Probabilistic Machine Learning

Lecture 1 | Fundamentals of Probabilistic Modeling

Amirabbas Asadi

amir.asadi78@sharif.edu

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Outline

- Probabilistic (generative) models
- Bayesian Learning
- Parametric Probabilistic models
- Nonparametric Probabilistic models
- Bayesian Inference
  - MCMC Inference
  - Variational Inference
  - Particle-Based Inference
- Scaling up probabilistic inference for real-world scenarios
- Discussed examples:
  - Probabilistic models for Time Series
  - Probabilistic models for Reinforcement Learning

Probabilistic Models

What is a probabilistic models?

Probabilistic Models

What is a probabilistic models?

$$p(X_1, X_2, ..., X_n)$$

Any representation of a joint distribution is a probabilistic model

Probabilistic Models

Why are we interested in designing probabilistic models?

Applications of Probabilistic Models

Sampling

$$x_1, x_2, ..., x_n \sim p(X_1, X_2, ..., X_n)$$

Applications of Probabilistic Models

Sampling

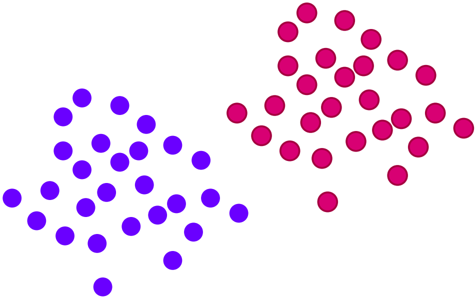
$$x_1, x_2, ..., x_n \sim p(X_1, X_2, ..., X_n)$$

Probabilistic Inference

$$p(Z, X_1, X_2, ..., X_n) \rightarrow p(Z|X_1, X_2, ..., X_n)$$

Applications of Probabilistic Models

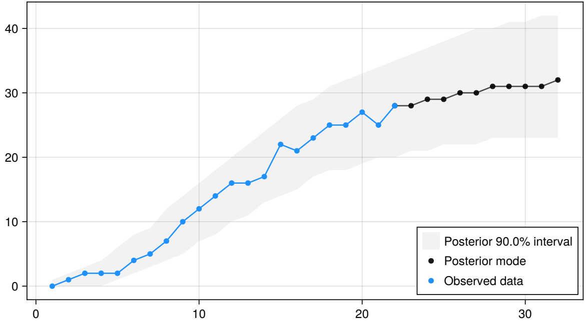
Classification : Discriminant Analysis



$$p_{C_1}(x) \quad p_{C_2}(x)$$

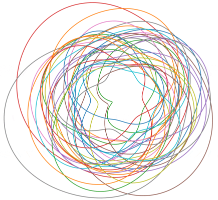
Applications of Probablistic Models

Forecasting and Regression



Applications of Probablistic Models

Uncertainty Quantification



Applications of Probablistic Models

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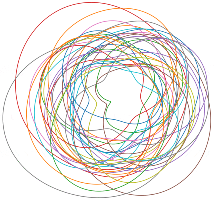

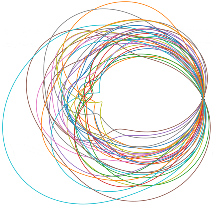

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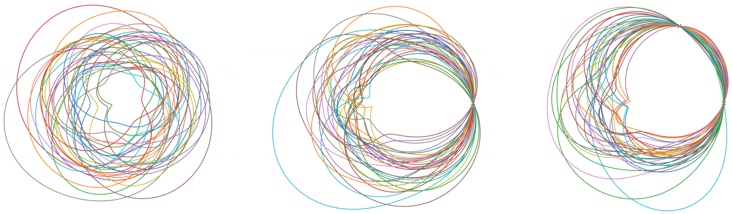

