Probabilistic Machine Learning with Julia

Lecture 7 | Decision-Making with Probabilistic Models

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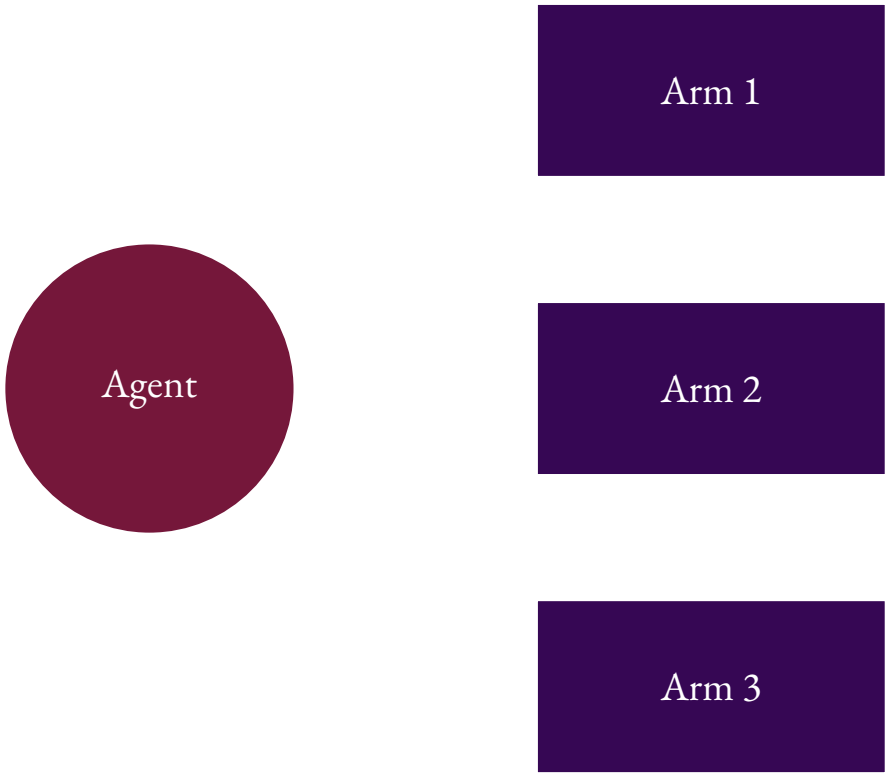
2025

Bandit Problems

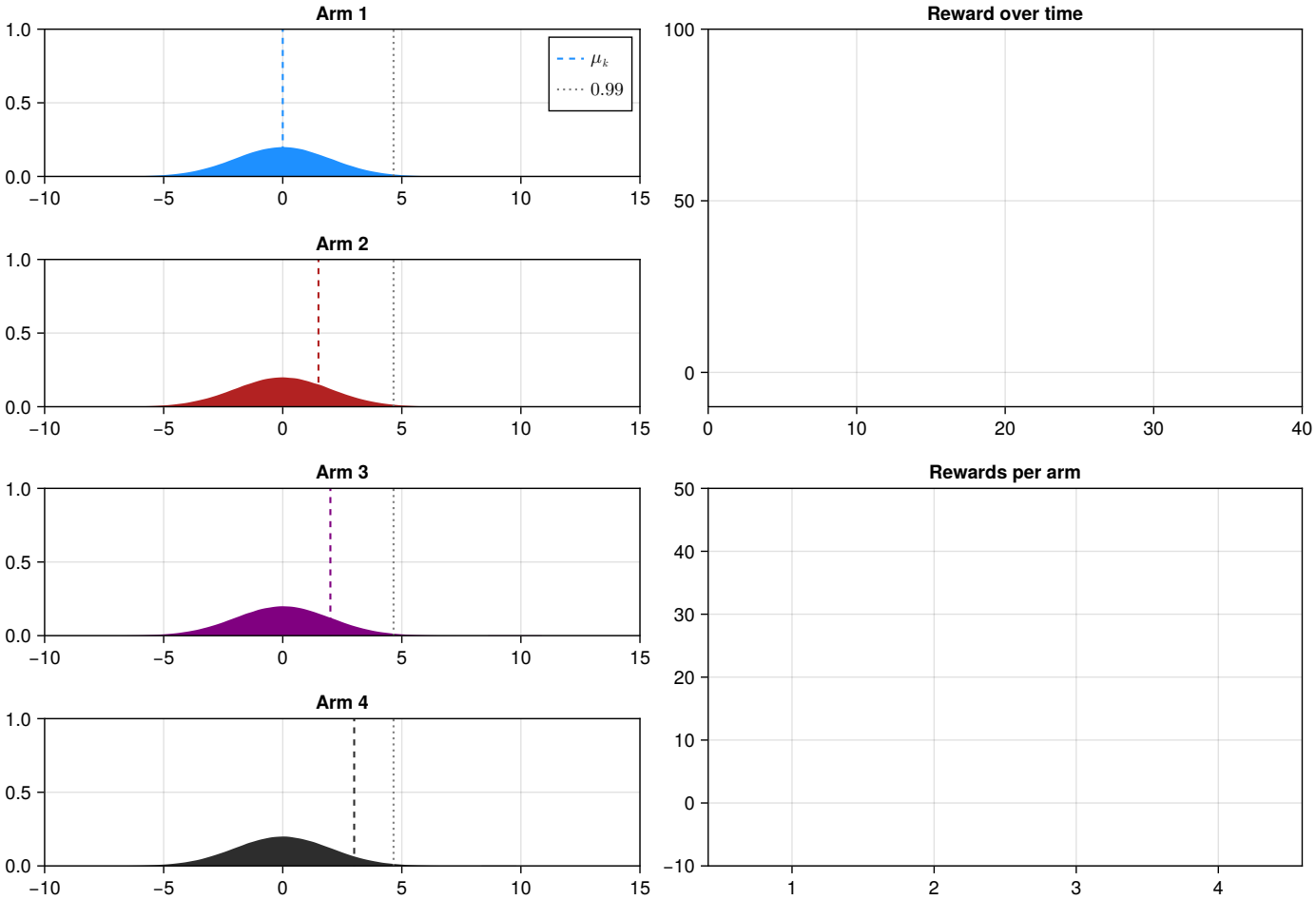


Basic Policies

We will discuss some basic policies for this problem:

- Uniform policy
- Explore-then-commit
- ϵ-greedy policy
- Upper Confidence Bound (UCB) policy

Upper Confidence Bound



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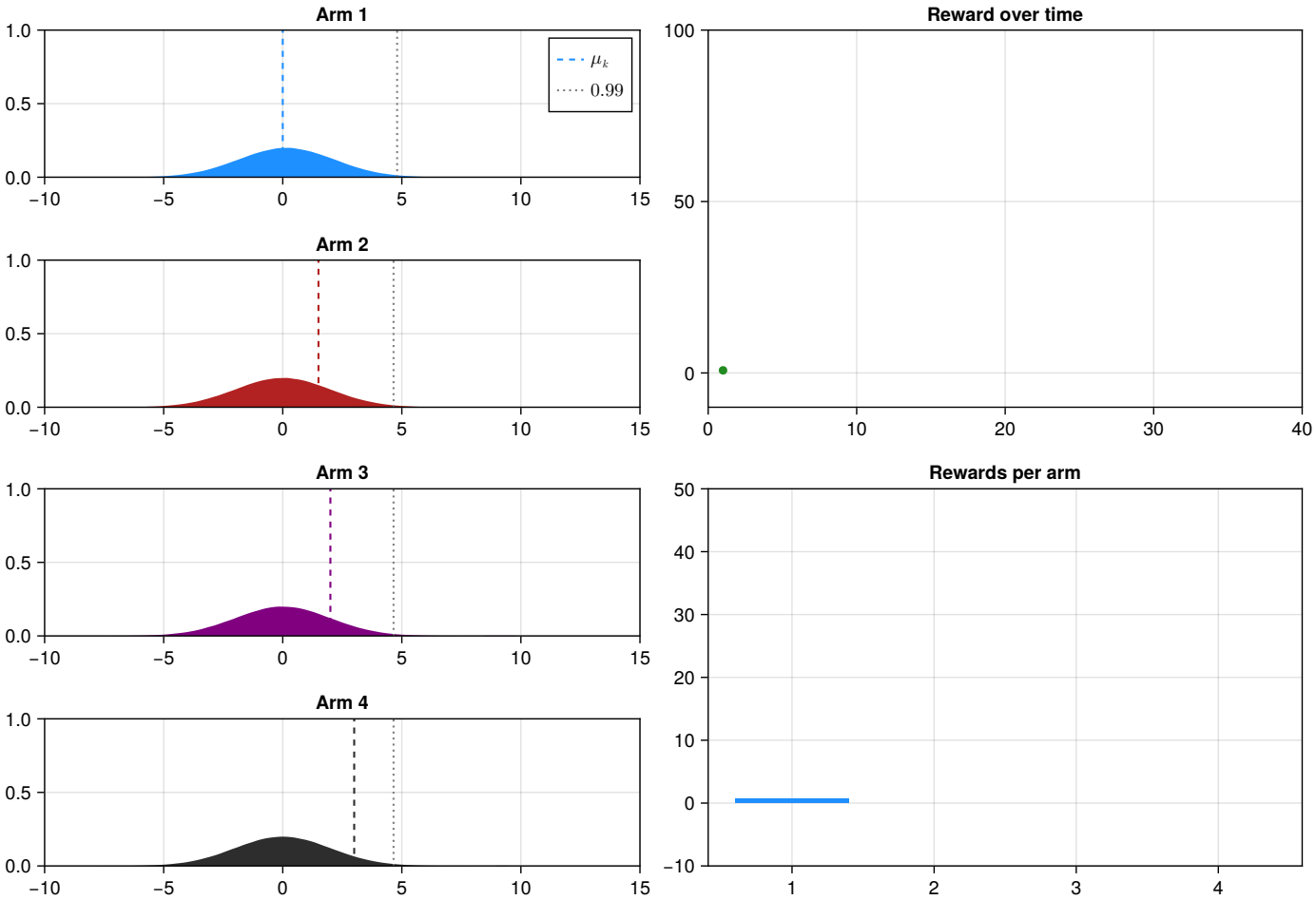

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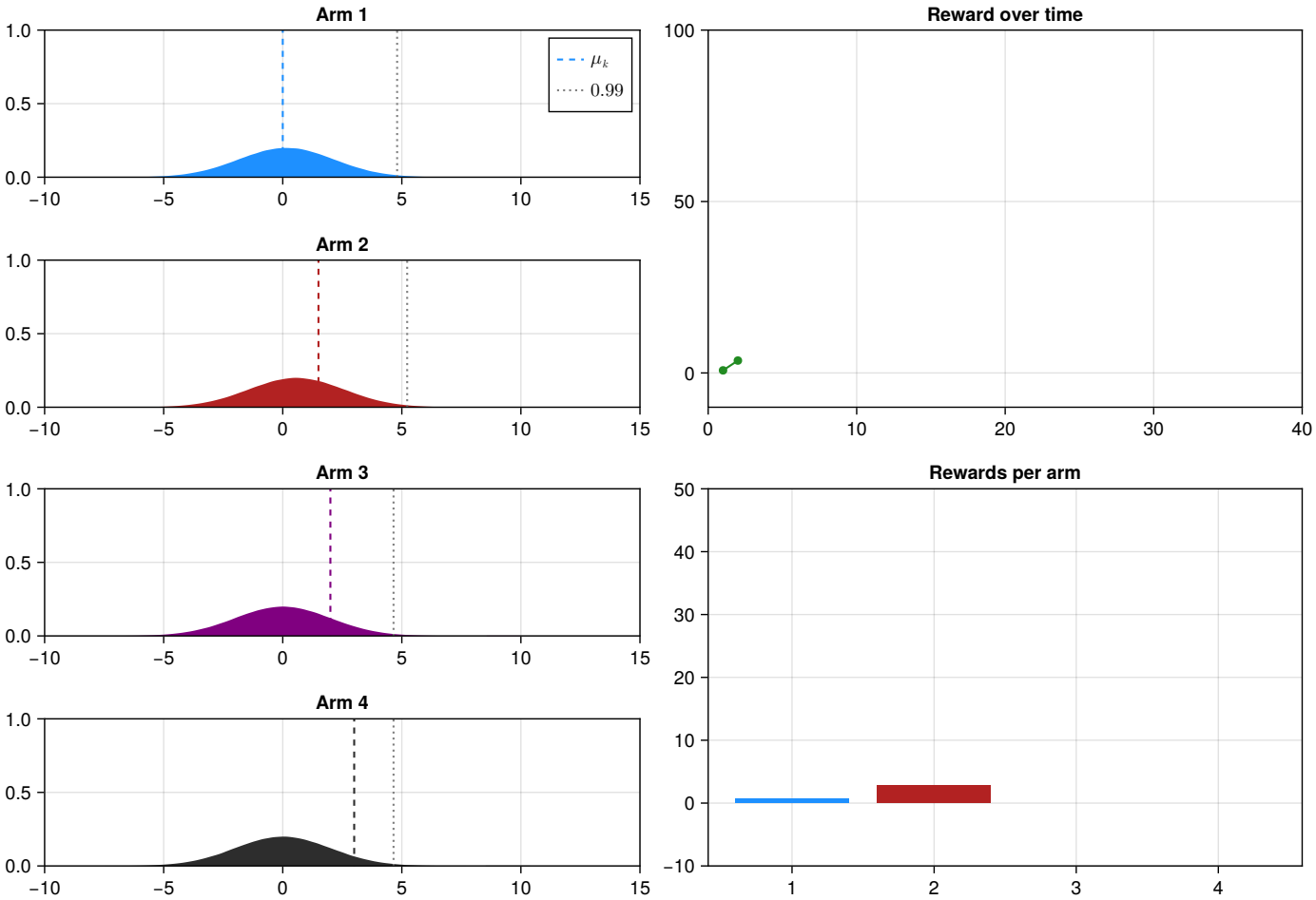

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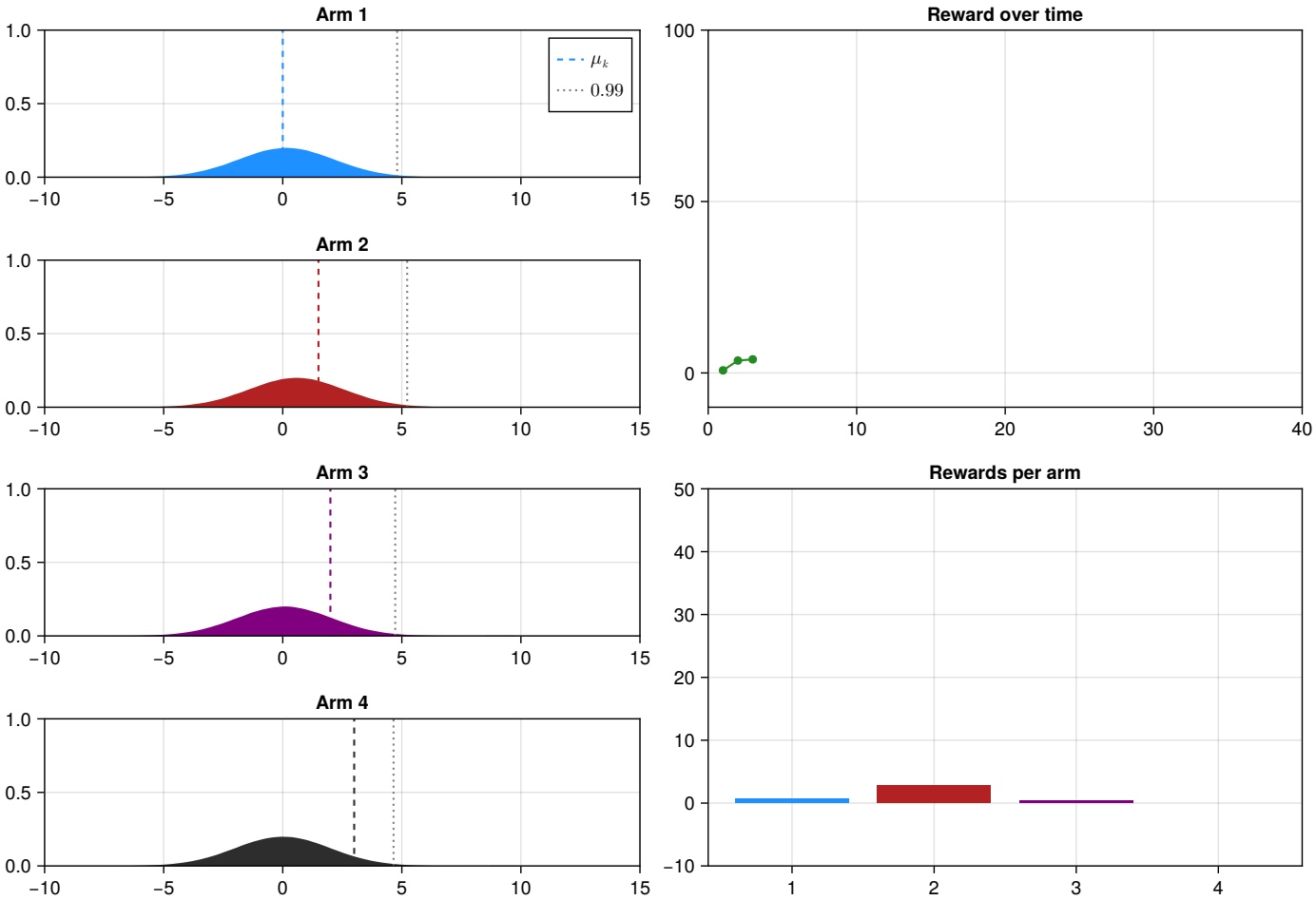

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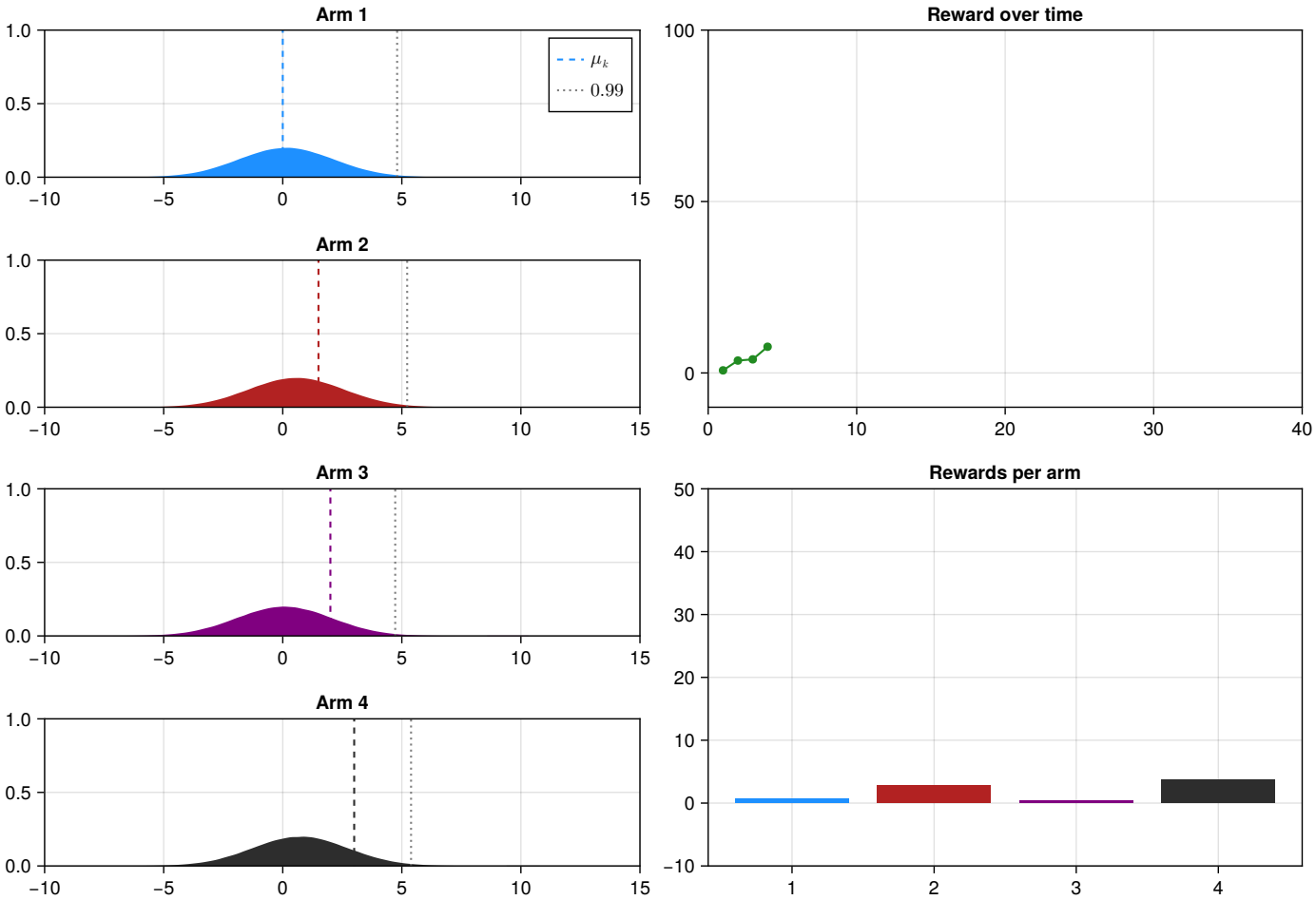

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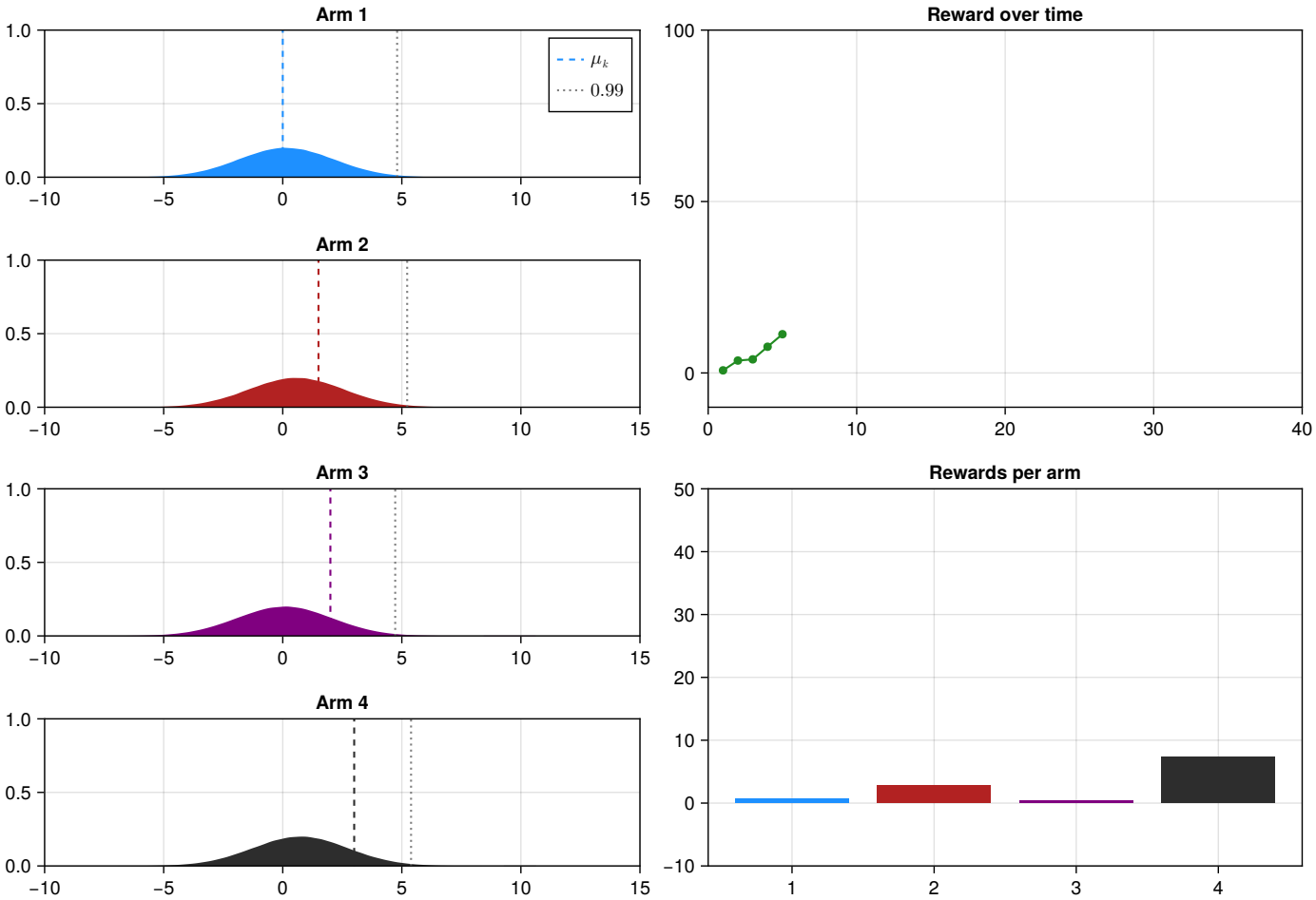

