Workshop Material: `https://tinyurl.com/yc4sz5vx`

# Bayesian Graphical Modeling

## Theory

*Summer School on Network Psychometrics*
*August 2025*
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UvA

BAYESIAN
Graphical Modeling Lab

Psych Systems

# Session setup

Part I: Theory

Part II: Tutorial

Part III: Practical

image: Flaticon.com

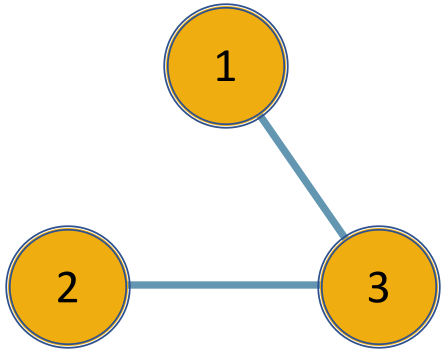
# Session setup

Part I: Theory



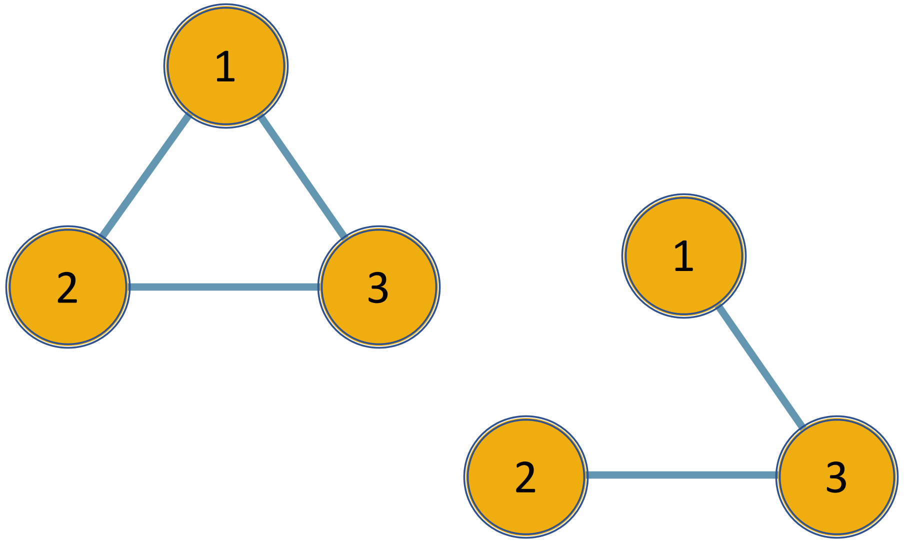
image: Flaticon.com

# Two fundamental questions in network analysis



# Two fundamental questions in network analysis

Is there an effect?



# Two fundamental questions in network analysis

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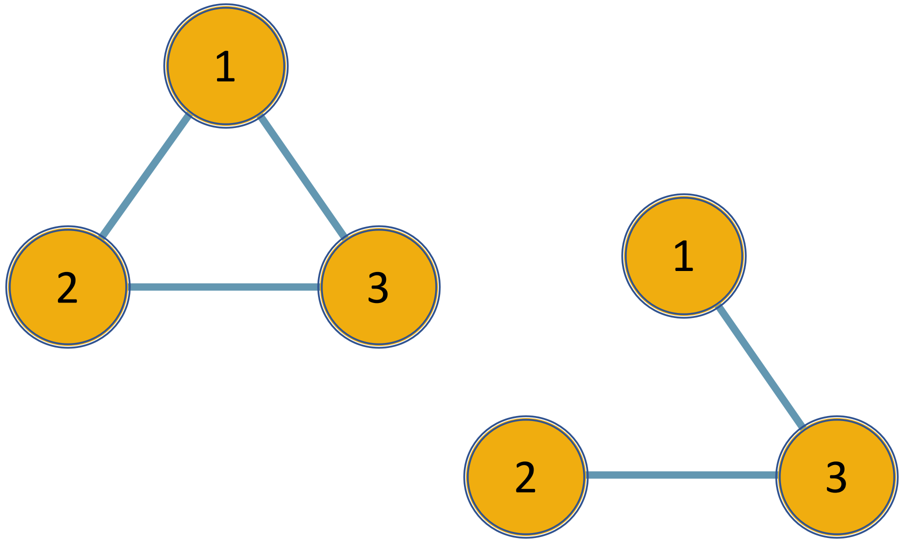
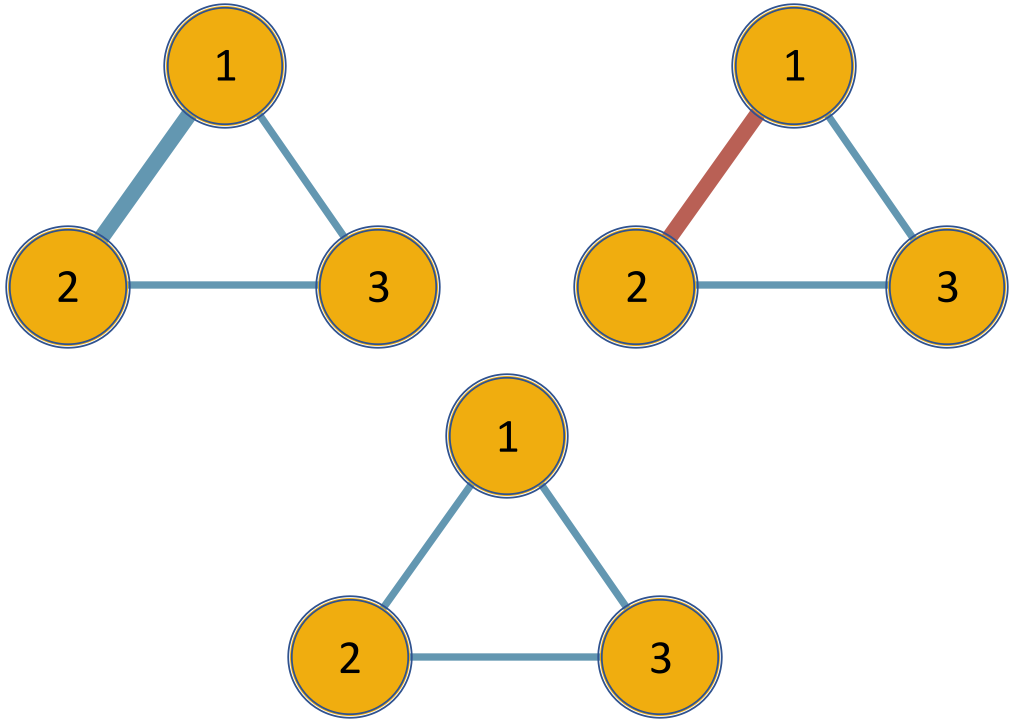
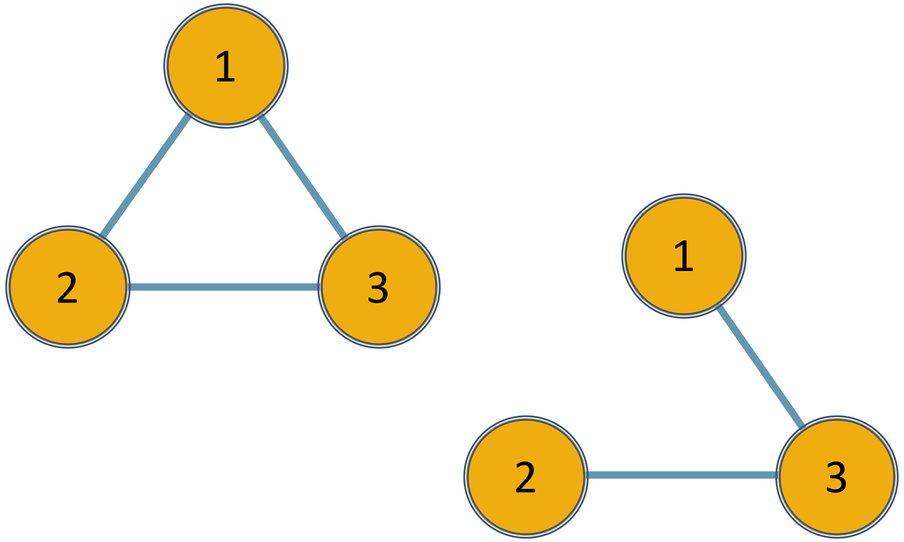


