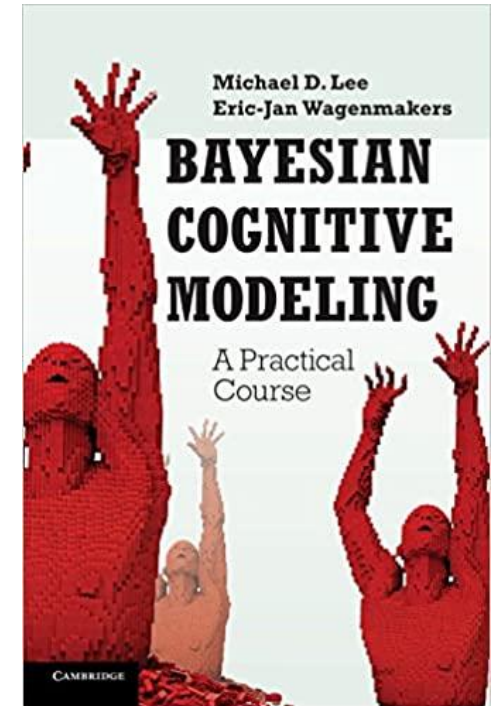
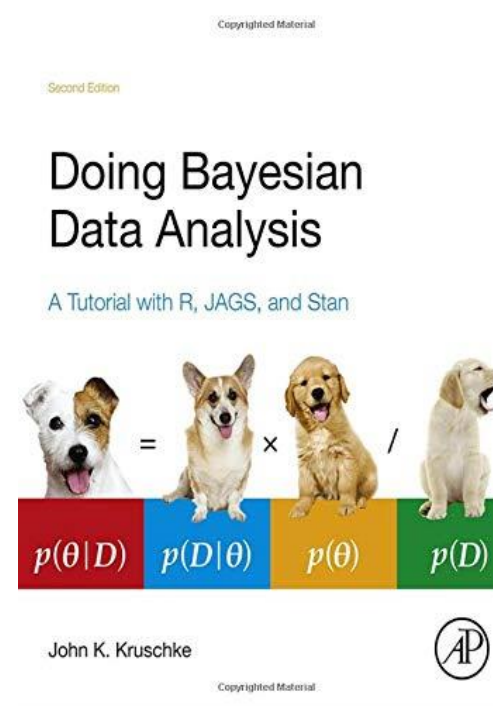
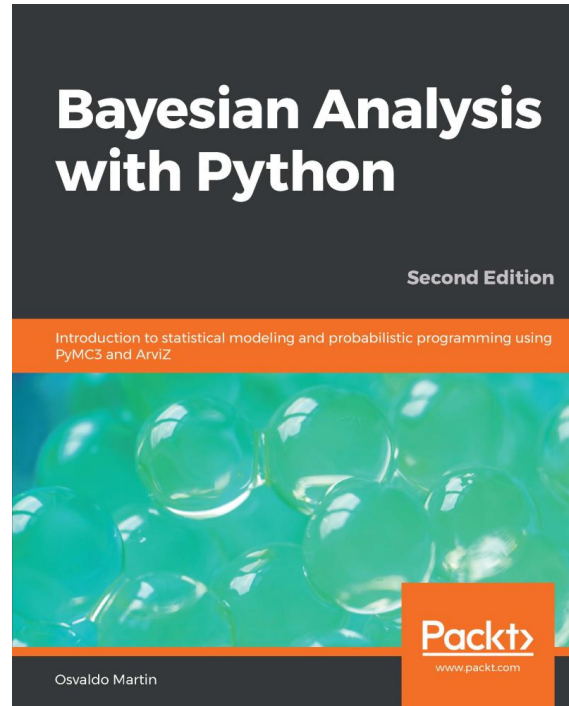
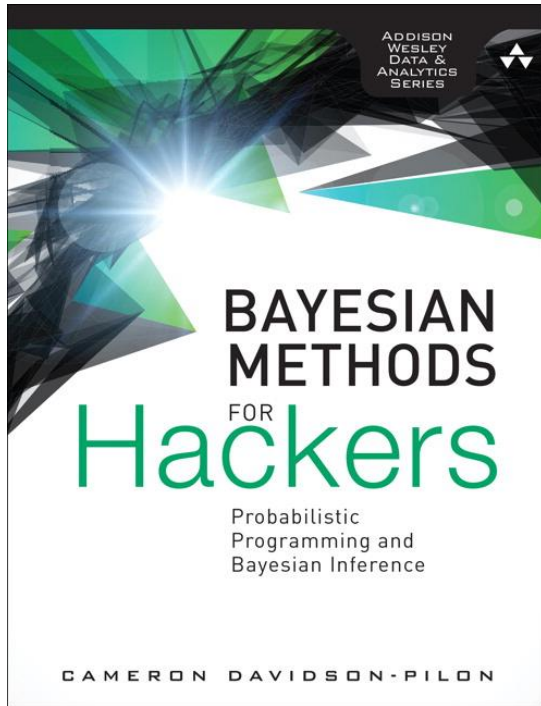


