PART II

Philosophical progress

The relativity revolution has left in its wake a topsy-turvy world of immense power and immense insecurity, and a sense of both progress and perplexity. We have learned the most profound secrets about space and time, only to be confronted by renewed mystery. Is matter a form of motion? Do the past and future exist now? Is there change? Why is the speed of light constant? Does the very length of a body depend on how it is measured? Do the past and future exist now? Is time travel possible?

From the eclipse expedition in 1919 through to today, this revolution has sent philosophers scurrying backwards to deepen our understanding of the nature of space and time. They have returned to the earliest debates of about 500BCE in ancient Greece and the great feuds over the new discoveries made during the scientific revolution in the 1600s. This research has substantially advanced our understanding of the origins of key concepts, and of the interpretation of Einstein's theories.

The following chapters pursue three key themes:

- *Paradox as a source of innovation*
  Long before experiments were conducted, key concepts emerged as solutions to philosophical problems.
- *The metaphysics of space*
  Do space or spacetime exist in their own right, as a kind of container, over and above bodies?
- *The rise of the relational worldview*
  Natural science has shifted the way philosophers think about relations, the glue that holds the cosmos together.

Throughout, the aim will be to show that insights accumulate. The lessons learned from studying the ancient paradoxes will provide fresh perspectives on the interpretation of general relativity and contemporary debates.

CHAPTER 8

Who invented space?

Some 5,000 or 6,000 years ago, early societies living in Turkey and Armenia spoke Indo-European: the language from which modern European languages have descended. Their vocabulary for concrete objects and simple actions paints a vivid picture of prehistoric life: "bear", "wolf", "monkey", "wheat", "apple", "wheel", "axle", "tree", "father", "carry", "see", "know", and so on. Words for less concrete aspects of the world were a long time coming. The adjective for "big" in Indo-European, for example, was "mega": the root of our "megabyte". This is an abstract word because it can apply to many different kinds of concrete objects; both bears and wheels can be big. Many centuries passed before humans were able to extend this to the very abstract concept of "bigness" or "size": a general noun for an abstract quality. In Homer, who composed his poems about 3,000 years ago, the noun "bigness" (*megathos*) refers only to the height of human bodies; the word is still tethered to specific and concrete objects.

Several centuries later, there was a breakthrough when the philosopher Zeno of Elea (*c.*490–*c.*430BCE) used "bigness" to mean something like the expanse or dimensionality of all existence: that is, he began to liberate extension from concrete things. Each time the word was stretched, and each advance toward greater abstraction was a tiny victory for poetic genius, and contributed to the richness and power of our language today. Every time we buy a shoe or a dress by asking for a certain "size" we draw on the slowly accumulating creations of these ancient word-artists.

But the concept of "space" was very different. It did not grow gradually by stretching earlier meanings over many centuries. It appeared as an act of deliberate creation in ancient Greece during the greatest philosophical controversy of the sixth century BCE. In fact, some historians assert that European philosophy itself was born in the heat of this debate, as it slowly emerged out of religion, mythology and folk history. The controversy centred on what is perhaps the oldest and most venerated problem in European philosophy: the famous *problem of change*. In this chapter, we study the problem not only because it is beautiful and deep, but because it puts the invention of "space" into a new perspective. By studying the way earlier cultures came to their concepts of space, we begin to see how fragile and strange our own concept is.

Ambitious students are sometimes advised to avoid the "hard old problems" that have been lying around for a long time. They may be advised that new advances and discoveries come not from beating dead horses but by entering quickly into the fray of contemporary controversy. But in philosophy there is a catch. The newest problems, if they are really deep and interesting, are often found to have the oldest problems lurking underneath. Progress in philosophy sometimes comes from piercing through a new puzzle and discovering its tangled relations to well-explored regions like the problem of change or the problem of universals. Thus the "hard old problems" are philosophy's hidden shoals; success depends on a talent for navigating through and around them. Records of past struggles with these problems are precious charts – treasure maps of diagnoses and solutions that every ambitious philosopher will master.

Certain sects of philosophers once dismissed these old debates in philosophy as mere "pseudo-problems". They believed that any problems that so stubbornly resisted solution must be mere confusions or artefacts of our inadequate language. But these views have now been discredited. Battles with the problem of change led to concepts that form the foundations of our modern physics. Today the ongoing research on these problems continues to generate new insights and deepen our appreciation of these precious resources.

The problem of change

It is something of a mystery why wealthy young Greeks, clad in their toga-like robes and sitting around on stone benches in the town square, began to argue over subtle philosophical questions. There was a general quickening in the pace of life. Growing cities were sending out small fleets across the Mediterranean to start new daughter cities. A new alphabetic writing had been learned a century or so earlier from Phoenician traders sailing from ancient Lebanon and Israel. The arts of sculpture, architecture, painting and poetry all began to flourish. The upper classes still owned slaves and women led severely restricted lives. Perhaps this is because these elites lacked the concepts of universal justice and human rights; it was they who began to invent these concepts some centuries later. In the midst of this general revival from about 600BCE onwards, some enquiring minds began debates that were to lay the foundations of European philosophy, science, law, politics, art and literature. We are all in their debt.

One of the reasons the problem of change is so beautiful is that change is all around us. It takes a very subtle mind to notice that something so ordinary and common conceals, just beneath the surface, a fundamental mystery. What is change? It is as everyday as a leaf turning from green to yellow, and as intimate as your eyes shifting along this line of text. Change involves *difference*: the leaf is first green and then not green. It involves *newness*: the yellow that comes to be did not exist beforehand. It thus involves succession in time: green is followed by yellow. But change is not mere replacement. If one green leaf is simply removed and another yellow leaf is substituted, the first green leaf has not changed. It has merely been replaced. Thus change involves *persistence or sameness across time*. And finally change itself is some *process* or *transition*: the green leaf changes when it *becomes* yellow.

The Greeks used the same word to describe both motion from one place to another and change in the properties or nature of a thing. Movements and changing colour – kissing and blushing – were both examples of change for the Greeks. The word they used, *kinesis*, is the root of our "cinema", where we watch moving pictures.

What could the problem with change be? The ancient Greeks saw that the process of change, the way something came to be something else, was puzzling. They were first struck by the newness that change produced. Where was the yellow before the leaf changed its colour? In general, *where did the new quality or state come from*? We may think this question has an easy answer, but let's pause to ponder the question for a moment. It has surprising depths.

Some philosophers have interpreted this first version of the problem of change as groping towards our laws of conservation, so let us label it the "problem of change from conservation". In philosophy, discussions can often be kept simple by creatively using labels for ideas, and we will cultivate the habit here. This problem about the newness that change involves is really just the idea that "no one can pull a rabbit out of a hat". But it is important enough to give it a fancy name and carefully examine the ideas involved:

The problem of change from conservation

A.	If there is change, then something new comes to exist that did not exist before.	(P)
B.	If something did not exist before, then it was nothing.	(P)
C.	So, if there is change, then something new comes to exist from nothing.	(A,B)
D.	But it is not the case that something new comes to exist from nothing.	(P)
E.	So, there is no change.	(C,D)

This is a startling and daring conclusion. How could anyone doubt the change that occurs all around us? If you are certain that there is change, and wish to rescue it from this attack, you must find some error in the above argument.

This little argument is valuable because it points at two profound issues. First, it led the Greeks to express the idea that "nothing comes from nothing" (as the slogan goes). Thus the fourth sentence, D, is a deep and far-reaching idea. Our experience tells us that objects just do not simply appear or disappear: rabbits do not emerge from empty hats; pink elephants do not materialize in my bedroom at night. It is the same reassuring habit of things that modern scientists rely on when they formulate their conservation principles. Here, however, accepting this idea leads to trouble. If we accept this, are we forced to agree that there is no change?

Secondly, this argument is valuable because it makes us question the first sentence, A. It exposes a *tension* in our concept of change. Does change really produce something new? In a sense it obviously does. But we do not believe that the new aspects emerge "out of nothing". So the results of change are new, but they were also there before in some way or some form. They are somehow new, but also somehow old. We must explain this to avoid the preposterous conclusion that there is no change.

The idea that "nothing comes from nothing" became important again about 1,000 years later. Philosophers were debating with members of the new Christian religion in the early centuries after the birth of Christ. The Christian doctrines proudly asserted that God created the world "from nothing" (in Latin, *creatio ex nihilo*). At times, the theologians used this as a dig against the pagans. Without revelation, it was said, the pagans could not imagine creation from nothing, and ignorantly supposed even a god could not perform such a miracle.

> The problem of change from conservation is important because it articulates early notions of conservation and because it exposes a strange tension, a new-oldness buried deep in the nature of change.

Another problem of change

As the ancient Greeks struggled to detail a theory of change they stumbled on a more subtle issue that might be called the problem of change from contradiction (Sorabji calls it "the problem of stopping and starting"). There are some curious illustrations of the problem.

Did you know it is impossible to jump off a bridge? If you are on the bridge, you have not yet jumped. If you are off the bridge, you have already jumped. But there is nothing "between" being on or being off the bridge. You cannot be both on and off the bridge, and you cannot be neither on nor off the bridge. So there is no such thing as jumping off bridges.

Likewise, it is impossible for trains to start. Before they start, they are at rest. After they start, they are in motion. But the train cannot be at rest and in motion, and it cannot be neither at rest nor in motion. So trains do not start – ever.

These may seem to be merely silly or irritating riddles, but their ancestors opened up a deep chasm that many philosophers, mathematicians and scientists have struggled to cross. They point to difficulties in giving a "micro-theory" of the process of becoming, or of the transition from one state to another. It is worth dissecting this problem more clearly to get at the underlying issues.

The problem turns on the idea of opposites like "being on" and "being off" or "being at rest" and "being in motion". The Greeks distinguished between different kinds of opposites. Some pairs of opposites are *mutually exhaustive*. That is, every relevant thing must be one or the other: there is no alternative or intermediary. An integer greater than zero is either odd or even; there are no other options. A

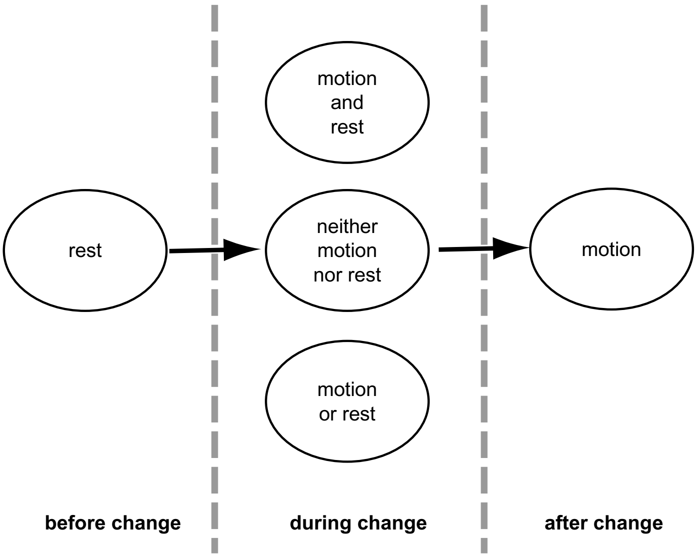

Figure 8.1 *The problem of change from contradiction. What is the transition between rest and motion? None of the intermediate states are satisfactory. The top two are ruled out by logic and the lowest is the same as either the initial or final state, and therefore is not change.*

train must be either at rest or in motion (i.e. not at rest). This kind of opposite is called a *contradictory*. Even and odd are contradictory properties among positive integers. The idea that something must have one or the other of two contradictory properties is called the *law of excluded middle*: there can be no third thing "between" two contradictories.

