CHAPTER 3

Reinvestment of Returns and Geometric Growth Concepts

TO REINVEST TRADING PROFITS OR NOT

Let's call the following system "System A." In it we have two trades—the first making 50%, the second losing 40%. Therefore, if we do not reinvest our returns, we make 10%. If we do reinvest, the same sequence of trades loses 10%.

System A

	No Reinvestment		With Reinvestment	
Trade No.	**P&L**	**Accum.**	**P&L**	**Accum.**
		100		100
1	50	150	50	150
2	−40	110	−60	90

Now let's look at System B, a gain of 15% and a loss of 5%, which also nets out 10% over two trades on a nonreinvestment basis, just like System A. But look at the results of System B with reinvestment. Unlike System A, it makes money.

System B

	No Reinvestment		With Reinvestment	
Trade No.	**P&L**	**Accum.**	**P&L**	**Accum.**
		100		100
1	15	115	15	115
2	−5	110	−5.75	109.25

An important characteristic of trading with reinvestment that must be realized is that *reinvesting trading profits can turn a winning system into a losing system but not vice versa!* A winning system is turned into a losing system in trading with reinvestment if the returns are not consistent enough. Further, *changing the order or sequence of trades does not affect the final outcome*. This is not only true on a nonreinvestment basis, but also true on a reinvestment basis (contrary to most people's misconception).

System A

	No Reinvestment		With Reinvestment	
Trade No.	**P&L**	**Accum.**	**P&L**	**Accum.**
		100		100
1	−40	60	−40	60
2	50	110	30	90

System B

	No Reinvestment		With Reinvestment	
Trade No.	**P&L**	**Accum.**	**P&L**	**Accum.**
		100		100
1	−5	95	−5	95
2	15	110	14.25	109.25

This is not just an aberration caused by a two-trade example. Let's take system A and add two more trades and then examine the results under all four possible sequences of trades.

First Sequence
(System A)

	No Reinvestment		With Reinvestment	
Trade No.	**P&L**	**Accum.**	**P&L**	**Accum.**
		100		100
1	−40	60	−40	60
2	50	110	30	90
3	−40	70	−36	54
4	50	120	27	81

Second Sequence
(System A)

	No Reinvestment		With Reinvestment	
Trade No.	**P&L**	**Accum.**	**P&L**	**Accum.**
		100		100
1	50	150	50	150
2	−40	110	−60	90
3	50	160	45	135
4	−40	120	−54	81

Third Sequence
(System A)

	No Reinvestment		With Reinvestment	
Trade No.	**P&L**	**Accum.**	**P&L**	**Accum.**
		100		100
1	50	150	50	150
2	50	200	75	225
3	−40	160	−90	135
4	−40	120	−54	81

Fourth Sequence
(System A)

	No Reinvestment		With Reinvestment	
Trade No.	**P&L**	**Accum.**	**P&L**	**Accum.**
		100		100
1	−40	60	−40	60
2	−40	20	−24	36
3	50	70	18	54
4	50	120	27	81

As can obviously be seen, the sequence of trades has no bearing on the final outcome, whether viewed on a reinvestment or nonreinvestment basis. What *are* affected, however, are the drawdowns. Listed next are the drawdowns to each of the sequences of trades listed above.

First Sequence	
No Reinvestment	Reinvestment
100 to 60 = 40 (40%)	100 to 54 = 46 (46%)

Second Sequence	
No Reinvestment	Reinvestment
150 to 110 = 40 (27%)	150 to 81 = 69 (46%)

Third Sequence	
No Reinvestment	Reinvestment
200 to 120 = 80 (40%)	225 to 81 = 144 (64%)

Fourth Sequence	
No Reinvestment	Reinvestment
100 to 20 = 80 (80%)	100 to 36 = 64 (64%)

Reinvestment trading is never the best based on absolute drawdown. One side benefit to trading on a reinvestment basis is that the drawdowns tend to be buffered. As a system goes into and through a drawdown period, each losing trade is followed by a trade with fewer and fewer contracts. That is why drawdowns as a percent of account equity are always less with reinvestment than with a nonreinvestment approach.

