Chapter 4

Inflationary cosmology and creation of matter in the universe

Andrei D Linde
Department of Physics, Stanford University, Stanford, USA

4.1 Introduction

The typical lifetime of a new trend in high-energy physics and cosmology is nowadays about 5–10 years. If it has survived for a longer time, the chances are that it will be with us for quite a while. Inflationary theory by now is 20 years old, and it is still very much alive. It is the only theory which explains why our universe is so homogeneous, flat and isotropic, and why its different parts began their expansion simultaneously. It provides a mechanism explaining galaxy formation and solves numerous different problems at the intersection between cosmology and particle physics. It seems to be in a good agreement with observational data and it does not have any competitors. Thus we have some reasons for optimism.

According to the standard textbook description, inflation is a stage of exponential expansion in a supercooled false vacuum state formed as a result of high-temperature phase transitions in Grand Unified Theories (GUTs). However, during the last 20 years inflationary theory has changed quite substantially. New versions of inflationary theory typically do not require any assumptions about initial thermal equilibrium in the early universe, supercooling and exponential expansion in the false vacuum state. Instead of this, we are thinking about chaotic initial conditions, quantum cosmology and the theory of a self-reproducing universe.

Inflationary theory was proposed as an attempt to resolve problems of the big bang theory. In particular, inflation provides a simple explanation of the extraordinary homogeneity of the observable part of the universe. But it can make the universe extremely inhomogeneous on a much greater scale. Now we believe that instead of being a single, expanding ball of fire produced in the big bang, the universe looks like a huge growing fractal. It consists of many inflating balls that produce new balls, which in turn produce more new balls, ad infinitum. Even now we continue learning new things about inflationary cosmology, especially about the stage of reheating of the universe after inflation.

In this chapter we will briefly describe the history of inflationary cosmology and then we will give a review of some recent developments.

4.2 Brief history of inflation

The first inflationary model was proposed by Alexei Starobinsky in 1979 [1]. It was based on investigation of conformal anomaly in quantum gravity. This model was rather complicated, it did not aim on solving homogeneity, horizon and monopole problems, and it was not easy to understand the beginning of inflation in this model. However, it did not suffer from the graceful exit problem and, in this sense, it can be considered the first working model of inflation. The theory of density perturbations in this model was developed in 1981 by Mukhanov and Chibisov [2]. This theory does not differ much from the theory of density perturbations in new inflation, which was proposed later by Hawking, Starobinsky, Guth, Pi, Bardeen, Steinhardt, Turner and Mukhanov [3, 4].

A much simpler model with a very clear physical motivation was proposed by Alan Guth in 1981 [5]. His model, which is now called 'old inflation', was based on the theory of supercooling during the cosmological phase transitions [6]. It was so attractive that even now all textbooks on astronomy and most of the popular books on cosmology describe inflation as exponential expansion of the universe in a supercooled false vacuum state. It is seductively easy to explain the nature of inflation in this scenario. False vacuum is a metastable state without any fields or particles but with a large energy density. Imagine a universe filled with such 'heavy nothing'. When the universe expands, empty space remains empty, so its energy density does not change. The universe with a constant energy density expands exponentially, thus we have inflation in the false vacuum.

Unfortunately this explanation is somewhat misleading. Expansion in the false vacuum in a certain sense is false: de Sitter space with a constant vacuum energy density can be considered either expanding, or contracting, or static, depending on the choice of a coordinate system [7]. The absence of a preferable hypersurface of decay of the false vacuum is the main reason why the universe after inflation in this scenario becomes very inhomogeneous [5]. After many attempts to overcome this problem, it was concluded that the old inflation scenario cannot be improved [8].

Fortunately, this problem was resolved with the invention of the new inflationary theory [9]. In this theory, just as in the Starobinsky model, inflation may begin in the false vacuum. This stage of inflation is not very useful, but it prepares a stage for the next stage, which occurs when the inflaton field ϕ driving inflation moves away from the false vacuum and slowly rolls down to the minimum of its effective potential. The motion of the field away from the false vacuum is of crucial importance: density perturbations produced during inflation are inversely proportional to $\dot{\phi}$ [2, 3]. Thus the key difference between the new inflationary scenario and the old one is that the useful part of inflation in the new scenario, which is responsible for homogeneity of our universe, does *not* occur in the false vacuum state.

The new inflation scenario was plagued by its own problems. This scenario works only if the effective potential of the field ϕ has a very flat plateau near $\phi = 0$, which is somewhat artificial. In most versions of this scenario the inflaton field originally could not be in a thermal equilibrium with other matter fields. The theory of cosmological phase transitions, which was the basis for old and new inflation, simply did not work in such a situation. Moreover, thermal equilibrium requires many particles interacting with each other. This means that new inflation could explain why our universe was so large only if it was very large and contained many particles from the very beginning. Finally, inflation in this theory begins very late, and during the preceding epoch the universe could easily collapse or become so inhomogeneous that inflation may never happen [7]. Because of all these difficulties no realistic versions of the new inflationary universe scenario have been proposed so far.

From a more general perspective, old and new inflation represented a substantial but incomplete modification of the big bang theory. It was still assumed that the universe was in a state of thermal equilibrium from the very beginning, that it was relatively homogeneous and large enough to survive until the beginning of inflation, and that the stage of inflation was just an intermediate stage of the evolution of the universe. At the beginning of the 1980s these assumptions seemed most natural and practically unavoidable. That is why it was so difficult to overcome a certain psychological barrier and abandon all of these assumptions. This was done with the invention of the chaotic inflation scenario [10]. This scenario resolved all the problems of old and new inflation. According to this scenario, inflation may occur even in the theories with simplest potentials such as $V(\phi) \sim \phi^n$. Inflation may begin even if there was no thermal equilibrium in the early universe, and it may start even at the Planckian density, in which case the problem of initial conditions for inflation can be easily resolved [7].

4.2.1 Chaotic inflation

To explain the basic idea of chaotic inflation, let us consider the simplest model of a scalar field ϕ with a mass m and with the potential energy density $V(\phi) = (m^2/2)\phi^2$, see figure 4.1. Since this function has a minimum at $\phi = 0$, one may expect that the scalar field ϕ should oscillate near this minimum. This is indeed the case if the universe does not expand. However, one can show that in a rapidly expanding universe the scalar field moves down very slowly, as a ball in a viscous liquid, viscosity being proportional to the speed of expansion.

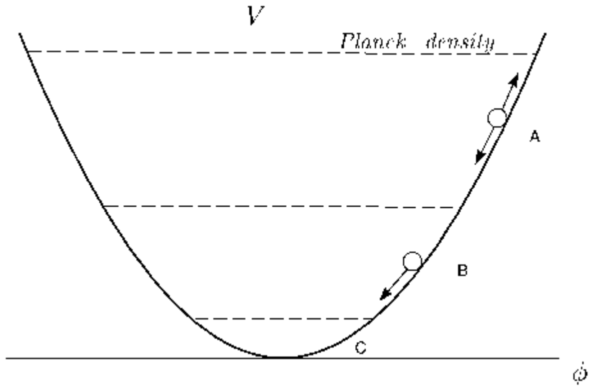

Figure 4.1. Motion of the scalar field in the theory with $V(\phi) = \frac{1}{2}m^2\phi^2$. Several different regimes are possible, depending on the value of the field ϕ. If the potential energy density of the field is greater than the Planck density $M_P^4 \sim 10^{94}$ g cm^{-3}, quantum fluctuations of spacetime are so strong that one cannot describe it in usual terms. Such a state is called spacetime foam. At a somewhat smaller energy density (region A: $mM_P^3 < V(\phi) < M_P^4$) quantum fluctuations of spacetime are small, but quantum fluctuations of the scalar field ϕ may be large. Jumps of the scalar field due to quantum fluctuations lead to a process of eternal self-reproduction of inflationary universe which we are going to discuss later. At even smaller values of $V(\phi)$ (region B: $m^2M_P^2 < V(\phi) < mM_P^3$) fluctuations of the field ϕ are small; it slowly moves down as a ball in a viscous liquid. Inflation occurs both in the region A and region B. Finally, near the minimum of $V(\phi)$ (region C) the scalar field rapidly oscillates, creates pairs of elementary particles, and the universe becomes hot.