Bayesian modeling

Thinking probabilistically

Introduction to PyMC3

Thinking probabilistically



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Thinking probabilistically

Frequentist

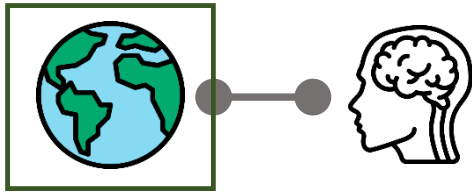


Bayesian

Probability is the long-run frequency of events.

Probability measure the *believability in an event*.

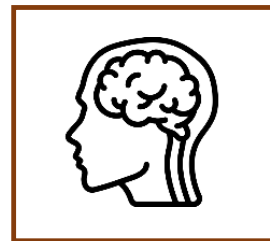
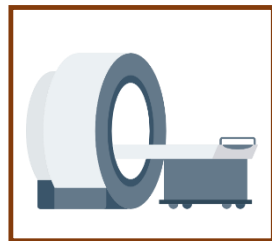
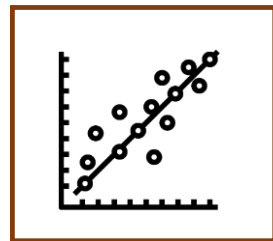
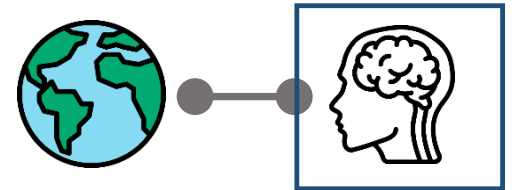
$f(y|\theta)$
↑
Fixed



$$p(\theta|y) = \frac{p(y|\theta)p(\theta)}{p(y)}$$

↑
Fixed

Bayesian inference is simply updating your beliefs after considering new evidence.



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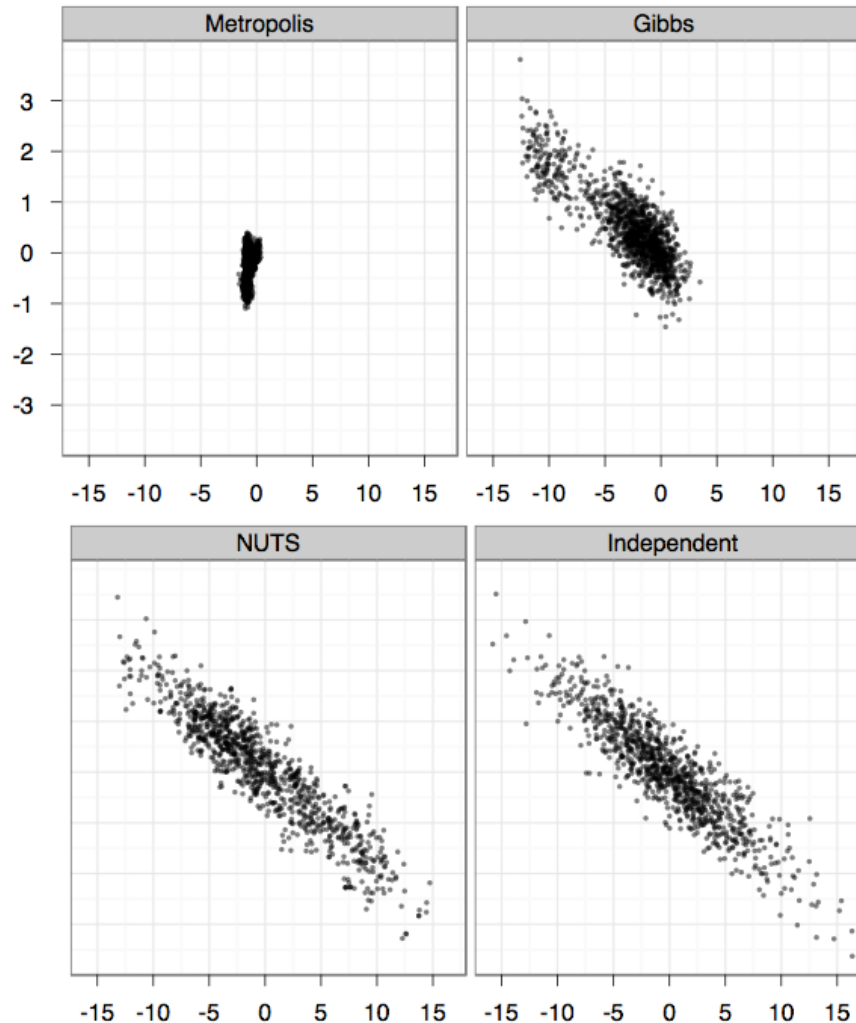