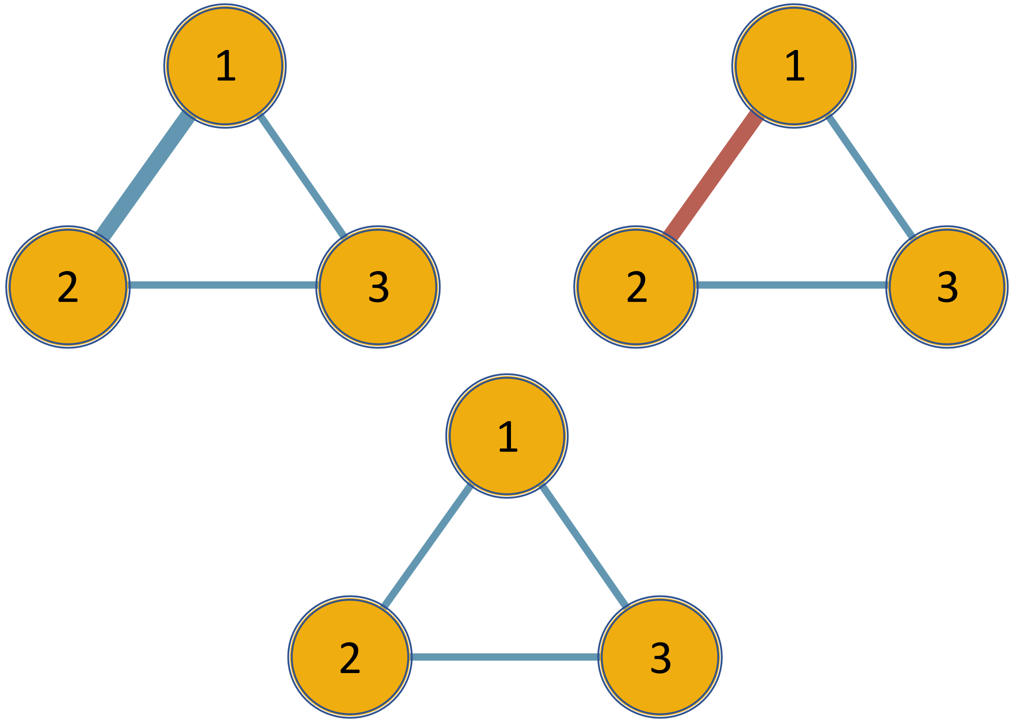




## What is the effect?

# Two fundamental questions in network analysis

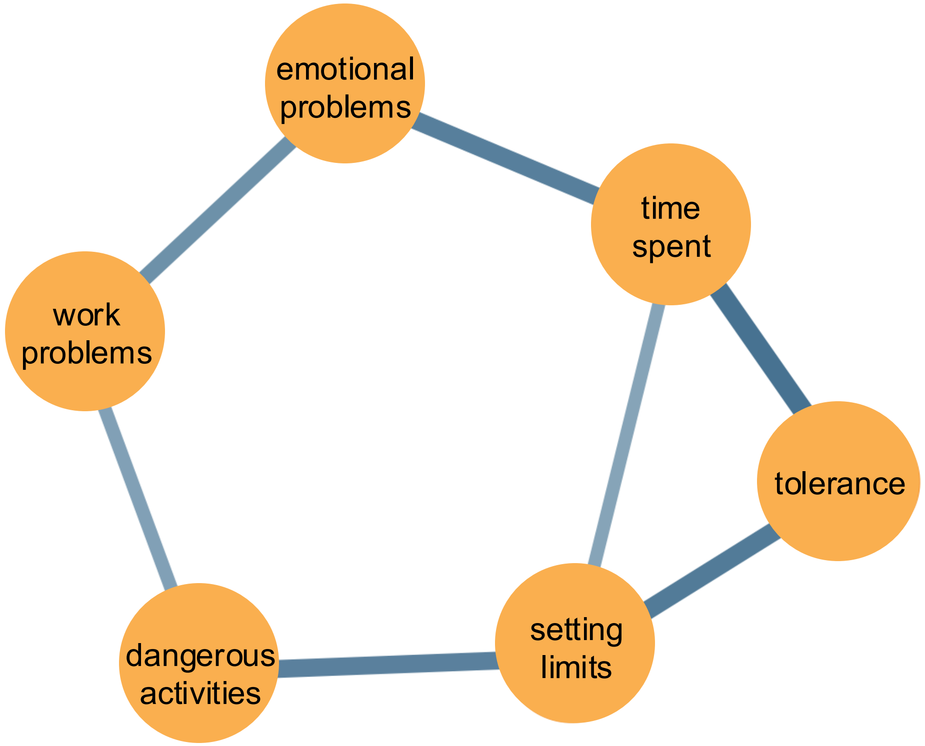
Is there an effect?




What is the effect?

# A network of alcohol disorder symptoms



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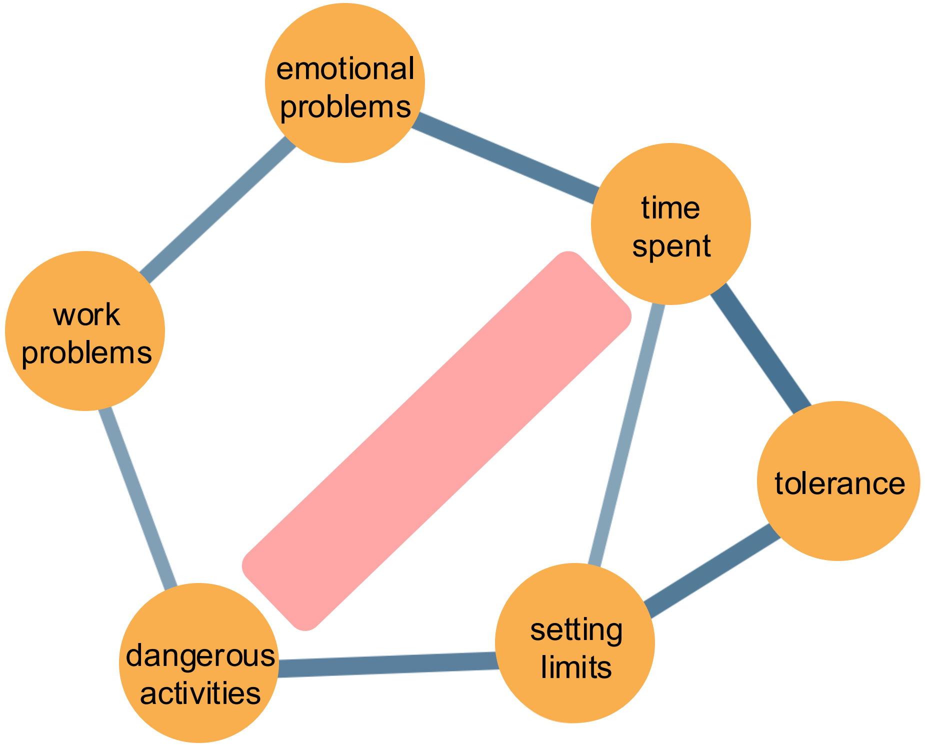


Why is the edge missing?

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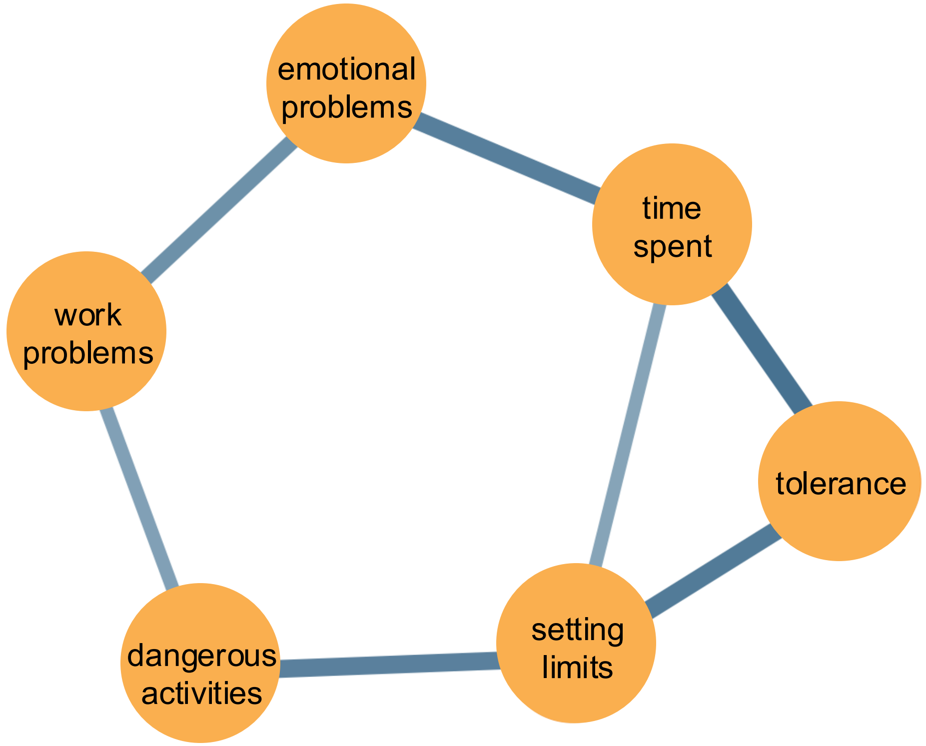
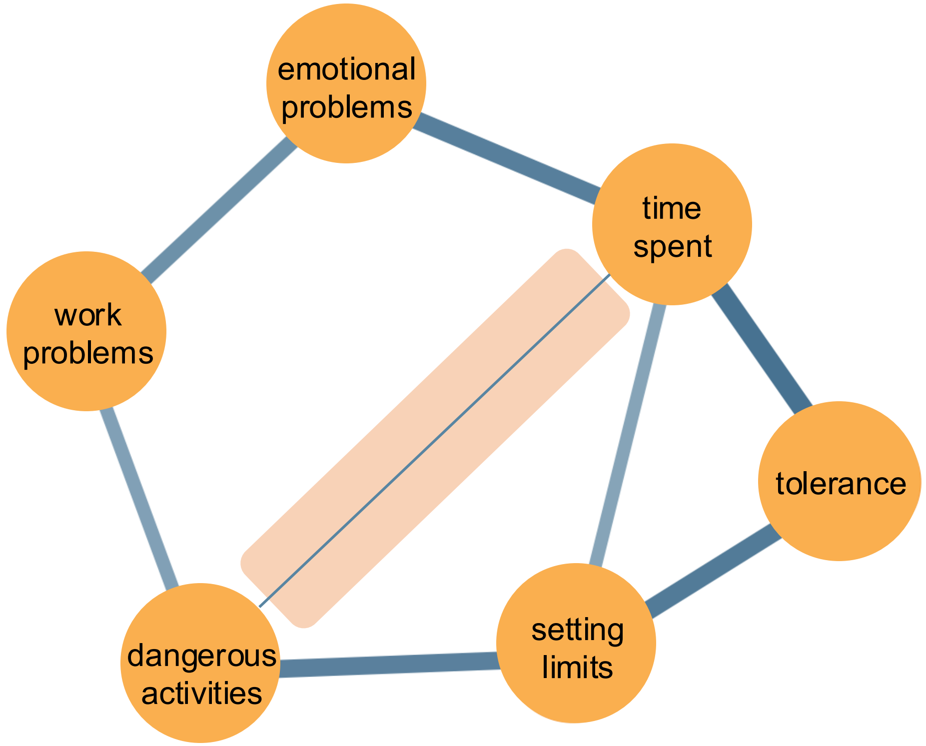



Why is the edge missing?

Two possible explanations:

# A network of alcohol disorder symptoms

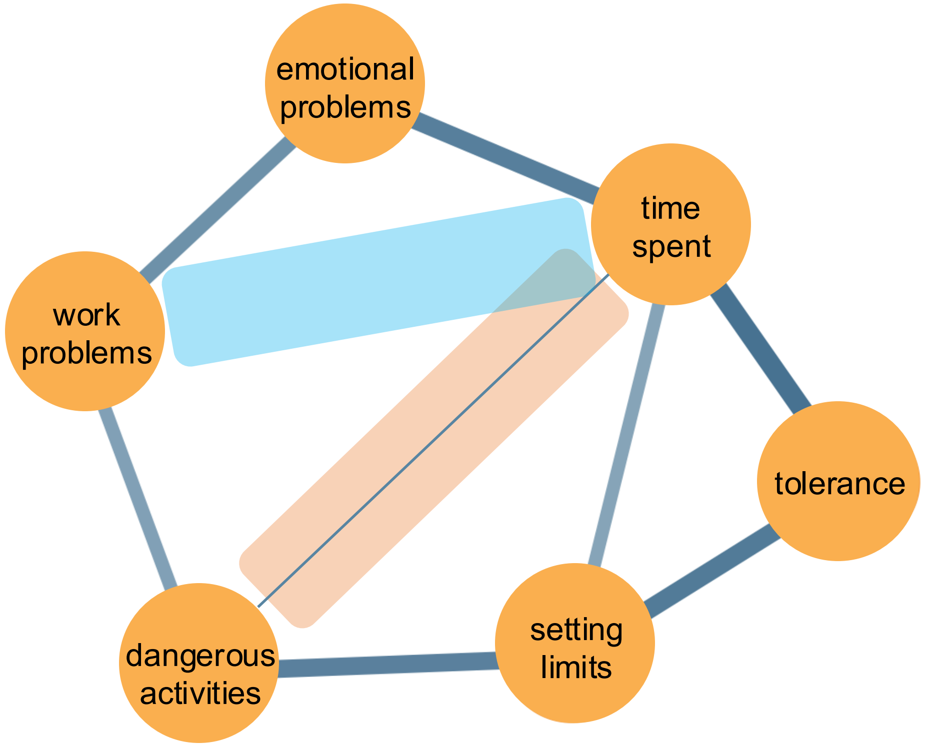



Why is the edge missing?

