In other states, it is not a priori clear how far the perturbative treatment extends. One indication for continued perturbativity is that according to the simple field theory model, every state gets heavy in the tachyon phase, including fluctuations of the dilaton, which may therefore be stuck at its bulk weak coupling value. It could be useful to employ AdS/CFT methods [11] to help decide this point.

3.5.1.2 *Discussion and Zoology*

Many timelike singularities are resolved in a way that involves new *light* degrees of freedom appearing at the singularity. In the examples reviewed in section 1, ordinary spacetime ends where the tachyon background becomes important. The tachyon at first constitutes a new light mode in the system, but its condensation replaces the would-be short-distance singularity with a phase where degrees of freedom ultimately become *heavy*. However, there are strong indications that there is a whole zoo of possible behaviors at cosmological spacelike singularities, including examples in which the GR singularity is replaced by a phase with *more* light degrees of freedom [12] (see [13] for an interesting null singularity where a similar behavior obtains).

Consider a spacetime with compact negative curvature spatial slices, for example a Riemann surface. The corresponding nonlinear sigma model is strongly coupled in the UV, and requires a completion containing more degrees of freedom. In supercritical string theory, the dilaton beta function has a term proportional to $D - D_{crit}$. The corresponding contribution in a Riemann surface compactification is $(2h-2)/V \sim 1/R^2$ where V is the volume of the surface in string units, h the genus and R the curvature radius in string units. This suggests that there are effectively $(2h-2)/V$ extra (supercritical) degrees of freedom in the Riemann surface case. Interestingly, this count of extra degrees of freedom arises from the states supported by the fundamental group of the Riemann surface.[10] For simplicity one can work at constant curvature and obtain the Riemann surface as an orbifold of Euclidean AdS_2, and apply the Selberg trace formula to obtain the asymptotic number density of periodic geodesics (as reviewed for example in [14]). This yields a density of states from a sum over the ground states in the winding sectors proportional to $e^{ml_s\sqrt{2h/V}}$ where m is proportional to the mass of the string state. Another check arises by modular invariance which relates the high energy behavior of the partition function to the lowest lying state: the system contains a light normalizable volume mode whose mass scales the right way to account for the modular transform of this density of states.

At large radius, the system is clearly two dimensional to a good approximation, and the 2d oscillator modes are entropically favored at high energy. It is interesting to contemplate possibility of cases where the winding states persist to the limit $V \to 1$, in which case the density of states from this sector becomes that of a $2h$

[10] I thank A. Maloney, J. McGreevy, and others for discussions on these points.

dimensional theory and the system crosses over to a very supercritical theory in which these states become part of the oscillator spectrum. In particular, states formed from the string wrapping generating cycles of the fundamental group in arbitrary orders (up to a small number of relations) constitute a $2h$-dimensional lattice random walk. At large volume, the lattice spacing is much greater than the string scale and the system is far from its continuum limit, so these states are a small effect. But at small Riemann surface volume it is an interesting possibility that these states cross over to the high energy spectrum of oscillator modes in $2h$ dimensions [12].

Of course as emphasized in [12], there are many possible behaviors at early times, including ones where the above states do not persist to small radius [3] and ones where they do persist but are part of a still larger system. One simple way to complete this sigma model is to extend it to a linear sigma model (containing more degrees of freedom) which flows to the Riemann surface model in the IR. Coupling this system to worldsheet gravity yields in general a complicated time dependent evolution, whose late time behavior is well described by the nonlinear sigma model on an expanding Riemann surface. If one couples this system to a large supercritical spectator sector, the time dependent evolution approaches the RG flow of the linear sigma model, which yields a controlled regime in which it is clear that at earlier times the system had more degrees of freedom.

