Chapter 6

Supergravity and cosmology

Renata Kallosh
Department of Physics, Stanford University, Stanford, USA

6.1 M/string theory and supergravity

Supergravity is a low-energy limit of a fundamental M/string theory. At present there is no well-established M/string theory cosmology. However, there are some urgent issues in cosmology which require a knowledge of the fundamental theory. Those issues are related to expanding universe, dark matter, inflation, creation of particles after inflation, etc. The basic problem is that general relativity which is required for explanation of the cosmology and an expanding universe is not yet combined with any relativistic quantum theory and particle physics to the extent in which a full description of the early universe would be possible. Superstring theory offers a consistent theory of quantum gravity at least at the level of the string theory perturbation theory in ten-dimensional target space. The non-perturbative string theory which includes the D-branes is much less understood, since these objects are charged under so-called Ramond–Ramond charges which can be incorporated only at the non-perturbative level. The main attempts during the last few years have been focused on understanding the M-theory, which represents a string theory at strong coupling, when an additional dimension is decompactified. M-theory has as a low-energy limit the 11-dimensional supergravity and has two types of extended objects: two-branes and five-branes.

The radical aspect of major attempts to construct quantum gravity is the concept that the spacetime $x^\mu = \{t, \boldsymbol{x}\}$ is not fundamental. The coordinates x^μ are not labels but fields which are defined by the dynamics of the the world-volume of a p-brane so that they depend on world-volume coordinates, $x^\mu(\sigma^0, \sigma^1, \dots, \sigma^p)$. A two-dimensional object, a string is an one-brane with $x^\mu(\sigma^0, \sigma^1)$, a two-brane is a three-dimensional object with $x^\mu(\sigma^0, \sigma^1, \sigma^2)$, a four-dimensional object called a three-brane and has $x^\mu(\sigma^0, \sigma^1, \sigma^2, \sigma^3)$, etc. M-theory/string theory includes a theory of branes of various dimensions. The fields $x^\mu(\sigma)$ have their own dynamics. The zero modes of the excitations of such extended objects are coordinates of spacetime, $x^\mu(\sigma) = x^\mu_{\text{constant}} + \cdots$. Thus the concept of spacetime is an approximation to a full quantum theory of gravity!

Supergravity (gravity + supersymmetry) may be viewed as an approximate effective description of a fundamental theory when the dependence on coordinates of the world-volume is ignored. The smallest theory of supergravity includes two types of fields, the graviton and the gravitino. Supergravity interacting with matter multiplets includes also scalars, spinors and vectors. All these fields are functions of the usual spacetime coordinates $t, \boldsymbol{x}$ in a four-dimensional spacetime. The fundamental M-theory, which should encompass both supergravity and string theory, at present experiences rapid changes. Over the last few years M-theory and string theory focused its main attention on the superconformal theories and adS/CFT (anti-de Sitter/conformal field theory) correspondence [1]. It has been discovered that IIB string theory on $adS_5 \times S^5$ is related to $SU(2, 2|4)$ superconformal symmetry. In particular, one finds the $SU(2, 2|1)$ superconformal algebra from the anti-de Sitter compactification of the string theory with one-quarter of the unbroken supersymmetry. These recent developments in M-theory and non-perturbative string theory suggest that we should take a *fresh look at the superconformal formulation underlying the supergravity*.

The 'phenomenological supergravity' based on the most general $N = 1$ supergravity [2] has an underlying superconformal structure. This has been known for a long time but only recently the complete most general $N = 1$ gauge theory superconformally coupled to supergravity was introduced [4]. The theory has local $SU(2, 2|1)$ symmetry and no dimensional parameters. The phase of this theory with spontaneously broken conformal symmetry gives various formulations of $N = 1$ supergravity interacting with matter, depending on the choice of the R-symmetry fixing.

The relevance of supergravity to cosmology is that it gives a framework of an effective field theory in the background of the expanding universe and time-dependent scalar fields. Let us remind here that the early universe is described by an FRW metric which can be written in a form which is conformal to a flat metric:

$$\mathrm{d}s^2 = a^2(\eta)[-\mathrm{d}\eta^2 + \gamma_{ij}\, \mathrm{d}x^i\, \mathrm{d}x^j]. \tag{6.1}$$

This fact leads to an interest in the superconformal properties of supergravity.

6.2 Superconformal symmetry, supergravity and cosmology

The most general four-dimensional $N = 1$ supergravity [2] describes a supersymmetric theory of gravity interacting with scalars, spinors and vectors of a supersymmetric gauge theory. It is completely defined by the choice of the three functions: the superpotential $W[\phi]$ and the vector coupling $f_{ab}[\phi]$ which are holomorphic functions of the scalar fields (depend on ϕ^i and do not depend on ϕ_i^*) and the Kähler potential $K[\phi, \phi*]$. These functions from the perspective of supergravity are arbitrary. One may hope that they will be defined eventually from the fundamental M/string theory.

