# ELORank

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## Introduction

ELORank is a C library providing structures and functions implementing the ELO ranking system in a version supporting several players per run with eventual ties.

It uses the PBErr, PBMath and GSet library.

### 1 Definitions

The ELO rank is calculated incrementally by updating the current ELO rank of each entity according to their result in an evaluation process independant from the ELO ranking system. Given a result of this evaluation process, each pair of winner/looser in this result is updated as follow:

$$\begin{cases}
E'_{w} = E_{w} + K * \left(1.0 - \frac{1.0}{1.0 + 10.0 \frac{E_{l} - E_{w}}{400.0}}\right) \\
E'_{l} = E_{l} - K * \left(\frac{1.0}{1.0 + 10.0 \frac{E_{w} - E_{l}}{400.0}}\right)
\end{cases}$$
(1)

where K = 8.0 and,  $E_w$  and  $E_l$  are respectively the current ELO of the winner and the current ELO of the looser and,  $E'_w$  and  $E'_l$  are respectively the new ELO of the winner and the new ELO of the looser.

Tie between two entities results in no changes in their respective ELO rank.

### 2 Interface

```
// ====== ELORANK.H ========
#ifndef ELORANK_H
#define ELORANK_H
// ========= Include =========
#include <stdlib.h>
#include <stdio.h>
#include <math.h>
#include <string.h>
#include <stdbool.h>
#include "pberr.h"
#include "gset.h"
#include "pbmath.h"
// ====== Define =======
#define ELORANK_K 8.0
#define ELORANK_STARTELO 0.0
// ======= Data structure =========
typedef struct ELOEntity {
 // Pointer toward user struct
 void* _data;
 // Number of evaluation
 int _nbRun;
} ELOEntity;
```

```
typedef struct ELORank {
  // ELO coefficient
  float _k;
  // Set of ELO entities
  GSet _set;
} ELORank;
// ====== Functions declaration =======
// Create a new ELORank
ELORank* ELORankCreate(void);
/*#if BUILDMODE == 0
ELORank ELORankCreateStatic(void);
#endif*/
// Free memory used by an {\tt ELORank}
void ELORankFree(ELORank** that);
// Free memory used by an ELOEntity
void ELOEntityFree(ELOEntity** that);
// Set the K coefficient of 'that' to 'k'
#if BUILDMODE != 0
inline
#endif
void ELORankSetK(ELORank* const that, const float k);
// Get the K coefficient of 'that'
#if BUILDMODE != 0
inline
#endif
float ELORankGetK(const ELORank* const that);
// Add the entity 'data' to 'that'
void ELORankAdd(ELORank* const that, void* const data);
// Remove the entity 'data' from 'that'
void ELORankRemove(ELORank* const that, void* data);
// Get the number of entity in 'that'
#if BUILDMODE != 0
inline
#endif
int ELORankGetNb(const ELORank* const that);
// Update the ranks in 'that' with results 'res' given as a GSet of
// pointers toward entities (_data in GSetElem equals _data in
// ELOEntity) in winning order
// The _sortVal of the GSet represents the score (and so position)
// of the entities for this update (thus, equal _sortVal means tie)
// The set of results must contain at least 2 elements
// Elements in the result set must be in the ELORank
void ELORankUpdate(ELORank* const that, const GSet* const res);
// Get the current rank of the entity 'data' (starts at 0)
int ELORankGetRank(const ELORank* const that, const void* const data);
// Get the current ELO of the entity 'data'
float ELORankGetELO(const ELORank* const that, const void* const data);
```

