Universidad Nacional Autónoma de México

Temas selectos de Ingeniería en Computación III

(Introducción a la Computación Cuántica)

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Tarea moral 1

Instrucciones: Realiza los siguientes ejercicios documentando y justificando todos los pasos para realizarlos. La entrega ser á en un archivo PDF.

1 Repaso números complejos y matrices

1. Resuelve las siguientes operaciones entre imaginarios escribiendo el procedimiento completo:

- $(3+2i)+(-2+9i) \Rightarrow (3+2i)+(-2+9i) = 3+2i-2+9i = 1+11i$
- $(9+3i)-(8+17i) \Rightarrow (9+3i)-(8+17i) = 9+3i-8-17i = 1-14i$
- $(3+5i)*(2i) \Rightarrow (3+5i)*(2i) = 3(2i)+5i(2i) = 6i+10(i)^2 = -10+6i$

2. Muestra que las siguientes matrices son unitarias:

- $R_x(\theta) = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}$

$$(R_x)(R_x^t)^* = \begin{pmatrix} \cos\frac{\theta}{2} & -i\sin\frac{\theta}{2} \\ -i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix}\begin{pmatrix} \cos\frac{\theta}{2} & i\sin\frac{\theta}{2} \\ i\sin\frac{\theta}{2} & \cos\frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} \cos\frac{\theta}{2}\cos\frac{\theta}{2} + \left(-i\sin\frac{\theta}{2}\right)i\sin\frac{\theta}{2} & \cos\frac{\theta}{2}i\sin\frac{\theta}{2} + \left(-i\sin\frac{\theta}{2}\right)\cos\frac{\theta}{2} \\ -i\sin\frac{\theta}{2}\cos\frac{\theta}{2} + \cos\frac{\theta}{2}i\sin\frac{\theta}{2} & -i\sin\frac{\theta}{2}i\sin\frac{\theta}{2} + \cos\frac{\theta}{2}\cos\frac{\theta}{2} \end{pmatrix}$$

$$= \begin{pmatrix} \cos^2\frac{\theta}{2} - (i)^2\sin^2\frac{\theta}{2} & 0 \\ 0 & -(i)^2\sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} \cos^2\frac{\theta}{2} + \sin^2\frac{\theta}{2} & 0 \\ 0 & \sin^2\frac{\theta}{2} + \cos^2\frac{\theta}{2} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

- $R_x(\theta) = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}$

$$(R_x)(R_x^t)^* = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta} \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & e^{-i\theta} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{i\theta}e^{-i\theta} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & e^{0} \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix} = I$$

2 Introducción a los qubits y medición

1. Verifique que los siguientes estados están bien definidos escribiendo el procedimiento completo:

- $|0\rangle - i|1\rangle \Rightarrow |1|^2 + \sqrt{0^2+(1)^2}^2 = 1+1 = 2 \neq 1$ **No**
- $\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle \Rightarrow (\cos\theta)^2 + \sqrt{0^2+(\sin\theta)^2}^2 = \cos^2\theta + \sin^2\theta = 1$ **Si**
- $\frac{1}{\sqrt{2}}|0\rangle - \frac{1}{\sqrt{2}}|1\rangle \Rightarrow \left(\frac{1}{\sqrt{2}}\right)^2 + \left(-\frac{1}{\sqrt{2}}\right)^2 = \frac{1}{2}+\frac{1}{2} = 1$ **Si**

2. Dados los estados cuánticos $|\phi\rangle = \frac{i}{\sqrt{2}}|0\rangle + \frac{i}{\sqrt{2}}|1\rangle$ y $|\psi\rangle = \frac{\sqrt{3}}{2}|0\rangle + \frac{1}{\sqrt{4}}|1\rangle$ calcula (escribiendo y justificando los paso):

