A likelihood that incorporates T_{eff} and $\log g$

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Suppose we have estimates of effective temperature and surface gravity for n stars. Following the notation of Stein et al. [2013], denote these estimates $(\hat{\phi}_{T_{\mathrm{eff}},i},\ \hat{\phi}_{\log g,i})$, $i = 1, \ldots, n$. Denote the (known) uncertainties in these estimates $(\sigma_{T_{\mathrm{eff}},i},\ \sigma_{\log g,i})$, $i = 1, \ldots, n$, and suppose that

$$\mathrm{cor}(\hat{\phi}_{T_{\mathrm{eff}},i}, \hat{\phi}_{\log g,i}) = \rho \text{ for all } i,$$

where ρ is also known. (Does ρ vary from star to star? If so, is each ρ_i known? It is not hard to accommodate this in the model.)

Assuming that the estimates $(\hat{\phi}_{T_{\mathrm{eff}},i},\ \hat{\phi}_{\log g,i})$ follow bivariate Gaussian distributions and are independent for different stars, the log likelihood for these observations is

$$\begin{aligned}\text{constant} - \frac{1}{2(1-\rho^2)} \sum_{i=1}^{n} \Bigg\{ &\left(\frac{\hat{\phi}_{T_{\mathrm{eff}},i} - \phi_{T_{\mathrm{eff}},i}}{\sigma_{T_{\mathrm{eff}},i}} \right)^2 + \left(\frac{\hat{\phi}_{\log g,i} - \phi_{\log g,i}}{\sigma_{\log g,i}} \right)^2 \\ &- 2\rho \left(\frac{\hat{\phi}_{T_{\mathrm{eff}},i} - \phi_{T_{\mathrm{eff}},i}}{\sigma_{T_{\mathrm{eff}},i}} \right) \left(\frac{\hat{\phi}_{\log g,i} - \phi_{\log g,i}}{\sigma_{\log g,i}} \right) \Bigg\},\end{aligned}$$

where the true $\phi_{T_{\mathrm{eff}},i}$ and $\phi_{\log g,i}$ for each star depend on the other physical parameters such as mass and age through the computer models (see equations (2)–(4) in Stein et al. [2013]).

This log likelihood can simply be added to the log likelihood for the photometric data to obtain the joint log likelihood for the full dataset.

References

N. M. Stein, D. A. van Dyk, T. von Hippel, S. DeGennaro, E. J. Jeffrey, and W. H. Jefferys. Combining computer models to account for mass loss in stellar evolution. *Statistical Analysis and Data Mining,*, 6:34–52, 2013.