#### 3 Code

#### 3.1 elorank.c

```
// ====== ELORANK.C =========
// ========= Include =========
#include "elorank.h"
#if BUILDMODE == 0
#include "elorank-inline.c"
// ========= Functions declaration =========
// Create a new ELOEntity
static ELOEntity* ELOEntityCreate(void* const data);
// Return the GSetElem in 'that'->_set for the entity 'data'
static GSetElem* ELORankGetElem(const ELORank* const that, const void* const data);
// ====== Functions implementation =========
// Create a new ELORank
ELORank* ELORankCreate(void) {
  // Allocate memory
  ELORank* that = PBErrMalloc(ELORankErr, sizeof(ELORank));
  // Set the default coefficient
  that->_k = ELORANK_K;
  \ensuremath{//} Create the set of entities
  that->_set = GSetCreateStatic();
  // Return the new ELORank
 return that;
// Free memory used by an ELORank
void ELORankFree(ELORank** that) {
  // Check the argument
  if (that == NULL || *that == NULL) return;
  // Empty the set of entities
  GSet* set = &((*that)->_set);
  while (GSetNbElem(set) > 0) {
   ELOEntity *ent = GSetPop(set);
   ELOEntityFree(&ent);
```

```
// Free memory
 free(*that);
  // Set the pointer to null
 *that = NULL;
// Free memory used by an ELOEntity
void ELOEntityFree(ELOEntity** that) {
 // Check the argument
 if (that == NULL || *that == NULL) return;
 // Free memory
 free(*that);
 // Set the pointer to null
 *that = NULL;
// Add the entity 'data' to 'that'
void ELORankAdd(ELORank* const that, void* const data) {
#if BUILDMODE == 0
 // Check arguments
 if (that == NULL) {
   ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'that' is null");
   PBErrCatch(ELORankErr);
 if (data == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'data' is null");
   PBErrCatch(ELORankErr);
 }
#endif
  // Create a new ELOEntity
 ELOEntity *ent = ELOEntityCreate(data);
  // Add the new entity to the set with a default score
 GSetAddSort(&(that->_set), ent, ELORANK_STARTELO);
// Create a new ELOEntity
static ELOEntity* ELOEntityCreate(void* const data) {
#if BUILDMODE == 0
 // Check argument
 if (data == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'data' is null");
   PBErrCatch(ELORankErr);
 }
#endif
  // Allocate memory
 ELOEntity *that = PBErrMalloc(ELORankErr, sizeof(ELOEntity));
 // Set properties
  that->_data = data;
 that->_nbRun = 0;
  // Return the new ELOEntity
 return that;
// Remove the entity 'data' from 'that'
void ELORankRemove(ELORank* const that, void* data) {
#if BUILDMODE == 0
 // Check arguments
 if (that == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
```

```
sprintf(ELORankErr->_msg, "'that' is null");
   PBErrCatch(ELORankErr);
 if (data == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'data' is null");
   PBErrCatch(ELORankErr);
 }
#endif
  // Search the entity
 GSetElem* elem = ELORankGetElem(that, data);
  // If we have found the entity
  if (elem != NULL) {
    // Free the memory
    ELOEntityFree((ELOEntity**)(&(elem->_data)));
    // Remove the element
    GSetRemoveElem(&(that->_set), &elem);
 }
// Return the GSetElem in 'that'->_set for the entity 'data'
static GSetElem* ELORankGetElem(const ELORank* const that, const void* const data) {
#if BUILDMODE == 0
  // Check arguments
 if (that == NULL) {
   ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'that' is null");
   PBErrCatch(ELORankErr);
 if (data == NULL) {
   ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'data' is null");
   PBErrCatch(ELORankErr);
 7
#endif
 // Search the element
 GSetElem* elem = that->_set._head;
 while (elem != NULL && ((ELOEntity*)(elem->_data))->_data != data)
    elem = elem->_next;
  // Return the element
 return elem;
// Update the ranks in 'that' with results 'res' given as a GSet of
// pointers toward entities (_data in GSetElem equals _data in
// ELOEntity) in winning order
// The _sortVal of the GSet represents the score (and so position)
// of the entities for this update (thus, equal _sortVal means tie)
// The set of results must contain at least 2 elements
// Elements in the result set must be in the ELORank
void ELORankUpdate(ELORank* const that, const GSet* const res) {
#if BUILDMODE == 0
  // Check arguments
 if (that == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'that' is null");
   PBErrCatch(ELORankErr);
  if (res == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'res' is null");
    PBErrCatch(ELORankErr);
```

```
if (GSetNbElem(res) < 2) {</pre>
    ELORankErr->_type = PBErrTypeInvalidArg;
    sprintf(ELORankErr->_msg,
      "Number of elements in result set invalid (%ld>=2)",
      GSetNbElem(res));
   PBErrCatch(ELORankErr);
  }
#endif
  // Declare a variable to memorise the delta of elo of each entity
  VecFloat* deltaElo = VecFloatCreate(GSetNbElem(res));
  // Calculate the delta of elo for each pair of entity
  GSetElem* elemA = res->_head;
  int iElem = 0;
  while (elemA != NULL) {
    GSetElem* elemAElo = ELORankGetElem(that, elemA->_data);
#if BUILDMODE == 0
    if (elemAElo == NULL) {
      ELORankErr->_type = PBErrTypeNullPointer;
      sprintf(ELORankErr->_msg,
        "Entity in the result set can't be found in the ELORank.");
      PBErrCatch(ELORankErr);
   }
#endif
    GSetElem* elemB = res->_head;
    while (elemB != NULL) {