The potential V of the scalar fields is given by

$$M_{\mathrm{P}}^{-2}\mathrm{e}^K[-3WW^* + (\mathcal{D}^i W)g^{-1}{}_i{}^j(\mathcal{D}_j W^*)] + \tfrac{1}{2}(\mathrm{Re}(f)_{\alpha\beta})D^\alpha D^\beta, \qquad (6.2)$$

here D^α are the D-components of the vector superfields, which may take some non-vanishing values. The metric of the Kähler space, $g_i j$ which depends on $\phi, \phi*$, is the metric of the moduli space which defines the kinetic term for the scalar fields:

$$g_i{}^j \partial_\mu \phi^i \partial^\mu \phi_j^*. \qquad (6.3)$$

The properties of the Kähler space in M/string theory are related to the Calabi–Yau spaces on which the theory is compactified to four dimensions.

One of the problems related to the gravitino is the issue of the conformal invariance of the gravitino and the possibility of non-thermal gravitino production in the early universe.

Many observable properties of the universe are, to a large extent, determined by the underlying conformal properties of the fields. One may consider inflaton scalar field(s) ϕ which drive inflation, inflaton fluctuations which generate cosmological metric fluctuations, gravitational waves generated during inflation, photons in the cosmic microwave background (CMB) radiation which (almost) freely propagate from the last scattering surface, etc. If the conformal properties of any of these fields were different, the universe would also look quite different. For example, the theory of the usual massless electromagnetic field is conformally invariant. This implies, in particular, that the strength of the magnetic field in the universe decreases as $a^{-2}(\eta)$. As a result, all vector fields become exponentially small after inflation. Meanwhile the theory of the inflaton field(s) should not be conformally invariant, because otherwise these fields would rapidly disappear and inflation would never happen.

Superconformal supergravity is particularly suitable to study the conformal properties of various fields, because in this framework all fields initially are conformally covariant; this invariance becomes spontaneously broken only when one uses a particular gauge which requires that some combination of scalar fields becomes equal to M_{P}^2.

The issue of conformal invariance of the gravitino remained rather obscure for a long time. One could argue that a massless gravitino should be conformally invariant. Once we introduce a scalar field driving inflation, the gravitino acquires a mass $m_{3/2} = \mathrm{e}^{K/2}|W|/M_{\mathrm{P}}^2$. Thus, one could expect that the conformal invariance of gravitino equations should be broken only by the small gravitino mass $m_{3/2}$, which is suppressed by the small gravitational coupling constant M_{P}^{-2}. This is indeed the case for the gravitino component with helicity $\pm 3/2$. However, breaking of conformal invariance for the gravitino component with helicity $\pm 1/2$, which appears due to the super-Higgs effect, is much stronger.

In the first approximation in the weak gravitational coupling, it is related to the chiral fermion mass scale [3].

This locally superconformal theory is useful for describing the physics of the early universe with a conformally flat FRW metric.

Superconformal theory underlying supergravity has no dimensional parameters and one extra chiral superfield, the conformon. This superfield can be gauged away using local conformal symmetry and S-supersymmetry. The mechanism can be explained using a simple example: an arbitrary gauge theory with Yang–Mills fields W_μ coupled to fermions λ and gravity:

$$S^{\text{conf}} = \int \mathrm{d}^4x\,\sqrt{g}\big(\tfrac{1}{2}(\partial_\mu\phi)(\partial_\nu\phi)g^{\mu\nu} - \tfrac{1}{12}\phi^2 R \\ - \tfrac{1}{4}\,\mathrm{Tr}\,F_{\mu\nu}g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma} - \tfrac{1}{2}\bar{\lambda}\gamma^\mu D_\mu\lambda\big). \tag{6.4}$$

The field ϕ is a conformon. The last two terms in the action represent super-Yang–Mills theory coupled to gravity. The action is conformal invariant under the following local transformations:

$$g'_{\mu\nu} = \mathrm{e}^{-2\sigma(x)}g_{\mu\nu}, \qquad \phi' = \mathrm{e}^{\sigma(x)}\phi, \qquad W'_\mu = W_\mu, \qquad \lambda' = \mathrm{e}^{\frac{3}{2}\sigma(x)}\lambda. \tag{6.5}$$