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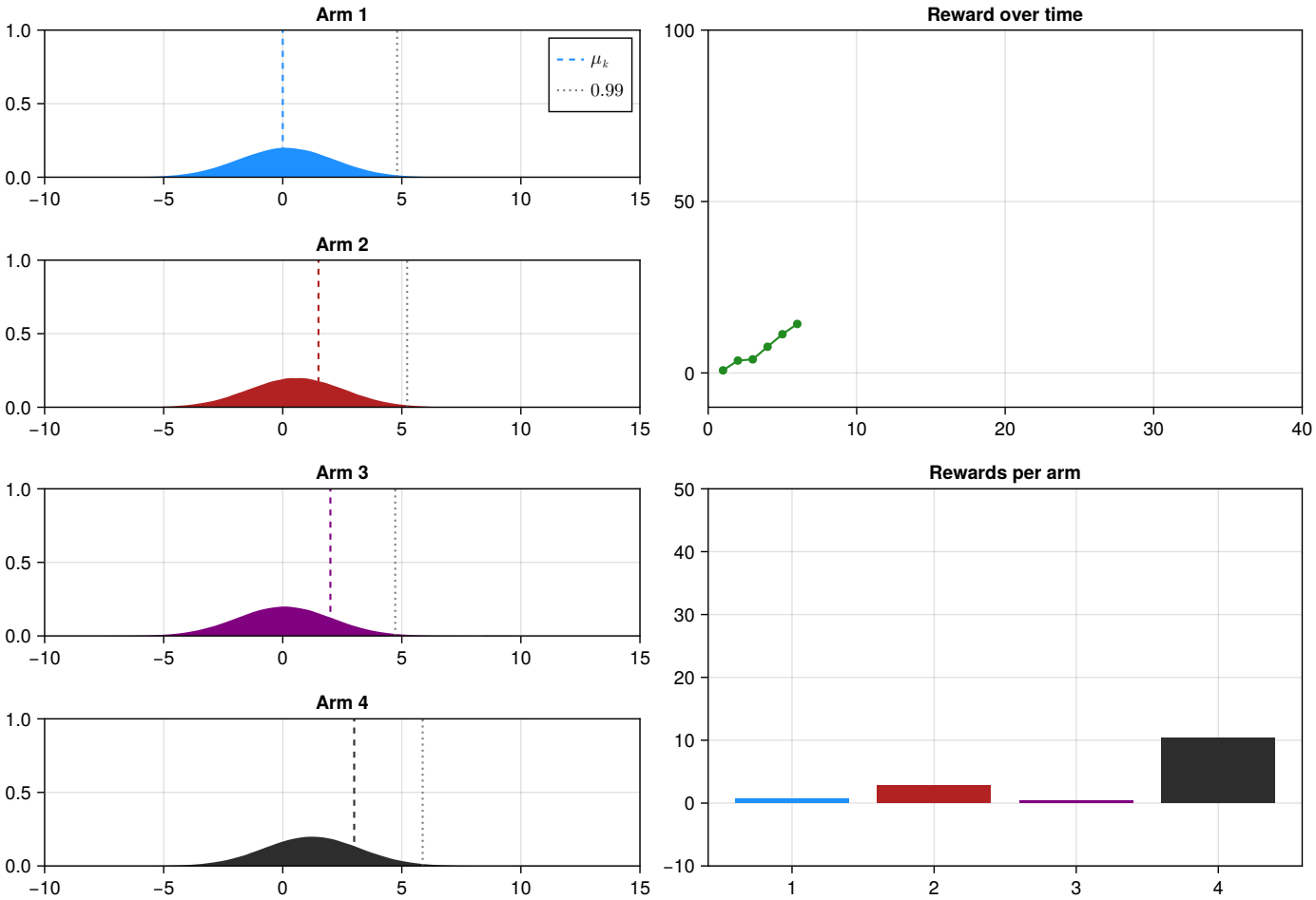

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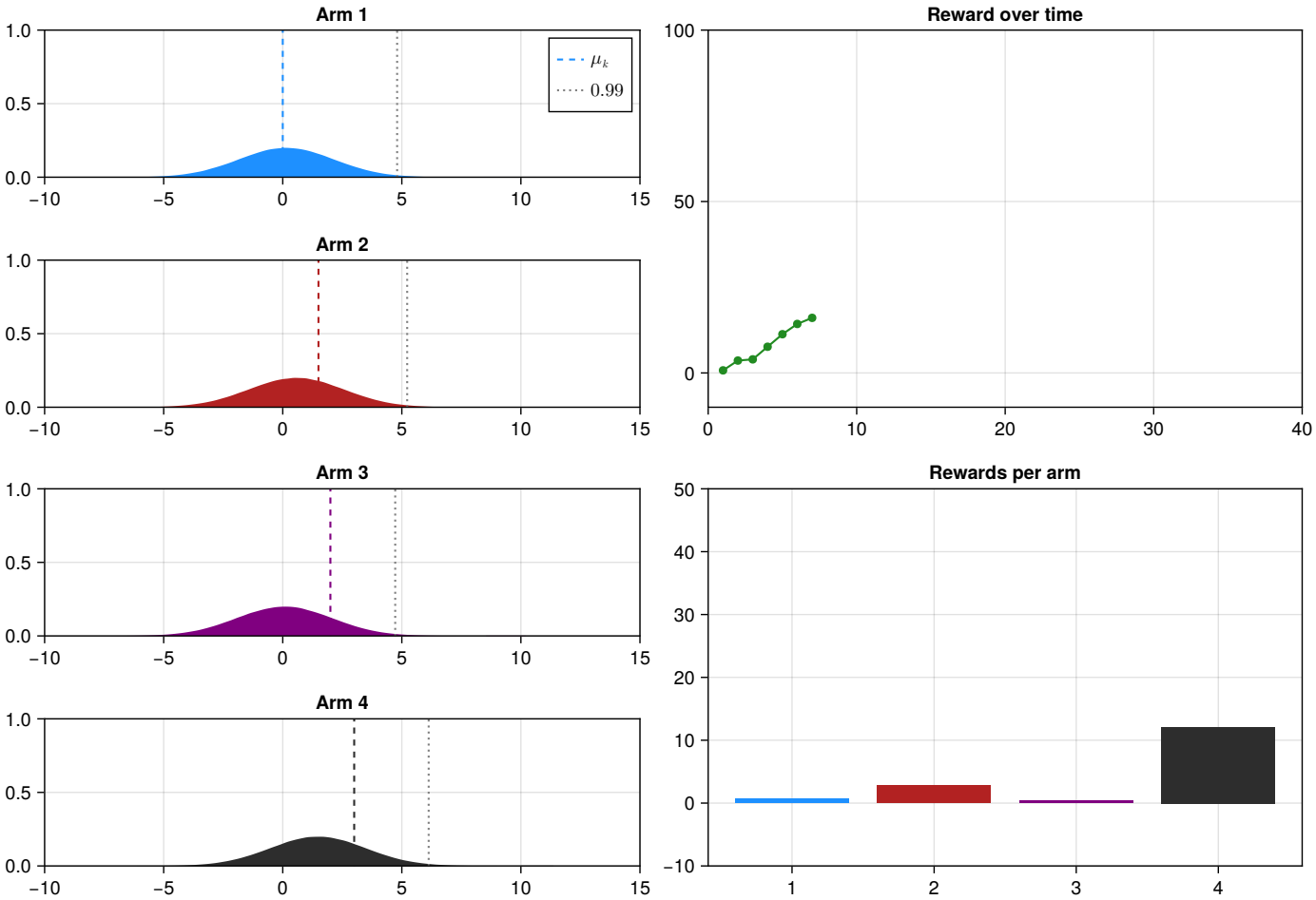

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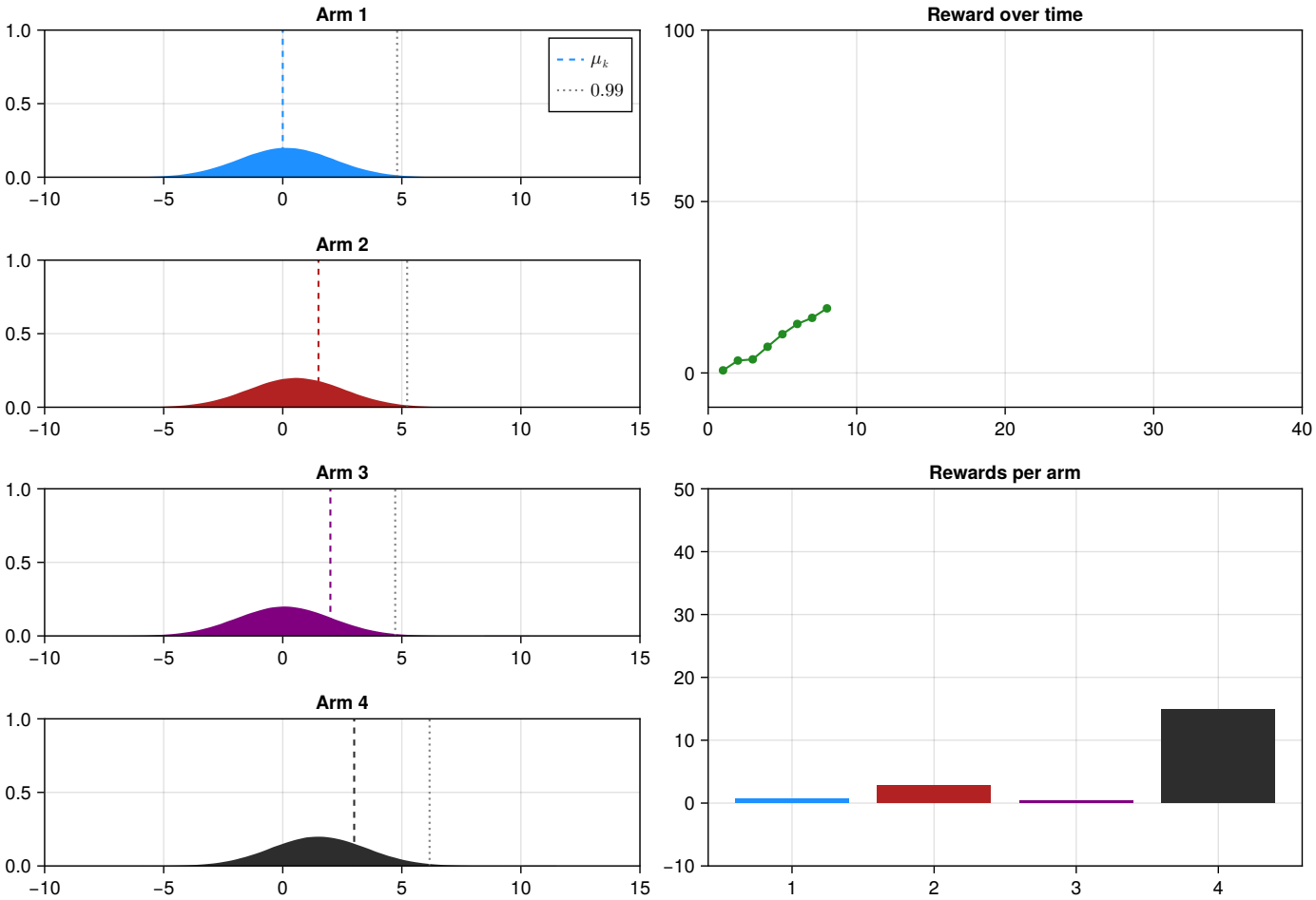

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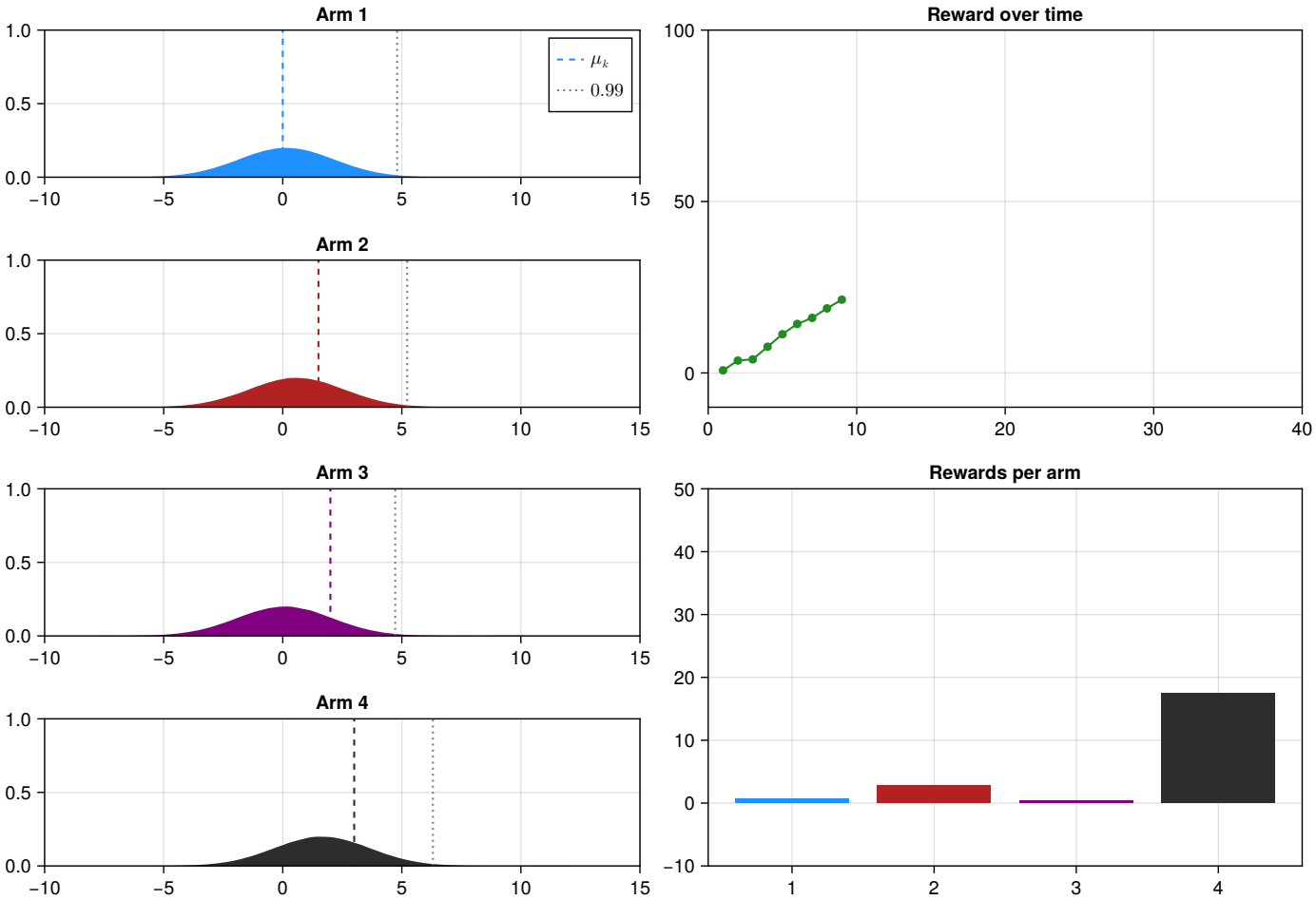

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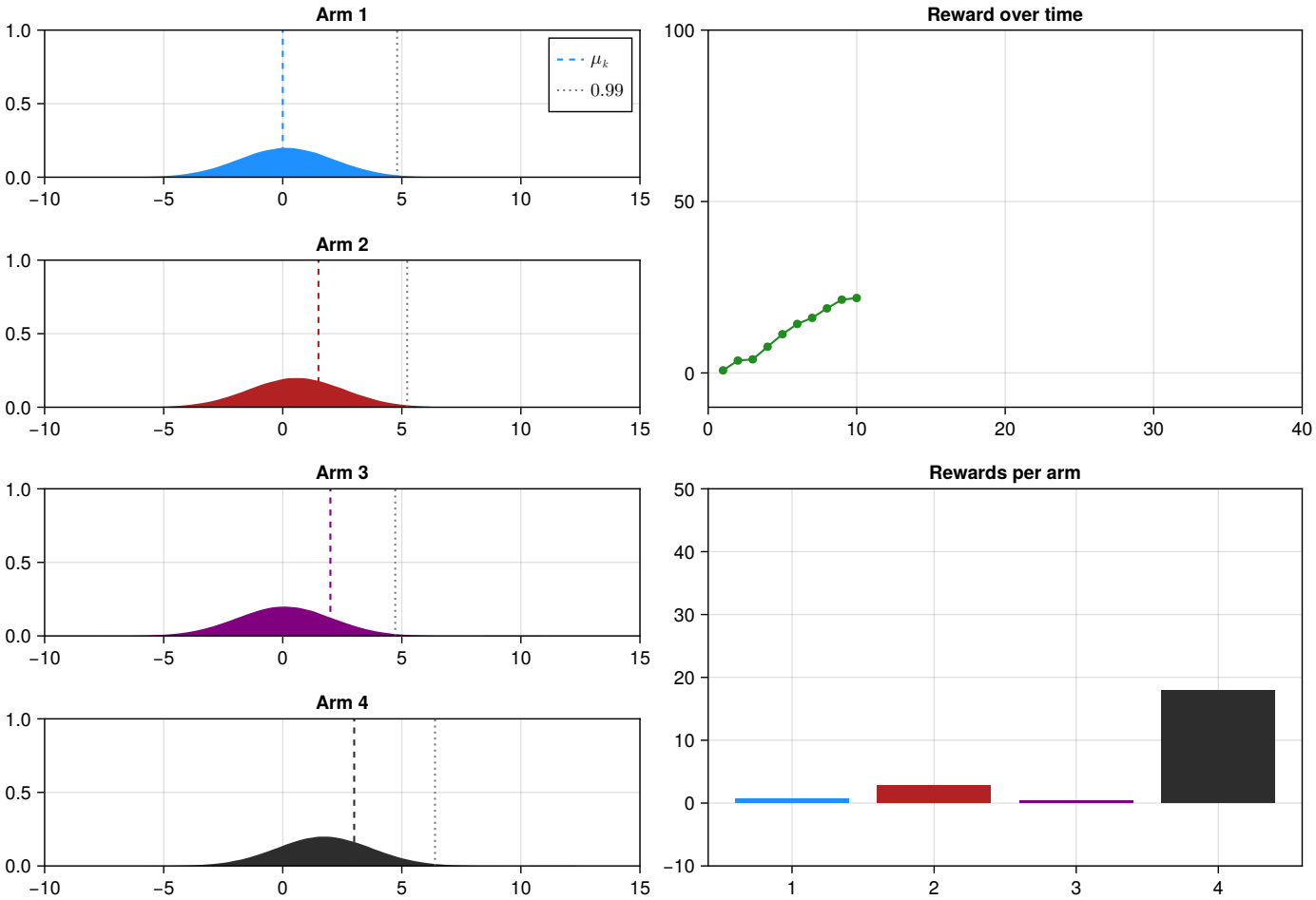

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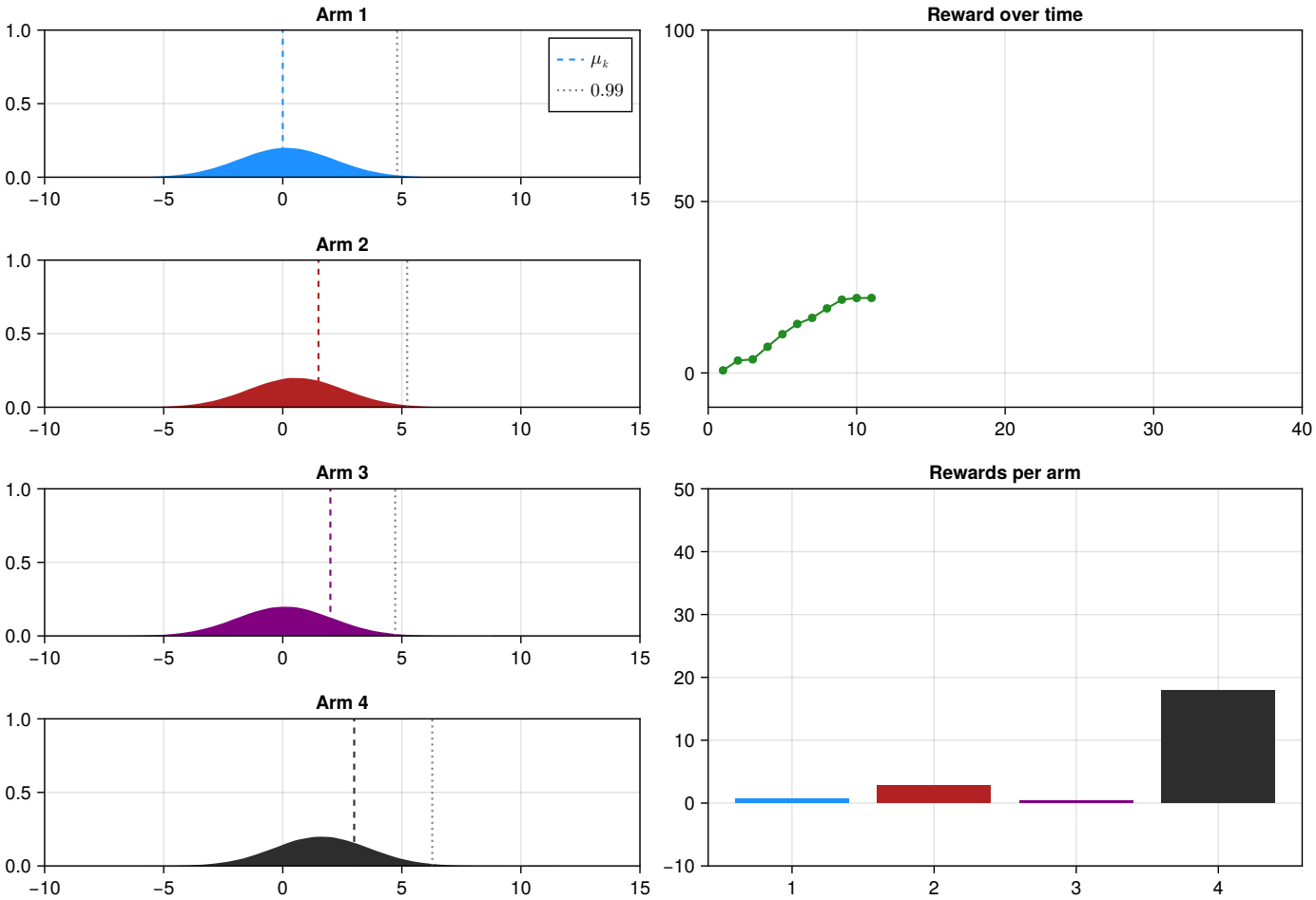

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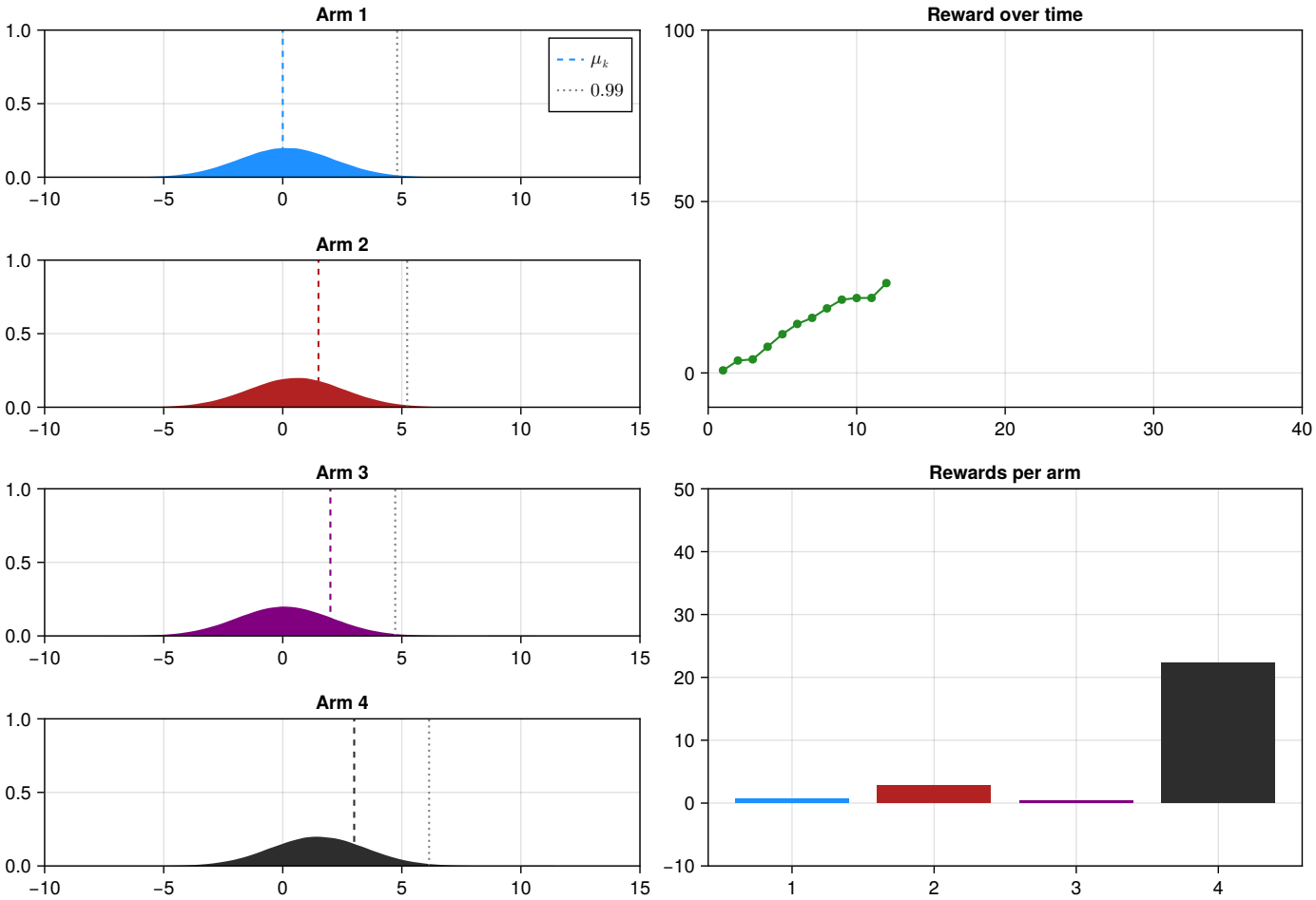