Clearly a priori this can happen in many ways. In addition to the landscape of metastable vacua of string theory I believe the conservative expectation is that there will be a zoo of possible cosmological histories with similar late time behavior; indeed inflationary cosmology already has this feature. While it may be tempting to reject this possibility out of hand in hopes of a unique prediction for cosmological singularities, this seems to me much more speculative. However there are various indications that gravity may simplify in large dimensions (see e.g. [15]) and it would be interesting to try to obtain from this an organizing principle or measure applying to the plethora of cosmological singularities of this type.[11]

In any case, the singularities discussed in section 1, which are replaced by a phase of tachyon condensate, are simpler, appear more constrained [10][6], and apply more directly to black hole physics. It would be interesting to understand if there is any relation between black hole singularities and cosmological singularities.

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[11] as mentioned for example in J. Polchinski's talk in the cosmology session.

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3.6 Discussion

T. Banks Generically, in your models you seem to have few initial states and many final states. What does unitarity mean in a context like that?

E. Silverstein Which model are you referring to specifically?

T. Banks The model where you use the Hartle-Hawking initial state.

E. Silverstein In that model, we merely determine what that state looks like in the bulk.

T. Banks I was under the impression that you were endorsing the fact that a Hartle-Hawking prescription picks out one initial state.

E. Silverstein I am not claiming that consistency picks out this one state, merely that perturbative time-dependent string theory with this initial state is well-defined.

T. Banks So there might be an infinite Hilbert space of consistent initial states?

E. Silverstein That is what I discussed in another part of my talk. There might only be a subset allowed by consistency.

3.7 Prepared Comments

3.7.1 *Thibault Damour: Cosmological Singularities and* E_{10}

Near spacelike singularity limit and a $SUGRA/[E_{10}/K(E_{10})]$ correspondence

The consideration of the *near horizon limit* (Maldacena) of certain black D-branes has greatly enriched our comprehension of string theory. In this limit, there emerges a *correspondence* between two seemingly different theories: 10-dimensional string theory in ADS spacetime on one side, and a lower-dimensional CFT on the other side. It is believed that this correspondence maps two different descriptions of the (continuation of the) same physics.

In recent years, the consideration of the *near spacelike singularity limit* of generic (classical) solutions of 10-dimensional string theories, or 11-dimensional supergravity, has suggested the existence of a *correspondence* between (say) 11-dimensional supergravity and a *one-dimensional* $E_{10}/K(E_{10})$ nonlinear σ model [1]. If this correspondence were confirmed, it might provide both the basis of a new definition of M-theory, and a description of the 'de-emergence' of space near a cosmological singularity (where the 10-dimensional spatial extension would be replaced by the infinite number of coordinates of the $E_{10}/K(E_{10})$ coset space).

Cosmological billiards

The first hint of a correspondence $SUGRA/[E_{10}/K(E_{10})]$ (or, for short, $SUGRA/E_{10}$) emerged through the study, à la Belinskii-Khalatnikov-Lifshitz (BKL), of the structure of cosmological singularities in string theory and $SUGRA_{11}$ [2]. Belinskii, Khalatnikov and Lifshitz [3] introduced an approximate way of discussing the structure of generic cosmological singularities. The basic idea is that, near a spacelike singularity, the time derivatives are expected to dominate over spatial derivatives. More precisely, BKL found that spatial derivatives introduce terms in the equations of motion for the metric which are similar to the "walls" of a billiard table [3]. In an Hamiltonian formulation [4], [5] where one takes as basic gravitational variables the *logarithms* of the diagonal components of the metric, say β^a, these walls are Toda-like potential walls, i.e. exponentials of linear combinations of the β's, say $\exp -2w(\beta)$, where $w(\beta) = \sum_a w_a \beta^a$. To each wall is therefore associated a certain *linear form* in β space, $w(\beta) = \sum_a w_a \beta^a$, and also a corresponding hyperplane $\sum_a w_a \beta^a = 0$. Ref. [2] found that the set of leading walls $w_i(\beta)$ entering the cosmological dynamics of $SUGRA_{11}$ or type-II string theories could be identified with the *Weyl chamber* of the hyperbolic Kac-Moody algebra E_{10} [6], i.e. the set of hyperplanes defined by the *simple roots* $\alpha_i(h)$ of E_{10}. Here h parametrizes a generic element of a Cartan subalgebra (CSA) of E_{10}, and the index i labels both the leading walls and the simple roots. [i takes r values, where r denotes the rank of the considered Lie algebra. For E_{10}, $r = 10$.] Let us also note that, for Heterotic and type-I string theories, the cosmological billiard is the Weyl chamber of another rank-10 hyperbolic Kac-Moody algebra, namely BE_{10}.