There are two equations which describe evolution of a homogeneous scalar field in our model, the field equation

$$\ddot{\phi} + 3H\dot{\phi} = -V'(\phi), \tag{4.1}$$

and the Einstein equation

$$H^2 + \frac{k}{a^2} = \frac{8\pi}{3M_P^2}\left(\frac{1}{2}\dot{\phi}^2 + V(\phi)\right). \tag{4.2}$$

Here $H = \dot{a}/a$ is the Hubble parameter in the universe with a scale factor $a(t)$, $k = -1, 0, 1$ for an open, flat or closed universe respectively, M_P is the Planck mass. In the case $V = m^2\phi^2/2$, the first equation becomes similar to the equation of motion for a harmonic oscillator, where instead of $x(t)$ we have $\phi(t)$, with a friction term $3H\dot{\phi}$:

$$\ddot{\phi} + 3H\dot{\phi} = -m^2\phi. \tag{4.3}$$

If the scalar field ϕ initially was large, the Hubble parameter H was large too, according to the second equation. This means that the friction term in the first equation was very large, and therefore the scalar field was moving very slowly, as a ball in a viscous liquid. Therefore at this stage the energy density of the scalar field, unlike the density of ordinary matter, remained almost constant, and expansion of the universe continued with a much greater speed than in the old cosmological theory. Due to the rapid growth of the scale of the universe and a slow motion of the field ϕ, soon after the beginning of this regime one has $\ddot{\phi} \ll 3H\dot{\phi}$, $H^2 \gg (k/a^2)$, $\dot{\phi}^2 \ll m^2\phi^2$, so the system of equations can be simplified:

$$3\frac{\dot{a}}{a}\dot{\phi} = -m^2\phi, \qquad \frac{\dot{a}}{a} = \frac{2m\phi}{M_{\mathrm{P}}}\sqrt{\frac{\pi}{3}}. \tag{4.4}$$

The last equation shows that the size of the universe in this regime grows approximately as e^{Ht}, where

$$H = \frac{2m\phi}{M_{\mathrm{P}}}\sqrt{\frac{\pi}{3}}.$$

More exactly, these equations lead to following solutions for ϕ and a:

$$\phi(t) = \phi_0 - \frac{mM_{\mathrm{P}}t}{\sqrt{12\pi}}, \tag{4.5}$$

$$a(t) = a_0 \exp\frac{2\pi}{M_{\mathrm{P}}^2}(\phi_0^2 - \phi^2(t)). \tag{4.6}$$

This stage of exponentially rapid expansion of the universe is called inflation. In realistic versions of inflationary theory its duration could be as short as 10^{-35} s. When the field ϕ becomes sufficiently small, viscosity becomes small, inflation ends, and the scalar field ϕ begins to oscillate near the minimum of $V(\phi)$. As any rapidly oscillating classical field, it loses its energy by creating pairs of elementary particles. These particles interact with each other and come to a state of thermal equilibrium with some temperature T. From this time on, the corresponding part of the universe can be described by the standard hot universe theory.

The main difference between inflationary theory and the old cosmology becomes clear when one calculates the size of a typical inflationary domain at the end of inflation. Investigation of this question shows that even if the initial size of inflationary universe was as small as the Plank size $l_{\mathrm{P}} \sim 10^{-33}$ cm, after 10^{-35} s of inflation the universe acquires a huge size of $l \sim 10^{10^{12}}$ cm!

This number is model-dependent, but in all realistic models the size of the universe after inflation appears to be many orders of magnitude greater than the size of the part of the universe which we can see now, $l \sim 10^{28}$ cm. This immediately solves most of the problems of the old cosmological theory.

Our universe is almost exactly homogeneous on large scale because all inhomogeneities were stretched by a factor of $10^{10^{12}}$. The density of primordial monopoles and other undesirable ‘defects’ becomes exponentially diluted by inflation. The universe becomes enormously large. Even if it was a closed universe of a size $\sim 10^{-33}$ cm, after inflation the distance between its 'South' and 'North' poles becomes many orders of magnitude greater than 10^{28} cm. We see only a tiny part of the huge cosmic balloon. That is why nobody has ever seen how parallel lines cross. That is why the universe looks so flat.

If one considers a universe which initially consisted of many domains with chaotically distributed scalar field ϕ (or if one considers different universes with different values of the field), then domains in which the scalar field was too small never inflated. The main contribution to the total volume of the universe will be given by those domains which originally contained large scalar field ϕ. Inflation of such domains creates huge homogeneous islands out of initial chaos. Each homogeneous domain in this scenario is much greater than the size of the observable part of the universe.

The first models of chaotic inflation were based on the theories with polynomial potentials, such as

$$V(\phi) = \pm\frac{m^2}{2}\phi^2 + \frac{\lambda}{4}\phi^4.$$

But the main idea of this scenario is quite generic. One should consider any particular potential $V(\phi)$, polynomial or not, with or without spontaneous symmetry breaking, and study all possible initial conditions without assuming that the universe was in a state of thermal equilibrium, and that the field ϕ was in the minimum of its effective potential from the very beginning [10]. This scenario strongly deviated from the standard lore of the hot big bang theory and was psychologically difficult to accept. Therefore during the first few years after invention of chaotic inflation many authors claimed that the idea of chaotic initial conditions is unnatural, and made attempts to realize the new inflation scenario based on the theory of high-temperature phase transitions, despite numerous problems associated with it. Gradually, however, it became clear that the idea of chaotic initial conditions is most general, and it is much easier to construct a consistent cosmological theory without making unnecessary assumptions about thermal equilibrium and high temperature phase transitions in the early universe.

Many other versions of inflationary cosmology have been proposed since 1983. Most of them are based not on the theory of high-temperature phase transitions, as in old and new inflation, but on the idea of chaotic initial conditions, which is the definitive feature of the chaotic inflation scenario.

4.3 Quantum fluctuations in the inflationary universe

The vacuum structure in the exponentially expanding universe is much more complicated than in ordinary Minkowski space. The wavelengths of all vacuum fluctuations of the scalar field ϕ grow exponentially during inflation. When the wavelength of any particular fluctuation becomes greater than H^{-1}, this fluctuation stops oscillating, and its amplitude freezes at some non-zero value

$\delta\phi(x)$ because of the large friction term $3H\dot{\phi}$ in the equation of motion of the field ϕ. The amplitude of this fluctuation then remains almost unchanged for a very long time, whereas its wavelength grows exponentially. Therefore, the appearance of such a frozen fluctuation is equivalent to the appearance of a classical field $\delta\phi(x)$ that does not vanish after averaging over macroscopic intervals of space and time.

Because the vacuum contains fluctuations of all wavelengths, inflation leads to the continuous creation of new perturbations of the classical field with wavelengths greater than H^{-1}, i.e. with momentum k smaller than H. One can easily understand on dimensional grounds that the average amplitude of perturbations with momentum $k \sim H$ is $O(H)$. A more accurate investigation shows that the average amplitude of perturbations generated during a time interval H^{-1} (in which the universe expands by a factor of e) is given by [7]

$$|\delta\phi(x)| \approx \frac{H}{2\pi}. \tag{4.7}$$

Some of the most important features of inflationary cosmology can be understood only with an account taken of these quantum fluctuations. That is why in this section we will discuss this issue. We will begin this discussion on a rather formal level, and then we will suggest a simple interpretation of our results.

First of all, we will describe inflationary universe with the help of the metric of a flat de Sitter space,

$$\mathrm{d}s^2 = \mathrm{d}t^2 - \mathrm{e}^{2Ht}\,\mathrm{d}\boldsymbol{x}^2. \tag{4.8}$$

We will assume that the Hubble constant H practically does not change during the process, and for simplicity we will begin with investigation of a massless field ϕ.

To quantize the massless scalar field ϕ in de Sitter space in the coordinates (4.8) in much the same way as in Minkowski space [11]. The scalar field operator $\phi(x)$ can be represented in the form

$$\phi(\boldsymbol{x}, t) = (2\pi)^{-3/2} \int \mathrm{d}^3 p\, [a_p^+ \psi_p(t)\mathrm{e}^{\mathrm{i}\boldsymbol{px}} + a_p^- \psi_p^*(t)\mathrm{e}^{-\mathrm{i}\boldsymbol{px}}], \tag{4.9}$$

where $\psi_p(t)$ satisfies the equation

$$\ddot{\psi}_p(t) + 3H\dot{\psi}_p(t) + \boldsymbol{p}^2\mathrm{e}^{-2Ht}\psi_p(t) = 0. \tag{4.10}$$

The term $3H\dot{\psi}_p(t)$ originates from the term $3H\dot{\phi}$ in equation (4.1), the last term appears because of the gradient term in the Klein–Gordon equation for the field ϕ. Note, that p is a comoving momentum, which, just like the coordinates x, does not change when the universe expands.

In Minkowski space, $\psi_p(t)\frac{1}{\sqrt{2p}}\mathrm{e}^{-\mathrm{i}pt}$, where $p = \sqrt{\boldsymbol{p}^2}$. In de Sitter space (4.8), the general solution of (4.10) takes the form

$$\psi_p(t) = \frac{\sqrt{\pi}}{2} H\eta^{3/2}[\mathrm{C}_1(p)H_{3/2}^{(1)}(p\eta) + \mathrm{C}_2(p)H_{3/2}^{(2)}(p\eta)], \tag{4.11}$$

where $\eta = -H^{-1}\mathrm{e}^{-Ht}$ is the conformal time, and the $H^{(i)}_{3/2}$ are Hankel functions:

$$H^{(2)}_{3/2}(x) = [H^{(1)}_{3/2}(x)]^* = -\sqrt{\frac{2}{\pi x}}\mathrm{e}^{-\mathrm{i}x}\left(1 + \frac{1}{\mathrm{i}x}\right). \tag{4.12}$$

Quantization in de Sitter space and Minkowski space should be identical in the high-frequency limit, i.e. $\mathrm{C}_1(p) \to 0$, $\mathrm{C}_2(p) \to -1$ as $p \to \infty$. In particular, this condition is satisfied† for $\mathrm{C}_1 \equiv 0$, $\mathrm{C}_2 \equiv -1$. In that case,

$$\psi_p(t) = \frac{\mathrm{i}H}{p\sqrt{2p}}\left(1 + \frac{p}{\mathrm{i}H}\mathrm{e}^{-Ht}\right)\exp\left(\frac{\mathrm{i}p}{H}\mathrm{e}^{-Ht}\right). \tag{4.13}$$

Note that at sufficiently large t (when $p\mathrm{e}^{-Ht} < H$), $\psi_p(t)$ ceases to oscillate, and becomes equal to $\mathrm{i}H/p\sqrt{2p}$.