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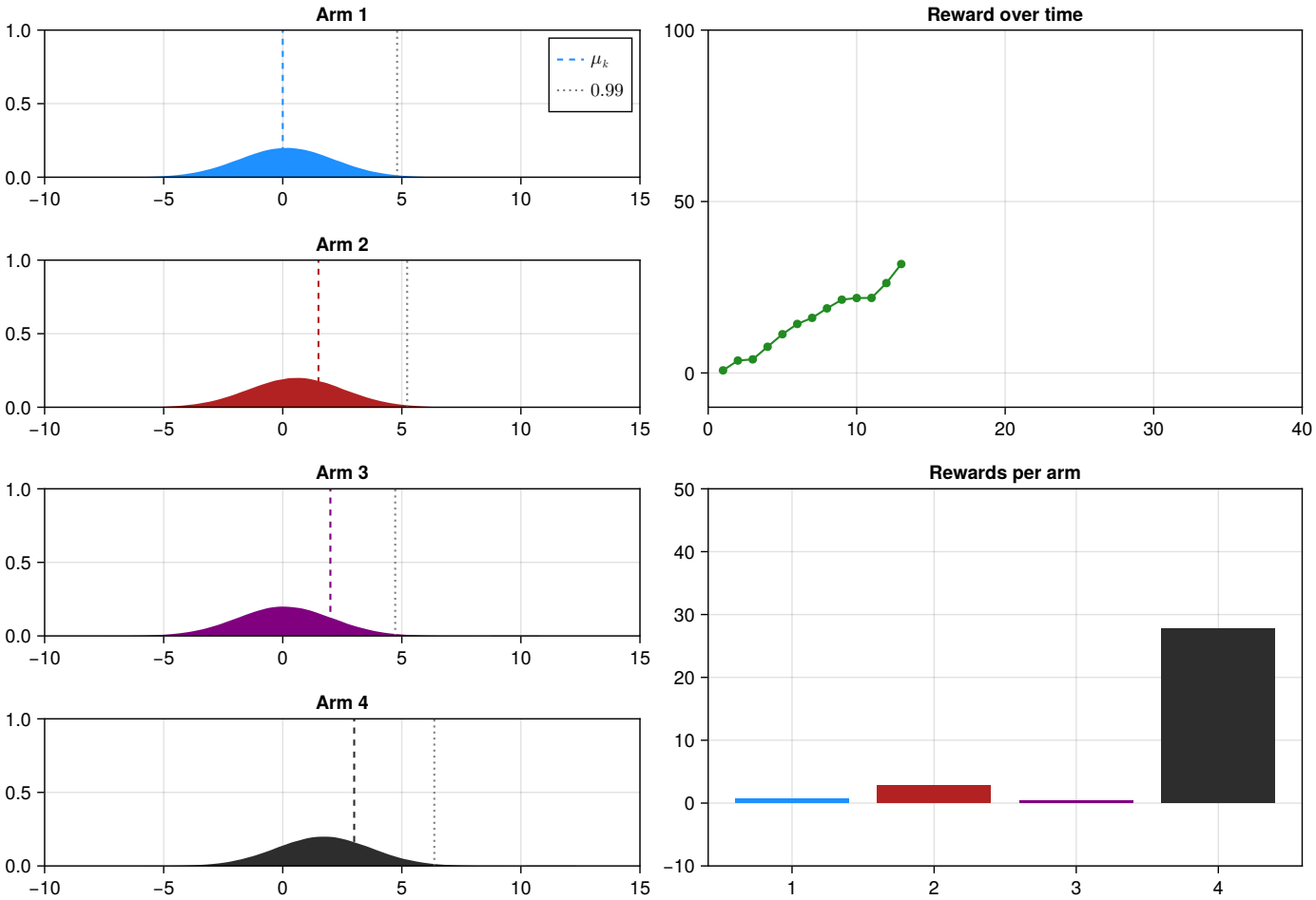

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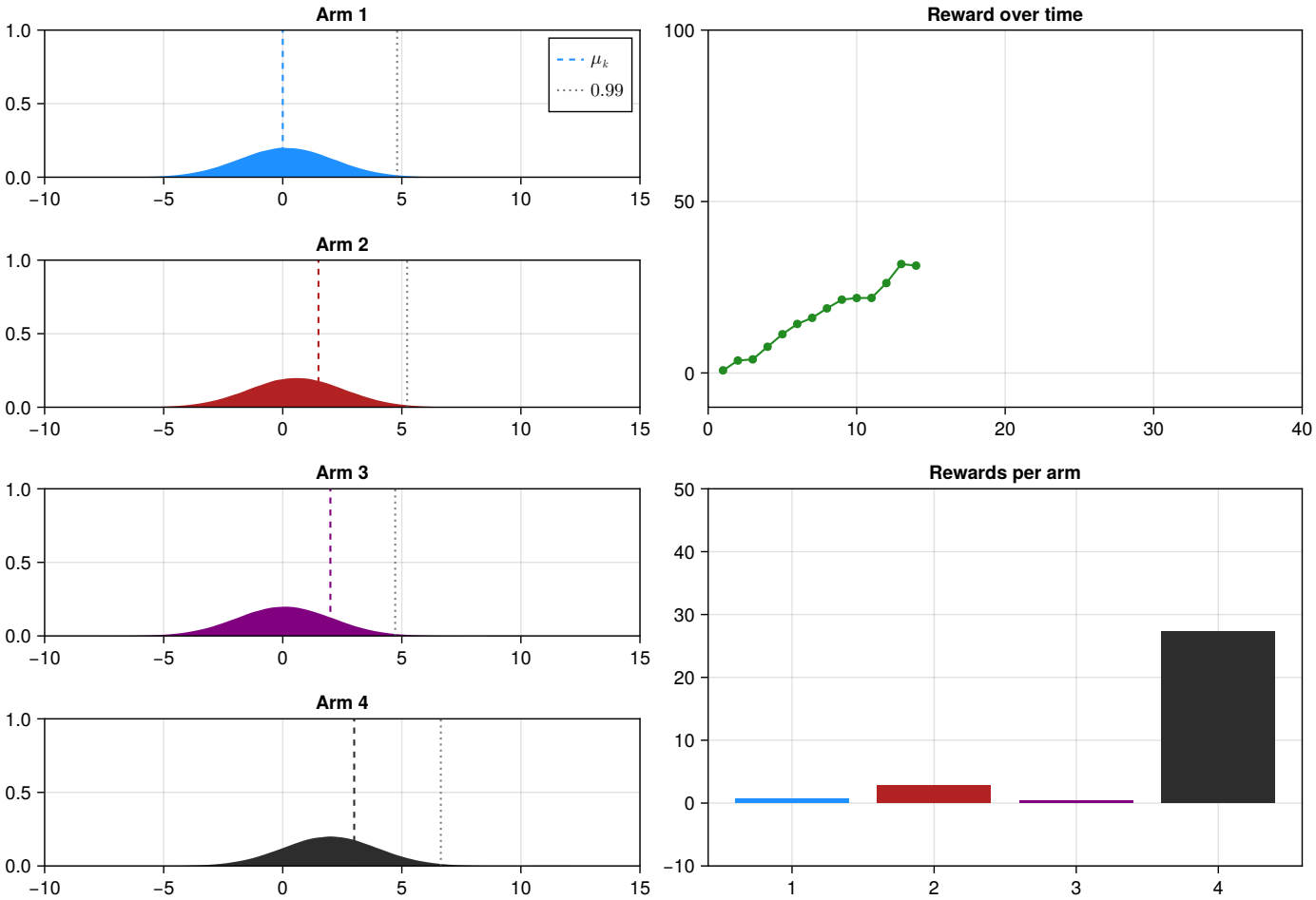

Example: Bayesian Optimization

In the standard multi-armed bandit, the arms are independent.
In many problems by choosing an arm we can gain information about the others.
We will see two such examples:

- Blackbox optimization : Bayesian Optimization
- Collaborative filtering : Probabilistic Matrix Factorization

Example: Bayesian Optimization

Consider the problem of maximizing a function $f : \mathbb{R}^n \to \mathbb{R}$ such that:

- f is a blackbox function
- f is expensive to call

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Can we formulate this as a bandit problem with infinite arms and apply something like UCB?

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- f is expensive to call

Can we formulate this as a bandit problem with infinite arms and apply something like UCB?

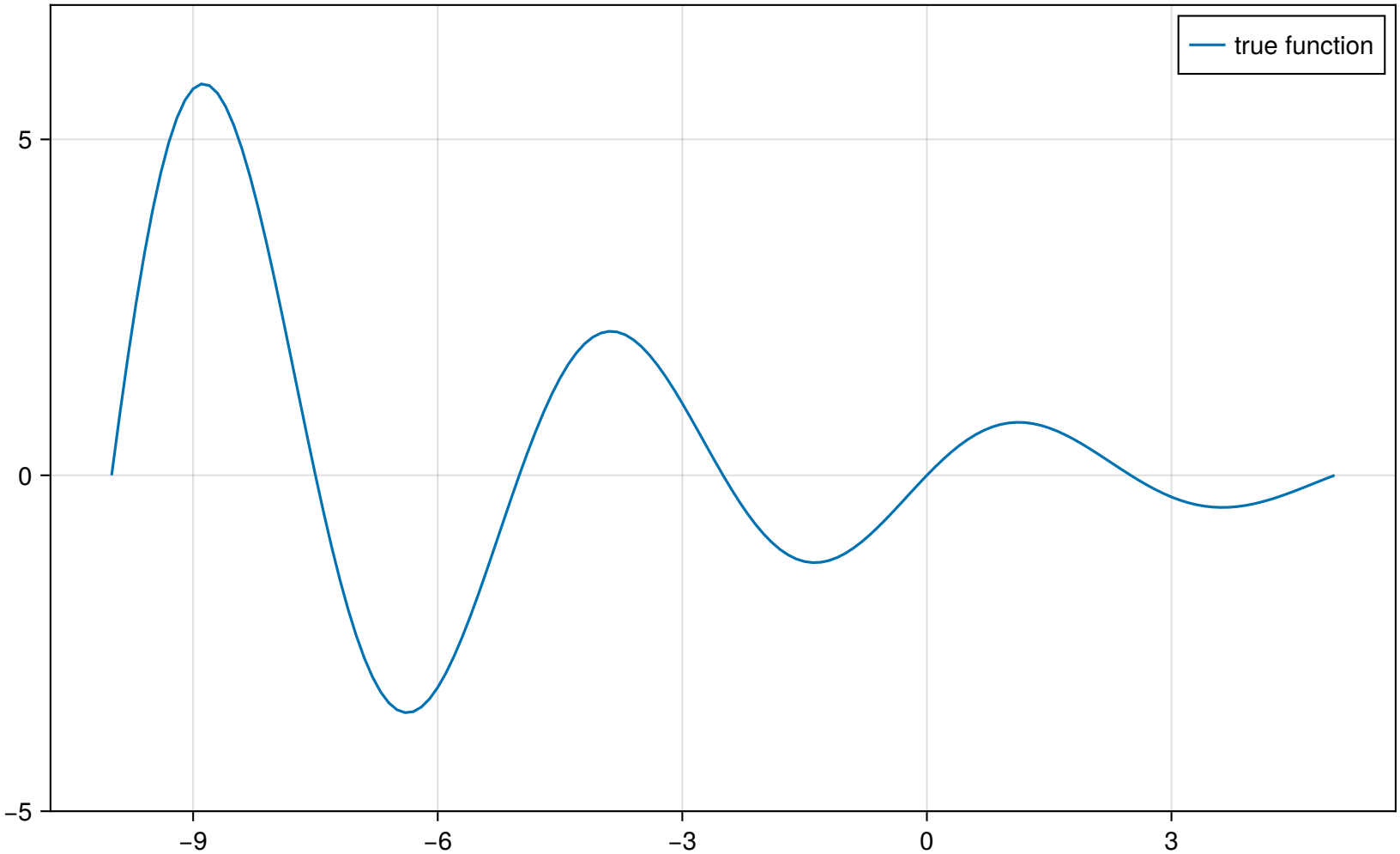
The are two main approaches to define probabilistic models with infinite latent variables:

- Nonparametric modeling
- Amortized inference

Example: Bayesian Optimization

We saw Gaussian Process in the previous lectures, so let's use it with a UCB policy

Example: Bayesian Optimization



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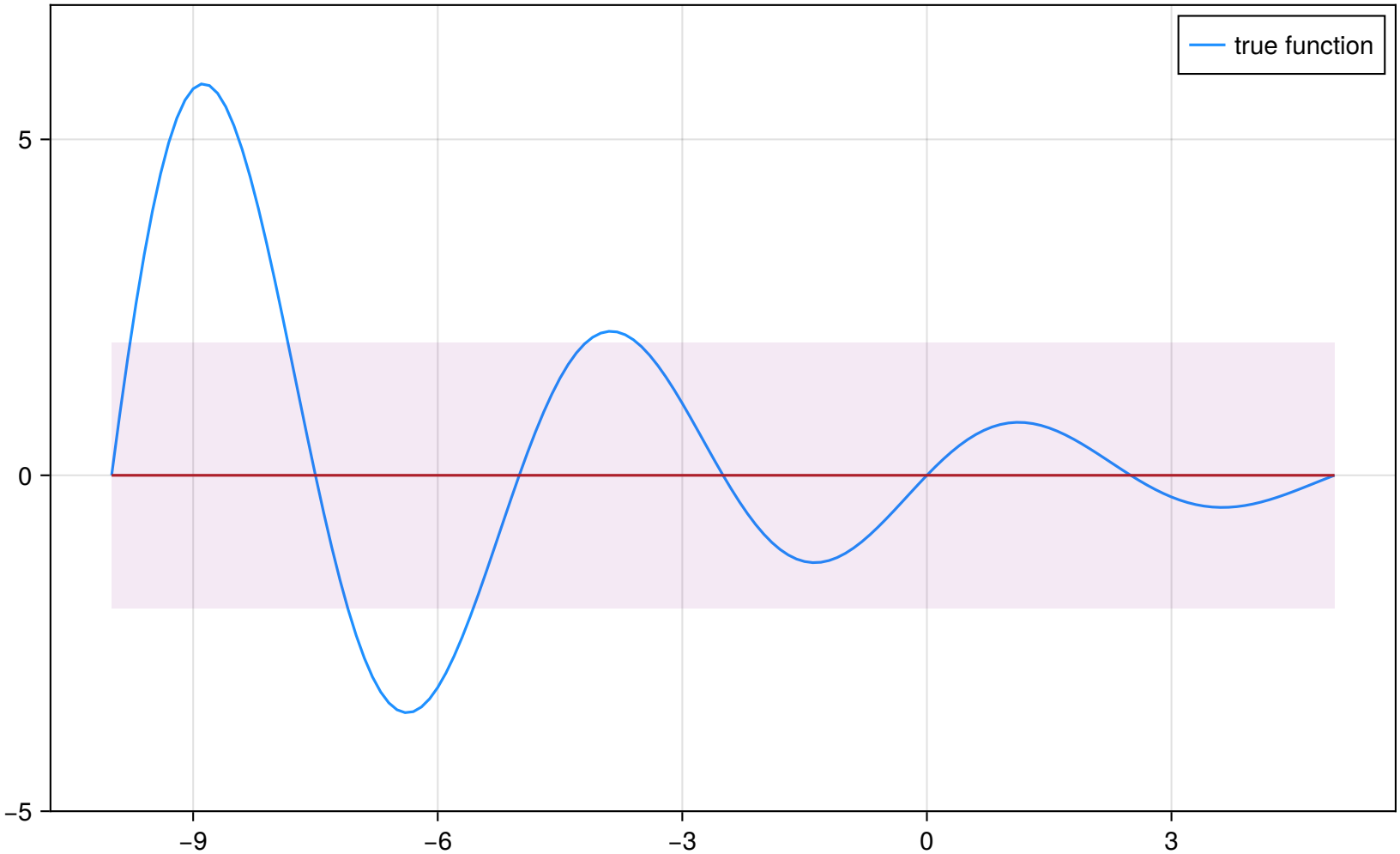

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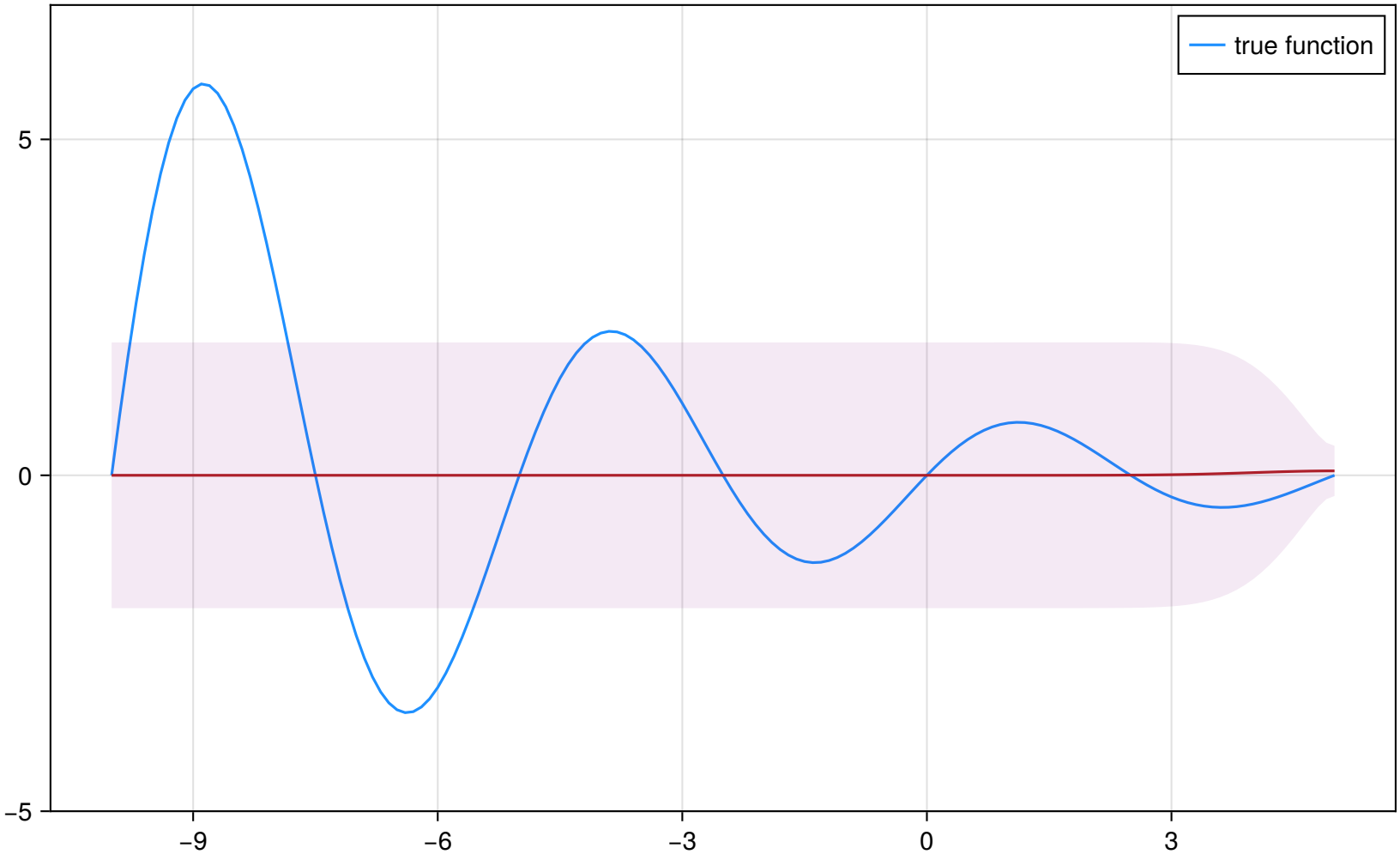

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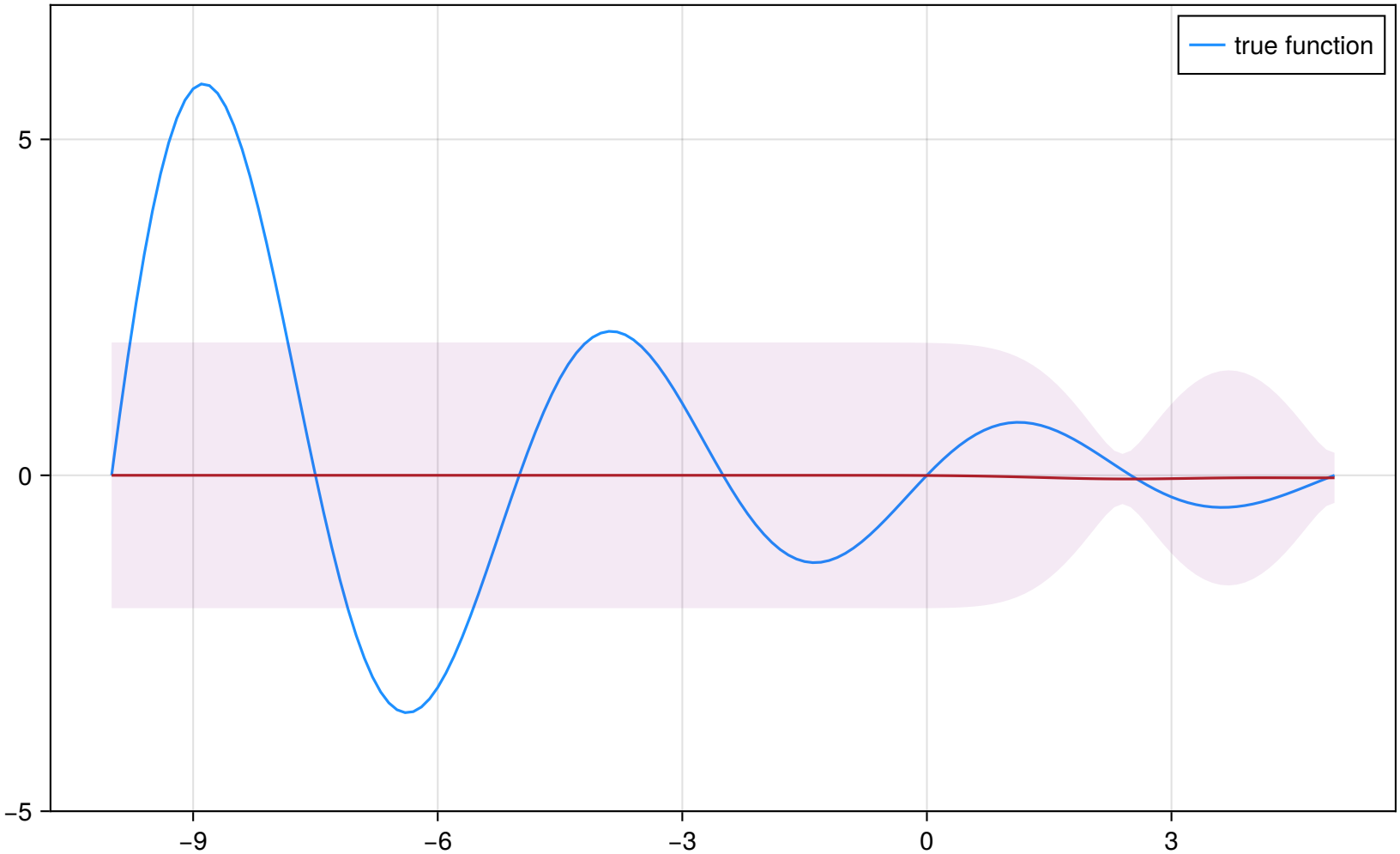

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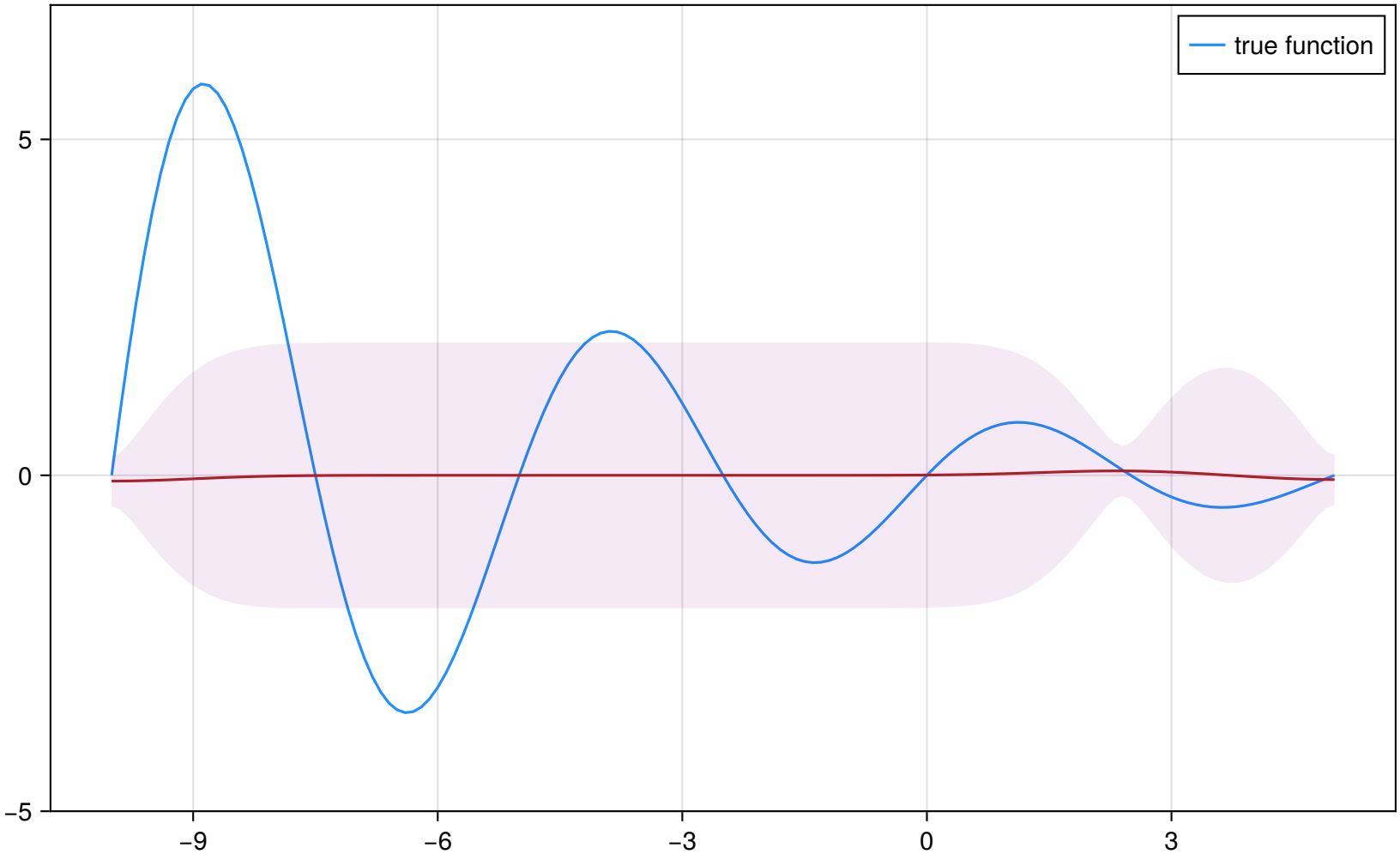