Two possible explanations:

1. Too little information.

# A network of alcohol disorder symptoms

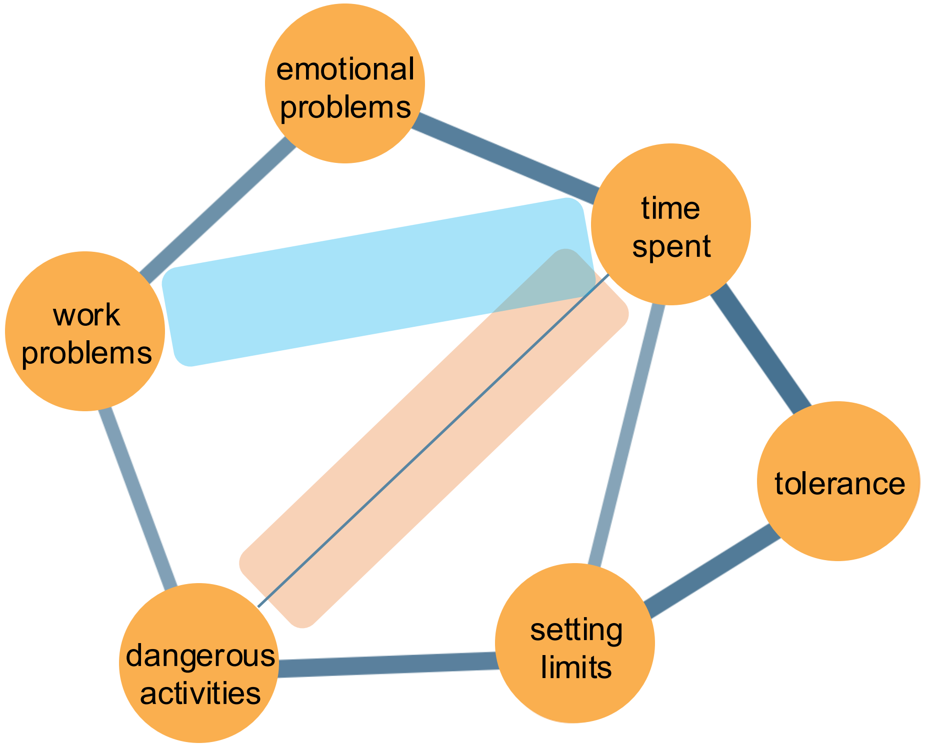



Why is the edge missing?

Two possible explanations:

1. Too little information.
2. True absence.

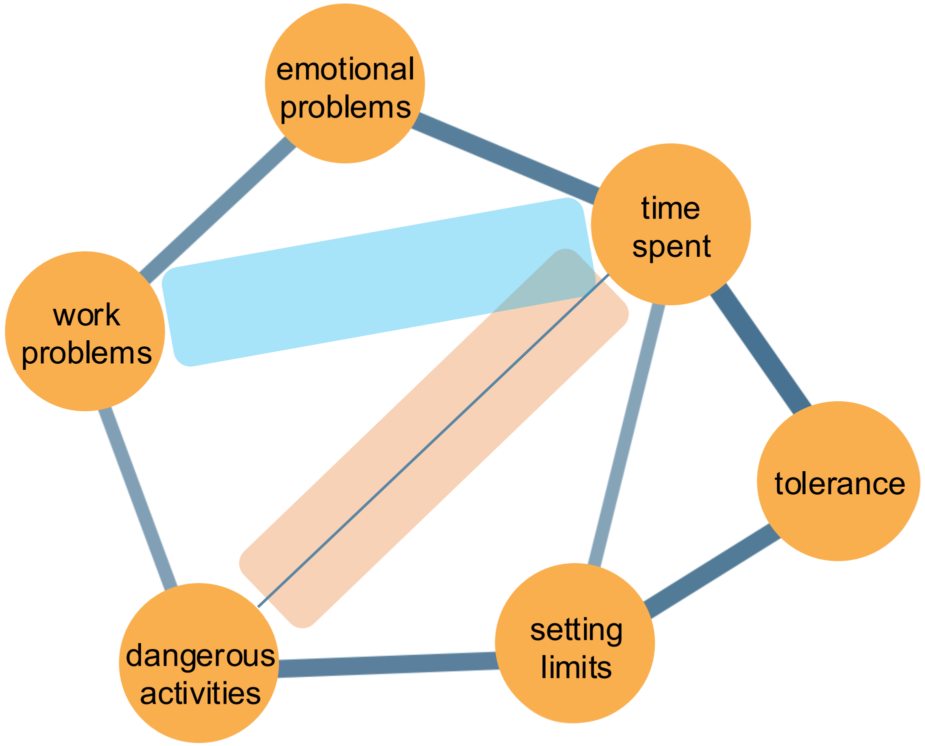
# Two take-home messages




Network estimation is insufficient to determine whether an edge should be included or excluded.

Inclusion tests need to distinguish between **evidence of absence** and **absence of evidence**.

# Two take-home messages

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# The Bayesian analysis of network structures

Evaluate the predictive success of the network structure.

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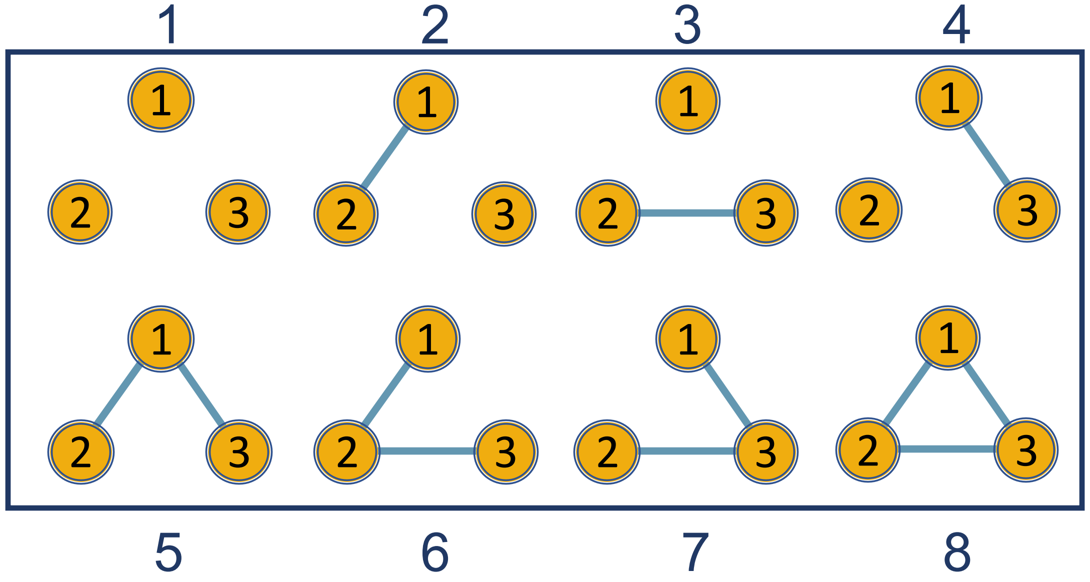

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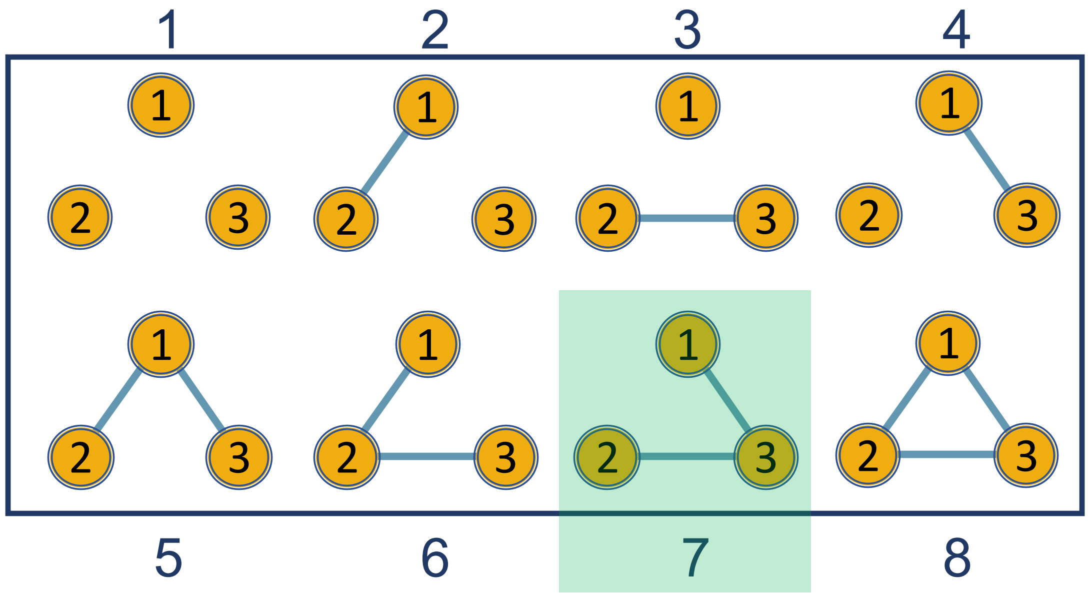
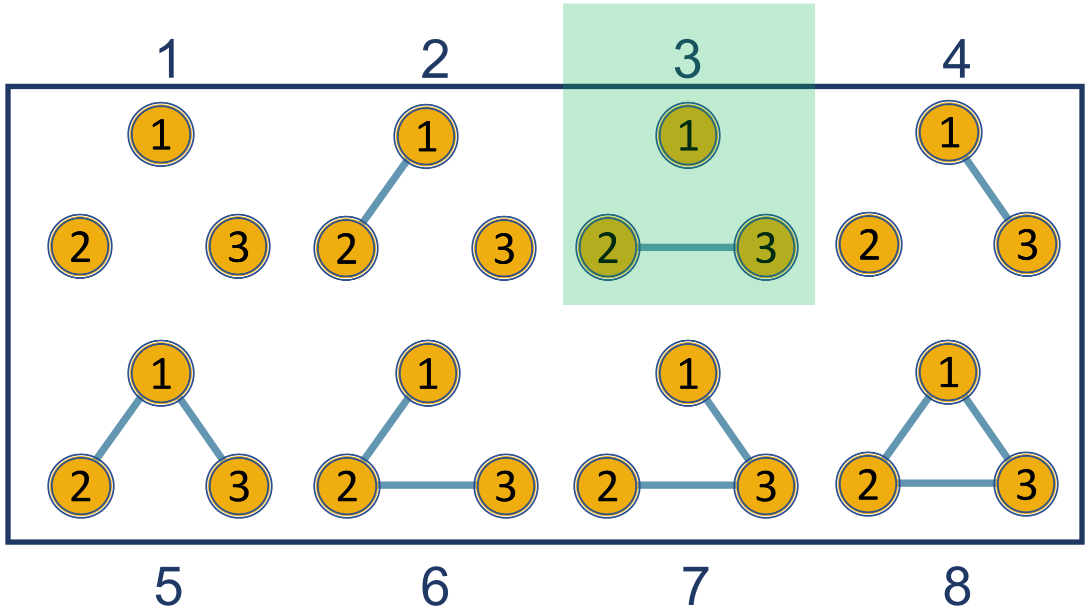


# The Bayesian analysis of network structures

Evaluate the predictive success of the network structure.

$$P(\text{data} \mid \text{structure } 3)$$

But which structure?