The quantity $\langle\phi^2\rangle$ may be simply expressed in terms of ψ_p:

$$\langle\phi^2\rangle = \frac{1}{(2\pi)^3}\int |\psi_p|^2\,\mathrm{d}^3p = \frac{1}{(2\pi)^3}\int\left(\frac{\mathrm{e}^{-2Ht}}{2p} + \frac{H^2}{2p^3}\right)\mathrm{d}^3p. \tag{4.14}$$

The physical meaning of this result becomes clear when one transforms from the conformal momentum p, which is time-independent, to the conventional physical momentum $k = p\mathrm{e}^{-Ht}$, which decreases as the universe expands:

$$\langle\phi^2\rangle = \frac{1}{(2\pi)^3}\int\frac{\mathrm{d}^3k}{k}\left(\frac{1}{2} + \frac{H^2}{2k^2}\right). \tag{4.15}$$

The first term is the usual contribution of vacuum fluctuations in Minkowski space with $H = 0$. This contribution can be eliminated by renormalization. The second term, however, is directly related to inflation. Looked at from the standpoint of quantization in Minkowski space, this term arises because of the fact that de Sitter space, apart from the usual quantum fluctuations that are present when $H = 0$, also contains ϕ-particles with occupation numbers

$$n_k = \frac{H^2}{2k^2}. \tag{4.16}$$

It can be seen from (4.15) that the contribution to $\langle\phi^2\rangle$ from long-wave fluctuations of the ϕ field diverges.

However, the value of $\langle\phi^2\rangle$ for a massless field ϕ is infinite only in eternally existing de Sitter space with $H =$ constant, and not in the inflationary universe, which expands (quasi)exponentially starting at some time $t = 0$ (for example, when the density of the universe becomes smaller than the Planck density).

† It is important that if the inflationary stage is long enough, all physical results are independent of the specific choice of functions $\mathrm{C}_1(p)$ and $\mathrm{C}_2(p)$ if $\mathrm{C}_1(p) \to 0$, $\mathrm{C}_2(p) \to -1$ as $p \to \infty$.

Indeed, the spectrum of vacuum fluctuations (4.15) strongly differs from the spectrum in Minkowski space when $k \ll H$. If the fluctuation spectrum before inflation has a cut-off at $k \leq k_0 \sim T$ resulting from high-temperature effects, or at $k \leq k_0 \sim H$ due to a small initial size $\sim H^{-1}$ of an inflationary region, then the spectrum will change at the time of inflation, due to exponential growth in the wavelength of vacuum fluctuations. The spectrum (4.15) will gradually be established, but only at momenta $k \geq k_0 \mathrm{e}^{-Ht}$. There will then be a cut-off in the integral (4.14). Restricting our attention to contributions made by long-wave fluctuations with $k \leq H$, which are the only ones that will subsequently be important for us, and assuming that $k_0 = O(H)$, we obtain

$$\begin{aligned}\langle\phi^2\rangle &\approx \frac{H^2}{2(2\pi)^3}\int_{H\mathrm{e}^{-Ht}}^{H}\frac{\mathrm{d}^3k}{k} = \frac{H^2}{4\pi^2}\int_{-Ht}^{0}\mathrm{d}\ln\frac{k}{H} \\ &\equiv \frac{H^2}{4\pi^2}\int_{0}^{Ht}\mathrm{d}\ln\frac{p}{H} = \frac{H^3}{4\pi^2}t. \end{aligned} \tag{4.17}$$

A similar result is obtained for a massive scalar field ϕ. In that case, long-wave fluctuations with $m^2 \ll H^2$ behave as

$$\langle\phi^2\rangle = \frac{3H^4}{8\pi^2 m^2}\left[1 - \exp\left(-\frac{2m^2}{3H}t\right)\right]. \tag{4.18}$$

When $t \leq 3H/m^2$, the term $\langle\phi^2\rangle$ grows linearly, just as in the case of the massless field (4.17), and it then tends to its asymptotic value

$$\langle\phi^2\rangle = \frac{3H^4}{8\pi^2 m^2}. \tag{4.19}$$

Let us now try to provide an intuitive physical interpretation of these results. First, note that the main contribution to $\langle\phi^2\rangle$ (4.17) comes from integrating over exponentially small k (with $k \sim H\exp(-Ht)$). The corresponding occupation numbers n_k (4.16) are then exponentially large. One can show that for large $l = |\boldsymbol{x} - \boldsymbol{y}|\mathrm{e}^{Ht}$, the correlation function $\langle\phi(x)\phi(y)\rangle$ for the massless field ϕ is

$$\langle\phi(\boldsymbol{x},t)\phi(\boldsymbol{y},t)\rangle \approx \langle\phi^2(\boldsymbol{x},t)\rangle\left(1 - \frac{1}{Ht}\ln Hl\right). \tag{4.20}$$

This means that the magnitudes of the fields $\phi(x)$ and $\phi(y)$ will be highly correlated out to exponentially large separations $l \sim H^{-1}\exp(Ht)$, and the corresponding occupation numbers will be exponentially large. By all these criteria, long-wave quantum fluctuations of the field ϕ with $k \ll H^{-1}$ behave like a weakly inhomogeneous (quasi)classical field ϕ generated during the inflationary stage.

Analogous results also hold for a massive field with $m^2 \ll H^2$. There, the principal contribution to $\langle\phi^2\rangle$ comes from modes with exponentially small momenta $k \sim H\exp(-3H^2/2m^2)$, and the correlation length is of order $H^{-1}\exp(3H^2/2m^2)$.

Later on we will develop a stochastic formalism which will allow us to describe various properties of the motion of the scalar field.

4.4 Quantum fluctuations and density perturbations

Fluctuations of the field ϕ lead to adiabatic density perturbations $\delta\rho \sim V'(\phi)\delta\phi$, which grow after inflation. The theory of inflationary density perturbations is rather complicated, but one can make an estimate of their post-inflationary magnitude in the following intuitively simple way: Fluctuations of the scalar field lead to a local delay of the end of inflation by the time $\delta t \sim \delta\phi/\dot{\phi}$. Density of the universe after inflation decreases as t^{-2}, so the local time delay δt leads to density contrast $|\delta\rho/\rho| \sim |2\delta t/t|$. If one takes into account that $\delta\phi \sim H/2\pi$ and that at the end of inflation $t^{-1} \sim H$, one obtains an estimate

$$\frac{\delta\rho}{\rho} \sim \frac{H^2}{2\pi\dot{\phi}}. \tag{4.21}$$

Needless to say, this is a very rough estimate. Fortunately, however, it gives a very good approximation to the correct result which can be obtained by much more complicated methods [2–4, 7]:

$$\frac{\delta\rho}{\rho} = C\frac{H^2}{2\pi\dot{\phi}}, \tag{4.22}$$

where the parameter C depends on equation of state of the universe. For example, $C = 6/5$ for the universe dominated by cold dark matter [4]. Then equations $3H\dot{\phi} = V'$ and $H^2 = 8\pi V/3M_{\mathrm{P}}^2$ imply that

$$\frac{\delta\rho}{\rho} = \frac{16\sqrt{6\pi}}{5}\frac{V^{3/2}}{V'}. \tag{4.23}$$

Here ϕ is the value of the classical field $\phi(t)$ (4), at which the fluctuation we consider has the wavelength $l \sim k^{-1} \sim H^{-1}(\phi)$ and becomes frozen in amplitude. In the simplest theory of the massive scalar field with $V(\phi) = \frac{1}{2}m^2\phi^2$ one has

$$\frac{\delta\rho}{\rho} = \frac{8\sqrt{3\pi}}{5}m\phi^2. \tag{4.24}$$

Taking into account (4.4) and also the expansion of the universe by about 10^{30} times after the end of inflation, one can obtain the following result for the density perturbations with the wavelength l (cm) at the moment when these perturbations begin growing and the process of the galaxy formation starts:

$$\frac{\delta\rho}{\rho} \sim m\ln l\ (\mathrm{cm}). \tag{4.25}$$

The definition of $\delta\rho/\rho$ used in [7] corresponds to COBE data for $\delta\rho/\rho \sim 5\times10^{-5}$. This gives $m \sim 10^{-6}$, in Planck units, which is equivalent to 10^{13} GeV.

An important feature of the spectrum of density perturbations is its flatness: $\delta\rho/\rho$ in our model depends on the scale l only logarithmically. For the theories with exponential potentials, the spectrum can be represented as

$$\frac{\delta\rho}{\rho} \sim l^{(1-n)/2}. \tag{4.26}$$

This representation is often used for the phenomenological description of various inflationary models. Exact flatness of the spectrum implies $n = 1$. Usually $n < 1$, but the models with $n > 1$ are also possible. In most of the realistic models of inflation one has $n = 1 \pm 0.2$.

Flatness of the spectrum of $\delta\rho/\rho$ together with flatness of the universe ($\Omega = 1$) constitute the two most robust predictions of inflationary cosmology. It is possible to construct models where $\delta\rho/\rho$ changes in a very peculiar way, and it is also possible to construct theories where $\Omega \neq 1$, but it is extremely difficult to do so.