Not all pairs of opposites are mutually exhaustive. Black and white are opposites but they are not mutually exhaustive because something may have a colour "between" black and white; namely, grey. Such an opposite is merely a *contrary* and not a contradictory.

The second version of the problem of change is strongest when phrased in terms of contradictory properties:

The problem of change from contradiction

A. If a thing is changing from one property to its contradictory property, then the thing has exactly one, or both, or neither of the properties. (P)

B. Such a changing thing does not have exactly one of the properties, because it is then either before or after the change. (P)
C. Such a changing thing does not have both of the properties, because they are opposites. (P)
D. Such a changing thing does not have neither of the properties, because they are contradictories and therefore mutually exhaustive. (P)
E. Therefore, it is not the case that a thing is changing from one property to its contradictory property. (from A–D)

Thus the would-be jumper cannot get from being on the bridge to being not on the bridge, and the train cannot start.

> When philosophers attempted to give a micro-theory of the process of change or becoming, they encountered "gaps" when neither property applied or "overlaps" when both did. But these are impossible.

Parmenides

The first and most famous solution to these problems of change was advanced by Parmenides of Elea, who lived in a Greek colony in southern Italy about 650BCE. Although his surviving writings are all fragments of poetry, he is now revered as the first philosopher in the European tradition. At the time, however, the word "philosopher" had probably not yet been invented, and there was no European culture.

Parmenides' solution was dramatic. The fury it triggered propelled many developments in philosophy, and it will not be reassuring to those frustrated with the problems above. In short, Parmenides caved in, accepted the arguments and declared that there was no change! He asserted that the everyday appearance of change around us is some sort of illusion. If we could penetrate through these appearances we would see that the world is really static and frozen without movement of any kind. Thus, according to Parmenides, there is no error in the above arguments. The only "problem" is our mistaken idea that change and movement are real.

More extremely, Parmenides not only rejected change through time, but also seemed to assert that there is no "change through space". That is, there are no differences between things. Everything is the same, completely homogeneous, and in fact *just one thing*. The world is like a geometric point, and has no parts or other distinguishable features. We are all one and the same person, and identical with the material world. This philosophy is known as *monism*, since it insists that everything is one.

For most people, Parmenides' solution is extreme and even insane, but he nonetheless remains enormously important. His solution grew out of his heroic commitment to clear thinking. After much study, when he could discover no error in the arguments against change, he instead rejected the plain evidence of his five senses. He championed clear, abstract thinking long before its value had been proved in science or mathematics. More deeply, his solution grew out of belief that the world must be intelligible, that it must be understandable. When the arguments above convinced him that change was contradictory, he threw out change. If change could not be understood, it could not be part of the world. Parmenides is a hero to philosophers for these commitments and for proclaiming them so vividly. In an odd way, he did much to inspire subsequent philosophy and science because he challenged everyone else to say clearly why his solution was wrong. We hope, however, to honour his principles and yet find some way to understand change.

Parmenides' story has a very surprising ending – he may have been right. His conception of a changeless world was different from the block universe debated above, but both reject motion and change. Did Einstein vindicate Parmenides?

The invention of space

The first recognizable concept of space was created as part of a solution to the problem of change. The solution is ingenuous and was indeed the product of great genius, and its distant cousins are taught in every secondary school today.

Several philosophers, now called the *atomists*, developed a comprehensive theory of reality to eliminate the problem of change. At the time it was remarkable for its clarity and depth, and served as a model for later scientific theories. The following contains the central doctrines of the three thinkers: Leucippus, Democritus and Epicurus.

Democritus is honoured as the main founder of atomism; his portrait appeared on modern Greek coins before the Euro was adopted. The atomists' theory had three principles:

The metaphysics of atomism

I. *Atoms and the void*: changeless atoms and a changeless void exist. Both are eternal; only they exist.
II. *Motion*: change is the motion of atoms through the void, that is, their rearrangement.
III. *Bonds*: the atoms have "hooks and barbs" and can clump together into stable, large bodies.

These ideas were so novel in 600BCE that the atomists needed to coin new words to express them. Their atoms were supposed to be very tiny, and impossible to break or cut. The first syllable of "atom" means "not", just as in the English word "amoral" and the second syllable "tom" is from the Greek verb "to cut". So "atom" literally meant the "uncut" or "uncuttable".

In this system, the change of a leaf from green to yellow was easily explained as the rearrangement of the atoms or chemicals within the leaf. This is, in essence, similar to the view of modern chemistry and biology. Thus, the atomists could brag, there was no mysterious emergence from nothing.

Their second new idea led to a furious controversy in which many of the themes of this book were voiced for the first time. What does it mean to say that the void exists? If a void is just emptiness, can emptiness exist? In struggling to answer these questions, atomists finally developed a concept of the void that was an important forerunner of our modern "space". The word the atomists used for void, *kenon*, was quite ordinary. It named the inside of an empty cup or container, which might later be filled. That is, it chiefly meant the "vacant inside of a container". But the atomists were stretching the meaning of the word in a very creative way. Their void was the whole universe. It was pure emptiness. It was not inside a container: it was the inside without the container. The atomists knew this was strange, but went out of their way to emphasize the new meaning they were giving to the word "void". Their language became quite vehement when they asserted that the void was "nothing" and that "nothingness exists".

Although radical, the atomists' metaphysics was a brilliant proposal for solving the problem of change. They argued that larger bodies were built up from atoms. The changes we see around us are mere appearance: a kind of optical illusion. In reality, there was no change at all. The atoms never change and the void, as a mere nothing, certainly cannot change. Moreover, since there was no real change, there was no problem of change. This solution also nicely captures our sense that change involves both something new and something old. The arrangements of the atoms are new, but the atoms are eternal and never come into existence. Thus the atomists can claim to have saved the principle that "nothing comes from nothing".

In a way, however, the atomists resembled Parmenides. Like him, they had so much regard for the problem of change that they saw no recourse but to deny the existence of change. Despite its attractions, atomist metaphysics struck many as an entirely absurd proposal. Who could believe that the void, a mere nothing, *exists*? Why were the atoms uncuttable and unchangeable? What prevented them from breaking down further or simply eroding away?

> The atomists may be interpreted as substituting one paradox for another: instead of the problem of change, we have a nothing that is a something.

Aristotle's common sense

Plato (*c*.428–*c*.348BCE) is held to be the greatest philosopher of ancient times. His devotees say he combined a mind as deep as Einstein's with the literary powers of a Shakespeare. But Plato's student Aristotle has been by far the most influential European philosopher. For many centuries, Plato's writings were almost entirely lost, and Aristotle came to dominate European thought. He was known simply and affectionately as "the Philosopher". His works were considered almost as true as the Bible, and anyone who claimed to find obscurities or mistakes in Aristotle was considered a poor interpreter. In a way, this was very fortunate. Aristotle is the great champion of common sense. He fought off the paradoxes and contradictions that seduced his predecessors, and grounded his philosophy firmly on everyday experience.

Aristotle was the last of the three great philosophers of ancient Greece: Socrates, Plato and Aristotle. Socrates (*c*.469–399BCE) was a poor stone-cutter who wandered barefoot through Athens posing uncomfortable ethical questions to his social betters. Traditionally compared to Jesus Christ, he was a charismatic and inspiring figure who transformed the history of philosophy, even though he left no writings. The Athenians finally voted to put him to death in 399BCE for "corrupting the youth". Plato, a wealthy aristocrat devoted to Socrates in his youth, went on to found the school known in Greek as the Academy, where philosophy, mathematics, language and astronomy were studied. This was perhaps Europe's first university and managed to survive in some form for a thousand years. Aristotle was Plato's most famous student, and studied in the Academy for about 20 years. When he left, he founded his own school, called the Lyceum. Aristotle's father had been court physician to the Emperor Phillip II, and Aristotle became tutor to Phillip's son, who is known to history as Alexander the Great: he conquered the known world, at least from Italy to India, before dying at the age of 33. Aristotle is regarded as the first scientific biologist because of his careful studies of animals and their development, but he is important here because of his emphatic rejection of the atomists' concept of space. Aristotle's arguments and his enormous authority meant that "space" was frequently considered to be an entirely incoherent notion during much of the next 2,000 years.

In response to the atomists and other early philosophers, Aristotle also advanced a comprehensive theory of the world. His metaphysics, only part of which we study here, was founded on the notion of substance:

Aristotle's metaphysics

I. *Substances and properties*: the basic things are the concrete bodies and objects we encounter in our ordinary experience: humans, horses, trees, rocks and so on. They have properties like "being rational", "being two-legged", "being an animal" and so on.

II. *Actuals versus potentials*: in addition to their actual properties, substances have potential properties "inside" them. Change occurs when a potential property becomes actual: a green leaf has potential yellowness, and changes colour when that potential becomes actual yellowness.

III. *Plenum*: all substances touch and are surrounded by other substances. There is no empty space. Like a fish moving through water, motion occurs when the substances ahead are shoved aside and other substances fill in behind.

Later, Latin translations of Aristotle's new terms became some of the most important in European philosophy. The word "substance" literally means whatever "stands under" a thing's properties. For example, an egg has the properties of whiteness, hardness and so on; these properties belong to its underlying substance, which holds them together. "Potential" means "could be or might be". It comes from the Latin translation for the Greek word for capability or power (as does the English "potent"). Potential properties have the power to become actual, but are not yet so. "Plenum" describes that which is full and has no gaps (like the English "plenty" or "plentiful").

It is important to see here how Aristotle thought he had solved the problem of change without invoking the idea of space. His strategy became a model for later philosophers who denied the existence of space. Aristotle argued for substances by appealing to common sense. We should base our philosophy on what seems most secure and irrefutable, namely the existence of the objects around us. We should not speculate about mysterious and invisible atoms, nor make them the foundation of our metaphysics.

Likewise, his common sense led him to agree that "nothing comes from nothing". The new end products of change had to exist in some form before change. They were new but also old. Thus he posited his potential properties. Aristotle does not say exactly what these are, but simply insists that they must be there to avoid the paradoxes. They are ghostly, shadowy properties that are there but that do not show or exemplify themselves. Potential yellowness is not coloured; it does not appear in any way until it becomes actual. Thus, Aristotle claims, the problem of change is solved. The new results of change do not come from nothing; they come from potential properties.