By inspection it would seem you are better off to trade on a nonreinvestment basis rather than to reinvest. This would seem so, since your probability of winning is greater. However, this is not a valid assumption, because in the real world we do not withdraw all of our profits and make up all of our losses by depositing new cash into an account. Further, the nature of investment or trading is predicated upon the effects of compounding. If we do away with compounding (as in the nonreinvestment plan), we can plan on doing little better in the future than we can today, no matter how successful our trading is between now and then. It is compounding that takes the linear function of account growth and makes it a geometric function.

Refer back to the statement that under a reinvestment plan a winning system can be turned into a losing system but not vice versa. Why, then, reinvest our profits into our trading? The sole reason is that by reinvestment, winning systems can be made to win far more than could ever be accomplished on a nonreinvestment basis.

The reader may still be inclined to prefer the nonreinvestment approach since an account that may not be profitable on a reinvestment basis may be profitable on a nonreinvestment basis. However, if a system is good enough, the profits generated on a reinvestment basis will be far greater than on a nonreinvestment basis, and that gap will widen as time goes by. If you have a system that can beat the market, it doesn't make any sense to trade it any other way than to increase your amount wagered as your stake increases.

There is another phenomenon that lures traders away from reinvestment-based trading. That phenomenon is that a losing trade, or losing streak, is inevitable after a prolonged equity run-up. This is true by inspection. The only way a streak of winning trades can end is by a losing trade. The only way a streak of profitable months can end is with a losing month. The problem with reinvestment-based trading is that when the inevitable losses come along you will have the most contracts on. Hence, the losses will be bigger. Similarly, after a losing streak, the reinvestment-basis trader will have fewer contracts on when the inevitable win comes along to break the streak.

This is not to say that there is any statistical reason to assume that winning streaks portend losing trades or vice versa. Rather, what is meant is: If you trade long enough, you will eventually encounter a loss. If you are trading on a reinvestment basis, that loss will be magnified, since, as a result of your winning streak, you will have more contracts on when the loss comes. Unfortunately, there is no way to avoid this—at least no way based on statistical fact in a stationary distribution environment, unless we are talking about a dependent trials process.

Therefore, assuming that the market system in question generates independent trades, there is no way to avoid this phenomenon. It is unavoidable when trading on a reinvestment basis, just as losses are unavoidable in trading under any basis. Losses are a part of the game. Since the goal of good money management is to exploit a profitable system to its fullest potential, the intelligent trader should realize that this phenomenon is part of the game and accept it as such to achieve the longer-term rewards that correct money-management techniques provide for.

MEASURING A GOOD SYSTEM FOR REINVESTMENT—THE GEOMETRIC MEAN

So far we have seen how a system can be sabotaged by not being consistent enough from trade to trade. Does this mean we should close up and put our money in the bank? Let's go back to System A, with its first two trades. For the sake of illustration we are going to add two winners of one point each.

System A

	No Reinvestment		With Reinvestment	
Trade No.	P&L	Accum.	P&L	Accum.
		100		100
1	50	150	50	150
2	−40	110	−60	90
3	1	111	0.9	90.9
4	1	112	0.909	91.809
Percent Wins		0.75		0.75
Avg. Trade		3		−2.04775
Profit Factor		1.3		0.86
Std. Dev.		31.88		39.00
Avg. Trade/Std. Dev.		0.09		−0.05

Now let's take System B and add two more losers of one point each.

System B

	No Reinvestment		With Reinvestment	
Trade No.	P&L	Accum.	P&L	Accum.
		100		100
1	15	115	15	115
2	−5	110	−5.75	109.25
3	−1	109	−1.0925	108.1575
4	−1	108	−1.08157	107.0759
Percent Wins		0.25		0.25
Avg. Trade		2		1.768981
Profit Factor		2.14		1.89
Std Dev.		7.68		7.87
Avg. Trade/Std. Dev.		0.26		0.22

Now, if consistency is what we're really after, let's look at a bank account, the perfectly consistent vehicle (relative to trading), paying 1 % per period. We'll call this series System C.

Notice that in reinvestment the standard deviation always goes up (and hence the Avg. Trade/Std. Dev. tends to come down). Furthermore, the Profit Factor[1] measure is never higher in reinvestment than it is in non-reinvestment trading.