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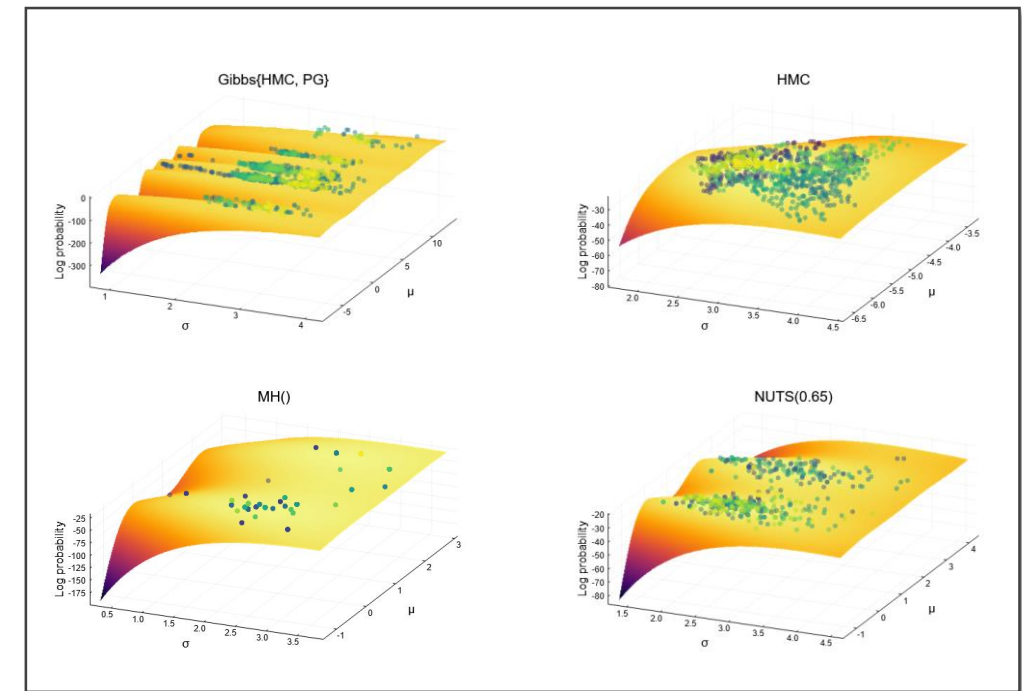
MCMC samplers

- Approximation
 - Variational inference
- Stochastic sampling
 - MCMC methods



Online demo:

<https://chi-feng.github.io/mcmc-demo/>



<https://turing.ml/dev/docs/using-turing/sampler-viz>



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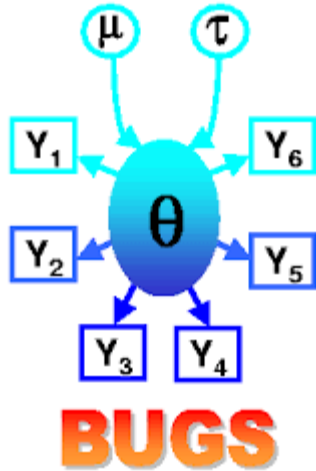
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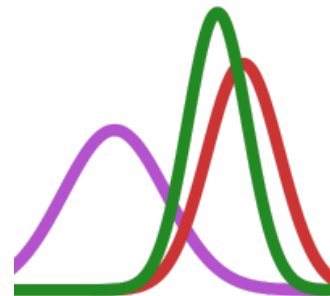
Probabilistic programming languages