4.5 From the big bang theory to the theory of eternal inflation

A significant step in the development of inflationary theory which I would like to discuss here is the discovery of the process of self-reproduction of inflationary universe. This process was known to exist in old inflationary theory [5] and in the new one [12], but it is especially surprising and leads to most profound consequences in the context of the chaotic inflation scenario [13]. It appears that in many models large scalar field during inflation produces large quantum fluctuations which may locally increase the value of the scalar field in some parts of the universe. These regions expand at a greater rate than their parent domains, and quantum fluctuations inside them lead to the production of new inflationary domains which expand even faster. This surprising behaviour leads to an eternal process of self-reproduction of the universe.

To understand the mechanism of self-reproduction one should remember that the processes separated by distances l greater than H^{-1} proceed independently of one another. This is so because during exponential expansion the distance between any two objects separated by more than H^{-1} is growing with a speed exceeding the speed of light. As a result, an observer in the inflationary universe can see only the processes occurring inside the horizon of the radius H^{-1}.

An important consequence of this general result is that the process of inflation in any spatial domain of radius H^{-1} occurs independently of any events outside it. In this sense any inflationary domain of initial radius exceeding H^{-1} can be considered as a separate mini-universe.

To investigate the behaviour of such a mini-universe, with an account taken of quantum fluctuations, let us consider an inflationary domain of initial radius

H^{-1} containing sufficiently homogeneous field with initial value $\phi \gg M_{\mathrm{P}}$. Equation (4.4) implies that during a typical time interval $\Delta t = H^{-1}$ the field inside this domain will be reduced by $\Delta\phi = M_{\mathrm{P}}^2/4\pi\phi$. By comparison this expression with

$$|\delta\phi(x)| \approx \frac{H}{2\pi} = \sqrt{\frac{2V(\phi)}{3\pi M_{\mathrm{P}}^2}} \sim \frac{m\phi}{3M_{\mathrm{P}}},$$

one can easily see that if ϕ is much less than

$$\phi^* \sim \frac{M_{\mathrm{P}}}{3}\sqrt{\frac{M_{\mathrm{P}}}{m}},$$

then the decrease of the field ϕ due to its classical motion is much greater than the average amplitude of the quantum fluctuations $\delta\phi$ generated during the same time. But for $\phi \gg \phi^*$ one has $\delta\phi(x) \gg \Delta\phi$. Because the typical wavelength of the fluctuations $\delta\phi(x)$ generated during the time is H^{-1}, the whole domain after $\Delta t = H^{-1}$ effectively becomes divided into $e^3 \sim 20$ separate domains (mini-universes) of radius H^{-1}, each containing almost homogeneous field $\phi - \Delta\phi + \delta\phi$. In almost a half of these domains the field ϕ grows by $|\delta\phi(x)| - \Delta\phi \approx |\delta\phi(x)| = H/2\pi$, rather than decreases. This means that the total volume of the universe containing *growing* field ϕ increases 10 times. During the next time interval $\Delta t = H^{-1}$ the situates repeats. Thus, after the two time intervals H^{-1} the total volume of the universe containing the growing scalar field increases 100 times, etc. The universe enters eternal process of self-reproduction.

This effect is very unusual. Its investigation still brings us new unexpected results. For example, for a long time it was believed that self-reproduction in the chaotic inflation scenario can occur only if the scalar field ϕ is greater than ϕ^* [13]. However, it was shown in [14] that if the size of the initial inflationary domain is large enough, then the process of self-reproduction of the universe begins for all values of the field ϕ for which inflation is possible (for $\phi > M_{\mathrm{P}}$ in the theory $2m^2\phi^2$). This result is based on the investigation of quantum jumps with amplitude $\delta\phi \gg H/2\pi$.

Until now we have considered the simplest inflationary model with only one scalar field, which had only one minimum of its potential energy. Meanwhile, realistic models of elementary particles propound many kinds of scalar fields. For example, in the unified theories of weak, strong and electromagnetic interactions, at least two other scalar fields exist. The potential energy of these scalar fields may have several different minima. This means that the same theory may have different 'vacuum states', corresponding to different types of symmetry breaking between fundamental interactions, and, as a result, to different laws of low-energy physics.

As a result of quantum jumps of the scalar fields during inflation, the universe may become divided into infinitely many exponentially large domains that have different laws of low-energy physics. Note that this division occurs even if the whole universe originally began in the same state, corresponding to one particular minimum of potential energy.

To illustrate this scenario, we present here the results of computer simulations of the evolution of a system of two scalar fields during inflation. The field ϕ is the inflaton field driving inflation; it is shown by the height of the distribution of the field $\phi(x, y)$ in a two-dimensional slice of the universe. The field χ determines the type of spontaneous symmetry breaking which may occur in the theory. We paint the surface black if this field is in a state corresponding to one of the two minima of its effective potential; we paint it white if it is in the second minimum corresponding to a different type of symmetry breaking, and therefore to a different set of laws of low-energy physics.

In the beginning of the process the whole inflationary domain was black, and the distribution of both fields was very homogeneous. Then the domain became exponentially large (but it has the same size in comoving coordinates, as shown in figure 4.1). Each peak of the mountains corresponds to nearly Planckian density and can be interpreted as a beginning of a new 'big bang'. The laws of physics are rapidly changing there, but they become fixed in the parts of the universe where the field ϕ becomes small. These parts correspond to valleys in figure 4.2. Thus quantum fluctuations of the scalar fields divide the universe into exponentially large domains with different laws of low-energy physics, and with different values of energy density.

If this scenario is correct, then physics alone cannot provide a complete explanation for all the properties of our part of the universe. The same physical theory may yield large parts of the universe that have diverse properties. According to this scenario, we find ourselves inside a four-dimensional domain with our kind of physical laws not because domains with different dimensionality and with alternate properties are impossible or improbable, but simply because our kind of life cannot exist in other domains.

This consideration is based on the anthropic principle, which was not very popular among physicists for two main reasons. First of all, it was based on the assumption that the universe was created many times until the final success. Second, it would be much easier (and quite sufficient) to achieve this success in a small vicinity of the solar system rather than in the whole observable part of our universe.

Both objections can be answered in the context of the theory of eternal inflation. First of all, the universe indeed reproduces itself in all its possible versions. Second, if the conditions suitable for the existence of life appear in a small vicinity of the solar system, then because of inflation the same conditions will exist in a domain much greater than the observable part of the universe. This means that inflationary theory for the first time provides real physical justification of the anthropic principle.

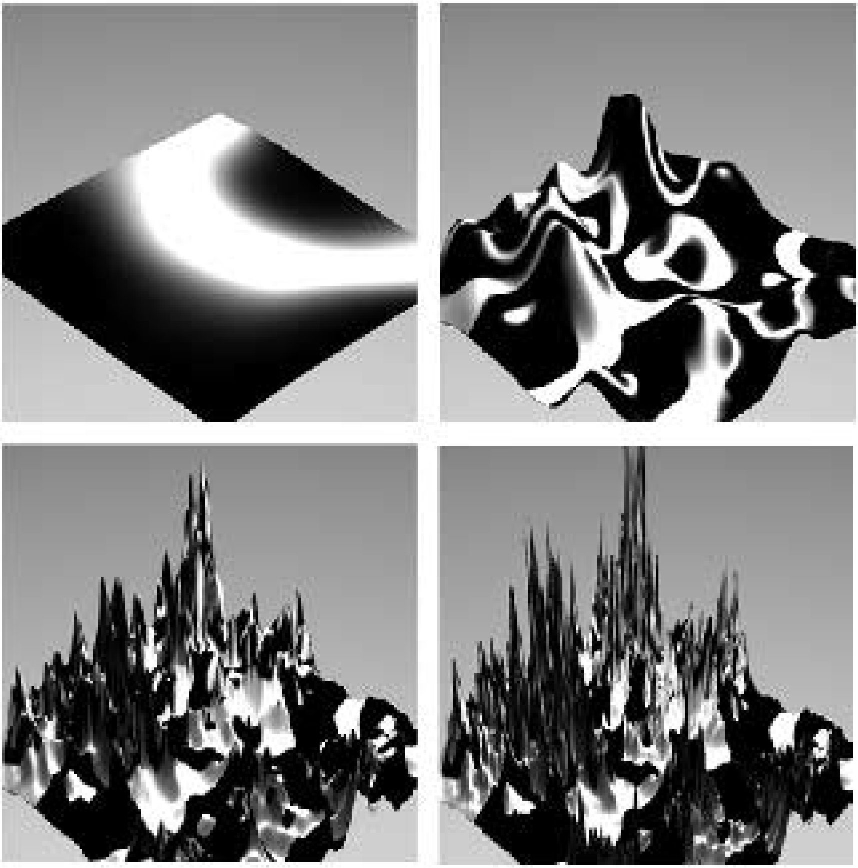

Figure 4.2. Evolution of scalar fields ϕ and χ during the process of self-reproduction of the universe. The height of the distribution shows the value of the field ϕ which drives inflation. The surface is painted black in those parts of the universe where the scalar field χ is in the first minimum of its effective potential, and white where it is in the second minimum. The laws of low-energy physics are different in the regions of different colour. The peaks of the 'mountains' correspond to places where quantum fluctuations bring the scalar fields back to the Planck density. Each such place in a certain sense can be considered as the beginning of a new big bang.