The appearance of E_{10} in the BKL behaviour of $SUGRA_{11}$ revived an old suggestion of B. Julia [7] about the possible role of E_{10} in a *one-dimensional reduction* of $SUGRA_{11}$. A posteriori, one can see the BKL behaviour as a kind of spontaneous reduction to one dimension (time) of a multidimensional theory. Note, however, that it is essential to consider a generic *inhomogeneous* solution (instead of a naively one-dimensionally reduced one) because the wall structure comes from the sub-leading ($\partial_x \ll \partial_t$) spatial derivatives.

Gradient expansion versus height expansion of the $E_{10}/K(E_{10})$ coset model

Refs. [1, 8] went beyond the leading BKL analysis of Ref. [2] by including the first three "layers" of spatial gradients modifying the zeroth-order *free billiard* dynamics defined by keeping only the time derivatives of the (diagonal) metric. This *gradient expansion* [5] can be graded by counting how many leading wall forms $w_i(\beta)$ are contained in the exponents of the sub-leading potential walls associated to these higher-order spatial gradients. As further discussed below, it was then found that this counting could be related (up to height 29 included) to the grading defined by the *height* of the roots entering the Toda-like Hamiltonian walls of the dynamics defined by the motion of a massless particle on the coset space $E_{10}/K(E_{10})$, with action

$$S_{E_{10}/K(E_{10})} = \int \frac{dt}{n(t)} (v^{\rm sym} | v^{\rm sym}). \tag{1}$$

Here, $v^{\rm sym} \equiv \frac{1}{2}(v + v^T)(\equiv P$ in [6]) is the symmetric part of the "velocity" $v \equiv (dg/dt)g^{-1}$ of a group element $g(t)$ running over E_{10}. The transpose operation

T is the negative of the Chevalley involution ω, so that the elements of the Lie algebra of $K(E_{10})$ are "T-antisymmetric", $k^T = -k$ (which is equivalent to them being fixed under ω: $\omega(k) = +k$).

Current tests of the $SUGRA/E_{10}$ correspondence

An E_{10} group element $g(t)$ is parametrized by an infinite number of coordinates. When decomposing (the Lie algebra of) E_{10} with respect to (the Lie algebra of) the $GL(10)$ subgroup defined by the horizontal line in the Dynkin diagram of E_{10}, the various components of $g(t)$ can be graded by their $GL(10)$ level ℓ. At the $\ell = 0$ level $g(t)$ is parametrized by a $GL(10)$ matrix k^i_j, to which is associated (in the coset space $GL(10)/SO(10)$) a symmetric matrix $g^{ij} = (e^k)^i_s (e^k)^j_s$. [The indices $i, j, \cdots$ take ten values.] At the level $\ell = 1$, one finds a 3-form A_{ijk}. At the level $\ell = 2$, a 6-form A_{ijklmn}, and at the level $\ell = 3$ a 9-index object A_9 with Young-tableau symmetry $\{8, 1\}$.