JAGS



Stan



Turing.jl



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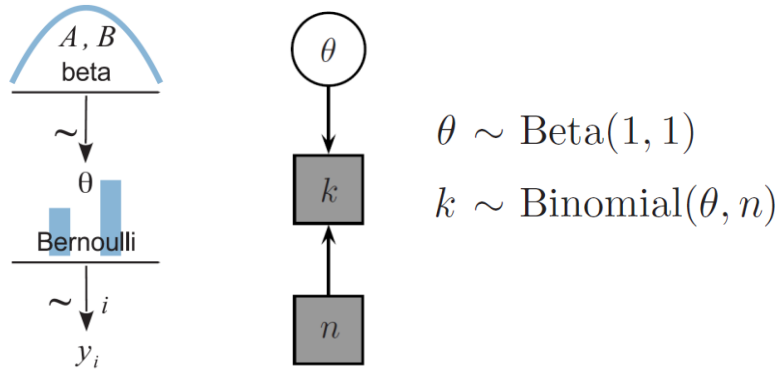


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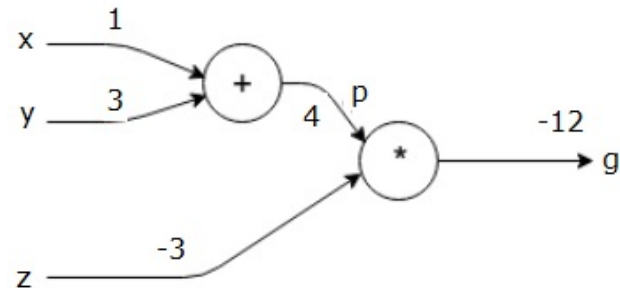


Probabilistic programming languages

Graphical model

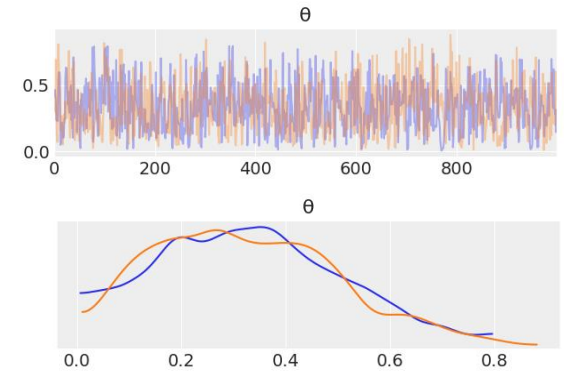


Computational graph



theano 

- Automatic differentiation
- GPU computing
- Optimizations

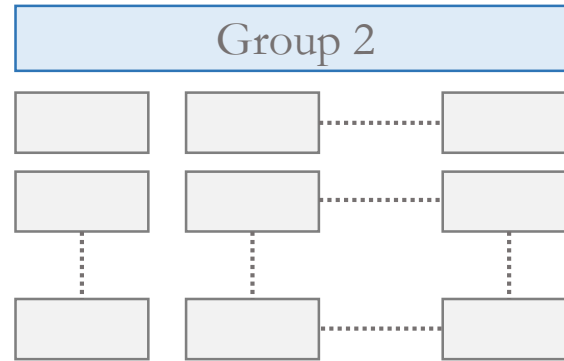
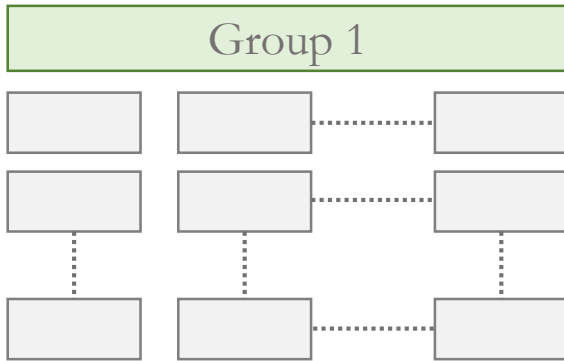


Sampling

```
with pm.Model() as our_first_model:
    theta = pm.Beta('theta', alpha=1., beta=1.)
    y = pm.Bernoulli('y', p=theta, observed=data)
    trace = pm.sample(1000, random_seed=123)
```