# The Bayesian analysis of network structures

Evaluate the predictive success of the network structure.

$$P(\text{data} \mid \text{structure } s)$$

We assign **prior** weights (probabilities) to the possible structures to reflect our uncertainty about the true structure.

$$P(\text{structure } s)$$

# The Bayesian analysis of network structures

We use data to update our **prior** probabilities to **posterior** probabilities – what we know after having seen the data

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$$P(\text{structure } 3 \mid \text{data}) = \frac{P(\text{data} \mid \text{structure } 3)\, P(\text{structure } 3)}{P(\text{data})}$$

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Predictive Updating Factor

$$P(\text{structure } 3 \mid \text{data}) = \frac{P(\text{data} \mid \text{structure } 3)\, P(\text{structure } 3)}{P(\text{data})}$$

# The Bayesian analysis of network structures

We use data to update our **prior** probabilities to **posterior** probabilities – <u>what we know after having seen the data</u>


$$P(\text{structure } 3 \mid \text{data}) = \frac{\boxed{P(\text{data} \mid \text{structure } 3)}\, P(\text{structure } 3)}{\sum_{s=1}^{8} \boxed{P(\text{data} \mid \text{structure } s)}\, P(\text{structure } s)}$$

# The Bayesian analysis of network structures

We use data to update our **prior** probabilities to **posterior** probabilities – <u>what we know after having seen the data</u>


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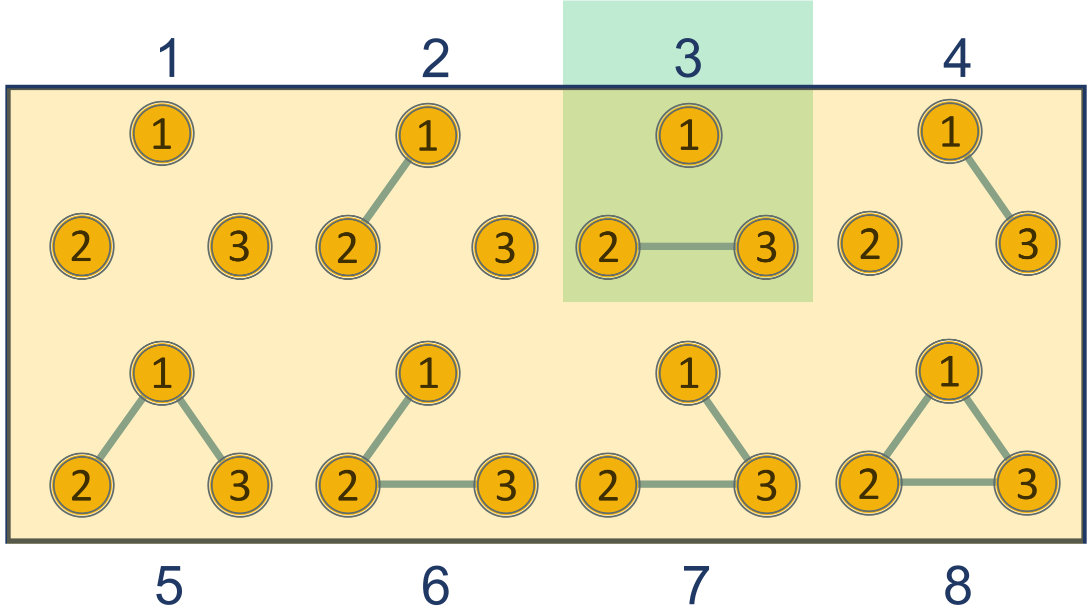
# The **prior** structure probabilities

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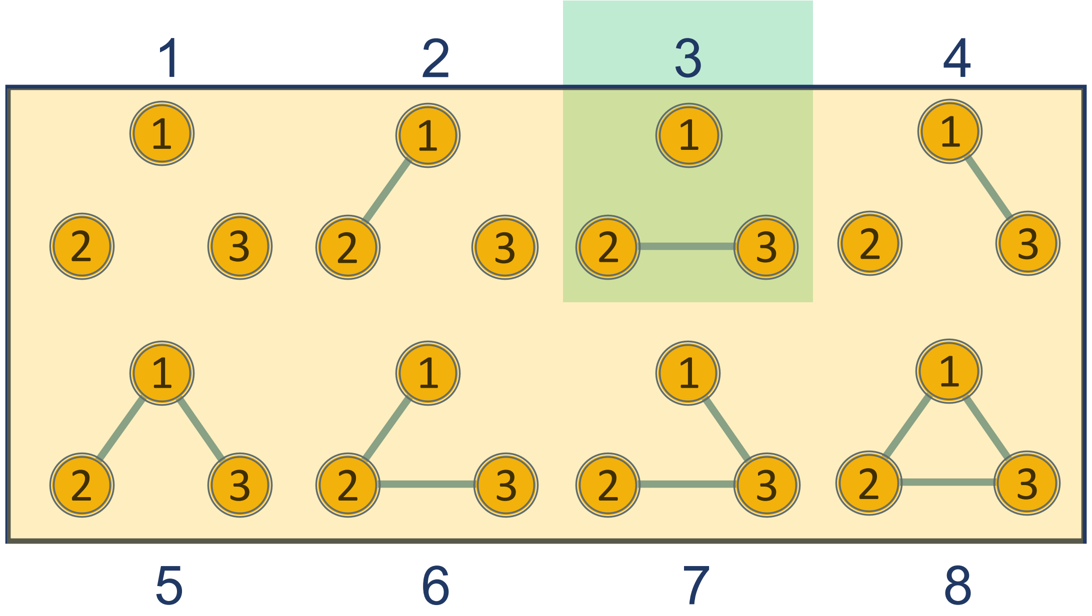


$$P(\text{structure } 3 \mid \text{data}) = \frac{P(\text{data} \mid \text{structure } 3)\ \frac{1}{8}}{\sum_{s=1}^{8} P(\text{data} \mid \text{structure } s)\ \frac{1}{8}}$$

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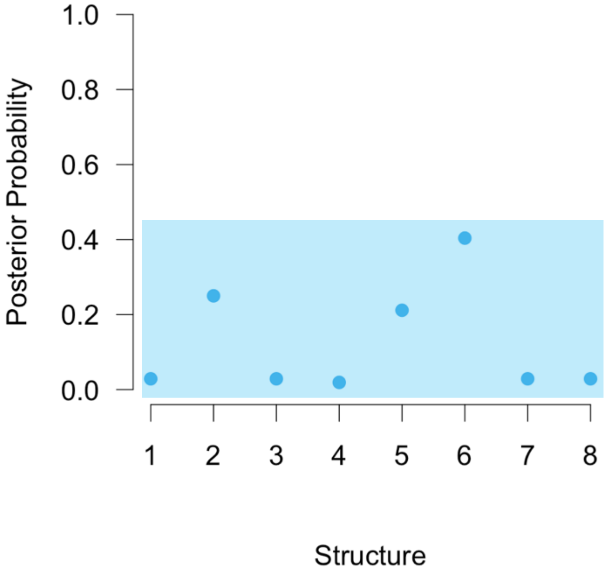



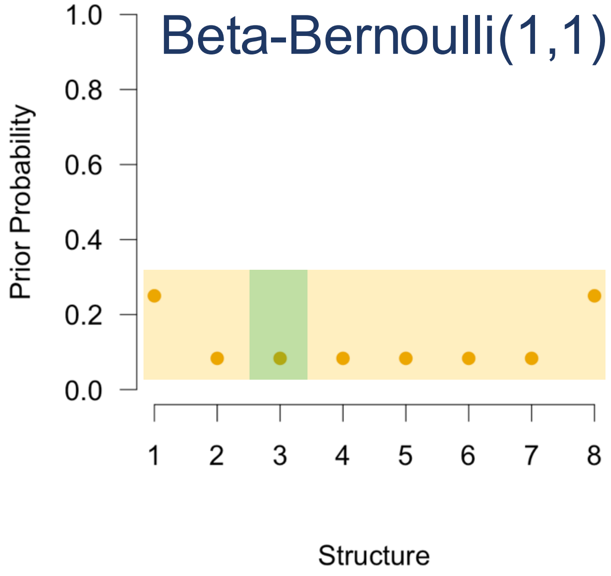


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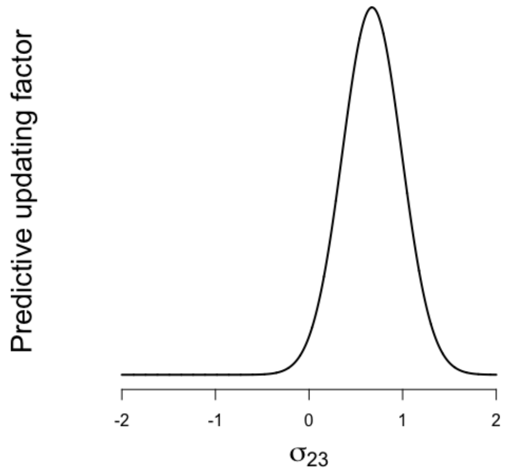
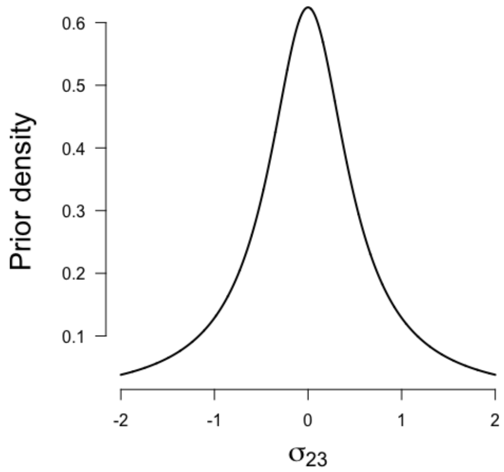
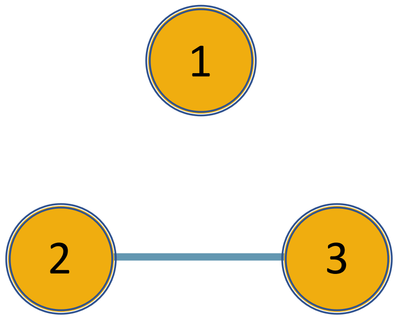
# The **marginal likelihood**



