CHAPTER 10

The Geometry of Leverage Space Portfolios

Just as everyone is at a value for *f* whether they acknowledge it or not, so too therefore is everyone in leverage space, at some point on the terrain therein, whether they acknowledge it or not. The consequences they must pay for this are not exorcised by their ignorance to this.

DILUTION

If we are trading a portfolio at the full optimal allocations, we can expect tremendous drawdowns on the entire portfolio in terms of equity retracement.

Even a portfolio of blue chip stocks, if traded at their geometric optimal portfolio levels, will show tremendous drawdowns. Yet, these blue chip stocks must be traded at these levels, as these levels maximize potential geometric gain relative to dispersion (risk), and also provide for attaining a goal in the least possible time. When viewed from such a perspective, trading blue chip stocks is no more risky than trading pork bellies, and pork bellies are no less conservative than blue chip stocks. The same can be said of a portfolio of commodity trading systems and a portfolio of bonds.

Typically, investors practice dilution, whether inadvertent or not. That is, if, optimally, one should trade a certain component in a portfolio at the *f*\$ level of, say, \$2,500, they may be trading it consciously at an *f*\$ level of, say, \$5,000, in a conscious effort to smooth out the equity curve and buffer drawdowns, or, unconsciously, at such a half-optimal *f* level, since all positions can be assigned an f value as detailed in earlier chapters. Often, people practice asset allocation is by splitting their equity into two subaccounts, an active subaccount and an inactive subaccount. These are not two separate accounts; rather, in theory, they are a way of splitting a single account.

The technique works as follows. First, you must decide upon an initial fractional level. Let's suppose that, initially, you want to emulate an account at the half f level. Therefore, your initial fractional level is .5 (the initial fractional level must be greater than 0 and less than 1). This means you will split your account, with .5 of the equity in your account going into the inactive subaccount and .5 going into the active subaccount. Let's assume we are starting out with a \$100,000 account. Therefore, \$50,000 is initially in the inactive subaccount and \$50,000 is in the active subaccount. It is the equity in the active subaccount that you use to determine how many units to trade. These subaccounts are not real; they are a hypothetical construct you are creating in order to manage your money more effectively. You always use the full optimal fs with this technique. Any equity changes are reflected in the active portion of the account. Therefore, each day, you must look at the account's total equity (closed equity plus open equity, marking open positions to the market) and subtract the inactive amount (which will remain constant from day to day). The difference is your active equity, and it is on this difference that you will calculate how many units to trade at the full f levels.

Let's suppose that the optimal f for market system A is to trade one contract for every \$2,500 in account equity. You come into the first day with \$50,000 in active equity and, therefore, you will look to trade 20 units. If you were using the straight half f strategy, you would end up with the same number of units on day one. At half f, you would trade one contract for every \$5,000 in account equity (\$2,500/.5) and you would use the full \$100,000 account equity to figure how many units to trade. Therefore, under the half f strategy, you would trade 20 units on this day as well.

However, as soon as the equity in the account changes, the number of units you will trade changes as well. Let's assume that you make \$5,000 this next day, thus pushing the total equity in the account up to \$105,000. Under the half f strategy, you will now be trading 21 units. However, under the split equity technique, you must subtract the now-constant inactive amount of \$50,000 from your total equity of \$105,000. This leaves an active equity portion of \$55,000, from which you will figure your contract size at the optimal f level of one contract for every \$2,500 in equity. Therefore, under the split equity technique, you will now look to trade 22 units.

The procedure works the same on the downside of the equity curve as well, with the split equity technique peeling off units at a faster rate than the fractional f strategy. Suppose we lost \$5,000 on the first day of trading, putting the total account equity at \$95,000. Under the fractional f strategy, you would now look to trade 19 units ($95,000/$5,000). However, under the split equity technique you are now left with $45,000 of active equity and, thus, you will look to trade 18 units ($45,000/$2,500).

Notice that with the split equity technique, the exact fraction of optimal *f* that we are using changes with the equity changes. We specify the fraction we want to start with. In our example, we used an initial fraction of .5. When the equity increases, this fraction of the optimal *f* increases, too, approaching 1 as a limit as the account equity approaches infinity. On the downside, this fraction approaches 0 as a limit at the level where the total equity in the account equals the inactive portion. This fact, that there is built-in portfolio insurance with the split equity technique, is a tremendous benefit and will be discussed at length later in this chapter.

Because the split equity technique has a fraction for *f* that moves, we will refer to it as a dynamic fractional *f* strategy, as opposed to the straight fractional *f* (which we will call a *static* fractional *f*) strategy.

Using the dynamic fractional *f* technique is analogous to trading an account full out at the optimal *f* levels, where the initial size of the account is the active equity portion.

So, we see that there are two ways to dilute an account down from the full geometric optimal portfolio. We can trade a static fractional or a dynamic fractional *f*. Although the two techniques are related, they also differ. Which is best?

To begin with, we need to be able to determine the arithmetic average HPR for trading n given scenario spectrums simultaneously, as well as the variance in those HPRs for those n simultaneously traded scenario spectrums, for given *f* values ($f_1 \ldots f_n$) operating on those scenario spectrums. These are given now as:

$$\text{AHPR}\,(f_1 \ldots f_n) = \frac{\sum_{k=1}^{m}\left[\left(1+\sum_{i=1}^{n}\left(fi^{*}\left(\frac{-PL_{k,i}}{BL_i}\right)\right)\right) * \text{Prob}_k\right]}{\sum_{k=1}^{m}\text{Prob}_k} \tag{10.01}$$

where: n = The number of scenario spectrums (market systems or portfolio components).

m = The possible number of combinations of outcomes between the various scenario spectrums (market systems) based on how many scenarios are in each set. m = The number of scenarios in the first spectrum * the number of scenarios in the second spectrum $*\ldots*$ the number of scenarios in the nth spectrum.

Prob = The sum of probabilities of all m of the HPRs for a given set of f values. Prob_k is the sum of the values in brackets {} in the numerator, for all m values of a given set of f values.

f_i = The f value being used for component i. f_i must be greater than 0, and can be infinitely high (i.e., it can be greater than 1.0).

$\text{PL}_{k,j}$ = The outcome profit or loss for the ith component (i.e., scenario spectrum or market system) associated with the kth combination of scenarios.

BL_i = The worst outcome of scenario spectrum (market system) i.

Thus, Prob_k in the equation is equal to Equation (9.03)

Equation (10.01) simply takes the coefficient of each HPR *times* its probability and sums these. The resultant sum is then divided by the sum of the probabilities.

The variance in the HPRs for a given set of multiple simultaneous scenario spectrums being traded at given f values can be determined by first taking the *raw coefficient* of the HPRs, the rawcoef:

$$\text{rawcoef}_k = 1 + \sum_{i=1}^{n} \left(f_i * \left(\frac{-\text{PL}_{k,i}}{\text{BL}_i} \right) \right) \tag{10.02}$$

Then, these raw coefficients are averaged for all values of k between 1 and m, to obtain arimeanrawcoef:

$$\text{arimeanrawcoef} = \frac{\left(\sum_{k=1}^{m} \text{rawcoef}_k \right)}{m} \tag{10.03}$$

Now, the variance V can be determined as:

$$V = \frac{\sum_{k=1}^{m} (\text{rawcoef}_k - \text{arimeanrawcoef})^2 * \text{Prob}_k}{\sum_{k=1}^{m} \text{Prob}_k} \tag{10.04}$$

Where again, Prob_k is determined by Equation (9.03).

If we know what the AHPR is, and the variance at a given f level (say the optimal f level for argument's sake), we can convert these numbers into what they would be trading at a level of dilution we'll call FRAC. And, since we are able to figure out the two legs of the right triangle, we can also figure the estimated geometric mean HPR at the diluted level. The formulas are now given for the diluted AHPR, called FAHPR, the diluted standard deviation (which is simply the square root of variance), called FSD, and the diluted geometric mean HPR, called FGHPR here:

$$FAHPR = (AHPR - 1) * FRAC + 1$$
$$FSD = SD * FRAC$$
$$FGHPR = \sqrt{FAHPR^2 - FSD^2}$$

where: FRAC = The fraction of optimal *f* we are solving for.
AHPR = The arithmetic average HPR at the optimal *f*.
SD = The standard deviation in HPRs at the optimal *f*.
FAHPR = The arithmetic average HPR at the fractional *f*.
FSD = The standard deviation in HPRs at the fractional *f*.
FGHPR = The geometric average HPR at the fractional *f*.

Let's assume we have a system where the AHPR is 1.0265. The standard deviation in these HPRs is .1211 (i.e., this is the square root of the variance given by Equation (10.04)); therefore, the estimated geometric mean is 1.019. Now, we will look at the numbers for a .2 static fractional *f* and a .1 static fractional *f*. The results, then, are:

	Full *f*	.2 *f*	.1 *f*
AHPR	1.0265	1.0053	1.00265
SD	.1211	.02422	.01211
GHPR	1.01933	1.005	1.002577

Here is what will also prove to be a useful equation, the time expected to reach a specific goal:

$$T = \frac{\ln(\text{goal})}{\ln(\text{geometric mean})}$$

where: T = The expected number of holding periods to reach a specific goal.
goal = The goal in terms of a multiple on our starting stake, a TWR.
ln ( ) = The natural logarithm function.

Now, we will compare trading at the .2 static fractional *f* strategy, with a geometric mean of 1.005, to the .2 dynamic fractional *f* strategy (20% as initial active equity) with a daily geometric mean of 1.01933. The time (number of days, since the geometric means are daily) required to double the static fractional f is given by Equation (5.07) as:

$$\frac{\ln(2)}{\ln(1.005)} = 138.9751$$

To double the dynamic fractional f requires setting the goal to 6. This is because, if you initially have 20% of the equity at work, and you start out with a \$100,000 account, then you initially have \$20,000 at work. The goal is to make the active equity equal \$120,000. Since the inactive equity remains at \$80,000, you will have a total of \$200,000 on your account that started at \$100,000. Thus, to make a \$20,000 account grow to \$120,000 means you need to achieve a TWR of 6. Therefore, the goal is 6 in order to double a .2 dynamic fractional f:

$$\frac{\ln(6)}{\ln(1.01933)} = 93.58634$$

Notice how it took 93 days for the dynamic fractional f versus 138 days for the static fractional f.

Now let's look at the .1 fraction. The number of days expected in order for the static technique to double is expected as:

$$\frac{\ln(2)}{\ln(1.002577)} = 269.3404$$

If we compare this to doubling a dynamic fractional f that is initially set to .1 active, you need to achieve a TWR of 11. Hence, the number of days required for the comparative dynamic fractional f strategy is:

$$\frac{\ln(11)}{\ln(1.01933)} = 125.2458$$

To double the account equity, at the .1 level of fractional f is, therefore, 269 days for our static example, compared to 125 days for the dynamic. The lower the fraction for f, the faster the dynamic will outperform the static technique.