Hierarchical/multilevel models



- Hyperpriors
- Hyperparameters

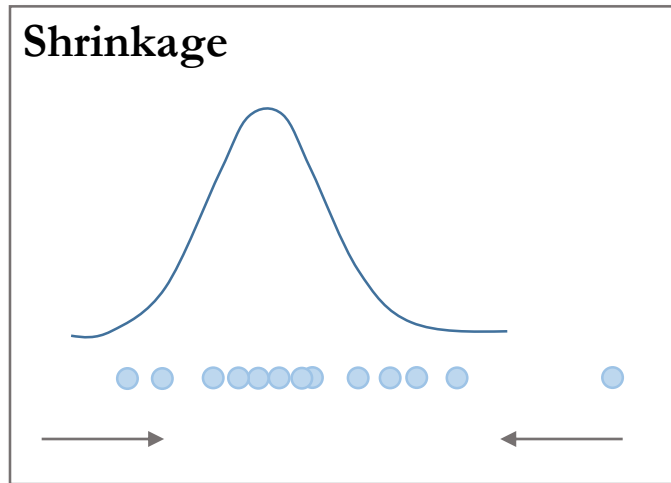


Plate notation

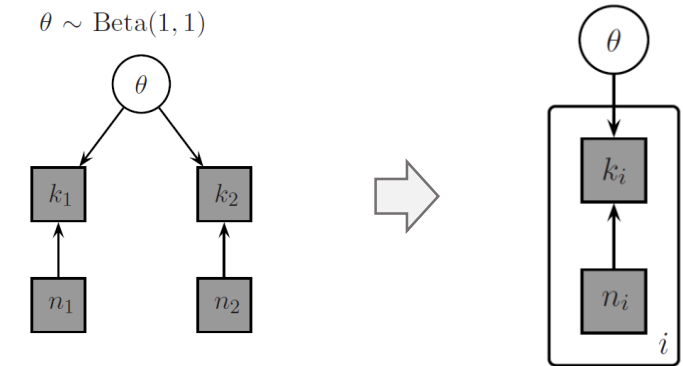
$$k_1 \sim \text{Binomial}(\theta, n_1)$$

$$k_2 \sim \text{Binomial}(\theta, n_2)$$

$$\theta \sim \text{Beta}(1, 1)$$

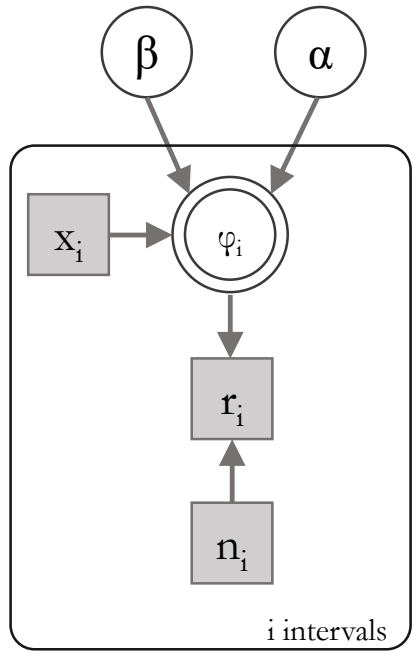
$$k_i \sim \text{Binomial}(\theta, n_i)$$