      // Ignore tie and match with itself
      if (ISEQUALF(elemA->_sortVal, elemB->_sortVal) == false) {
        GSetElem* elemBElo = ELORankGetElem(that, elemB->_data);
#if BUILDMODE == 0
        if (elemBElo == NULL) {
          ELORankErr->_type = PBErrTypeNullPointer;
          sprintf(ELORankErr->_msg,
            "Entity in the result set can't be found in the ELORank.");
          PBErrCatch(ELORankErr);
#endif
        // If elemA has won
        if (elemA->_sortVal > elemB->_sortVal) {
          float winnerELO = elemAElo->_sortVal;
          float looserELO = elemBElo->_sortVal;
          float a =
            1.0 / (1.0 + pow(10.0, (looserELO - winnerELO) / 400.0));
          VecSetAdd(deltaElo, iElem, that->_k * (1.0 - a));
        // Else, if elemA has lost
        } else {
          float winnerELO = elemBElo->_sortVal;
          float looserELO = elemAElo->_sortVal;
          float b =
            1.0 / (1.0 + pow(10.0, (winnerELO - looserELO) / 400.0));
          VecSetAdd(deltaElo, iElem, -1.0 * that->_k * b);
      }
      elemB = elemB->_next;
    elemA = elemA->_next;
    ++iElem;
  // Apply the delta of elo and update the number of run
  GSetElem* elem = res->_head;
  iElem = 0:
  while (elem != NULL) {
```

```
GSetElem* elemElo = ELORankGetElem(that, elem->_data);
#if BUILDMODE == 0
    if (elemElo == NULL) {
      ELORankErr->_type = PBErrTypeNullPointer;
      sprintf(ELORankErr->_msg,
        "Entity in the result set can't be found in the ELORank.");
      PBErrCatch(ELORankErr);
    }
#endif
    elemElo->_sortVal += VecGet(deltaElo, iElem);
    ++(((ELOEntity*)(elemElo->_data))->_nbRun);
    ++iElem;
    elem = elem->_next;
  // Free memory
  VecFree(&deltaElo);
  // Sort the ELORank
  GSetSort(&(that->_set));
// Get the current rank of the entity 'data' (starts at 0)
int ELORankGetRank(const ELORank* const that, const void* const data) {
#if BUILDMODE == 0
  // Check arguments
  if (that == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'that' is null");
   PBErrCatch(ELORankErr);
  if (data == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'data' is null");
   PBErrCatch(ELORankErr);
  }
#endif
  \ensuremath{//} Declare a variable to memorize the rank
  int rank = 0;
  // Search the element
  GSetElem* elem = that->_set._tail;
  while (elem != NULL && ((ELOEntity*)(elem->_data))->_data != data) {
    elem = elem->_prev;
    ++rank;
#if BUILDMODE == 0
  if (elem == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg,
      "Entity requested can't be found in the ELORank.");
   PBErrCatch(ELORankErr);
  }
#endif
  // Return the element
  return rank;
// Get the current ELO of the entity 'data'
float ELORankGetELO(const ELORank* const that, const void* const data) {
#if BUILDMODE == 0
  // Check arguments
  if (that == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'that' is null");
```

```
PBErrCatch(ELORankErr);
  }
  if (data == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'data' is null");
   PBErrCatch(ELORankErr);
#endif
  // Declare a variable to memorize the ELO
  float elo = ELORANK_STARTELO;
  // Search the element
  GSetElem* elem = that->_set._head;
  while (elem != NULL && ((ELOEntity*)(elem->_data))->_data != data) {
   elem = elem->_next;
  if (elem != NULL) {
   elo = elem->_sortVal;
#if BUILDMODE == 0
  } else {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg,
      "Entity requested can't be found in the ELORank.");
    PBErrCatch(ELORankErr);
#endif
  // Return the element
 return elo;
// Get the 'rank'-th entity according to current ELO of 'that'
// (starts at 0)
const ELOEntity* ELORankGetRanked(const ELORank* const that, const int rank) {
#if BUILDMODE == 0
  // Check arguments
  if (that == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'that' is null");
    PBErrCatch(ELORankErr);
  if (rank < 0 || rank >= GSetNbElem(&(that->_set))) {
    ELORankErr->_type = PBErrTypeInvalidArg;
    sprintf(ELORankErr->_msg, "'rank' is invalid (0<=%d<%ld)", rank,
     GSetNbElem(&(that->_set)));
   PBErrCatch(ELORankErr);
 }
#endif
  GSetElem* elem = that->_set._tail;
  for (int i = rank; i--;)
    elem = elem->_prev;
 return (ELOEntity*)(elem->_data);
3.2
        elorank-inline.c
// ====== ELORANK-INLINE.C ========
// ======= Functions implementation ==========
// Set the K coefficient of 'that' to 'k'
#if BUILDMODE != 0
```

```
inline
#endif
void ELORankSetK(ELORank* const that, const float k) {
#if BUILDMODE == 0
  // Check arguments
  if (that == NULL) {
   ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'that' is null");
   PBErrCatch(ELORankErr);
#endif
 that->_k = k;
// Get the K coefficient of 'that'
#if BUILDMODE != 0
inline
#endif
float ELORankGetK(const ELORank* const that) {
#if BUILDMODE == 0
  // Check argument
  if (that == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'that' is null");
   PBErrCatch(ELORankErr);
#endif
 return that->_k;
// Get the number of entity in 'that'
#if BUILDMODE != 0
inline
#endif
int ELORankGetNb(const ELORank* const that) {
#if BUILDMODE == 0
  // Check argument
  if (that == NULL) {
    ELORankErr->_type = PBErrTypeNullPointer;
    sprintf(ELORankErr->_msg, "'that' is null");
   PBErrCatch(ELORankErr);
 }
#endif
 return GSetNbElem(&(that->_set));
```