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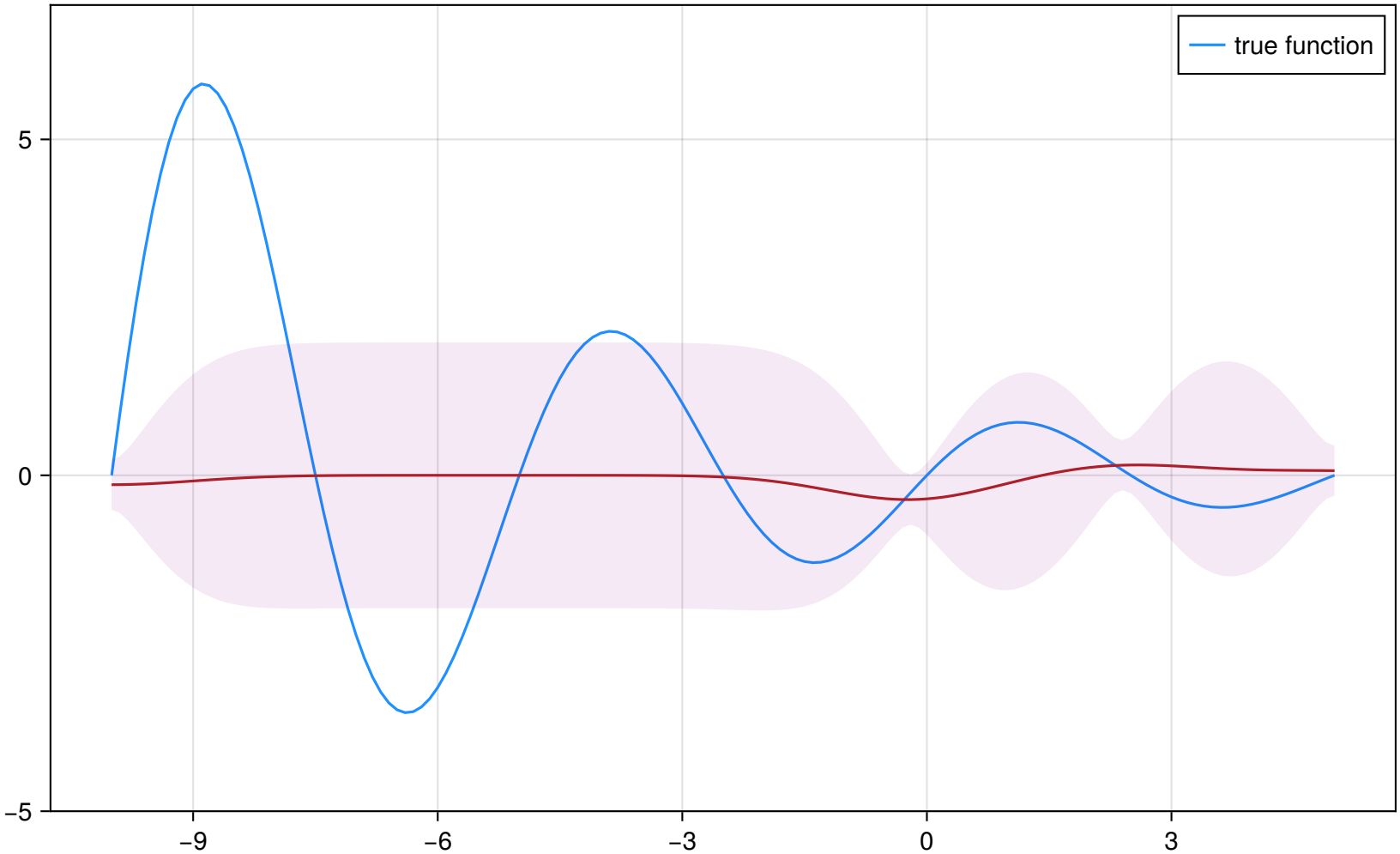

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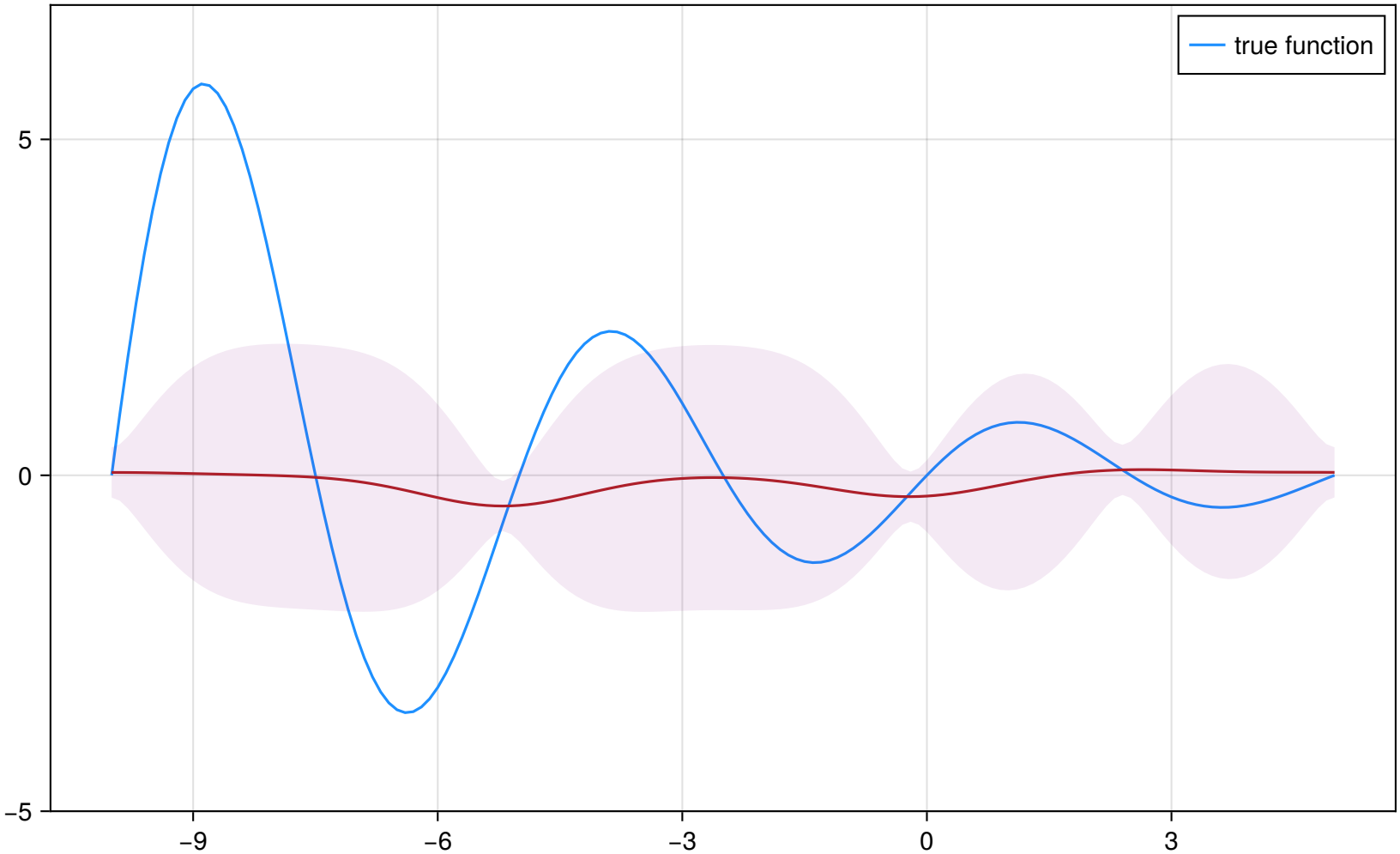

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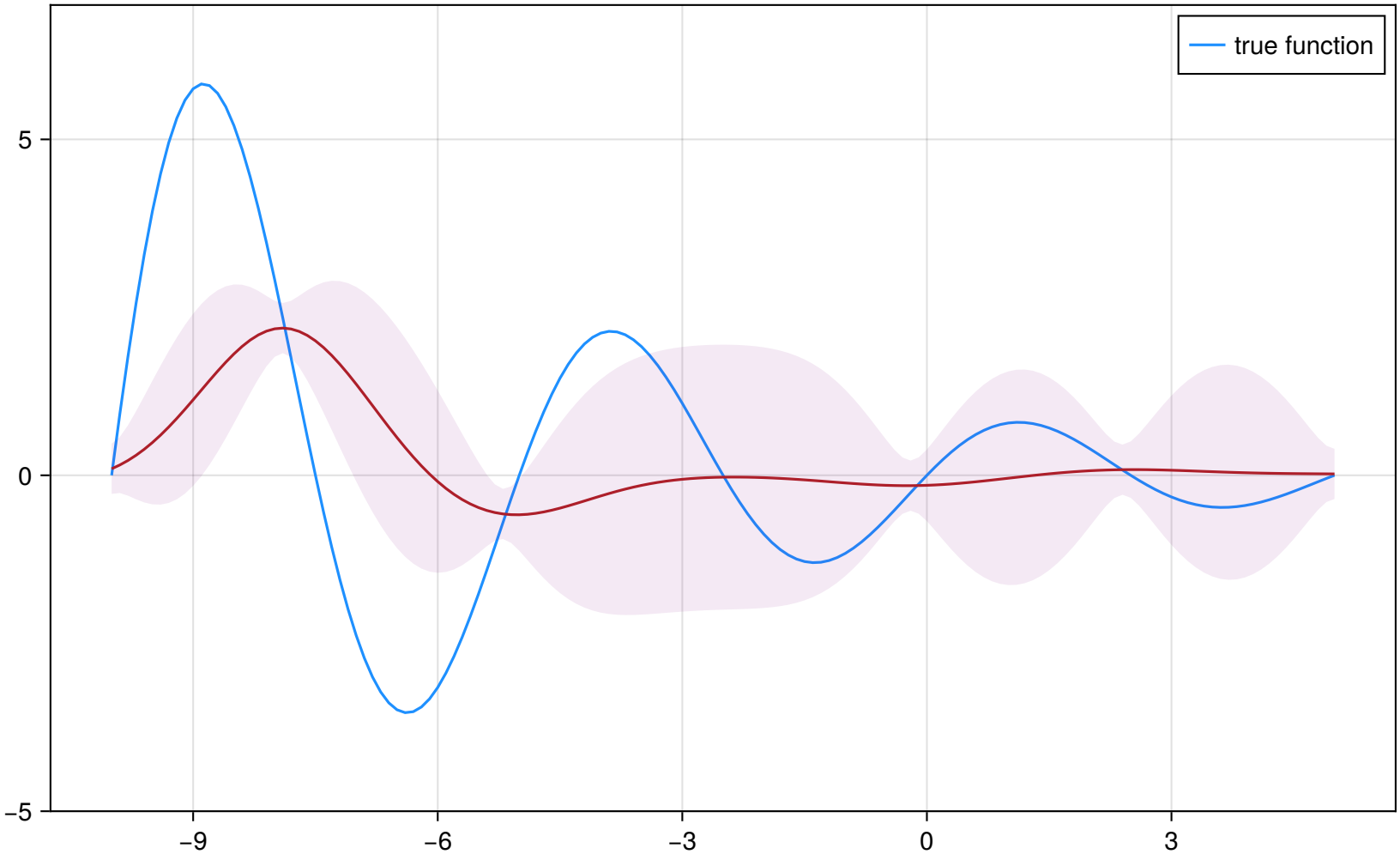

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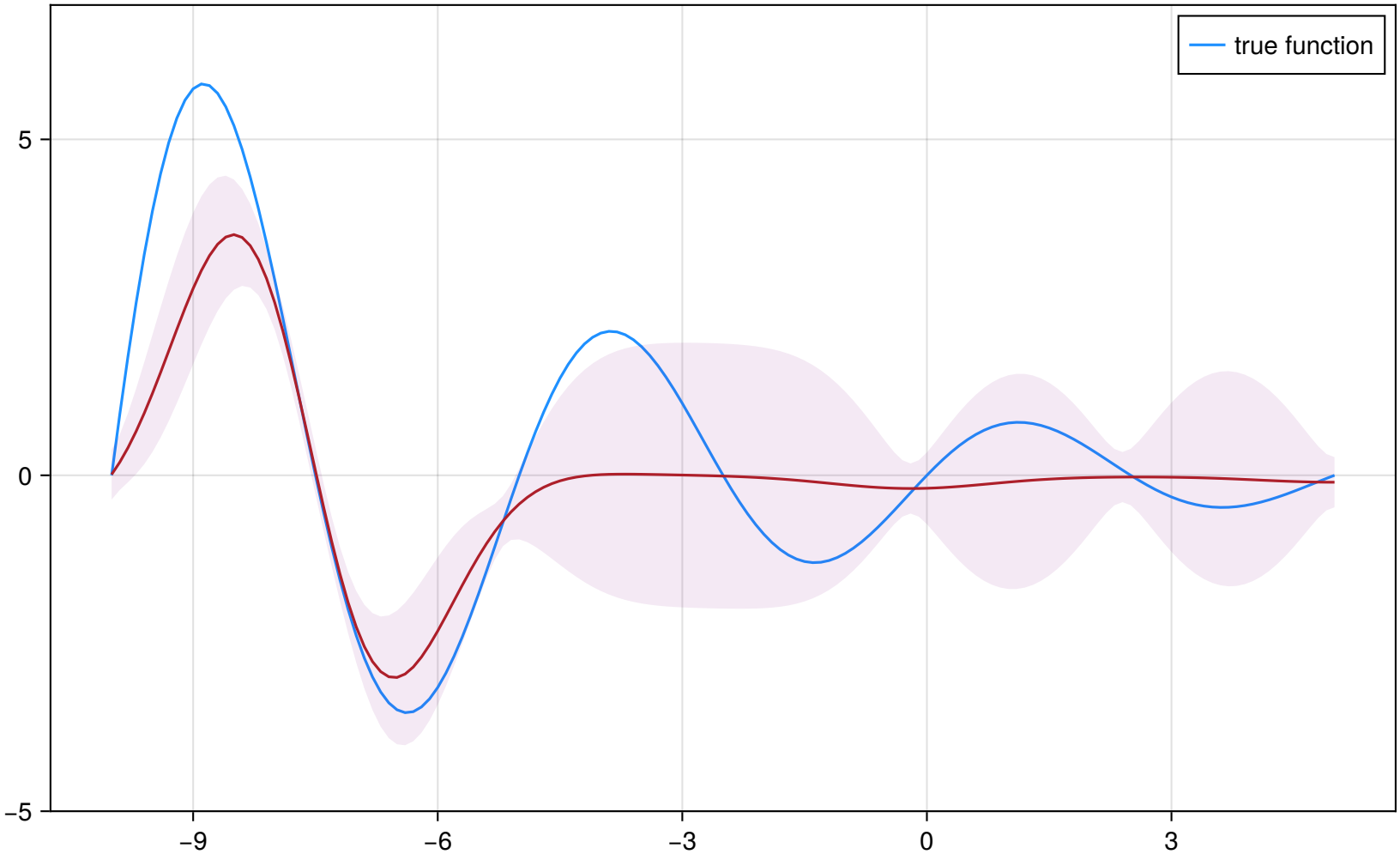

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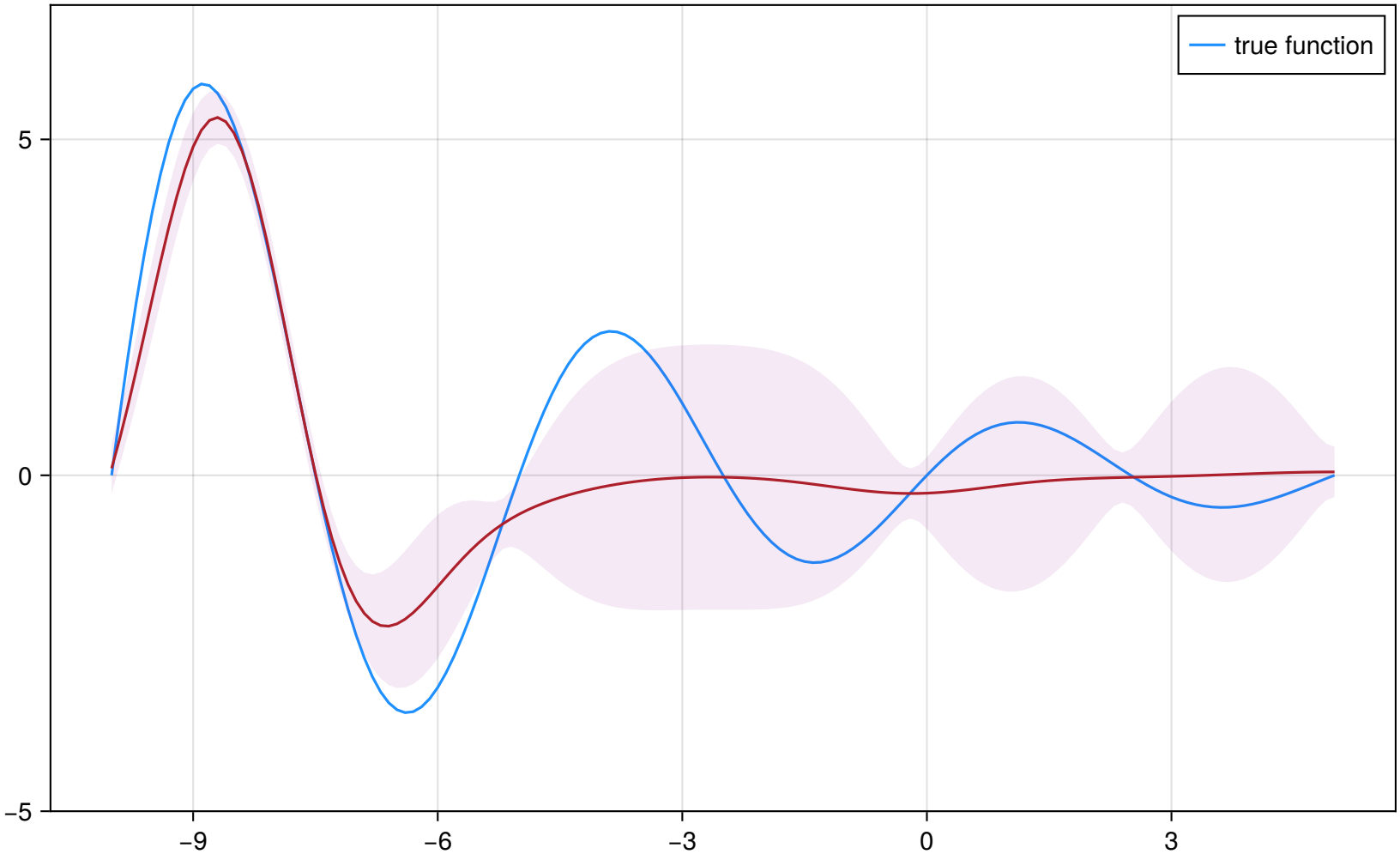


Example: Recommendation with Collaborative Filtering

Another example of structured bandit problems arises in recommender systems (movies, music, ...). Observing a user's preferences can give us information about other similar users likewise about movies.

To apply a policy like UCB we need a probabilistic approach to collaborative filtering.

Probabilistic Matrix Factorization

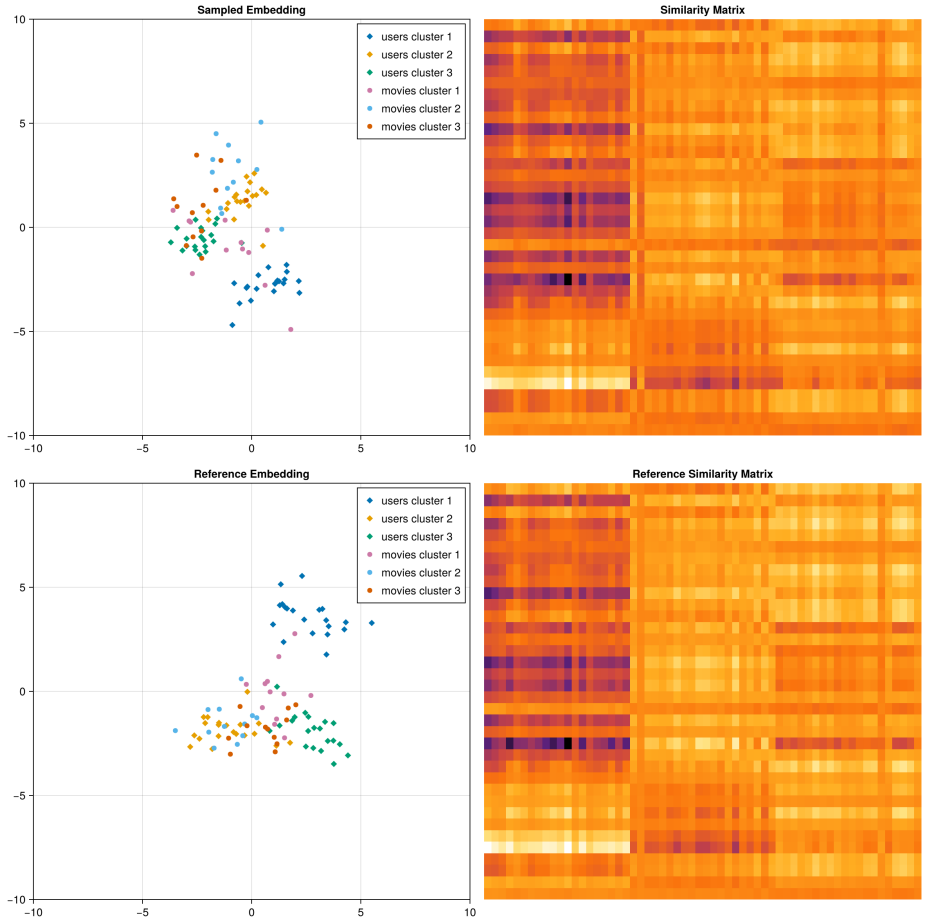
$$u_i \sim \text{MvNormal}(0, I)$$

$$m_j \sim \text{MvNormal}(0, I)$$

$$p_{ij} = \sigma(u_i . m_j)$$

$$x_{ij} \sim \text{Bernoulli}(p_{ij})$$

Probabilistic Matrix Factorization



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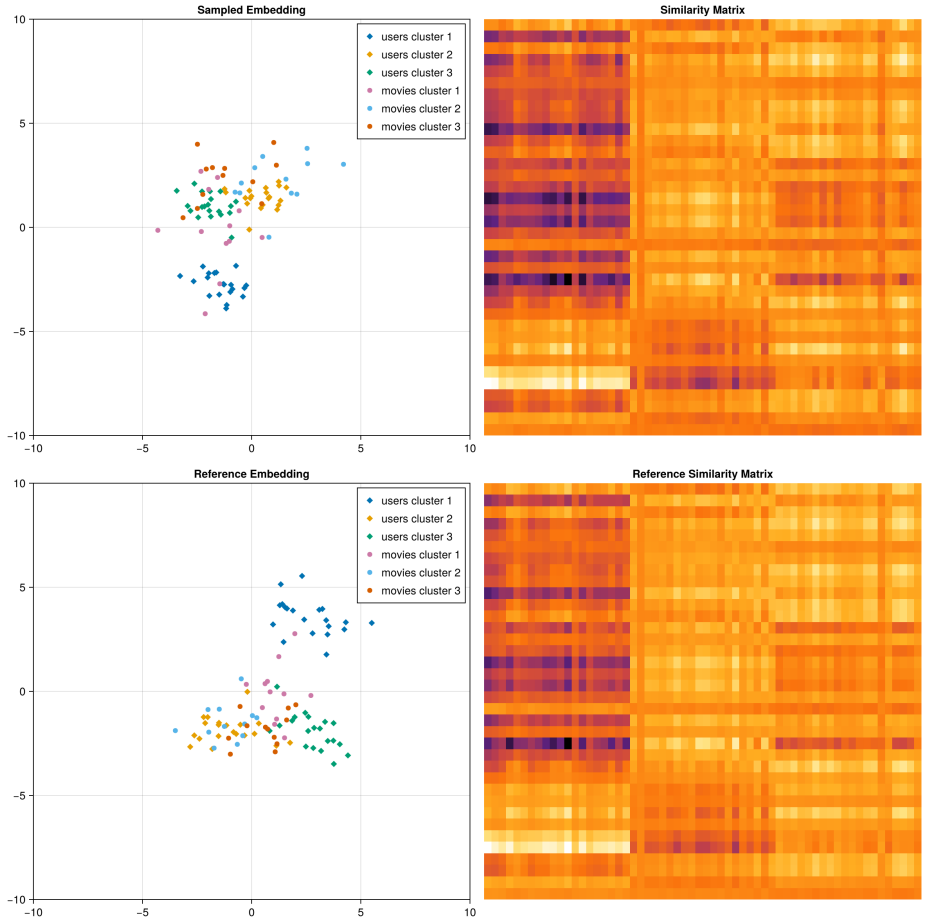