$$\theta \sim \text{Beta}(1, 1)$$

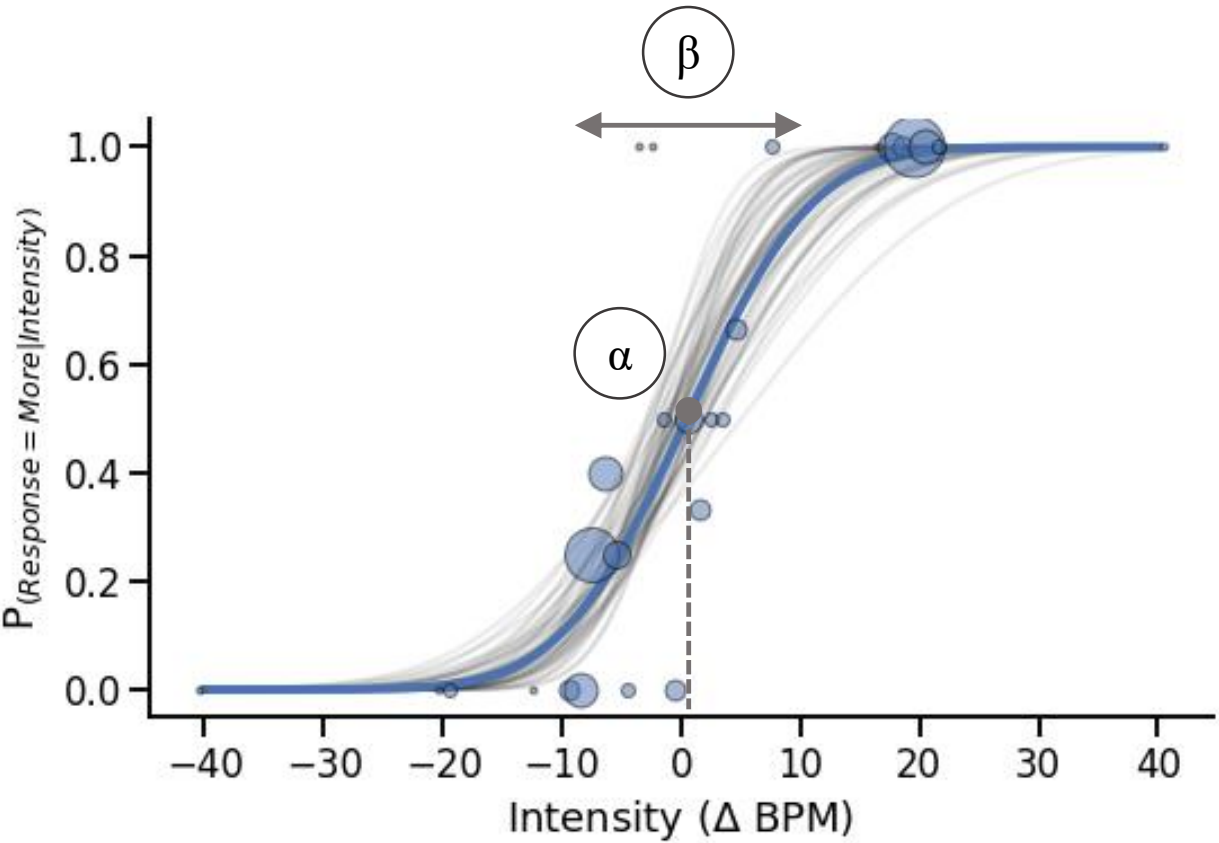


Psychophysics

Psychophysics is concerned with measuring how external physical stimuli cause internal psychological sensations.