Let's take a look at tripling the .2 fractional f. The number of days expected by static technique to triple is:

$$\frac{\ln(3)}{\ln(1.005)} = 220.2704$$

This compares to its dynamic counterpart, which requires:

$$\frac{\ln(11)}{\ln(1.01933)} = 125.2458$$

To make 400% profit (i.e., a goal or TWR, of 5) requires of the .2 static technique:

$$\frac{\ln(5)}{\ln(1.005)} = 322.6902$$

Which compares to its dynamic counterpart:

$$\frac{\ln(21)}{\ln(1.01933)} = 1590201$$

It takes the dynamic almost half the time it takes the static to reach the goal of 400% in this example. However, if you look out in time 322.6902 days to where the static technique doubled, the dynamic technique would be at a TWR of:

$$\begin{aligned} &= .8 + 1.01933^{322.6902} * .2 \\ &= .8 + 482.0659576 * .2 \\ &= 97.21319 \end{aligned}$$

This represents making over 9,600% in the time it took the static to make 400%.

We can now amend Equation (5.07) to accommodate both the static and fractional dynamic f strategies to determine the expected length required to achieve a specific goal as a TWR. To begin with, for the static fractional f, we can create Equation (5.07b):

$$T = \frac{\ln(\text{goal})}{\ln(\text{FGHPR})}$$

where:

T = The expected number of holding periods to reach a specific goal.

goal = The goal in terms of a multiple on our starting stake, a TWR.

FGHPR = The adjusted geometric mean. This is the geometric mean, run through Equation (5.06) to determine the geometric mean for a given static fractional f.

ln( ) = The natural logarithm function.

For a dynamic fractional f, we have Equation (5.07c):

$$T = \frac{\ln\left(\left(\frac{(\text{goal}-1)}{\text{FRAC}}\right) + 1\right)}{\ln(\text{geometric mean})}$$

where: T = The expected number of holding periods to reach a specific goal.
goal = The goal in terms of a multiple on our starting stake, a TWR.
FRAC = The initial active equity percentage.
geometric mean = the raw geometric mean HPR at the optimal f; there is no adjustment performed on it as there is in Equation (5.07b)
ln( ) = The natural logarithm function.

Thus, to illustrate the use of Equation (5.07c), suppose we want to determine how long it will take an account to double (i.e., TWR = 2) at .1 active equity and a geometric mean of 1.01933:

$$\begin{aligned}
T &= \frac{\ln\left(\left(\frac{(\text{goal}-1)}{\text{FRAC}}\right)+1\right)}{\ln(\text{geometric mean})} \\
&= \frac{\ln\left(\left(\frac{(2-1)}{.1}\right)+1\right)}{\ln(1.01933)} \\
&= \frac{\ln\left(\frac{(1)}{.1}+1\right)}{\ln(1.01933)} \\
&= \frac{\ln(10+1)}{\ln(1.01933)} \\
&= \frac{\ln(11)}{\ln(1.01933)} \\
&= \frac{2.397895273}{.01914554872} \\
&= 125.2455758
\end{aligned}$$

Thus, if our geometric means are determined off scenarios which have a daily holding period basis, we can expect about 125 1/4 days to double. If our scenarios used months as holding period lengths, we would have to expect about 125 1/4 months to double.

As long as you are dealing with a T large enough that Equation (5.07c) is greater than Equation (5.07b), then you are benefiting from dynamic fractional f trading. This can, likewise, be expressed as Equation (10.05):

$$\text{FGHPR}^T <= \text{geometric mean}^T * \text{FRAC} + 1 - \text{FRAC} \tag{10.05}$$

Thus, you must iterate to that value of T where the right side of the equation exceeds the left side—that is, the value for T (the number of holding periods) at which you should wait before reallocating; otherwise, you are better off to trade the static fractional f counterpart.

Figure 10.1 illustrates this graphically. The arrow is that value for T at which the left-hand side of Equation (10.05) is equal to the right-hand side.

Thus, if we are using an active equity percentage of 20% (i.e., FRAC = .2), then FGHPR must be figured on the basis of a .2f. Thus, for the case where our geometric mean at full optimal f is 1.01933, and the .2 f (FGHPR) is 1.005, we want a value for T that satisfies the following:

$$1.005^T <= 1.01933^T \ * \ .2 + 1 - .2$$

We figured our geometric mean for optimal f and, therefore, our geometric mean for the fractional f (FGHPR) on a daily basis, and we want to see if one quarter is enough time. Since there are about 63 trading days per quarter, we want to see if a T of 63 is enough time to benefit by dynamic fractional f. Therefore, we check Equation (10.05) at a value of 63 for T:

$$\begin{aligned}
1.005^{63} &<= 1.01933^{63} \ * \ .2 + 1 - .2 \\
1.369184237 &<= 3.340663933 \ * \ .2 + 1 - .2 \\
1.369184237 &<= .6681327866 + 1 - .2 \\
1.369184237 &<= 1.6681327866 - .2 \\
1.369184237 &<= 1.4681327866
\end{aligned}$$

The equation is satisfied, since the left side is less than or equal to the right side of the equation. Thus, we can reallocate on a quarterly basis under the given values and benefit from using dynamic fractional f.

Figure 10.1 demonstrates the relationship between trading at a static versus a dynamic fractional f strategy over a period of time.

This chart shows a 20% initial active equity, traded on both a static and a dynamic basis. Since they both start out trading the same number of units, that very same number of units is shown being traded straight through as a constant contract. The geometric mean HPR, at full f used in this chart, was 1.01933; therefore, the geometric mean at the .2 static fractional f was 1.005, and the arithmetic average HPR at full f was 1.0265.

All of this leads to a couple of important points, that the *dynamic fractional* f *will outpace the static fractional f faster, the lower the fraction and the higher the geometric mean.* That is, using an initial active equity percentage of .1 (for both dynamic and static) means that the dynamic will overtake the static faster than if you used a .5 fraction for both. Thus, generally, the dynamic fractional f will overtake its static counterpart faster, the lower the portion of initial active equity. In other words, a portfolio with an initial active equity of .1 will overcome its static counterpart faster than a portfolio with an initial active equity allocation of .2 will overtake

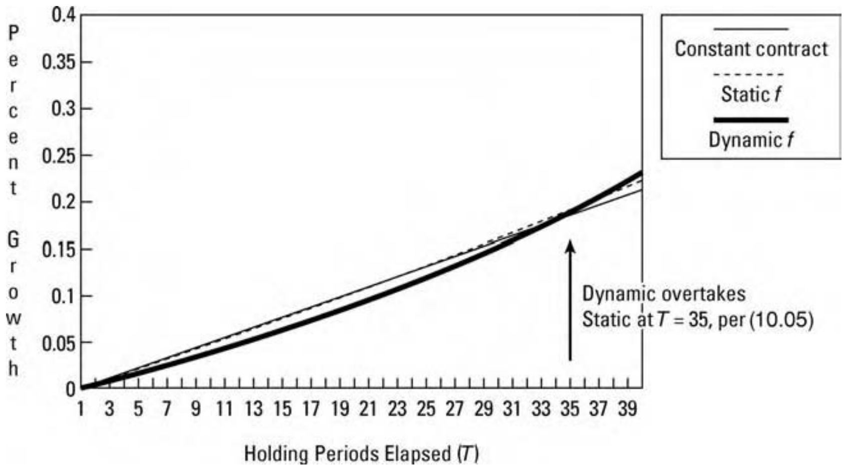

FIGURE 10.1 Percent growth per period for constant contract, static, and dynamic f

its static counterpart. At an initial active equity allocation of 100% (1.0), the dynamic never overtakes the static fractional f (rather, they grow at the same rate). Also affecting the rate at which the dynamic fractional f overtakes its static counterpart is the geometric mean of the portfolio itself. The higher the geometric mean, the sooner the dynamic will overtake its static counterpart. At a geometric mean of 1.0, the dynamic never overtakes its static counterpart.

The more time that elapses, the greater the difference between the static fractional f and the dynamic fractional f strategy. Asymptotically, the dynamic fractional f strategy has infinitely greater wealth than its static counterpart.

One last important point about Figure 10.1. The constant contract line crosses the other two lines before they cross over each other.

In the long run, you are better off to practice asset allocation with a dynamic fractional f technique. That is, you determine an initial level—a percentage—to allocate as active equity. The remainder is inactive equity. The day-to-day equity changes are reflected in the active portion only. The inactive dollar amount remains constant. Therefore, each day you subtract the constant inactive dollar amount from your total account equity. This difference is the active portion, and it is on this active portion that you will figure your quantities to trade in, based on the optimal f levels.

Now, when the margin requirement is calculated for the positions, it will not be exactly the same as your active equity. It can be more or less; it doesn't matter. Thus, unless your margin requirement is for 100% of the equity in the account, you will have some unused cash in the account on any given holding period. Thus, you are almost always inadvertently allocating something to cash (or cash equivalents). So you can see that there isn't any need for a scenario spectrum for cash or cash equivalents—they already get their proper allocation when you do the active and inactive equity split.

REALLOCATION

Notice in Figure 10.1 that, trading at a dynamic fractional f, eventually the active portion of your equity will dwarf the inactive portion, and you will be faced with a portfolio that is far too aggressive for your blood—the same situation you faced in the beginning when you looked at trading the portfolio at the full optimal f amount. Thus, at some point in time in the future, you will want to *reallocate* back to some level of initial active equity.

For instance, you start out at a 10% initial active equity on a \$100,000 account. You, therefore, have \$10,000 active equity—equity that you are trading full out at the optimal f level. Each day, you will subtract \$90,000 from the equity on the account. The difference is the active equity, and it is on the active equity that you trade at the full optimal f levels.

Now, assume that this account got up to \$1 million equity. Thus, subtracting the constant dollar inactive amount of \$90,000 leaves you at an active equity of \$910,000, which means you are now at 91% active equity. Thus, you face those gigantic drawdowns that you sought to avoid initially, when you diluted f and started trading at a 10% initial active equity.

Consider the case of reallocating after every trade or every day. Such is the case with static fractional f trading. Recall again Equation (10.08a), the time required to reach a specific goal.

Let's return to our system that we are trading with a .2 active portion and a geometric mean of 1.01933. We will compare this to trading at the static fractional .2 f, where the resultant geometric mean is 1.005. Now, if we are starting out with a \$100,000 account, and we want to reallocate at \$110,000 total equity, the number of days (since our geometric means here are on a per-day basis) required by the static fractional .2 f is:

$$\frac{\ln(1.1)}{\ln(1.005)} = 19.10956$$

This compares to using \$20,000 of the \$100,000 total equity at the full f amount, and trying to get the total account up to \$110,000. This would represent a goal of 1.5 times the \$20,000:

$$\frac{\ln(1.5)}{\ln(1.01933)} = 21.17807$$

At lower goals, the static fractional f strategy grows faster than its corresponding dynamic fractional f counterpart. As time elapses, the dynamic overtakes the static until, eventually, the dynamic is infinitely further ahead. Figure 10.1 graphically displays this relationship between the static and dynamic fractional f's.

If you reallocate too frequently, you are only shooting yourself in the foot, as the technique would be inferior to its static fractional f counterpart. Therefore, since you are better off, in the long run, to use the dynamic fractional f approach to asset allocation, you are also better off to reallocate funds between the active and inactive subaccounts as infrequently as possible. Ideally, you will only make this division between active and inactive equity once, at the outset of the program.

It is not beneficial to reallocate too frequently. Ideally, you will never reallocate. Ideally, you will let the fraction of optimal f you are using keep approaching 1 as your account equity grows. In reality, however, you most likely will reallocate at some point in time. Hopefully, you will not reallocate so frequently that it becomes a problem.

