Concordia University
ELEC 342 Discrete Time Signals and Systems
Final Exam – Winter 2023

Students are allowed three 8.5*11 inch formula sheets. Anything can be written on these sheets and both sides may be written on. The ENCS calculator, pens, pencils, erasers and straightedges are also allowed.

If you have difficulty you may try making REASONABLE assumptions. State the assumption and how the assumption limits your answer.

Show all your work and justify all your answers. Marks are given for how an answer is arrived at not just the answer itself.

1. Consider a causal LTI system that is implemented with the following LCCDE

$$y[k] + \frac{3}{4}y[k-1] + \frac{1}{8}y[k-2] = x[k-1] + x[k-2]$$

a) Write down the transfer function H(z) of this system. Be sure to state the ROC. **(3 marks)**
b) Is this system stable? Justify your response. **(2 marks)**
c) Write down the impulse response of this system. **(5 marks)**
d) Write down the frequency response of this system. **(2 marks)**
e) If the input is $x[k] = j^k$ write down the output y[k]. **(4 marks)**

2. Given

$$x[k] = 0.3^k(u[k] - u[k-5])$$

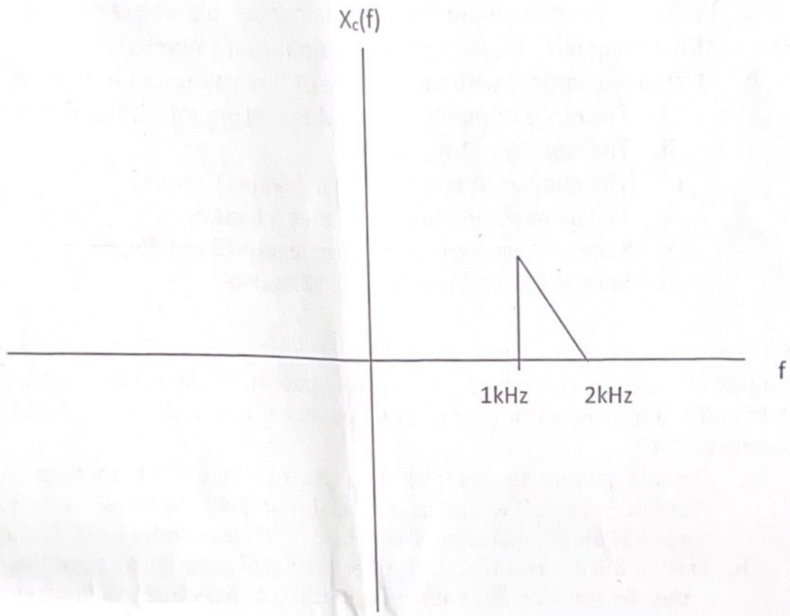
And

$$h[k] = \begin{cases} 1 & 1 \le k \le 7 \\ 0 & otherwise \end{cases}$$

find y[k]=x[k]*h[k] where "*" means convolution. This question <u>**must**</u> be done by using convolution in the time domain. **(8 marks)**

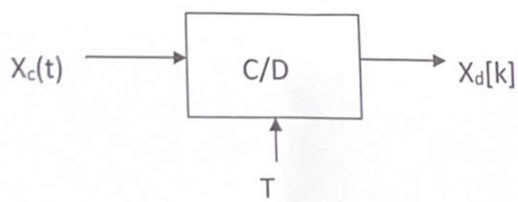
3. You are given a graph or plot of the spectrum of a signal that you don't know much about.
   a. Can you tell from the spectrum whether the signal is discrete time or continuous time? If yes explain how you would tell which one it is. **(2 marks)**
   b. Assuming the signal is a discrete time signal, can you tell from spectrum whether the signal is periodic in the time domain or not? If yes explain how you would tell that. **(2 marks)**
   c. Assuming the signal is a discrete time signal, can you tell from the spectrum whether the signal is real in the time domain? If yes explain how you would tell that. **(2 marks)**
   d. Assuming the signal is a discrete time signal could you tell from the spectrum whether the time signal is absolutely summable or not? If so explain how you would tell that. **(2 marks)**