At times Aristotle thought of potentials not as ghostly properties but as the powers or capacities or "dispositions" to do or become something. The green leaf must *be* capable of turning yellow; otherwise it could not. Thus this capability exists in its substance. The results of change emerge, not from nothingness, but from capacities.

Although there is something correct in Aristotle's solution to the problem of change, he has in fact shifted the question. The earlier philosophers were led to doubt and deny the very existence of change, and challenged Aristotle to defend it. Aristotle just sets this aside. He argues that not everything can be proved. Instead, he assumes it is obvious that change occurs and asks what follows from this. Thus, he argues, *since* change occurs, *then* potential properties or capacities must exist to avoid the paradoxes. As a cautious and conservative philosopher, Aristotle constructed theories to redeem common sense and simply found the paradox-mongers distasteful.

> Aristotle says that substances are the common-sense things we see around us; but his concept of a substance, which seamlessly unifies its actual and potential properties, is not commonsensical.

Criticism

The problem of change spawned a great rivalry at the heart of the metaphysical tradition in Europe. It led to the invention of the first concepts of space, to the assertions that "nothing exists" and to the forceful reaction embodied in Aristotle's common-sense substances. *Both space and substance were thus born as solutions to the problem of change.* The following chapters will trace the fortunes of these two antagonistic worldviews. Both solutions failed to resolve the problem of change. It is important to see why.

By making the atoms and the void eternal and unchanging, the atomists attempted to solve the problem of change by eliminating change altogether. Instead, however, they simply concealed change. The atomists gave no name to the relation between the atom and the bit of the void it sits in, but it later become known as the *occupancy relation*. They might have imagined that this relation was nothing at all. It appears, for example, that no concrete tie of any sort relates an ordinary stone to the ground it sits on. But this relation is real; it is where change in the atomists' system lies concealed.

One argument for the reality of the occupancy relation relies on the *Truthmaker Principle*, which simply asserts that if some sentence is true, then something makes it so. This is a useful way to express the transition from truths we know to assertions about reality, that is, from epistemology to metaphysics. Thus:

Argument for the reality of the occupancy relation

A. If some sentence is true, then something makes it so. (P)
B. It is true that "an atom is related to the bit of space it occupies: it is *in* a place". (P)
C. Thus something relates an atom to space. (from A,B)

The argument does not tell us much about the structure of this occupancy relation, but simply that it exists. The atomists simply overlooked it, and trusted to their intuition that things simply sit in places without any real tie between them. The argument here is an antidote to such naivety. Once we recognize the reality of the relations between atoms and the void, whatever these relations may be, it becomes obvious that this is the place or *locus* of change in the atomists' system. When atoms move through the void and are rearranged, what changes are the occupancy relations. That is, the relational tie between an atom and its place is broken, and new ties in new places are formed.

If it is true that occupancy relations change, then the problem of change is resurrected within the atomists' system. The new occupancy relations are new, and therefore they emerge from "nothing", or did not emerge at all and so on. As Barnes says, atomism is "fundamentally a flop, it does not answer Parmenides".

Thus the initial persuasiveness of atomism depends on a confusion about relations. Our ordinary experience with moving objects suggests that no palpable thing relates a body to its place. The relation is invisible and seems almost nothing at all. The atomists hid real change in this relation; they swept the problem under the rug, and thus did not solve but only suppressed the problem of change.

The fatal flaw in Aristotle's solution to the problem of change is remarkably similar. In his plenum of substances jostling against each other, Aristotle takes it for granted that substances will touch. But he does not stop to analyse this relation, called the *contiguity relation* ("contiguity" is "the quality of touching"). It has obvious problems.

Intuitively, when two things touch, no third thing is brought into existence; there seems to be no "touching" relation over and above the things that touch. This intuition is again based on our ordinary experience; we never see some new entity created by touching. The truthmaker principle, however, again tells us that something must make it true that things are touching. What could this be?

The atomists might answer that touching occurs when two atoms occupy adjacent places, and these places are held together in a unified space. Thus the touching relation is really two occupancy relations plus whatever holds space together. But touching is very difficult to explain within Aristotle's system. If only substances exist, is the touching relation another substance? If not, and touching is not a new entity, what "holds" substances next to each other? What does it mean for them to be "next to each other"? Aristotle cannot say that they sit in adjacent bits of space, because he denies the existence of space. This difficulty came to the fore in the philosophy of Leibniz, a later Aristotelian, discussed below.

Just as the atomists hid real change in occupancy relations, Aristotle hides change or motion in these contiguity relations between substances. He does not spell out what touching is, nor how touching relations change during movement within the plenum. He relies instead on ordinary intuitions about touching. Thus Aristotle too merely conceals and suppresses the problem of change.

There is a similar difficulty with Aristotle's notion of potential properties, which are meant to explain change within a substance. Aristotle does not and cannot account for the *inherency relation* between a substance and its properties, and understandably resists calling this a relation at all. But change in properties involves a change in inherency relations, and thus triggers another, new version of the problem of change. In sum, Aristotle hides the problem of change in two new sorts or relations, contiguity and inherency, but never explains how these are immune to the problem of change.

The problem of change survived these first onslaughts. The dragon waits to be slain.

CHAPTER 9

Zeno's paradoxes: is motion impossible?

Counting things was the beginning of mathematics. The integers came first: 1, 2, 3, ... Later the need to measure straight lines and flat areas led to the study of geometry in ancient Egypt and India. But mathematics stumbled when it came to curves, spheres, continuous quantities and smooth changes. Early mathematics could not grasp our more fluid world, could not bring its changes and subtleties to life. Mathematics had to learn about change and infinity. It had to enter the labyrinth of the continuum.

Zeno's famous paradoxes may seem to be merely teasing riddles or bewildering games, but they are much, much more than that. They provoked the first great debates over infinity in the European tradition. Two thousand years later, students were still immersed in study of the paradoxes, and one of them, the Englishman Isaac Newton, grew up to create a new kind of mathematics of change: the infinitesimal calculus. Today, the jets we fly in, the bridges we cross and the devices that play our music were all designed using Newton's calculus.

Since space, motion and time are often thought of as continuous and infinite, Zeno's paradoxes were also the first deep enquiry into their structures. Philosophers, however, have tended to study Zeno's paradoxes of motion as if they were primarily about space, motion and time. Plato portrayed Zeno as Parmenides' younger lover, and historians have tended to agree that Zeno's paradoxes were an indirect defence of his friend's strange philosophy. Analysis of the paradoxes confirms this, and opens up broader ways of thinking about Zeno's work.

The arrow and the dichotomy

Zeno argued that an arrow shot from a bow never moves during its entire flight. This has become known as the *paradox of the arrow*. His idea is deceptively simple:

The paradox of the arrow

A. At each instant of its flight, the arrow is in a place exactly its size. (P)
B. If a thing is in a place exactly its size, it is motionless. (P)
C. So, at each instant, the arrow is motionless. (from A,B)
D. If something is true for each instant during a period, then it is true for the entire period. (P)
E. So, for the entire period of its flight, the arrow is motionless. (C,D)

(Zeno's paradox did not mention instants of time, and the above is only one interpretation of his words.)

Although almost everyone agrees Zeno's conclusion is daft, there is no agreement about why. Each of the premises has been attacked.

Some philosophers have thought that the problem occurs at the outset with the idea of "instants". They have denied that time is composed of separate instants. Instead, they supposed that time had to be composed of small stretches of time. But there are difficulties here. If an arrow moves during the smallest stretch of time, then the stretch has parts, and is not the smallest.

Others rejected that idea that "to be in a place exactly its size" implies that a thing is motionless (B). But during motion a thing passes through different places; if it is in motion, it cannot be in just one place.

The shift from each instant to the entire period (E) is also suspicious. There are many cases when something that is true of the parts is not true of the whole, and vice versa. Each individual in a human population is a person, but the population is not a person. Objects composed of atoms are coloured, but no atom is coloured. With the arrow, though, Zeno seems safe. If the arrow is motionless in each instant, when could any motion occur?

These considerations immediately show the value of Zeno's paradoxes. They forced philosophers to develop ideas about the nature of time and its continuous structure. Before giving a deeper diagnosis of the paradox, let's examine another famous paradox about infinity. According to Zeno's paradox of the dichotomy, you cannot walk to the nearby wall you are facing. Here "dichotomy" means "cutting in two" ("dicho" means "in two" and "tom" means "cut" – as in "atom"). The argument runs as follows:

The paradox of the dichotomy

A. If a runner reaches the end-point of a distance, the runner also visits its mid-point. (P)
B. If the runner visits the mid-point, then the runner visits a point half the way to the mid-point. (P)
C. Thus, if a runner reaches the end-point, the runner visits an infinity of points. (from A,B, induction)
D. But it is impossible to visit an infinity of points. (P)
E. Therefore, the runner does not reach the end-point of the distance. (C,D)

This is obviously a general argument and, if it holds at all, implies that you cannot walk to a nearby wall. In fact, you cannot even move an inch in that direction.

The last premise, D, seems very suspicious, but the idea behind it is simple. "Infinite" literally means "without end" (Latin "in" means "not" and "finis" means "end"). So to finish visiting an infinity of points would mean coming to the end of what does not have an end.

Deeper questions lurk in the transition to the third sentence. How do we get from the idea that "each interval has a mid-point" to an infinity of points? This is a leap. Today logicians and mathematicians usually consider this leap acceptable and call it "mathematical induction" (but there is still a leap of some sort).

> The problem is not that Zeno's conclusions are true, but that there is no agreement about why they are false.

Aristotle banishes infinity

Before getting further tangled in Zeno's paradoxes, we should pause at the brink to consider what infinity might be. For many classical thinkers, at least from the time of Aristotle, there was no such thing. Infinities were pathologies: signs that a theory or line of reasoning had gone wrong somewhere.

Aristotle defined orthodox thought about infinity for some 2,000 years. He first admitted that there were powerful reasons for supposing that infinities existed. The integers seem to be infinite, and the universe may be infinite in extent. Time seems to go on and on, and thus would also be infinite. But infinity was notoriously paradoxical. Even in classical times there were a number of well-known puzzles. As above, to say that an infinity existed, and was wholly present, seemed to assert that something without end had come to an end. But the incomplete could not be complete, and thus there could be no infinity. There were also problems with parts and wholes. Consider the integers, 1, 2, 3, . . . The even integers, 2, 4, 6, . . ., are only a part of all the integers, yet for every integer there is one corresponding even integer: its double. Thus there is a one-to-one correspondence between integers and even integers, and there is the same number of each. But then the part is equal to the whole! Since this cannot be, there cannot be such a thing as an infinity.