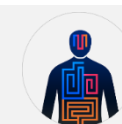
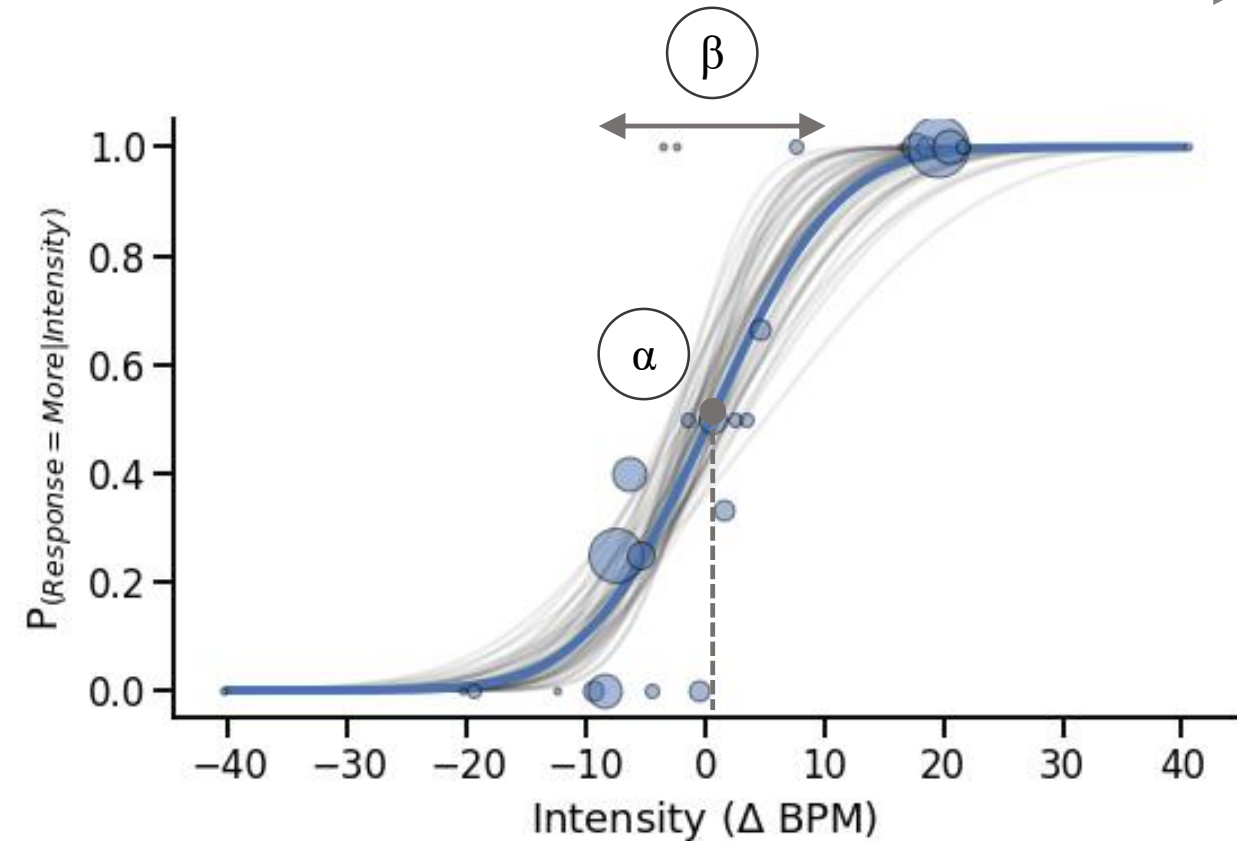
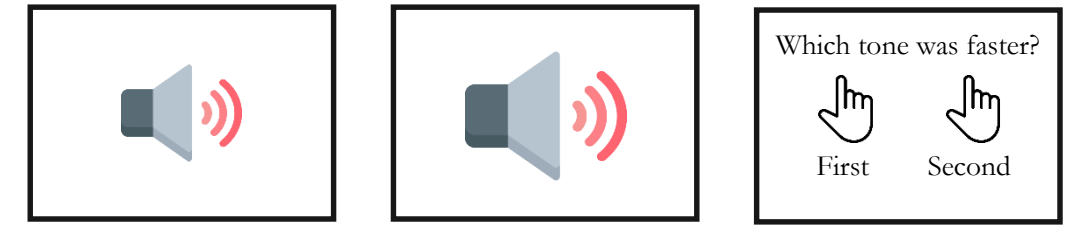
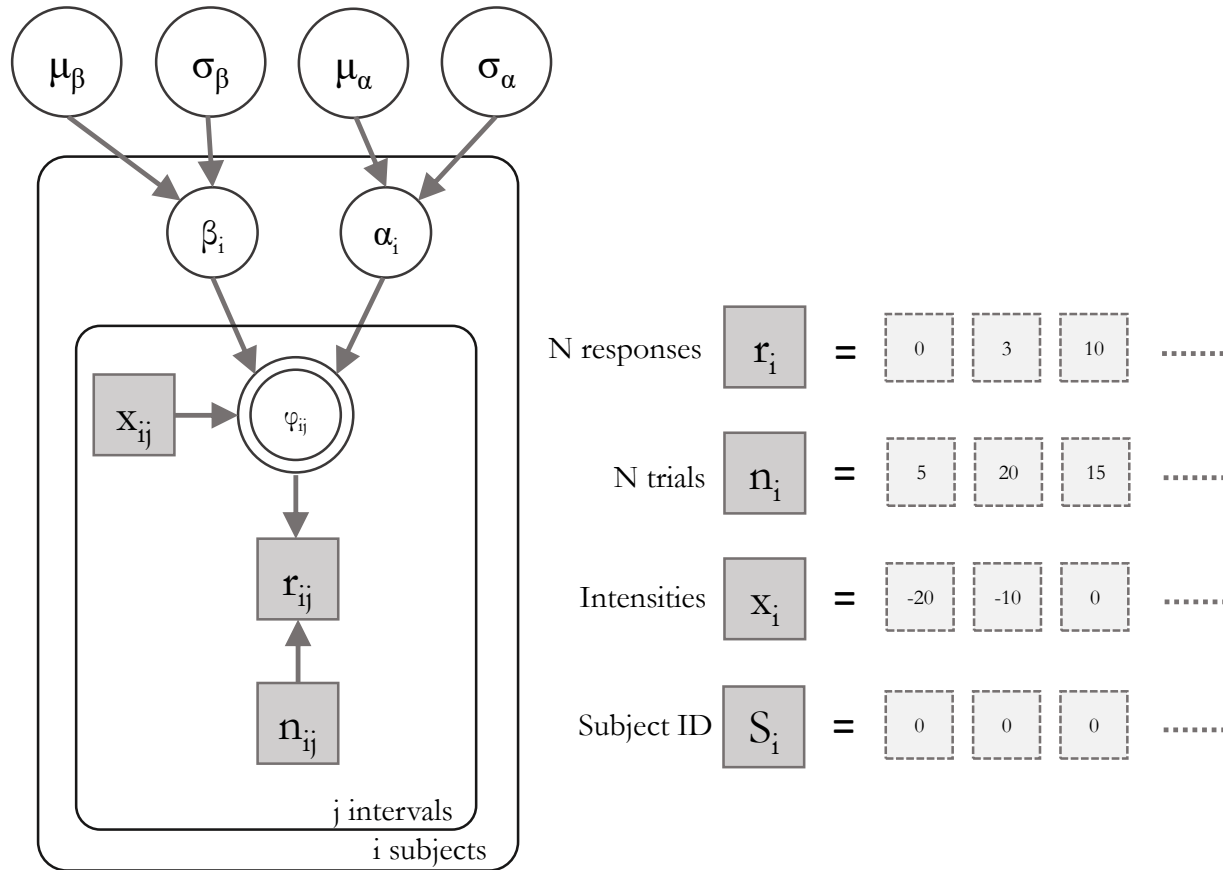


