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Applied Linear Algebra Test 1 , Variant 43

1. $$P = \begin{bmatrix} -2 & -1 & 2 & 3 \\ -4 & 20 & -10 & -17 \end{bmatrix} \begin{matrix} \to x_i \\ \to y_i \end{matrix}$$

Solution:

By Lagrange Formula

$$p(x) = \sum_{i=1}^{n} y_i \prod_{\substack{i=1 \\ j\neq i}}^{n} \frac{x-x_j}{x_i-x_j}$$; n: the number of points

$$p(x) = -4\frac{(x+1)(x-2)(x-3)}{(-2+1)(-2-2)(-2-3)} + 20\frac{(x+2)(x-2)(x-3)}{(-1+2)(-1-2)(-1-3)}$$

$$-10\frac{(x+2)(x+1)(x-3)}{(2+2)(2+1)(2-3)} - 17\frac{(x+2)(x+1)(x-2)}{(3+2)(3+1)(3-2)}$$

$$-4\frac{(x+1)(x^2-5x+6)}{(-1)(-4)(-5)} = \frac{1}{5}(x^3-5x^2+6x+x^2-5x+6)$$
$$= \frac{1}{5}(x^3-4x^2+x+6) \quad — 1$$

$$20\frac{(x-3)(x^2-4)}{(1)(-3)(-4)} = \frac{5}{3}(x^3-4x-3x^2+12) \quad — 2$$

1 + 2

$$\frac{3}{15}(x^3-4x^2+x+6) + \frac{25}{3}(x^3-4x-3x^2+12)$$

$$= \frac{28}{15}x^3 - \frac{87}{15}x^2 - \frac{97}{15}x + \frac{318}{15}$$

$\boxed{2}$

- $-10\,\dfrac{(x+2)(x^2-2x-3)}{(4)(3)(-1)} = \dfrac{5}{6}(x^3-2x^2-3x+2x^2-4x-6)$

$= \dfrac{5}{6}(x^3-7x-6)$ — 3

- $-17\,\dfrac{(x^2-4)(x+1)}{(5)(4)(1)} = (x^3+x^2-4x-4)\left(\dfrac{-17}{20}\right)$ — 4

3 + 4

$= \dfrac{50}{60}(x^3-7x-6) - \dfrac{51}{60}(x^3+x^2-4x-4)$

$-\dfrac{1}{60}x^3 - \dfrac{51}{60}x^2 - \dfrac{146}{60}x - \dfrac{96}{60}$ — ★

$p(x) = ★ + ★$

$\dfrac{28}{15}x^3 - \dfrac{87}{15}x^2 - \dfrac{97}{15}x + \dfrac{318}{15} - \dfrac{1}{60}x^3 - \dfrac{51}{60}x^2 \dfrac{146}{60}x - \dfrac{96}{60}$

$\rightarrow p(x) = \dfrac{111}{60}x^3 - \dfrac{399}{60}x^2 - \dfrac{534}{60}x + \dfrac{1176}{60}$

Answer $\Rightarrow p(x) = \dfrac{37}{20}x^3 - \dfrac{133}{20}x^2 - \dfrac{89}{10}x + \dfrac{196}{10}$

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2. Bezier curve

$$P = \left[\begin{array}{cccc} 1 & 3 & 5 & 7 \\ 8 & 8 & 3 & 4 \end{array}\right] \begin{array}{l} \to x_i \\ \to y_i \end{array}$$

$$B(t) = \sum_{k=0}^{n-1} P_n^k(t) A_k \quad ; \quad P_n^k = \binom{n}{k}(1-t)^{n-k} t^k$$

- $B_x(t) = (1-t)^3(1) + 3t(1-t)^2(3) + 3t^2(1-t)(5) + t^3(7)$

$= (-t^3+3t^2-3t+1) + 9t(t^2-2t+1) + 15t^2(1-t) + 7t^3$

$= -t^3 + 3t^2 - 3t + 1 + 9t^3 - 18t^2 + 9t + 15t^2 - 15t^3 + 7t^3$

$B_x(t) = 6t + 1$

- $B_y(t) = (1-t)^3(8) + 3t(1-t)^2(8) + 3t^2(1-t)(3) + t^3(4)$

$= (-t^3+3t^2-3t+1)(8) + 24t(t^2-2t+1) + 9t^2(1-t) + 4t^3$

$= -8t^3 + 24t^2 - 24t + 8 + 24t^3 - 48t^2 + 24t + 9t^2 - 9t^3 + 4t^3$

$B_y(t) = 11t^3 - 15t^2 + 8$

$$x = 6t + 1 \implies t = \frac{x-1}{6}$$

$B(x) = 11\left(\frac{x-1}{6}\right)^3 - 15\left(\frac{x-1}{6}\right)^2 + 8$

$= \frac{11}{216}(x-1)^3 - \frac{15}{36}(x-1)^2 + 8$

$= \frac{11}{216}(x^3 - 3x^2 + 3x - 1) - \frac{15}{36}(x^2 - 2x + 1) + 8$

$$\boxed{B(x) = \frac{11}{216}x^3 - \frac{123}{216}x^2 + \frac{213}{216}x + \frac{1627}{216}}$$

