PBDataAnalysis

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Contents

Introduction

PBDataAnalysis is a C library providing structures and functions to perform various data analysis.

It implements the following algorithms:

- K-means clustering (random and Forgy seeds)

It uses the `PBErr`, `PBMath`, `GSet` libraries.

1 Definitions

1.1 K-means clustering

The goal of the K-means clustering algorithm is to find K Voronoi cells which clusters a data set in a way that for each cells the center of this cell is the nearest possible to the average value of the input data inside this cell.

The K-means algorithm is as follow, where 'seed' defines the way we initialise the algorithm: 'random' or 'Forgy'.

```
if seed = random
  init the center of each cluster with a random value inside the bounds
  of the input data
else if seed = Forgy
  init the center of each cluster with one of the input data randomly
  choosen, one given input data can't be choosen twice
loop until clusters' center have all converged
  init K empty sets S[]
  for each input data I
    ID = get the cluster containing I
    add I to S[ID]
  for each set S[I]
    AVG = calculate the average value of the input in S[I]
    if AVG is equal to the center of the I-th cluster
      the center of the I-th cluster has converged
    else
      set the center of the I-th cluster to AVG
```

2 Interface

```
// ============ PBDATAANALYSIS.H ================

#ifndef PBDATAANALYSIS_H
#define PBDATAANALYSIS_H

// ================= Include =================

#include <stdlib.h>
#include <stdio.h>
#include <stdbool.h>
#include <execinfo.h>
#include <errno.h>
#include <string.h>
#include "pberr.h"
#include "pbmath.h"
#include "gset.h"

// ================= Define ==================

// ================= Data structure ====================

typedef enum KMeansClustersSeed {
```

```
  KMeansClustersSeed_Random, KMeansClustersSeed_Forgy
} KMeansClustersSeed;

typedef struct KMeansClusters {
  GSetVecFloat _centers;
  KMeansClustersSeed _seed;
} KMeansClusters;

// ================ Functions declaration ====================

// Create a KMeansClusters for a K-means clustering initialized
// using the 'seed' technique
KMeansClusters KMeansClustersCreateStatic(const KMeansClustersSeed seed);

// Free the memory used by a PBDAKMeansClusters
void KMeansClustersFreeStatic(KMeansClusters* const that);

// Create the set of 'K' clusters clustering the 'input' data according
// to the K-means algorithm
// 'K' must be inferior or equal to the number of input data
// srandom() must have been called before using this function
void KMeansClustersSearch(KMeansClusters* const that,
  const GSetVecFloat* const input, const int K);

// Get the set of clusters' center for the KMeansClusters 'that'
#if BUILDMODE != 0
inline
#endif
const GSetVecFloat* KMeansClustersCenters(
  const KMeansClusters* const that);

// Get the 'iCluster'-th cluster's center for the KMeansClusters 'that'
#if BUILDMODE != 0
inline
#endif
const VecFloat* _KMeansClustersCenterFromId(
  const KMeansClusters* const that, const int iCluster);

// Get the seed of the KMeansClusters 'that'
#if BUILDMODE != 0
inline
#endif
KMeansClustersSeed KMeansClustersGetSeed(const KMeansClusters* const that);

// Set the seed of the KMeansClusters 'that' to 'seed'
#if BUILDMODE != 0
inline
#endif
void KMeansClustersSetSeed(KMeansClusters* const that,
  const KMeansClustersSeed seed);

// Get the center of the cluster including the 'input' data for the
// KMeansClusters 'that'
#if BUILDMODE != 0
inline
#endif
const VecFloat* _KMeansClustersCenterFromPos(
  const KMeansClusters* const that, const VecFloat* input);

// Get the index of the cluster including the 'input' data for the
// KMeansClusters 'that'
int KMeansClustersGetId(const KMeansClusters* const that,
```

```
  const VecFloat* input);

// Get the seed of the KMeansClusters 'that'
#if BUILDMODE != 0
inline
#endif
int KMeansClustersGetK(const KMeansClusters* const that);

// ================= Polymorphism ==================

#define KMeansClustersCenter(Cluster, Input) _Generic(Input, \
  VecFloat*: _KMeansClustersCenterFromPos, \
  const VecFloat*: _KMeansClustersCenterFromPos, \
  int: _KMeansClustersCenterFromId, \
  const int: _KMeansClustersCenterFromId, \
  default: PBErrInvalidPolymorphism)(Cluster, Input)

// ================ Inliner ====================

#if BUILDMODE != 0
#include "pbdataanalysis-inline.c"
#endif

#endif
```