[1]Profit Factor = Avg Win/Avg Loss × Percent Winners/(1-Percent Winners). (3.01)

System C

	No Reinvestment		With Reinvestment	
Trade No.	**P&L**	**Accum.**	**P&L**	**Accum.**
		100		100
1	1	101	1	101
2	1	102	1.01	102.01
3	1	103	1.0201	103.0301
4	1	104	1.030301	104.0604
Percent Wins		1.00		1.00
Avg. Trade		1		1.015100
Profit Factor		Infinite		Infinite
Std. Dev.		0.00		0.01
Avg. Trade/Std. Dev.		Infinite		89.89

Our aim is to maximize our profits under reinvestment trading. With that as the goal, we can see that our best reinvestment sequence came from System B. How can we have known that, given only information regarding nonreinvestment trading? By percent of winning trades? By total dollars? Average trade? The answer to these questions is no, since that would have us trading System A (but this is the solution most futures traders opt for). What if we opted for most consistency (i.e., highest ratio of Avg. Trade/Std. Dev. or lowest standard deviation). How about highest profit factor or lowest drawdown? This is not the answer, either. If it were, we should put our money in the bank and forget about trading.

System B has the right mix of profitability and consistency. Systems A and C do not. That is why System B performs the best under reinvestment trading. How best to measure this "right mix"? It turns out there is a formula that will do just that: the *geometric mean*. This is simply the Nth root of the Terminal Wealth Relative (TWR), where N is the number of periods (trades). The TWR is simply what we've been computing when we figure what the final cumulative amount is under reinvestment. In other words, the TWRs for the three systems we just saw are:

SYSTEM	**TWR**
System A	91.809
System B	107.0759
System C	104.0604

Since there are four trades in each of these, we take the TWRs to the fourth root to obtain the geometric mean:

SYSTEM	GEO. MEAN
System A	0.978861
System B	1.017238
System C	1.009999

$$\text{TWR} = \prod_{i=1}^{N} \text{HPR}_i \tag{3.02}$$

$$\text{Geometric Mean} = \text{TWR}^{1/N} \tag{3.03}$$

where: N = Total number of trades.
HPR = Holding period returns (equal to 1 plus the rate of return).

For example, an HPR of 1.10 means a 10% return over a given period/bet/trade. TWR shows the number of dollars of value at the end of a run of periods/bets/trades per dollar of initial investment, assuming gains and losses are allowed to compound. Here is another way of expressing these variables:

TWR = Final stake / Starting stake

Geometric Mean = Your growth factor per play, or
Final stake / starting stake)$^{1/\text{number of plays}}$.

or

$$\text{Geometric Mean} = \exp((1/N) * \log(\text{TWR})) \tag{3.03a}$$

where: N = Total number of trades.
log(TWR) = The log base 10 of the TWR.
exp = The exponential function.

Think of the geometric mean as the "growth factor" of your stake, per play. The system or market with the highest geometric mean is the system or market with the highest utility to the trader trading on a reinvestment of returns basis. A geometric mean < 1 means that the system would have lost money if you were trading it on a reinvestment basis. Furthermore, it is vitally important that you use realistic slippage and commissions in calculating geometric means in order to have realistic results.

ESTIMATING THE GEOMETRIC MEAN

There exists a simple technique of finding the geometric mean, whereby you do not have to take the product of all HPRs to the Nth root. The geometric mean squared can be very closely approximated as the arithmetic mean of the HPRs squared minus the population standard deviation of HPRs squared. So the way to approximate the geometric mean is to square the average HPR, then subtract the squared population standard deviation of those HPRs. Now take the square root of this answer and that will be a very close approximation of the actual geometric mean. As an example, assume the following HPRs over four trades:

	1.00
	1.50
	1.00
	.60
Arithmetic Mean	1.025
Population Standard Deviation	.3191786334
Estimated Geometric Mean	.9740379869
Actual Geometric Mean	.9740037464

Here is the formula for finding the estimated geometric mean (EGM):

$$\text{EGM} = \sqrt{\text{Arithmetic Mean}^2 - \text{Pop. Std. Dev.}^2} \tag{3.04}$$

The formula given in Chapter 1 to find the standard deviation of a Normal Probability Function is not what you use here. If you already know how to find a standard deviation, skip this section and go on to the next section entitled "How Best to Reinvest."