The gauge symmetry (6.5) with one local gauge parameter can be gauge fixed. If we choose the $\phi = \sqrt{6}M_{\mathrm{P}}$ gauge, the ϕ-terms in (6.4) reduce to the Einstein action, which is no longer conformally invariant:

$$S^{\text{conf}}_{\text{g.f.}} \sim \int \mathrm{d}^4x\,\sqrt{g}\big(-\tfrac{1}{2}M_{\mathrm{P}}^2 R - \tfrac{1}{4}F_{\mu\nu}g^{\mu\rho}g^{\nu\sigma}F_{\rho\sigma} + \tfrac{1}{2}\bar{\lambda}\gamma^\mu D_\mu\lambda\big). \tag{6.6}$$

Here $M_{\mathrm{P}} \equiv M_{\text{Planck}}/\sqrt{8\pi} \sim 2\times 10^{18}$ GeV. In this action, the transformation (6.5) no longer leaves the Einstein action invariant. The R-term transforms with derivatives of $\sigma(x)$, which in the action (6.4) were compensated by the kinetic term of the compensator field. However, the actions of the Yang–Mills sector of the theory, i.e. spin-$\frac{1}{2}$ and spin-1 fields interacting with gravity, remain conformally invariant. Only the conformal properties of the gravitons are affected by the removal of the compensator field. A supersymmetric version of this mechanism requires adding a few more symmetries, so that the $SU(2,2|1)$ symmetric theory is constructed. The non-conformal properties of the gravitino can be followed from this starting point, as shown in [4].

Few applications of superconformal theory to cosmology include the study of (i) particle production after inflation, in particular the study of the non-conformal helicity $\pm 1/2$ states of gravitino; (ii) the super-Higgs effect in cosmology and the derivation of the equations for the gravitino interacting with any number of chiral and vector multiplets in the gravitational background with varying scalar fields; and (iii) the weak coupling limit of supergravity $M_{\mathrm{P}} \to \infty$ and gravitino–goldstino equivalence. This explains why gravitino production in the early universe in general is not suppressed in the limit of weak gravitational coupling.

6.3 Gravitino production after inflation

During the last couple of years there has been a growing interest in understanding gravitino production in the early universe [3, 14]. The general consensus is that gravitinos can be produced during pre-heating after inflation due to a combined effect of interactions with an oscillating inflaton field and because the helicity $\pm 1/2$ gravitino have equations of motion which break conformal invariance. In general the probability of gravitino production is *not* suppressed by the small gravitational coupling. This may lead to a copious production of gravitinos after inflation. The efficiency of the new non-thermal mechanism of gravitino production is very sensitive to the choice of the underlying theory. This may put strong constraints on certain classes of inflationary models.

A formal reason why the effect may be strong even at $M_{\mathrm{P}} \to \infty$ is the following: in Minkowski space the constraint which the massive gravitino satisfies has the form

$$\gamma^{\mu}\psi_{\mu} = 0. \tag{6.7}$$

In an expanding universe, the analogue of equation (6.7) looks as follows:

$$\gamma^{0}\psi_{0} - \hat{A}\gamma^{i}\psi_{i} = 0 \tag{6.8}$$

where, in the limit $M_{\mathrm{P}} \to \infty$,

$$\hat{A} = \frac{p}{\rho} + \gamma_{0}\frac{2\dot{W}}{\rho}, \qquad |\hat{A}|^{2} = 1. \tag{6.9}$$

Matrix $\hat{A}$ rotates twice during each oscillation of the field ϕ. The non-adiabaticity of the gravitino field ψ_0 (related to helicity $\pm 1/2$ is determined not by the mass of the gravitino but by the mass of the chiral fermion $\mu = W_{\phi\phi}$. This equation was obtained in the framework of a simple model of the supergravity theory interacting with one chiral multiplet. The gauge-fixing of the spontaneously broken supersymmetry was relatively easy, the only one available in the model chiral fermion, a goldstino field, was chosen to vanish and the massive gravitino was described by helicity $\pm 3/2$ as well as helicity $\pm 1/2$ states.

A physical reason for gravitino production is a gravitino–goldstino equivalence theorem which, however, had to be properly understood in the cosmological context.

One of the major problems with studies of gravitino production after inflation was to consider the theories with few chiral multiplets. It become clear that one cannot simply apply the well-known super-Higgs mechanism of supergravity in the flat background to the situation in which we have a curved metric of the early universe.

6.4 Super-Higgs effect in cosmology

We would like to choose a gauge in which a goldstino equals zero. The question is which field is this goldstino: we start with the gravitino ψ_μ and some number of left- and right-handed chiral fermions χ^i, χ_i. In the past, this has been sought for constant backgrounds [2], but in cosmological applications the scalar fields are time-dependent in the background. Therefore we need a modification.