## 4 Makefile

```
# Build mode
# 0: development (max safety, no optimisation)
# 1: release (min safety, optimisation)
# 2: fast and furious (no safety, optimisation)
BUILD_MODE?=1
all: main
# Makefile definitions
MAKEFILE_INC=../PBMake/Makefile.inc
```

```
include $(MAKEFILE_INC)

# Rules to make the executable
repo=elorank
$($(repo)_EXENAME): \
$($(repo)_EXENAME).o \
$($(repo)_EXE_DEP) \
$($(repo)_DEP)
$(COMPILER) 'echo "$($(repo)_EXE_DEP) $($(repo)_EXENAME).o" | tr ' ' '\n' | sort -u' $(LINK_ARG) $($(repo)_LINK_ARG)
$($(repo)_EXENAME).o: \
$($(repo)_DIR)/$($(repo)_EXENAME).c \
$($(repo)_INC_H_EXE) \
$($(repo)_EXE_DEP)
$(COMPILER) $(BUILD_ARG) $($(repo)_BUILD_ARG) 'echo "$($(repo)_INC_DIR)" | tr ' ' '\n' | sort -u' -c $($(repo)_DIR)/$
```

#### 5 Unit tests

```
#include <stdlib.h>
#include <stdio.h>
#include <time.h>
#include <string.h>
#include <time.h>
#include <unistd.h>
#include <sys/time.h>
#include "elorank.h"
#include "pberr.h"
#include "pbmath.h"
#define RANDOMSEED 2
typedef struct Player {
  int _id;
} Player;
void UnitTestCreateFree() {
  ELORank* elo = ELORankCreate();
  if (elo == NULL || elo->_k != ELORANK_K) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankCreate failed");
    PBErrCatch(ELORankErr);
  ELORankFree(&elo);
  if (elo != NULL) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankFree failed");
    PBErrCatch(ELORankErr);
printf("UnitTestCreateFree OK\n");
}
void UnitTestSetGetK() {
  ELORank* elo = ELORankCreate();
  float k = 1.0;
  ELORankSetK(elo, k);
  if (ISEQUALF(elo->_k, k) == false) \{
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankSetK failed");
```

```
PBErrCatch(ELORankErr);
  }
  if (ISEQUALF(ELORankGetK(elo), k) == false) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankGetK failed");
    PBErrCatch(ELORankErr);
  ELORankFree(&elo);
 printf("UnitTestSetGetK OK\n");
void UnitTestAddRemoveGetNb() {
  ELORank* elo = ELORankCreate();
  Player *playerA = PBErrMalloc(ELORankErr, sizeof(Player));
  Player *playerB = PBErrMalloc(ELORankErr, sizeof(Player));
  ELORankAdd(elo, playerA);
  if (ELORankGetNb(elo) != 1) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankAdd failed");
    PBErrCatch(ELORankErr);
  if (((ELOEntity*)(elo->_set._head->_data))->_data != playerA) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankAdd failed, _data invalid");
    PBErrCatch(ELORankErr);
  if (((ELOEntity*)(elo->_set._head->_data))->_nbRun != 0) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
sprintf(ELORankErr->_msg, "ELORankAdd failed, _nbRun invalid");
    PBErrCatch(ELORankErr);
  if (ISEQUALF(elo->_set._head->_sortVal, ELORANK_STARTELO) == false) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankAdd failed, _sortVal invalid");
    PBErrCatch(ELORankErr);
  ELORankAdd(elo, playerB);
  if (ELORankGetNb(elo) != 2) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankAdd failed");
   PBErrCatch(ELORankErr);
  if (((ELOEntity*)(elo->_set._head->_next->_data))->_data !=