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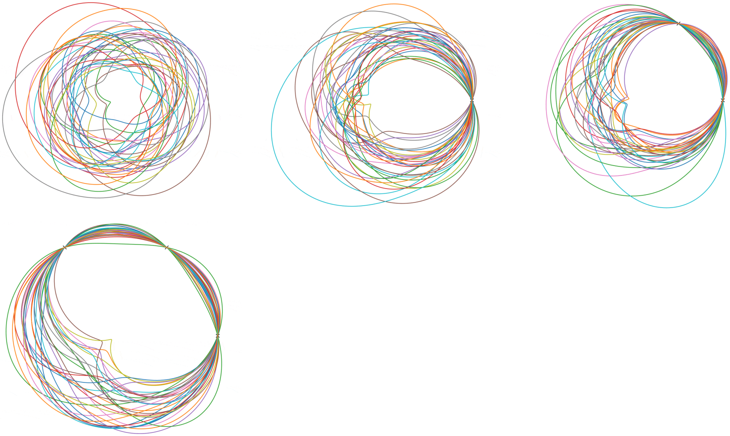

Applications of Probablistic Models

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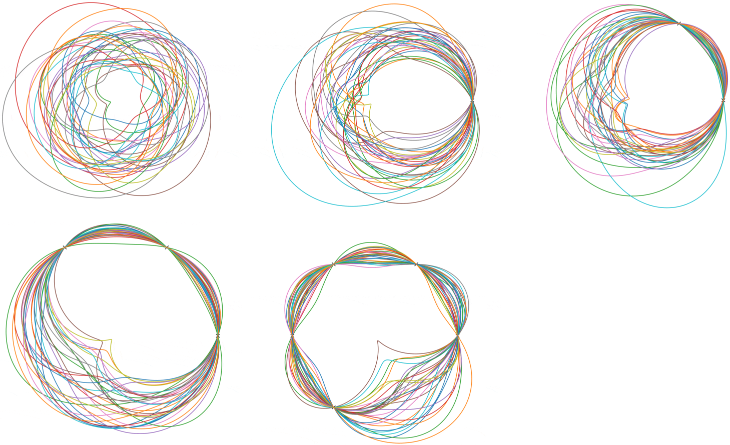

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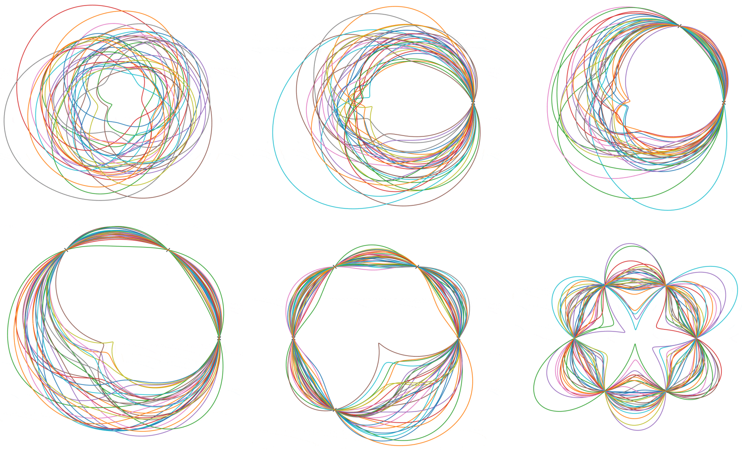
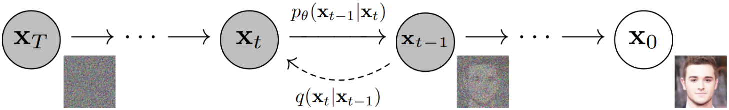