4 Answer:

$$B(t) = \begin{pmatrix} 6t + 1 \\ 11t^3 - 15t^2 + 8 \end{pmatrix}$$

$$\cancel{B(x) = \frac{11}{216}} t^3$$

$$B(x) = \frac{11}{216}x^3 - \frac{123}{216}x^2 + \frac{213}{216}x + \frac{1627}{216}$$

5 3.

$$A = \begin{pmatrix} 18 & -4 & -4 \\ 9 & 2 & 2 \\ 0 & 8 & 8 \\ -9 & 14 & 14 \end{pmatrix}$$

Solution:

Find a full rank decomposition by applying Gaussian

$$A = \begin{pmatrix} 18 & -4 & -4 \\ 9 & 2 & 2 \\ 0 & 8 & 8 \\ -9 & 14 & 14 \end{pmatrix} \overset{\frac{1}{4} I_3}{\sim} \begin{pmatrix} 18 & -4 & -4 \\ 9 & 2 & 2 \\ 0 & 2 & 2 \\ -9 & 14 & 14 \end{pmatrix} \underset{I_1 = 2I_3 + I_1}{\overset{I_2 = -I_3 + I_2}{\sim}} \begin{pmatrix} 18 & 0 & 0 \\ 9 & 0 & 0 \\ 0 & 2 & 2 \\ -9 & 14 & 14 \end{pmatrix}$$

$$\overset{I_1 = -2I_2 + I_1}{\sim} \begin{pmatrix} 0 & 0 & 0 \\ 9 & 0 & 0 \\ 0 & 2 & 2 \\ -9 & 14 & 14 \end{pmatrix} \underset{I_4 = -7I_3 + I_4}{\overset{I_4 = I_2 + I_4}{\sim}} \begin{pmatrix} 0 & 0 & 0 \\ 9 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \end{pmatrix} \underset{I_2 \leftrightarrow I_3}{\overset{I_1 \leftrightarrow I_2}{\sim}} \begin{pmatrix} 9 & 0 & 0 \\ 0 & 2 & 2 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \end{pmatrix}$$

$$\underset{I_2 = \frac{1}{2} I_2}{\overset{I_1 = \frac{1}{9} I_1}{\sim}} \begin{pmatrix} \boxed{\begin{matrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{matrix}}^{\,G} \\ \begin{matrix} 0 & 0 & 0 \\ 0 & 0 & 0 \end{matrix} \end{pmatrix}$$

$\boxed{6}$

$$A = FG$$

$$F = (A^1 \; A^2) = \begin{pmatrix} 18 & -4 \\ 9 & 2 \\ 0 & 8 \\ -9 & 14 \end{pmatrix}$$

$$A = \overset{F}{\begin{pmatrix} 18 & -4 \\ 9 & 2 \\ 0 & 8 \\ -9 & 14 \end{pmatrix}} \cdot \overset{G}{\begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix}}$$

Now we will find pesudoinverses for both F and G

$$A^+ = G^+ F^+$$

$$G^+ = G^*(GG^*)^{-1}$$

$$F = (F^* F)^{-1} F^*$$

$$GG^* = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 1 \end{pmatrix} \cdot \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} = \begin{pmatrix} 1 & 0 \\ 0 & 2 \end{pmatrix}$$

$$(GG^*)^{-1} = \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} \quad ; \quad det = 2$$

$$G^+ = G^*(GG^*)^{-1} = \frac{1}{2} \begin{pmatrix} 1 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \cdot \begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix} = \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix}$$

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$$F^+ = (F^*F)^{-1}F^*$$

$$F^*F = \begin{pmatrix} 18 & 9 & 0 & -9 \\ -4 & 2 & 8 & 14 \end{pmatrix} \cdot \begin{pmatrix} 18 & -4 \\ 9 & 2 \\ 0 & 8 \\ -9 & 14 \end{pmatrix} = \begin{pmatrix} 486 & -180 \\ -180 & 280 \end{pmatrix}$$