In the action there are a few terms where gravitinos mix with the other fermions, and these as well as the supersymmetry transformations should give us the possibility of finding the correct goldstino in the cosmological time-dependent background. We want to obtain a combination whose variation is always non-zero for spontaneously broken supersymmetry. This leads to the following definition of a goldstino:

$$\upsilon = \xi^{\dagger i}\chi_i + \xi_i^\dagger\chi^i + \tfrac{1}{2}\mathrm{i}\gamma_5 D_\alpha\lambda^\alpha, \tag{6.10}$$

where the λ^α are gauginos, the D_α are auxiliary fields from the vector multiplets and

$$\xi^{\dagger i} \equiv \mathrm{e}^{K/2}D^i W - \gamma_0 g_j{}^i\dot{\phi}^j, \qquad \xi_i^\dagger \equiv \mathrm{e}^{K/2}D_i W - \gamma_0 g_i{}^j\dot{\phi}_j. \tag{6.11}$$

The goldstino defined here differs from the one in the flat background by the presence of the time-dependent derivatives of the scalar fields.

Goldstino is non-vanishing in the vacuum supersymmetry transformation:

$$\delta\upsilon = -\tfrac{3}{2}(H^2 + m_{3/2}^2)\epsilon. \tag{6.12}$$

Here H is the Hubble 'constant':

$$\left(\frac{\dot{a}}{a}\right)^2 = H^2 = \frac{\rho}{3M_\mathrm{P}^2}. \tag{6.13}$$

This has important implications. First of all, it shows that, in a conformally flat universe (6.1), the parameter α is strictly positive. To avoid misunderstandings, we should note that, in general, one may consider situations in which the energy density ρ is negative. The famous example is anti-de Sitter space with a negative cosmological constant. However, in the context of inflationary cosmology, the *energy density never can turn negative*, so anti-de Sitter space cannot appear. The reason is that inflation makes the universe almost exactly flat. As a result, the term k/a^2 drops out from the Einstein equation for the scale factor independently of whether the universe is closed, open or flat. Then gradually the energy density decreases, but it can never become negative even if a negative cosmological constant is present, as in anti-de Sitter space. Indeed, the equation

$$\left(\frac{\dot{a}}{a}\right)^2 = \frac{\rho}{3M_\mathrm{P}^2}$$

implies that as soon as the energy density becomes zero, expansion stops. Then the universe recollapses, and the energy density becomes positive again. This implies that supersymmetry is *always broken*. The symmetry breaking is associated, to an equal extent, with the expansion of the universe and with the non-vanishing gravitino mass (the term $(H^2 + m_{3/2}^2)$. This is an interesting result because usually supersymmetry breaking is associated with the existence of the gravitino mass. Here we see that, in an expanding universe, the Hubble parameter H plays an equally important role.

The progress achieved in understanding the super-Higgs effect in an expanding universe has allowed us to find the equations for the gravitino in the most general theory of supergravity interacting with chiral and vector multiplets [4]. Analysis of these equations in various inflationary models and the estimates of the scale of gravitino production remains to be done.

Consider, for example, the hybrid inflation model. In this model all coupling constants are of order 10^{-1}, so there should be no suppression of the production of chiral fermions as compared to the other particles. One can expect, therefore, that

$$\frac{n_{3/2}}{s} \sim 10^{-1}\text{–}10^{-2}. \tag{6.14}$$

This would violate the cosmological bound by 13 orders of magnitude! However, one should check whether these gravitinos will survive until the end or turn into the usual fermions.

Thus supergravity theory and its underlying superconformal structures provide the framework for studies of the production of particles in supersymmetric theories in the early universe.

6.5 $M_P \to \infty$ limit

The complete equations of motion for the gravitino in a cosmological background were derived in [4] with an account of the gravitational effects. However, in [11] some part of these equations, corresponding to the vanishing Hubble constant and vanishing gravitino mass, was derived in the framework of a gauge theory, i.e. from rigid supersymmetric theory without gravity. To find the relation between these two equations one has to understand how to take the limit $M_P \to \infty$ in supergravity. This is a very subtle issue, if one starts with the fields of phenomenological supergravity. One has to do various rescaling of the fields with different powers of the M_P to be able to compare these two sets of equations. Surprisingly, the full set of rescalings reproduces exactly the fields of the underlying superconformal theory. These are the fields which survive in the weak coupling limit of supergravity.

Thus at present there are indications that a description of the cosmology of the early universe may be achieved in the framework of superconformal theory only after the gauge-fixing of conformal symmetry is equivalent to supergravity. The super-Higgs mechanism in cosmology and the goldstino–gravitino equivalence theorem have a clear origin in this $SU(2, 2|1)$ symmetric theory of gravity.

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