4.6 (P)reheating after inflation

The theory of the universe reheating after inflation is the most important application of the quantum theory of particle creation, since almost all matter constituting the universe was created during this process.

At the stage of inflation all energy is concentrated in a classical slowly moving inflaton field ϕ. Soon after the end of inflation this field begins to oscillate near the minimum of its effective potential. Eventually it produces many elementary particles, they interact with each other and come to a state of thermal equilibrium with some temperature T_r.

Elementary theory of this process was developed many years ago [15]. It was based on the assumption that the oscillating inflaton field can be considered as a collection of non-interacting scalar particles, each of which decays separately in accordance with perturbation theory of particle decay. However, it was recently understood that in many inflationary models the first stages of reheating occur in a regime of a broad parametric resonance. To distinguish this stage from the subsequent stages of slow reheating and thermalization, it was called *pre-heating* [16]. The energy transfer from the inflaton field to other bose fields and particles during pre-heating is extremely efficient.

To explain the main idea of the new scenario we will consider first the simplest model of chaotic inflation with the effective potential $V(\phi) = \frac{1}{2}m^2\phi^2$, and with the interaction Lagrangian $-\frac{1}{2}g^2\phi^2\chi^2$. We will take $m = 10^{-6}M_{\rm P}$, as required by microwave background anisotropy [7] and, in the beginning, we will assume for simplicity that χ particles do not have a bare mass, i.e. $m_\chi(\phi) = g|\phi|$.

In this model inflation occurs at $|\phi| > 0.3M_{\rm P}$ [7]. Suppose for definiteness that initially ϕ is large and negative, and inflation ends at $\phi \sim -0.3M_{\rm P}$. After that the field ϕ rolls to $\phi = 0$, and then it oscillates about $\phi = 0$ with a gradually decreasing amplitude.

For the quadratic potential $V(\phi) = \frac{1}{2}m\phi^2$ the amplitude after the first oscillation becomes only $0.04M_{\rm P}$, i.e. it drops by a factor of ten during the first oscillation. Later on, the solution for the scalar field ϕ asymptotically approaches the regime

$$\begin{aligned} \phi(t) &= \Phi(t)\sin mt \\ \Phi(t) &= \frac{M_{\rm P}}{\sqrt{3\pi}mt} \sim \frac{M_{\rm P}}{2\pi\sqrt{3\pi}N}. \end{aligned} \tag{4.27}$$

Here $\Phi(t)$ is the amplitude of oscillations, N is the number of oscillations since the end of inflation. For simple estimates which we will make later one may use

$$\Phi(t) \approx \frac{M_{\rm P}}{3mt} \approx \frac{M_{\rm P}}{20N}. \tag{4.28}$$

The scale factor averaged over several oscillations grows as$a(t) \approx a_0(t/t_0)^{2/3}$. Oscillations of ϕ in this theory are sinusoidal, with the decreasing amplitude

$$\Phi(t) = \frac{M_{\rm P}}{3}\left(\frac{a_0}{a(t)}\right)^{3/2}.$$

The energy density of the field ϕ decreases in the same way as the density of non-relativistic particles of mass m:

$$\rho_\phi = \tfrac{1}{2}\dot{\phi}^2 + \tfrac{1}{2}m^2\phi^2 \sim a^{-3}.$$

Hence the coherent oscillations of the homogeneous scalar field correspond to the matter-dominated effective equation of state with vanishing pressure.

We will assume that $g > 10^{-5}$ [16], which implies $gM_{\mathrm{P}} > 10^2 m$ for the realistic value of the mass $m \sim 10^{-6} M_{\mathrm{P}}$. Thus, immediately after the end of inflation, when $\phi \sim M_{\mathrm{P}}/3$, the effective mass $g|\phi|$ of the field χ is much greater than m. It decreases when the field ϕ moves down, but initially this process remains adiabatic, $|\dot{m}_\chi| \ll m_\chi^2$.

Particle production occurs at the time when the adiabaticity condition becomes violated, i.e. when $|\dot{m}_\chi| \sim g|\dot{\phi}|$ becomes greater than $m_\chi^2 = g^2\phi^2$. This happens only when the field ϕ rolls close to $\phi = 0$. The velocity of the field at that time was $|\dot{\phi}_0| \approx m M_{\mathrm{P}}/10 \approx 10^{-7} M_{\mathrm{P}}$. The process becomes non-adiabatic for $g^2\phi^2 < g|\dot{\phi}_0|$, i.e. for $-\phi_* < \phi < \phi_*$, where $\phi_* \sim \sqrt{|\dot{\phi}_0|/g}$ [16]. Note that for $g \gg 10^{-5}$ the interval $-\phi_* < \phi < \phi_*$ is very narrow: $\phi_* \ll M_{\mathrm{P}}/10$. As a result, the process of particle production occurs nearly instantaneously, within the time

$$\Delta t_* \sim \frac{\phi_*}{|\dot{\phi}_0|} \sim (g|\dot{\phi}_0|)^{-1/2}. \tag{4.29}$$

This time interval is much smaller than the age of the universe, so all effects related to the expansion of the universe can be neglected during the process of particle production. The uncertainty principle implies in this case that the created particles will have typical momenta $k \sim (\Delta t_*)^{-1} \sim (g|\dot{\phi}_0|)^{1/2}$. The occupation number n_k of χ particles with momentum k is equal to zero all the time when it moves toward $\phi = 0$. When it reaches $\phi = 0$ (or, more exactly, after it moves through the small region $-\phi_* < \phi < \phi_*$) the occupation number suddenly (within the time Δt_*) acquires the value [16]

$$n_k = \exp\left(-\frac{\pi k^2}{g|\dot{\phi}_0|}\right), \tag{4.30}$$

and this value does not change until the field ϕ rolls to the point $\phi = 0$ again.

To derive this equation one should first represent quantum fluctuations of the scalar field $\hat{\chi}$ minimally interacting with gravity in the following way:

$$\hat{\chi}(t, \boldsymbol{x}) = \frac{1}{(2\pi)^{3/2}} \int \mathrm{d}^3k\, (\hat{a}_k \chi_k(t) \mathrm{e}^{-\mathrm{i}\boldsymbol{kx}} + \hat{a}_k^+ \chi_k^*(t) \mathrm{e}^{\mathrm{i}\boldsymbol{kx}}), \tag{4.31}$$

where $\hat{a}_k$ and $\hat{a}_k^+$ are annihilation and creation operators. In general, one should write equations for these fluctuations taking into account expansion of the universe. However, in the beginning we will neglect expansion. Then the functions χ_k obey the following equation:

$$\ddot{\chi}_k + (\boldsymbol{k}^2 + g^2\phi^2(t))\chi_k = 0. \tag{4.32}$$

Equation (4.32) describes an oscillator with a variable frequency $\omega_k^2 = k^2 + g^2\phi^2(t)$. If ϕ does not change in time, then one has the usual solution $\chi_k =$

$e^{-i\omega_k t}/\sqrt{2\omega_k}$. However, when the field ϕ changes, the solution becomes different, and this difference can be interpreted in terms of creation of particles χ.

The number of created particles is equal to the energy of particles $\frac{1}{2}|\dot{\chi}_k|^2 + \frac{1}{2}\omega_k^2|\chi_k|^2$ divided by the energy ω_k of each particle:

$$n_k = \frac{\omega_k}{2}\left(\frac{|\dot{\chi}_k|^2}{\omega_k^2} + |\chi_k|^2\right) - \frac{1}{2}. \tag{4.33}$$

The subtraction $\frac{1}{2}$ is needed to eliminate vacuum fluctuations from the counting. To calculate this number, one should solve equation (4.32) and substitute the solutions to equation (4.33). One can easily check that for the usual quantum fluctuations $\chi_k = e^{-i\omega_k t}/\sqrt{2\omega_k}$ one finds $n_k = 0$. In the case described earlier, when the particles are created by the rapidly changed field ϕ in the regime of strong violation of adiabaticity condition, one can solve equation (4.32) analytically and find the number of produced particles given by equation (4.30).

One can also solve equations for quantum fluctuations and calculate n_k numerically. In figure 4.3 we show the growth of fluctuations of the field χ and the number of particles χ produced by the oscillating field ϕ in the case when the mass of the field ϕ (i.e. the frequency of its oscillations) is much smaller than the average mass of the field χ given by $g\phi$.

The time evolution in figure 4.3 is shown in units $m/2\pi$, which corresponds to the number of oscillations N of the inflaton field ϕ. The oscillating field $\phi(t) \sim \Phi \sin mt$ is zero at integer and half-integer values of the variable $mt/2\pi$. This allows us to identify particle production with time intervals when $\phi(t)$ is very small.

During each oscillation of the inflaton field ϕ, the field χ oscillates many times. Indeed, the effective mass $m_\chi(t) = g\phi(t)$ is much greater than the inflaton mass m for the main part of the period of oscillation of the field ϕ in the broad resonance regime with $q^{1/2} = g\Phi/2m \gg 1$. As a result, the typical frequency of oscillation $\omega(t) = \sqrt{k^2 + g^2\phi^2(t)}$ of the field χ is much higher than that of the field ϕ. That is why during the most of this interval it is possible to talk about an adiabatically changing effective mass $m_\chi(t)$. But this condition breaks at small ϕ, and particles χ are produced there.