4. Consider a continuous time signal with spectrum as shown below.



Note that f is frequency and is in Hz. **Assume the height of the spectrum is 1.**

$x_c(t)$ is to be put through the following system:



a) What is the maximum frequency in $x_c(t)$? **(1 mark)**

b) State a sampling rate so that $x_c(t)$ could be recovered from $x_d[k]$. **(2 marks)**

c) State the sampling period corresponding to the sampling rate in b). **(1 mark)**

d) Let T = 1/5000 seconds. Draw the spectrum $X_d(\Omega)$. Label all points of interest. **(4 marks)**

e) Let T = 1/10000 seconds. Draw the spectrum $X_d(\Omega)$. Label all points of interest. **(4 marks)**

f) Can $x_c(t)$ be recovered from the $x_d[k]$ in part d)? Explain. **(2 marks)**

g) Can $x_c(t)$ be recovered from the $x_d[k]$ in part e)? Explain. **(3 marks)**

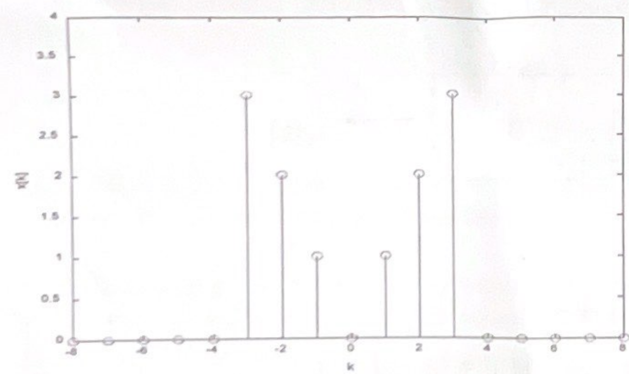
5. A continuous time signal that varies between 0 and 15 volts is to be sampled at 10kHz and the then quantized to form a digital signal. This digital signal along with 999 other similar signals are packaged together and sent on a 60 Megabit/sec channel.

   a. In the above scenario how many bits per sample would be available for quantizing one of these signals? Explain your calculations. **(3 marks)**
   b. Design a quantizer with the number of bits you found in a) for this signal.
      i. The range of inputs that would yield granular noise for your quantizer. **(1 mark)**
      ii. The step size **(2 marks)**
      iii. The number of reconstruction levels **(1 mark)**
      iv. The quantization noise variance. **(1 mark)**
      v. State at least 6 reconstruction levels. **(2 marks)**
      vi. State at least 6 break points. **(2 marks)**

6. We wish to design an IIR Low pass filter using the impulse invariance method and the Butterworth continuous time filter. The low pass filter should have a passband between 0 and 0.4π and a stop band from 0.6π to π. Its passband ripple should 0.15 and its stop band ripple should be 0.12.

   a. Draw a picture to describe this specification. Your drawing should mark clearly "forbidden zones" where the designed filter magnitude response should not go into. Be sure to mark all interesting frequencies and magnitudes on your diagram. **(3 marks)**
   b. Design the discrete time LPF filter. Show all your work. Your final answer should be a transfer function H(z) of the designed Low pass filter. **(8 marks)**

7. Consider the discrete time signal shown below. **Each part below is worth 2 marks.**



a) Draw a picture of $y_1[k] = x[k]\delta[k-2]$. Be sure to mark all interesting points.
b) Draw a picture of $y_2[k] = x[k](u[k+1] - u[k-2])$. Be sure to mark all interesting points.
c) Draw a picture of $y_3[k] = x[k] * (\delta[k+6] + \delta[k-5])$ here * means convolution. Be sure to mark all interesting points.
d) Draw a picture of $y_4[k] = x[k-5]$ Be sure to mark all interesting points.
e) Draw a picture of $y_5[k] = x[-k+2]$. Be sure to mark all interesting points.

Total **82 marks.**