3 Code

3.1 pbdataanalysis.c

```
// ============ PBDATAANALYSIS.C ================

// ================= Include =================

#include "pbdataanalysis.h"
#if BUILDMODE == 0
#include "pbdataanalysis-inline.c"
#endif

// ================= Define ==================

// ================ Functions declaration ====================

void KMeansClustersInitRandom(KMeansClusters* const that,
  const GSetVecFloat* const input);

void KMeansClustersInitForgy(KMeansClusters* const that,
  const GSetVecFloat* const input);

// ================ Functions implementation ====================

// Create a KMeansClusters for a K-means clustering initialized
// using the 'seed' technique
// srandom() must have been called before using this function
KMeansClusters KMeansClustersCreateStatic(const KMeansClustersSeed seed) {
  // Declare the KMeansClusters
  KMeansClusters clusters;
  // Init the properties
  clusters._seed = seed;
```

```
  clusters._centers = GSetVecFloatCreateStatic();
  // Return the KMeansClusters
  return clusters;
}

// Free the memory used by a PBDAKMeansClusters
void KMeansClustersFreeStatic(KMeansClusters* const that) {
#if BUILDMODE == 0
  if (that == NULL) {
    PBDataAnalysisErr->_type = PBErrTypeNullPointer;
    sprintf(PBDataAnalysisErr->_msg, "'that' is null");
    PBErrCatch(PBDataAnalysisErr);
  }
#endif
  // Free the memory used by the cluster centers
  while (GSetNbElem(&(that->_centers)) > 0) {
    VecFloat* v = GSetPop((GSetVecFloat*)(KMeansClustersCenters(that)));
    free(v);
  }
}

// Create the set of 'K' clusters clustering the 'input' data according
// to the K-means algorithm
// 'K' must be inferior or equal to the number of input data
// srandom() must have been called before using this function
void KMeansClustersSearch(KMeansClusters* const that,
  const GSetVecFloat* const input, const int K) {
#if BUILDMODE == 0
  if (that == NULL) {
    PBDataAnalysisErr->_type = PBErrTypeNullPointer;
    sprintf(PBDataAnalysisErr->_msg, "'that' is null");
    PBErrCatch(PBDataAnalysisErr);
  }
  if (input == NULL) {
    PBDataAnalysisErr->_type = PBErrTypeNullPointer;
    sprintf(PBDataAnalysisErr->_msg, "'input' is null");
    PBErrCatch(PBDataAnalysisErr);
  }
  if (K < 1) {
    PBDataAnalysisErr->_type = PBErrTypeInvalidArg;
    sprintf(PBDataAnalysisErr->_msg, "'K' is invalid (%d>=1)", K);
    PBErrCatch(PBDataAnalysisErr);
  }
  if (GSetNbElem(input) < K) {
    PBDataAnalysisErr->_type = PBErrTypeInvalidArg;
    sprintf(PBDataAnalysisErr->_msg, "'K' is invalid (%d<=%ld)", K,
      GSetNbElem(input));
    PBErrCatch(PBDataAnalysisErr);
  }
#endif
  // If there are alreay computed clusters,
  // free the memory used by the cluster centers
  while (GSetNbElem(&(that->_centers)) > 0) {
    VecFloat* v = GSetPop((GSetVecFloat*)(KMeansClustersCenters(that)));
    free(v);
  }
  // Allocate an array of sets used for computation
  GSetVecFloat* inputsByCluster =
    PBErrMalloc(PBDataAnalysisErr, sizeof(GSetVecFloat) * K);
  // Get the dimension of the input
  long dim = VecGetDim(GSetGet(input, 0));
  // Allocate memory for the clusters' center and sets used for
```

```
  // computation
  for (int iCenter = K; iCenter--;) {
    // The dimension of the means is the same as the one of the input
    // data
    VecFloat* v = VecFloatCreate(dim);
    GSetAppend((GSetVecFloat*)(KMeansClustersCenters(that)), v);
    inputsByCluster[iCenter] = GSetVecFloatCreateStatic();
  }
  // Initialise the means according to the seed
  switch(KMeansClustersGetSeed(that)) {
    case KMeansClustersSeed_Random:
      KMeansClustersInitRandom(that, input);
      break;
    case KMeansClustersSeed_Forgy:
      KMeansClustersInitForgy(that, input);
      break;
    default:
      break;
  }
  // Create a vector used for computation
  VecFloat* w = VecFloatCreate(dim);
  // Loop until the clusters' center are aligned with their
  // real center according to input data
  float shift = 1.0;
  while (shift > PBMATH_EPSILON) {
    // Reset the shift
    shift = 0.0;
    // Loop on input data
    GSetIterForward iterInput = GSetIterForwardCreateStatic(input);
    do {
      // Get the input data
      VecFloat* v = GSetIterGet(&iterInput);
      // Get the id of the cluster containing this data
      int clusterId = KMeansClustersGetId(that, v);
      // Add the input to the corresponding set
      GSetAppend(inputsByCluster + clusterId, v);
    } while(GSetIterStep(&iterInput));
    // For each cluster
    for (int iCenter = K; iCenter--;) {
      // Reset the vector used for computation
      VecSetNull(w);
      // Memorize the number of input associated to this cluster
      int nbInput = GSetNbElem(inputsByCluster + iCenter);
      // If there are data in this cluster
      if (nbInput > 0) {
        // Loop on the input contained by this cluster
        GSetIterForward iterSet =