    playerB) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankAdd failed, _data invalid");
    PBErrCatch(ELORankErr);
  ELORankRemove(elo, playerA);
  if (ELORankGetNb(elo) != 1) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankRemove failed");
    PBErrCatch(ELORankErr);
  if (((ELOEntity*)(elo->_set._head->_data))->_data != playerB) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankRemove failed, _data invalid");
    PBErrCatch(ELORankErr);
  ELORankFree(&elo);
  free(playerA);
  free(playerB);
```

```
printf("UnitTestAddRemoveGetNb OK\n");
void UnitTestUpdateGetRankGetElo() {
  srandom(RANDOMSEED);
  ELORank* elo = ELORankCreate();
  Player *players[3] = {NULL};
  GSet res = GSetCreateStatic();
  Gauss gausses[3];
  for (int i = 3; i--;) {
    players[i] = PBErrMalloc(ELORankErr, sizeof(Player));
    players[i]->_id = i;
    ELORankAdd(elo, players[i]);
    gausses[i] = GaussCreateStatic(3 - i, 1.0);
  int nbRun = 100;
  FILE* f = fopen("./elorank.txt", "w");
  for (int iRun = nbRun; iRun--;) {
    GSetFlush(&res);
    for (int i = 3; i--;)
      GSetAddSort(&res, players[i], GaussRnd(gausses + i));
    ELORankUpdate(elo, &res);
fprintf(f, "%d %f %f %f\n", (nbRun - iRun),
      ELORankGetELO(elo, players[0]),
      ELORankGetELO(elo, players[1]),
      ELORankGetELO(elo, players[2]));
  fclose(f);
  for (int i = 3; i--;) {
    if (ELORankGetRank(elo, players[i]) != i) {
      ELORankErr->_type = PBErrTypeUnitTestFailed;
      sprintf(ELORankErr->_msg, "ELORankUpdate failed");
      PBErrCatch(ELORankErr);
   }
  }
  const ELOEntity *winner = ELORankGetRanked(elo, 0);
  if (winner->_data != players[0]) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "ELORankGeRanked failed");
    PBErrCatch(ELORankErr);
  if (winner->_nbRun != nbRun) {
    ELORankErr->_type = PBErrTypeUnitTestFailed;
    sprintf(ELORankErr->_msg, "nbRun invalid");
    PBErrCatch(ELORankErr);
  ELORankFree(&elo);
  GSetFlush(&res);
  for (int i = 3; i--;)
    free(players[i]);
 printf("UnitTestUpdateGetRankGetElo OK\n");
void UnitTestAll() {
  UnitTestCreateFree();
  UnitTestSetGetK();
  UnitTestAddRemoveGetNb();
  UnitTestUpdateGetRankGetElo();
 printf("UnitTestAll OK\n");
int main() {
```

```
UnitTestAll();
  // Return success code
  return 0;
}
```

## 6 Unit tests output

UnitTestCreateFree OK UnitTestSetGetK OK UnitTestAddRemoveGetNb OK UnitTestUpdateGetRankGetElo OK UnitTestAll OK

#### 7 ELORank.txt

```
1 8.000000 -8.000000 0.000000
2 15.723836 -15.723836 -0.000000
3 15.181863 -7.181863 -8.000000
4 22.658251 -6.934165 -15.724085
5 29.878273 -14.695467 -15.182804
6 36.852001 -14.190701 -22.661297
7 43.589993 -21.704742 -21.885246
8 50.101974 -20.963814 -29.138155
9 48.398232 -12.250955 -36.147270
10 46.752392 -3.834395 -42.917988
11 53.164032 -3.704153 -49.459869
12 59.366798 -11.578976 -47.787811
13 65.365189 -11.188716 -54.176464
14 71.173401 -10.813753 -60.359642
15 76.800728 -10.453563 -66.347153
16 82.256058 -10.107615 -72.148430
17 79.547867 -1.775375 -77.772476
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