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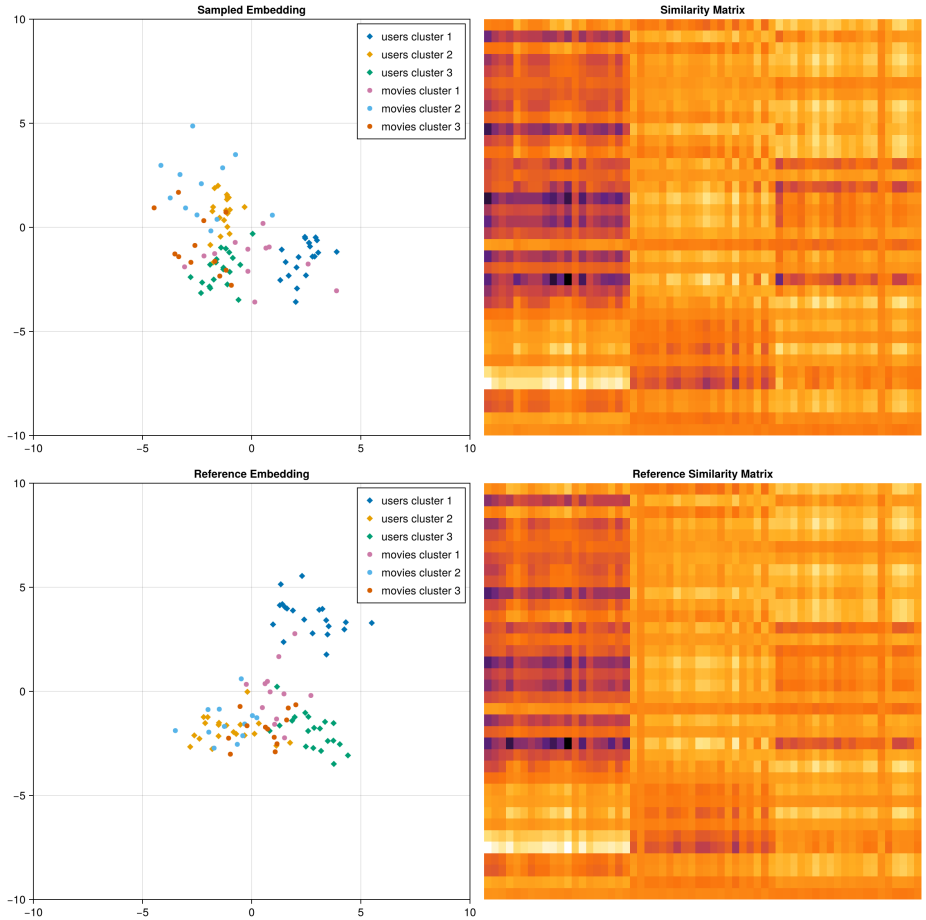

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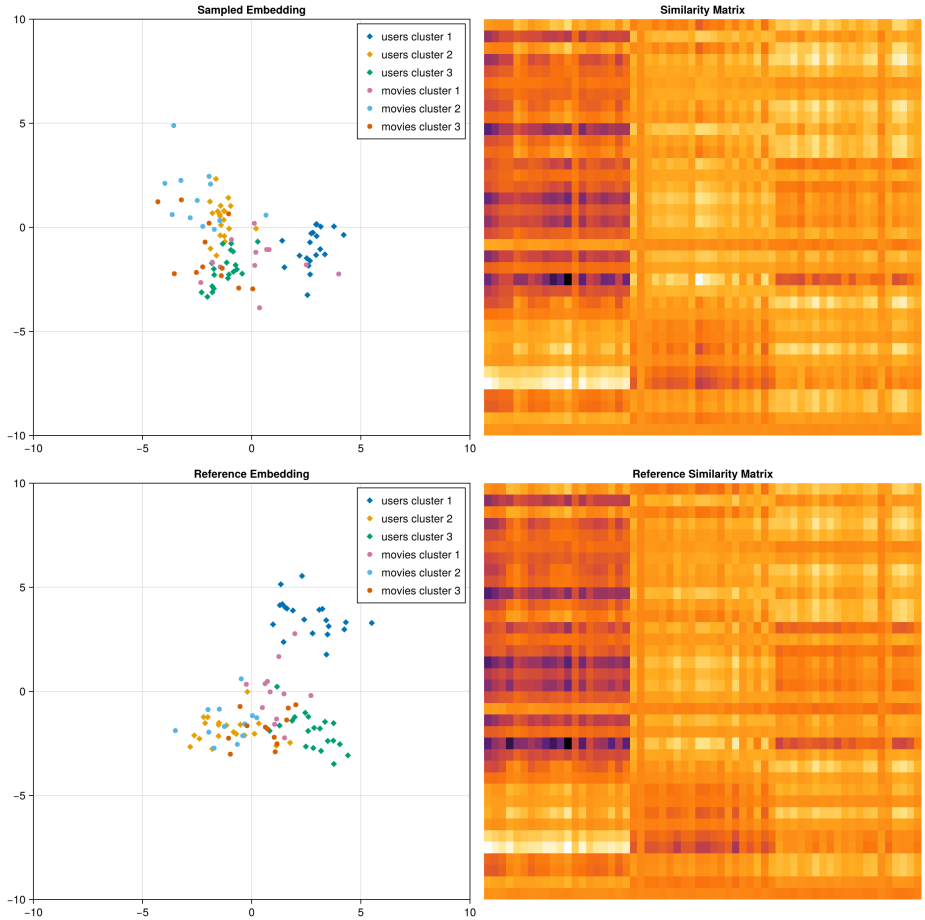

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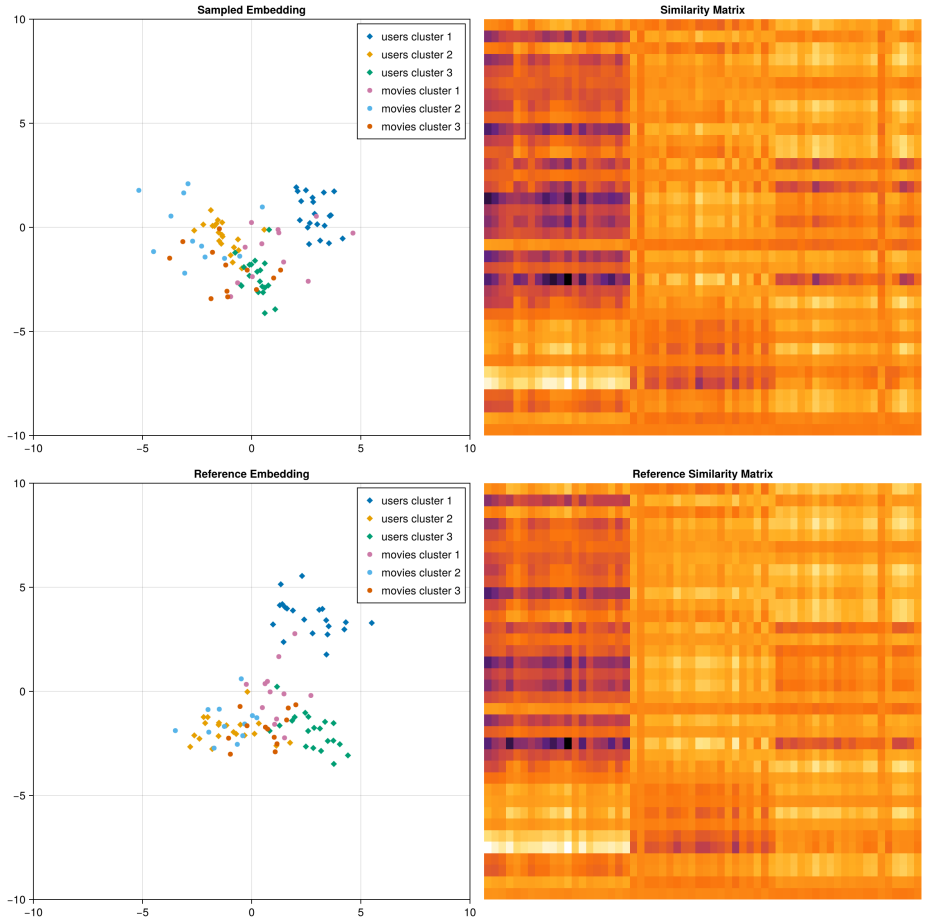

Probabilistic Matrix Factorization

Application: Ad click prediction and recommendation with Bayesian Upper Confidence Bound

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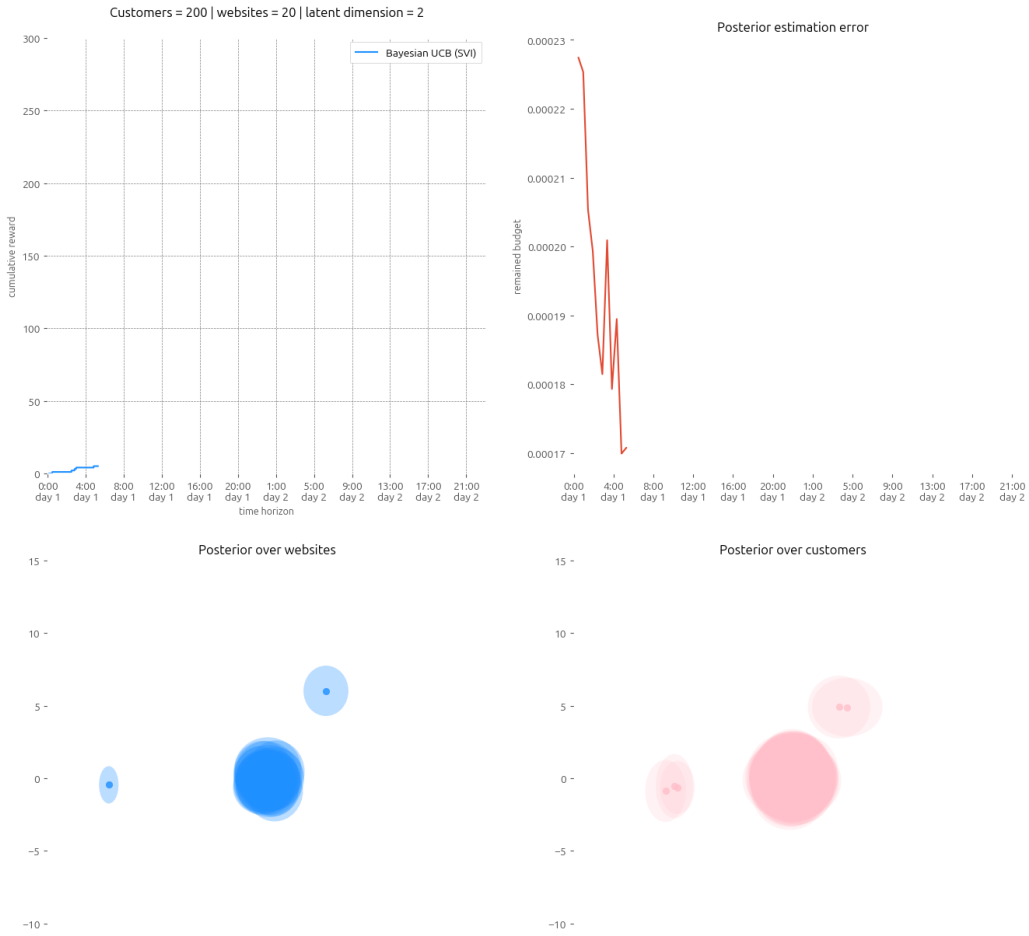

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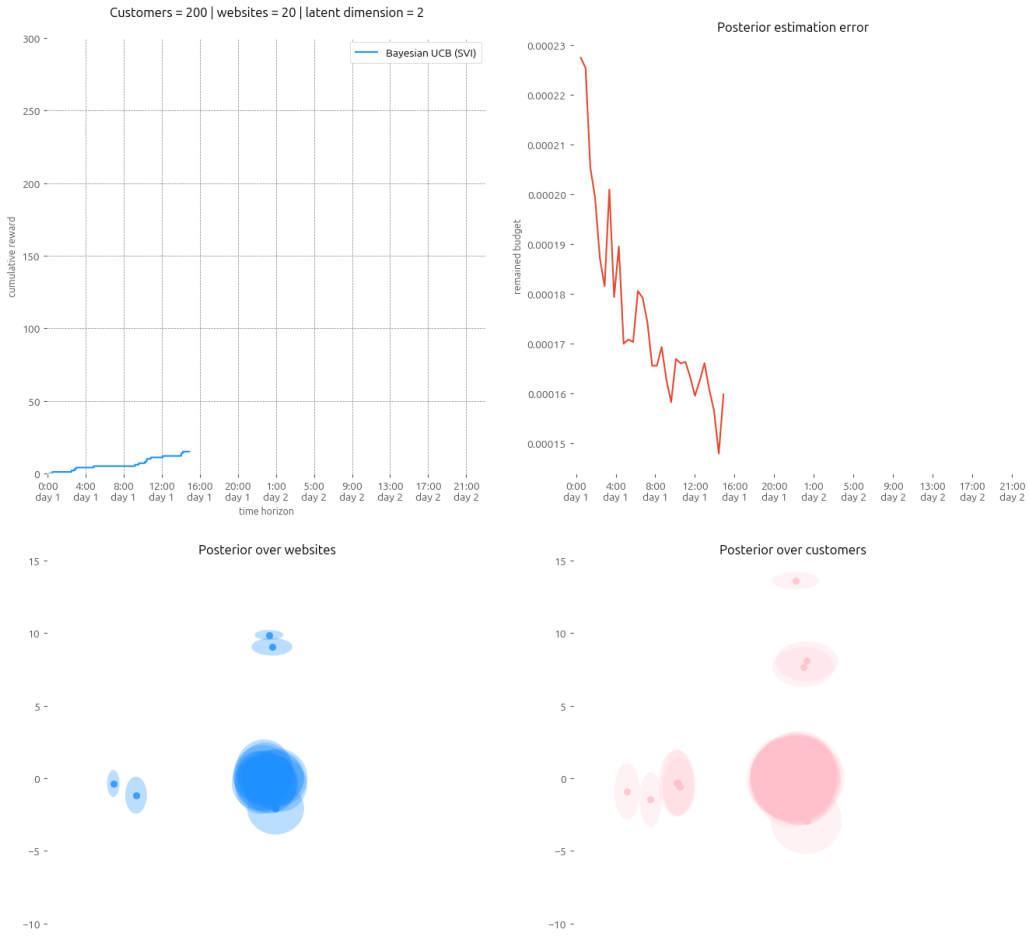

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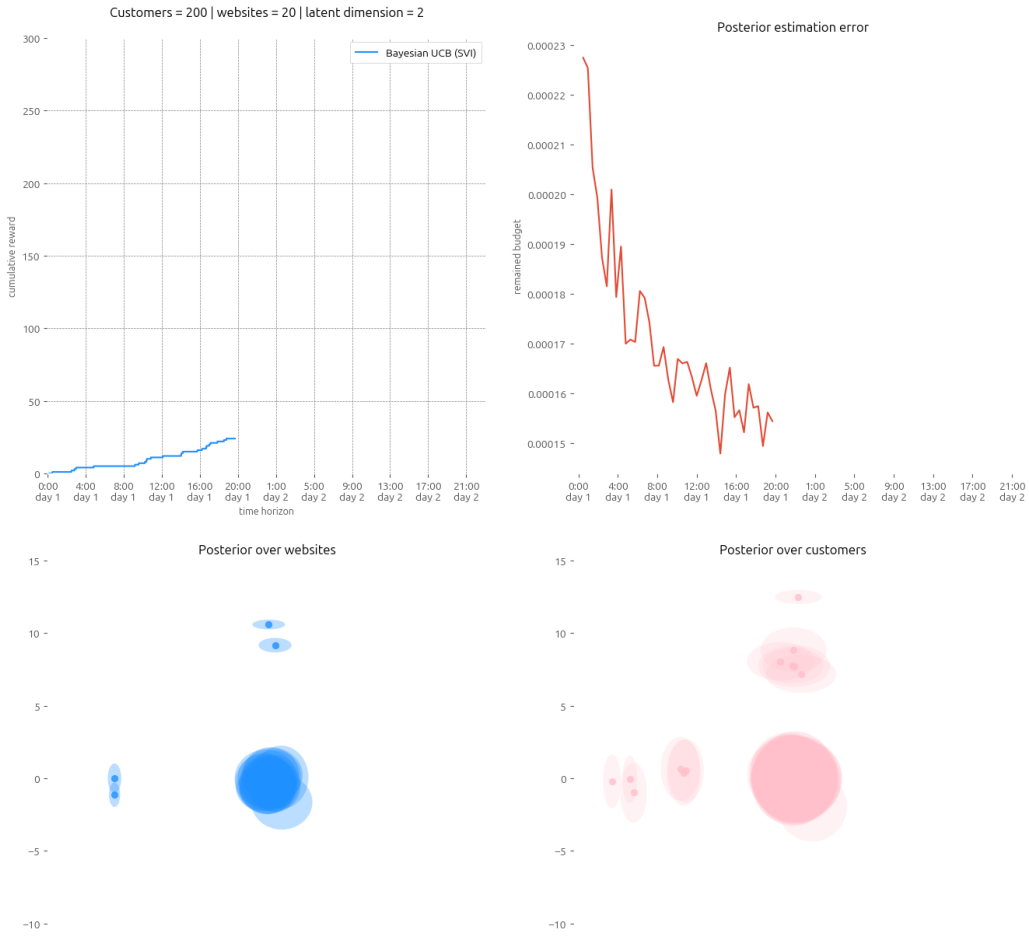

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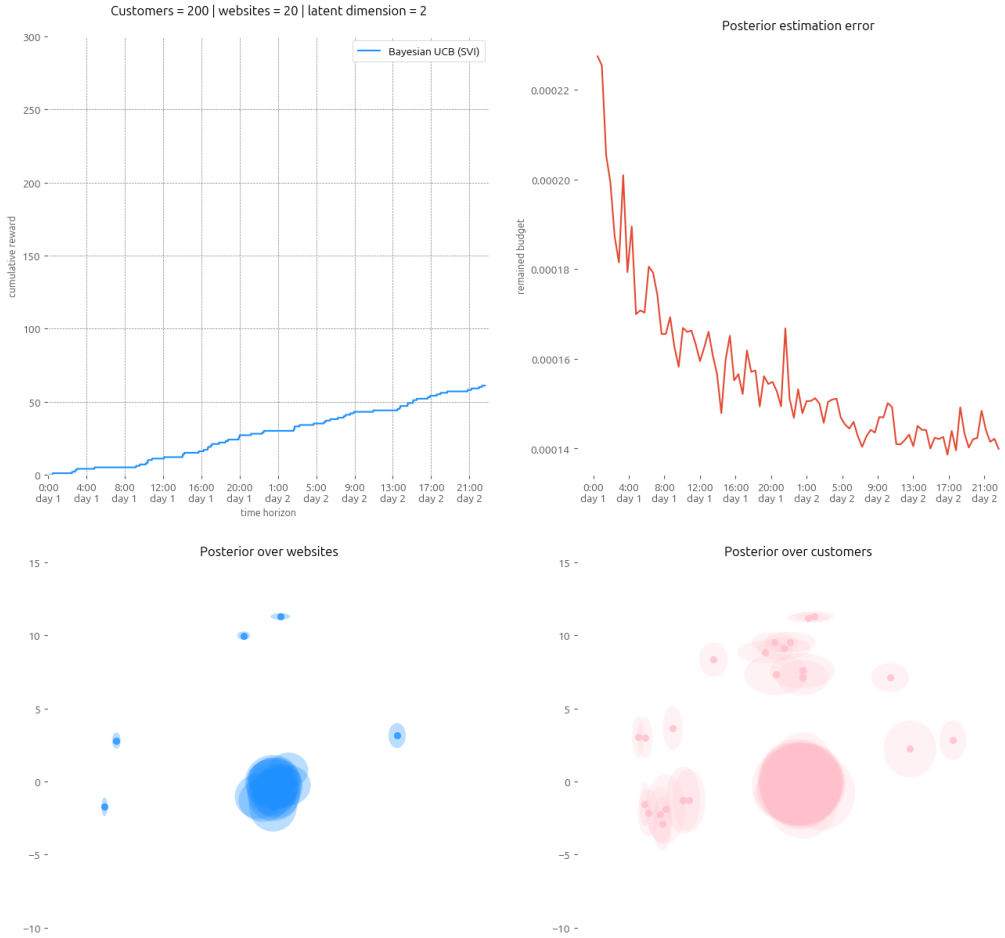
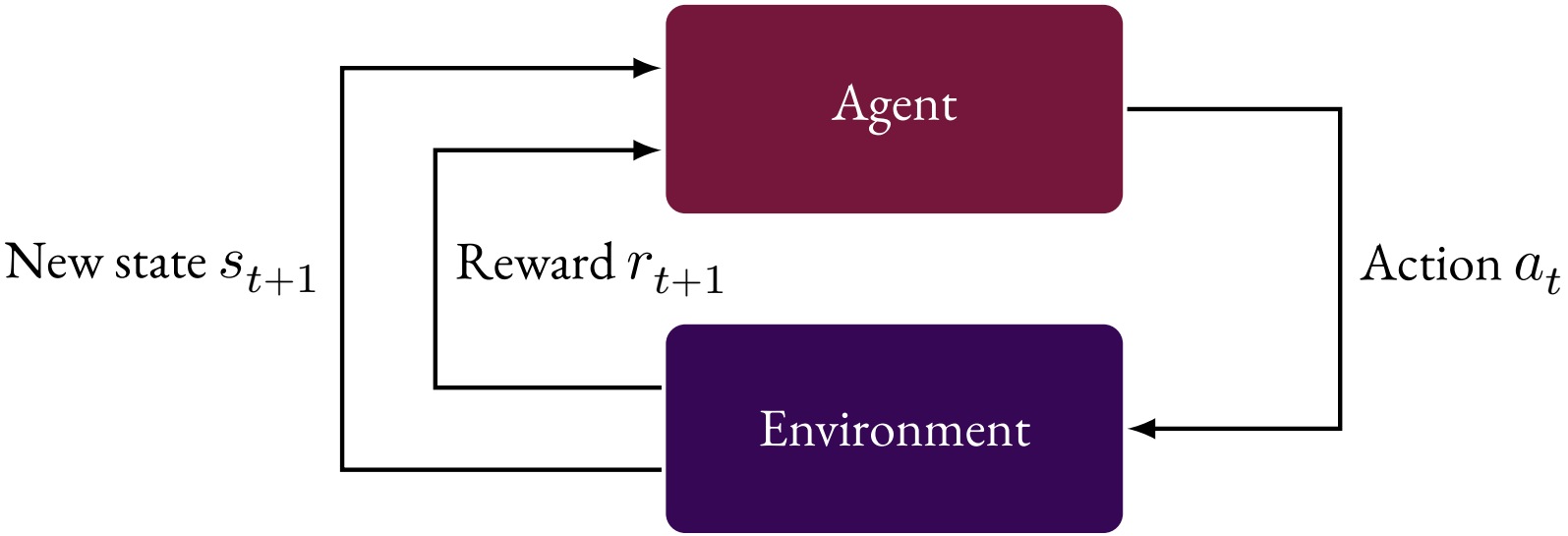


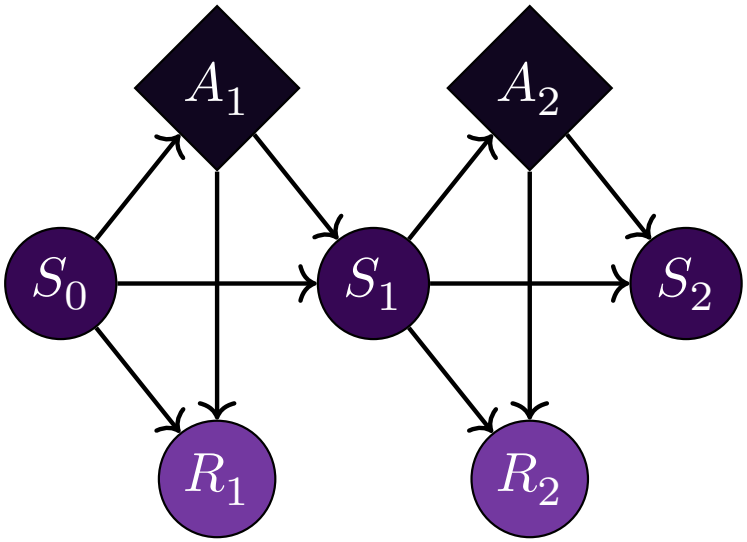
Reinforcement Learning

Sometimes, environment is not static and the decisions can affect it too

Reinforcement Learning



Markov Decision Process



...