          GSetIterForwardCreateStatic(inputsByCluster + iCenter);
        do {
          // Get the input data
          VecFloat* v = GSetIterGet(&iterSet);
          // Sum it to the temporary vector
          VecOp(w, 1.0, v, 1.0);
        } while(GSetIterStep(&iterSet));
        // Get the center (average) of the input contained by this
        // cluster
        VecScale(w, 1.0 / (float)nbInput);
        // Update the shift
        shift += VecDist(KMeansClustersCenter(that, iCenter), w);
        // Update the cluster center with the center of the input
//VecPrint(KMeansClustersCenter(that, iCenter), stdout);printf(" ");
        VecCopy((VecFloat*)KMeansClustersCenter(that, iCenter), w);
```

```
        // Reset the sets of input data
        GSetFlush(inputsByCluster + iCenter);
      }
    }
//printf("\n");
  }
  // Free the memory used by the vector and sets used for computation
  free(inputsByCluster);
  VecFree(&w);
}

void KMeansClustersInitRandom(KMeansClusters* const that,
  const GSetVecFloat* const input) {
#if BUILDMODE == 0
  if (that == NULL) {
    PBDataAnalysisErr->_type = PBErrTypeNullPointer;
    sprintf(PBDataAnalysisErr->_msg, "'that' is null");
    PBErrCatch(PBDataAnalysisErr);
  }
  if (input == NULL) {
    PBDataAnalysisErr->_type = PBErrTypeNullPointer;
    sprintf(PBDataAnalysisErr->_msg, "'input' is null");
    PBErrCatch(PBDataAnalysisErr);
  }
#endif
  // Get the bounds of the input data
  GSetVecFloat bounds = GSetGetBounds(input);
  VecFloat* boundMin = GSetGet(&bounds, 0);
  VecFloat* boundMax = GSetGet(&bounds, 1);
  // Get the dimension of the data
  int dim = VecGetDim(GSetGet(input, 0));
  // For each cluster's center
  for (int iCenter = KMeansClustersGetK(that); iCenter--;) {
    // Get the iCenter-th cluster center
    VecFloat* center = (VecFloat*)KMeansClustersCenter(that, iCenter);
    // Initialize randomly the components of the iCenter-th cluster's
    // center
    for (int iDim = dim; iDim--;) {
      VecSet(center, iDim, VecGet(boundMin, iDim) +
        rnd() * (VecGet(boundMax, iDim) - VecGet(boundMin, iDim)));
    }
  }
  // Free memory
  GSetFlush(&bounds);
  VecFree(&boundMin);
  VecFree(&boundMax);
}

void KMeansClustersInitForgy(KMeansClusters* const that,
  const GSetVecFloat* const input) {
#if BUILDMODE == 0
  if (that == NULL) {
    PBDataAnalysisErr->_type = PBErrTypeNullPointer;
    sprintf(PBDataAnalysisErr->_msg, "'that' is null");
    PBErrCatch(PBDataAnalysisErr);
  }
  if (input == NULL) {
    PBDataAnalysisErr->_type = PBErrTypeNullPointer;
    sprintf(PBDataAnalysisErr->_msg, "'input' is null");
    PBErrCatch(PBDataAnalysisErr);
  }
#endif
```

```
  // Create a set to select the seeds
  GSetVecFloat forgyInp = GSetVecFloatCreateStatic();
  // Get the number of inputs
  long nbInput = GSetNbElem(input);
  // Declare a variable to pick one input randomly
  VecFloat* inp = NULL;
  // For each cluster's center
  for (int iCenter = KMeansClustersGetK(that); iCenter--;) {
    // Pick an input while avoiding picking twice the same
    // Supposes K is far less than the number of inputs
    do {
      inp = GSetGet(input, (long)round(rnd() * (nbInput - 1)));
    } while (GSetFirstElem(&forgyInp, inp) != NULL);
    // Set the center of the iCenter-th cluster to this input
    VecCopy((VecFloat*)KMeansClustersCenter(that, iCenter), inp);
  }
  // Empty the set used to select the seeds
  GSetFlush(&forgyInp);
}

// Get the index of the cluster including the 'input' data for the
// KMeansClusters 'that'
int KMeansClustersGetId(const KMeansClusters* const that,
  const VecFloat* input) {
#if BUILDMODE == 0
  if (that == NULL) {
    PBDataAnalysisErr->_type = PBErrTypeNullPointer;
    sprintf(PBDataAnalysisErr->_msg, "'that' is null");
    PBErrCatch(PBDataAnalysisErr);
  }
  if (input == NULL) {
    PBDataAnalysisErr->_type = PBErrTypeNullPointer;
    sprintf(PBDataAnalysisErr->_msg, "'input' is null");
    PBErrCatch(PBDataAnalysisErr);
  }
#endif
  // Declare variables to memorize the id and distance to the input
  float dist = 0.0;
  int id = -1;
  int iCenter = 0;
  // Loop on the clusters' center
  GSetIterForward iter =
    GSetIterForwardCreateStatic(KMeansClustersCenters(that));
  do {
    // Get the center of the cluster
    VecFloat* center = GSetIterGet(&iter);
    // Calculate the distance to the input
    float d = VecDist(center, input);
    // If it's the first considered cluster or
    // if the distance is nearer
    if (id == -1 || dist > d) {
      id = iCenter;
      dist = d;
      // TODO: we can stop if the distance is less than half the
      // shortest distance between two clusters' center to make
      // this function faster
    }
    // Increment the center index
    ++iCenter;
  } while (GSetIterStep(&iter));
  // Return the id
  return id;
```

```
}
```