Reallocation seems to do just the opposite of what we want to do, in that reallocation trims back after a run up in equity, or adds more equity to the active portion after a period in which the equity has been run down.

Reallocation is a compromise. It is a compromise between the theoretical ideal and the real-life implementation. The techniques discussed allow us to make the most of this compromise. Ideally, you would never reallocate. Your humble little \$10,000 account, when it grew to \$10 million, would never go through reallocation. Ideally, you would sit through the drawdown which took your account down to \$50,000 from the \$10 million mark before it shot up to \$20 million. Ideally, if your active equity were depleted down to one dollar, you would still be able to trade a fractional contract (a *microcontract?*). In an ideal world, all of these things would be possible. In real life, you are going to reallocate at some point on the upside or the downside. Given that you are going to do this, you might as well do it in a systematic, beneficial way.

In reallocating—compromising—you *reset* things back to a state where you would be if you were starting the program all over again, only at a different equity level. Then, you let the outcome of the trading dictate where the fraction of f floats to by using a dynamic fractional f in between reallocations. Things can get levered up awfully fast, even when starting out with an active equity allocation of only 5%. Remember, you are using the full optimal f on this 5%, and if your program does modestly well, you'll be trading in substantial quantities relative to the total equity in the account in short order.

The first, and perhaps most important, thing to realize about reallocation, can be seen in Figure 10.1. Note the arrow in the figure, which is identified as that T where Equation (10.09) is equal. This amount of time, T, is critical. If you reallocate before T, you are doing yourself harm in trading the dynamic, rather than the static, fractional f.

The next critical thing to realize about reallocation is that you have some control over the maximum drawdown in terms of percentage equity retracements. Notice that you are trading the active portion of an account as though it were an account of exactly that size, full out at the optimal levels. Since you should expect to see nearly 100% equity retracements when trading at the full optimal f levels, you should expect to see 100% of the active equity portion wiped out at any one time.

Further, many traders who have been using the fractional dynamic f approach over the last couple of years relate what appears to be a very good rule of thumb: *Set your initial active equity at one half of the maximum drawdown you can tolerate.* Thus, if you can take up to a 20% drawdown, set your initial active equity at 10% (however, if the account is profitable and your active equity begins to exceed 20%, you are very susceptible to seeing drawdowns in excess of 20%).

There is a more accurate implementation of this very notion. Notice, that for portfolios, you must use the sum of all f in determining exposure. That is, you must sum the f values up across the components. This is important in that, suppose you have a portfolio of three components with f values determined, respectively, of .5, .7, and .69. The total of these is 1.89. That is the f you are working with in the portfolio, as a whole. Now, if each of these components saw the worst-case scenario manifest, the account would see a 189% drawdown on active equity! When working with portfolios, you should be very careful to be ever-vigilant for such an event, and to bear this in mind when determining initial active equity allocations.

The third important notion about reallocation pertains to the concept of portfolio insurance and its relationship to optimal f.

PORTFOLIO INSURANCE AND OPTIMAL *f*

Assume for a moment that you are managing a stock fund. Figure 10.2 depicts a typical portfolio insurance strategy, also known as dynamic hedging. The floor in this example is the current portfolio value of 100 (dollars per share). The typical portfolio will follow the equity market one for one. This is represented by the unbroken line. The insured portfolio is depicted by the dotted line. You will note that the dotted line is below the unbroken

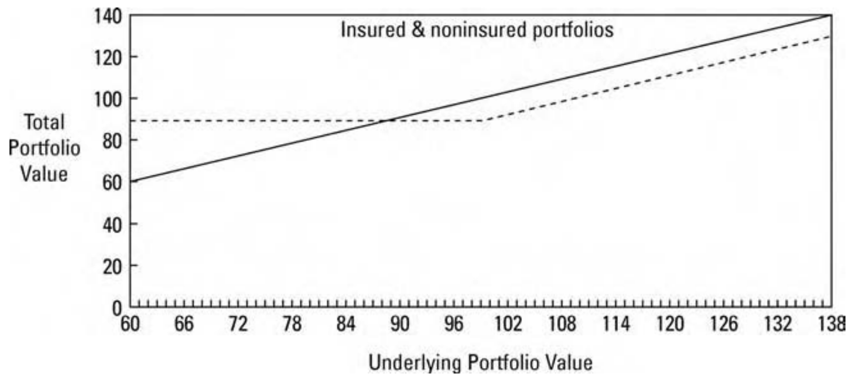

FIGURE 10.2 Portfolio insurance

line when the portfolio is at its initial value (100) or greater. This difference represents the cost of performing the portfolio insurance. Otherwise, as the portfolio falls in value, portfolio insurance provides a floor on the value of the portfolio at a desired level (in this case, the present value of 100) minus the cost of performing the strategy.

In a nutshell, portfolio insurance is akin to buying a put option on the portfolio. Let's suppose that the fund you are managing consists of only one stock, which is currently priced at \$100. Buying a put option on this stock, with a strike price of \$100, at a cost of \$10, would replicate the dotted line in Figure 10.2. The worst that could happen to your portfolio of one stock and a put option on it is that you could exercise the put, which sells your stock at \$100, and you lose the value of the put, \$10. Thus, the worst that this portfolio can be worth is \$90, regardless of how far down the underlying stock goes.

On the upside, your insured portfolio suffers somewhat, in that the value of the portfolio is always reduced by the cost of the put.

Now, consider that being long a call option will give you the same profile as being long the underlying and long a put option with the same strike price and expiration date as the call option. When we speak of the same profile, we mean an equivalent position in terms of the risk/reward characteristics at different values for the underlying. Thus, the dotted line in Figure 10.2 can also represent a portfolio composed of simply being long the \$100 call option at expiration.

Here is how *dynamic hedging* works to provide portfolio insurance. Suppose you buy 100 shares of this single stock for your fund, at a price of \$100 per share. Now, you will replicate the call option by using this underlying stock. The way you will do this is by determining an initial floor for the stock. The floor you choose is, say, 100. You also determine an expiration date for this hypothetical option which you are going to create. Let's say that the expiration date you choose is the date on which this quarter ends.

Now, you will figure the delta (the instantaneous rate of change in the price of a call option relative to the change in price of the underlying instrument) for this 100 call option with the chosen expiration date. Suppose the delta is .5. This means that you should be 50% invested in the given stock. Thus, you would have only 50 shares of stock rather than the 100 shares you would have if you were not practicing portfolio insurance. As the value of the stock increases, so, too, will the delta, and likewise the number of shares you hold. The upside limit is a delta at 1, where you would be 100% invested. In our example, at a delta of 1, you would have 100 shares.

As the stock decreases, so, too, does the delta, and likewise the size of the position in the stock decreases. The downside limit is at a delta of 0, where you wouldn't have any position in the stock.

Operationally, stock fund managers have used *noninvasive methods* of dynamic hedging. Such a technique involves not having to trade the cash portfolio. Rather, the portfolio as a whole is adjusted to what the current delta should be, as dictated by the model by using stock index futures, and, sometimes, put options. One benefit of a technique using futures is that futures have low transactions cost.

Selling short futures against the portfolio is equivalent to selling off part of the portfolio and putting it into cash. As the portfolio falls, more futures are sold, and as it rises, these short positions are covered. The loss to the portfolio, as it goes up and the short futures positions are covered, is what accounts for the portfolio insurance cost, the cost of the replicated put options. Dynamic hedging, though, has the benefit of allowing us to closely estimate this cost at the outset. To managers trying to implement such a strategy, it allows the portfolio to remain untouched, while the appropriate asset allocation shifts are performed through futures trades. This noninvasive technique of using futures permits the separation of asset allocation and active portfolio management.

To someone implementing portfolio insurance, you must continuously adjust the portfolio to the appropriate delta. This means that, say, each day, you must input into the option pricing model the current portfolio value, time until expiration, interest rate levels, and portfolio volatility, to determine the delta of the put option you are trying to replicate. Adding this delta (which is a number between 0 and -1) to 1 will give you the corresponding call's delta. This is the hedge ratio, the percentage that you should be investing in the fund.

Suppose your hedge ratio for the present moment is .46. Let's say that the size of the fund you are managing is the equivalent of 50 S&P futures units. Since you want to be only 46% invested, it means you want to be

54% disinvested. Fifty-four percent of 50 units is 27 units. Therefore, at the present price level of the fund at this point in time, for the given interest rate and volatility levels, the fund should be short 27 S&P units along with its long position in cash stocks.

Because the delta needs to be recomputed on an ongoing basis, and portfolio adjustments must be constantly monitored, the strategy is called a dynamic hedging strategy.

One problem with using futures in the strategy is that the futures market does not exactly track the cash market. Further, the portfolio you are selling futures against may not exactly follow the cash index upon which the futures market is traded. These tracking errors can add to the expense of a portfolio insurance program. Furthermore, when the option being replicated gets very near to expiration, and the portfolio value is near the strike price, the gamma of the replicated option goes up astronomically. Gamma is the instantaneous rate of change of the delta or hedge ratio. In other words, gamma is the delta of the delta. If the delta is changing very rapidly (i.e., if the replicated option has a high gamma), portfolio insurance becomes increasingly more cumbersome to perform. There are numerous ways to work around this problem, some of which are very sophisticated. One of the simplest involves the concept of a perpetual option. For instance, you can always assume that the option you are trying to replicate expires in, say, three months. Each day you will move the replicated option's expiration date ahead by a day. Again, this high gamma usually becomes a problem only when expiration draws near and the portfolio value and the replicated option's strike price are very close.

There is a very interesting relationship between optimal f and portfolio insurance. When you enter a position, you can state that f percent of your funds are invested. For example, consider a gambling game where your optimal f is .5, biggest loss -1, and bankroll is \$10,000. In such a case, you would bet one dollar for every two dollars in your stake since -1, the biggest loss, divided by $-.5$, the negative optimal f, is 2. Dividing \$10,000 by 2 yields \$5,000. You would, therefore, bet \$5,000 on the next bet, which is f percent (50%) of your bankroll. Had we multiplied our bankroll of \$10,000 by f (.5), we would have arrived at the same \$5,000 result. Hence, we have bet f percent of our bankroll.

Likewise, if our biggest loss were \$250 and everything else the same, we would be making one bet for every \$500 in our bankroll since $-\$250/-.5 = \$500$. Dividing \$10,000 by \$500 means that we would make twenty bets. Since the most we can lose on any one bet is \$250, we have thus risked f percent, 50% of our stake in risking \$5,000 ($\$250 * 20$).

Therefore, we can state that f equals the percentage of our funds at risk, or f equals the hedge ratio. Remember, when discussing portfolios, we are discussing the sum of the f values of the components. Since f is only applied on the active portion of our portfolio in a dynamic fractional f strategy, we can state that the hedge ratio of the portfolio, H, equals:

$$H = \left(\sum_{i=1}^{n} f_i\right) * \frac{\text{active\$}}{\text{total equity}} \tag{10.06a}$$

where: H = The hedge ratio of the portfolio.
f_i = The f value of the ith component in the portfolio.
active$ = The active portion of funds in an account.