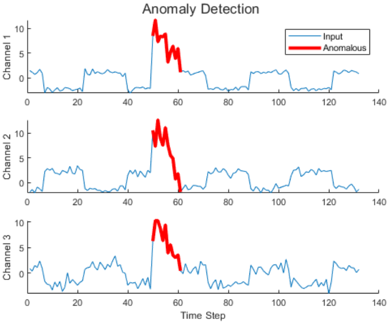
Applications of Probabilistic Models

Data Generation



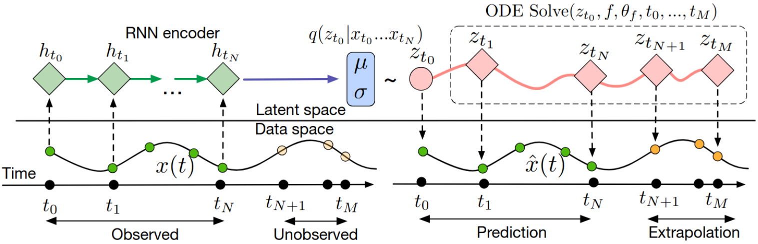
Applications of Probabilistic Models

Anomaly Detection



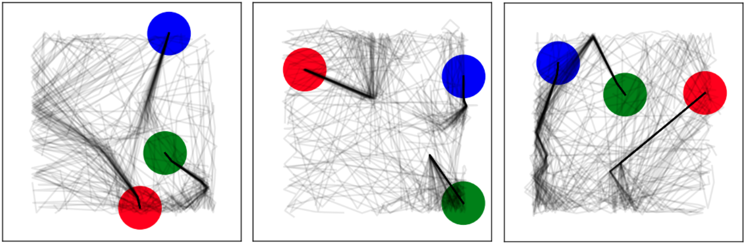
Applications of Probabilistic Models

Scientific Discovery



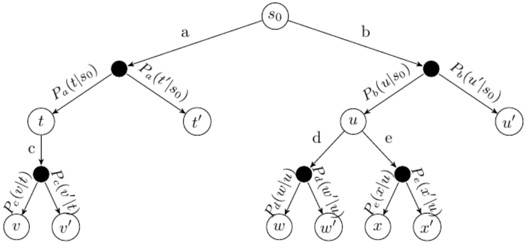
Applications of Probabilistic Models

Data-driven Reasoning



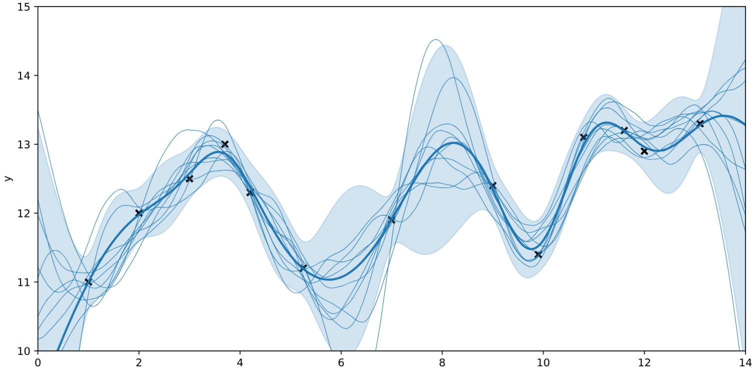
Applications of Probabilistic Models

Reinforcement Learning : Planning



Applications of Probabilistic Models

Guiding Exploration



Challenges : Fantasy vs Reality

$$p(x_1, x_2, ..., x_n)$$

Having this object, we can solve a wide range of problems

p has encoded anything we know about $x_1, ..., x_n$ and their interactions

Finding it is not going to be easy!

Challenges : Representation

What does a probabilistic model look like?

Challenges : Inference

Suppose we eventually found a model

$$p(x, z)$$

How can we perform inference?

$$p(z|x)$$

Challenges : Inference

$$p(z|x) = \frac{p(x|z)p(z)}{p(x)}$$

Challenges : Inference

$$p(z|x) = \frac{p(x|z)p(z)}{p(x)}$$

$$p(x) = \int p(x, z)dz$$

Challenges : Inference

$$p(z|x) = \frac{p(x|z)p(z)}{p(x)}$$

$$p(x) = \int p(x,z)dz$$

Now imagine what does $p(x)$ look like if we have used something like neural networks inside the model!

Challenges : Inference

If we choose likelihood and prior carefully, Exact Inference is possible.

Likelihood	**Conjugate Prior**
Bernoulli	Beta
Binomial	Beta
Poisson	Gamma
Categorical	Dirichlet
Uniform	Pareto

Challenges

For a long time it was believed that Bayesian methods are

- limited to small data

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- practically not applicable to complex models

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For a long time it was believed that Bayesian methods are

- limited to small data
- practically not applicable to complex models
- time-consuming and consequestly not suitable for online applications

Milestones

So what has changed now!?

- Powerful frameworks for representation

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- Advances in inference methods

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- Deep Learning and Differentiable Programming

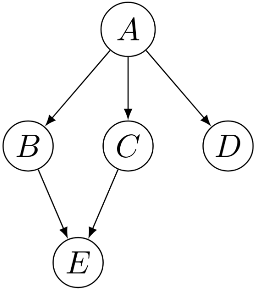
Milestones

So what has changed now!?

- Powerful frameworks for representation
- Advances in inference methods
- Deep Learning and Differentiable Programming
- Advances in hardware (CPU, GPU, TPU, ...)