4 Makefile

```
# Build mode
# 0: development (max safety, no optimisation)
# 1: release (min safety, optimisation)
# 2: fast and furious (no safety, optimisation)
BUILD_MODE?=0

all: pbmake_wget main

# Automatic installation of the repository PBMake in the parent folder
pbmake_wget:
if [ ! -d ../PBMake ]; then wget https://github.com/BayashiPascal/PBMake/archive/master.zip; unzip master.zip; rm -f

# Makefile definitions
MAKEFILE_INC=../PBMake/Makefile.inc
include $(MAKEFILE_INC)

# Rules to make the executable
repo=pbdataanalysis
$($(repo)_EXENAME): \
$($(repo)_EXENAME).o \
$($(repo)_EXE_DEP) \
$($(repo)_DEP)
$(COMPILER) `echo "$($(repo)_EXE_DEP) $($(repo)_EXENAME).o" | tr ' ' '\n' | sort -u` $(LINK_ARG) $($(repo)_LINK_ARG)

$($(repo)_EXENAME).o: \
$($(repo)_DIR)/$($(repo)_EXENAME).c \
$($(repo)_INC_H_EXE) \
$($(repo)_EXE_DEP)
$(COMPILER) $(BUILD_ARG) $($(repo)_BUILD_ARG) `echo "$($(repo)_INC_DIR)" | tr ' ' '\n' | sort -u` -c $($(repo)_DIR)/$
```