Aristotle laid down the orthodox doctrine. He had used the distinction between potential and actual to solve the problems of change. It explained how the results of change could be both new and old. Now he proposed to use the same distinction again to resolve tensions surrounding infinity: we had some reasons to suppose it existed, but other reasons to deny its existence altogether.

According to Aristotle, *there is no actual infinity*: no existing thing is infinite. Thus the paradoxes are avoided. However, there may be *potential infinities*. If we can always add 1 to an integer to produce the next integer, then the integers can potentially go on and on. If one day can always be followed by the next, then days are potentially infinite. No integer and no day, no existing thing, is ever infinite.

This famous doctrine had wide ramifications for Aristotle's metaphysics. He denied, for example, that the universe was infinite. It also explains in part his hostility to Zeno's paradoxes. His distinction did not, however, lead him to a satisfactory resolution. He appears to have changed his mind and gave incompatible solutions to the paradoxes in different places.

Aristotle's insistence that there are no actual infinities makes us even more suspicious of Zeno's reasoning in the paradox of the dichotomy, but leaves the enigma intact.

Paradoxes of plurality

Bertrand Russell, the most prominent Anglo-American philosopher of the twentieth century, said that every generation solves Zeno's paradoxes, and every following generation feels the need to solve them again. Rather than advancing one more supposed solution, perhaps we can develop a deeper perspective on why they are so difficult to disentangle. What issues lie beneath Zeno's paradoxes?

Plato portrays Zeno as indirectly defending Parmenides. Instead of arguing positively that all reality is one and unchanging, Zeno argues negatively that reality could not be many things. Most historians agree that this interpretation of Zeno is correct, but philosophers still tend to treat the above paradoxes as if they were merely puzzles about motion through space and time. According to this view in Plato, Zeno's deeper concern was more general, and focused on aspects of the so-called *problem of the one and the many*. Aristotle mentions that this was already an "ancient" problem in his time, and the problem of unity in diversity continued to dominate Greek metaphysics for the next thousand years after Plato and Aristotle. Can we interpret Zeno's paradoxes as manifestations of this more fundamental tension between unity and plurality?

To be one thing and to be many things are contradictories: something cannot be both one and many in the same respect and at the same time. But as we look around us, almost everything we see is both one and many. An egg is one and yet is many properties, such as hardness and whiteness. Space is one and yet is made up of many places. An army is one and yet is many soldiers. How do all these manage to be both one and many?

The first, obvious answer is that they are one and many *in different respects*. An egg is one as substance, but has many properties. A space is one as a whole but has places as parts. An army is one as an army, but as soldiers many. This simple response, however, merely pushes the problem down one level, for how can one thing comprise many "respects" and yet remain one? This is a tough question. The quick answers fail to dissolve the tension between the contradictories, and simply push the problem around.

This central tension between one-ness and many-ness is the core of the problem, but like other deep problems it can be manifested in a bewildering variety of ways. The phrase "problem of the one and the many" has come to stand for this hornets' nest of related problems. Some philosophers use it to refer to the conflict between two basic sorts of ontologies: pluralism says that there are many things or many kinds of elements, while monism says that there is only one thing or one kind of thing. But the one–many conflict is more general than this, and infects all metaphysical questions.

Plato's dialogues return to these problems of unity and diversity again and again, and at times they portray it as the central problem of metaphysics. Although only fragments of Zeno's writings have survived as quotations in other books, several of these fragments are *paradoxes of plurality*, and make it clear that the many-ness of things was one of Zeno's main targets. Indeed, Plato says in the *Phaedrus* that Zeno made "the same things seem like and unlike, and one and many, and at rest and in motion". As one example, we find Zeno advancing the following paradox. This passage, from a book by Simplicius, is thought to be a genuine quotation, and may be the earliest, recorded "argument" in European philosophy:

> In proving once again that if there are many things, the same things are limited and unlimited, Zeno's very own words are as follows.
>
> "If there are many things, it is necessary that they are just as many as they are, and neither more nor less than that. But if they are as many as they are, they will be limited.
>
> If there are many things, the things that are are unlimited; for there are always others between the things that are, and again others between those. And thus the things that are are unlimited."

Zeno's point is that if many things are both unlimited and limited, they are contradictory. But, since there are no contradictions, there are not many things and reality must be just one thing. The thinking behind the second half of this argument probably goes as follows. If things are many, they cannot seamlessly touch or overlap. Thus they must be somehow separated from each other, say by boundaries or by intervening things. And these separators too must be distinct from the things they separate (if not, there is no separation), and are therefore just more things among the many. But then they in turn must be separated from the others, and an infinite regress follows: they are unlimited in number. (This puzzling argument has some strength. Note that we cannot answer it by assuming that things or their separators are wholes made of parts or that space is continuous, because these presume some sort of many-ness.)

Without examining the depths of this argument, it is clear evidence that Zeno was attacking plurality. Is it possible to show that the above paradoxes of motion are also rooted in similar puzzles about unity and diversity? Zeno's paradoxes are very controversial and have been interpreted in many ways. Here we can only suggest that this is a fruitful way of thinking about the underlying issues.

The nub of the paradox of the dichotomy is the infinity of points that must be traversed. What is the source of this infinity? Why do we agree there is an infinity of points before the end-point? Why is there a mid-point between any two points? One answer is that there could not be *nothing* between the two points, because then they would not be separated, and would not be *two* points. This sort of answer shows an immediate link to the paradox of plurality above, and suggests that the paradoxes of motion are related to the deeper problem of the one and the many. Indeed, Simplicius quoted the passage above to make that point.

Today, we might say there is an infinity of points because the intervening space is smooth and *continuous*. However, any attempt to spell out what we mean by this would again encounter the one–many conflict. Even in modern mathematics, theories of the continuum have been beset by difficulties, and some suggest these are technical analogues of the problem of the one and the many.

The paradox of the arrow can be understood in the following way. A motion seems to involve both diversity and unity. It is easy to see how diversity is involved because the motion traverses many places or takes many moments. It is less easy to see the role of unity. A motion can be a smooth and unbroken whole, as Aristotle thought, and it traverses its whole route during a whole period of time. But it is hard to see how the unity combines with the diversity of places and times to constitute a motion.

Since the role of unity is hard to express or even articulate, it is easy for Zeno's argument (a body does not move in a place its size, etc.) to conclude that there is no unity at all. But stripped of unity, we have no motion at all. When a motion is broken down into many instants without connection, into a "pure diversity", then motion itself disappears. The strength of the paradox does not lie, therefore, in the rather dodgy argument for breaking down motion into static instants, but in the difficulty we have in articulating how motion can be both a one and a many. Unless we have a clear idea of how motion unites its diverse instants, we will fall back into a picture of motion as pure diversity and thus into Zeno's lair.

The German philosopher Hegel made a bizarre suggestion in the early 1800s, which may help to illustrate the kind of solution which would fend off Zeno's attack on motion. Hegel said that a moving body is actually in two places at once. Perhaps he meant the two places were almost entirely overlapping, and that motion just was a strange ability to unite the diverse. If so, Hegel can answer Zeno by saying that a body in a place exactly its size can be moving; namely, when it is also in another place at the same instant! Whether or not this makes sense, Hegel at least aims at a concept of motion that explicitly fuses the many into one, and this is the sort of picture that is needed to rebut the paradox of the arrow.

Russell gave a solution to the paradox of the arrow that was influenced by mathematics and relativity theory. In essence, he gave in to Zeno, and agreed that motion just was many motionless instants. If time really flows, then his solution is very much like the films, mentioned earlier, where a sequence of still photographs constitutes motion (sometimes called "cinematic motion" or "the staccato universe").

> Zeno's paradoxes can be interpreted as manifestations of the problem of the one and the many. If many things are somehow unified, a relation does the work.

What are relations?

Space and time are composed of relations, and so they are in a way the subject of this book. The thrust of this chapter and Chapter 8 is that the struggle to conceptualize change, motion, space and time stumbled upon deeper issues with the ontology of relations. The atomists and Aristotle suppressed the problem of change only by sweeping the real change in their systems into occupancy and inherency relations. Zeno exploded notions of space and time by teasing out the implications of the one–many-ness residing within all real relations. Before our investigation of space and time continues, we should consider some general issues and views about the nature of relations.

The primitive tendency to think that everything that exists is an individual object obstructed enquiry into relations. Many philosophers have even denied that relations have any existence of their own, over and above the things they relate. Crudely put, those who think of relations as real tend to think of them as something like a great stone bridge stretching between two cliffs, and somehow connecting or uniting them. In this picture, a relation connects two particular things, the cliffs, but has some extra being of its own, the mortar and stones in the arch. Russell called these "real relations", and meant thereby that relations had some reality distinct from what they related. Thus this view can be called *realism* about relations.

Other philosophers have maintained that only particular things exist and would favour pictures like the following. One tree is taller than another tree because the first has a height of 10 metres and the second has a height of 15 metres. That is, the relation "is taller than" is just an awkward way of talking about the properties belonging to individual trees. The only things that exist are the individual trees and the properties in them; nothing stretches between the trees when one is taller than the other. This is sometimes described as *reductionism* since the relations are "reduced to" particular things and their individual properties.

Despite prominent exceptions, it is fair to say that for some 2,000 years, from Aristotle to Russell, reductionist views dominated among philosophers. Aristotle rejected realism and his extraordinary fame lent this great weight. But even apart from this, there were three main philosophical reasons for favouring reductionism.

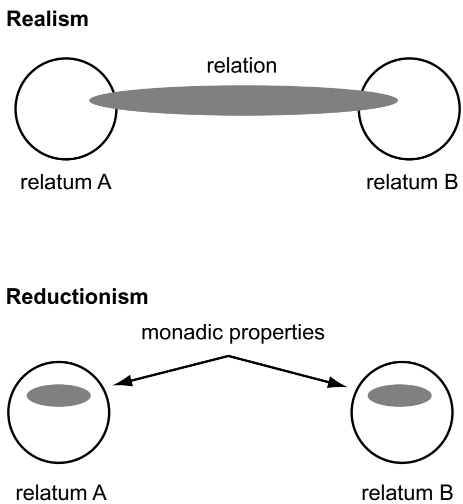

Figure 9.1 *Two views of the ontology of relations. According to realism, the relation stretches between the two relata. According to reductionism, the relation is merely an ordinary one-place property inside each of the two relata.*

First, as mentioned above, our experience with ordinary objects encourages the view that they are primary. Bodies seem to move without being caught in a web of changing real relations. Talk of anything over and above particulars seems preposterous to common-sense philosophers. (They overlook, however, that we always encounter objects involved in relations: in contact, in the world, in space, etc.)