Each time the field ϕ approaches the point $\phi = 0$, new χ particles are being produced. Bose statistics implies that the number of particles produced each time will be proportional to the number of particles produced before. This leads to explosive process of particle production out of the state of thermal equilibrium. We called this process *pre-heating* [16].

This process does not occur for all momenta. It is most efficient if the field ϕ comes to the point $\phi = 0$ in phase with the field χ_k, which depends on k; see phases of the field χ_k for some particular values of k for which the process is most efficient on the upper panel of figure 4.3. Thus we deal with the effect of the exponential growth of the number of particles χ due to parametric resonance.

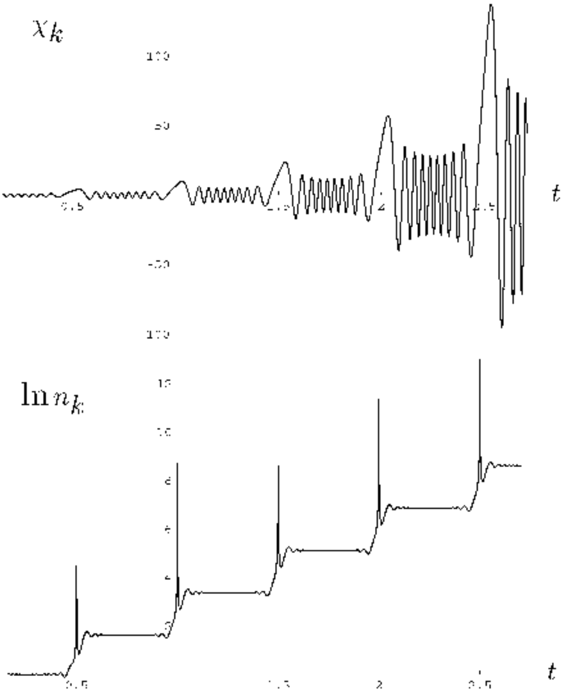

Figure 4.3. Broad parametric resonance for the field χ in Minkowski space in the theory $\frac{1}{2}m^2\phi^2$. For each oscillation of the field $\phi(t)$ the field χ_k oscillates many times. Each peak in the amplitude of the oscillations of the field χ corresponds to a place where $\phi(t) = 0$. At this time the occupation number n_k is not well defined, but soon after that time it stabilizes at a new, higher level, and remains constant until the next jump. A comparison of the two parts of this figure demonstrates the importance of using proper variables for the description of pre-heating. Both χ_k and the integrated dispersion $\langle\chi^2\rangle$ behave erratically in the process of parametric resonance. Meanwhile n_k is an adiabatic invariant. Therefore, the behaviour of n_k is relatively simple and predictable everywhere except at the short intervals of time when $\phi(t)$ is very small and the particle production occurs.

Expansion of the universe modifies this picture for many reasons. First of all, expansion of the universe's redshifts produced particles, making their momenta smaller. More importantly, the amplitude of oscillations of the field ϕ decreases because of the expansion. Therefore the frequency of oscillations of the field χ also decreases. This may destroy the parametric resonance because it changes, in an unpredictable way, the phase of the oscillations of the field χ each moment that ϕ becomes close to zero.

That is why the number of created particles χ may either increase or decrease each time when the field ϕ becomes zero. However, a more detailed investigation

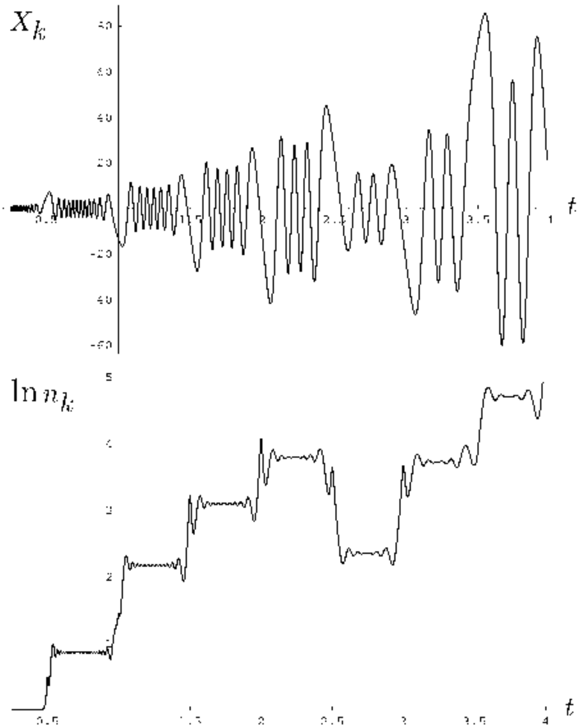

Figure 4.4. Early stages of parametric resonance in the theory $\frac{1}{2}m^2\phi^2 2$ in an expanding universe with scale factor $a \sim t^{2/3}$ for $g = 5 \times 10^{-4}$, $m = 10^{-6} M_{\rm P}$. Note that the number of particles n_k in this process typically increases, but it may occasionally decrease as well. This is a distinctive feature of stochastic resonance in an expanding universe. A decrease in the number of particles is a purely quantum mechanical effect which would be impossible if these particles were in a state of thermal equilibrium.

shows that it increases three times more often than it decreases, so the total number of produced particles grows exponentially, though in a rather specific way, see figure 4.4. We called this regime *stochastic resonance*.

In the course of time the amplitude of the oscillations of the field ϕ decreases, and when $g\phi$ becomes smaller than m, particle production becomes inefficient and their number stops growing.

In reality the situation is even more complicated. First of all, created particles change the frequency of oscillations of the field ϕ because they give a contribution $\sim g^2\langle\chi^2\rangle$ to the effective mass squared of the inflaton field [16]. Also, these particles scatter on each other and on the oscillating scalar field ϕ, which leads to additional particle production. As a result, it becomes extremely difficult to describe analytically the last stages of the process of the parametric resonance,

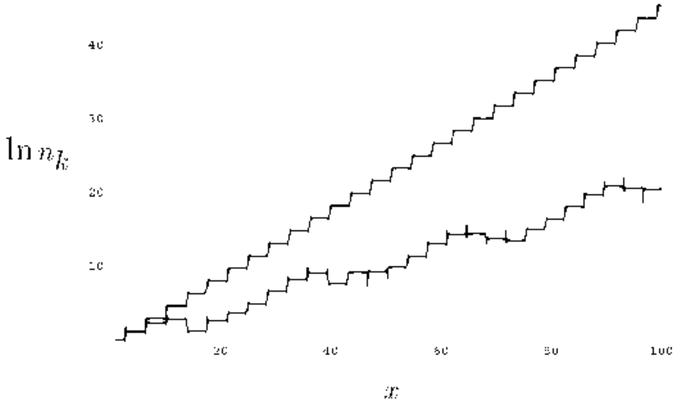

Figure 4.5. Development of the resonance in the theory $\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$ for $g^2/\lambda = 5200$. The upper curve corresponds to the massless theory, the lower curve describes stochastic resonance with a theory with a mass m which is chosen to be much smaller than $\sqrt{\lambda}\phi$ during the whole period of calculations. Nevertheless, the presence of a small mass term completely changes the development of the resonance.

even though in many cases it is possible to estimate the final results. In particular, one can show that the whole process of parametric resonance typically takes only few dozen of oscillations, and the final occupation numbers of particles grow up to $n_k \sim 10^2 g^{-2}$ [16]. But a detailed description of the last stages of pre-heating requires lattice simulations, as proposed by Khlebnikov and Tkachev [18].

The theory of pre-heating is very sensitive to the choice of the model. For example, in the theory $\frac{1}{4}\lambda\phi^4+\frac{1}{2}g^2\phi^2\chi^2$ the resonance does not become stochastic despite expansion of the universe. However, if one adds to this theory even a very small term $m^2\phi^2$, the resonance becomes stochastic [17].

This conclusion is illustrated by figure 4.5, where we show the development of the resonance both for the massless theory with $g^2/\lambda \sim 5200$, and for the theory with a small mass m. As we see, in the purely massless theory the logarithm of the number density n_k for the leading growing mode increases linearly in time x, whereas in the presence of a mass m, which we took to be much smaller than $\sqrt{\lambda}\phi$ during the whole process, the resonance becomes stochastic.

In fact, the development of the resonance is rather complicated even for smaller g^2/λ. The resonance for a massive field with $m \ll \sqrt{\lambda}\phi$ in this case is not stochastic, but it may consist of stages of regular resonance separated by the stages without any resonance, see figure 4.6.

Thus we see that the presence of the mass term $\frac{1}{2}m^2\phi^2$ can modify the nature of the resonance even if this term is much smaller than $\frac{1}{4}\lambda\phi^4$. This is a rather unexpected conclusion, which is an additional manifestation of the non-perturbative nature of pre-heating.

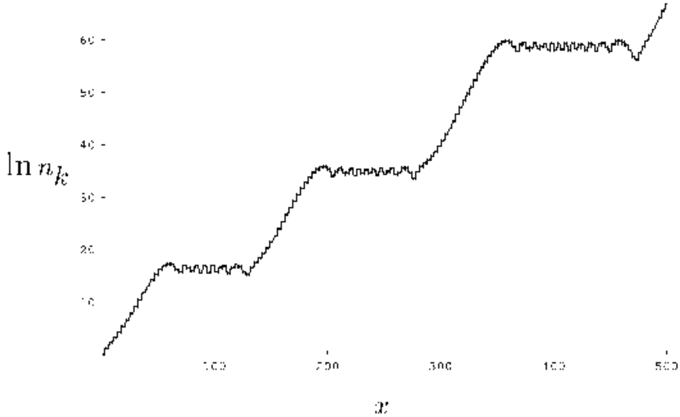

Figure 4.6. Development of the resonance in the theory $\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$ with $m^2 \ll \lambda\phi^2$ for $g^2/\lambda = 240$. In this particular case the resonance is not stochastic. As time x grows, the relative contribution of the mass term to the equation describing the resonance also grows. This shifts the mode from one instability band to another.