The coset action (1) then defines a coupled set of equations of motion for $g_{ij}(t), A_{ijk}(t), A_{i_1 i_2 \cdots i_6}(t), A_{i_1 i_2 \cdots i_9}(t), \cdots$. By explicit calculations, it was shown that these coupled equations of motion could be identified, modulo terms which correspond to potential walls of height at least 30, to the $SUGRA_{11}$ equations of motion. This identification between the coset dynamics and the $SUGRA_{11}$ one is obtained by means of a *dictionary* which maps: (1) $g_{ij}(t)$ to the spatial components of the 11-dimensional metric $G_{ij}(t, \mathbf{x}_0)$ in a certain coframe (Ndt, θ^i), (2) $\dot{A}_{ijk}(t)$ to the mixed temporal-spatial ('electric') components of the 11-dimensional field strength $\mathcal{F} = d\mathcal{A}$ in the same coframe, (3) the conjugate momentum of $A_{i_1 i_2 \cdots i_6}(t)$ to the dual (using $\epsilon^{i_1 i_2 \cdots i_{10}}$) of the spatial ('magnetic') frame components of $\mathcal{F} = d\mathcal{A}$, and (4) the conjugate momentum of $A_{i_1 i_2 \cdots i_9}(t)$ to the ϵ^{10} dual (on jk) of the structure constants C^i_{jk} of the coframe θ, i.e. $d\theta^i = \frac{1}{2} C^i_{jk} \theta^j \wedge \theta^k$. Here all the SUGRA field variables are considered at some fixed (but arbitrary) spatial point $\mathbf{x}_0$.

The fact that at levels $\ell = 2$ (A_6), and $\ell = 3$ (A_9) the dictionary between SUGRA and coset variables is such that the first spatial gradients of the SUGRA variables $G, \mathcal{A}$ are mapped onto (time derivatives of) coset variables suggested the conjecture that the infinite tower of coset variables could fully encode all the spatial derivatives of the SUGRA variables, thereby explaining how a one-dimensional coset dynamics could correspond to an 11-dimensional one. Some evidence for this conjecture comes from the fact that among the infinite number of generators of E_{10} there do exist towers of generators that have the appropriate $GL(10)$ index structure for representing the infinite sequence of spatial gradients of the various SUGRA variables.

It is not known how to extend this dictionary beyond the level $\ell = 3$ (corresponding to A_9). The difficulty in extending the dictionary might be due (similarly to what happens in the ADS/CFT case) to the non-existence of a common domain of validity for the two descriptions.

However, Ref. [9] found evidence for a nice compatibility between some high-level contributions in the coset action, corresponding to *imaginary* roots ($(\alpha, \alpha) < 0$, by contrast to the roots that entered the checks [1, 8] which were all "real" with $(\alpha, \alpha) = +2$), and M-theory one-loop corrections to $SUGRA_{11}$, notably the terms quartic in the curvature tensor. This finding suggests new ways of testing the conjecture by looking at the structure of higher loop terms. [See also [10] for a different approach to the possible role of the imaginary roots of E_{10}.]

Two recent studies of the fermionic sector of $SUGRA_{11}$ have also found a nice compatibility between $SUGRA_{11}$ and the extension of the (bosonic) massless particle action (1) to an action describing the (supersymmetric) dynamics of a *massless spinning particle* on $E_{10}/K(E_{10})$ [11, 12]. In this extension $K(E_{10})$ plays the role of a generalized 'R symmetry'.

Conclusion

Much work, and probably new tools, are needed to establish the conjectured correspondence between $SUGRA_{11}$, or hopefully M-theory, and the dynamics of a (quantum) massless spinning particle on the coset space $E_{10}/K(E_{10})$. It is, however, interesting to speculate that, as one approaches a cosmological singularity, space 'de-emerges' in the sense that the 11-dimensional description of $SUGRA_{11}/M$-theory gets replaced (roughly when the curvature exceeds the – 11-dimensional– Planck scale) by a 1-dimensional $E_{10}/K(E_{10})$ coset model (where the only remaining dimension is timelike).

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3.8 Discussion

J. Harvey There is a five-dimensional supergravity formally similar to eleven-dimensional supergravity, with a one-form potential with Chern-Simons term. Do you know whether there is a similar algebraic structure associated to that supergravity in this particular limit?

T. Damour Yes, Kac-Moody algebras can generically be associated to supergravities, and even to ordinary gravitational theories in various dimensions.