S : State

A : Action

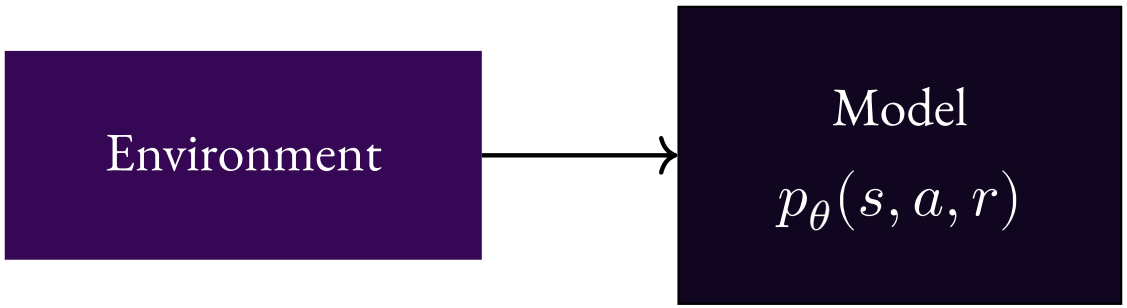
R : Reward

$\gamma \in (0, 1)$: Discount factor

The goal is to maximize $\mathbb{E}_\pi[G]$ the expected total reward $G = \sum_{t=1}^{\infty} \gamma^i R_i$

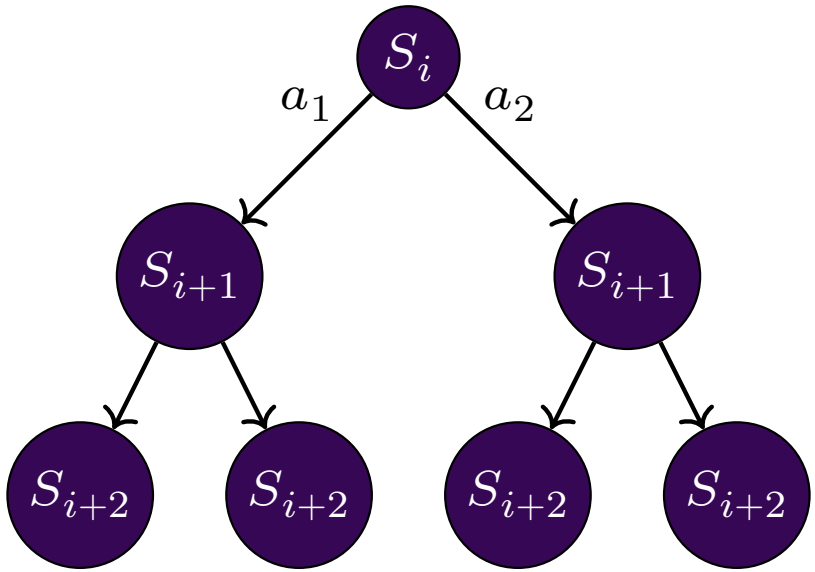
Model-Based RL

Main idea : designing a model of environment and using it for planning.



Model-Based RL

The model can be used to expand a search tree over future actions in order to perform approximate dynamic programming



Example: Monte Carlo Tree Search

Challenges with RL

- RL is not sample efficient.
- RL agents are difficult to train in problems with sparse reward.
- RL framework is too much dependent on the notion of extrinsic reward.
- RL agents are not modular.
- Robustness in non-stationery problems
- Generalizability

Active Inference

Is it possible to formulate decision-making as probabilistic inference?

Active Inference

$$p(s|o) = \frac{p(o|s)p(s)}{p(o)}$$

$$\text{Surprise} := -\log p(o) = -\log \sum_{s'} p(o, s')$$

Active Inference

How to avoid getting surprised?

- Understanding the environment better
- Affecting the surprising observations using actions

Active Inference

Variational Free Energy

$$
\begin{aligned}
-\log p(o) &= -\log \sum_{s'} p(o, s') \\
&= -\log \sum \frac{p(o, s')q(s')}{q(s')} \\
&= -\log \mathbb{E}_{s' \sim q}[\frac{p(o, s')}{q(s')}] \\
&\leq -\mathbb{E}_{s' \sim q}[\log \frac{p(o, s')}{q(s')}] \\
&= \mathbb{E}_{s' \sim q}[\log \frac{q(s')}{p(o, s')}]
\end{aligned}
$$

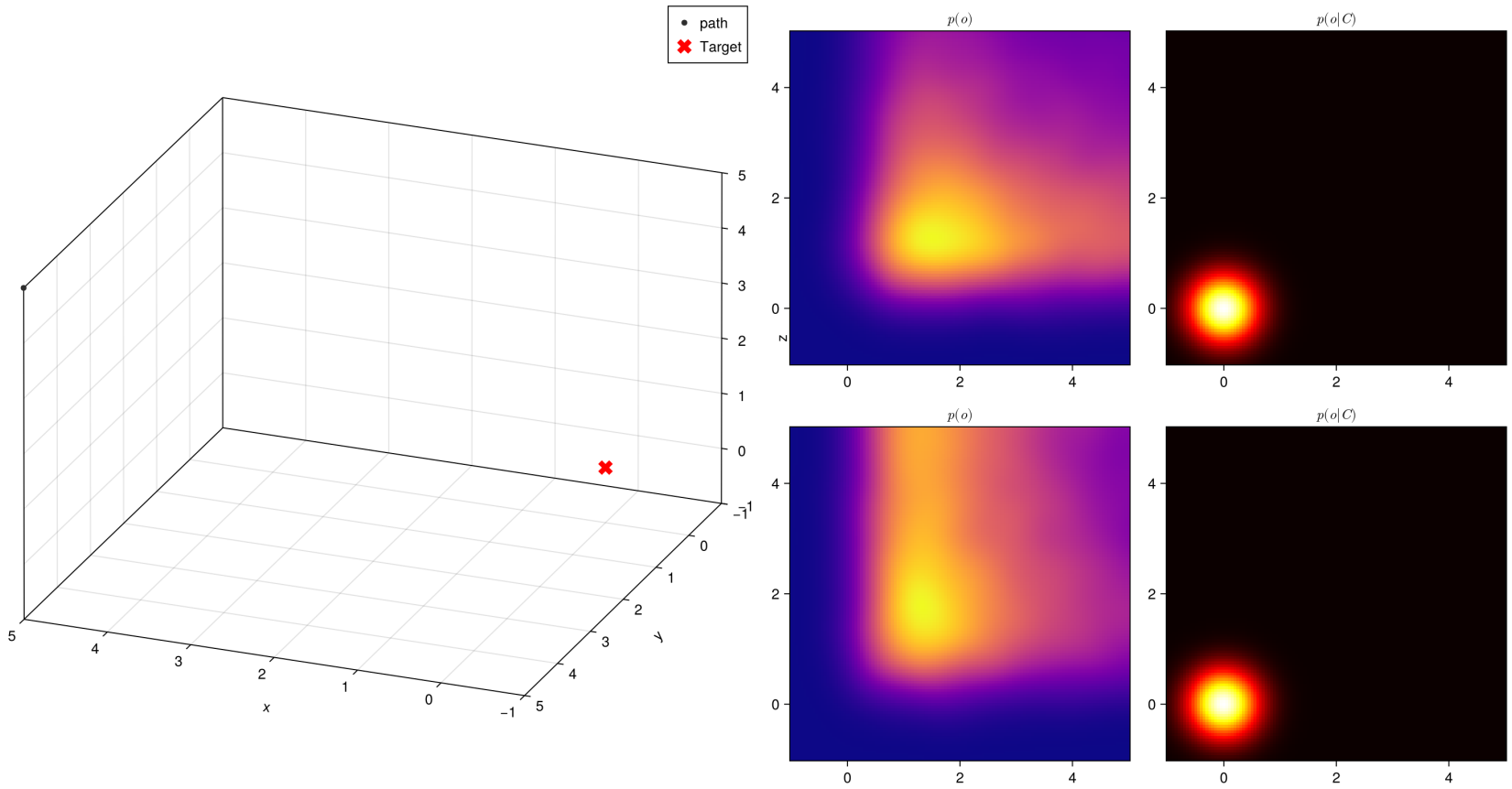
Active Inference

Instead of a scalar reward signal, In Active Inference we design a prior distribution $p(o|C)$ concentrated on the prefered outcomes. This way, deviation from $p(o|C)$ decreases evidence and consequently increases surprise.
So agent can decrease surprise by:

- Learning more about the environment dynamics, improving the variational approximation $q(s)$
- Choosing actions that do not lead to surprising states, reducing $D_{KL}[q(o)||p(o|C)]$

Active Inference

Example: Zero-shot navigation with uncertain dynamics



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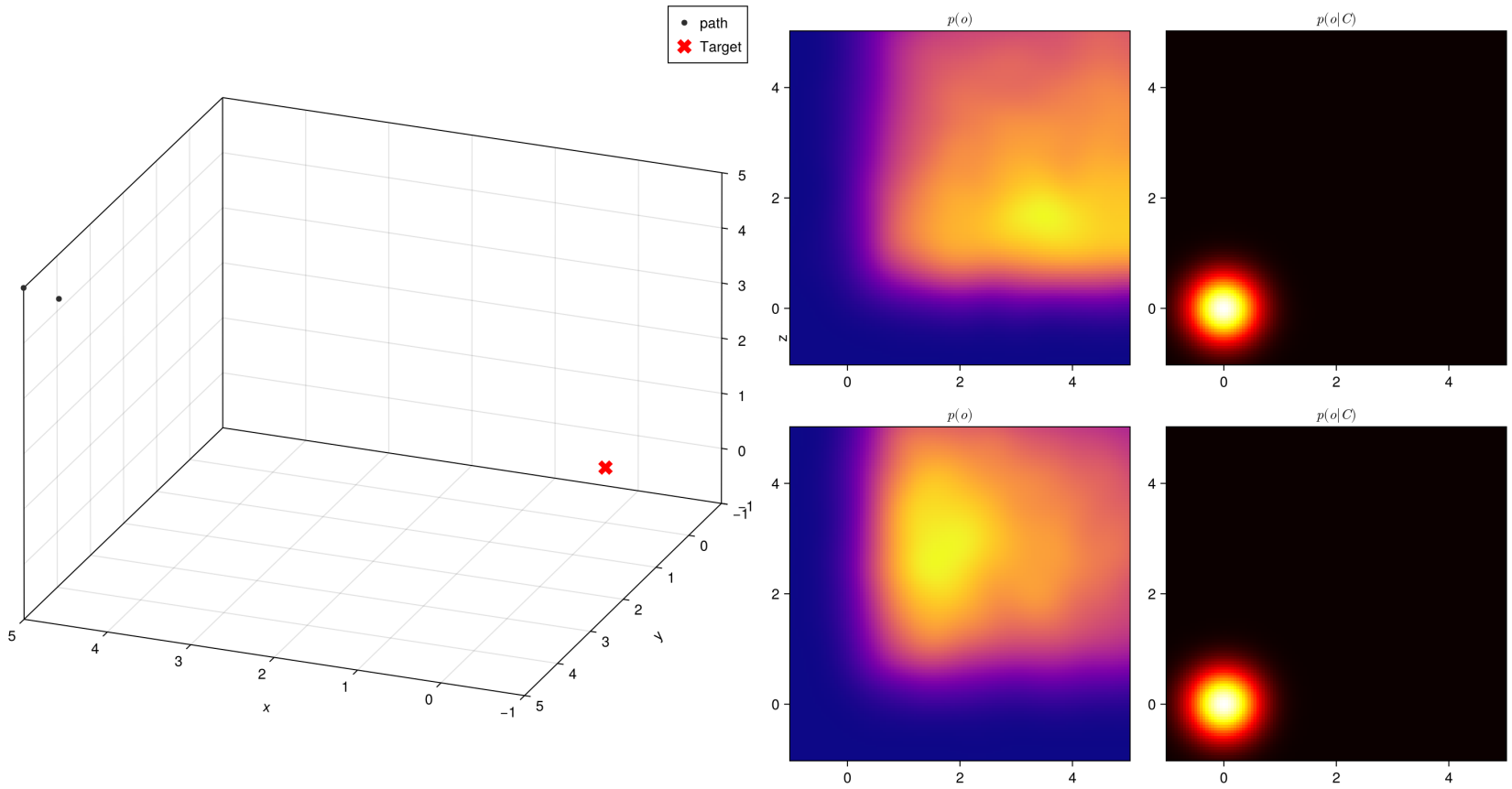

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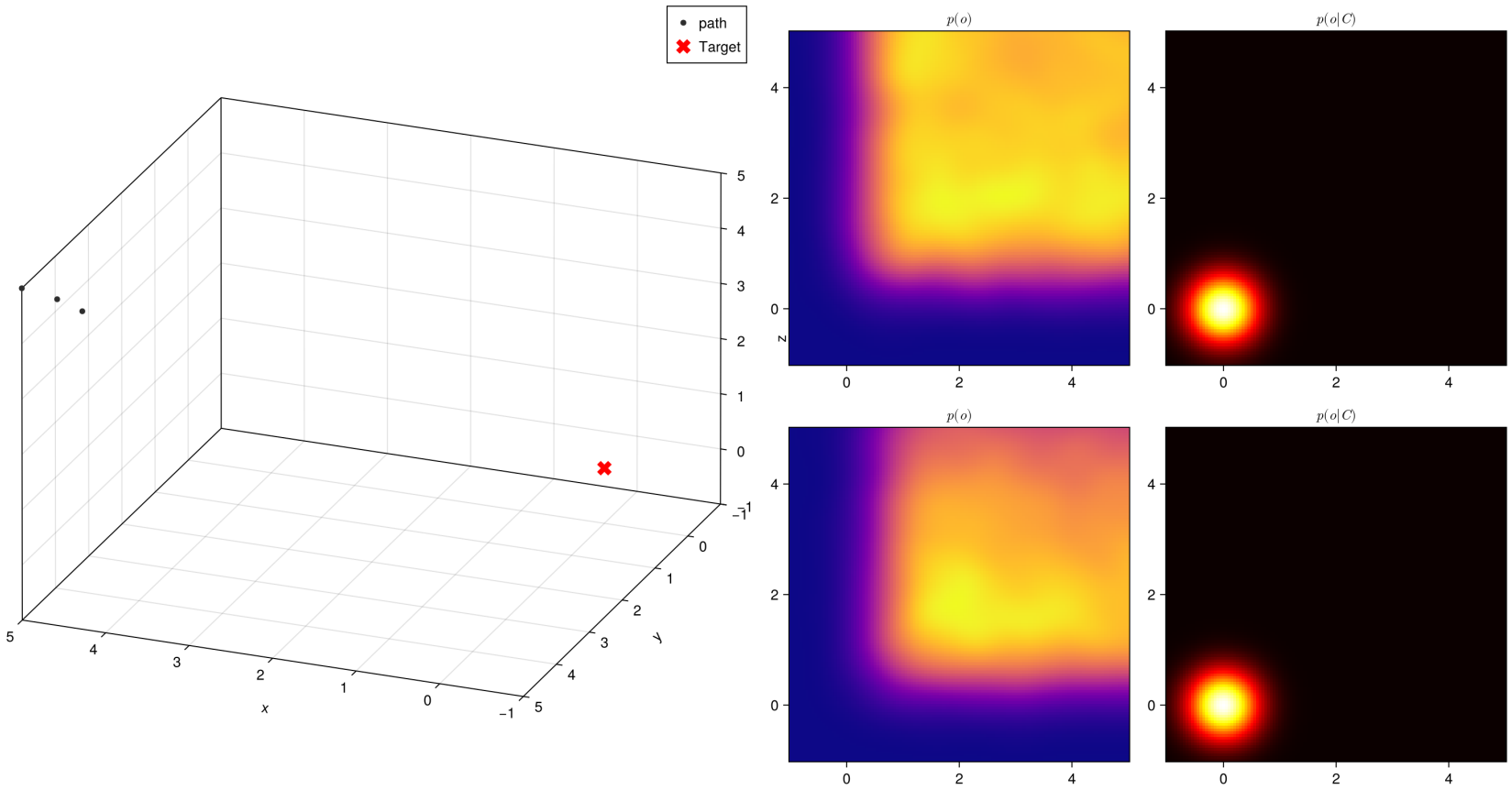

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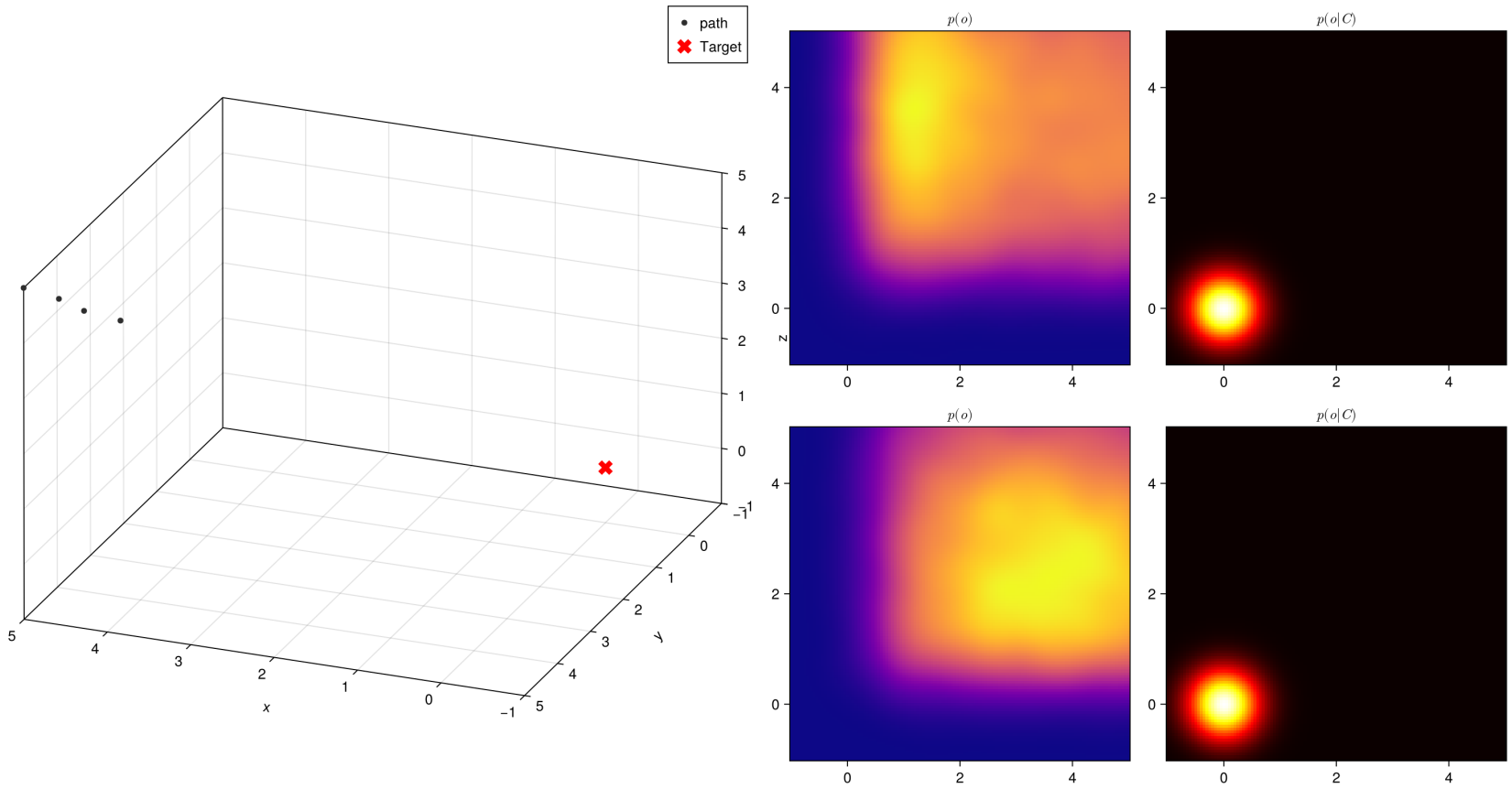
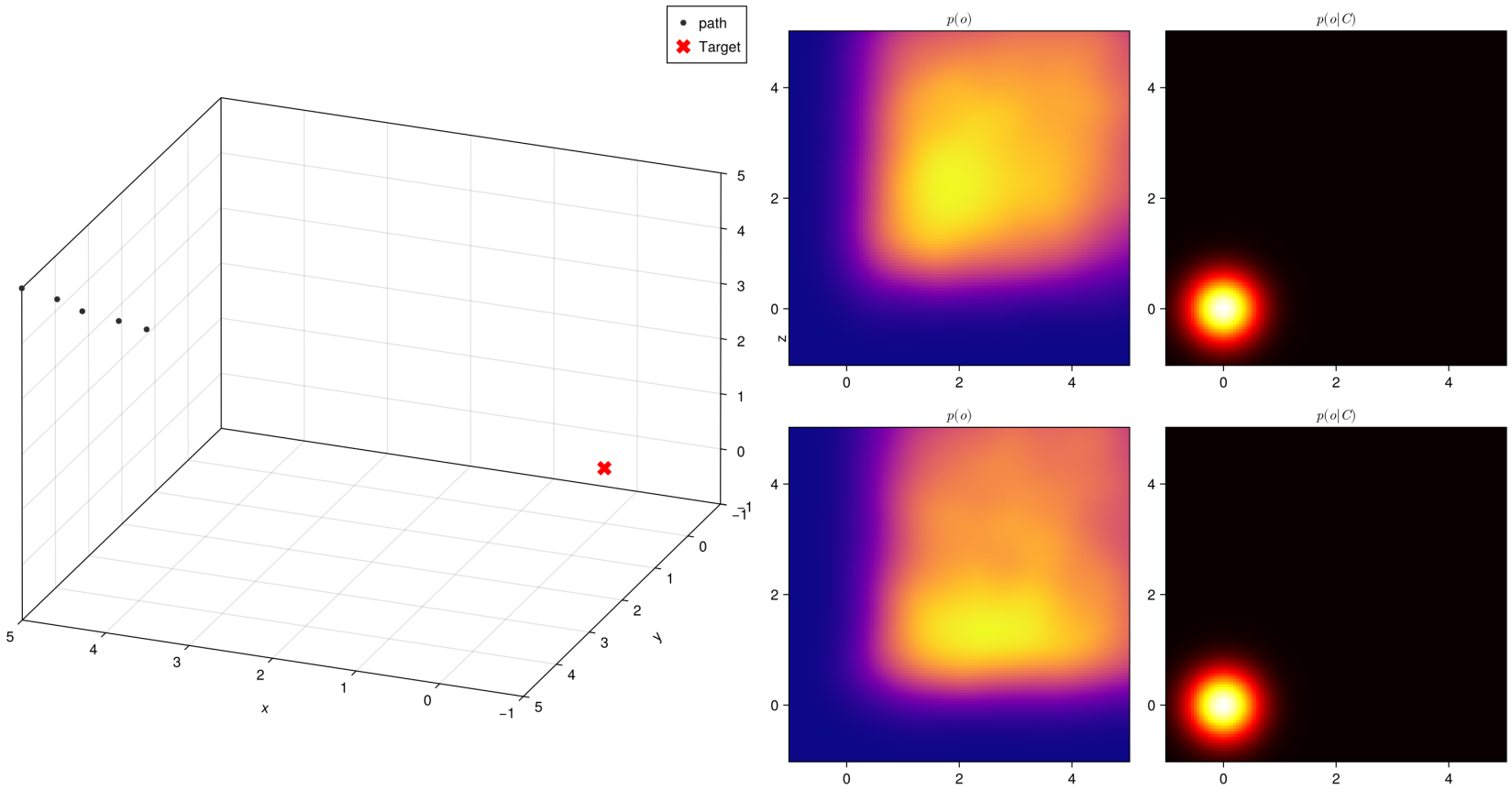