Equation (10.06a) gives us the hedge ratio for a portfolio being traded on a dynamic fractional f strategy. Portfolio insurance is also at work in a static fractional f strategy, only the quotient active$/total equity equals 1, and the value for f (the optimal f) is multiplied by whatever value we are using for the fraction of f. Thus, in a static fractional f strategy, the hedge ratio is:

$$H = \left(\sum_{i=1}^{n} f_i\right) * \text{FRAC} \tag{10.06b}$$

We can state that in trading an account on a dynamic fractional f basis, we are performing portfolio insurance. Here, the floor is known in advance and is equal to the initial inactive equity plus the cost of performing the insurance. However, it is often simpler to refer to the floor of a dynamic fractional f strategy as the initial inactive equity of an account.

We can state that Equation (10.06a) or (10.06b) equals the delta of the call option of the terms used in portfolio insurance. Further, we find that this delta changes much the way a call option, which is deep out of the money and very far from expiration, changes. Thus, by using a constant inactive dollar amount, trading an account on a dynamic fractional f strategy is equivalent to owning a put option on the portfolio which is deep in the money and very far out in time. Equivalently, we can state that trading a dynamic fractional f strategy is the same as owning a call option on the portfolio which doesn't expire for a very long time and is very far out of the money.

However, it is also possible to use portfolio insurance as a reallocation technique to steer performance somewhat. This steering may be analogous to trying to steer a tanker with a rowboat oar, but this is a valid reallocation technique. The method initially involves setting parameters for the program. First, you must determine a floor value. Once chosen, you must decide upon an expiration date, volatility level, and other input parameters to the particular option model you intend to use. These inputs will give you the option's delta at any given point in time. Once the delta is known, you can determine what your active equity should be. Since the delta for the account, the variable H in Equation (10.06a), must equal the delta for the call option being replicated:

$$H = \left(\sum_{i=1}^{n} f_i\right) * \frac{\text{active\$}}{\text{total equity}}$$

Therefore:

$$\frac{H}{\sum_{i=1}^{n} f_i} = \frac{\text{active\$}}{\text{total equity}} \qquad \text{if } H < \sum_{i=1}^{n} f_i \tag{10.07}$$

Otherwise:

$$H = \frac{\text{active\$}}{\text{total equity}} = 1$$

Since active\$/total equity is equal to the percentage of active equity, we can state that the percentage of funds we should have in active equity, of the total account equity, is equal to the delta on the call option divided by the sum of the f values of the components. However, you will note that if H is greater than the sum of these f values, then it is suggesting that you allocate greater than 100% of an account's equity as active. Since this is not possible, there is an upper limit of 100% of the account's equity that can be used as active equity.

Portfolio insurance is great in theory, but poor in practice. As witnessed in the 1987 stock market crash, the problem with portfolio insurance is that, when prices plunge, there isn't any liquidity at any price. This does not concern us here, however, since we are looking at the relationship between active and inactive equity, and how this is mathematically similar to portfolio insurance.

The problem with implementing portfolio insurance as a reallocation technique, as detailed here, is that reallocation is taking place constantly. This detracts from the fact that a dynamic fractional f *strategy will asymptotically dominate a static fractional* f *strategy. As a result, trying to steer performance by way of portfolio insurance as a dynamic fractional f reallocation strategy probably isn't such a good idea. However, anytime you use fractional f, static or dynamic, you are employing a form of portfolio insurance.*

UPSIDE LIMIT ON ACTIVE EQUITY AND THE MARGIN CONSTRAINT

Even if you are trading only one market system, margin considerations can often be a problem. Consider that the optimal f in dollars is very often less than the initial margin requirement for a given market. Now, depending on what fraction of f you are using at the moment, whether you are using a static or dynamic fractional f strategy, you will encounter a margin call if the fraction is too high.

When you trade a portfolio of market systems, the problem of a margin call becomes even more likely.

What is needed is a way to reconcile how to create an optimal portfolio within the bounds of the margin requirements on the components in the portfolio. This can very easily be found. The way to accomplish this is to find what fraction of f you can use as an upper limit. This upper limit, L, is given by Equation (10.08):

$$L = \frac{\underset{i=1}{\overset{n}{MAX}}(f_i\$)}{\sum_{k=1}^{n}\left(\left(\underset{i=1}{\overset{n}{MAX}}(f_i\$)/f_k\$\right) * \text{ margin}_k\right)} \tag{10.08}$$

where: L = The upside fraction of f. At this particular fraction of f, you are trading the optimal portfolio as aggressively as possible without incurring an initial margin call.
$f_k\$$ = The optimal f in dollars for the kth market system.
$\text{margin}_k\$$ = The initial margin requirement of the kth market system.
n = The total number of market systems in the portfolio.

Equation (10.08) is really much simpler than it appears. For starters, in both the numerator and the denominator, we find the expression $\underset{i=1}{\overset{n}{MAX}}$, which simply means to take the greatest f \$ of all of the components in the portfolio.

Let's assume a two-component portfolio, which we'll call Spectrums A and B. We can arrange the necessary information for determining the upside limit on active equity in a table as follows:

Component	*f*\$	Margin	Greatest *f*\$/*f*\$
Spectrum A	\$2,500	\$11,000	2500/2500 = 1
Spectrum B	\$1,500	\$2,000	2500/1500 = 1.67

Now we can plug these values into Equation (10.08). Notice that $\overset{n}{\underset{i=1}{MAX}}$ is \$2,500, since the only other f\$ is \$1,500, which is less. Thus:

$$L = \frac{2500}{1*11000 + 1.67*2000} = \frac{2500}{11000+3340} = \frac{2500}{14{,}340} = 17.43375174\%$$

This tells us that 17.434% should be our maximum upside percentage.

Now, suppose we had a \$100,000 account. If we were at 17.434% active equity, we would have \$17,434 in active equity. Thus, assuming we can trade in fractional units for the moment, we would buy 6.9736 (17,434/2,500) of Spectrum A and 11.623 (17,434/1,500) of Spectrum B. The margin requirements on this would then be:

$$\begin{aligned} 6.9726 \ * \ 11{,}000 &= 76{,}698.60 \\ 11.623 \ * \ 2{,}000 &= \underline{23{,}245.33} \\ \text{Total Margin Requirement} &= \$99{,}943.93 \end{aligned}$$

If, however, we are still employing a static fractional f strategy (despite this author's protestations), then the highest you should set that fraction is 17.434%. This will result in the same margin call as above.

Notice that using Equation (10.08) yields the highest fraction for f without incurring an initial margin call that gives you the same ratios of the different market systems to one another.

Earlier in the text we saw that adding more and more market systems (scenario spectrums) results in higher and higher geometric means for the portfolio as a whole. However, there is a trade-off in that each market system adds marginally less benefit to the geometric mean, but marginally more detriment in the way of efficiency loss due to simultaneous rather than sequential outcomes. Therefore, we have seen that you do not want to trade an infinitely high number of scenario spectrums. What's more, theoretically optimal portfolios run into the real-life application problem of margin constraints. In other words, you are usually better off to trade three scenario spectrums at the full optimal f levels than to trade 10 at dramatically reduced levels as a result of Equation (10.08). Usually, you will find that the optimal number of scenario spectrums to trade in, particularly when you have many orders to place and the potential for mistakes, is but a handful.

f SHIFT AND CONSTRUCTING A ROBUST PORTFOLIO

There is a polymorphic nature to the n + 1 dimensional landscape; that is, the landscape is undulating—the peak in the landscape tends to move around as the markets and techniques we use to trade them change in character.

This f shift is doubtless a problem to all traders. Oftentimes, if the f shift is toward zero for many axes—that is, as the scenario spectrums weaken—it can cause what would otherwise be a winning method on a constant unit basis to be a losing program because the trader is beyond the peak of the f curve (to the right of the peak) to an extent that he is in a losing position.

f shift exists in all markets and approaches. It frequently occurs to the point at which many scenario spectrums get allocations in one period in an optimal portfolio construction, then no allocations in the period immediately following. This tells us that the performance, out of sample, tends to greatly diminish. The reverse is also true. Markets that appear as poor candidates in one period where an optimal portfolio is determined, then come on strong in the period immediately following, since the scenarios do not measure up.

When constructing scenarios and scenario sets, you should pay particular attention to this characteristic: Markets that have been performing well will tend to underperform in the next period and vice versa. Bearing this in mind when constructing your scenarios and scenario spectrums will help you to develop more robust portfolios, and help alleviate f shift.

TAILORING A TRADING PROGRAM THROUGH REALLOCATION

Often, money managers may opt for the dynamic f, as opposed to the static, even when the number of holding periods is less than that specified by Equation (10.05) simply because the dynamic provides a better implementation of portfolio insurance.

In such cases, it is important that the money manager not reallocate until Equation (10.05) is satisfied—that is, until enough holding periods elapse that the dynamic can begin to outperform the static counterpart.

A real key to tailoring trading programs to fit the money manager's goals in these instances is by reallocating on the upside. That is, at some upside point in active equity, you should reallocate to achieve a certain goal, yet that point is beyond some minimum point in time (i.e., number of elapsed holding periods).

Returning to Figure 10.1, Equation (10.05) gives us T, or where the crossing of the static f line by the dynamic f line occurs with respect to the horizontal coordinate. That is the point, in terms of number of elapsed holding periods, at which we obtain more benefit from trading the dynamic f rather than the static f. However, once we know T from Equation (10.05), we can figure the Y, or vertical, axis where the points cross as:

$$\text{Y} = \text{FRAC} * \text{Geometric Mean}^{T} - \text{FRAC} \qquad (10.09)$$

where: T = The variable T derived from Equation (10.05).
FRAC = The initial active portion of funds in an account.
Geometric Mean = The raw geometric mean HPR; there is no adjustment performed on it as there is in Equation (5.07b).

Example:

Initial Active Equity Percentage = 5% (i.e., .05)
Geomean HPR per period = 1.004171
$T = 316$

We know at 316 periods, on average, the dynamic will begin to outperform the corresponding static f for the same value of f, per Equation (10.05). This is the same as saying that, starting at an initial active equity of 5%, when the account is up by 13.63% ($.05 * 1.004171^{316} - .05$), the dynamic will begin to outperform the corresponding static f for the same value of f.

So, we can see that there is a minimum number of holding periods which must elapse in order for the dynamic fractional f to overtake its static counterpart (*prior to which, reallocation is harmful if implementing the dynamic fractional* f, *and, after which, it is harmful to trade the static fractional* f), which can also be converted from a horizontal point to a vertical one. That is, rather than a minimum number of holding periods, a minimum profit objective can be used.

Reallocating when the equity equals or exceeds this target of active equity will generally result in a much smoother equity curve than reallocating based on T, the horizontal axis. That is, most money managers will find it advantageous to reallocate based on upside progress rather than elapsed holding periods.

What is most interesting here is that for a given level of initial active equity, the upside target will always be the same, regardless of what values you are using for the geometric mean HPR or T *! Thus, a 5% initial active equity level will always see the dynamic overtake the static at a 13.63% profit on the account!*

Since we have an optimal upside target, we can state that there is, as well, an optimal delta on the portfolio on the upside. So, what is the formula for the optimal upside delta? This can be discerned by Equations (10.06a) and (10.06b), where FRAC equals that fraction of active equity which would be seen by satisfying Equation (10.09). This is given as:

$$\text{FRAC} = \frac{(\text{Initial Active Equity} + \text{Upside Target})}{1 + \text{Upside Target}} \tag{10.10}$$

Thus, if we start out with an initial active equity of 5%, then 13.63% is the upside point where the dynamic would be expected to overtake the static, and we would use the following for FRAC in Equations (10.10a) and (10.10b) in determining the hedge ratio at the upside point, Y, dictated by Equation (10.13):

$$\begin{aligned}
\text{FRAC} &= \frac{(0.5 + 1.363)}{(1 + .1363)} \\
&= \frac{.1863}{1.1363} \\
&= .1639531814
\end{aligned}$$

Thus, when we have an account which is up 13.63%, and we start with a 5% initial active equity, we know that the active equity is then 16.39531814%.

