ELEC 342

Midterm Oct 2023

1a) Period $k_0 = 6$

b) $\Omega_0 = \frac{2\pi}{k_0} = \frac{2\pi}{6} = \frac{\pi}{3}$

c)

$$D_n = \frac{1}{K_0} \sum_{K=\langle K_0 \rangle} x[k] e^{-jn\Omega_0 k}$$

we will use interval $K = 0 \ldots 5$

$$D_n = \frac{1}{6}\left[e^{-jn\Omega_0 2} + 3e^{-jn\Omega_0 3} + 2e^{-jn\Omega_0 4} \right]$$

$n = 0, 1, 2 \ldots 5$

Note $D_n = D_{n+6}$

d) D_n not real

e) We didn't expect D_n to be real because $x[k]$ was not conjugate symmetric

f) $$x[k] = \sum_{n=0}^{5} D_n e^{jn\Omega_0 k}$$

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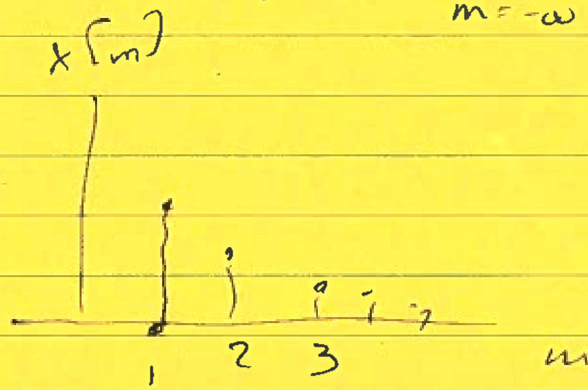
2.

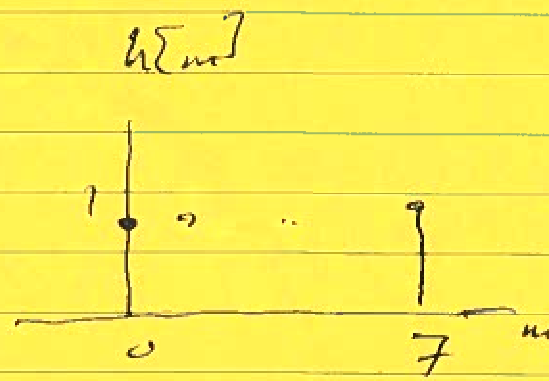
$$h[k] = u[k]\,u[7-k]$$

$$x[k] = (0.25)^{k-1}\,u[k-1]$$

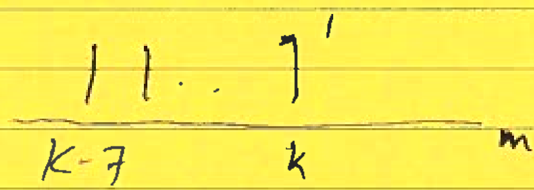
We will use

$$y[k] = \sum_{m=-\infty}^{\infty} x[m]\,h[k-m]$$





h[k-m]



3 Cases

Case 1 $\quad k < 1 \quad$ "no overlap"

$$y[k] = 0$$

Case 2 "running on"

$k \geq 1 \qquad k-7 < 1$

$k < 8$

$$1 \leq k < 8$$

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③

$$y[k] = \sum_{m=1}^{k} \left(\frac{1}{4}\right)^{m-1}$$

geometric

1st term $A = 1$
factor $r = 1/4$
terms $k-1+1 = k$

$$\therefore\ y[k] = \frac{1-\left(\frac{1}{4}\right)^k}{1-1/4} = \frac{4}{3}\left(1-\left(\frac{1}{4}\right)^k\right)$$

Case 3 "fully on" $k-7 \geq 1$
$k \geq 8$

Here

$$y[k] = \sum_{m=k-7}^{k} \left(\frac{1}{4}\right)^{m-1}$$

geometric

1st term $A = \left(\frac{1}{4}\right)^{k-7-1} = \left(\frac{1}{4}\right)^{k-8}$

$r = \frac{1}{4}$

terms $k-(k-7)+1 = 8$

$$\therefore\ y[k] = \left(\frac{1}{4}\right)^{k-8} \frac{1-\left(\frac{1}{4}\right)^8}{1-1/4} = \frac{4}{3}\left[\left(\frac{1}{4}\right)^{k-8} - \left(\frac{1}{4}\right)^k\right]$$

$$y[k] = \begin{cases} 4/3\left(1-\left(\frac{1}{4}\right)^k\right) & 1 \leq k < 8 \\ 4/3\left[\left(\frac{1}{4}\right)^{k-8} - \left(\frac{1}{4}\right)^k\right] & k \geq 8 \\ 0 & \text{o.w} \end{cases}$$

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(4)

3

a) we note that $h[k]$ is non zero for some negative k. for ex

$$h[-1] = 2^{-1} = 0.5$$

$\therefore$ not causal

b) Check abs summability

$$\sum_{k=-\infty}^{\infty} |h[k]| = \sum_{k=-\infty}^{\infty} 2^{k}$$

↑ blows up

$\therefore$ ~~not~~ $h[k]$ is <u>not</u> absolutely summable

$\therefore$ $h[k]$ is <u>not</u> stable

4.

$$h[k] = (0.5)^k u[k] \quad \leftarrow \text{LTI}$$

$$x[k] = 2\left(\frac{1}{3}\right)^k u[k]$$

a) $$H(\Omega) = \frac{1}{1-0.5e^{-j\Omega}}$$ note $|0.5|<1$

b) $$X(\Omega) = \frac{2}{1-\frac{1}{3}e^{-j\Omega}}$$ $|\frac{1}{3}|<1$

$$Y(\Omega) = X(\Omega)\,H(\Omega)$$

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$$\therefore \; Y(\Omega) = \frac{2}{\left(1-\frac{1}{2}e^{-j\Omega}\right)\left(1-\frac{1}{3}e^{-j\Omega}\right)}$$ ← spectrum $Y(\Omega)$

c) P.F. simple roots in denom, deg of num less than deg of denom

$$Y(\Omega) = \frac{\frac{2}{(1-\frac{1}{3}2)}}{1-\frac{1}{2}e^{-j\Omega}} + \frac{\frac{2}{(1-\frac{1}{2}(3))}}{1-\frac{1}{3}e^{-j\Omega}}$$

$$= \frac{6}{1-\frac{1}{2}e^{-j\Omega}} + \frac{-4}{1-\frac{1}{3}e^{-j\omega}}$$

check const. yes it is true

$$\therefore \; y[k] = 6\left(\frac{1}{2}\right)^k u[k] - 4\left(\frac{1}{3}\right)^k u[k]$$