Using output from `sampleWDMass`

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The output from `sampleWDMass` appears in two files, a `.massSamples` file and a `.massSamples.membership` file. The first k columns of the `.massSamples` file are individual draws of the parameter vector $\mathbf{\Theta}$, where k is the dimension of $\mathbf{\Theta}$ and $\mathbf{\Theta}$ includes IFMR parameters, if those are being sampled. The next n_{WD} (n_{WD} = number of white dwarfs in the data set) columns of `.massSamples` are sampled primary masses for each white dwarf. The mass $M_i^{(j)}$ of the ith star in the jth row is sampled from the distribution

$$p(M_i \mid Z_i = 1, \mathbf{\Theta}^{(j)}, \boldsymbol{X}),$$

where $\mathbf{\Theta}^{(j)}$ is the parameter vector in the jth row, $\boldsymbol{X}$ is the observed data, and Z_i is the indicator for whether star i is a cluster member ($Z_i = 1$ if cluster member, $Z_i = 0$ if field star). Thus, we sample a white dwarf mass conditional on that star being a cluster member.

The `.membership` file contains n_{WD} columns. The entry at column i and row j is

$$p_i^{(j)} := \Pr(Z_i = 1 \mid \mathbf{\Theta}^{(j)}, \boldsymbol{X}).$$

Suppose the `.massSamples` and `.membership` files have J rows. We can estimate the conditional posterior expectation of a white dwarf mass, given that it is a cluster member, by weighting the samples $M_i^{(j)}$ by the probabilities $p_i^{(j)}$:

$$\hat{E}(M_i \mid Z_i = 1, \boldsymbol{X}) = \frac{\sum_{j=1}^J p_i^{(j)} M_i^{(j)}}{\sum_{j=1}^J p_i^{(j)}}.$$

This can be viewed as an importance sampling estimator of

$$E(M_i \mid Z_i = 1, \boldsymbol{X}) = E\left\{E(M_i \mid Z_i = 1, \mathbf{\Theta}, \boldsymbol{X}) \mid Z_i = 1, \boldsymbol{X}\right\},$$

using the fact that

$$\frac{p(\mathbf{\Theta}^{(j)} \mid Z_i = 1, \boldsymbol{X})}{p(\mathbf{\Theta}^{(j)} \mid \boldsymbol{X})} = \frac{p_i^{(j)}}{\Pr(Z_i = 1 \mid \boldsymbol{X})} \propto p_i^{(j)}.$$

If we want to estimate quantiles of $p(M_i \mid Z_i = 1, \boldsymbol{X})$, one simple way would be to use Sampling Importance Resampling (SIR). We resample values

$$M_i^{*(1)}, \ldots, M_i^{*(L)}$$

with replacement from $M_i^{(1)}, \ldots, M_i^{(J)}$ with probabilities proportional to $p_i^{(1)}, \ldots, p_i^{(J)}$. We can then use $M_i^{*(1)}, \ldots, M_i^{*(L)}$ directly as approximate draws from $p(M_i \mid Z_i = 1, \boldsymbol{X})$.