GRADIENT TRADING AND CONTINUOUS DOMINANCE

We have seen throughout this text, that trading at the optimal f for a given market system or scenario spectrum (or the set of optimal fs for multiple simultaneous scenario spectrums or market systems) will yield the greatest growth asymptotically, that is, in the long run, as the number of holding periods we trade for gets greater and greater. However, we have seen in Chapter 5, with "Threshold to Geometric," and in Chapter 6, that if we have a finite number of holding periods and we know how many holding periods we are going to trade for, what is truly optimal is somewhat more aggressive even than the optimal f values; that is, it is those values for f which maximize expected average compound growth (EACG).

Ultimately, each of us can only trade a finite number of holding periods—none of us will live forever. Yet, in all but the rarest cases, we do not know the exact length of that finite number of holding periods, so we use the asymptotic limit as the next best approximation.

Now you will see, however, a technique that can be used in this case of an unknown, but finite, number of holding periods over which you are going to trade at the asymptotic limit (i.e., the optimal f values), which, if you are trading any kind of a diluted f (static or dynamic), allows for dominance not only asymptotically, but for *any given holding period in the future.*

That is, we will now introduce a technique for a diluted f (which nearly all money managers must use in order to satisfy the real-world demands of clients pertaining to drawdowns) that not only ensures that an account will be at the highest equity in the very long

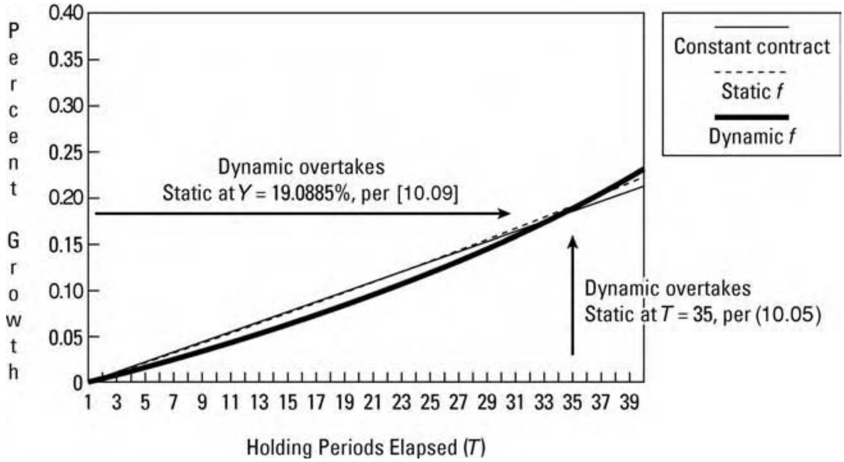

FIGURE 10.3 Points where one method overtakes another can be viewed with respect to time or return

run sense, but ensures that it will be at the highest equity at any point in time, however near or far into the future that is! No longer must someone adhering to optimal f (or, in a broader sense, this new framework) reconcile themselves with the notion that it will be dominant in the long run. Rather, the techniques about to be illustrated seek dominance at all points in time!

Everyone is at an f value whether they acknowledge it or not. Since nearly everyone is diluting what their optimal f values are—either intentionally or unintentionally through ignorance—these techniques always maximize the profitability of an account in cases of diluted f values, not just, as has always been the case with geometric mean maximization, in the very long run.

Again, we must turn our attention to growth functions and rates. Look at Figure 10.3 where growth (the growth functions) is represented as a percentage of our starting stake. Now consider Figure 10.4, which shows the growth rate as a percentage of our stake.

Again, these charts show a 20% initial active equity, traded on both a static and a dynamic basis. Since they both start out trading the same number of units, that very same number of units is shown being traded straight through as a constant contract. The geometric mean HPR (at full f) used in this chart was 1.01933; therefore, the geometric mean at the .2 static fractional f was 1.005, and the arithmetic average HPR at full f was 1.0265.

Notice that by always trading that technique which has the highest gradient at the moment, we ensure the probability of the account being at its greatest equity at any point in time. Thus, we start out trading on a

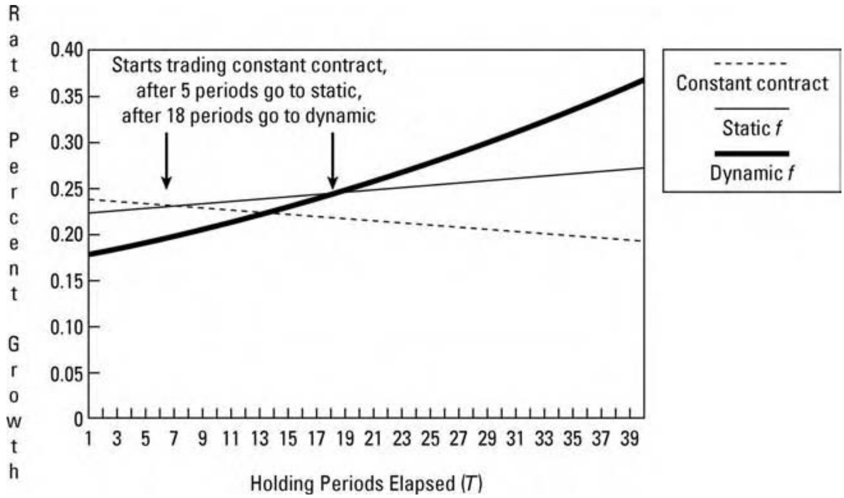

FIGURE 10.4 Growth rate as a percentage of stake

constant contract basis, with the number of units being determined as that number which would be traded initially if we were trading a fractional f.

Next, the static f gradient dominates, at which point in time (or on the upside in equity) we switch to trading the static f Finally, the dynamic gradient dominates, at which point we switch to trading on a dynamic f basis, Notice that by always trading that technique which has the highest gradient at that moment means you will be on the highest of the three lines in Figure 10.3.

The growth function, Y, for the constant contract technique is now given as:*

$$Y = 1 + (\text{AHPR} - 1) * \text{FRAC} * T \tag{10.11}$$

*Just as Equation (10.05) gave us that point where the dynamic overtakes the static with respect to the horizontal axis T, we can determine from Equations (10.11) and (10.12) where the static overtakes a constant contract as that value of T where Equation (10.12) equals Equation (10.11).

$$1 + (\text{AHPR} - 1) * \text{FRAC} * T => \text{FGHPR}^T$$

Likewise, this can be expressed in terms of the Y coordinate, to tell us at what percentage of profit, on the total equity in the account, we should switch from a constant contract to static f trading:

$$Y = \text{FGHPR}^T - 1$$

The value for T used in the preceding equation is derived from the one above it.

The growth functions are taken from Equation (10.05). Thus, the static f growth function is the left side of (10.05) and the dynamic f is the right side. Thus, the growth function for static f is:

$$Y = \text{FGHPR}^T \tag{10.12}$$

And for dynamic f, it is:

$$Y = \text{geometric mean}^T * \text{ FRAC} + 1 - \text{FRAC} \tag{10.13}$$

Equations (10.11) through (10.13) give us the growth function as a multiple of our starting stake, at a given number of elapsed holding periods, T. Thus, by subtracting 1 from Equations (10.11) through (10.13), we obtain the percent growth as depicted in Figure 10.3.

The gradients, depicted in Figure 10.4, are simply the first derivatives of Y with respect to T, for Equations (10.11) through (10.13). Thus, the gradients are given by the following.

For constant contract trading:

$$\frac{dY}{dT} = \frac{((\text{AHPR} - 1) * \text{FRAC})}{(1 + \text{AHPR}-) * \text{FRAC} * T} \tag{10.14}$$

For static fractional f:

$$\frac{dY}{dT} = \text{FGHPR}^T * \ln(\text{FGHPR}) \tag{10.15}$$

And finally for dynamic fractional f:

$$\frac{dY}{dT} = \text{geometric mean}^T * \ln(\text{geometric mean}) * \text{FRAC} \tag{10.16}$$

where:
T = The number of holding periods.
FRAC = The initial active equity percentage.
geometric mean = The raw geometric mean HPR at the optimal f.
AHPR = The arithmetic average HPR at full optimal f.
FGHPR = The fractional f geometric mean HPR given by Equation (5.06).
ln( ) = The natural logarithm function.

The way to implement these equations, especially as your scenarios (scenario spectrums) and joint probabilities change from holding period to holding period, is as follows. Recall that just before each holding period we must determine the optimal allocations. In the exercise of doing that, we derive all of the necessary information to get the values for the variables listed above (for FRAC, geometric mean, AHPR, and the inputs to Equation

(5.06) to determine the FGHPR) Next we plug these values into Equations (10.14), (10.15), and (10.16). Whichever of these three equations results in the greatest value is the technique we go with.

To illustrate by way of an example, we now return to our familiar two-to-one coin toss. Let's assume that this is our only scenario set, comprising the two scenarios heads and tails. Further, suppose we are going to trade it at a .2 fraction (i.e., one-fifth optimal f). Thus, FRAC is .2, the geometric mean is 1.06066, and the AHPR is 1.125. To figure the FGHPR, from Equation (5.06) we already have FRAC and AHPR; we need only SD, the standard deviation in HPRs, which is .375. Thus, the FGHPR is:

$$1.022252415 = \left(\sqrt{((1.125 - 1) * .2 + 1)^2 - (.375 * .2)^2}\right)$$

Plugging these values into the three gradient functions, Equations (10.14) through (10.16), gives us the following table:

	Eq. (10.14)	**Eq. (10.15)**	**Eq. (10.16)**
T	**Constant Contract**	**Static *f***	**Dynamic *f***
1	0.024390244	0.022498184	0.012492741
2	0.023809524	0.022998823	0.013250551
3	0.023255814	0.023510602	0.014054329
4	0.022727273	0.02403377	0.014906865
5	0.022222222	0.024568579	0.015811115
6	0.02173913	0.025115289	0.016770217
7	0.021276596	0.025674165	0.017787499
8	0.020833333	0.026245477	0.018866489
9	0.020408163	0.026829503	0.02001093
10	0.02	0.027426524	0.021224793
11	0.019607843	0.02803683	0.022512289
12	0.019230769	0.028660717	0.023877884
13	0.018867925	0.029298488	0.025326317
14	0.018518519	0.02995045	0.026862611
15	0.018181818	0.030616919	0.028492097
16	0.017857143	0.03129822	0.030220427
17	0.01754386	0.031994681	0.032053599
18	0.017241379	0.03270664	0.03399797
19	0.016949153	0.033434441	0.036060287
20	0.016666667	0.034178439	0.038247704

We find that we are at the greatest gradient for the first two holding periods by trading on a constant contract basis, and that on the third period, we should switch to static f. On the seventeenth period, we should switch