The second barrier to realism was a phenomenon known nowadays as a "Cambridge change". Suppose the top of the taller tree is lopped off, and only 8 metres remain. If a relation were some kind of bridge from one tree to the other, then trimming one tree would change the other. During the transformation of the untouched tree from shorter to taller tree it would be violently disconnected from any bridge-like relation that might connect the two trees. This seems odd. How could a change in one tree instantaneously affect a distant tree? In fact, all trees everywhere would suddenly alter their relations to the trimmed tree. This sort of spooky, invisible change has made the very idea of real relations seem ridiculous.

The third barrier to realism has been the problem of the one and the many discussed above. It is very difficult to say how a bridge-like relation connects or unites the things it relates. Are they one or many? If they are strongly united and become one new individual, then there is no relation, just a particular thing with properties. If they remain many, then there is no relation – just isolated things. Somehow relations are both one and many, but these are contradictories.

During the early 1900s there was a great shift among Anglo-American philosophers who reversed 2,000 years of prejudice and overcame these obstacles to recognizing the reality of relations. This was Russell's great achievement.

Two great, countering pressures outweighed the scepticism about real relations. First, in the 1600s, European mathematicians began to adopt Hindu numerals and Arabic algebra, and thus began to write mathematics in a new language: they used "equations". Earlier mathematics had been written as a series of ratios or proportions or in ordinary language. But writing formulas with an equals sign made them look like representations of relations. When physicist-philosophers such as Descartes adopted this new language and used it to describe the world, it became hard to avoid accepting relations as real. What else were the equations of physics describing?

Russell's victory, however, was especially helped by the invention of the new symbolic logics around 1900. These aped the language of mathematics, and gave philosophers new tools for analysing and studying the various types of relations. Relations were promoted from nonentities to the subject of a new science. Thus this symbolic technology for logic brought with it a profound shift in the basic ontological views of Anglo-American philosophers.

The above objections to realism are now problems about relations and not reasons for rejecting real relations altogether.

The acceptance of real relations leaves the problem of the one and the many as a deep problem about the structure of relations, which heightens the significance of Zeno's attacks on plurality. If his paradoxes were devices for exposing one–many tensions, then they are relevant to the new realism about relations. They are especially important to any deep theory of spatial or temporal relations.

Zeno and the mathematicians

> From Zeno to our own day, the finest intellects of each generation in turn attacked the problems, but achieved, broadly speaking, nothing. In our own time, however, three mathematicians – Weierstrass, Dedekind, and Cantor – have not merely made advances on Zeno's problems, but have completely solved them. The solutions are so clear as to leave no longer the slightest doubt or difficulty. This achievement is probably the greatest of which our age has to boast; and I know of no age (except perhaps the golden age of Greece) which has a more convincing proof to offer of the transcendent genius of its great men.
>
> (Russell, 1901)

Russell used his enormous reputation among philosophers to propagandize for mathematical solutions of Zeno's paradoxes. Russell's views still have a residual influence among today's philosophers, many of whom believe that "mathematics have solved all that". But Russell was wrong, and his views have been overtaken by history.

In essence, there was tremendous optimism during the late 1800s and early 1900s that set theory and the new mathematics of infinity would sweep away the ancient paradoxes and furnish a crystal-clear foundation for all of mathematics. But this revolution quickly discovered that it too was beset by paradoxes, and soon became bogged down in attempts to repair its own premises in ways that would evade a complete collapse. These movements still have some prestige, and set theory is still taught to students, but they really have failed to satisfy the hopes of the pioneers. Now a variety of approaches compete for attention, and there is no consensus that Zeno's problems have been solved; instead, they have simply been transformed into more technical paradoxes. Zeno lives.

CHAPTER 10

Philosophers at war: Newton vs. Leibniz

The Englishman Sir Isaac Newton, one of the greatest physicists of all time, has been reinvented. During the past 30 or so years, historians like Betty Jo Dobbs began to uncover the human behind the scientist and made discoveries that have surprised the world of science.

Newton has often been idolized, but at such a distance that he seemed a cold, remote and austere figure, like the marble statues that depict him. He was famous for basing his science strictly on what he could observe and measure. He mocked other philosophers whose premises and hypotheses were spun out of their own brains, and proudly hissed "I feign no hypotheses". As a professor and later head of the Royal Mint in London, he seemed a ready-made, secular saint for science.

In the 1930s, boxes of Newton's unpublished papers were discovered in an attic and sold at auction. Their surprising contents led to some talk of a cover-up by his family and followers. As historians began to investigate these and other scattered papers, a new Newton emerged. They showed that he spent much of his time working, not on physics, but on alchemy: the magical search for a way to produce gold. Newton was in fact something of a transitional figure. He was half-wizard stoking his furnace and half-scientist covering pages of parchment with his sprawling mathematical calculations. Other papers showed that Newton suffered bouts of insanity. Researchers found that his alchemical notebooks recorded not only the colour, weight and other properties of the chemicals he was mixing, but also their *taste*. Tests on surviving locks of his hair reportedly showed mercury poisoning.

Other investigations showed that Newton, fearing imprisonment or worse, concealed his more radical activities. He befriended the philosopher John Locke before Locke had to flee to Holland. Newton was a Christian heretic, but hid his rejection of the Trinity. Like many scholars at the time he remained unmarried, but his biographer, Richard Westfall, believed the evidence shows that he was homosexual. Newton was a stock-market investor, and lost a fortune when a speculative bubble burst. The new Newton is spectacular: not a secular saint but a richer, fascinating, multi-dimensional human being. None of this detracts from the man or the science. It does represent a real discovery by the historians: the eccentric genius beneath the marble myth.

In the philosophy of space and time, Newton made two monumental contributions:

- *Revival of the atomists' void*
  Newton put the concept of the void at the centre of his new physical theory. Its success eventually reversed the Aristotelian consensus that the concept of the void was incoherent.
- *Invention of a mathematics of change*
  The Greek problem of change and Zeno's paradoxes had shown how difficult it was to form a workable concept of change. In his struggles to predict the motions of the planets, Newton invented a new way to handle change that has had enormous repercussions for philosophical debates about change.

Our goal in this chapter is to penetrate to the core of Newton's arguments for his innovations. Everyone believed that Einstein had finally overthrown Newton's views on space and time but, as we shall see, Newton is making a comeback.

The geometrization of space

When Newton was a student he studied the works of René Descartes (1596–1650), who was something like the Einstein of the 1600s. Descartes was the chief founder of modern post-Aristotelian philosophy, a mathematician and a scientist, who left France for the relative intellectual freedom of Holland. There is a lovely story about Descartes's first stay there when he was just out of the army and still in his twenties. He met another ambitious fellow in the street and they started talking.

Their friendship was cemented when they discovered each shared an interest in combining physics and mathematics. Astronomers had always used mathematics but, until the Scientific Revolution in the seventeenth century, physics was largely qualitative and conceptual. It was a branch of philosophy that emphasized explanations and not precise predictions. Descartes's new friend wrote in his diary: "Physico-mathematicians are very rare, and Descartes says he has never met any one other than myself who pursues his studies in the way I do, combining Physics and Mathematics in an exact way. And for my part, I have never spoken with anyone apart from him who studies in this way". In a nutshell, then, the two new friends excitedly discussed the novel idea that "physics should use mathematics". Descartes went on to discover one of the first mathematical laws of physics and, with figures like Kepler and Galileo, is honoured as a pioneer of mathematical physics.

Descartes could not, however, extend his law into a complete system of physics. His book *The Principles of Philosophy* (1644) was an advance over Aristotle's still-dominant metaphysics but it remained conceptual and qualitative. In fact, Descartes had cooked up a strange mixture of previous systems. He circumspectly advocated the fashionable atomism of the newer philosophers, but also insisted that atoms comprised a plenum like that of Aristotle.

Thus before Newton it was not clear that using mathematics in physics was going to be a successful direction for research. The programme was gathering strength, but its prophets remained a minority. Newton had bragged to his friends in the new coffeehouses that he could calculate the paths of the planets. Working feverishly night after night, however, he began to realize he had more than he thought. His book grew. He added chapters, pestered astronomers for more data and began new calculations. Westfall's biography captures the mood of Newton's almost hysterical energies as he began to suspect that he had *everything*. He had a system of the world. He had God's blueprint for the cosmos. He could predict the orbits of the planets around the Sun, the eerie streaks of the comets, the waxing and waning of the tides, the rise and fall of cannonballs, the dropping of an apple: he could calculate and predict all known motions.

In 1687, Newton's *Mathematical Principles of Natural Philosophy* appeared. The title was a put-down aimed at Descartes's followers, and set the agenda for the future of physics. Unlike Descartes's *Principles of Philosophy*, Newton's method was rigorous and mathematical and carefully limited to the study of material nature. It avoided all discussion of souls and psychology.

This book not only changed our conception of space and time, but it changed for ever humanity's vision of the universe. It showed that the same laws that governed the motions of the planets through outer space governed motions here on Earth and even in our own bodies. It showed that these laws were mathematical, and thus lent themselves to precise measurement and prediction. It showed that the human mind could reach out and comprehend nature, that the mystery and magic of the living world concealed a rational system. In sum, Newton redeemed the faith of the Greek philosophers that reason was at home in the world: that the world was ultimately intelligible. In the long history of the human race, Newton's may prove to be the most important book ever written.

Newton boiled down his entire system into the three compact laws below, from which all the rest followed. They are worth memorizing. The first law was discovered earlier by Galileo, Descartes and others. This was an enormous breakthrough. Most of the motions here on Earth peter out and come to an end: a marble will eventually stop rolling. It took real insight to see that the marble would have naturally continued *for ever* if the force of friction did not slow it. The second law is about "forces", which are just pushes and pulls. It says that a stronger push will create a faster change in movement, and also that a heavier body will react more slowly to a push. The third law is here labelled the "conservation of energy", but the modern concepts of energy and momentum had not yet been invented in Newton's time. He expressed the idea by saying that when one body pushes a second body and gives it motion, the first body loses the same amount of motion in the same direction: "equal and opposite reactions".

Newton's laws of motion

I. *Inertia*: a moving body will follow straight lines at the same speed unless changed by forces.
II. *Force*: equals mass times acceleration.
III. *Conservation of energy*: for every action there is an equal and opposite reaction.

These are the core of Newton's system, and are some of the most precious words ever written.

As Newton developed his theories of motion, he discovered that the world could not be a mere plenum as conceived by Aristotle and Descartes. It is not hard to see that these laws demanded the existence of some "space" over and above the bodies moving in it. The law of inertia says that bodies will follow straight lines unless deflected by some force. If the theory is correct, these straight lines must exist. There must be, in addition to moving material bodies, a geometric space that houses them. According to Newton, the lines in space tell bodies which way to move. Since the theory best explained all known motions, Newton concluded that the lines and space must be real.

Philosophers call this kind of argument an *inference to best explanation*. If a theory gives the best explanation of something that occurs, then we are entitled to infer that the theory is a correct description. This kind of argument is often used in science and in everyday life, but it is not very secure. Mystery writers construct their surprise endings by arranging for the best explanation of a crime to be overthrown by some last-minute piece of evidence. A claim that rests on inference to best explanation alone is never completely secure.