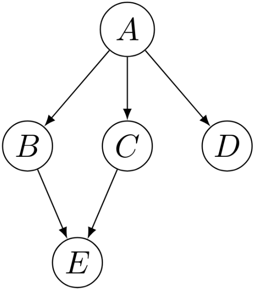
Representing Probabilistic Models

Directed Graphical Models



Representing Probabilistic Models

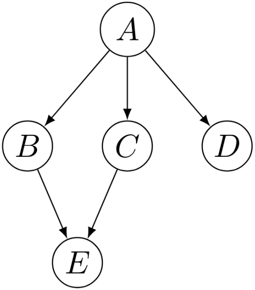
Directed Graphical Models



$$p(A, B, C, D, E) =$$

Representing Probabilistic Models

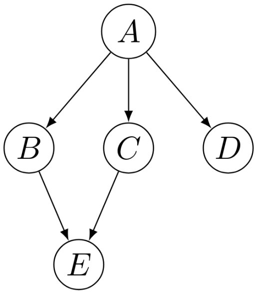
Directed Graphical Models



$$p(A, B, C, D, E) = p(E|B, C)$$

Representing Probabilistic Models

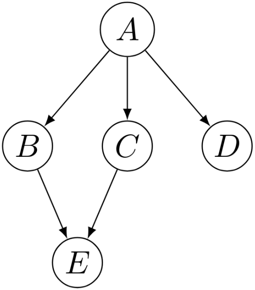
Directed Graphical Models



$$p(A, B, C, D, E) = p(E|B, C)p(B|A)$$

Representing Probabilistic Models

Directed Graphical Models



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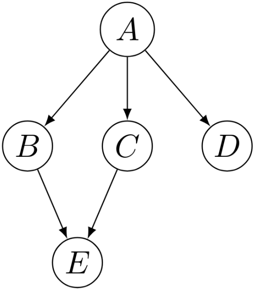

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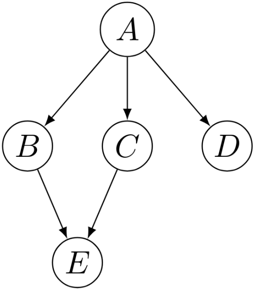
Representing Probabilistic Models

Directed Graphical Models



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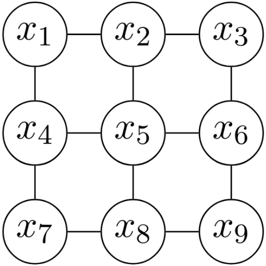
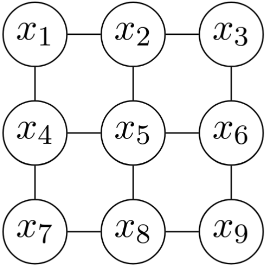
Representing Probabilistic Models

Undirected Graphical Models



Representing Probabilistic Models

Undirected Graphical Models



$$p() = \frac{1}{Z} \prod_{c \in C} \psi_c(x_c)$$

Representing Probabilistic Models

Can we represent a probabilistic model using an algorithm?

Representing Probabilistic Models

$$p(\theta, X)$$

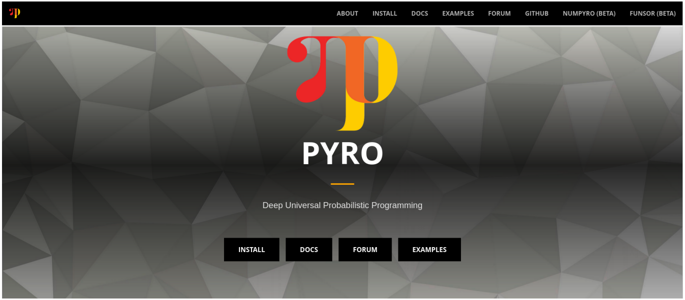
Representing Probabilistic Models

$$p(\theta, X)$$

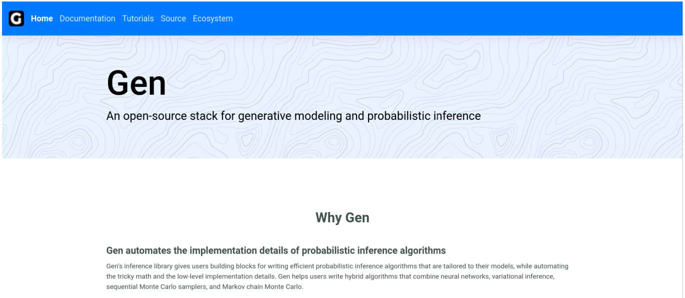
```
θ ~ Beta(1.0, 1.0)

for i in 1:N
    X[i]  ~ Bernoulli(θ)
end
```