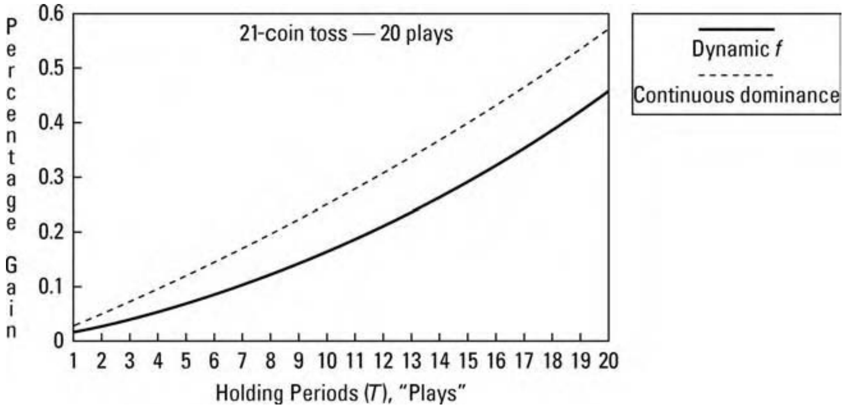

FIGURE 10.5 Continuous dominance vs. dynamic *f*

to dynamic *f*. If we were to do this, Figure 10.5 shows how much better we would have fared, on average, over the first 20 plays or holding periods, than by simply trading a dynamic fractional *f* strategy:

Notice that, at every period, an account traded this way has a higher expected value than even the dynamic fractional *f*. Further, from period 17 on, where we switched from static to dynamic, both lines are forevermore on the same gradient. That is, the dynamic line will never be able to catch up to the continuous dominance line. Thus, the principle of always trading the highest gradient to achieve continuous dominance helps a money manager maximize where an account will be at any point in the future, not just in an asymptotic sense.

To clarify by carrying the example further, suppose we play this two-to-one coin-toss game, and we start out with an account of $200. Our optimal *f* is .25, and a .2 *f*, one-fifth of this, means we are trading an *f* value of .05, or we bet $1 for every $20 in our stake. Therefore, on the first play we bet $10. Since we are trading constant contract, regardless of where the account equity is thereafter, we bet $10 on each subsequent play until we switch over to static *f*. This occurs on the third play. So, on the third bet, we take where our stake is and bet $1 for every $20 we have in equity. We proceed as such through play 16, where, going into the seventeenth play, we will switch over to dynamic. Thus, as we go into every play, from play 3 through play 16, we divide our total equity by $20 and bet that many dollars, thus performing a static fractional *f*.

So, assume that after the second play we have $210 in our stake. We would now bet $10 on the next play (since 210/20 = 10.5, and we must round down to the integer). We keep doing this going into each play through the sixteenth play.

On the seventeenth play, we can see that the dynamic f gradient overtakes the others, so we must now switch over to trading on a dynamic f basis. Here is how. When we started, we decided that we were going to trade a 20% active equity, in effect (because we decided to trade at one-fifth the full optimal f). Since our starting stake was \$200, then it means we would have started out, going into play 1, with \$40 active equity. We would therefore have \$160 inactive equity.

So, going into play 17, where we want to switch over to dynamic, we subtract \$160 from whatever is our equity. The difference we then divide by \$4, the optimal f\$, and that is how many bets we make on play 17. We proceed by doing this before each play, ad infinitum.

Therefore, let's assume our stake stood at \$292 after the sixteenth play. We subtract \$160 from this, leaving us with \$132, which we then divide by the optimal f\$, which is \$4, for a result of 33. We would thus make 33 bets on the seventeenth play (i.e., bet \$33).

If you prefer, you can also figure these continuous dominance breakpoints as an upside percentage gain which must be made before switching to the next level. This is the preferred way. Just as Equation (10.09) gives us the vertical, or Y, coordinate corresponding to Equation (10.05)'s horizontal coordinate, we can determine the vertical coordinates corresponding to Equations (10.14) through (10.16). Since you move from a constant contract to static f at that value of T whereby Equation (10.15) is greater than Equation (10.14), you can then plug that T into Equation (10.12) and subtract 1 from the answer. This is the percentage gain on your starting equity required to switch from a constant contract to static f.

Since you move to dynamic f from static f at that value of T whereby Equation (10.16) is greater than Equation (10.15), you can then plug that value for T into Equation (10.13), subtract 1 from the answer, and that is the percentage profit from your starting equity to move to trading on a dynamic f basis.

IMPORTANT POINTS TO THE LEFT OF THE PEAK IN THE $n + 1$-DIMENSIONAL LANDSCAPE

We continue this discussion that is directed towards most money managers, who will trade a diluted f set (whether they know it or not), that is, they will trade at less aggressive levels than optimal for the different scenario spectrums or market systems they are employing. We refer to this as being to the *left*, a term which comes from the idea that, if we were looking at trading one scenario spectrum, we would have one curve drawn out in two-dimensional space, where being to the left of the peak corresponds to having less units on a trade than is optimal. If we are trading two scenario spectrurms, we have a topographical map in three-dimensional space, where such money managers would restrict themselves to that real estate which is to the left of the peak when looking from south to north at the landscape, and left of the peak when looking from east to west at the landscape. We could carry the thought into more dimensions, but the term *to the left*, is irrespective of the number of dimensions; it simply means at less than full optimal with respect to each axis (scenario spectrum).

Money managers are *not* wealth maximizers. That is, their utility function or, rather, the utility functions imposed on them by their clients and their industry, their $U''(\mathrm{x})$, is less than zero. They are, therefore, to the left of the peaks of their optimal *f*s.

Thus, given the real-world constraints of smoother equity curves than full optimal calls for, as well as the realization that a not-so-typical drawdown at the optimal level will surely cause a money manager's clients to flee, we are faced with the prospect of where, to the left, is an opportune point (to satisfy their $U''(x)$)? Once this opportune point is found, we can then exercise continuous dominance. In so doing, we will be ensuring that by trading at this opportune point to the left, we will have the highest expected value for the account at any point thereafter. It does not mean, however, that it will outpace an account traded at the full optimal f set. It will not.

Now we actually begin to work with this new framework. Hence, the point of this section is twofold: first, to point out that there are possible advantageous points to the left, and, second, but more importantly, to show you, by way of examples, how the new framework can be used.

There are a number of advantageous points to the left of the peak, and what follows is not exhaustive. Rather, it is a starting place for you.

The first point of interest to the left pertains to constant contract trading, that is, always trading in the same unit size regardless of where equity runs up or shrinks. This should not be dismissed as overly simplistic by candidate money managers for the following reason: *Increasing your bet size after a loss maximizes the probability of an account being profitable at any arbitrary future point. Varying the trading quantity relative to account equity attempts to maximize the profitability (yet it does not maximize the probability of being profitable).*

The problem with trading the same constant quantity is that it not only puts you to the left of the peak, but, as the account equity grows, you are actually migrating toward zero on the various f axes.

For instance, let's assume we are playing the two-to-one coin toss game. The peak is at $f = .25$, or making one bet for every \$4 in account equity. Let's say we have a twenty dollar account, and we plan to always make two bets, that is, to always bet \$2 regardless of where the equity goes. Thus, we start out (fortunately, this is a two-dimensional case since we are only discussing one scenario spectrum) trading at an $f\$$ of \$10, which is an f of .1, since $f\$ = -\mathrm{BL}/f$, it follows that $f = -\mathrm{BL}/f\$$. Now, let us assume that we continue to always bet \$2; that if the account were to get to \$30 total equity, our f, given that we are still only betting \$2, corresponding to an $f\$$ of \$15, has migrated to .067. As the account continues to make money, the f we are employing would continues to migrate left. However, it also works in reverse—that, if we are losing money, the f we are employing migrates right, and at some point may actually round over the peak of the landscape. Thus, the peak represents where a constant contract trader should stop constant contract trading on the downside. Thus, the f is migrating around, passing through other points in the landscape, some of which are still to be discussed.

Another approach is to begin by defining the worst case drawdown the money manager can afford, in terms of percentage equity retracements, and use that in lieu of the optimal f in determining $f\$$.

$$f\$ = \frac{\text{abs(Biggest Loss Scenario)}}{\text{Maximum Drawdown Percent}} \tag{10.17a}$$

Thus, if the maximum tolerable drawdown to a money manager is 20%, and the worst-case scenario calls for a loss of −\$1,000:

$$f\$ = \frac{\$1{,}000}{.2} = \$5{,}000$$

He should thus use \$5,000 for his $f\$$. In doing so, he still does not restrict his worst-case drawdown to 20% retracement on equity. Rather, what he has accomplished is that the drawdown to be experienced with the manifestation of the single catastrophic event is defined in advance.

Note that in using this technique, the money manager must make certain that the maximum drawdown percent is not greater than the optimal f, or this technique will put him to the right of the peak. For instance, if the optimal f is actually .1, but the money manager uses this technique with a .2 value for maximum drawdown percentage, he will then be trading an $f\$$ of \$5,000 when he should be trading an $f\$$ of \$10,000 at the optimal level! Trouble is certain to befall him.

Further, the example given only shows for trading one scenario spectrum. If you are trading more than one scenario spectrum, you must change your denominator to reflect this, dividing the maximum drawdown percent by n, the number of scenario spectrums:

$$f\$ = \frac{\text{abs(Biggest Loss Scenario)}}{\left(\dfrac{\text{Maximum Drawdown Percent}}{n}\right)} \tag{10.17b}$$

where: $n =$ The number of components (scenario spectrums or market systems) in the portfolio.

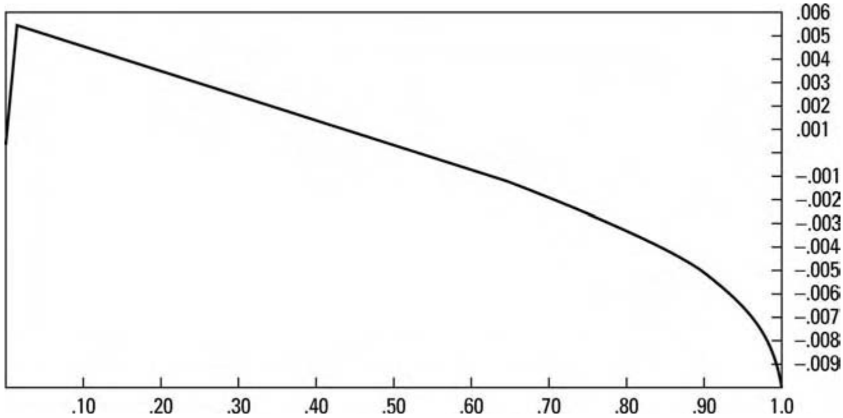

FIGURE 10.6 Two-to-one coin toss, GRR at $T = 1$

Notice that by doing this, if each scenario spectrum realizes its worst-case scenario simultaneously, you will still have defined the maximum drawdown percent for the entire portfolio.