A. Ashtekar If I consider the coset space dynamics as being fundamental, where is $\hbar$?

T. Damour It is encoded in the eleven-dimensional Planck length.

A. Ashtekar It appears in the metric?

T. Damour: Yes, for instance R^4 terms in the action are made dimensionless using the Planck length.

F. Englert I am wondering whether your theory is not simply valid near a space-like singularity for different reasons, namely that near a space-like singularity you essentially only have the time-direction. After all, if you dimensionally reduce gravity to one dimension you expect E_{10} as a symmetry. Near a singularity this is verified because the problem becomes essentially one-dimensional.

T. Damour The singularity is used here as a tool to reveal a symmetry structure which exists independently. Recall that here indeed one find E_{10} starting from any dimension. One never sees E_7, or other duality groups appear.

Prepared comment by N. Turok.

E. Silverstein Wound strings might not get blue-shifted at the singularity, but other modes will?

N. Turok Other modes will be non-perturbative states like black holes, etc. If perturbative gravity goes through the singularity, I would already be delighted. Let us deal with the non-perturbative states later. If you work out are going to be created at the crunch, the creation rate is proportional to θ_0 which can be taken small.

E. Silverstein They will not be created, but what if you send one in?

N. Turok If you send a black hole in ...

E. Silverstein Just send a perturbative mode in.

Answer by N. Turok An initial particle? A pragmatic answer is that the density of particles going in will be negligeable. Still one should worry.

A. Polyakov There is a comment I wanted to make, concerning singularities. When we try to treat the Big Bang, Big Crunch singularities in string theory, you write down a sigma-model and then try a one-loop approximation and you find the Einstein equation with Friedman type solution. From the point of view of the sigma-model, the singularity in such a solution is nothing but the Landau-pole, which follows from the renormalization group flow at one loop. In many cases, the singularity can be resolved as e.g. in the $O(3)$ sigma-model (which becomes massive). The Big Bang would then not be so big after all.

3.9 Prepared Comments

3.9.1 *Abhay Ashtekar: Singularities: quantum nature of the big bang in loop quantum gravity*

3.9.1.1 *Introduction*

A central feature of general relativity is that gravity is encoded in the very geometry of space-time. Loop quantum gravity is a non-perturbative approach to unifying general relativity with quantum physics which retains this interplay between geometry and gravity [1, 2]. There is no background space-time; matter as well as geometry are 'quantum from birth'. Effects of quantum geometry are negligible under ordinary circumstances but they dominate near singularities. There, quantum space-time is dramatically different from the smooth continuum of general relativity. In particular, quantum geometry effects have led to a natural resolution of space-like singularities in a number of mini and midi-superspaces. These encompass both black hole and cosmological contexts.

In the cosmological setting, there are several long-standing questions that have been relegated to quantum gravity. Examples are:

• How close to the big-bang does a smooth space-time of general relativity make sense? In particular, can one show from first principles that this approximation is valid at the onset of inflation?
• Is the Big-Bang singularity naturally resolved by quantum gravity? Or, is some external input such as a new principle or a boundary condition at the Big Bang essential?
• Is the quantum evolution across the 'singularity' deterministic? One needs a fully non-perturbative framework to answer this question in the affirmative. (In the Ekpyrotic and Pre-Big-Bang scenarios, for example, the answer is in the negative.)
• If the singularity is resolved, what is on the 'other side'? Is there just a *quantum foam* far removed from any classical space-time, or, is there another large, classical universe?

Using loop quantum gravity, these and related questions have been answered in detail in several models by combining analytic and numerical methods.

3.9.1.2 *Novel features of loop quantum cosmology*

Quantum cosmology is an old subject. It was studied extensively in the framework of geometrodynamics where quantum states are taken to be functions of 3-geometries and matter fields. In the cosmological context, the wave functions $\Psi(a, \phi)$ depend on the scale factor a and the matter field ϕ. They are subject to a quantum constraint, called the Wheeler-DeWitt equation. Initially, it was hoped that the quantum evolution dictated by this equation would resolve classical singularities. Unfortunately, this hope was not realized. For example in the simplest of homogeneous isotropic models, if one begins with a semi-classical state at late times and evolves it back via Wheeler DeWitt equation, one finds that it just follows the classical trajectory into the big bang singularity.