Newton himself was not satisfied with this kind of argument, and began his book with a long note or "Scholium" on space and time. These famous pages were the most important development in the philosophy of space and time since Plato and Aristotle 2,000 years earlier. In essence, Newton revived the atomists' conception of space and radically transformed it to serve his own theories.

Atomist doctrines were in the air, and creating quite a scandal. In 1633, Galileo had been prosecuted and placed under house arrest in part for his flirtations with atomism; Descartes had to deny that he would even dabble in views so associated with atheism. A generation earlier, in 1600, the philosopher Giordano Bruno had been burned at the stake in Rome for refusing to recant his heresies. In the century after the Protestant churches had broken away from Roman Catholicism, authorities everywhere were in a panic about subversive and divisive philosophies. Atomism was resolutely materialist and seemed to challenge all religions. Newton's embrace of the atomists' void was in keeping with his other radical views.

> Newton's *Principia* ignored the paradoxes about change, motion and space, and therefore smuggled them into the foundations of all contemporary physics.

Absolute space and absolute time

> I don't know what I may seem to the world, but, as to myself, I seem to have been only like a boy playing on the sea shore, and diverting myself in now and then finding a smoother pebble or a prettier shell than ordinary, whilst the great ocean of truth lay all undiscovered before me.
>
> (Newton, shortly before his death in 1726)

For philosophers of space and time, Newton's Scholium is the Old Testament. Even after Einstein revealed the New Testament of relativity theory, the deep framework of Newton's vision remains basic to all of physics. It was Newton who made the key terms "relative" and "absolute" central to classical theories of space and time. The word "absolute" means "independent" in the sense that a thing is absolute when it does not depend on other things, is free from interference and makes itself what it is ("to absolve" means "to set free"). A king has absolute power when he exercises it himself, independently of a constitution, legislature or foreign allies.

Invariance is, here, a property of appearances. If all measurements of a spacetime interval yield the same result, then the observed interval is invariant. For Newton, "absolute" is a metaphysical term, and describes the reality behind appearances. A thing is absolute when it exists in its own right, when no other thing can alter it.

Newton began the Scholium with a definition of absolute time:

> Absolute, true, and mathematical time, of itself, and from its own nature, flows equably without relation to anything external, and by another name is called duration; relative, apparent, and common time, is some sensible and external measure of duration . . .

Newton's metaphor here is very important. He says that time "flows", perhaps as a river does. He further asserts that the passage of time has a constant "speed", that is, that it flows "equably" and uniformly. This seems reassuring, but is also very puzzling. A river flows past its banks. What does time flow past? The speed of a water flow can be measured in units of, say, kilometres per hour. At what speed does time flow? One hour per hour? Newton does not answer these questions.

For Newton, a "relative space" depends on something else. The hold of a ship is an enclosed volume that moves with the ship and depends on it, and is thus merely a relative space. By contrast, Newton thought that empty outer space did not depend on anything, and was therefore an "absolute" space. Relative spaces can move; absolute spaces are immobile:

> Absolute space . . . remains always the same and immovable. Relative space is some movable dimension or measure of the absolute spaces, which our senses determine by its position to other bodies . . . Absolute and relative spaces are the same in shape and size; but they do not remain always one and the same. For if the earth, for instance, moves, a space of our air . . . will at one time be one part of the absolute space, and at another time it will be another part of the same. And so, absolutely understood, it will be continually changed.

Thus a relative space is part of absolute space. But if the boundaries of the relative space move, then the relative space moves with them. An absolute space is not dependent on anything.

Bodies move in space from one place to another, that is, from one part of space to another. But because there are two kinds of spaces, there are two kinds of motions. In a key paragraph, Newton says:

> Absolute motion is the translation of a body from one absolute place into another; and relative motion, the translation from one relative place into another. Thus in a ship under sail, the relative place of a body is that part of the ship or the hold which the body fills, and which therefore moves with the ship. But real, absolute rest is the continuance of the body in the same part of immovable, absolute space.

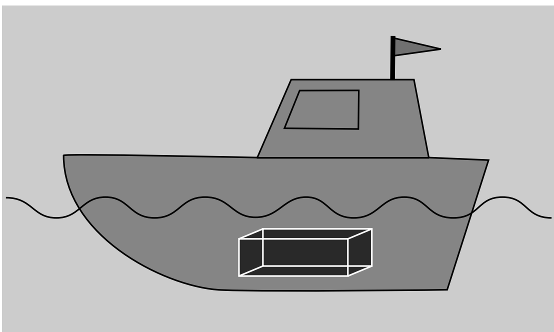

Figure 10.1 *Relative versus absolute space. The space within the ship's hold moves with the ship and is therefore a relative space. The shaded exterior space does not move with the ship and is absolute.*

This can be a bit tricky. All motion is in or through or towards or away from something else. All motion is relative to something. *Thus absolute motion is actually a kind of relative motion. Absolute motion is motion relative to absolute space.*

Newton loudly takes a stand in the Scholium that was at odds with his old reputation. He was often hailed as the father of strict empiricism. This philosophy insists on limiting research to what can be observed and measured. Thus empiricists refuse to investigate God or angels, what makes a poem beautiful or what happened before the beginning of the universe. These are not open to observation and measurement and therefore, they say, should not be part of science.

Modern empiricism has been so enormously fruitful that its advocates sometimes pushed the idea to an extreme. They rejected discussion of anything that could not be *directly* observed, and attacked those philosophers who championed conceptual or linguistic investigations. In these controversies, Newton was used as an emblem of strict empiricism, as a scientist whose great discoveries stemmed from his adoption of empiricism. This image is, of course, completely outdated now. What is peculiar is that even Newton's published papers proclaimed that he was a moderate empiricist. He made a great contribution by emphasizing observation and measurement, and even constructed his own telescopes and other instruments. But Newton thought of himself as a philosopher and also balanced his empiricism with a sense of its limitations.

The Scholium contains a passage in which Newton suggests that science must go beyond what can be directly observed, must go beyond strict empiricism:

> But because the parts of absolute space cannot be seen, or distinguished from one another by our senses, we use sensible measures of them. And so, instead of absolute places and motions, we use relative ones; and that without any inconvenience in common affairs; but in philosophical disquisitions, we ought to abstract from our senses, and consider things in themselves, distinct from what are only sensible measures of them. For it may be that there is no body really at rest, to which the places and motions of others may be referred.
>
> Those who confound real quantities with their relations and sensible measures defile the purity of mathematical and philosophical truths.

Relative spaces are directly observable because their boundaries are (the ship); absolute spaces are not. Relative spaces appear to us;

absolute spaces are the reality behind the appearances. Newton emphasizes that his science cannot be limited to what is directly observable because that would exclude absolute space. Readers of Kant will recognize his distinction between phenomena and noumena: between appearances and things-in-themselves. Early in his career, Kant was a vigorous Newtonian and defender of absolute space, as we shall see.

> Remember the slogans: "absolute motion is motion relative to absolute space" and "relative motion is motion relative to bodies or their surroundings".

The bucket argument

Newton was not satisfied with mere definitions of absolute space. He concluded the Scholium with a virtuoso performance, and suggested experiments and arguments *proving* the existence of absolute space. These have had an extraordinary afterlife. They were a direct inspiration to Einstein as he struggled with his theory of gravitation. In different guises, they remain today at the centre of debates over space and time among philosophers. But students sometimes giggle when they read Newton's proposals: they seem silly, almost crude and rustic. These first impressions are wrong. A satellite built by NASA and Stanford University (see Appendix D for their websites) is a high-tech version of Newton's proposals, as will be explained below.

Newton's famous *bucket argument* is now considered a philosophical classic. As their studies progress, philosophers sometimes fall in love with arguments the way others might with a favourite novel, a breathtaking mountain or a moving symphony. As connoisseurs, philosophers hope for arguments with beauty, depth, simplicity – and a bit of mystery. When they find these together, they return again and again to the argument, hoping each time to learn a little more, to push it a bit farther. For all its simplicity, Newton's bucket illuminates the deepest issues in relativity theory and few can resist its allure.

The strategy of Newton's proof is to show that there are certain observable effects that *could only* be caused by absolute space. Newton begins disarmingly. Even though absolute space is not directly observable, he proposes to prove its existence with a bucket. Suppose, he says, that our wooden bucket is nearly full of water and is

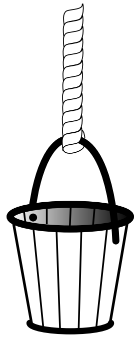

Figure 10.2 *Newton's famous bucket.*

suspended by a good, flexible rope from the tree branch overhead. Suppose we rotate the bucket and twist the rope up as far as it will go without tangling. If we now gently release the bucket, the calm, flat waters will at first remain still within the rotating bucket. As the rope unwinds and the bucket begins to whirl more quickly, the water will gradually be affected by the movement of the bucket and start to spin. As it does, the surface of the water will become *concave*: it will be lower in the middle as the water crowds outwards towards the sides of the bucket.

All this is straightforward, but Newton's genius now notices a subtlety. As the rope smoothly unwinds, the spinning concave water in the bucket will finally catch up with the rotating bucket. Soon they will *rotate at the same speed*. Newton saw here an argument for the existence of absolute space. (Do you?) The argument turns on two key facts: the surface of the rotating water is concave, and the bucket and water eventually rotate at the same speed. In this state, *there is no relative motion between the water and the bucket* – just as a child on a spinning merry-go-round is moving around along with the horses, and thus not moving relative to the horses. Newton's argument can be interpreted as follows:

Newton's bucket argument

A. The motion causes the concavity. (P)
B. Motion is either relative or absolute. (P)
C. Thus, either relative or absolute motion causes the concavity. (from A,B)
D. But relative motion does not cause the concavity. (P)

E.	So absolute motion causes the concavity.	(C,D)
F.	If something is a cause, then it exists.	(P)
G.	So absolute motion exists.	(E,F)
H.	If absolute motion exists, then absolute space exists.	(P)
I.	So absolute space exists.	(G,H)

Several of the assumptions here are straightforward. For example, motion causes the concavity, A, because the surface of the water is flat before the bucket is released and begins to turn. Newton actually leaps from E to I, but later debates have shown that it is important to fill in these steps.

Laying out the argument carefully like this, however, exposes the biggest assumption. Why does Newton think it is obvious that relative motion does not cause the concavity (premise D)? The Scholium defends this in one key sentence. The concavity persists when "the water rested relatively in the bucket", he says, "and therefore this does not depend upon any movement of the water relative to the ambient bodies". How does Newton make the connection between the water resting in the bucket and the independence from surrounding bodies? Much rests on the answer to this question. If Newton's assumption is sound, absolute space exists and Einstein was wrong.