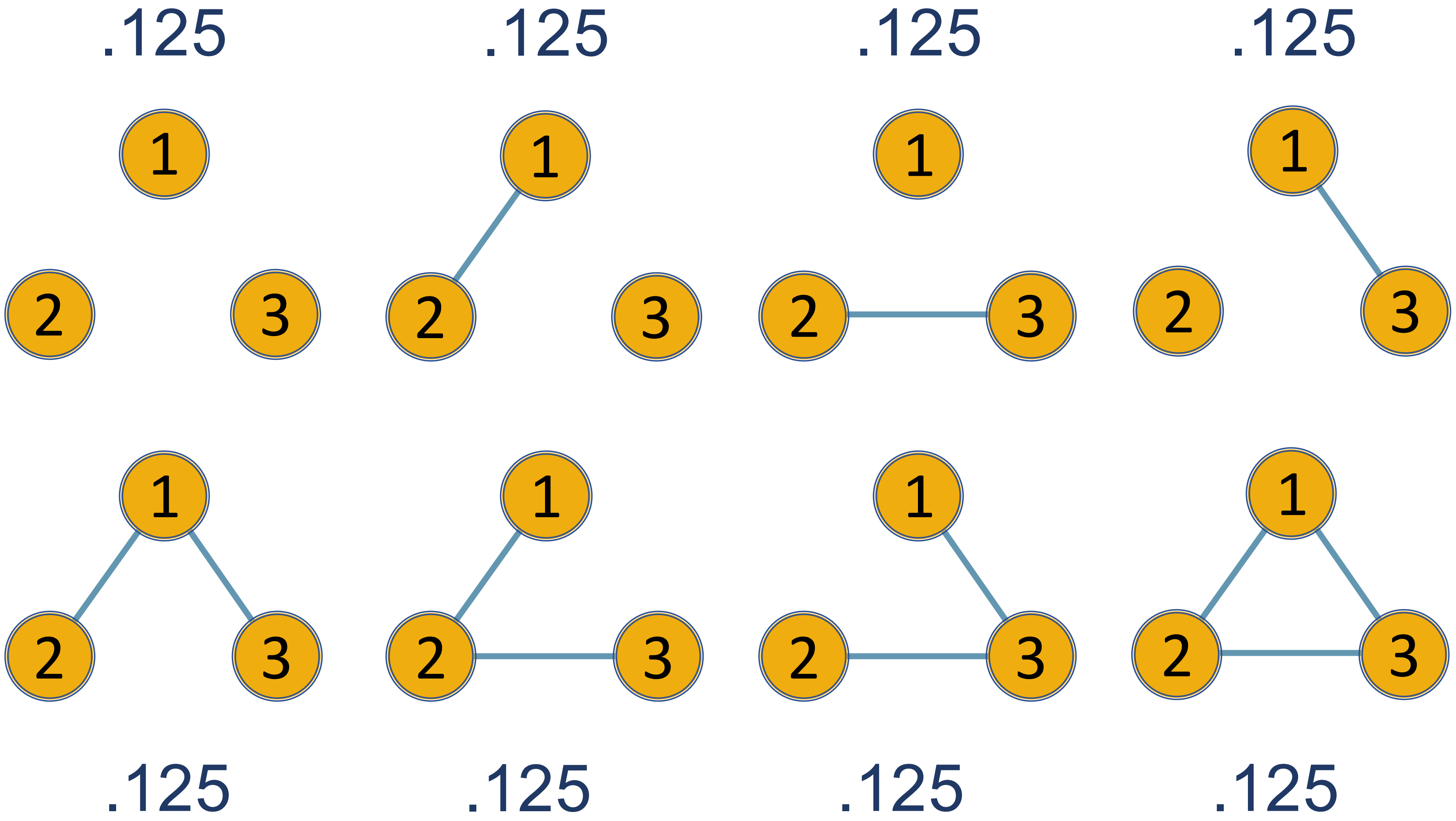



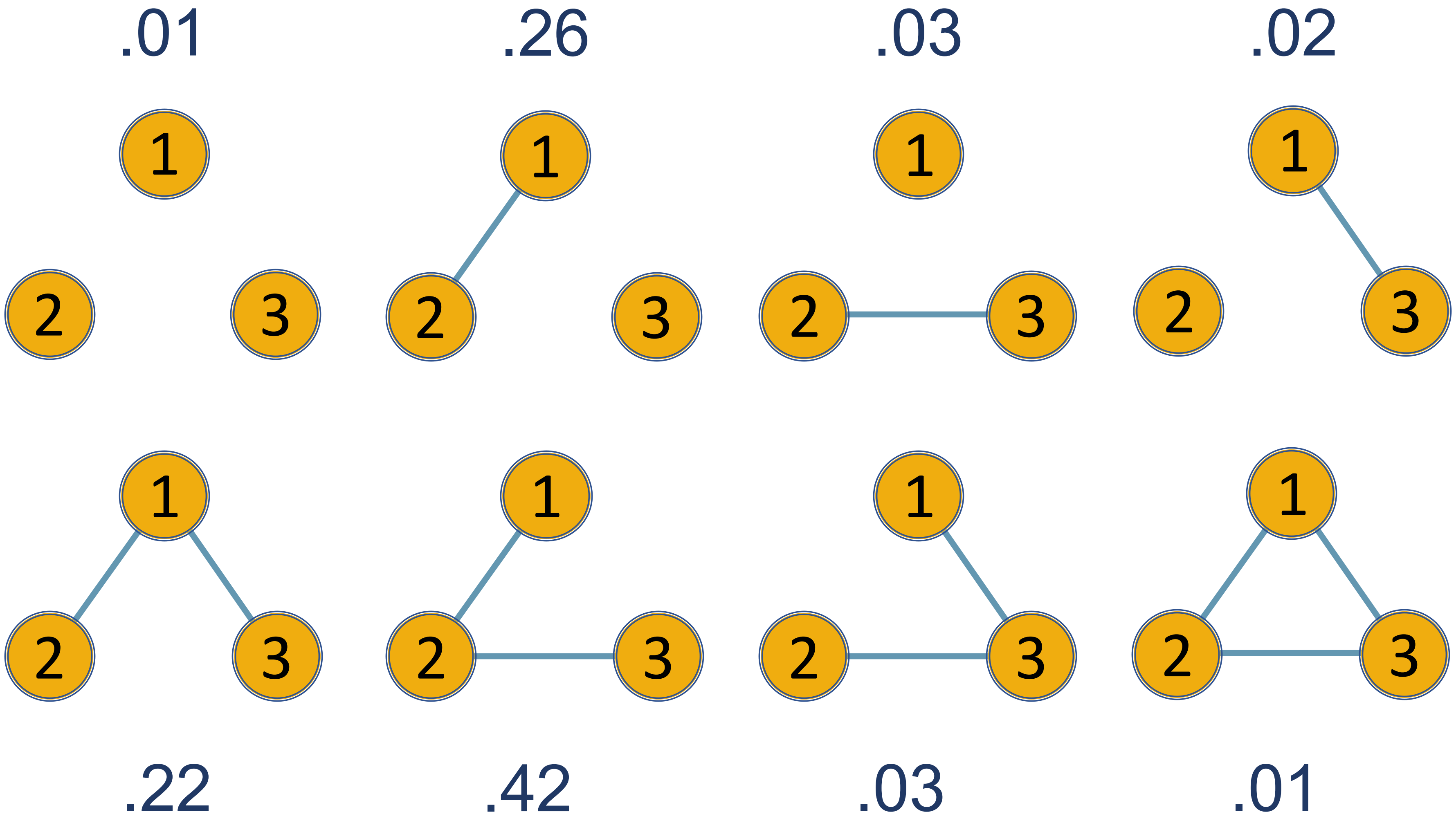

$$\boxed{P(\text{data} \mid \text{structure } 3)} = \int_{\mathbb{R}} p(\text{data} \mid \sigma_{23})\, p(\sigma_{23})\, \mathrm{d}\sigma_{23}$$

$$P(\text{structure } 3 \mid \text{data}) = \frac{\boxed{P(\text{data} \mid \text{structure } 3)}\, \frac{1}{8}}{\sum_{s=1}^{8} P(\text{data} \mid \text{structure } s)\, \frac{1}{8}}$$