The standard deviation is simply the square root of the variance:

$$\text{Variance} = (1/(N-1)) \sum_{i=1}^{N} (X_i - \bar{X})^2$$

where: $\bar{X}$ = The average of the data points.
X_i = The i'th data point.
N = The total number of data points.

This will give you what is called the *sample* variance. To find what is called the *population* variance you simply substitute the term (N − 1) with (N).

Notice that if we take the square root of the sample variance, we obtain the sample standard deviation. If we take the square root of the population variance, we will obtain the population standard deviation. Now, let's run through an example using the four data points:

$$\begin{array}{r} 1.00 \\ 1.50 \\ 1.00 \\ .60 \end{array}$$

1. Find the average of the data points. In our example this is:

$$\begin{aligned} \bar{X} &= (1.00 + 1.50 + 1.00 + .6)/4 \\ &= 4.1/4 \\ &= 1.025 \end{aligned}$$

2. For each data point, take the difference between that data point and the answer just found in step 1 (the average). For our example this would be:

$$\begin{aligned} 1.00 - 1.025 &= -.025 \\ 1.50 - 1.025 &= .475 \\ 1.00 - 1.025 &= -.025 \\ .60 - 1.025 &= -.425 \end{aligned}$$

3. Square each answer just found in step 2. Note that this will make all answers positive numbers:

$$\begin{aligned} -.025 * -.025 &= .000625 \\ .475 * .475 &= .225625 \\ -.025 * -.025 &= .000625 \\ -.425 * -.425 &= .180625 \end{aligned}$$

4. Sum up all of the answers found in step 3. For our example:

$$\begin{array}{r} .000625 \\ .225625 \\ .000625 \\ +.180625 \\ \hline .4075 \end{array}$$

5. Multiply the answer just found in step 4 by (1/N). If we were looking to find the sample variance, we would multiply the answer just found in step 4 by (1/(N − 1)). Since we eventually want to find the population

standard deviation of these four HPRs to find the estimated geometric mean, we will therefore multiply our answer to step 4 by the quantity (1/N).

$$\begin{aligned}\text{Population Variance} &= (1/\text{N}) * (.4075)\\ &= (1/4) * (.4075)\\ &= .25 * .4075\\ &= .101875\end{aligned}$$

6. To go from variance to standard deviation, take the square root of the answer just found in step 5. For our example:

$$\begin{aligned}\text{Population Standard Deviation} &= \sqrt{.101875}\\ &= .3191786334\end{aligned}$$

Now, let's suppose we want to figure our estimated geometric mean for our example:

$$\begin{aligned}\text{EGM} &= \sqrt{\text{Arithmetic Mean}^2 - \text{Pop. Std. Dev.}^2}\\ &= \sqrt{1.025^2 - .3191786334^2}\\ &= \sqrt{1.050625 - .101875}\\ &= \sqrt{.94875}\\ &= .9740379869\end{aligned}$$

This compares to the actual geometric mean for our example data set of:

$$\begin{aligned}\text{Geometric Mean} &= \sqrt[4]{1.00 * 1.50 * 1.00 * .6}\\ &= \sqrt[4]{.9}\\ &= .9740037464\end{aligned}$$

As you can see, the estimated geometric mean is very close to the actual geometric mean—so close, in fact, that we can use the two interchangeably throughout the text.

HOW BEST TO REINVEST

Thus far, we have discussed reinvestment of returns in trading whereby we reinvest 100% of our stake on all occasions. Although we know that in order to maximize a potentially profitable situation we must use reinvestment, a 100% reinvestment is rarely the wisest thing to do.

Take the case of a coin toss. Someone is willing to pay you \$2 if you win the toss, but will charge you \$1 if you lose. You can figure what you should make, on average, per toss by the mathematical expectation formula:

$$\text{Mathematical Expectation} = \sum_{i=1}^{N} (P_i * A_i)$$

where: P = Probability of winning or losing.
A = Amount won or lost.
N = Number of possible outcomes.

In the given example of the coin toss:

$$\begin{aligned}\text{Mathematical Expectation} &= (2 * .5) + (1 * (-.5)) \\ &= 1 - .5 \\ &= .5\end{aligned}$$

In other words, you would expect to make 50 cents per toss, on average. This is true of the first toss and all subsequent tosses, provided you do not step up the amount you are wagering. But in an independent trials process, that is exactly what you should do. As you win, you should commit more and more to each trade.