Loop quantum gravity is based on spin-connections rather than metrics and is thus closer in spirit to gauge theories. The basic dynamical variables are holonomies h of a gravitational spin-connection A and electric fields E canonically conjugate to these connections. However, the E's now have a dual, geometrical interpretation: they represent orthonormal triads which determine the Riemannian geometry. Thanks to the contributions from 2 dozen or so groups since the mid-nineties, the subject has reached a high degree of mathematical precision [1]. In particular, it has been shown that the fundamental quantum algebra based on h's and E's admits a *unique* diffeomorphism covariant representation [3]. From the perspective of Minkowskian field theories, this result is surprising and brings out the powerful role of the requirement of diffeomorphism covariance (i.e., background independence). In this representation, there are well-defined holonomy operators $\hat{h}$ but *there is no operator $\hat{A}$ corresponding to the connection itself.* The second key feature is that Riemannian geometry is now quantized: there are well-defined operators corresponding to, say, lengths, areas and volumes, and all their eigenvalues are discrete.

In quantum cosmology, one deals with symmetry reduced models. However, in loop quantum cosmology, quantization is carried out by closely mimicking the procedure used in the full theory, and the resulting theory turns out to be qualitatively different from the Wheeler DeWitt theory. Specifically, because only the holonomy operators are well-defined and there is no operator corresponding to the connection itself, the von-Neumann uniqueness theorem is by-passed. A new representation of the algebra generated by holonomies and triads becomes available. *We have new quantum mechanics.* In the resulting theory, the Wheeler-DeWitt differential equation is replaced by a *difference* equation (Eq (1) below), the size of the step being dictated by the first non-zero area eigenvalue —i.e., the 'area gap'— in quantum geometry. Qualitative differences from the Wheeler-DeWitt theory emerge precisely near the big-bang singularity. Specifically, the evolution does not follow the classical trajectory. Because of quantum geometry effects, gravity becomes *repulsive* near the singularity and there is a quantum bounce.

3.9.1.3 *A Simple model*

I will now illustrate these general features through a simple model: Homogeneous, isotropic $k = 0$ cosmologies with a zero rest mass scalar field. Since there is no potential in this model, the big-bang singularity is inevitable in the classical theory. The momentum p_ϕ of the scalar field is a constant of motion and for each value of

p_ϕ, there are two trajectories: one starting out at the Big Bang and expanding and the other contracting in to a Big Crunch, each with a singularity.

Classical dynamics suggests that here, as well as in the closed models, one can take the scalar field as an *internal clock* defined by the system itself —unrelated to any choice of coordinates or a background space-time. This idea can be successfully transported to quantum theory because the Hamiltonian constraint equation

$$\frac{\partial^2\Psi}{\partial\phi^2} = C^+(v)\Psi(v+4,\phi) + C^o(v)\Psi(v,\phi) + C^-(v)\Psi(v-4,\phi) \qquad (1)$$

'evolves' the wave functions $\Psi(v,\phi)$ with respect to the internal time ϕ. (Here v is the oriented volume of a fixed fiducial cell in Planck units, so $v \sim \pm(\text{scale factor})^3$, and $C^\pm, C^o$ are simple algebraic functions on v.) The detailed theory is fully compatible with this interpretation. Thus, this simple model provides a concrete realization of the *emergent time* scenario, discussed in another session of this conference.