# Bayesian estimation: **prior** probabilities



# Bayesian estimation: **posterior** probabilities



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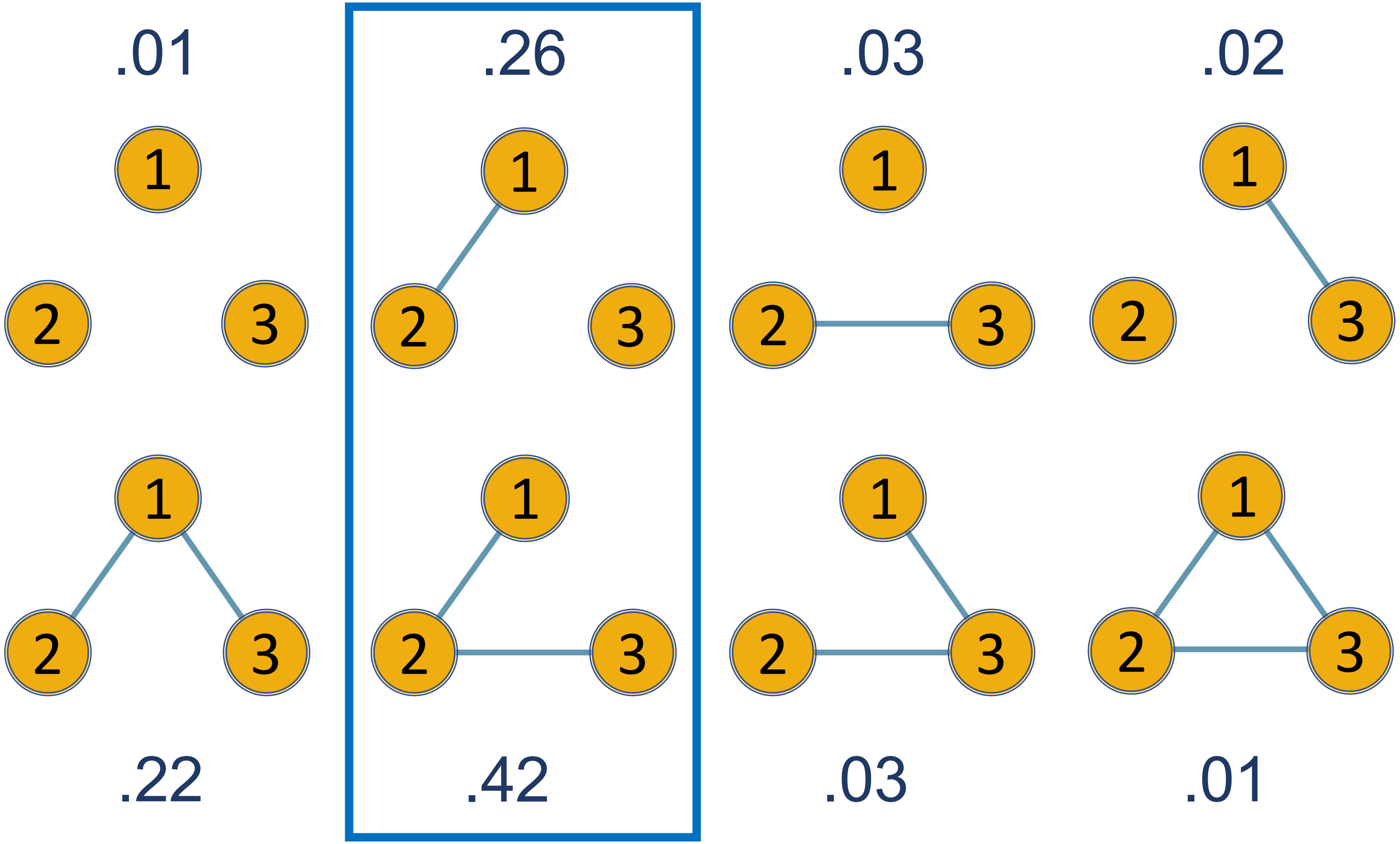

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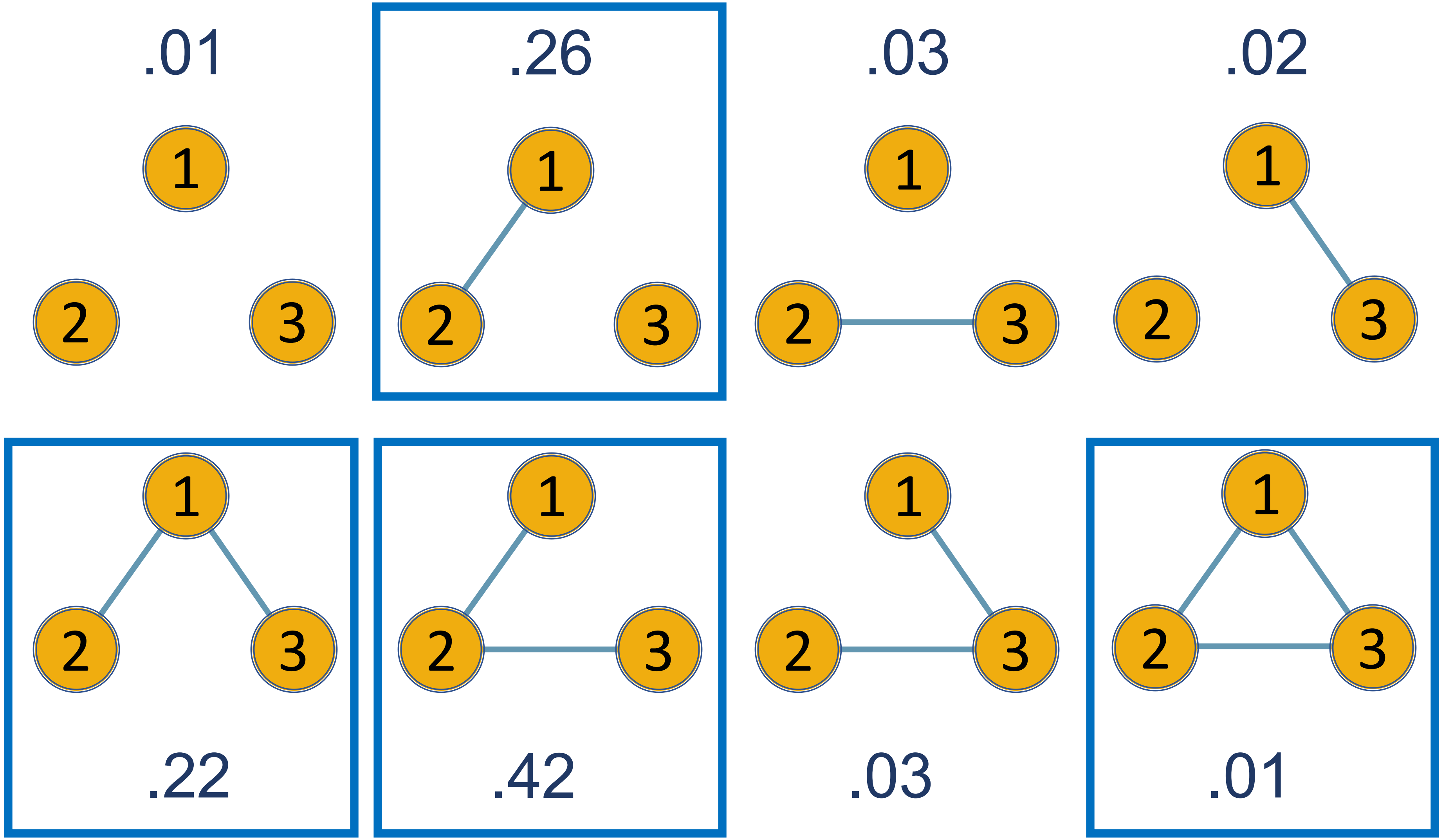

# Bayesian hypothesis testing

$$\mathcal{H}_0 : \text{There is no edge between variables 1 and 2}$$

$$\mathcal{H}_1 : \text{There is an edge between variables 1 and 2}$$

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$$\underbrace{\frac{P(\mathcal{H}_1 \mid \text{data})}{P(\mathcal{H}_0 \mid \text{data})}}_{\text{Posterior odds}} = \underbrace{\text{BF}_{10}}_{\text{Bayes factor}} \times \underbrace{\frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)}}_{\text{Prior odds}}$$

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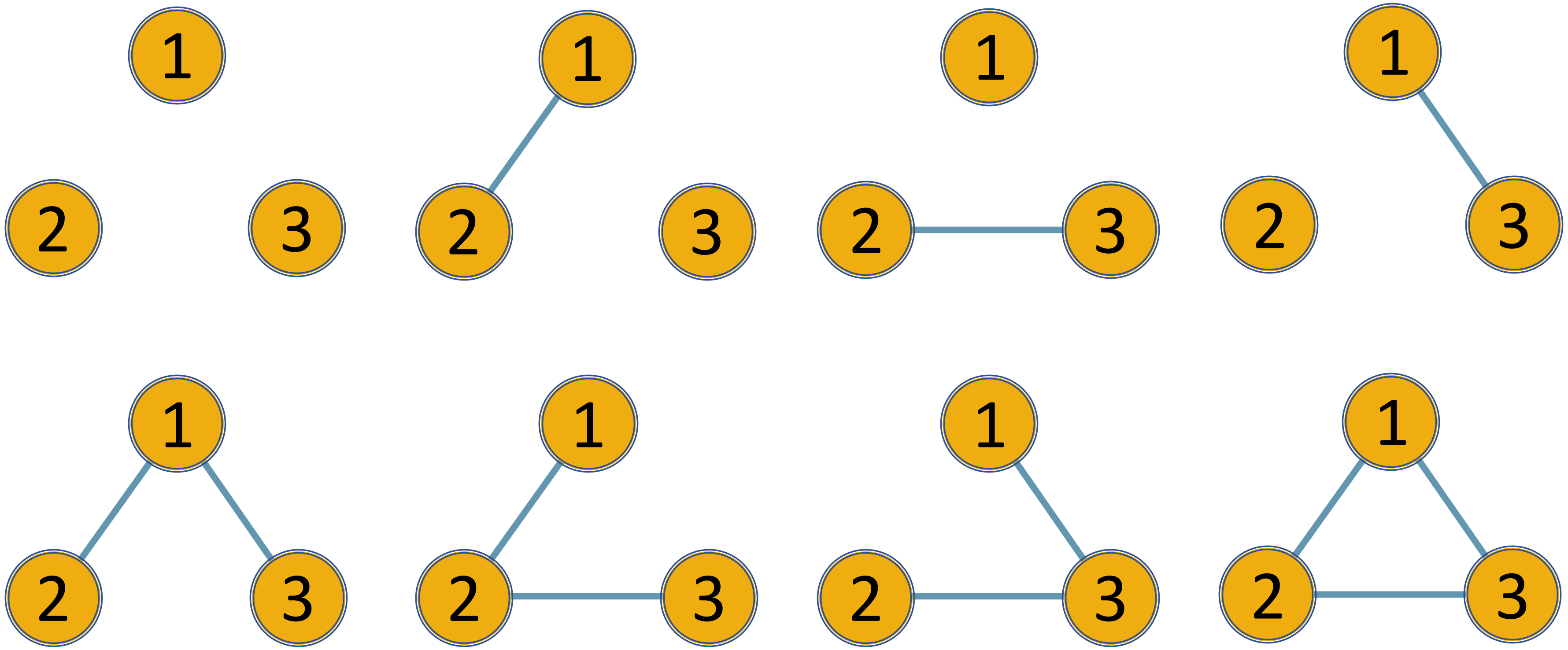
$$\underbrace{\text{BF}_{10}}_{\text{Bayes factor}} = \underbrace{\frac{P(\mathcal{H}_1 \mid \text{data})}{P(\mathcal{H}_0 \mid \text{data})}}_{\text{Posterior odds}} \Bigg/ \underbrace{\frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)}}_{\text{Prior odds}}$$