Next, we move on to another important point to the left, which may be of importance to certain money managers: the *growth risk ratio*, or GRR (Figure 10.6). If we take the TWR as the growth, the numerator, and the f used (or the sum of the f values used for portfolios) as representing risk, since it represents the percentage of your stake you would lose if the worst case scenarios(s) manifest, then we can write the growth risk ratio as:

$$\text{GRR}_T = \frac{\text{TWR}_T}{\sum_{i=1}^{n} f_i} \tag{10.18}$$

This ratio is exactly what its name implies, the ratio of growth (TWR_T, the expected multiple on our stake after T plays) to risk (sum of the f values, which represent the total percentage of our stake risked). If TWR is a function of T, then too is the GRR. That is, as T increases, the GRR moves from that point where it is an infinitesimally small value for f, towards the optimal f (see Figure 10.7). At infinite T, the GRR equals the optimal f. Much like the EACG, you can trade at the f value to maximize the GRR if you know, a priori, what value for T you are trying to maximize for.

The migration from an infinitesimally small value for f at $T = 1$ to the optimal f at $T =$ infinity happens with respect to all axes, although in Figures 10.6 and 10.7 it is shown for trading one scenario spectrum. If you were trading two scenario spectrums simultaneously, the peak of the GRR would migrate through the three-dimensional landscape as T increased,

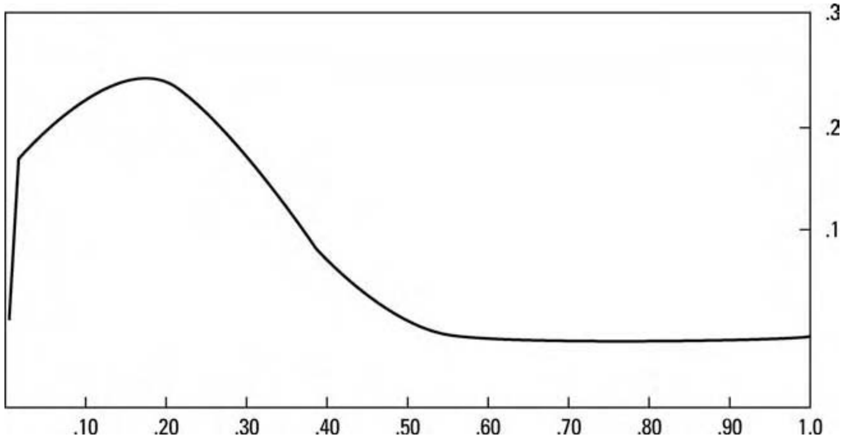

FIGURE 10.7 Two-to-one coin toss, GRR at $T = 30$

from nearly 0,0 for both values of f, to the optimal values for f (at .23,.23 in the two-to-one coin toss).

Discerning the GRR for more than one scenario spectrum traded simultaneously is simple, using Equation (10.18), regardless of how many multiple simultaneous scenario spectrums we are looking at.

The next and final point to be covered to the left, which may be quite advantageous for many money managers, is the point of inflection in the TWR with respect to f.

Refer again to Figure 9.2 in Chapter 9. Notice that as we approach the peak in the optimal f from the left, starting at 0, we gain TWR (vertical) at an ever-increasing rate, up to a point. We are thus getting greater and greater benefit for a linear increase in risk. However, at a certain point, the TWR curve gains, but at a slower and slower rate for every increase in f. This point of changeover, called *inflection*, because it represents where the function goes from concave up to concave down, is another important point to the left for the money manager. The point of inflection represents the point where the marginal increase in gain stops increasing and actually starts to diminish for every marginal increase in risk. Thus, it may be an extremely important point for a money manager, and may even, in some cases, be optimal in the eyes of the money manager in the sense of what it does in fact, maximize.

However, recall that Figure 9.2 represents the TWR after 40 plays. Let's look at the TWR after one play for the two-to-one coin loss, also simply called the geometric mean HPR, as shown in Figure 10.8.

Interestingly, there isn't any point here where the function goes from concave up to concave down, or vice versa. There aren't any points of inflection. The whole thing is concave down.

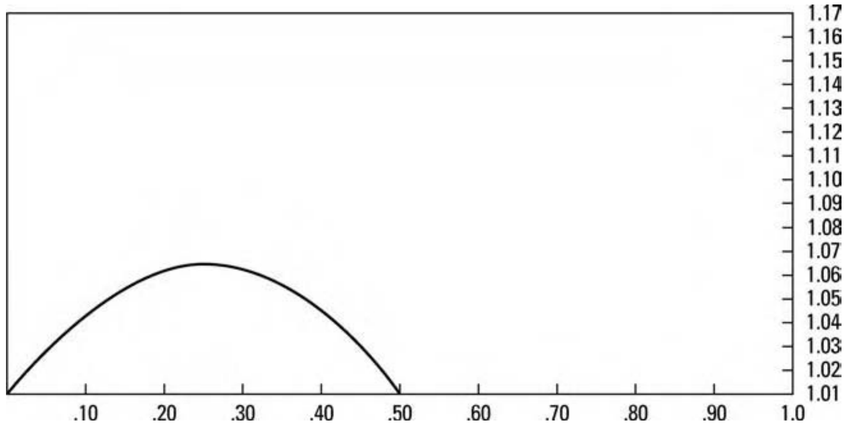

FIGURE 10.8 Geometric mean HPR two-to-one coin toss (= TWR at $T = 1$)

For a positive arithmetic expectation, the geometric mean does not have any points of inflection. However, the TWR, if $T > 1$, has two points of inflection, one to the left of the peak and one to the right. The one which concerns us is, of course, the one to the left of the peak.

The left point of inflection is nonexistent at $T = 1$ and, as T increases, it approaches the optimal f from the left (Figures 10.9 and 10.10). When T is infinite, the point of inflection converges upon optimal f.

Unfortunately, the left point of inflection migrates toward optimal f as T approaches infinity, just like with the GRR. Again, just like EACG, if you

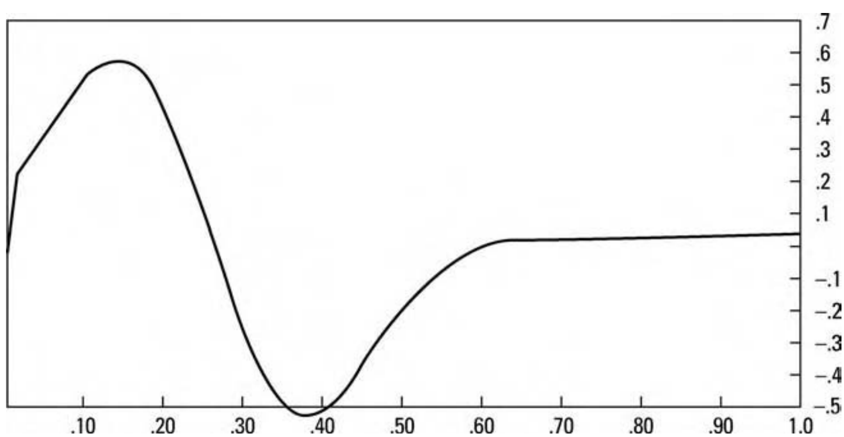

FIGURE 10.9 $d\mathrm{TWR}/df$ for 40 plays ($T = 40$) of the two-to-one coin toss. The peak to the left and the trough to the right are the points of inflection

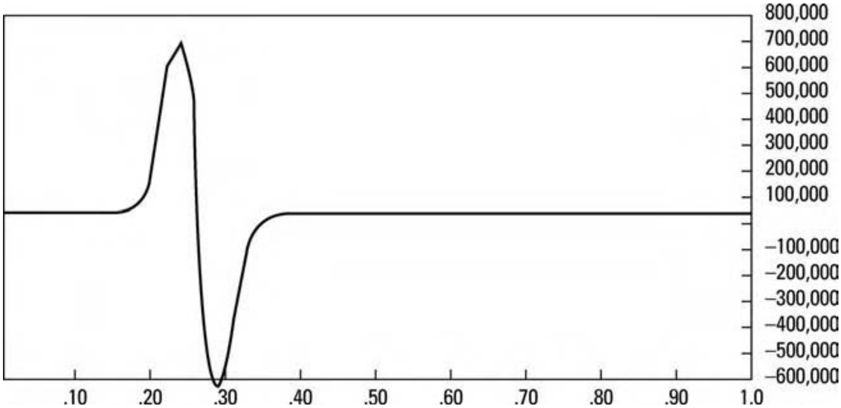

FIGURE 10.10 dTWR/df for 800 plays ($T = 800$) of the two-to-one coin toss. The peak to the left and the trough to the right are the points of inflection. The left peak is at $f = .23$

knew how many finite T you were to trade before you started trading, you could maximize the left point of inflection.*

To recap how the left point of inflection migrates towards optimal f, the following table depicts the numbers for the two-to-one coin-toss game:

2:1 Coin Toss

# plays (T)	f inflection left
1	0
30	.12
40	.13
80	.17
800	.23

Thus, we again see that, as more time elapses, as T increases, missing the optimal f carries with it a steep penalty. Asymptotically, nearly everything is maximized, whether it is EACG, GRR, or the left point of inflection. As T increases, they all converge on optimal f. Thus, as T

*Interestingly, though, if you were trying to maximize the EACG for a given T, you would be seeking a point to the *right* of the peak of the f curve, as the f value which maximizes EACG migrates toward the optimal f as T approaches infinity from the *right.*

increases, the distance between these advantageous points and optimal f diminishes.

Suppose a money manager uses daily HPRs and wants to be optimal (with respect to inflection or GRR) over the course of the current quarter (63 days). He would use a value of 63 for T and set himself at those coordinates to be optimal for each quarter.

When we begin working in more than two dimensions, that is, when we are dealing with more than one scenario spectrum, we enter an altogether more complicated problem.

The solution can be expressed mathematically as that point where the second partial derivatives of the TWR [Equation (9.04), raised to the power of T, the number of holding periods at which we are seeking the points of inflection] with respect to a each particular f equals zero, and each point is to the left (on its axis) of the peak. This becomes ever more complicated in that such a point, where the second partials of the TWR with respect to each f equaling zero may not, depending upon the parameters of the scenario spectrums themselves and how high or low T is, exist. If T equals one, the TWR equals the geometric mean HPR, which is upside down parabolic—it doesn't have any points of inflection! Yet as T approaches infinity, the point(s) of inflection approach the optimal f(s)! Shy of infinite T, there may not be in most cases, such a conveniently common point of inflection with respect to all axes.*

All of this brings us right back to the notion of the $n+1$ dimensional terrain in leverage space, if you will, the axes of which correspond to the f values of the different scenario sets, is to act as a *framework* for analyzing portfolio construction and quantity determination through time. There is so much more to be done in working with this new framework. This chapter is not the end-all on the subject. Rather, it is a mere introduction to an altogether new and, I believe, better way of determining asset allocation. Almost certainly, portfolio strategists, applied mathematicians, asset allocators, and programmers have much new fertile ground to work. Truly, there is a great deal to be done in analyzing, working with, and adding to this new framework, the rewards of which cannot yet even be determined. More importantly, whether one attempts to actively employ the Leverage Space Model, the tenets of The New Framework, as expressed here, are at work and apply to him regardless.

*Remember that the primary thing gained by diversification, that is, trading more than one scenario spectrum, or working in more than two dimensions, is that you increase T, the number of holding periods in a given period of time—you do not reduce risk. In light of this, someone looking to maximize the marginal increase in gain to a marginal increase in *risk*, may well opt to trade only one scenario spectrum.