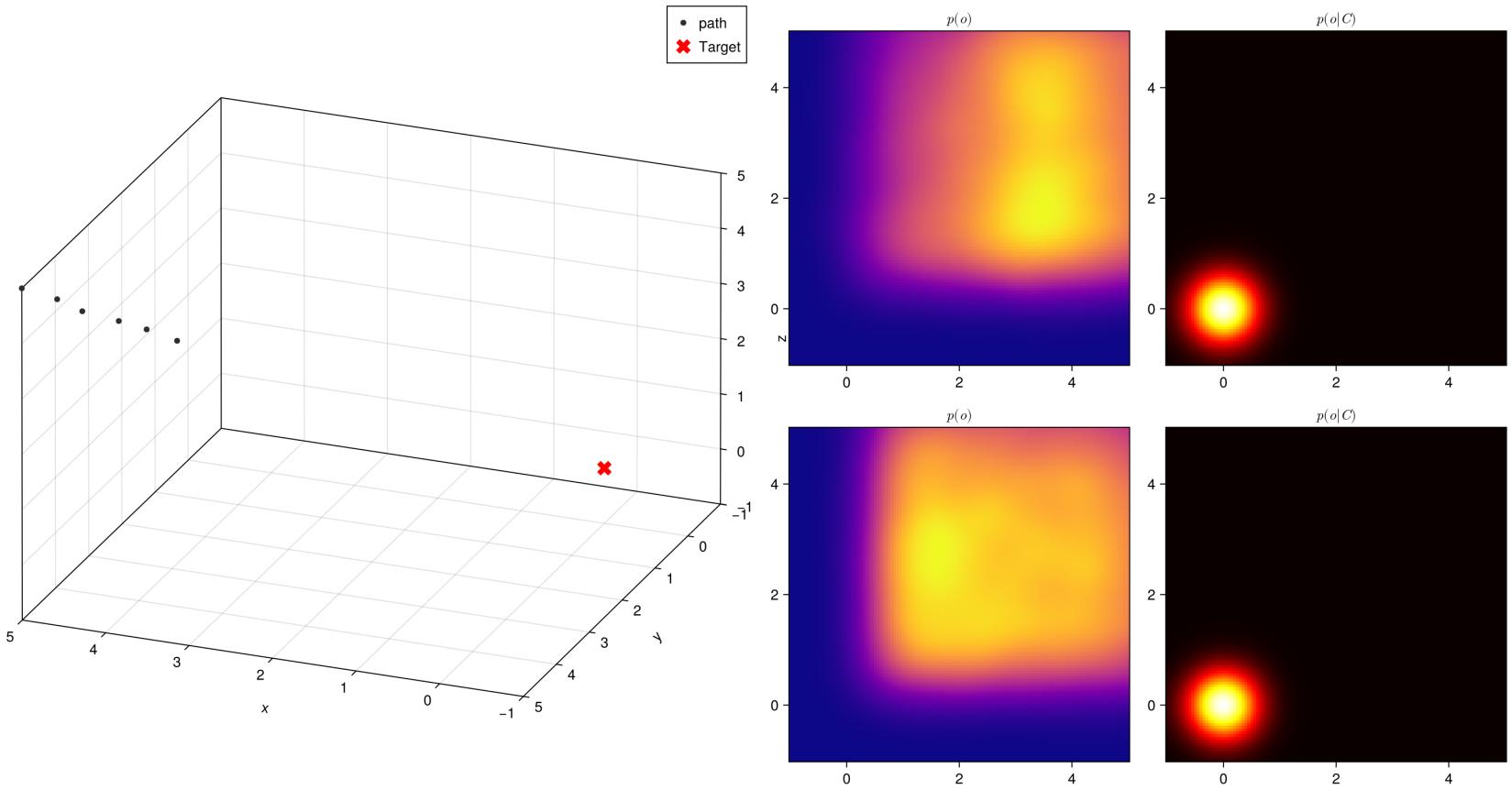
Active Inference

Example: Zero-shot navigation with uncertain dynamics



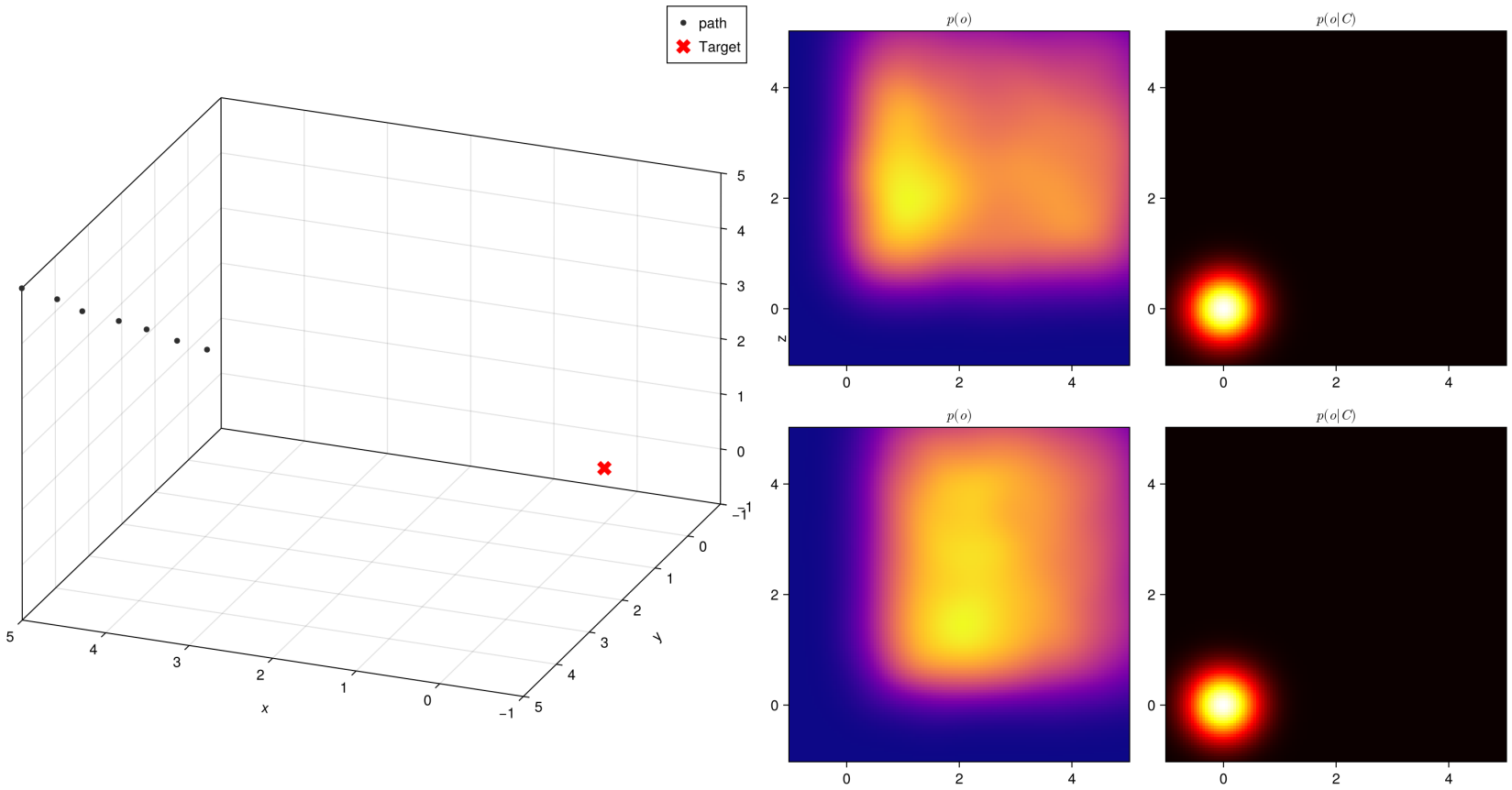
Active Inference

Example: Zero-shot navigation with uncertain dynamics



Active Inference

Example: Zero-shot navigation with uncertain dynamics



Active Inference

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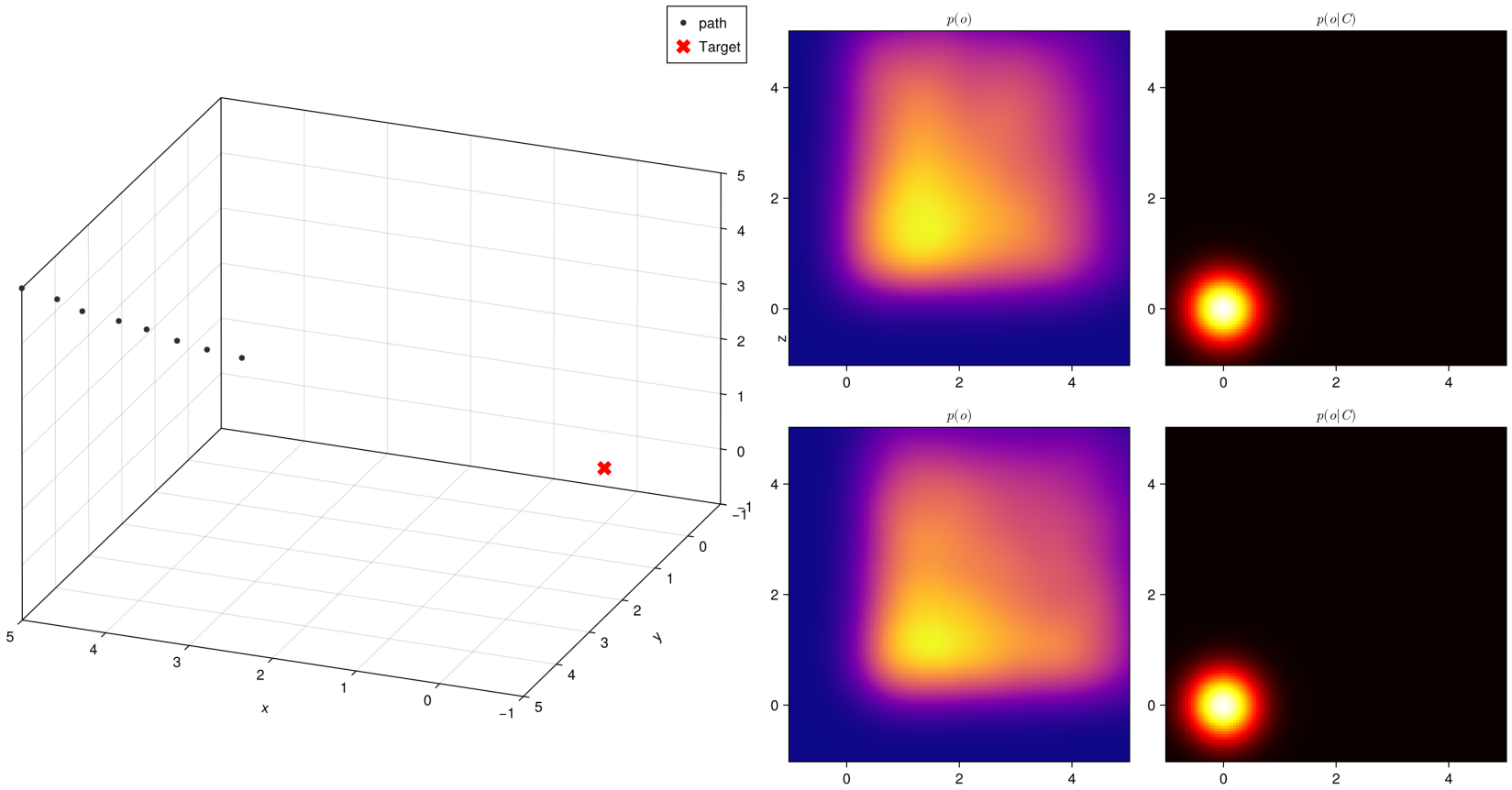
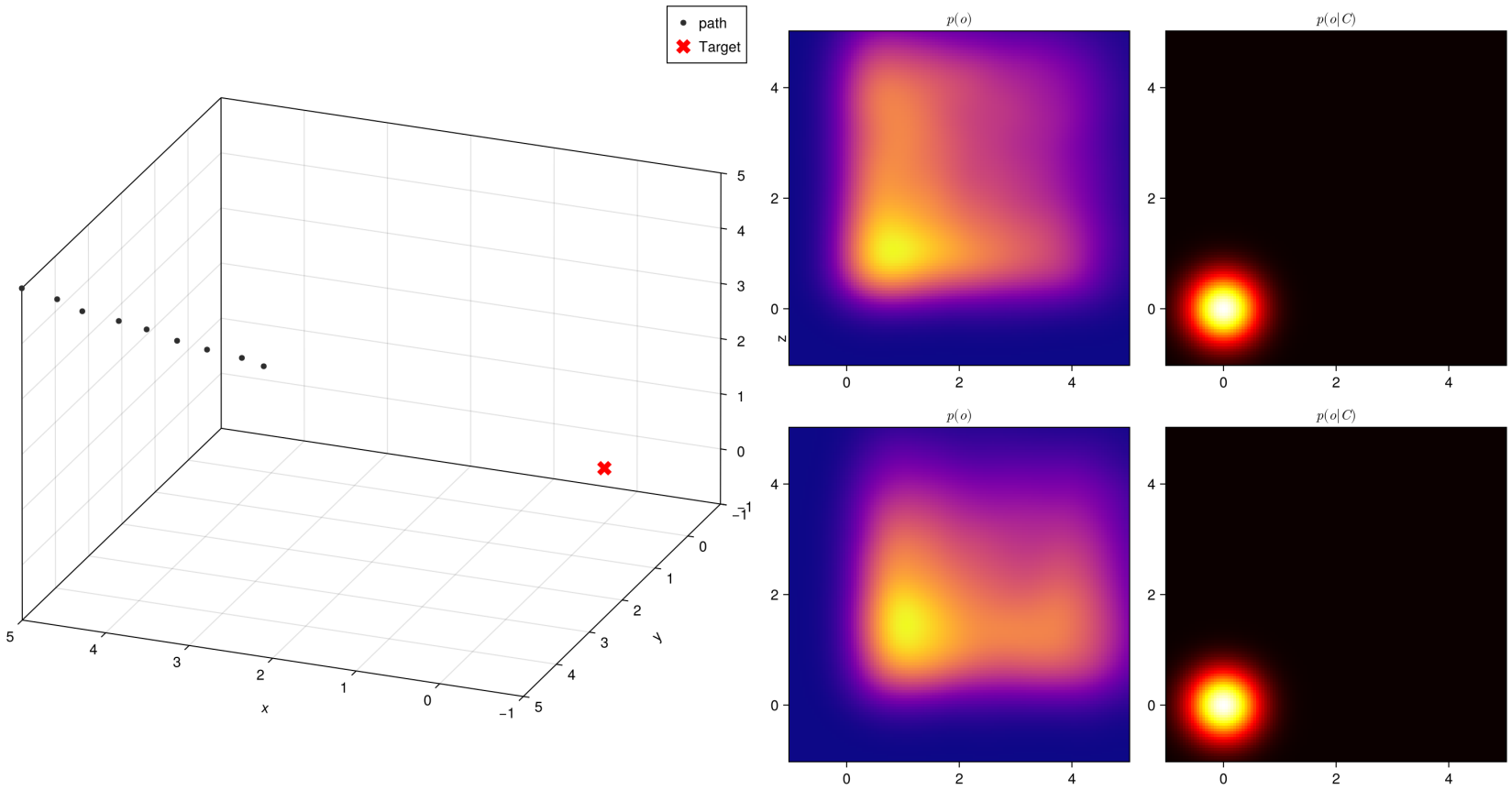


Active Inference

Example: Zero-shot navigation with uncertain dynamics



Active Inference

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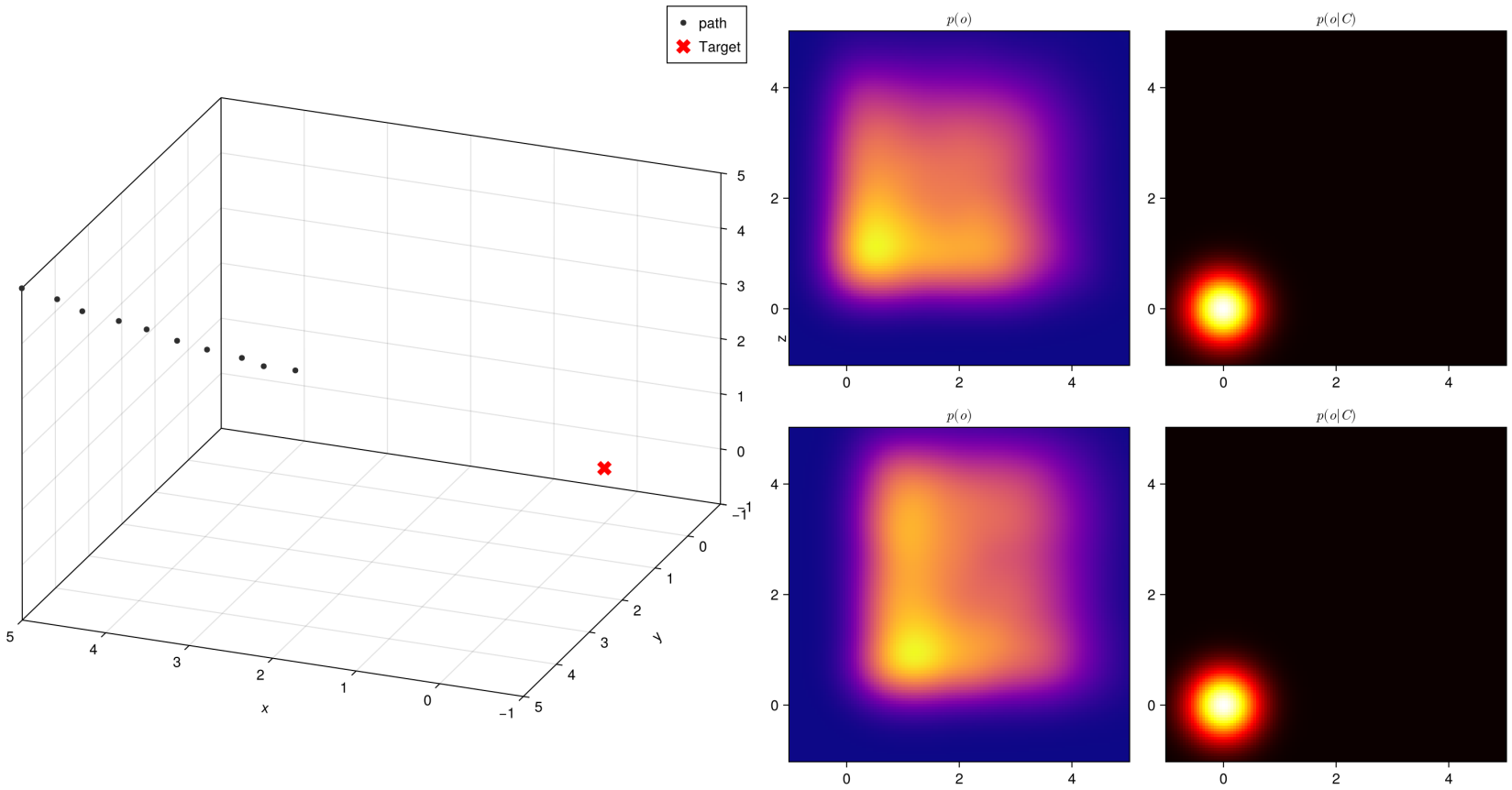
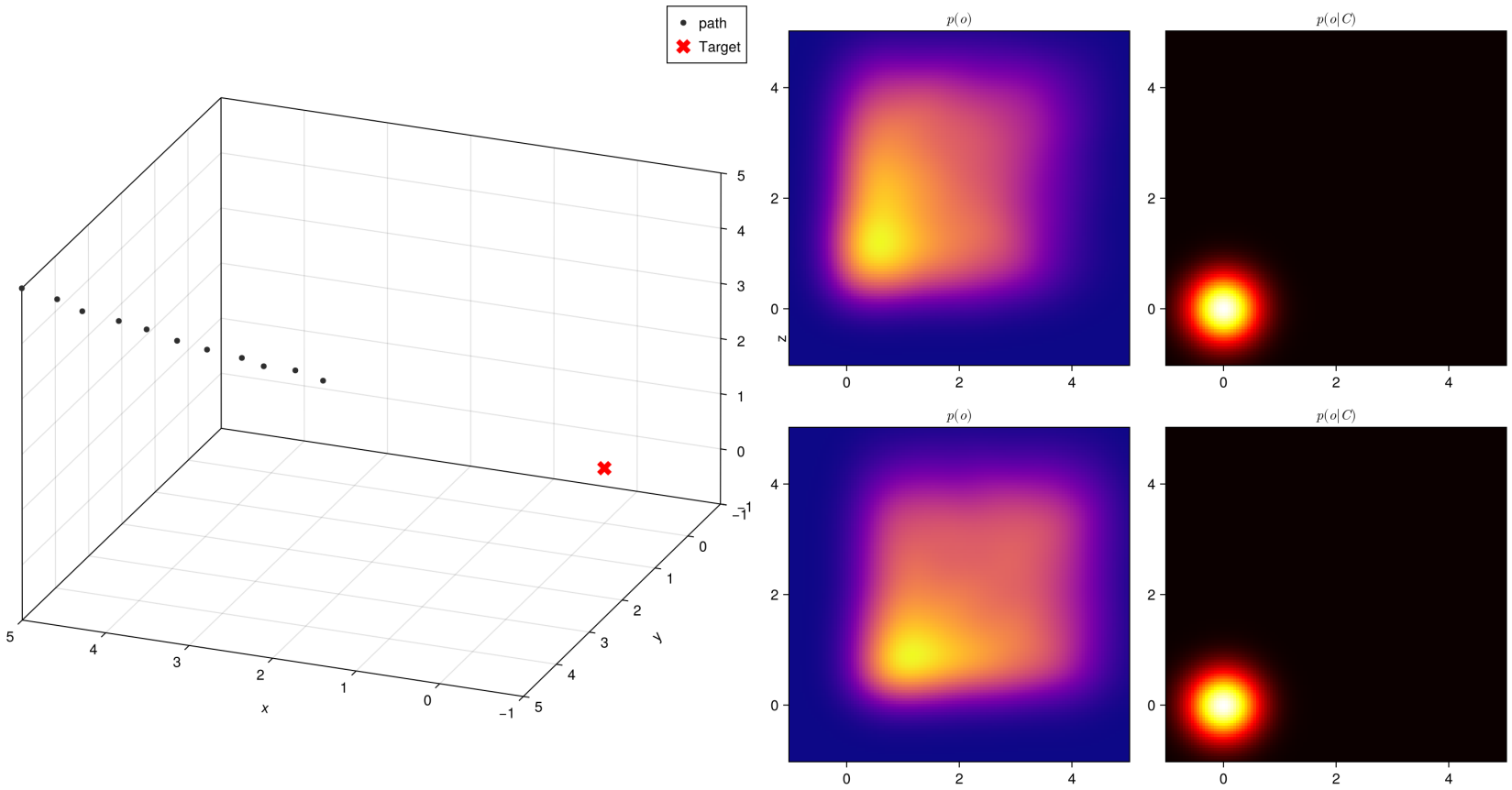


