3. $\because (J^TJ+uI)\Delta x_{lm} = -F'(x)^T$ and $J^TJ = V\Lambda V^T$

$\therefore (V\Lambda V^T + uI)\Delta x_{lm} = -F'(x)^T$

$\because$ V is a orthonormal matrix . $V^T = V^{-1}$

$\therefore \quad V^TV = I = VV^T$

$(V\Lambda V^T + uVV^T)\Delta x_{lm} = -F'(x)^T$

$(V(\Lambda + uI)V^T)\Delta x_{lm} = -F'(x)^T$

$(\Lambda + uI)V^T\Delta x_{lm} = -V^T F'(x)^T$

$V^T\Delta x_{lm} = -(\Lambda + uI)^{-1} V^T F'(x)^T$

$\Delta x_{lm} = -V(\Lambda + uI)^{-1} V^T F'(x)^T$

$$\Delta x_{lm} = -\sum_{j=1}^{n} \frac{V_j^T F'^T}{\lambda_j + \mu} V_j$$

2. $f_{15} = \dfrac{\partial \delta \alpha_{b_{k+1}}}{\partial \delta b_k^g}$

$$\alpha_{b_i b_{k+1}} = \alpha_{b_i b_k} + \beta_{b_i b_k} \delta t + \boxed{\frac{1}{2} a \delta t^2}$$

$\because$ $\alpha_{b_i b_k}$, $\beta_{b_i b_k} \delta t$ is not releuant with b_k^g

$$f_{15} = \frac{\partial \frac{1}{2} a \delta t^2}{\partial \delta b_k^g}$$

$$a = \frac{1}{2}\left(q_{b_i b_k}(a^{b_k} - b_k^a) + \boxed{q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2} w \delta t \end{bmatrix} (a^{b_{k+1}} - b_k^a)} \right)$$

Only the red part is relevant with b_k^g

$\because$ $w = \frac{1}{2}((w^{b_k} - b_k^g) + (w^{b_{k+1}} - b_k^g))$

$$= \frac{1}{2}(w^{b_k} + w^{b_{k+1}}) - b_k^g$$

$$\therefore \frac{\partial \delta \alpha_{b_{k+1}}}{\partial \delta b_k^g} = \frac{1}{4} \frac{\partial \, q_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2} w \delta t \end{bmatrix} \otimes \begin{bmatrix} 1 \\ -\frac{1}{2} \delta b_k^g \delta t \end{bmatrix} (a^{k+1} - b_k^a) \delta t^2}{\partial \delta b_k^g}$$

$$= \frac{1}{4} \frac{R_{b_i b_{k+1}} \exp([-\delta b_k^g \delta t]_\times)(a^{k+1} - b_k^a) \delta t^2}{\delta b_k^g}$$

$$= \frac{1}{4} \frac{R_{b_i b_{k+1}} (I + [-\delta b_k^g \delta t]_\times)(a^{k+1} - b_k^a) \delta t^2}{\delta b_k^g}$$

$$= \frac{1}{4} \frac{-R_{b_i b_{k+1}} [a^{k+1} - b_k^a]_\times (-\delta b_k^g \delta t)}{\delta b_k^g}$$

$$= \frac{1}{4} - R_{b_i b_{k+1}} [a^{k+1} - b_k^a]_\times (-\delta t)$$

$$g_{12} = \frac{\partial\, \delta a_{b_{k+1}}}{\partial\, \delta n_k^g}$$

$$= \frac{\partial\, \delta\left(a_{b_k} + \beta_{b_i b_k}\delta t + \frac{1}{2} a \delta t^2\right)}{\partial\, \delta n_k^g}$$

$$= \partial\, \delta\left(\frac{1}{2}\cdot\frac{1}{2}\left(g_{b_i b_k}(\bar{a}^{b_k} + n_k^a - b_k^a) + g_{b_i b_{k+1}}(\bar{a}^{b_{k+1}} + n_{k+1}^a - b_k^a)\right)\delta t^2\right.$$

$$= \partial\, \delta\, \frac{1}{4}\, g_{b_i b_{k+1}}(\bar{a}^{b_{k+1}} + n_{k+1}^a - b_k^a))\delta t^2$$

$$= \frac{\partial\, \delta\, \frac{1}{4}\, g_{b_i b_k}}{\partial\, \delta n_k^g} \otimes \begin{bmatrix} 1 \\ \frac{1}{2} w \delta t \end{bmatrix} (\bar{a}^{b_{k+1}} + n_{k+1}^a - b_k^a))\delta t^2$$

$$= \frac{\partial\, \delta\, \frac{1}{4}\, g_{b_i b_k} \otimes \begin{bmatrix} 1 \\ \frac{1}{2} w \delta t \end{bmatrix} \begin{bmatrix} 1 \\ \frac{1}{4} n_k^g \delta t \end{bmatrix} (a^{b_{k+1}} - b_k^a)\delta t^2}{\partial\, \delta n_k^g}$$

$$= \frac{\partial\, R_{b_i b_{k+1}} \exp\left([\frac{1}{2}\delta n_k^g \delta t]_\times\right)(a^{b_{k+1}} - b_k^a)\delta t^2}{4\, \partial\, \delta n_k^g}$$

$$= \partial\, R_{b_i b_{k+1}} \frac{1}{4} \left[\frac{\cancel{I +} [\frac{1}{2}\delta n_k^g \delta t]_\times}{\partial\, \delta n_k^g}\right] (a^{b_{k+1}} - b_k^a)\delta t^2$$

$$= -\partial\, R_{b_i b_{k+1}} \frac{1}{4} \frac{[(a^{b_{k+1}} - b_k^a)\delta t^2]_\times (\frac{1}{2}\delta n_k^g \delta t)}{\partial\, \delta n_k^g}$$

$$g_{12} = -\frac{1}{4}\left(R_{b_i b_{k+1}} [(a^{b_{k+1}} - b_k^a)]_\times \delta t^2\right)\left(\frac{1}{2}\delta t\right)$$