Probabilistic Programming Languages



Probabilistic Programming Languages



Probabilistic Programming Languages

Turing.jl Get Started Tutorials Libraries News Team

Turing.jl: Bayesian inference with probabilistic programming.

Intuitive

Turing models are easy to write and communicate — syntax is close to mathematical notations.

General-purpose

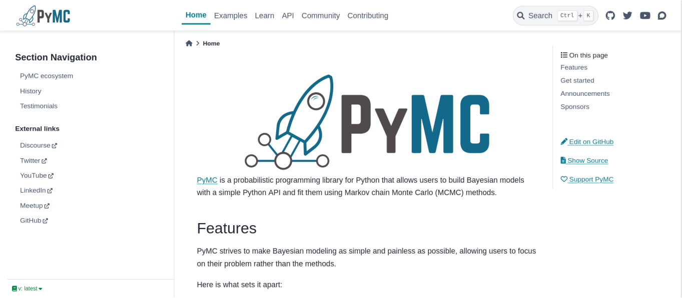
Turing supports models with discrete parameters and stochastic control flow.

Modular and composable

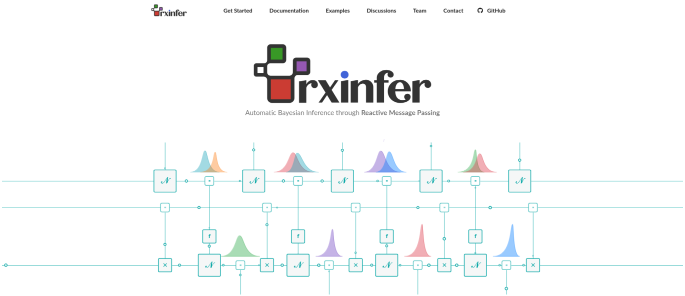
Turing is modular, written entirely in Julia, and is interoperable with the powerful Julia ecosystem.

Get Started

Probabilistic Programming Languages

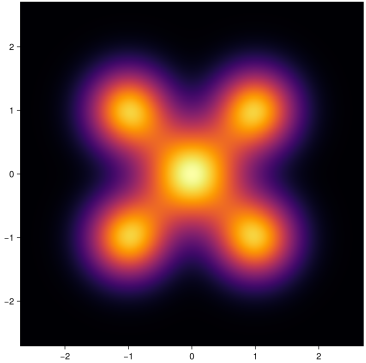


Probabilistic Programming Languages



Approximate Inference

Exact Inference is not possible for most of the models



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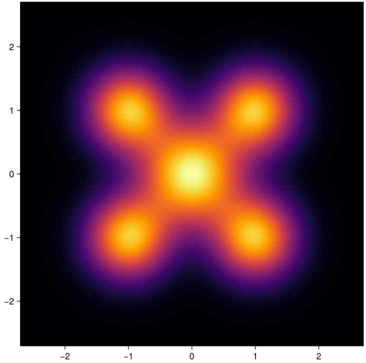
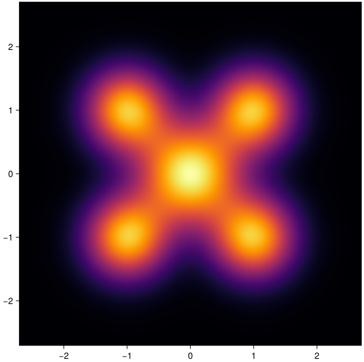