; det = 103680

$$(B^*B)^{-1} = \frac{1}{103680} \begin{pmatrix} 280 & 180 \\ 180 & 486 \end{pmatrix}$$

$$F^+ = (F^*F)^{-1}F^* = \frac{1}{103680} \begin{pmatrix} 280 & 180 \\ 180 & 486 \end{pmatrix} \cdot \begin{pmatrix} 18 & 9 & 0 & -9 \\ -4 & 2 & 8 & 14 \end{pmatrix}$$

$$= \frac{1}{103680} \begin{pmatrix} 4320 & 2880 & 1440 & 0 \\ 1296 & 2592 & 3888 & 6642 \end{pmatrix}$$

$$A^+ = G^+ F^+$$

$$= \frac{1}{2} \begin{pmatrix} 2 & 0 \\ 0 & 1 \\ 0 & 1 \end{pmatrix} \cdot \frac{1}{103680} \begin{pmatrix} 4320 & 2880 & 1440 & 0 \\ 1296 & 2592 & 3888 & 6642 \end{pmatrix}$$

$$\boxed{A^+ = \frac{1}{2} \times \frac{1}{103680} \begin{pmatrix} 8640 & 5760 & 2880 & 0 \\ 1296 & 2592 & 3888 & 6642 \\ 1296 & 2592 & 3888 & 6642 \end{pmatrix}}$$

$\boxed{9}$ 4. minimal length least squres Solution

$$3x + 0y + 5z + 6t = 7$$

$$9x + 2y + 2z + 0t = 9$$

$$8x + 8y + 3z + 4t = 2$$

$$10x + 4y + 1z - 4t = 9$$

$$A = \begin{pmatrix} 3 & 0 & 5 & 6 \\ 9 & 2 & 2 & 0 \\ 8 & 8 & 3 & 4 \\ 10 & -4 & 1 & -4 \end{pmatrix} \qquad B = \begin{pmatrix} 7 \\ 9 \\ 2 \\ 9 \end{pmatrix}$$

$$\hat{X} = A^{+}B$$

· First we will find a full rank decomposition of A to find A^{+}

$$A \sim \begin{pmatrix} 1 & 0 & 5/3 & 2 \\ 0 & 2 & -13 & -18 \\ 0 & 0 & \frac{125}{3} & 60 \\ 0 & 0 & -\frac{125}{3} & -60 \end{pmatrix} \sim \begin{pmatrix} 1 & 0 & 5/3 & 2 \\ 0 & 1 & -13/2 & -9 \\ 0 & 0 & 1 & 36/25 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

$\boxed{10}$

$$\sim \begin{pmatrix} 1 & 0 & 0 & -2/5 \\ 0 & 1 & 0 & 9/25 \\ 0 & 0 & 1 & 36/25 \\ 0 & 0 & 0 & 0 \end{pmatrix}$$

G

rank = 3

$$F = \begin{pmatrix} 3 & 0 & 5 \\ 9 & 2 & 2 \\ 8 & 8 & 3 \\ 10 & -4 & 1 \end{pmatrix} \qquad G = \begin{pmatrix} 1 & 0 & 0 & -2/5 \\ 0 & 1 & 0 & 9/25 \\ 0 & 0 & 1 & 36/25 \end{pmatrix}$$

$$A^+ = G^+ F^+$$

$$G^+ = G^*(GG^*)^{-1} \qquad F = (F^*F)^{-1}F^*$$

$$G^+ = \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \\ -2/5 & 9/25 & 36/25 \end{pmatrix} \cdot \begin{pmatrix} 1001/1051 & 45/1051 & 180/1051 \\ 45/1051 & 2021/2102 & -162/1051 \\ 180/1051 & -162/1051 & 403/1051 \end{pmatrix}$$

$$= \begin{pmatrix} 1001/1051 & 45/1051 & 180/1051 \\ 45/1051 & 2021/2102 & -162/1051 \\ 180/1051 & -162/1051 & 403/1051 \\ -125/1051 & 225/2102 & 450/1051 \end{pmatrix}$$

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III $F^+ = \begin{pmatrix} 9/1250 & -1/12500 & -77/6250 \\ -1/12500 & 5417/375000 & -547/62500 \\ -77/6250 & -547/62500 & 1631/31250 \end{pmatrix} \begin{pmatrix} 3 & 9 & 8 & 10 \\ 0 & 2 & 8 & -4 \\ 5 & 2 & 3 & 1 \end{pmatrix}$

$= \begin{pmatrix} -1/25 & 1/25 & 1/50 & 3/50 \\ -11/250 & 4/375 & 133/1500 & -101/1500 \\ 28/125 & -3/125 & -3/250 & -9/250 \end{pmatrix}$

$A^+ = G^+ F^+$

$A^+ = \begin{pmatrix} -17/10510 & 181/5255 & 437/21020 & 1011/21020 \\ -1651/21020 & 247/15765 & 11093/126120 & -7141/126120 \\ 451/5255 & -21/5255 & -78/5255 & 36/5255 \\ 2017/21020 & -73/5255 & 83/42040 & -1251/42040 \end{pmatrix}$