5 Unit tests

```
#include <stdlib.h>
#include <stdio.h>
#include <time.h>
#include <string.h>
#include <math.h>
#include "pbdataanalysis.h"

void UnitTestKMean() {
  srandom(3);
  GSetVecFloat input = GSetVecFloatCreateStatic();
  int K = 3;

  float mean = 0.0;
  float sigma = 3.0;
  Gauss gauss = GaussCreateStatic(mean, sigma);
  int nbData = 100;
  VecFloat2D tgtMean[3];
  FILE* csvFile = fopen("./kmeancluster.csv", "w");
  for (int iInput = 0; iInput < K; ++iInput) {
    tgtMean[iInput] = VecFloatCreateStatic2D();
```

```
  VecSet(tgtMean + iInput, 0, rnd() * 50.0);
  VecSet(tgtMean + iInput, 1, rnd() * 50.0);
  printf("Target #%d: ", iInput);
  VecPrint(tgtMean + iInput, stdout);
  printf("\n");
  fprintf(csvFile, "%f %f\n",
    VecGet(tgtMean + iInput, 0),
    VecGet(tgtMean + iInput, 1));
}
fprintf(csvFile, "\n");
for (int iData = 0; iData < nbData; ++iData) {
  for (int iInput = 0; iInput < K; ++iInput) {
    VecFloat* vFloat = VecFloatCreate(2);
    GSetAppend(&input, vFloat);
    VecSet(vFloat, 0, VecGet(tgtMean + iInput, 0) +
      GaussRnd(&gauss));
    VecSet(vFloat, 1, VecGet(tgtMean + iInput, 1) +
      GaussRnd(&gauss));
    fprintf(csvFile, "%f %f ",
      VecGet(vFloat, 0), VecGet(vFloat, 1));
  }
  fprintf(csvFile, "\n");
}
fprintf(csvFile, "\n");

printf("--- Seed: random\n");
KMeansClustersSeed seed = KMeansClustersSeed_Random;
KMeansClusters clusters = KMeansClustersCreateStatic(seed);
KMeansClustersSearch(&clusters, &input, K);

if (GSetNbElem(KMeansClustersCenters(&clusters)) != K) {
  PBMathErr->_type = PBErrTypeUnitTestFailed;
  sprintf(PBMathErr->_msg, "_GetKMeansClusterFloat NOK");
  PBErrCatch(PBDataAnalysisErr);
}
for (int iCenter = 0; iCenter < K; ++iCenter) {
  const VecFloat* vFloat = KMeansClustersCenter(&clusters, iCenter);
  printf("Cluster #%d: ", iCenter);
  VecPrint(vFloat, stdout);
  printf("\n");
  fprintf(csvFile, "%f %f\n",
    VecGet(vFloat, 0), VecGet(vFloat, 1));
}
KMeansClustersFreeStatic(&clusters);

printf("--- Seed: forgy\n");
seed = KMeansClustersSeed_Forgy;
clusters = KMeansClustersCreateStatic(seed);
KMeansClustersSearch(&clusters, &input, K);

if (GSetNbElem(KMeansClustersCenters(&clusters)) != K) {
  PBMathErr->_type = PBErrTypeUnitTestFailed;
  sprintf(PBMathErr->_msg, "_GetKMeansClusterFloat NOK");
  PBErrCatch(PBDataAnalysisErr);
}
for (int iCenter = 0; iCenter < K; ++iCenter) {
  const VecFloat* vFloat = KMeansClustersCenter(&clusters, iCenter);
  printf("Cluster #%d: ", iCenter);
  VecPrint(vFloat, stdout);
  printf("\n");
  fprintf(csvFile, "%f %f\n",
    VecGet(vFloat, 0), VecGet(vFloat, 1));
```

```
  }

  KMeansClustersFreeStatic(&clusters);
  fclose(csvFile);
  while (GSetNbElem(&input) > 0) {
    VecFloat* v = GSetPop(&input);
    VecFree(&v);
  }

  printf("UnitTestKMean OK\n");
}

void UnitTestAll() {
  UnitTestKMean();
}

int main(void) {
  UnitTestAll();
  return 0;
}
```