Different regimes of parametric resonance in the theory

$$\tfrac{1}{2}m^2\phi^2 + \tfrac{1}{4}\lambda\phi^4 + \tfrac{1}{2}g^2\phi^2\chi^2$$

are shown in figure 4.7. We suppose that immediately after inflation the amplitude Φ of the oscillating inflaton field is greater than $m/sqrt\lambda$. If $g/\sqrt{\lambda} < \sqrt{\lambda}M_{\mathrm{P}}/m$, the χ-particles are produced in the regular stable resonance regime until the amplitude $\Phi(t)$ decreases to $m/\sqrt{\lambda}$, after which the resonance occurs as in the theory $\frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$ [16]. The resonance never becomes stochastic.

If $g\sqrt{/}\lambda > \sqrt{\lambda}M_{\mathrm{P}}/m$, the resonance originally develops as in the conformally invariant theory $\frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$, but with a decrease of $\Phi(t)$ the resonance becomes stochastic. Again, for $\Phi(t) < m/\sqrt{\lambda}$ the resonance occurs as in the theory $\frac{1}{2}m^2\phi^2 + \frac{1}{2}g^2\phi^2\chi^2$. In all cases the resonance eventually disappears when the field $\Phi(t)$ becomes sufficiently small. Reheating in this class of models can be complete only if there is a symmetry breaking in the theory, i.e. $m^2 < 0$, or if one adds interaction of the field ϕ with fermions. In both cases the last stages of reheating are described by perturbation theory [17].

Adding fermions does not alter substantially the description of the stage of parametric resonance. Meanwhile the change of sign of m^2 does lead to substantial changes in the theory of pre-heating, see figure 4.8. Here we will briefly describe the structure of the resonance in the theory $-\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$ for various g^2 and λ neglecting effects of backreaction.

First of all, at $\Phi \gg m/\sqrt{\lambda}$ the field ϕ oscillates in the same way as in the massless theory $\frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$. The condition for the resonance to be

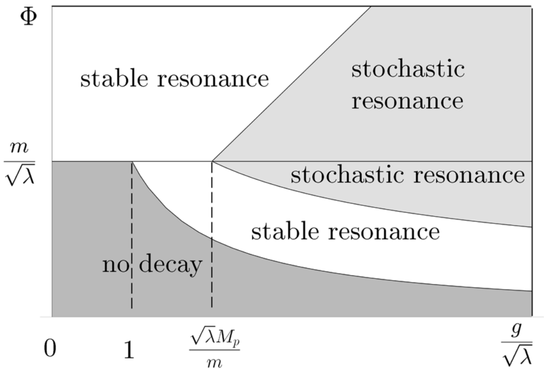

Figure 4.7. Schematic representation of different regimes which are possible in the theory $\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$ for $m/\sqrt{\lambda} \ll 10^{-1}M_{\mathrm{P}}$ and for various relations between g^2 and λ in an expanding universe. The theory developed in this chapter describes the resonance in the white area above the line $\Phi = m/\sqrt{\lambda}$. The theory of pre-heating for $\Phi < m/\sqrt{\lambda}$ is given in [16]. A complete decay of the inflaton is possible only if additional interactions are present in the theory which allow one inflaton particle to decay to several other particles, for example, an interaction with fermions $\bar{\psi}\psi\phi$.

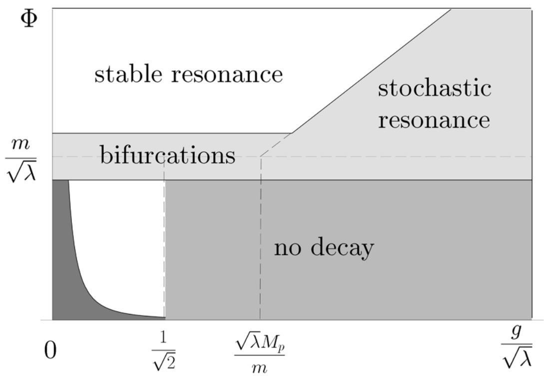

Figure 4.8. Schematic representation of different regimes which are possible in the theory $-\frac{1}{2}m^2\phi^2 + \frac{1}{4}\lambda\phi^4 + \frac{1}{2}g^2\phi^2\chi^2$. White regions correspond to the regime of a regular stable resonance, a small dark region in the left-hand corner near the origin corresponds to the perturbative decay $\phi \to \chi\chi$. Unless additional interactions are included (see figure 4.7), a complete decay of the inflaton field is possible only in this small area.

stochastic is

$$\Phi < \frac{g}{\sqrt{\lambda}}\frac{\pi^2 m^2}{3\lambda M_{\mathrm{P}}}.$$

However, as soon as the amplitude Φ drops down to $m/\sqrt{\lambda}$, the situation changes dramatically. First of all, depending on the values of parameters the field rolls to one of the minima of its effective potential at $\phi = \pm m/\sqrt{\lambda}$. The description of this process is rather complicated. Depending on the values of parameters and on the relation between $\sqrt{\langle\phi^2\rangle}$, $\sqrt{\langle\chi^2\rangle}$ and $\sigma \equiv m/\sqrt{\lambda}$, the universe may become divided into domains with $\phi = \pm\sigma$, or it may end up in a single state with a definite sign of ϕ. After this transitional period the field ϕ oscillates near the minimum of the effective potential at $\phi = \pm m/\sqrt{\lambda}$ with an amplitude $\Phi \ll \sigma = m/\sqrt{\lambda}$. These oscillations lead to parametric resonance with χ-particle production. For definiteness we will consider here the regime $\lambda^{3/2}M_{\mathrm{P}} < m \ll \lambda^{1/2}M_{\mathrm{P}}$. The resonance in this case is possible only if $g^2/\lambda < \frac{1}{2}$. Using the results of [16] one can show that the resonance is possible only for

$$\frac{g}{\sqrt{\lambda}} > \left(\frac{m}{\sqrt{\lambda}M_{\mathrm{P}}}\right)^{1/4} .$$

(The resonance may terminate somewhat earlier if the particles produced by the parametric resonance give a considerable contribution to the energy density of the universe.) However, this is not the end of reheating, because the perturbative decay of the inflaton field remains possible. It occurs with the decay rate $\Gamma(\phi \to \chi\chi) = g^4 m/8\pi\lambda$. This is the process which is responsible for the last stages of the decay of the inflaton field. It occurs only if one ϕ-particle can decay into two χ-particles, which implies that $g^2/\lambda < \frac{1}{2}$.

Thus we see that pre-heating is an incredibly rich phenomenon. Interestingly, complete decay of the inflaton field is not by any means guaranteed. In most of the models not involving fermions the decay never completes. Efficiency of pre-heating and, consequently, efficiency of baryogenesis, depends in a very non-monotonic way on the parameters of the theory. This may lead to a certain 'unnatural selection' of the theories where all necessary conditions for creation of matter and the subsequent emergence of life are satisfied.

Bosons produced at that stage are far away from thermal equilibrium and have enormously large occupation numbers. Explosive reheating leads to many interesting effects. For example, specific non-thermal phase transitions may occur soon after pre-heating, which are capable of restoring symmetry even in the theories with symmetry breaking on the scale $\sim 10^{16}$ GeV [19]. These phase transitions are capable of producing topological defects such as strings, domain walls and monopoles [20]. Strong deviation from thermal equilibrium and the possibility of production of superheavy particles by oscillations of a relatively light inflaton field may resurrect the theory of GUT baryogenesis [21] and may considerably change the way baryons are produced in the Affleck–Dine scenario [22], and in the electroweak theory [23].

Usually only a small fraction of the energy of the inflaton field $\sim 10^{-2}g^2$ is transferred to the particles χ when the field ϕ approaches the point $\phi = 0$ for the first time [24]. The role of the parametric resonance is to increase this energy exponentially within several oscillations of the inflaton field. But suppose that the particles χ interact with fermions ψ with the coupling $h\bar{\psi}\psi\chi$. If this coupling is strong enough, then χ particles may decay to fermions before the oscillating field ϕ returns back to the minimum of the effective potential. If this happens, parametric resonance does not occur. However, something equally interesting may occur instead of it: the energy density of the χ particles at the moment of their decay may become much greater than their energy density at the moment of their creation. This may be sufficient for a complete reheating.

Indeed, prior to their decay the number density of χ particles produced at the point $\phi = 0$ remains practically constant [16], whereas the effective mass of each χ particle grows as $m_\chi = g\phi$ when the field ϕ rolls up from the minimum of the effective potential. Therefore their total energy density grows. One may say that χ particles are 'fattened', being fed by the energy of the rolling field ϕ. The fattened χ particles tend to decay to fermions at the moment when they have the greatest mass, i.e. when ϕ reaches its maximal value $\sim 10^{-1} M_{\mathrm{P}}$, just before it begins rolling back to $\phi = 0$.