At this point it is important to realize the keystone rule to money-management systems, which states: *In an independent trials process, if the mathematical expectation is less than or equal to 0, no money-management technique, betting scheme, or progression can turn it into a positive expectation game.*

This rule is applicable to trading one market system only. When you begin trading more than one market system, you step into a strange environment where it is possible to include a market system with a negative mathematical expectation as one of the market being traded, and actually have a net mathematical expectation higher than the net mathematical expectation of the group before the inclusion of the negative expectation system! Further, it is possible that the net mathematical expectation for the group with the inclusion of the negative mathematical expectation market system can be higher than the mathematical expectation of any of the individual market systems!

For the time being, we will consider only one market system at a time, and therefore we must have a positive mathematical expectation in order for the money-management techniques to work.

Refer again to the two-to-one coin-toss example (which is a positive mathematical expectation game). Suppose you begin with an initial stake of $1. Now suppose you win the first toss and are paid $2. Since you had your entire stake ($1) riding on the last bet, you bet your entire stake ($3 now) on the next toss as well. However, this next toss is a loser and your entire $3 stake is gone. You have lost your original $1 plus the $2 you had won. If you had won the last toss, it would have paid you $6, since you had three full $1 bets on it. The point is that if you are betting 100% of your stake, then as soon as you encounter a losing wager (an inevitable event), you'll be wiped out.

If you were to replay the previous scenario and bet on a non-reinvestment basis (i.e., a constant bet size) you would make $2 on the first bet and lose $1 on the second. You would now be ahead $1 and have a total stake of $2. Somewhere between these two scenarios lies the optimal betting approach.

Now, consider four desirable properties of a money-management strategy. First, you want to make as much as mathematically possible, given a favorable game. Second, the trade-off between the potential rate of growth of your stake and its security should be considered as well (this may not be possible given the first property, but it should at least be considered.[2] Third, the likelihood of winning should be taken into consideration. Fourth and finally, the amounts you can win and the amounts you can lose should influence the bet size as well. If you know you have an edge over N bets, but you do not know which of those N bets will be winners, or for how much, and which will be losers, and for how much, you are best off (in the long run) treating each bet exactly the same in terms of what percentage of your total stake is at risk.

Let's go back to the coin toss. Suppose we have an initial stake of $2. We toss a coin three times; twice it comes up heads (whereby we win $1 per $1 bet) and once it comes up tails (whereby we lose $1 per every $1 bet). Also, assume this coin is flawed in that it always comes up heads two out of three times and comes up tails one out of three times. Let's further say that this flawed coin can never come up HHH or TTT on any three-toss sequence. Since we know that this coin is flawed in these ways, but do not know where that loss will come in, how can we maximize this situation? The three possible exact sequences (the sample space), because of the flaws, are:

H	H	T
H	T	H
T	H	H

[2]In the final sections of the text, where we look at real world implementation, this vital caveat is addressed.

Here is our dilemma: We know we will win 66% of the time, but we do not know when we will lose, and we want to maximize what we make out of this situation.

Suppose now that rather than bet an equal fraction of our stake—which optimally is one-third of our stake on each bet (more on how to calculate this later)—we arbitrarily bet \$2 on the first bet and \$1 on each bet thereafter. Our \$2 stake would grow to \$4 at the end of both the HHT and the HTH sequences. However, for the THH sequence we would have been tapped out on the first bet. Since there are three exact sequences, and two of them resulted in profits of \$2 and one resulted in a complete loss, we can say that the sum of all sequences was \$4 gained $(2 + 2 + 0)$. The average sequence was therefore a gain of \$1.33 (4/3).

You can try any other combination like this for yourself. Ultimately, you will find that, since you do not know where the loss is due to crop up, you are best to bet the same fraction of your stake on each bet. Optimally, this fraction is one-third, or 33%, whereby you would make a profit of about \$1.41 on each sequence, regardless of sequence(!), for a sum of all sequences of \$4.23 gained $(1.41 + 1.41 + 1.41)$. The average sequence was therefore a gain of \$1.41 (4.23/3).

