Additional mathematical considerations

1 Sample size for the estimation of posterior

The nonparametric estimation of the density function of a probability distribution from an i.i.d. sample is a well studied problem [Goldenshluger and Lepski, 2013]. The mean squared error minimax rate of estimation depends on the smoothness class of the density function. For Nikol'skii classes of function of smoothness s, which generalize Holder spaces for density functions, McDonald [2017] shows that the minimax rate of estimation for the L_2 norm is $\left(\frac{n}{d^d}\right)^{\frac{1}{2s+d}}$. This implies that to maintain a similar quality of estimation for each dimension, the log of the number of samples should behave as $\log(d) * d * (2s + d) - c(2s + d)$, indicating over exponential behavior with the number of dimensions.

References

A. Goldenshluger, O. Lepski, On adaptive minimax density estimation on $\mathbb{R}^d$, Probability Theory and Related Fields 159 (2013) 479–543. URL: https://doi.org/10.1007/s00440-013-0512-1.

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