A standard ('group averaging') procedure enables one to introduce a natural Hilbert space structure on the space of solutions to the Hamiltonian constraint. There are complete sets of Dirac observables using which one can rigorously construct semi-classical states and follow their evolution. Since we do not want to prejudice the issue by stating at the outset what the wave function should do at the singularity, let us specify the wave function at late time —say *now*— and take it to be sharply peaked at a point on the expanding branch. Let us use the Hamiltonian constraint to evolve the state backwards towards the classical singularity. Computer simulations show that the state remains sharply peaked on the classical trajectory till very early times, when the density becomes comparable to the Planck density. The fluctuations are all under control and we can say that the *the continuum space-time of general relativity is an excellent approximation till* this very early epoch. In particular, space-time can be taken to be classical at the onset of standard inflation. But in the Planck regime the fluctuations are significant and there is no unambiguous classical trajectory. This is to be expected. But then something unexpected happens. The state re-emerges on the other side again as a semi-classical state, now peaked on a contracting branch. Thus, in the Planck regime, although there are significant quantum fluctuations, we do not have a quantum foam on the other side. Rather, there is a quantum bounce. *Quantum geometry in the Planck regime serves as a bridge between two large classical universes.* The fact that the state is again semi-classical in the past was unforeseen and emerged from detailed numerical simulations [4]. However, knowing that this occurs, one can derive an effective modification of the Friedmann equation: $(\dot{a}/a)^2 = (8\pi G/3)\,\rho\,[1-\rho/\rho_\star] +$ higher order terms, where ρ is the matter density and $\rho_\star$, the critical density, is given by $\rho_\star = \text{const}\,(1/8\pi G\Delta)$, Δ being the smallest non-zero eigenvalue of the area operator. The key feature is that, without any extra input, the quantum geometry correction naturally comes with a *negative* sign making gravity repulsive in the Planck regime, giving rise to the bounce. The correction is completely negligible when the matter density is very small compared to the Planck density, i.e., when the universe is large. Finally, a key consequence of (1) is that the quantum evolution is deterministic across the 'quantum bridge'; no new input was required to 'join' the two branches. This is because, thanks to quantum geometry, one can treat the Planck regime fully non-perturbatively, without any need of a classical background geometry.

The singularity resolution feature is robust for the mini and midi-superspace models we have studied so far *provided* we use background independent description and quantum geometry. For example, in the anisotropic case, the evolution is again non-singular if we treat the full model non-perturbatively, using quantum geometry. But if one treats anisotropies as perturbations using the standard, Wheeler-DeWitt type Hilbert spaces, the perturbations blow up and the singularity is not resolved. Finally, the Schwarzschild singularity has also been resolved. This resolution suggests a paradigm for the black hole evaporation process which can explain why there is no information loss in the setting of the physical, Lorentzian space-times [5].

To summarize, quantum geometry effects have led to a resolution of a number of space-like singularities showing that quantum space-times can be significantly larger than their classical counterparts. These results have direct physical and conceptual ramifications. I should emphasize however that so far the work has been restricted to mini and midi superspaces and a systematic analysis of generic singularities of the full theory is still to be undertaken.

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3.10 Discussion

S. Kachru Through much of your discussion, you worked with a truncated Hilbert space. What are your reasons for believing that approximation in the region where the curvature is large?

A. Ashtekar In some models that we have completely worked through, one can indeed see that one can go through the singularity, but, yes, in general your question is very relevant.

T. Banks The loop quantum gravity equation shares with the Wheeler-De Witt equation its hyperbolic nature. The set of solutions then does not allow for a positive definite metric and yet you talk of Hilbert spaces. Could you comment?

A. Ashtekar: In this simple model, there is a Hilbert space and the analogue of the Wheeler-De Witt equation looks like a Klein-Gordon equation. One treats this theory then similarly to the Klein-Gordon equation, and one can define as in that case, a positive definite inner product.

T. Banks Is the evolution in your model unitary?

A. Ashtekar In the initial region ϕ can be chosen as time and the evolution is unitary in time. In the intermediate (crunch) region time is not well-defined, but one can still choose the value of ϕ as denoting time. If one does this, the evolution can still be considered unitary.

General Discussion on Singularities starts.

M. Douglas Two questions I would like to pose are: 1. For infinite time scenarios, why does entropy not increase eternally? and 2. Does the bounce not give less predictivity than an initial state?

G. Veneziano On question 1: the entropy after the Big Bang satisfies the holographic upper bound. A pre-Big Bang seems to be what is necessary to satisfy the bound, and otherwise, this amount of entropy would be difficult to understand. Initial entropy would not seem to be a problem for bouncing cosmologies.