DRAWDOWN MANAGEMENT AND THE NEW FRAMEWORK

Drawdowns occur from one of three means. The first of these, the most common, is a cataclysmic loss on one trade. I started in this business as a margin clerk where my job was to oversee hundreds of accounts. I have worked as a programmer and consultant to many of the largest traders in the world. I have been trading and working around the trading arena for my entire adult life, often with a bird's-eye view of the way people operate in and around the markets. I have witnessed many people being obliterated over the course of a single trade. I have plenty of firsthand experience in getting destroyed on a single trade as well.

The common denominator of every single occasion when this has happened has been a lack of liquidity in the market. The importance of liquidity cannot be overemphasized. Liquidity is not something I have been able to quantify. It isn't simply a function of open interest and volume. Further, liquidity need not dry up for long periods of time in order to do tremendous harm. The U.S. Treasury Bond futures were the most liquid contract in the world in 1987. Yet, that, too, was a very arid place for a few days in October of 1987. You must be ever vigilant regarding liquidity.

The second way people experience great drawdowns is the common, yet even more tragic, means of not knowing their position until the market has moved ferociously against them. This is tragic because, in all cases, this can be avoided. Yet it is a common occurrence. You must always know your position in every market.

The third cause of drawdowns is the most feared, although the consequences are the same as with the first two causes. This type of drawdown is characterized by a protracted losing streak, maybe with some occasional winning trades interspersed between many losers. This is the type of drawdown most traders live in eternal fear of. It is this type of drawdown that makes systems traders question whether or not their systems are still working. However, this is exactly the type of drawdown that can be managed and greatly buffered through the new framework.

The new framework in asset allocation concerns itself with growth optimality. However, the money management community, as a general rule, holds growth optimality as a secondary concern. The primary concern for the money management community is *capital preservation.*

This is true not only of money managers, but of most investors as well. Capital preservation is predicated upon reducing drawdowns. The new framework presented allows us for the first time to reduce the activity of drawdown minimization to a mathematical exercise. This is one of the many fortuitous consequences of—and the great irony of—the new framework.

Everything I have written of in the past and in this book pertains to growth optimality. Yet, in constructing a framework for viewing things in a growth optimal sense, we are able to view things in a drawdown optimal sense within the same framework. The conclusions derived therefrom are conclusions which otherwise would not have been arrived at.

The notion of optimal f, which has evolved into this new framework in asset allocation, can now go beyond the theoretical formulations and concepts and into the real-world implementation to achieve the goals of money managers and individual investors alike.

The older mean-variance models were ill-equipped to handle the notion of drawdown management. The first reason for this is that risk is reduced to the simplified notion that dispersion in returns constitutes risk. It is possible, in fact quite common, to reduce dispersion in returns yet not reduce drawdowns.

Imagine two components that have a correlation to each other that is negative. Component 1 is up on Monday and Wednesday, but down on Tuesday and Thursday. Component 2 is exactly the opposite, being down on Monday and Wednesday, but up on Tuesday and Thursday. On Friday, both components are down. Trading both components together reduces the dispersion in returns, yet on Friday the drawdown experienced can actually be worse than just trading one of the two components alone. *Ultimately, all correlations reduce to one.* The mean variance model does not address drawdowns, and simply minimizing the dispersion in returns, although it may buffer many of the drawdowns, still leaves you open to severe drawdowns.

To view drawdowns under the new framework, however, will give us some very useful information. Consider for a moment that drawdown is minimized by not trading (i.e., at $f = 0$). Thus, if we are considering two simultaneous coin-toss games, each paying two-to-one, growth is maximized at an f value of .23 for each game, while drawdown is minimized at an f value of zero for both games.

The first important point to recognize about drawdown optimality (i.e., minimizing drawdowns) is that it can be *approached* in trading. The optimal point, unlike the optimal growth point, cannot be achieved unless we do not trade; however, it can be approached. Thus, to minimize drawdowns, that is, to approach drawdown optimality, requires that we use as small a value for f, for each component, as possible. In other words, to approach drawdown optimality, you must hunker down in the corner of the landscape where all f values are near zero.

In something like the two-to-one coin-toss games, depicted in Figure 10.11, the peak does not move around. It is a theoretical ideal, and, in itself, can be used as a superior portfolio model to the conventional models.

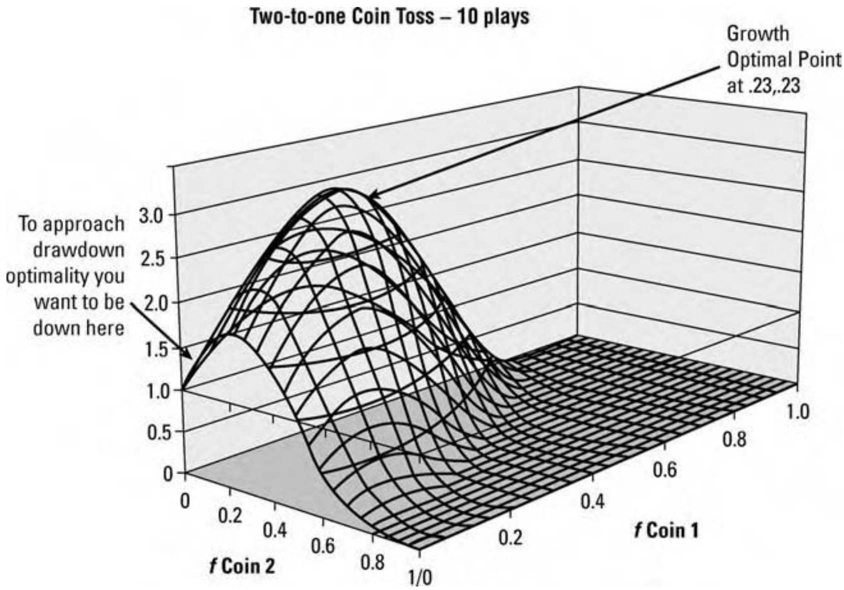

FIGURE 10.11 Drawdown optimality is approached at a different point on the landscape than the growth optimal point

However, as was mentioned earlier in this text, in the real world of trading, the markets do not conform so neatly to the theoretical ideal. The problem is that, unlike the two-to-one coin-toss games shown, the distribution of returns changes through time as market conditions change. The landscape is polymorphic, moving around as market conditions change. The closer you are to where the peak is, the more dramatic the negative effects will be on you when it moves, simply because the landscape is the steepest in the areas nearest the peak. If we were to draw a landscape map, such as the one in Figure 10.11, but only incorporating data over a period when both systems were losing, the landscape (the altitude or TWR) would be at 1.0 at the f coordinates 0,0, and then it would slide off, parabolically, from there.

We approach drawdown optimality by hunkering down in those f values near zero for all components. In Figure 10.11 we would want to be tucked down in the upper-left corner, near zero for all f values. The reason for this is that, as the landscape undulates, as the peak moves around, the negative effects on those in that corner are very minimal. In other words, as market conditions *change*, the effect on a trader down in this corner is minimized.

The seeming problem, then, is that growth is sacrificed and this sacrifice in growth occurs with an exponential difference. However, the solution to this problem can be found by the fundamental equation for trading. Since growth—that is, TWR—is the geometric mean holding period return to the power T, the number of plays is given by:

$$\text{TWR} = G^T \tag{10.19}$$

By hiding out in the corner, we have a much smaller G. However, by increasing T, that is, the number of trades, the effect of an exponential decrease in growth is countered, by itself an exponential function.

In short, if a trader must minimize drawdowns, he or she is far better off to trade at a very low f value and get off many more holding periods in the same span of time.

For example, consider playing only one of the two-to-one coin-toss games. After 40 holding periods, at the optimal f value of .25, the geometric mean HPR is 1.060660172, and the TWR is 10.55. If we were to play this same game with an f value of .01, our geometric mean HPR would be 1.004888053, which crosses 10.55 when raised to the power of 484. Thus, if you can get off 484 plays (holding periods) in the same time it would take you to get off 40 plays, you would see equivalent growth, with a dramatic reduction in drawdowns. Further, you would have insulated yourself tremendously from changes in the landscape. That is, you would also have insulated yourself a great deal from changing market conditions.

It may appear that you want to trade more than one component (i.e., scenario spectrum) simultaneously. That is, to increase T, you want to trade many more components simultaneously. This is counter to the idea presented earlier in discussing the points of inflection that you may be better off to trade only one component. However, by increasing the number of components traded simultaneously, you increase the composite f of the portfolio. For example, if you were to trade 20 scenario spectrums simultaneously, each with a .005 value of f, you would have a composite f of the entire portfolio of 0.1. At such a level, if the worstcase scenarios were to manifest simultaneously, you would experience a 10% drawdown on equity. By contrast, you are better off to trade only one scenario spectrum whereby you can get off the equivalent of 20 holding periods in the same spame of time. This may not be possible, but it is the direction you want to be working in to minimize drawdowns.

Finally, when a trader seeks to approach drawdown minimization, he or she can use the continuous dominance notion in doing so. Continuous dominance is great in the theoretical ideal model. However, it is extremely sensitive to changes in the landscape. That is, as the scenarios used as input change to accommodate changing market characteristics, continuous dominance begins to run into trouble. In a gambling game where the conditions do not change from one period to the next, continuous dominance is ideal.

In the real world of trading, you must insulate yourself from the undulations in the landscape. Thus, drawdown minimization under the new framework lends itself very well to implementing continuous dominance.

So we have now gone full circle, from discerning the landscape of leverage space and finding the growth optimal point on it to retreating away from that point to approach the real-world primary constraint of drawdown minimization and capital preservation. By simply increasing the exponent, by whatever means available to us, we achieve growth. We can possibly achieve equivalent growth if we can get a high enough T, a high enough exponent. Since the exponent is the number of holding periods in a given span of time, we want to get as many holding periods in a given span of time as possible. This does not necessarily mean, however, to trade as many components as possible. All correlations revert to one. Further, we must always assume that worst-case scenarios will manifest simultaneously for all components traded. We must consider that the composite f, the sum of the f values for all components being simultaneously traded, is a drawdown that we will, therefore, experience. This suggests that, in seeking to approach drawdown optimality, yet still striving for equivalent growth as at the growth optimal point, we trade as few components as possible, with as small an f for each component as possible, while managing to get as many holding periods in a given span of time as possible.

The growth optimal point is a dangerous place to be. However, if we hit it just right, that is, if we are at the place where the peak will be, we can see tremendous growth. Even so, we will endure severe drawdowns. However, the leverage space framework allows us to formulate a plan, a place to be on the map of leverage space, to achieve drawdown minimization. It further allows us an alternate avenue to achieve growth, by increasing T, the exponent, by whatever means necessary. This strategy is not so mathematically obvious when viewed under the earlier frameworks.

This is but one means, and a very crude one at that, for mitigating drawdowns in the Leverage Space Model. In Chapter 12, we will see how the terrain of leverage space is “pock-marked,” by “holes,” where the probability of a given drawdown is too high for an investor’s utility preference.

When viewed in the sense to be presented in Chapter 12, the drawdown mitigation technique just mentioned, virtually insures the investor will not be within a pock-marked-out area of the terrain. However, he pays a steep price here for being way to the left of the peak of all curves in leverage space. In Chapter 12, we will see that, although the $0, \ldots 0$ edge point in leverage space is never pock-marked out, we can determine other, far more favorable areas in the terrain.