Fortunately there are some methods for approximate inference

Approximate Inference : MCMC

posterior $p(z|x)$ is intractable



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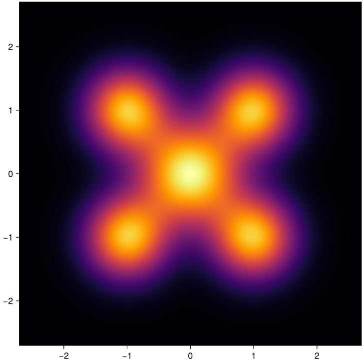
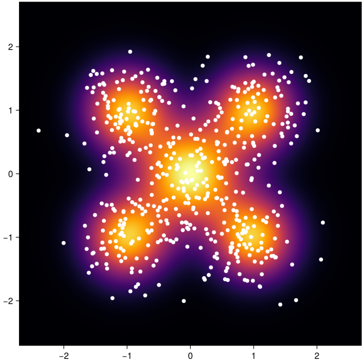
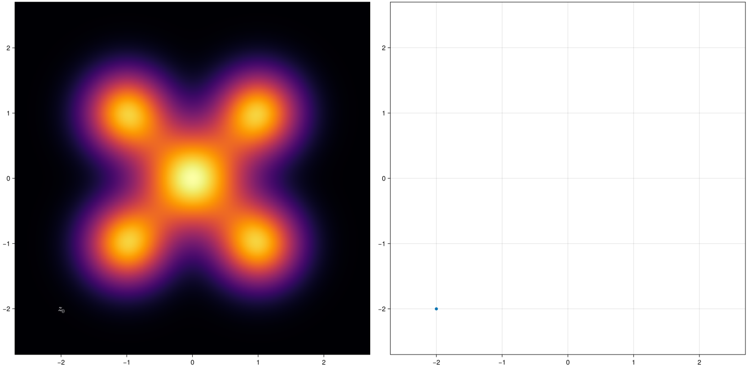




If we somehow generate enough number of samples from $p(z|x)$ then we can estimate our desired quantities.

Approximate Inference : MCMC



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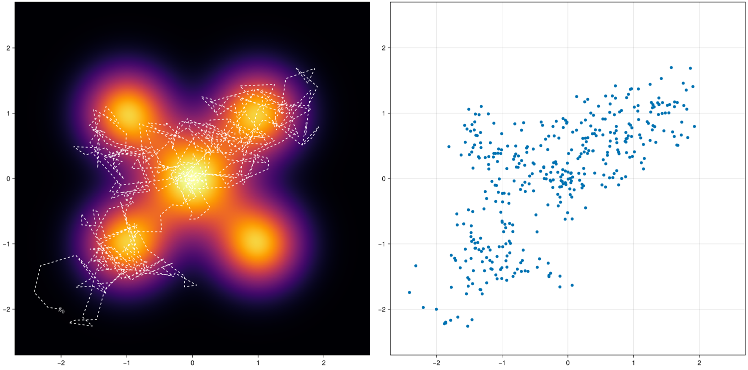

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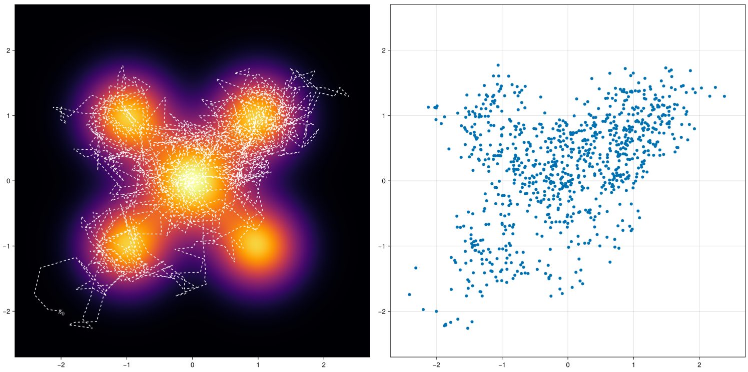