A. Strominger A closely related issue is degrees of freedom. We would like to think that there is one degree of freedom per Planck volume and not an infinite number as in field theory. A Big Bang could be thought of as a point before which there are no degrees of freedom. A Big Bounce seems to say that there are only few degrees of freedom finally. Having a universe of microscopic size would not change that conclusion much. So what do people advancing a Big Bounce have to say about having a large number of degrees of freedom today?

G. Horowitz You are assuming a closed universe when referring to it as being small?

A. Strominger Yes, I guess one could allow for an infinite volume bounce.

T. Banks I do not understand the issue of having a small number of degrees of freedom at the Big Bang. One can have a unitary evolution that stops at a given time if one has a time-dependent Hamiltonian. The issue with the Big Bang is that there is a particle horizon and that only a small number of degrees of freedom are correlated and interact with each other, but one can allow for many more, non-interacting degrees of freedom. I do believe there is an issue (raised by R. Penrose) with why the initial state had little entropy and why the thermodynamical and cosmological arrow of time coincide.

Question by S. Weinberg (asking for a prediction of the ekpyrotic universe)

N. Turok A prediction of our [ekpyrotic] model (compared to inflation) is that non-gaussianities are very highly suppressed.

S. Weinberg So, non-gaussian perturbations would rule out the model?

N. Turok Yes, as would tensor perturbations.

P. Steinhardt The tensor perturbations would not be precisely zero, just exponentially suppressed. The reason is that the Hubble constant at the time the perturbations are generated is exponentially small compared to today. This is possible because the fluctuations are not caused by rapid expansion or contraction, as they are for inflation, but, instead, by a different instability that occurs in a modulus field as it rolls down its exponentially steep potential.

S. Weinberg That confuses me. In the usual inflationary scenario the Hubble constant sets the scale of the perturbations we see today and we believe the primordial non-gaussianity is small because we see small perturbations now and therefore the Hubble constant in Planck units at the time of horizon exit in the usual picture is about 10^{-5}. So, in your scenario, are not the perturbations themselves very small?

P. Steinhardt No, because they are not produced by gravity itself, but by the potential term in the model.

S. Weinberg That is clear.

A. Ashtekar Is it possible to have a heuristic picture of which singularities are resolved in string theory?

G. Horowitz There are time-like, space-like and null singularities which have been resolved in string theory, with different mechanisms. I do not have a criterium to say which singularities will be resolved and which not.

A. Ashtekar Is there a heuristic, or an intuition? For example, is it the case that in the resolved cases the total energy was always positive?

G. Horowitz Yes, in the usual asymptotically flat context that is so.

A. Linde From my perspective, the theory of a pre-Big Bang is a kind of inflation, but not sufficiently good to solve flatness and horizon problems. Perhaps it works, but not by itself, but perhaps by adding on usual inflation afterwards. But then, who cares about the bounce? On the ekpyrotic scenario, I would like to say that we studied it. It may be possible to have a tachyonic instability before the singularity to produce fluctuations, but producing fluctuations is not the main difficulty. The main problem is how to make the universe isotropic, homogeneous, flat, ... In order to achieve this, the ekpyrotic scenario also uses a long stage of exponential expansion, as in inflation. What is different in inflation is that inflationary theory protects one from having to think about the singularity, whether or not there was a bounce.

P. Steinhardt Your description of the situation is a misconception. The accelerated expansion that occurs in the cyclic model is not necessary to make the universe smooth and flat. In fact, it may be that there are only a few e-folds of dark energy domination in our future before the contraction begins. The key in our scenario is the slowly contracting phase with very high pressure, which, it turns, smoothes and flattens the universe as well. So, we have learned from these studies that there are two ways of obtaining a flat, homogeneous and isotropic universe: a rapidly expanding phase with w near -1, conventional inflation, or a slowly contracting phase with $w > 1$.

W. Fischler It is always said that inflation solves the homogeneity and flatness problems, however, that is put in by hand. At the moment where inflation starts, homogeneity of the inflaton at scales larger than the causal region at that point in time, is fed into the model.

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