- $\langle\phi|\psi\rangle \Rightarrow \begin{pmatrix} -\frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{2}} \end{pmatrix}\begin{pmatrix} \frac{\sqrt{3}}{2} \\ \frac{1}{\sqrt{4}} \end{pmatrix} = -\frac{i}{\sqrt{2}}\frac{\sqrt{3}}{2} - \frac{i}{\sqrt{2}}\frac{1}{\sqrt{4}} = -\frac{i\sqrt{3}+i}{2\sqrt{2}} = -i\frac{\sqrt{3}+1}{2\sqrt{2}}$
- $\langle\psi|\phi\rangle \Rightarrow \begin{pmatrix} \frac{\sqrt{3}}{2} & \frac{1}{\sqrt{4}} \end{pmatrix}\begin{pmatrix} \frac{i}{\sqrt{2}} \\ \frac{i}{\sqrt{2}} \end{pmatrix} = \frac{\sqrt{3}}{2}\frac{i}{\sqrt{2}} + \frac{1}{\sqrt{4}}\frac{i}{\sqrt{2}} = \frac{i\sqrt{3}+i}{2\sqrt{2}} = i\frac{\sqrt{3}+1}{2\sqrt{2}}$

- $\langle\psi|\psi\rangle \Rightarrow \begin{pmatrix}\frac{\sqrt{3}}{2} & \frac{1}{\sqrt{4}}\end{pmatrix}\begin{pmatrix}\frac{\sqrt{3}}{2}\\ \frac{1}{\sqrt{4}}\end{pmatrix} = \frac{\sqrt{3}}{2}\frac{\sqrt{3}}{2} + \frac{1}{\sqrt{4}}\frac{1}{\sqrt{4}} = \frac{3}{4} + \frac{1}{4} = 1$
- $\langle\phi|\phi\rangle \Rightarrow \begin{pmatrix}-\frac{i}{\sqrt{2}} & -\frac{i}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{i}{\sqrt{2}}\\ \frac{i}{\sqrt{2}}\end{pmatrix} = -\frac{i}{\sqrt{2}}\frac{i}{\sqrt{2}} - \frac{i}{\sqrt{2}}\frac{i}{\sqrt{2}} = -2\frac{i}{\sqrt{2}}\frac{i}{\sqrt{2}} = -\frac{2i^2}{2} = 1$

3. Escribe y justifica todos los pasos para calcular lo siguiente:

- $|+\rangle\langle+| \Rightarrow \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1\\ 1 & 1\end{pmatrix}$
- $|+\rangle\langle-| \Rightarrow \begin{pmatrix}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\\ \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & -1\\ 1 & -1\end{pmatrix}$
- $|-\rangle\langle-| \Rightarrow \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & -1\\ -1 & 1\end{pmatrix}$
- $|-\rangle\langle+| \Rightarrow \begin{pmatrix}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\end{pmatrix}\begin{pmatrix}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\end{pmatrix} = \begin{pmatrix}\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\\ -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}}\frac{1}{\sqrt{2}}\end{pmatrix} = \frac{1}{2}\begin{pmatrix}1 & 1\\ -1 & -1\end{pmatrix}$

4. Dado el estado $|\boldsymbol{\phi}\rangle = \cos\boldsymbol{\theta}\,|\mathbf{0}\rangle + \boldsymbol{i}\sin\boldsymbol{\theta}\,|\mathbf{1}\rangle$ escribe todos los pasos para calcular los siguiente:

Sol. En 2.1.2. ya demostramos que, si cumple con ser un estado bien normalizado,

- $|\phi\rangle_{pm}^{0},\ \{a_0\} = \{0\} \Rightarrow \{\hat{M}_0\} = \{|0\rangle\langle 0|\} \Rightarrow |\phi\rangle_{pm}^{0} = \frac{\hat{M}_0|\phi\rangle}{\sqrt{P(0)}} = \frac{\hat{M}_0|\phi\rangle}{\sqrt{\langle\phi|\hat{M}_0^*\hat{M}_0|\phi\rangle}}$

$$\hat{M}_0 = |0\rangle\langle 0| = \begin{pmatrix}1\\0\end{pmatrix}\begin{pmatrix}1 & 0\end{pmatrix} = \begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}$$