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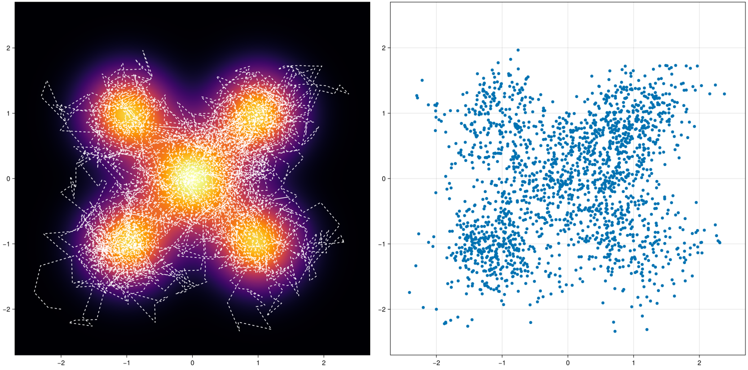
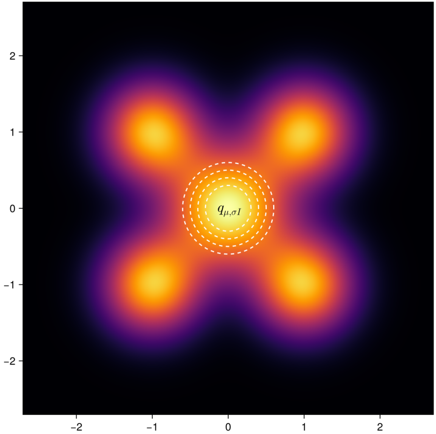
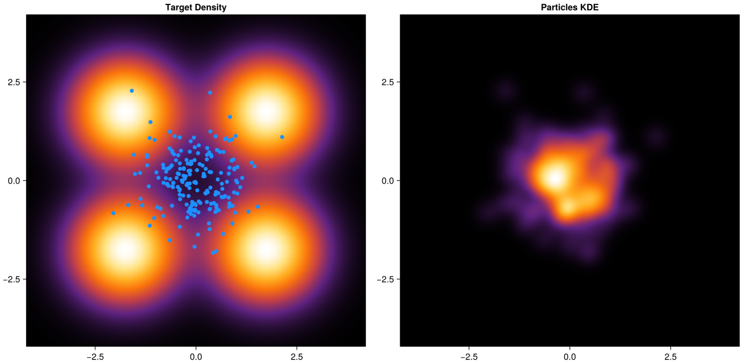


Approximate Inference : Variational Inference

Another idea is trying to find a tractable density $q(z;\lambda)$ as close as possible to the posterior.



Approximate Inference : Particle-Based Inference



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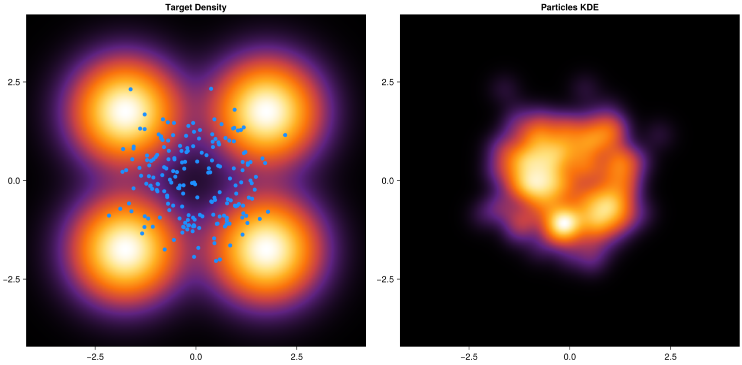

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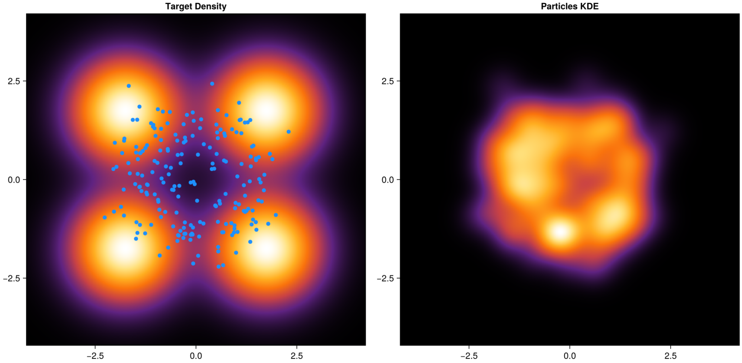

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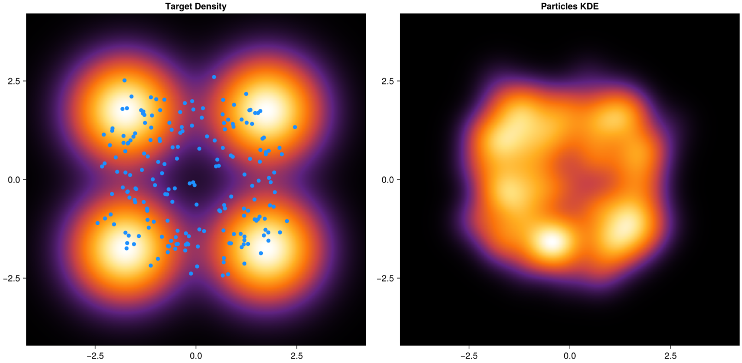