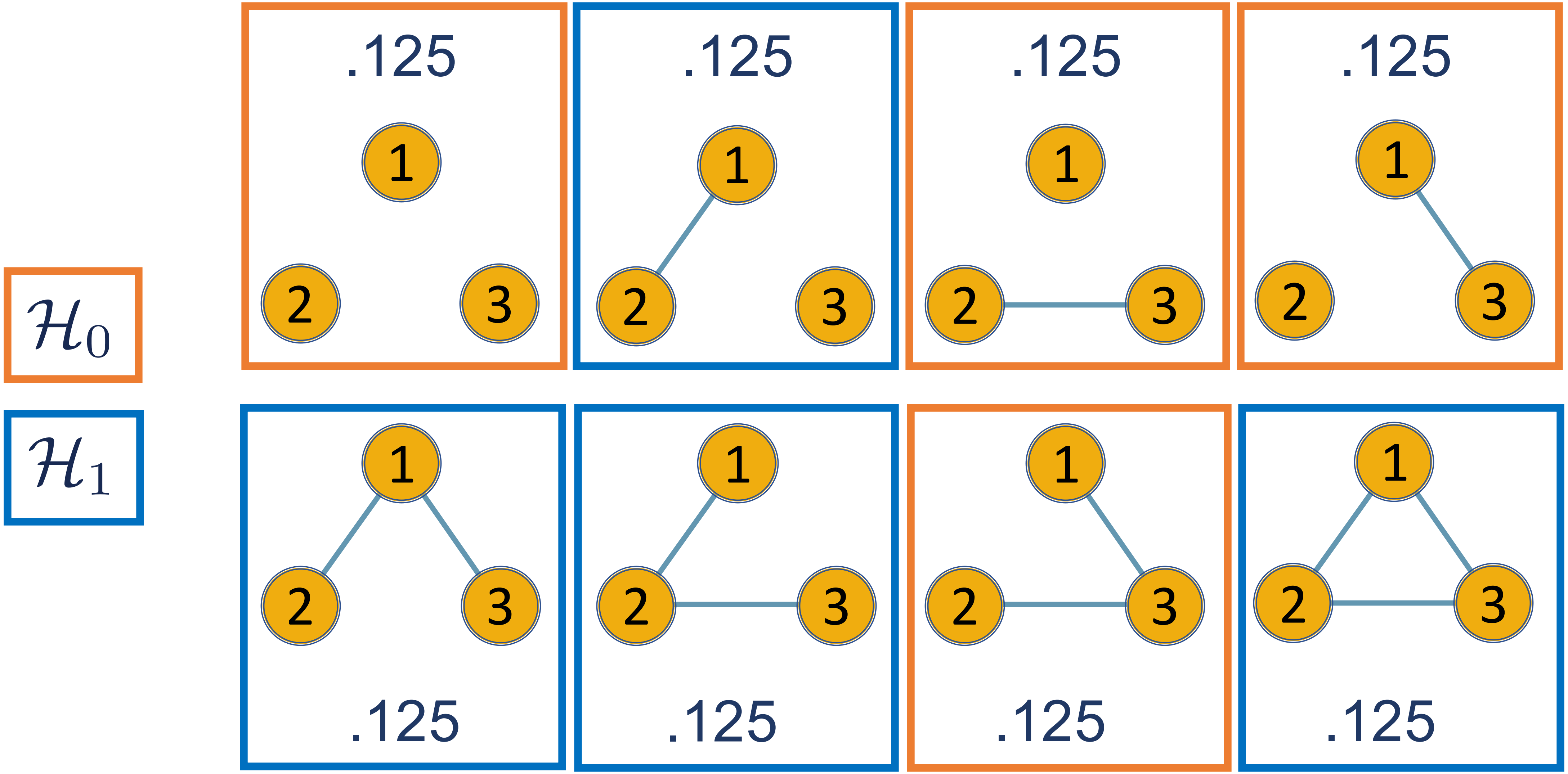
# Bayesian hypothesis testing: **prior odds**

.125 .125 .125 .125



.125 .125 .125 .125

# Bayesian hypothesis testing: **prior odds**



# Bayesian hypothesis testing: **prior odds**

| .125 | .125 | .125 | .125 |
|---|---|---|---|

$$p(\mathcal{H}_0) = .125 + .125 + .125 + .125 = .5$$

$$p(\mathcal{H}_1) = .125 + .125 + .125 + .125 = .5$$

| .125 | .125 | .125 | .125 |
|---|---|---|---|

# Bayesian hypothesis testing: **prior odds**

| | | | |
|---|---|---|---|
| .125 | .125 | .125 | .125 |

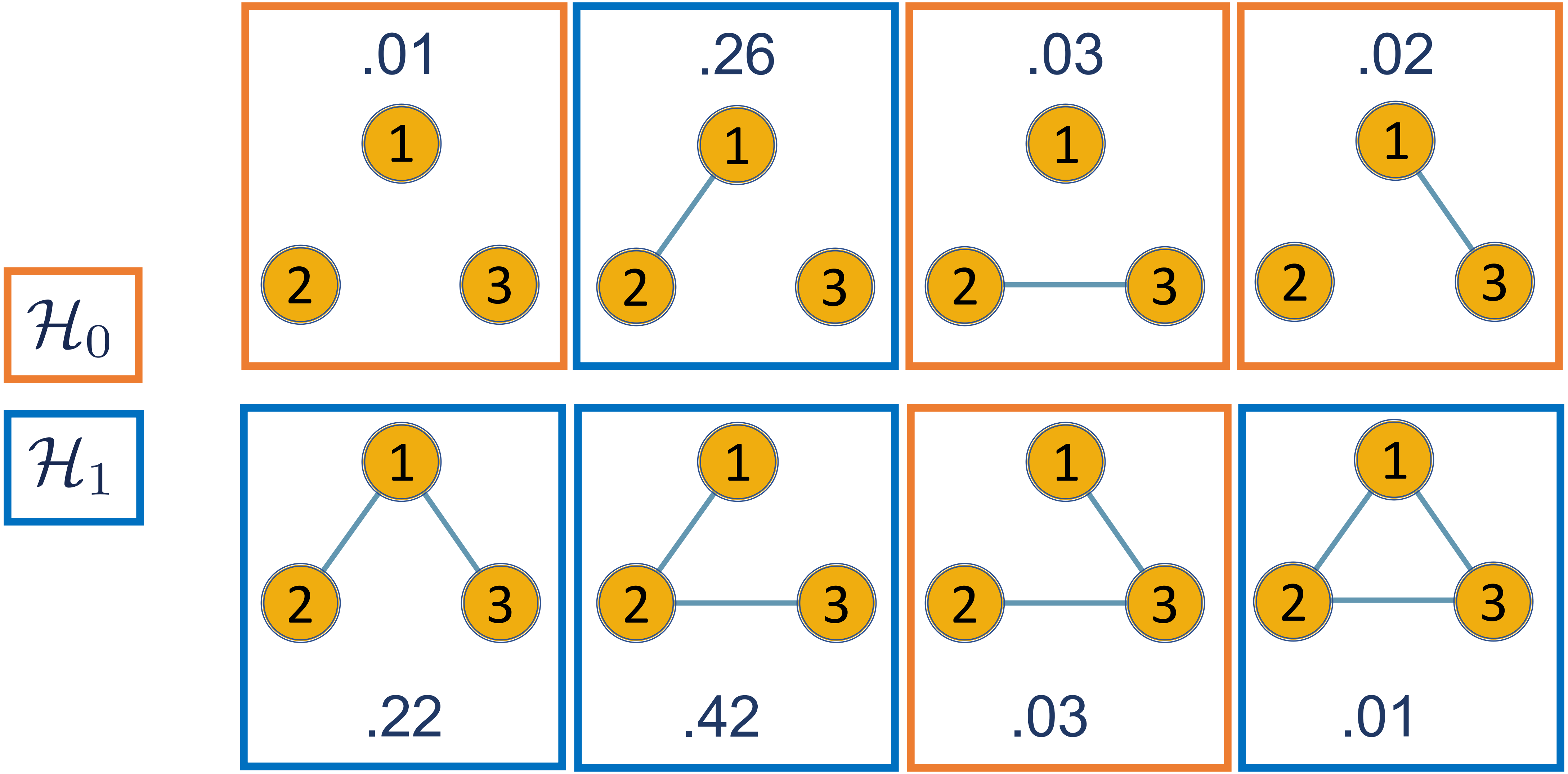
$$\boxed{P(\mathcal{H}_0) = .5}$$

$$\boxed{P(\mathcal{H}_1) = .5}$$

$$\frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)} = 1$$

| | | | |
|---|---|---|---|
| .125 | .125 | .125 | .125 |

# Bayesian hypothesis testing: **posterior odds**



# Bayesian hypothesis testing: **posterior odds**

| | | | |
|---|---|---|---|
| .01 | .26 | .03 | .02 |

$\mathcal{H}_0$ $$P(\mathcal{H}_0 \mid \text{data}) = .01 + .03 + .02 + .03 = .09$$

$\mathcal{H}_1$ $$P(\mathcal{H}_1 \mid \text{data}) = .26 + .22 + .42 + .01 = .91$$

| | | | |
|---|---|---|---|
| .22 | .42 | .03 | .01 |

# Bayesian hypothesis testing: **posterior odds**

| | | | |
|---|---|---|---|
| .01 | .26 | .03 | .02 |

$$\boxed{P(\mathcal{H}_0 \mid \text{data}) = .09}$$

$$\boxed{P(\mathcal{H}_1 \mid \text{data}) = .91}$$

$$\frac{P(\mathcal{H}_1 \mid \text{data})}{P(\mathcal{H}_1 \mid \text{data})} = \frac{.91}{.09} = 10.11$$

| | | | |
|---|---|---|---|
| .22 | .42 | .03 | .01 |

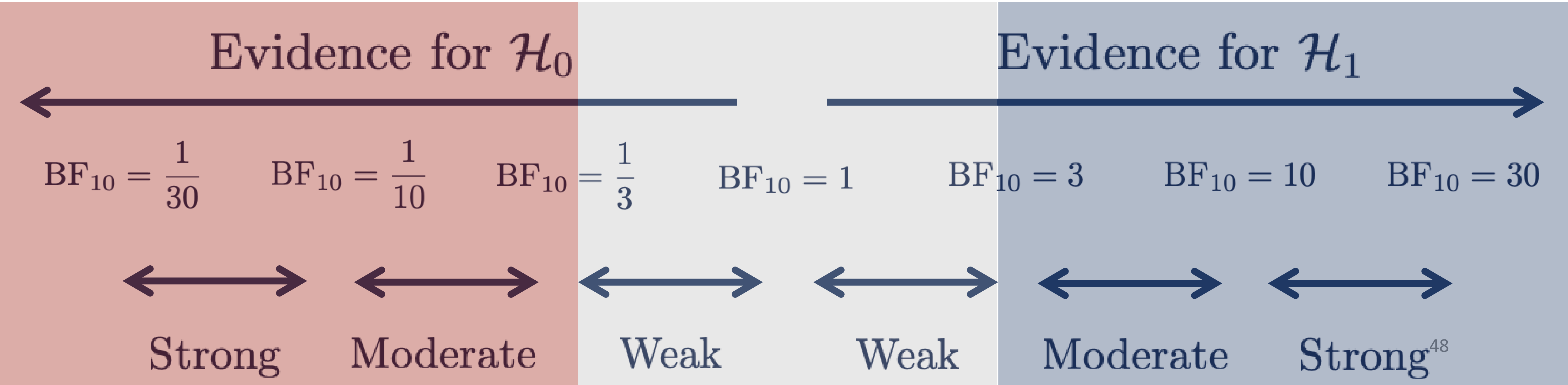
# Bayesian hypothesis testing: **Bayes factor**

$$\mathrm{BF}_{10} = \frac{P(\mathcal{H}_1 \mid \mathrm{data})}{P(\mathcal{H}_0 \mid \mathrm{data})} \Bigg/ \frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)} = 10.11$$

The data are 4.6 times more likely under structures that include an edge between variables 1 and 2!