N responses	r_i	=	0	3	10
N trials	n_i	=	5	20	15
Intensities	x_i	=	-20	-10	0



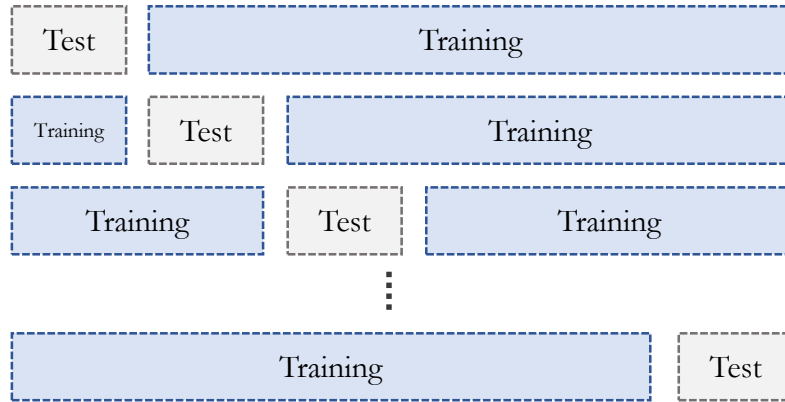
Hierarchical psychophysics

Psychophysics is concerned with measuring how external physical stimuli cause internal psychological sensations.



Model comparison

Cross-validation



- Akaike information criterion

If we have two or more equivalent explanations for the same phenomenon, we should choose the simpler one... but also the more accurate.

$$AIC = -2 \sum_{i=1}^n \log p(y_i | \theta_{mle}) + 2p_{AIC}$$

How well the model fits the data
Penalizes complex models

- Widely applicable information Criterion

$$WAIC = -2l_{ppd} + 2p_{WAIC}$$

How well the model fits the data
Penalizes complex models

Information criteria

How well models fit the data while taking into account their complexity through a penalization term

- Mean square error (MSE)

$$\frac{1}{n} \sum_{i=1}^n (y_i - E(y_i | \theta))^2$$

- Log-likelihood

$$\sum_{i=1}^n \log p(y_i | \theta)$$

- Deviance

$$-2 \sum_{i=1}^n \log p(y_i | \theta)$$



Resources

Chris Fonnesbeck - An introduction to Markov Chain Monte Carlo using PyMC3 | PyData London 2019

https://www.youtube.com/watch?v=SS_pqgFziAg

van de Schoot, R., Depaoli, S., King, R., Kramer, B., Märtens, K., Tadesse, M. G., Vannucci, M., Gelman, A., Veen, D., Willemssen, J., & Yau, C. (2021). Bayesian statistics and modelling. *Nature Reviews Methods Primers*, 1(1).
<https://doi.org/10.1038/s43586-020-00001-2>

Kruschke, J. (2015). *Doing Bayesian data analysis : a tutorial with R, JAGS, and Stan*. Boston: Academic Press.



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