Newton's idea is this. We know that the bucket itself did not cause the concavity because the water was in the bucket long before it became concave. We also know that the motion of the water *relative to the bucket* did not cause the concavity; this motion disappears when the rotating water catches up with and rests in the rotating bucket. What about the water's motion relative to other bodies? Newton thinks it would be preposterous to assert that the water's motion relative to the tree overhead, the grassy meadow or the distant twinkling stars caused the concavity. First, we could have hung the bucket on a different tree or in a different meadow and still found exactly the same concavity. The concavity does not depend on *which* distant bodies the water is moving relative to. Secondly, there are no means, no causal pathways, for other bodies to affect the water. How could the nearby stream cause the surface of the water in the bucket to curve? Thirdly, the water in the bucket is moving relatively to many, perhaps infinitely many, other bodies. It is moving around the Sun and through the galaxy; it is moving relative to passing birds and ships sinking off the coast of Madagascar. It is absurd to think that this crazy patchwork of relative motions might cause the concavity. These considerations are so obvious to Newton that he feels his assumption D is safe.

We will see in Chapter 15 that Einstein and his sometime mentor Ernst Mach strongly disagreed with Newton, and this led to extraordinary experiments testing Newton's assumption. Could the surrounding bodies have caused the concavity?

> Critics say absolute space is not observable, but Newton argued that its effects are observable. All things are observed only through their effects.

Leibniz's attack

If Newton was the greatest mind in Britain in the late-seventeenth century, W. G. Leibniz (1646–1716) was the greatest mind on the Continent. He is remembered as a philosopher, one of the last defenders of Aristotle's metaphysics, and as a scientist who played a key role in discovering the conservation of energy. He was co-inventor of the calculus and is given credit for inventing the binary number systems used in all our computers today. He was also a visionary logician, pushing forwards towards more rigorous approaches. But Newton and Leibniz hated each other. At a time when men in the upper classes still wore swords on formal occasions and fought duels to avenge any offence to their honour, Newton and Leibniz came as close as one could to coming to blows in a scholarly dispute.

Leibniz was a brilliant opportunist. He had a knack for turning to fashionable subjects and transforming them in unexpected and original ways. During his time at university, he rejected the Aristotelianism of his teachers and briefly became an atomist. But he finally found the atomists' idea of space too absurd and once again took up the idea of Aristotelian substances. He visited Paris in the mid 1670s and was introduced to avant-garde circles of thinkers and writers. At their high-society dinners for wealthy socialites and witty intellectuals, Leibniz quickly learned that the bright young lights in philosophy were studying mathematics. Descartes had died in 1650, but one of his students urged Leibniz to learn a little mathematics. Leibniz read a few essays, including some mathematical work of Newton's, returned home to scribble out some ideas of his own, and two years later invented the calculus.

Leibniz's stunning breakthrough infuriated Newton, who had fully developed calculus years before but was notoriously slow to disclose his ideas and had left them unpublished. He felt, however, that the short essay Leibniz had read contained enough hints, and that Leibniz had therefore plagiarized his ideas. This led to a great long-running controversy on both sides of the English Channel. The English rallied around Newton after a committee report from the new Royal Society in 1711 upheld his claims to priority, but Leibniz's defenders on the Continent were unpersuaded. (It turned out that Newton had secretly written the Royal Society report.)

This celebrated dispute made the second great clash between these titans even nastier. Many years after the publication of his system of calculus, Leibniz publicly attacked Newtonian ideas. He insinuated that Newton was a heretic whose doctrines were destroying religion. Leibniz then turned to the ideas about space and time expressed in the scholium, and levelled a blistering barrage of arguments against the absurdity of an infinite and invisible absolute space. In later years, these attacks made Leibniz a hero to many. He was correct that Newton and the rise of modern science would contribute to the decline of religion in Europe. Moreover, after Einstein overthrew Newton's absolute space in the early 1900s, Leibniz's reputation soared. It seemed he had anticipated Einstein's criticisms, and was simply 200 years ahead of his time.

Leibniz launched his attack because of his friendship with a princess. Leibniz was often at the Court of Berlin where Queen Sophia was his friend and student. There he met the young Princess Caroline, one of the most educated women in Europe, who played a role in leading controversies of the day. Caroline also became a student of Leibniz's and used her power and influence to spread his religious views. She married well. Her husband, a powerful German aristocrat who was the elector of Hanover, later became King George II of Great Britain and Ireland in 1727. From the time she first arrived in England, she again became a chief propagandist for Leibniz's views, as these edited extracts from her letters to Leibniz show:

> We are thinking very seriously of getting your book on theology translated into English. Dr Clarke is the most suitable but is too opposed to your opinions – he is too much of Sir Isaac Newton's opinion and I am myself engaged in a dispute with him. I implore your help. I can only ever believe what would conform to the perfection of God. I have found this much more perfect in your system than in that of Mr Newton, where in effect, God has to be always present to readjust his machine [the universe] because he was not able to do it at the beginning.

> I am in despair that persons of such great learning as you and Newton are not reconciled. The public would profit immensely if this could be brought about, but great men are like women, who never give up their lovers except with the utmost chagrin and mortal anger. And that, gentlemen, is where your opinions have got you.

Caroline went on to invite Leibniz to set out his differences with Newton in an open letter. In those days, magazines and academic journals were quite new and scholars often circulated their ideas as letters addressed to a prominent colleague. When Leibniz penned his courtly condemnation of Newton for Caroline, he knew all of Europe would soon be reading it.

Always shy of public controversy, Newton himself refused to reply to Leibniz. Instead, Newton's friend, disciple and sometime spokesman, the Anglican Bishop Samuel Clarke, accepted the Queen's request to defend Newton's ideas in the court of public opinion. Clarke and Leibniz each sent five alternating letters to Caroline. When they were collected and published as a book in 1717 it was a bestseller throughout Europe. Despite the undercurrent of acrimony, this Leibniz–Clarke correspondence is the most valuable discussion of the concept of space between the scholium and the time of Einstein.

Leibniz's vision of philosophy differed fundamentally from Newton's. For Leibniz, our metaphysics should be grounded in deep, clear and simple truths. The philosopher's task was to penetrate down to these fundamental principles and show that all other truths about the world flowed from them. Above all, Leibniz relied on reason, both to find his ideas and principles and to judge arguments based on them. As such, Leibniz is sometimes known as a defender of *rationalism*.

Newton's physical theories, however, were based first and foremost on experience and general patterns found in experience. Newton did not claim to understand the basic causes of these patterns and, indeed, emphasized the modesty of his achievement. He had found formulas that described and predicted experience but did not explain it. Newton desired to know the principles beneath these patterns but was satisfied with the limited (and glorious) progress his creations represented.

Einstein once quipped that most scientists accepted a theory when it was confirmed by experimental data, but that he never accepted the data until it was confirmed by theory. This was the spirit that Leibniz brought to his physics: distrust appearances until reason is satisfied. Leibniz's many arguments were thus designed to expose Newton's theories as philosophical nonsense, and therefore to demonstrate that they could not be fundamental, could not be a final theory. One theme of his attacks was that Newton's theories were incompatible with a now famous principle that Leibniz made the bedrock of his own philosophy. The idea is that "everything has an explanation, nothing is really left to chance":

> *Principle of sufficient reason:* For anything that is, there is a reason why it is so and not otherwise.

The key word here is "reason". Classical philosophers thought of reasons in broad terms. They included sentences, facts, conditions, causes and beliefs. Thus the reason for some flood was that the rains caused the dam to break, and the reason for a prejudice was that they believed in the stereotype. In this sense, reasons are aspects of our inner or outer worlds that lead on to other aspects.

A key distinction in philosophy is that between "necessary" and "sufficient". For example, a cause is necessary to its effect if the cause must be present for the effect to occur (although other factors may also contribute to the effect). A cause is sufficient for its effect if it is powerful enough to bring its effect about (although other causes may independently produce the same effect). The presence of oxygen is a necessary cause of an ordinary fire. Given the presence of oxygen, striking a match is merely a sufficient cause for a fire, because the fire could also have been started by a spark.

Leibniz claimed that there is some sufficient reason for everything that happens, and he built his own metaphysical theories on this principle. This is an assertion about the world. It says that the web of cause and effect is seamless. There are no uncaused events; there is no true chance or randomness. This principle is also an assertion of the world's intelligibility. Since we understand and explain events by discovering their reasons, Leibniz asserts that understanding is always possible: there is always a reason to discover.

It is surprising that such a basic and general principle could be used with deadly effect against Newton's absolute space. Leibniz noticed that the very existence of absolute space contradicted his principle. He agreed with Newton that, if it existed, such a space would be utterly smooth, uniform and homogeneous: the same in every place and in every direction. Newton had both conceptual and physical reasons for this commitment. Conceptually, Newton believed that space was nothing or at least not a body, and could have no properties. These belonged to substances, that is, to the contents of space. Physically, if inertia drove bodies smoothly forwards along straight lines for ever, there could be no bumps or rough patches in space to disturb them. Leibniz, however, saw that the uniformity of space created a problem.

Suppose that God grasped all the matter in the universe, that is, all the contents of infinite space, and shifted it around so that what lay in the east was now west and vice versa. If space was everywhere exactly the same, then this shift would produce no difference at all. Leibniz saw his opening there:

Symmetry argument against absolute space

A.	If absolute space is uniform, then there is no reason for the universe's orientation.	(P)
B.	Absolute space is uniform.	(P)
C.	Thus, there is no reason for the universe's orientation.	(from A,B)
D.	But, there is a reason for everything.	(P)
E.	Thus, there is a contradiction.	(between C,D)

Note that the argument itself does not depend on the existence of God. The last premise, D, is a restatement of the principle of sufficient reason.

For Leibniz, the source of the problem and its solution were clear. He believed that the concept of an absolute space was absurd. The argument just exposed a problem with absolute space, and was a reason to reject Newton's doctrine altogether. Thus Leibniz insisted that the existence of a uniform absolute space, premise B, must be denied.

Clarke and Newton were desperate. To save absolute space they had to call in God. They were impressed with the success of Newton's theories, and were determined to defend their uniform absolute space. Thus they insisted on the truth of the second premise. But they were also inclined to agree with the principle of sufficient reason. Like Leibniz, they were committed to the rational intelligibility of the world. Thus they accepted the last premise, and this meant that the first had to go. It was the only way to rebut Leibniz.

Clarke and Newton suggested that even in an entirely uniform space there would be a reason for the universe's orientation; namely, God's will. That is, they suggested that God could, in his wisdom, simply have decided to put some galaxies in the east of the universe and others in the west. This divine preference itself, they asserted, was the sufficient reason. It was the first premise in the argument, A, that should be thrown out to remove the contradiction. Leibniz was horrified at this manoeuvre. He countered that without a ground for choosing an orientation, God's decision was mere whim. Would God have been playing dice during the creation of the universe?