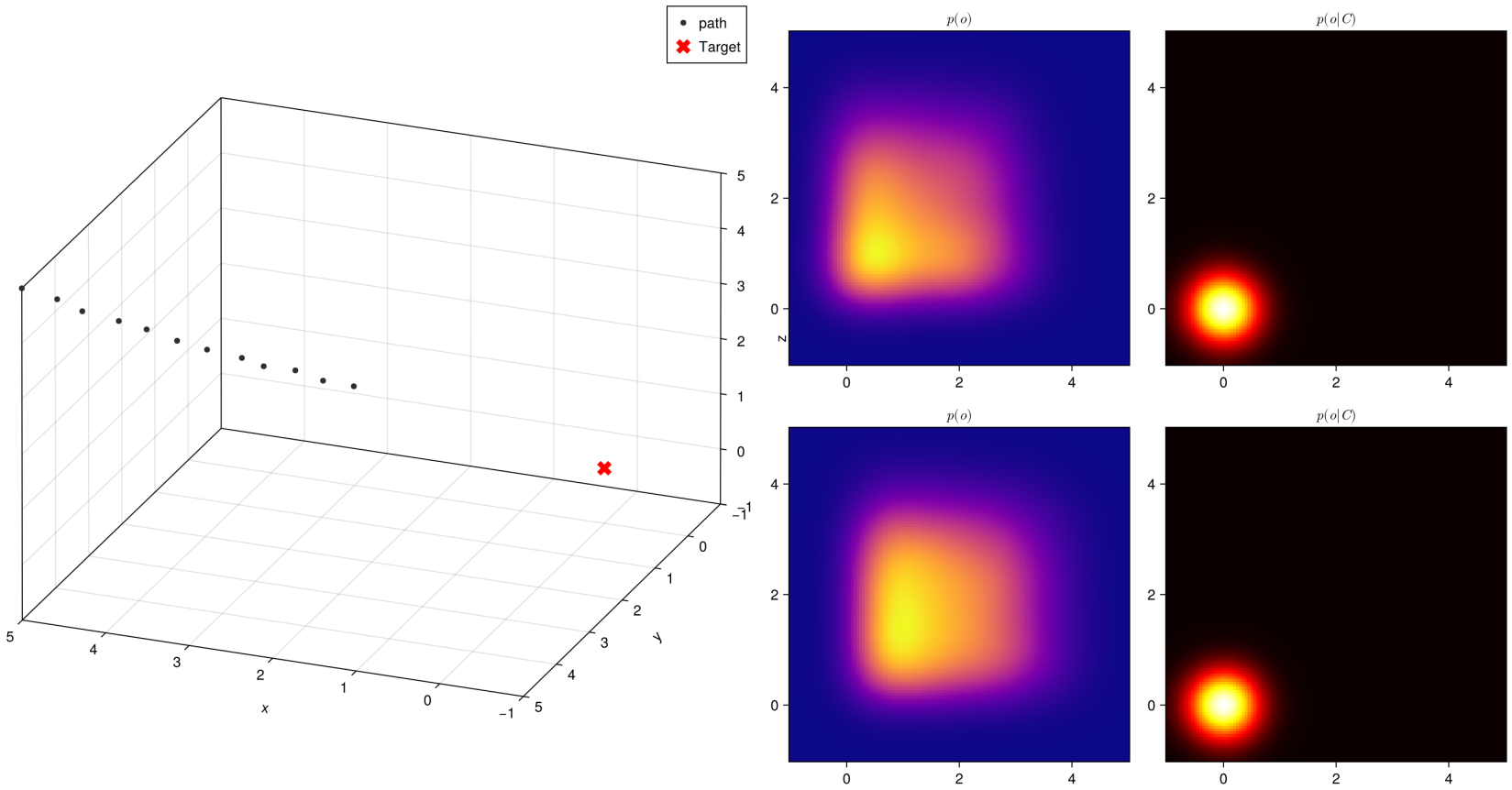
Active Inference

Example: Zero-shot navigation with uncertain dynamics



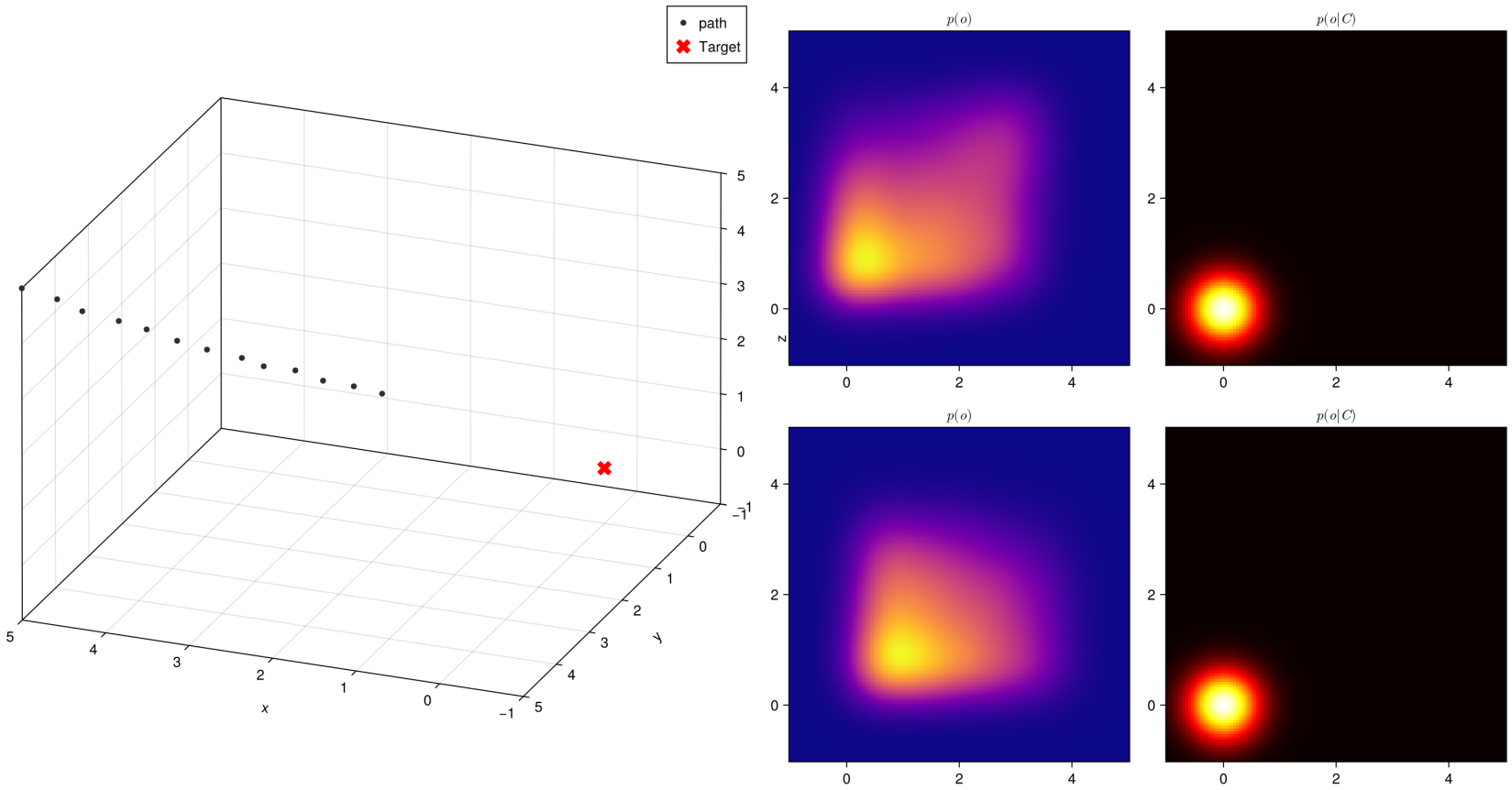
Active Inference

Example: Zero-shot navigation with uncertain dynamics



Active Inference

Example: Zero-shot navigation with uncertain dynamics



Active Inference

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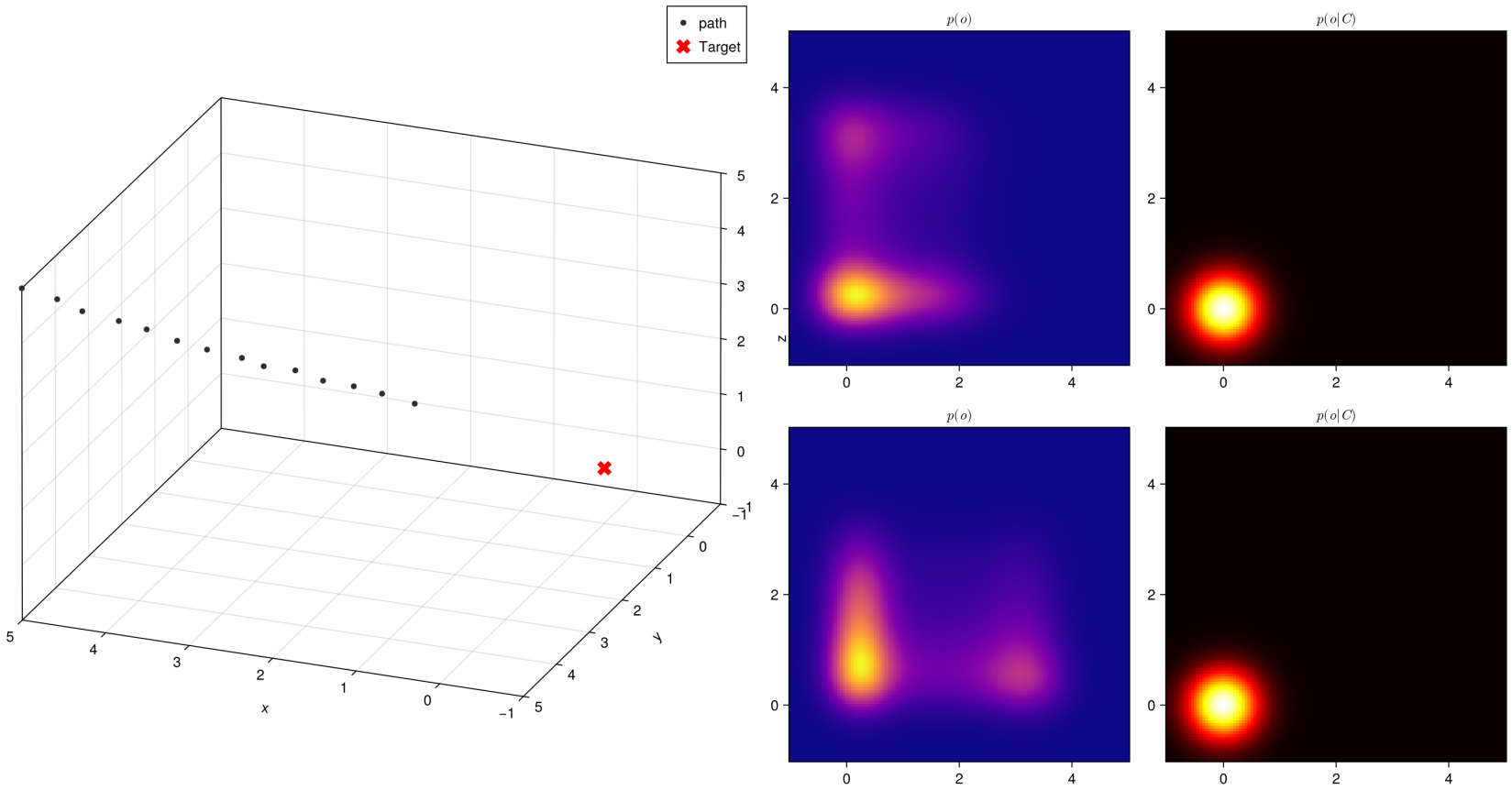
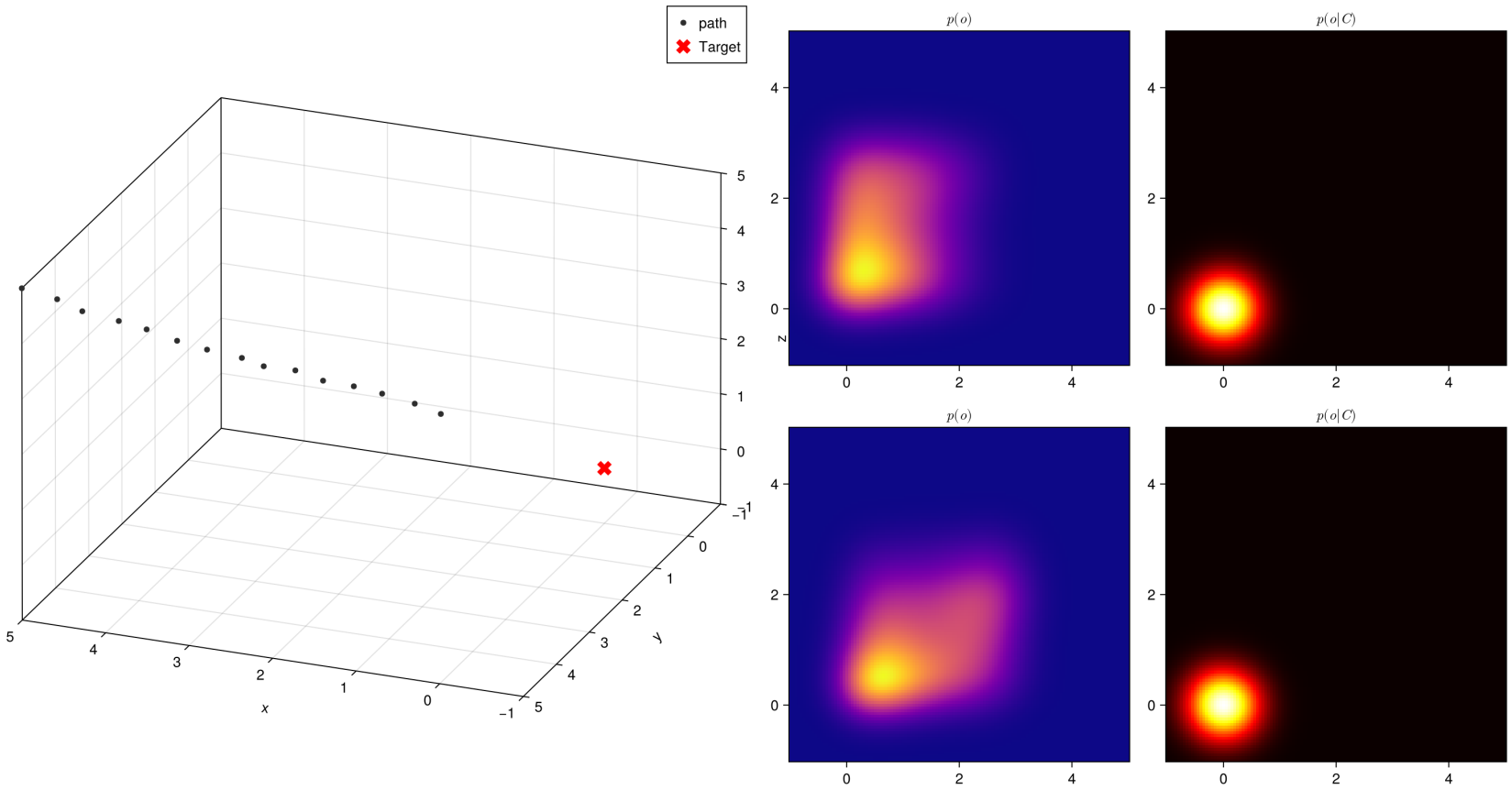


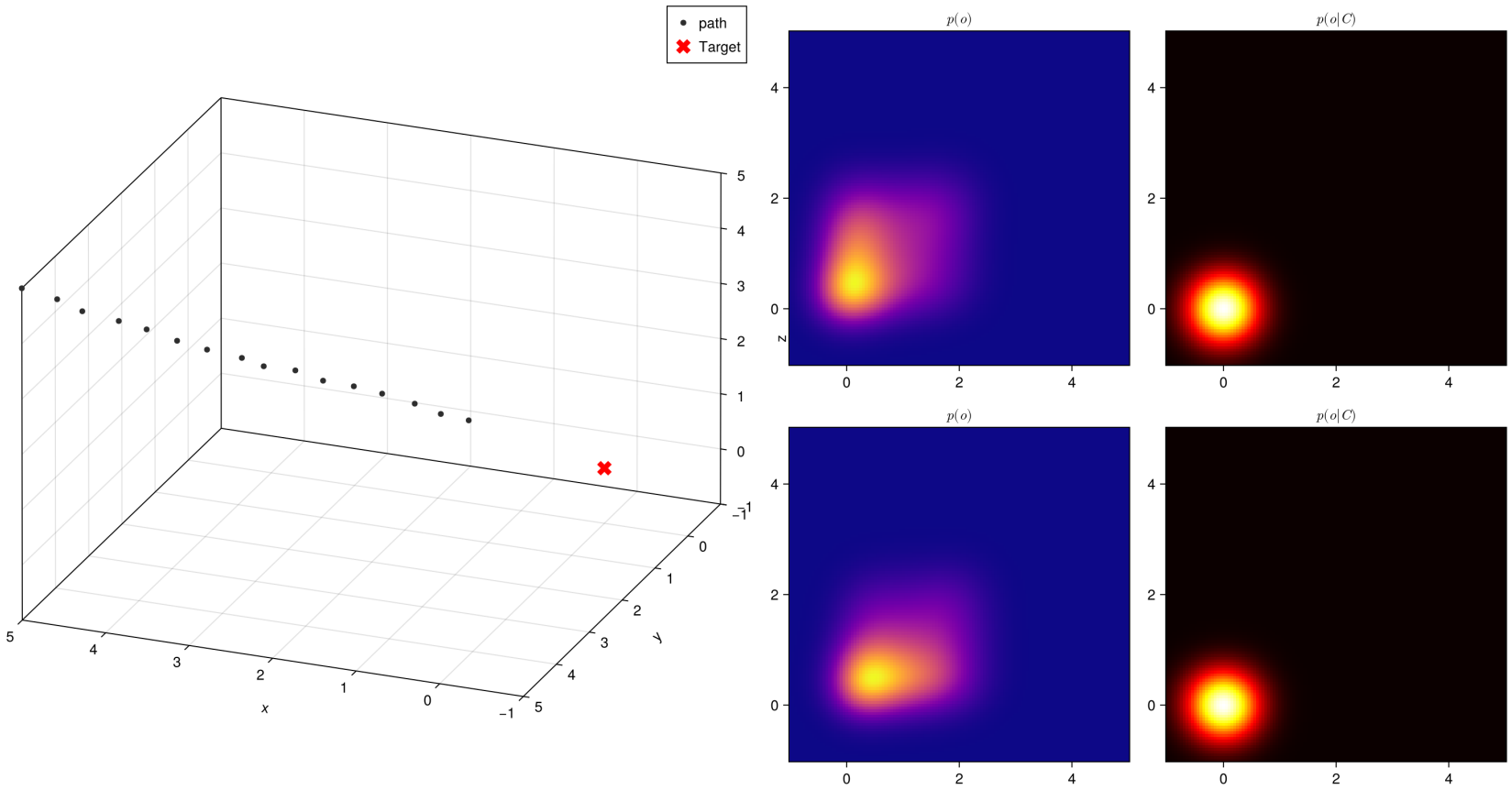
Active Inference

Example: Zero-shot navigation with uncertain dynamics



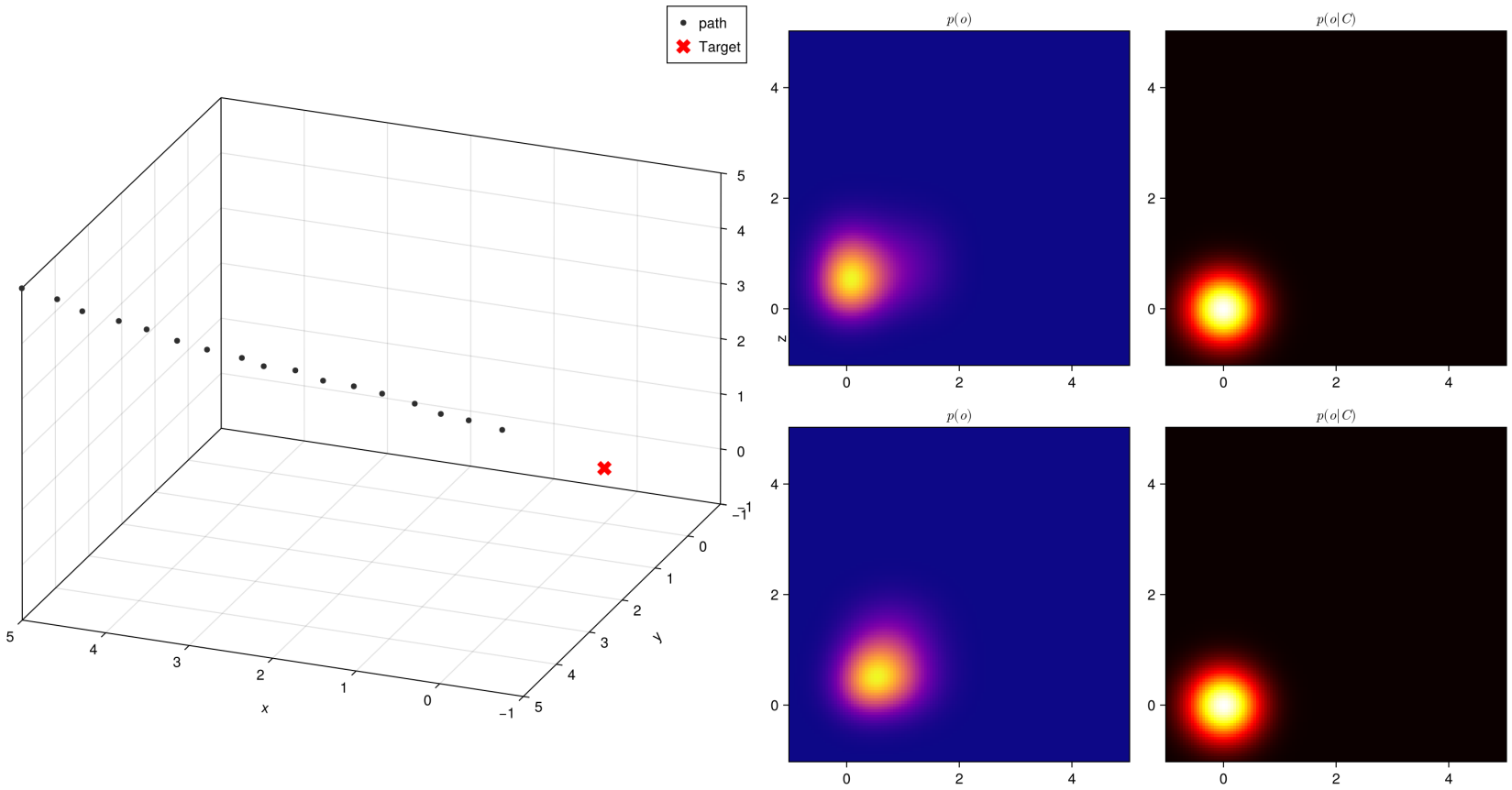
Active Inference

Example: Zero-shot navigation with uncertain dynamics



Active Inference

Example: Zero-shot navigation with uncertain dynamics



Active Inference

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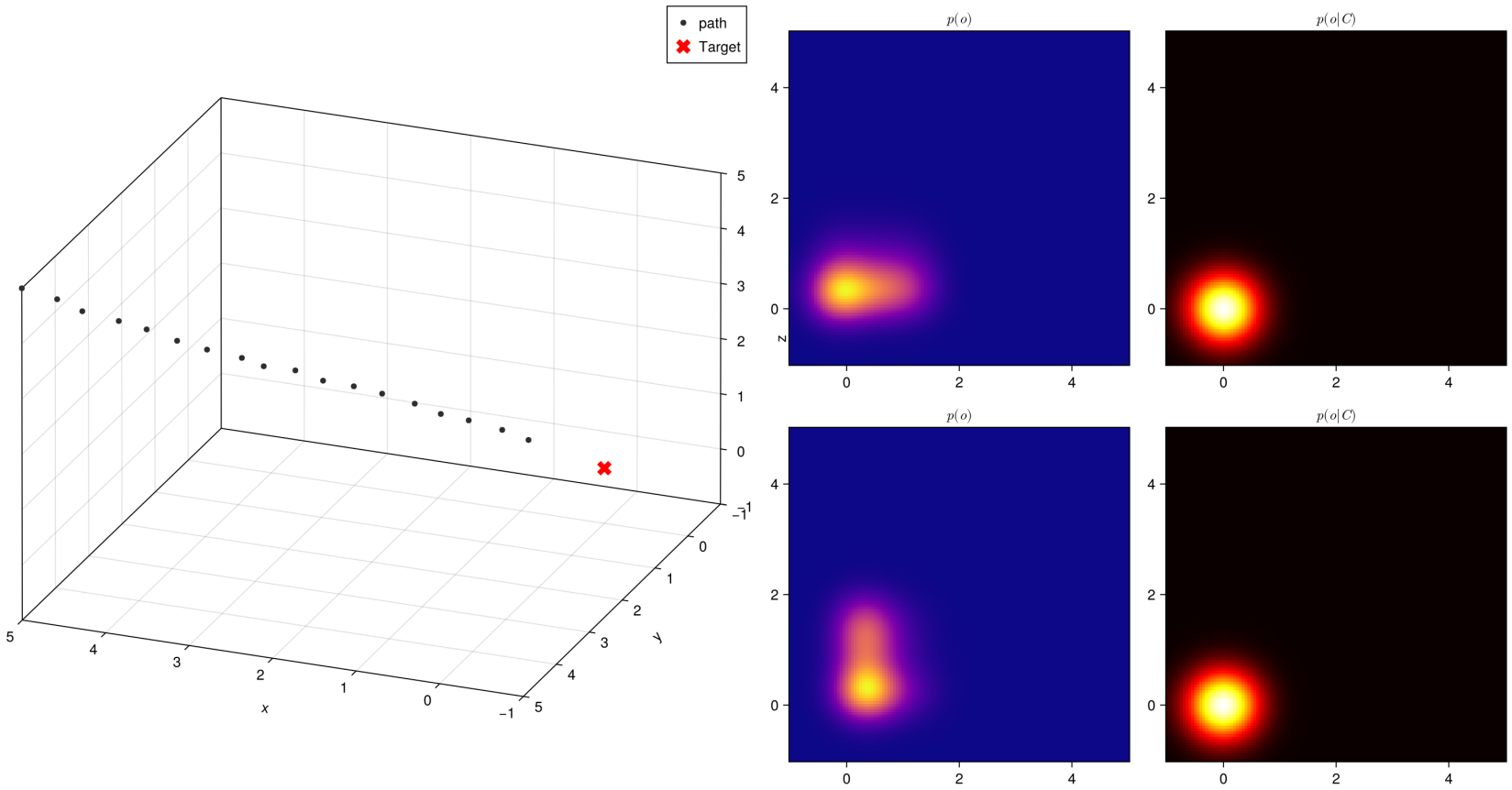

Active Inference

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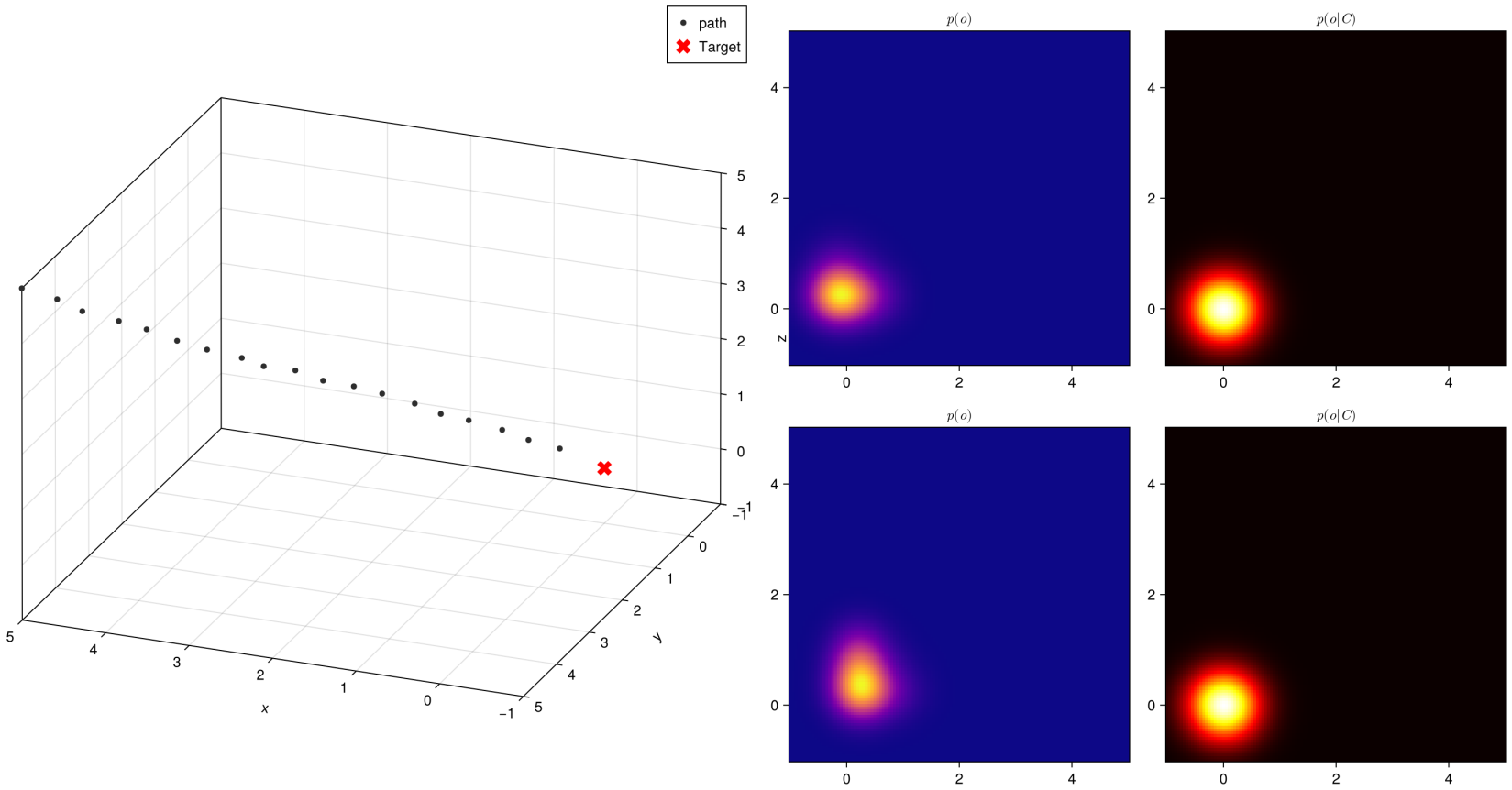

Active Inference

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