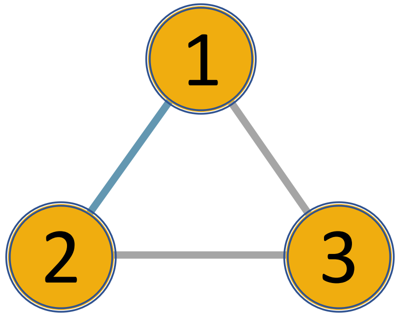
# Bayesian hypothesis testing: **Bayes factor**

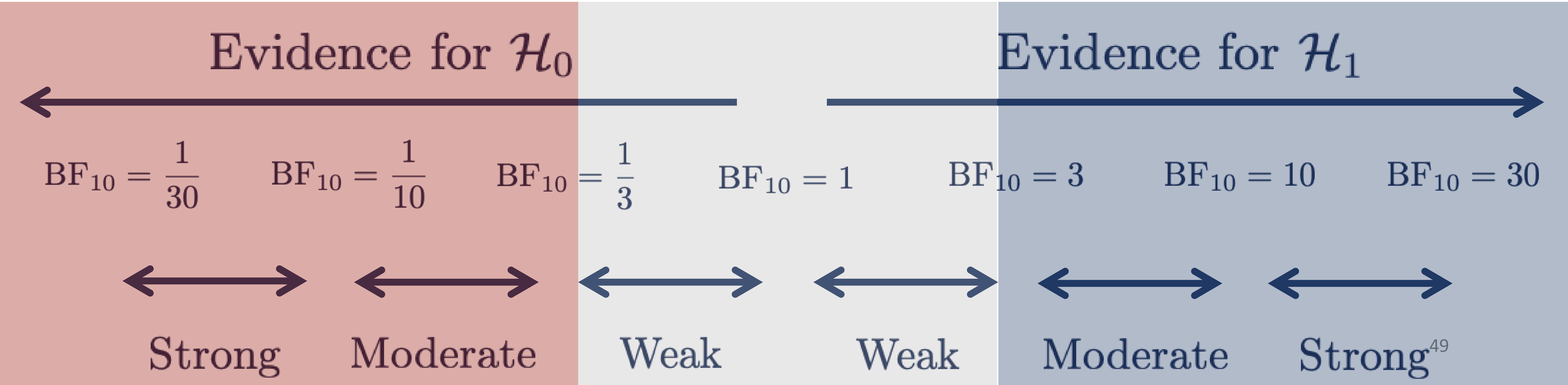
$$\mathrm{BF}_{10} = \frac{P(\mathcal{H}_1 \mid \text{data})}{P(\mathcal{H}_0 \mid \text{data})} \Bigg/ \frac{P(\mathcal{H}_1)}{P(\mathcal{H}_0)} = 10.11$$



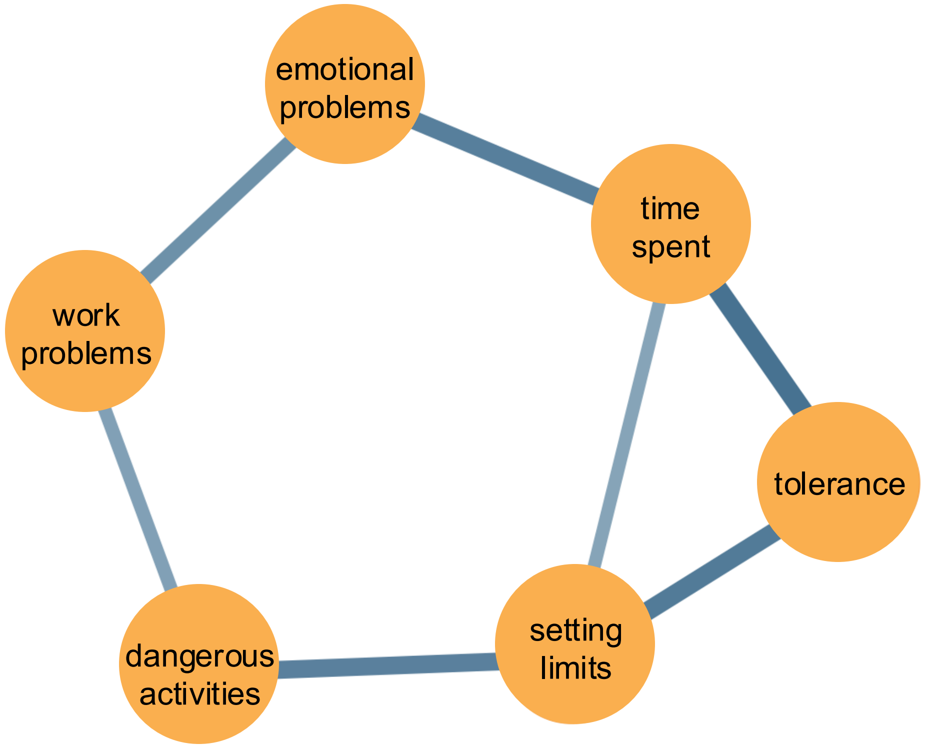
# Bayesian hypothesis testing: **Bayes factor**

$$\mathrm{BF}_{10}(1-2) = 10.11$$
$$\mathrm{BF}_{10}(1-3) = 0.39$$
$$\mathrm{BF}_{10}(2-3) = 0.96$$



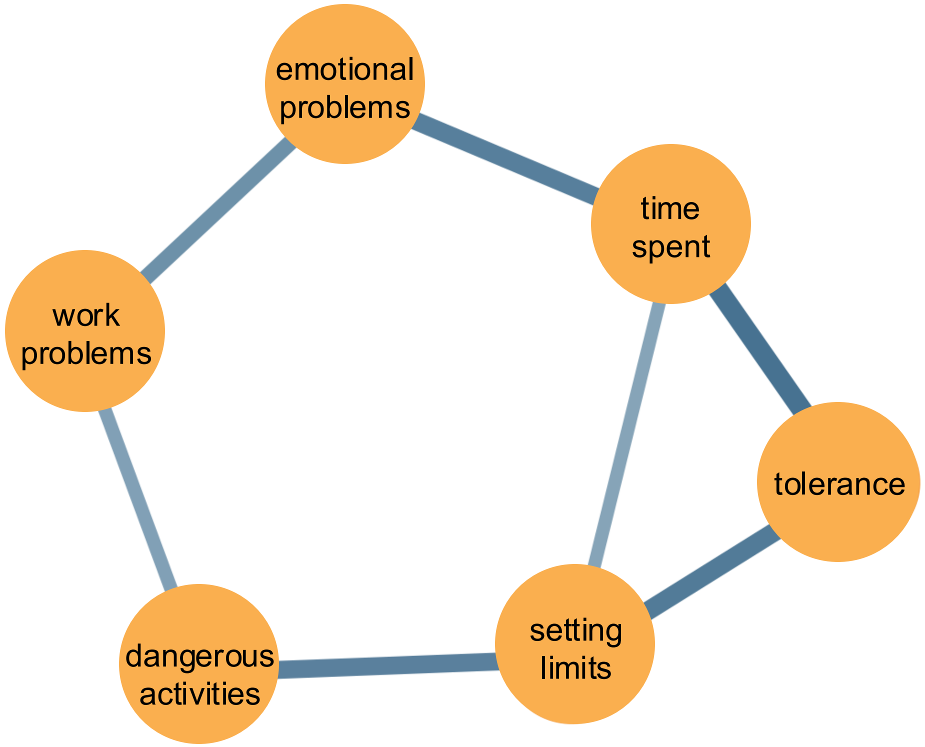
# Quiz: Why are edges missing?

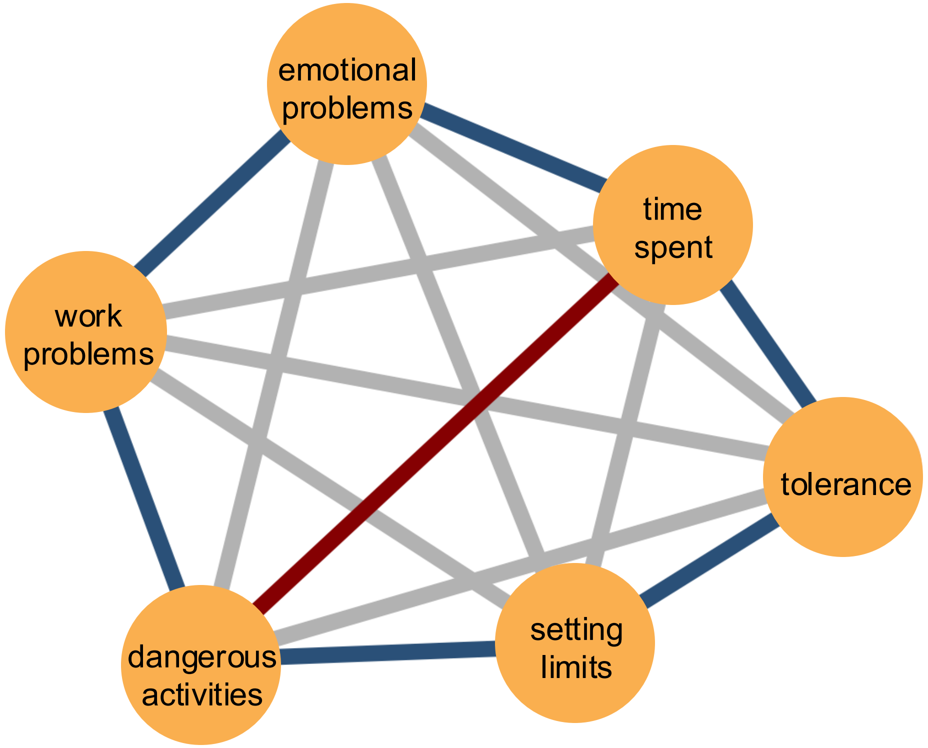


# Solution: Evidence plot

Absence of evidence

Evidence of absence



# End of Part I

Part I: Theory

Part II: Tutorial

## Before we take a break:

```
install.packages("easybgm")
```

```
https://jasp-stats.org/download/
```

image: Flaticon.com