Today, physicists regard Leibniz as the winner of this argument. The technique that he used in the above argument has become one of the most common and powerful strategies of reasoning in advanced physics. Leibniz is honoured as a pioneer for recognizing the importance of this strategy (although the idea of shifting the entire universe was common in medieval debates over space).

Leibniz claimed, in modern terms, that Newton's theory "had a symmetry". Its predictions about the behaviour of bodies remained the same even if their position in absolute space were different. If all bodies were shifted in tandem to a different place ten metres to the left, their behaviour would be the same. The idea of position in absolute space was useless, Leibniz concluded, and should be abandoned.

As we have seen, Einstein practically based his career on arguments about symmetry. He showed physicists that they were important tools for constructing new theories. They could be used to test which elements of a theory were doing work and which were merely excess baggage. Suppose that a theory includes names for several variable quantities (names like "length", "mass", etc.), and that when a theory is used numbers are substituted for these variables (the mass is 3 kg, etc.). In some cases, we may discover that the predictions of a theory remain the same when different numbers are substituted for some particular variable. Einstein said that such a symmetry meant that the variable was useless, was "superfluous structure", and should be cut from the theory. Thus, hunting for symmetries became a useful tool for identifying the useless bits in a theory (such as absolute position or absolute velocity).

More interestingly, symmetries could be used to find new theories. Suppose physicists knew that a variable in an incomplete theory had no effect on its predictions. As they extended the theory to cover more cases, the meaningless variable should remain meaningless. Thus symmetries restrict the ways theories can be extended, they narrow the choices available. As such, they throw a powerful searchlight into the infinite realm of possible theories, and have even led physicists to stumble towards the correct theory. The standard model, the deepest theory of matter at present, was discovered by Glashow, Salam and Weinberg using just these kinds of symmetry arguments, and won them the Nobel prize in 1979.

> Leibniz's principle of sufficient reason says, as Parmenides did, that the world is ultimately intelligible: "reasons" are understandable causes.

Leibniz's alternative vision: the monadology

Leibniz is a hero to those physicists and philosophers who celebrate his rejection of absolute space. But what positive, alternative theory did Leibniz offer? What was his conception of space and how did it evade the criticisms he levelled against Newton? Surprisingly, Leibniz saw so deeply into the perplexing nature of space and time that he violently rejected their existence altogether. For him, space and time were sorts of illusions staged by God: mere phantasms within our souls. His own metaphysics, therefore, ranks among the most bizarre ever defended by a major philosopher.

The key evolution in Leibniz's thought began during his 1672–76 trip to Paris. There he discussed the ontology of relations with other leading philosophers and read Plato's dialogue *The Phaedo*. His surviving notebooks reveal his struggles with the relational paradoxes. Soon after returning to Germany, he laid the foundations for his mature metaphysics, the famous *Monadology*. The key feature of this system was his denial of relations between and outside substances. What does this mean?

For Newton any two places in space were linked by an intervening stretch of space, and the distance between the places was just the length of the space between them. Metaphorically speaking, this stretch of space forms a "bridge" between the places: it is real, outside the places and links them together. As above, call this kind of relation, which has a real existence outside its relata but partially overlaps them, a "real relation". As Leibniz grew to feel the urgency of the problem of the one and the many, he found the idea of bridge-like, real relations absurd and was driven back to a pure substance ontology without any relations at all. Like Aristotle, Leibniz concluded that only substances and their properties existed. In a well-known passage in his fifth letter to Clarke and Newton, he poked fun at the very idea of relations spanning the gaps between substances:

> It cannot be said that both of them, the two relata together, are the subject of a single relation; for if so, we should have a relation in two substances, with one leg in one, and the other in the other; which is contrary to the notion of all properties inhering in substances. Therefore we must say that this relation is indeed out of the substances; but being neither a substance, nor a property, it must be unreal, a merely mental thing, the consideration of which is nevertheless useful.

Thus Leibniz's deeper objection to absolute space is that it is incompatible with a substance ontology. As a network of bridge-like real relations, space is neither a body nor a property of a body.

Leibniz penetrated more deeply than Aristotle into the implications of a substance ontology. Aristotle rejected the space of the atomists, and argued for the existence of a plenum. Leibniz saw clearly that the plenum too was problematic. Aristotle relied on common sense. It was obvious that things touched each other, rested next to each other, and bumped into each other. Likewise, it was obvious to Aristotle that there could be no gaps between things, no stretches of existent nothingness. Thus there was a plenum: a close-knit world of substances nestled in next to each other. Crucially, Leibniz saw that touching and adjacency were *relations*. Suppose that Jack and Jill exist, and in addition they touch. What makes this additional fact true? What is its truth-maker? It could not be Jack and Jill, because they also exist without touching. So touching must be something additional, some sort of relation over and above its relata. But it is neither a substance nor a property. Thus Leibniz concluded, there is no touching.

Leibniz's saw clearly the implications of excluding all real relations from his metaphysics. His substances were lonely and utterly isolated. Each probably contained a soul but was, for Leibniz, completely simple and contained no inner relations and, in fact, no inner differences or distinctions at all. Moreover, each was in effect a tiny universe. There was nothing outside each substance: no empty space, no plenum, no "nothing". Leibniz's substances were not "together" in any sense. They did not reside in the same space, they were not near or far from each other, and they could not influence each other at all. There were no real relations at all.

To reconcile this stark vision with appearances was difficult. We feel that we move through the world, and feel that other things touch us. Leibniz recognized this, of course, and explained that each ensouled substance had a series of images unfolding within it – as if, perhaps, it had a film playing in its mind. These inner "phenomena" contained impressions of touching, movement and causation, but these were, for Leibniz, simply illusions of some sort. Their only reality was as perceptions within the substance. Just as Aristotle thought that substances could seamlessly unify their actual and potential properties, Leibniz asserted that these shifting phenomena within his substances did not compromise their unity. His substances were changing in some sense but nonetheless utterly simple and the same. For both Aristotle and Leibniz, this ability to be both one and many was perhaps the most important characteristic of a substance.

Strangely, Leibniz did accept that there was some coordination between substances. When one of us talks, another would hear. This was not, however, causation. Rather he believed that God has so arranged the "film" playing within each substance that there was a correlation between causes and effects. Continuing the metaphor, this is as if, in a multiplex cinema, a gun was fired in one film and in the theatre next door a gangster fell groaning to the ground. God created "films" that were synchronized. This is Leibniz's famous doctrine of *pre-established harmony*. It is an attempt to reconcile the appearance of cause and effect with the absence of real causal relations. Leibniz thought this was a virtue of his system. In effect, he had argued that, since there were no real relations, there must be a God to pre-arrange all causes and effects. He had proved the existence of God in a new way. Other philosophers have thought this doctrine a desperate and absurd attempt to save his system.

We can learn from Leibniz's metaphysics a philosophical lesson that is extremely important today. The central mystery for us is how a great mind could have found all this plausible? How could a mathematician and scientist deny the reality of the physical world we perceive around us?

Leibniz found his metaphysics plausible because he was horrified by the problem of the one and the many. Any real relation must somehow unite its many relata, and therefore harbours a tension between unity and diversity. Leibniz saw no way to escape contradiction, and banished real relations altogether. Thus spatial, temporal and causal relations all disappear from his metaphysics, and God must be summoned to rig together appearances.

Leibniz was a deeper, and more clear-sighted philosopher than Newton. Leibniz saw difficulties that Newton was happy to brush aside as he constructed his theories. The success of Newton's theories bludgeoned later philosophers into accepting his views on space and time, and overlooking the ontological difficulties that so bothered critics like Leibniz. As we shall see below, these difficulties have been resurrected.

The monadology was the last great gasp of substance ontologies in the philosophy of space and time. Aristotle's core concept of substance continued to be influential for some time after Leibniz, and even survived until the 1800s, but was gradually expelled from mainstream science. For many philosophers, the concept of substance has become a symbol of the medieval scholastic philosophy that preceded modern science: a symbol of obscurity and intellectual stagnation. Leibniz's insight that the concept of substance was needed to suppress the paradoxes of change and the problem of the one and the many was largely lost.

As Leibniz famously put it, the monads had "no windows": there was no outside for them to look out upon, there was not even nothing to relate them.

The mathematics of change

> In retrospect, whereas the idea of change and variability had been banned from Greek mathematics because it led to Zeno's paradoxes, it was precisely this concept which, revived in the later Middle Ages and represented geometrically, led in the seventeenth century to the calculus . . . The objections raised in the eighteenth century to the calculus were in large measure unanswered in terms of the conceptions of the time. Their arguments were in the last analysis equivalent to those which Zeno had raised well over two thousand years previously and were based on questions of infinity and continuity.
>
> (C. Boyer, historian of mathematics)

Mathematics and geometry were long thought to deal with static, unchanging and eternal structures – the forms that are ultimate reality. In Plato's dialogues, geometry is held up as an ideal of eternal and divine knowledge. Although some geometers did study the curves traced out by moving objects (spirals, etc.), generally speaking mathematics could not handle continuous change until Newton's invention of the calculus in 1666. His groundbreaking essay was called "To Resolve Problems by Motion These Following Propositions are Sufficient". The success of Newton's physics has persuaded many that his calculus had somehow resolved Zeno's paradoxes, or at least shown that they were unimportant. This section briefly discusses why that is an error.

The Greeks did not have practical problems with the way they *talked* about motion. They could say that a journey by chariot took three hours and covered ten kilometres; they could agree to meet in the marketplace at noon. Their problems were with *understanding* change, with creating a contradiction-free theory of change. Remember that Zeno argued that we could not walk to a wall because we could not traverse the infinity of intervening points. It is not an adequate solution to insist "But we do reach the wall!" Zeno might agree with this; the problem is to explain or understand how we reach the wall.

In a sense, Newton invented a thoroughly practical way to describe continuous change in a mathematical language. He enabled mathematics to make successful predictions of astronomical and terrestrial motions. Crudely put, however, Newton accomplished this just by making the assumption "But we do reach the wall!"

At the core of Newton's calculus is the notion of a *limit*. This is the idea that, if we add together a half, and a quarter, an eighth and so on to infinity, we will exactly *reach* the sum of one. Zeno would say that the addition will never total to one because an infinity of numbers would have to be added, and no infinity can ever be completed. But Newton's calculus simply assumes that the series does add to one exactly. It assumes that we do reach the wall. The Greek philosophers did not have practical problems with talking about change, and Newton effectively taught mathematics to talk in a practical way about change and continuity. He does not solve Zeno's theoretical problems with change. Newton just assumes the truth of what Zeno questioned.

A *pragmatist* is a philosopher who believes that only practice matters, and might therefore argue that Newton had solved Zeno's paradoxes by showing that they had no practical consequences. But this argument has two premises in need of defence: pragmatism itself, and the claim that the paradoxes will not rear their head in practical ways in a physics deeper than Newton's.

> The calculus did not solve, but rather suppressed, Zeno's paradoxes.