Many "staking"systems have been created by gamblers throughout history. One, the martingale, has you double your bet after each loss until ultimately, when a win does occur, you are ahead by one unit. However, the martingale strategy can have you making enormous bets during a losing streak. On the surface, this would appear to be the ultimate betting progression, as you will always come out ahead by one unit if you can follow the progression to termination. Of course, if you have a positive mathematical expectation, there is no need to use a scheme such as this. Yet it seems this should work for an even-money game as well as for a game where you have a small negative expectancy.

Yet, as we saw in Chapter 1, the sum of a series of negative expectancy bets must be a negative expectation. Suppose you are betting à la martingale. You lose the first 10 bets in succession. Going into the eleventh bet, you are now betting 1,024 units. The probabilities of winning are again the same as if you were betting one unit (assuming an independent trials process). Your mathematical expectation therefore, as a percentage, is the same as in the first bet, but in terms of units it is 1,024 times greater than the first bet. If you are betting with any kind of a negative expectation, it is now multiplied 1,024 times over.

"It doesn't matter," you, the martingale bettor, reply, "since I'll just double up for the twelfth bet if I lose the eleventh, and eventually I will come out ahead one unit." What eventually stymies the martingale bettor is a ceiling on the amount that may be bet, either by a house limit or inadequate capital to continue the progression on the bettor's part.

Theoretically, if you are gambling in a situation with no house limit, it would seem you could work this progression successfully if you had unlimited capital. Yet who has unlimited capital?

Ultimately, the martingale bettor has a maximum bet size, imposed by either the house (as in casino gambling) or his capitalization (as in the markets). Eventually, the bettor will bet and lose this maximum bet size and thus go bust. Furthermore, this will happen regardless of mathematical expectation—that is why the martingale is completely foolish if you have a positive mathematical expectation, and just futile if you have an even-money game or a negative expectation. True, betting à la martingale you will most often walk away from the tables a winner. However, when you lose, it will be for an amount that will more than compensate the casino for letting you walk away a winner the vast majority of the time.

It is not the maximum bet size that stymies the martingale as much as it is the number of bets required to reach the maximum bet size (this is also one of the reasons why there are house minimums). To overcome this, gamblers have tried what is known as the small martingale—a somewhat watered-down version of the martingale.

The small martingale tries to provide survival for the bettor by increasing the number of bets required to reach the maximum bet size. Ultimately, the small martingale tries to win one unit per cycle. Since the system rules are easier to demonstrate than to describe, I will show this system through the use of examples. In the small martingale you keep track of a "progression list," and bet the amount that is the sum of the first and last values on the list. When a win is encountered, you cross off the first and last values on the list, thus obtaining new first and last values, giving you a new amount to wager on the next bet. The list starts at simply the number 1. When a loss is encountered, the next number is added on to the end of the list (i.e., 2, 3, 4, etc.). A cycle ends when one unit is won. If a list is ever composed of just the number 2, then convert it to a list of 1, 1. The following examples of four different cycles should make the progression clear:

Bet Number	List	Bet Size	Win/Loss
1	1	1	W

Bet Number	List	Bet Size	Win/Loss
1	1	1	L
2	1, 1	2	W

Bet Number	List	Bet Size	Win/Loss
1	1	1	L
2	1, 1	2	L
3	1, 1, 2	3	W
4	1	1	W

Bet Number	List	Bet Size	Win/Loss
1	1	1	L
2	1, 1	2	L
3	1, 1, 2	3	L
4	1, 1, 2, 3	4	W
5	1, 2	3	L
6	1, 2, 3	4	W
7	1, 1	2	L
8	1, 1, 2	...and continuing until the bettor is ahead by 1 unit.	

The small martingale is ultimately a loser, too, for the same reasons that the martingale system is a loser. A sum of negative expectancy bets must have a negative expectancy.

Another system, the antimartingale, is just the opposite of the martingale (as its name implies). Here, you increase your bet size after each win. The idea is to hit a streak of winning bets and maximize the gains from that streak. Of course, just as the martingale ultimately makes one unit, the antimartingale will ultimately lose all of its units (even in a positive mathematical expectation game) once a loss is incurred, if 100% of the stake is placed on each bet.