$$\langle\phi|\hat{M}_0^*\hat{M}_0|\phi\rangle = \begin{pmatrix}\cos\theta & -i\sin\theta\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\cos\theta\\ i\sin\theta\end{pmatrix} = \begin{pmatrix}\cos\theta & 0\end{pmatrix}\begin{pmatrix}\cos\theta\\ 0\end{pmatrix} = \cos^2\theta$$

$$|\phi\rangle_{pm}^{0} = \frac{\begin{pmatrix}1 & 0\\ 0 & 0\end{pmatrix}\begin{pmatrix}\cos\theta\\ i\sin\theta\end{pmatrix}}{\sqrt{\cos^2\theta}} = \frac{\begin{pmatrix}\cos\theta\\ 0\end{pmatrix}}{\cos\theta} = \begin{pmatrix}1\\0\end{pmatrix} = |0\rangle$$

- $|\phi\rangle_{pm}^{+},\ \{a_+\} = \{+\} \Rightarrow \{\hat{M}_+\} = \{|+\rangle\langle +|\} \Rightarrow |\phi\rangle_{pm}^{+} = \frac{\hat{M}_+|\phi\rangle}{\sqrt{P(+)}} = \frac{\hat{M}_+|\phi\rangle}{\sqrt{\langle\phi|\hat{M}_+^*\hat{M}_+|\phi\rangle}}$

$$\begin{aligned}
\langle\phi|\hat{M}_+^*\hat{M}_+|\phi\rangle &= (\cos\theta\,\langle 0| - i\sin\theta\,\langle 1|)(|+\rangle\langle +||+\rangle\langle +|)(\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle)\\
&= (\cos\theta\,\langle 0| - i\sin\theta\,\langle 1|)\left(\frac{|0\rangle + |1\rangle}{\sqrt{2}}\,\frac{\langle 0| + \langle 1|}{\sqrt{2}}\frac{|0\rangle + |1\rangle}{\sqrt{2}}\frac{\langle 0| + \langle 1|}{\sqrt{2}}\right)(\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle)\\
&= \frac{1}{\sqrt{2}^4}(\cos\theta\,\langle 0| - i\sin\theta\,\langle 1|)\big((|0\rangle + |1\rangle)\,(\langle 0| + \langle 1|)(|0\rangle + |1\rangle)(\langle 0| + \langle 1|)\big)(\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle)\\
&= \frac{1}{4}(\cos\theta\,\langle 0| - i\sin\theta\,\langle 1|)\big((|0\rangle + |1\rangle)(1+1)(\langle 0| + \langle 1|)\big)(\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle)\\
&= \frac{2}{4}(\cos\theta\,\langle 0| - i\sin\theta\,\langle 1|)(|0\rangle + |1\rangle)(\langle 0| + \langle 1|)(\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle)\\
&= \frac{1}{2}[\cos\theta\,\langle 0||0\rangle - i\sin\theta\,\langle 1||1\rangle][\langle 0|\cos\theta\,|0\rangle + \langle 1|i\sin\theta\,|1\rangle]\\
&= \frac{1}{2}[\cos\theta - i\sin\theta][\cos\theta + i\sin\theta]
\end{aligned}$$

$$= \frac{1}{2}[\cos^2\theta + i\sin^2\theta]$$
$$= \frac{1}{2}$$

$$\begin{aligned}
|\phi\rangle_{pm}^{+} &= \frac{|+\rangle\langle+|(\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle)}{\frac{1}{\sqrt{2}}} \\
&= \frac{\frac{|0\rangle + |1\rangle}{\sqrt{2}}\,\frac{\langle 0| + \langle 1|}{\sqrt{2}}(\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle)}{\frac{1}{\sqrt{2}}} \\
&= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)(\langle 0| + \langle 1|)(\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle) \\
&= \frac{1}{\sqrt{2}}(|0\rangle + |1\rangle)(\langle 0|\cos\theta\,|0\rangle + \langle 1|i\sin\theta\,|1\rangle) \\
&= \frac{1}{\sqrt{2}}(\cos\theta\,|0\rangle + i\sin\theta\,|1\rangle) \\
&= e^{i\theta}|x\rangle
\end{aligned}$$