At that moment χ particles can decay to two fermions with mass up to $m_\psi \sim \frac{1}{2} g 10^{-1} M_{\mathrm{P}}$, which can be as large as 5×10^{17} GeV for $g \sim 1$. This is five orders of magnitude greater than the masses of the particles which can be produced by the usual decay of ϕ particles. As a result, the chain reaction $\phi \to \chi \to \psi$ considerably enhances the efficiency of transfer of energy of the inflaton field to matter.

More importantly, superheavy particles ψ (or the products of their decay) may eventually dominate the total energy density of matter even if in the beginning their energy density was relatively small. For example, the energy density of the oscillating inflaton field in the theory with the effective potential $\frac{1}{4}\lambda\phi^4$ decreases as a^{-4} in an expanding universe with a scale factor $a(t)$. Meanwhile the energy density stored in the non-relativistic particles ψ (prior to their decay) decreases only as a^{-3}. Therefore their energy density rapidly becomes dominant even if originally it was small. A subsequent decay of such particles leads to a complete reheating of the universe.

Thus in this scenario the process of particle production occurs within less than one oscillation of the inflaton field. We called it *instant pre-heating* [24]. This mechanism is very efficient even in the situation when all other mechanisms fail. Consider, for example, models where the post-inflationary motion of the inflaton field occurs along a flat direction of the effective potential. In such theories the standard scenario of reheating does not work because the field ϕ does not oscillate. Until the invention of the instant pre-heating scenario the only mechanism of reheating discussed in the context of such models was based on the gravitational production of particles [25]. The mechanism of instant pre-heating in such models is typically much more efficient. After the moment when χ particles are produced their energy density grows due to the growth of the field ϕ. Meanwhile the energy density of the field ϕ moving along a flat direction of $V(\phi)$ decreases extremely rapidly, as $a^{-6}(t)$. Therefore very soon all energy becomes concentrated in the particles produced at the end of inflation, and reheating completes.

As we see, the theory of creation of matter in the universe is much more interesting and complicated than we expected few years ago.

4.7 Conclusions

During the last 20 years inflationary theory gradually became the standard paradigm of modern cosmology. In addition to resolving many problems of the standard big bang theory, inflation made several important predictions. In particular:

(1) The universe must be flat. In most models $\Omega_{\rm total} = 1 \pm 10^{-4}$.
(2) Perturbations of the metric produced during inflation are adiabatic. (Topological defects produce isocurvature perturbations.)
(3) These perturbations should have flat spectrum. In most of the models one has $n = 1 \pm 0.2$.
(4) These perturbations should be Gaussian. (Topological defects produce non-Gaussian perturbations.)
(5) There should be no (or almost no) vector perturbations after inflation. (They may appear in the theory of topological defects.)

At the moment all of these predictions seem to be in a good agreement with observational data [26], and no other theory is available that makes all of these predictions.

This does not mean that all difficulties are over and we can relax. First of all, inflation is still a scenario which changes with every new idea in particle theory. Do we really know that inflation began at Planck density 10^{94} g cm$^{-}3$? What if our space has large internal dimensions, and energy density could never rise above the electroweak density 10^{25} g cm$^{-}3$? Was there any stage before inflation? Is it possible to implement inflation in string theory/M-theory?

We do not know which version of inflationary theory will survive ten years from now. It is absolutely clear than new observational data are going to rule out 99% of all inflationary models. But it does not seem likely that they will rule out the basic idea of inflation. Inflationary scenario is very versatile, and now, after 20 years of persistent attempts of many physicists to propose an alternative to inflation, we still do not know any other way to construct a consistent cosmological theory. For the time being, we are taking the position suggested long ago by Sherlock Holmes: 'When you have eliminated the impossible, whatever remains, however improbable, must be the truth' [27]. Did we really eliminate the impossible? Do we really know the truth? It is for you to find the answer.

References

[1] Starobinsky A A 1979 *JETP Lett.* **30** 682

Starobinsky A A 1980 *Phys. Lett.* B **91** 99
[2] Mukhanov V F and Chibisov G V 1981 *JETP Lett.* **33** 532
[3] Hawking S W 1982 *Phys. Lett.* B **115** 295
Starobinsky A A 1982 *Phys. Lett.* B **117** 175
Guth A H and Pi S-Y 1982 *Phys. Rev. Lett.* **49** 1110
Bardeen J, Steinhardt P J and Turner M S 1983 *Phys. Rev.* D **28** 679
[4] Mukhanov V F 1985 *JETP Lett.* **41** 493
Mukhanov V F, Feldman H A and Brandenberger R H 1992 *Phys. Rep.* **215** 203
[5] Guth A H 1981 *Phys. Rev.* D **23** 347
[6] Kirzhnits D A 1972 *JETP Lett.* **15** 529
Kirzhnits D A and Linde A D 1972 *Phys. Lett.* B **42** 471
Kirzhnits D A and Linde A D 1974 *Sov. Phys.–JETP* **40** 628
Kirzhnits D A and Linde A D 1976 *Ann. Phys.* **101** 195
Weinberg S 1974 *Phys. Rev.* D **9** 3320
Dolan L and Jackiw R 1974 *Phys. Rev.* D **9** 3357
[7] Linde A D 1990 *Particle Physics and Inflationary Cosmology* (Chur, Switzerland: Harwood)
[8] Guth A H and Weinberg E J 1983 *Nucl. Phys.* B **212** 321
[9] Linde A D 1982 *Phys. Lett.* B **108** 389
Linde A D 1982 *Phys. Lett.* B **114** 431
Linde A D 1982 *Phys. Lett.* B **116** 335, 340
Albrecht A and Steinhardt P J 1982 *Phys. Rev. Lett.* **48** 1220
[10] Linde A D 1983 *Phys. Lett.* B **129** 177
[11] Vilenkin A and Ford L H 1982 *Phys. Rev.* D **26** 1231
Linde A D 1982 *Phys. Lett.* B **116** 335
Starobinsky A A 1982 *Phys. Lett.* B **117** 175
[12] Steinhardt P J 1982 *The Very Early Universe* ed G W Gibbons, S W Hawking and S Siklos (Cambridge: Cambridge University Press) p 251
Linde A D 1982 *Nonsingular Regenerating Inflationary Universe* (Cambridge: Cambridge University Press)
Vilenkin A 1983 *Phys. Rev.* D **27** 2848
[13] Linde A D 1986 *Phys. Lett.* B **175** 395
Goncharov A S, Linde A and Mukhanov V F 1987 *Int. J. Mod. Phys.* A **2** 561
[14] Linde A D, Linde D A and Mezhlumian A 1994 *Phys. Rev.* D **49** 1783
[15] Dolgov A D and Linde A D 1982 *Phys. Lett.* B **116** 329
Abbott L F, Fahri E and Wise M 1982 *Phys. Lett.* B **117** 29
[16] Kofman L A, Linde A D and Starobinsky A A 1994 *Phys. Rev. Lett.* **73** 3195
Kofman L, Linde A and Starobinsky A A 1997 *Phys. Rev.* D **56** 3258–95
[17] Greene P B, Kofman L, Linde A D and Starobinsky A A 1997 *Phys. Rev.* D **56** 6175–92 (hep-ph/9705347)
[18] Khlebnikov S and Tkachev I 1996 *Phys. Rev. Lett.* **77** 219
Khlebnikov S and Tkachev I 1997 *Phys. Lett.* B **390** 80
Khlebnikov S and Tkachev I 1997 *Phys. Rev. Lett.* **79** 1607
Khlebnikov S and Tkachev I 1997 *Phys. Rev.* D **56** 653
Prokopec T and Roos T G 1997 *Phys. Rev.* D **55** 3768
Greene B R, Prokopec T and Roos T G 1997 *Phys. Rev.* D **56** 6484
[19] Kofman L A, Linde A D and Starobinsky A A 1996 *Phys. Rev. Lett.* **76** 1011
Tkachev I 1996 *Phys. Lett.* B **376** 35

[20] Tkachev I, Kofman L A, Linde A D, Khlebnikov S and Starobinsky A A in preparation

[21] Kolb E W, Linde A and Riotto A 1996 *Phys. Rev. Lett.* **77** 4960

[22] Anderson G W, Linde A D and Riotto A 1996 *Phys. Rev. Lett.* **77** 3716

[23] García-Bellido J, Grigorev D, Kusenko A and Shaposhnikov M 1999 *Phys. Rev.* D **60** 123504
García-Bellido J and Linde A 1998 *Phys. Rev.* D **57** 6075

[24] Felder G, Kofman L A and LindeA D 1999 *Phys. Rev.* D **59** 123523

[25] Ford L H 1987 *Phys. Rev.* D **35** 2955
Spokoiny B 1993 *Phys. Lett.* B **315** 40
Joyce M 1997 *Phys. Rev.* D **55** 1875
Joyce M and Prokopec T 1998 *Phys. Rev.* D **57** 6022
Peebles P J E and Vilenkin A 1999 *Phys. Rev.* D **59** 063505

[26] Jaffe A H *et al* 2001 Cosmology from Maxima-1, Boomerang and COBE/DMR CMB observations *Phys. Rev. Lett.* **86** 3475

[27] Conan Doyle A *The Sign of Four* ch 6 (http://www.litrix.com/sign4/signo006.htm)