Notice, however, that fixed fractional trading is actually a small antimartingale! Recall our flawed-coin example earlier in this chapter. In that example we saw how our "best" strategy was the small antimartingale. In the final analysis, fixed fractional trading, the small antimartingale, is the optimal betting system—provided you have a positive mathematical expectation.[3]

Another famous system is the reserve strategy. Here, you trade a base bet plus a fraction of your winnings. However, in the reserve strategy, if the last bet was a winner, then you bet the same amount on the next bet as you did the last. Suppose you encounter the sequence of win \$1, win \$1, lose \$1, then win \$1 for every \$1 bet. If you are betting \$1 plus 50% of winnings (in the reserve strategy), you would bet \$1 on the first bet. Since it was a winner, you would still bet \$1 on the second bet—which was also a winner, boosting your total winnings to \$2. Since the second bet was also a winner, you would not increase your third bet; rather, you would still bet \$1. The

[3]This is critical. Optimal fixed fractional trading therefore possesses those characteristics, pro and con, of the small antimartingale. It *will* maximize growth, *but* it will cause you to endure severe and protracted drawdowns. Just as the martingale strategy has you leave the tables a winner most of the time, the antimartingale, and to a lesser extent, the small antimartingale (i.e., "fixed fractional trading") has you leave the tables—or a performance period—a loser more frequently than you would have betting on a constant-bet-size or martingale basis.

third bet, being a loss of $1, lowers your total winnings to $1. Since you encountered a loss, however, you recapitalize your bet size to the base bet ($1) plus 50% of your winnings (.5 * $1) and hence bet $1.50 on the fourth bet. The fourth bet was a winner, paying 1 to 1, so you made $1.50 on the fourth bet, bringing your total winnings to $2.50. Since this last bet was a winner, you will not recapitalize (step up or down) your bet size into the fifth bet; instead, you stay with a bet size of $1.50 into the fifth bet.

On the surface, the reserve strategy seems like an ideal staking system. However, like all staking systems, its long-term performance falls short of the simple fixed fraction (small antimartingale) approach. Another popular idea of gamblers/traders has been the base bet plus square root strategy, whereby you essentially are always betting the same amount you started with plus the square root of any winnings. As you can see, the possibilities of staking systems are endless.

Many people seem to be partial, for whatever reason, to adding contracts after a losing trade, a streak of losing trades, or a drawdown. Over and over again in computer simulations (by myself and others) this turns out to be a very poor method of money management. It is akin to the martingale and small martingale. Since we have determined that trading is largely an independent trials process, the past trades have no effect on the present trade. It doesn't matter whether the last 20 trades were all winners or all losers.

It is interesting to note that those computer tests that have been performed all bear out the same thing. In an independent trials process where you have an edge, you are better off to increase your bet size as your stake increases, and the bet size optimally is a fixed fraction of your entire stake. Time and again authors have performed studies that take a very long stream of independent outcomes with a net positive result, and have applied various staking systems to betting/trading on these outcomes. In a nutshell, the outcomes of every study of this type reach the same conclusion: that you are better off using a staking system that increases the size of the bet in direct proportion to the size of the total stake.

In another study, William T. Ziemba demonstrated in the June 1987 issue of *Gambling Times* magazine that proportional betting was superior to any other staking strategy.[4] Furthermore, Ziemba's article demonstrated how the optimal proportion (determined correctly by the Kelly formula in this study) far outperforms any other proportion. The study simulated 1,000 seasons of betting on 700 horse races, starting you out with an initial stake of $1,000. The test looked at such outcomes as how many seasons would

[4]Ziemba, William T., "A Betting Simulation, The Mathematics of Gambling and Investment," *Gambling Times*, pp. 46–47, 80, June 1987.

have tapped you out, how many seasons were profitable, how many made more than $5,000, $10,000, $100,000, and so on, as well as what the minimum, maximum, mean, and median terminal amounts were. The results of this test, too, were quite clear—betting a fixed fraction of your bankroll is far and away the best staking system.

"Wait," you say. "Aren't staking systems foolish to begin with? Didn't we see in Chapter 1 that they do not overcome the house advantage; rather, all they do is increase our total action?"

This is absolutely true for a situation with a negative mathematical expectation. For a positive mathematical expectation it is a different story altogether. In a positive expectancy situation the trader/gambler is posed with the question of how best to exploit the positive expectation.