$\hat{X} = A^+ B$

$\Rightarrow \hat{X} = \begin{pmatrix} 16251/21020 \\ -93641/126120 \\ 3136/5255 \\ 11889/42040 \end{pmatrix} \begin{matrix} \rightarrow x \\ \rightarrow y \\ \rightarrow z \\ \rightarrow t \end{matrix}$

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$r = \|A\hat{X} - B\|$

$$= \left\| \begin{pmatrix} 7 \\ 20/3 \\ 19/6 \\ 61/6 \end{pmatrix} - \begin{pmatrix} 7 \\ 9 \\ 2 \\ 9 \end{pmatrix} \right\| = \left\| \begin{pmatrix} 0 \\ -2.33 \\ 1.16 \\ 1.16 \end{pmatrix} \right\|$$

$$\rightarrow r = \sqrt{(0)^2 + (-2.33)^2 + (1.16)^2 + (1.16)^2}$$

$$r = 2.84957$$

13 5. $p(x) = x^3 + 3x^2 - 4x + 4 \qquad [-2,5]$

$T_3(x) = 4x^3 - 3x$ Chebyshev polynomial deg = 3

$$\overline{T}_3(x) = \frac{(b-a)^n}{2^{2n-1}} T_n\left(\frac{2x-(b+a)}{b-a}\right)$$

; $a = -2$
$b = 5$
$n = 3$

$$\overline{T}_3(x) = \frac{7^3}{32} T_3\left(\frac{2x-3}{7}\right)$$

$$\rightarrow \frac{7^3}{32}\left(4\left(\frac{2x-3}{7}\right)^3 - 3\left(\frac{2x-3}{7}\right)\right)$$

$$\overline{T}_3(x) = x^3 - \frac{9}{2}x^2 - \frac{78}{32}x - \frac{549}{32}$$

$$\|p(x) - q(x)\|_\infty \rightarrow \min \iff \|r(x)\|_\infty \rightarrow \min$$

$$x^3 + 3x^2 - 4x + 4 - q(x) = \overline{T}_3(x)$$

$$x^3 + 3x^2 - 4x + 4 - q(x) = x^3 - \frac{9}{2}x^2 - \frac{78}{32}x - \frac{549}{32}$$

$$q(x) = +\frac{15}{2}x^2 - \frac{50}{32}x + \frac{677}{32}$$

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14 6. $2x^2 + y^2(4q+1) + yz(-2q+6) + z^2(4q+3) = 1$

$v^0(1, 1, 1)$

Solution:

By Minkowski theorem.

To ensure that this surface is bounded we should check positive definitness of the respective symmetric matrix using Sylvester criterion.

$$B = \begin{pmatrix} 2 & 0 & 0 \\ 0 & 1+4q & 6-2q \\ 0 & 6-2q & 3+4q \end{pmatrix}$$

$$\Delta_1 = |2| = 2 > 0$$

$$\Delta_2 = \begin{vmatrix} 2 & 0 \\ 0 & 1+4q \end{vmatrix} = 2 + 8q > 0$$

$$\Delta_3 = \begin{vmatrix} 2 & 0 & 0 \\ 0 & 1+4q & 6-2q \\ 0 & 6-2q & 3+4q \end{vmatrix} = 24q^2 + 80q - 66 > 0$$

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$$\begin{cases} 2+8q>0 \\ 24q^2+80q-66>0 \end{cases} \Longleftrightarrow \begin{cases} q>-\frac{1}{4} \\ q<-\frac{25}{6} \\ q>\frac{5}{6} \end{cases} \Longleftrightarrow q\in\left(\frac{5}{6},+\infty\right)$$

- $24q^2+80q-66=0$

$\sqrt{D}=4.7\simeq 5$

$q_1=\frac{-25}{6}$

$q_2=\frac{5}{6}$

$$\Rightarrow q\in\left(\frac{5}{6},+\infty\right)$$

To calculate the $N_q((1,1,1)^T)$ we should find a Codirectional vector that lies on the discribed above unit circle.

Let $v=(1,1,1)^T$, $v^o\,(t,t,t)^T$ such that $N_q(v^o)=1$

$$2t^2+t^2(4q+1)+t^2(-2q+6)+t^2(4q+3)=1$$

$$t^2(6q+12)=1 \qquad t^2=\frac{1}{6q+12}$$

$$\Rightarrow t=\frac{1}{\sqrt{6q+12}}$$

$$v^o=\begin{pmatrix} 1/\sqrt{6q+12} \\ 1/\sqrt{6q+12} \\ 1/\sqrt{6q+12} \end{pmatrix} \qquad N_q(v)=\sqrt{6q+12}$$

$$q\in\left(\frac{5}{6},+\infty\right), N_q((1,1,1)^T)=\sqrt{6q+12}$$

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