Universidad Nacional Autónoma de México

Temas selectos de Ingeniería en Computación III

(Introducción a la Computación Cuántica)

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Tarea moral 2

1. **Dada la definición de $\boldsymbol{R_x(\theta)}$ vista en clase, resuelve lo siguiente:**

- $R_x(\theta)|0\rangle \Rightarrow R_x(\theta)|0\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix}\begin{pmatrix} 1 \\ 0 \end{pmatrix} = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) \end{pmatrix} = \cos\left(\frac{\theta}{2}\right)|0\rangle - \sin\left(\frac{\theta}{2}\right)|1\rangle$
- $R_x(\theta)|1\rangle \Rightarrow R_x(\theta)|1\rangle = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} -i\sin\left(\frac{\theta}{2}\right) \\ \cos\left(\frac{\theta}{2}\right) \end{pmatrix} = -i\sin\left(\frac{\theta}{2}\right)|0\rangle + \cos\left(\frac{\theta}{2}\right)|1\rangle$
- $R_x(\theta)(\alpha|0\rangle + \beta|1\rangle) \Rightarrow R_x(\theta)(\alpha|0\rangle + \beta|1\rangle) = \begin{pmatrix} \cos\left(\frac{\theta}{2}\right) & -i\sin\left(\frac{\theta}{2}\right) \\ -i\sin\left(\frac{\theta}{2}\right) & \cos\left(\frac{\theta}{2}\right) \end{pmatrix}\begin{pmatrix} \alpha \\ \beta \end{pmatrix} = \begin{pmatrix} \alpha\cos\left(\frac{\theta}{2}\right) - i\beta\sin\left(\frac{\theta}{2}\right) \\ -i\alpha\sin\left(\frac{\theta}{2}\right) + \beta\cos\left(\frac{\theta}{2}\right) \end{pmatrix}$

$$= \left[\alpha\cos\left(\frac{\theta}{2}\right) - i\beta\sin\left(\frac{\theta}{2}\right)\right]|0\rangle + \left[-i\alpha\sin\left(\frac{\theta}{2}\right) + \beta\cos\left(\frac{\theta}{2}\right)\right]|1\rangle$$

2. **Calcula lo siguiente ocupando las definiciones de las puertas cuánticas vistas en clase:**

- $\sigma_y H|1\rangle \Rightarrow \sigma_y H|1\rangle = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 0 \\ 1 \end{pmatrix} = \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}|-\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} i \\ i \end{pmatrix} = \frac{i|0\rangle - i|1\rangle}{\sqrt{2}}$
- $\sigma_x H|-\rangle \Rightarrow \sigma_x H|-\rangle = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix}\begin{pmatrix} 0 \\ 2 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 2 \\ 0 \end{pmatrix} = \begin{pmatrix} 1 \\ 0 \end{pmatrix} = |0\rangle$

3. **Siguiendo la definición de producto tensorial visto en clase; calcula lo siguiente:´**

- $\sigma_x \otimes \sigma_y \Rightarrow \sigma_x \otimes \sigma_y = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} \otimes \begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} = \begin{pmatrix} 0\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & 1\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \\ 1\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} & 0\begin{pmatrix} 0 & -i \\ i & 0 \end{pmatrix} \end{pmatrix} = \begin{pmatrix} 0 & 0 & 0 & -i \\ 0 & 0 & i & 0 \\ 0 & -i & 0 & 0 \\ i & 0 & 0 & 0 \end{pmatrix}$
- $H \otimes \sigma_z \Rightarrow H \otimes \sigma_z = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} & 1\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \\ 1\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} & -1\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & -1 & 0 & -1 \\ 1 & 0 & -1 & 0 \\ 0 & -1 & 0 & 1 \end{pmatrix}$
- $H \otimes |1\rangle \Rightarrow H \otimes |1\rangle = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} \otimes \begin{pmatrix} 0 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1\begin{pmatrix} 0 \\ 1 \end{pmatrix} & 1\begin{pmatrix} 0 \\ 1 \end{pmatrix} \\ 1\begin{pmatrix} 0 \\ 1 \end{pmatrix} & -1\begin{pmatrix} 0 \\ 1 \end{pmatrix} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 0 & 0 \\ 1 & 1 \\ 0 & 0 \\ 1 & -1 \end{pmatrix}$
- $|0\rangle \otimes |+\rangle \Rightarrow |0\rangle \otimes |+\rangle = \begin{pmatrix} 1 \\ 0 \end{pmatrix} \otimes \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1\begin{pmatrix} 1 \\ 1 \end{pmatrix} \\ 0\begin{pmatrix} 1 \\ 1 \end{pmatrix} \end{pmatrix} = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 \\ 1 \\ 0 \\ 0 \end{pmatrix}$

4. **Demuestra la siguiente igualdad: $\boldsymbol{H\sigma_z H = \sigma_x}$**

$$H\sigma_z H = \frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix}\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\frac{1}{\sqrt{2}}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{\sqrt{2}^2}\begin{pmatrix} 1 & -1 \\ 1 & 1 \end{pmatrix}\begin{pmatrix} 1 & 1 \\ 1 & -1 \end{pmatrix} = \frac{1}{2}\begin{pmatrix} 0 & 2 \\ 2 & 0 \end{pmatrix} = \begin{pmatrix} 0 & 1 \\ 1 & 0 \end{pmatrix} = \sigma_x$$

$$\therefore H\sigma_z H = \sigma_x \quad \blacksquare$$