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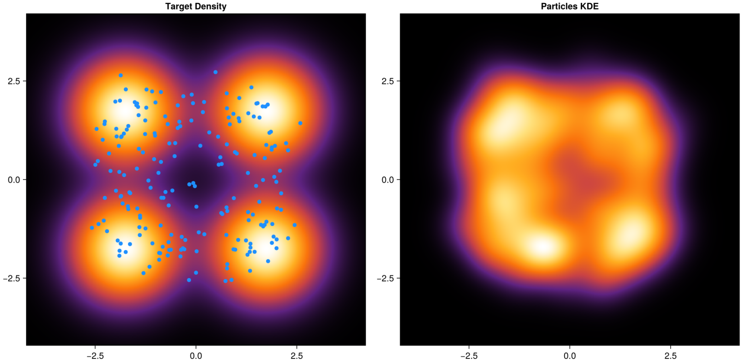

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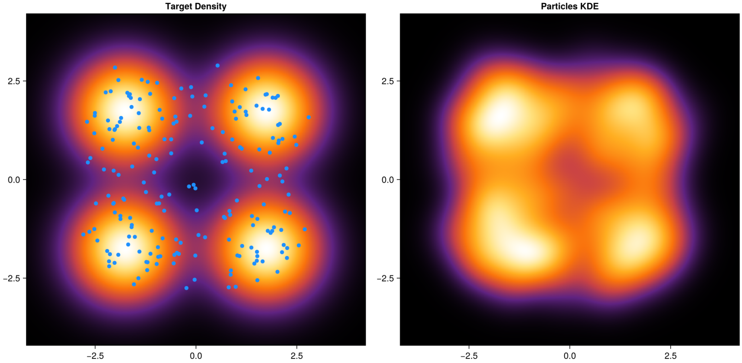

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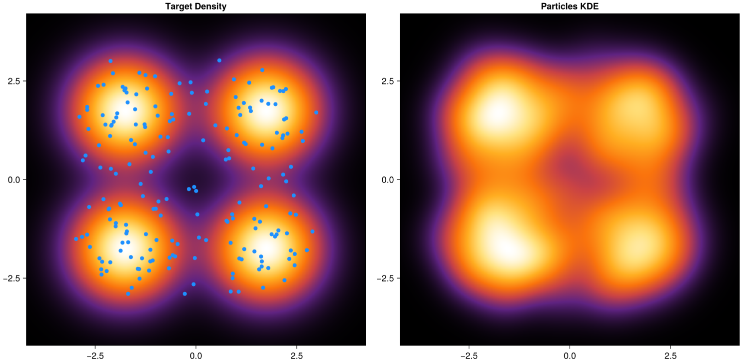

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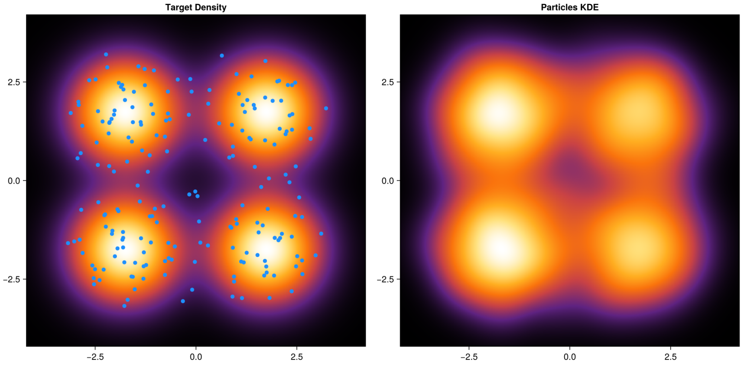

Conclusion

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- Apply Bayesian methods to large data

Conclusion

Thanks to advances in modeling and inference methods, Now we can:

- Explore a wide range of models
- Apply Bayesian methods to large data
- Use Bayesian methods efficiently even for streaming data