6 Unit tests output

```
Target #0: <28.069,11.249>
Target #1: <19.655,22.197>
Target #2: <14.252,7.239>
--- Seed: random
Cluster #0: <19.957,21.695>
Cluster #1: <13.881,7.343>
Cluster #2: <28.442,11.917>
--- Seed: forgy
Cluster #0: <28.442,11.917>
Cluster #1: <19.957,21.695>
Cluster #2: <13.881,7.343>
UnitTestKMean OK
```

kmeancluster.csv:

```
28.069008 11.249166
19.654589 22.196920
14.252063 7.239053

28.633823 12.292907 19.235598 18.823492 12.592678 3.948273
26.052284 19.234270 18.743282 24.418528 10.458279 5.078588
26.130377 12.850240 22.535856 21.906000 10.845601 13.083977
26.455097 8.574337 18.511803 20.898256 14.739879 4.883637
28.733614 10.599995 23.665422 25.562561 14.357016 6.407752
28.550997 10.792174 15.249710 18.156441 20.529245 7.550764
29.843285 10.751228 20.759161 25.732025 9.862355 9.238699
29.386349 10.359576 22.475607 19.885624 14.946790 11.217808
28.227690 11.757756 22.867130 25.741074 14.902649 10.882385
28.538338 19.813602 18.626719 26.148878 11.498481 5.557252
23.947056 8.374683 20.750635 24.407253 13.187488 6.842857
28.845385 11.675117 22.477848 25.658518 16.712572 6.494166
28.240320 6.265765 18.270994 16.920401 12.141680 9.390920
24.723717 8.320883 22.146622 22.992455 11.246556 7.748080
26.633812 6.446450 13.803344 20.392096 18.239250 9.086374
```

```
30.953035 9.323502 21.829857 21.131969 16.259020 7.211296
31.116953 12.879289 23.409569 22.789968 14.732009 6.460821
20.959503 12.602968 17.141291 23.852451 14.999218 8.711921
27.956499 13.609019 18.733019 22.014162 15.527146 10.999743
31.872864 8.943518 18.358654 24.653770 8.360085 10.208816
26.097677 16.264061 24.531662 23.721464 12.468403 6.878686
26.388678 11.445029 20.422182 25.463303 14.714397 3.323504
24.513933 7.059963 18.509027 22.554037 18.467571 4.957978
29.366932 10.861627 17.426935 22.157187 14.995366 8.656789
31.595938 14.901086 19.876196 26.168688 12.279941 6.107559
27.047731 18.721298 17.749092 27.143915 17.660381 4.754192
24.668179 13.485799 21.459522 22.727383 13.219142 7.327538
30.785263 15.390684 23.087008 21.896502 11.119055 6.156090
25.330399 12.283073 18.255594 22.551985 12.758935 11.746021
24.778790 12.417658 15.977521 20.013983 17.043211 8.389001
31.640587 12.987787 17.524673 20.441366 14.605970 4.453372
29.095758 11.579761 22.153564 18.537165 14.559906 8.431289
32.779095 15.358913 20.257149 25.212011 13.068298 10.726461
35.030907 6.639762 21.238081 25.346832 16.072897 6.441304
26.896448 16.172876 14.700457 27.644926 8.433315 7.357404
32.441208 13.260548 18.330910 24.867006 14.426729 3.567520
19.391371 12.997995 15.806515 19.831161 15.215398 5.320875
28.895748 9.814054 20.318626 23.252077 7.718215 7.693003
27.068031 10.696910 16.536890 22.733471 15.396071 5.993648
32.208397 16.384439 25.096729 21.963247 11.924290 10.051946
26.491650 6.049950 20.415474 22.120098 10.869869 5.957185
26.589045 10.089059 18.382231 22.843460 14.501829 10.078506
26.485561 12.192287 19.841579 19.588705 13.052887 9.544924
24.927652 12.940019 17.671413 26.235970 15.761093 6.421089
31.238827 18.318321 19.240604 25.089767 11.164451 6.369447
28.939318 12.142779 18.180834 20.816853 12.375098 5.505887
27.630131 10.561348 22.605457 19.463205 14.375712 7.359557
27.850735 13.993544 15.209973 21.950428 14.728844 5.261156
30.143599 11.337689 15.328620 22.779089 8.273642 8.736053
30.769220 12.207006 19.299946 24.585787 17.362402 4.612987
32.244652 10.282434 17.320114 19.385283 12.901281 2.647302
24.538588 11.602779 17.704199 17.823856 18.038988 7.197000
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random seed in grey, Forgy seed in black: