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Linking urban sprawl and income segregation – Findings from a stylized agent-based model

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Abstract

Urban sprawl and income segregation are two undesired urban patterns that occur during urban development. Empirical studies show that income level and inequality are positively correlated with urban sprawl and income segregation, respectively. However, the relationship between urban sprawl and income segregation is not only rarely investigated but also shows ambiguous empirical results when it is. Therefore, in this study, we built a stylized agent-based model with individual behaviours based on Alonso's bid rent theory and ran simulations with different combinations of income level and income inequality. We measured the overall emergent patterns with indicators for urban sprawl and income segregation. The model confirms the established positive correlations between income level and urban sprawl and between income inequality and segregation. Furthermore, the model shows a negative correlation between urban sprawl and income segregation under free market conditions. The model indicates that without any policy implementation, a city will either suffer from urban sprawl or income segregation. Thus, this study serves as a starting point to study the effects of different urban planning policies on these two urban problems.

Keywords

Agent-based modelling, income segregation, urban sprawl

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Introduction

Urbanization is one of the most striking global changes currently. According to United Nations estimates, two-thirds of the population worldwide will be living in cities by the year 2050 (United Nations, 2014). Some urban problems related to ongoing urbanization such as urban sprawl are associated more with the physical structure of cities, while others, such as spatial segregation of households according to income, are related to social and economic conditions. Due to undesired effects from these two urban problems on city development, city planners aim to eliminate problems such as sprawl and segregation in the city (Torrens, 2008). These two issues, urban sprawl and income segregation, are also considered to be inter-related. For example, Pendall and Carruthers (2003) stated that a fundamental goal of many smart growth efforts is to promote greater socioeconomic equity through more compact development. Interestingly, from an empirical point of view, the interaction between these two phenomena is not clear; the literature provides contradictory empirical findings on how sprawl and segregation are related (Jargowsky, 2002; Wheeler, 2006). These differing findings motivated us to further explore this relationship.

State of research in urban sprawl and income segregation

Urban sprawl has become a subject of particular interest among planners and policy makers. The simple view is that urban sprawl involves low-density residential developments (Frenkel and Ashkenazi, 2007) over a large area. The negative impacts of urban sprawl include a lack of scale economies, enlarged traffic volume, loss of open space and thus damage to ecosystems (Dieleman and Wegener, 2004; Frenkel and Ashkenazi, 2007). Indicators to describe urban sprawl include the total residential area and the population density gradient, which describe the decline of the population density from the centre to the outskirts.

Income segregation is the uneven spatial distribution of income groups within a certain area. The negative impacts of income segregation include increased economic disadvantages of low-income families when compared to high-income families. For example, income segregation concentrates political power and influence in a small number of local areas (Bischoff and Reardon, 2013). The uneven distribution of collective goods, for instance, has more influence on children than on adults because these groups spend more time in the neighbourhood, and they need more crucial public amenities, such as schools, parks, and so on (Bischoff and Reardon, 2013). Comparing the income structure of each district in the city with the overall income structure of the city is the most common approach to measure income segregation. The Neighbourhood Sorting Index (Jargowsky, 1996) and Centile Gap Index (Watson, 2009) are examples of these indicators.

Empirical evidence suggests a positive relationship between income level and urban sprawl. From the view of urban-economics, urban sizes are the result of orderly market equilibrium and rising levels of income increase housing demand, which leads to a larger city and hence urban sprawl (Brueckner and Fansler 1983). For instance, Brueckner and Fansler (1983) found a positive contribution of mean income to urban area in 40 US urbanized areas in the 1970s. Margo (1992) suggested that as much as half of the increase in suburbanization between 1950 and 1980 can be explained by rising incomes. Bhatta et al. (2010) concluded that the expansion of the economic base (including increases in income) encourages urban sprawl. Additionally, theoretical considerations based on the model suggested by Alonso (1964) and later expanded by Muth (1969) and Mills (1972) concerned with urban land use and market land prices confirm this relationship. Modelling shows that market processes lead to increasing city size with rising income levels, where each household takes its housing location, rent, transport cost and composite good into account and maximizes its utility (Fujita, 1989).

The relationship between income inequality and income segregation has also received scientific attention. Tiebout (1956) provided a possible theoretical explanation for the contribution of income distribution to income segregation: As inequality increases, rich and poor households are less likely to have the same willingness to pay for a given set of neighbourhood amenities (Watson, 2009). Three recent studies provided empirical evidence that increasing inequality contributes to increasing income segregation. Watson (2009) analysed the change in income inequality and segregation between 1970 and 2000 for 216 Metropolitan Areas in the US and found that an increase in inequality by one standard deviation drove up income segregation by 0.4 to 0.9 standard deviations. Reardon and Bischoff (2011) estimated that, over a 30-year period, growing inequality accounted for approximately 60% of the increase in income segregation across the 100 largest metro areas in the US. Scarpa (2014) confirmed the same cause of increasing segregation based on data for Malmö, Sweden. The studies based on US cities found that the connection between rising inequality and increasing income segregation is largely driven by the fact that families at the top of the income distribution are geographically separating themselves as they become more affluent.

Interestingly, however, there is minimal literature directly relating income segregation and urban sprawl. Moreover, these studies report ambiguous empirical results. Wheeler (2006) stated that little evidence supports the idea that urban sprawl is systematically associated with greater income segregation in American metropolitan areas. Jargowsky (2002) argued that sprawl and central city decline are both a manifestation of a metropolitan development process that leads to a higher level of economic segregation and provides empirical evidence for US metropolitan areas. In contrast, Glaeser and Kahn (2004) stated that urban sprawl may reduce income segregation in car-based edge cities. They found a negative correlation between urban growth and level of income segregation in US cities. Because of these contradictory findings, we further investigate the relationship between income segregation and urban sprawl in the remainder of this paper.

Overview of related agent-based models (ABMs)

ABM is a modelling method that considers preferences and behavioural rules of individual agents to explore the effects of the interactions between agents within a particular environment over time (Gilbert, 2004). As a bottom-up approach, ABM has an advantage and is popular in urban residential mobility research. Compared to other modelling methods that uniformly apply aggregated rules across system components, ABMs model individuals and their behaviours in a more direct way (Joshi et al., 2006; Yin and Muller, 2008). Due to their ability to represent an individual's decision-making process and interaction from the bottom up, ABMs have become popular in the field of urban land-use-change simulation (Huang et al., 2014). Constructing stylized models based on classical theories together with data-driven models and models combining theories and empirical findings are common approaches for urban ABM (Filatova et al., 2009; Lemoy et al., 2013). The monocentric urban model of Alonso (1964) with its assumption of a free market is one of the classical theories that is often applied in urban simulation (Parker and Filatova, 2008). Building upon Alonso's basic assumptions about location choice, existing models following this approach cover land market dynamics (Filatova et al., 2009), the effect of spatial policies (Kim and Batty, 2011), or extensions to the monocentric model (Lemoy et al., 2013). The family of simulation models building on Schelling's approach (Benenson and Hatna, 2011; Crooks, 2010; Torrens, 2007) is based on different population groups and the effects of preferences for living close to one's own group. Spielman and Harrison (2014) investigated the effects of the built environment on segregation in such a framework and found a positive relationship between sprawl and segregation. These classic-model-based ABMs belong to the family of stylized models following the "Keep It Simple Stupid" (KISS) approach rather than the "Keep it Descriptive Stupid" (KIDS) approach (Edmonds and Moss, 2005). We chose the KISS approach as it has the advantage of being simple and straight-forward and also the potential to produce generalized results.

Aim and organisation of the study

In this study, we explore what the relationship between urban sprawl and income segregation. To this end, we developed a stylized monocentric urban ABM based on Alonso's theory, which means that we do not include preferences regarding living close to specific types of agents as would be the case in the Schelling framework. Thus, all segregation patterns emerging in our model emerge from location choices solely due to prices and availability of living space. We describe this model in section "Model description". The indicators to evaluate model outputs are given in section "Output evaluation". The setup of our simulation experiments is summarised in section "Simulation experiments", and section "Overview of simulation runs" provides an overview of the simulation runs. To validate our stylized model, we used observed empirical patterns of a system, which is a suggested method for validating emergent phenomena (Grimm et al., 2005). More specifically, we use positive correlations between income level and urban sprawl (section "Validation 1: Income level and sprawl") as well as between income inequality and income segregation (section "Validation 2: Income inequality and income segregation") to validate our model. Section "The relationship between sprawl and income segregation" provides the results for our main research question, namely the relationship between urban sprawl and income segregation, and the results of a sensitivity analysis are summarised in section "Sensitivity analysis". The results are discussed in section "Discussion", and conclusions are drawn in section "Conclusion".

Methods

Model description

Overview. [1] In this model, agents represent households who use their income for housing (rent), transportation and composite goods without saving. The income of households is generated randomly during initialization (section "Initialization of household income") with different income and inequality levels. The city is represented in a spatially explicit way and follows a monocentric structure, as in Alonso's model (section "The city landscape"). During the course of a simulation, households re-locate within the city to maximize their utility (section "Households' decision making"). Simple rules mimic market dynamic and adjust land rents according to supply and demand (section "Update of land rents"). The overall schematic of the model is given in Figure 1, and initialisation values are summarised in section "Simulation experiments". Time is modelled as discrete steps.

The city landscape. A 25×25 grid represents the artificial urban landscape in the model. The centre is located in the middle patch. The artificial city is divided into 25 districts (5 by 5 grid cells per district, see Figure 2). Except for the central business district (CBD) patch, all other patches provide 100 units of living area for households (see also section "Simulation experiments" for the relationship of household number and city size). In the monocentric

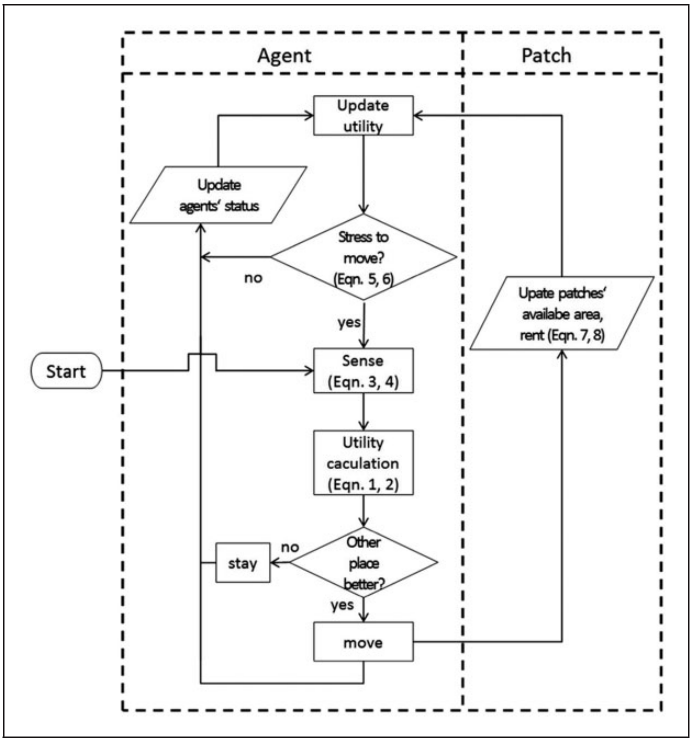

Figure 1. Flowchart of the model.

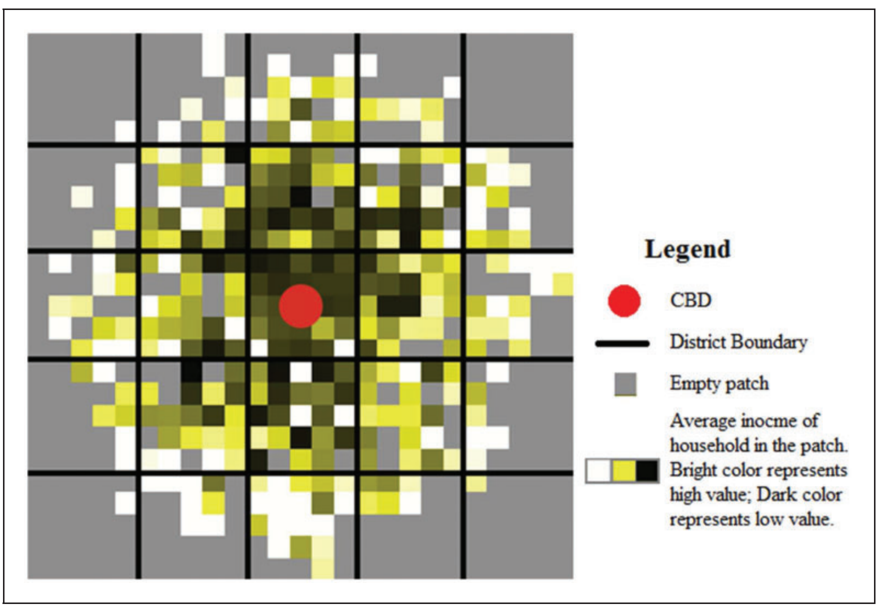

Figure 2. Average income per patch in one exemplary simulation run for mean income 1200 and Gini coefficient 0.3 (for other parameters see section "Simulation experiments").

model, all household activities, e.g. working, shopping, are assumed to take place in the CBD. Households occupy a certain amount of the living area in the patch. There is no fixed-area building entity in the landscape. Therefore, households can flexibly take the amount of living area as needed. In contrast to most other conceptual models following Alonso, in this model a patch can accommodate more than one household. This allows for a variability of population density, both per patch and per district. The rent of a patch changes in the simulation but is never lower than the initial rent (section "Update of land rents").

Initialization of household income. We applied a generalized beta distribution, which is widely used as a distribution of income (McDonald, 1984; Thurow, 1970), to initialize household income. We varied the level of income (average income) and inequality (measured with the Gini coefficient) for different simulations. The income of one household in a single simulation is constant.

Households' decision making. Following Alonso's model, the households face a trade-off between living area, location and composite goods. The composite goods can be seen as the income remaining after rent and transportation costs, which is assumed to be spent in a way that directly "creates utility" for the household. We use a utility function based on a Cobb–Douglas function for composite goods and living area to realize this trade-off (equation (1)) (Kim and Batty, 2011). It is constrained by the income of the households, which is spent on housing (renting), transportation and composite goods at each time step (equation (2)). The transportation cost is calculated by the distance to CBD times the travel cost per distance unit, which is constant throughout the simulation. The preference of households regarding income spent on the living area or composite goods is controlled by the weight in equation (1). Because that preference is not the focal point of this study, we set it to 0.5 as an unweighted default.

$$Utility = composite\ goods^{\alpha} \times living\ area^{\beta},\ with\ \alpha + \beta = 1 \tag{1}$$

$$income = living\ area \times rent + distance\ to\ CBD \times transport\ cost\ perunit + composite\ goods \tag{2}$$

At the initial stage, households are not located anywhere. The aim of households in the simulation is to find a location to maximize their utility. Households make decisions and move based on the order of their income from high to low at each time step, reflecting a better access to the housing market, and higher probability of success to obtain housing of more wealthy people. In each step, households search only 10 random locations to represent the limited time and resource that one has in the search process. Households calculate their potential utility for each candidate location. Households can take a flexible amount of living area depending on their demand. The optimum living area (OLA) is the area that the household should take from a specific patch to have the highest utility in the candidate patch (equation (3)). If there is not enough area in the candidate patch for the OLA, the household will consider taking all the area that is still available in the patch (available living area, ALA), which provides them the highest utility in that patch in the current occupancy situation (equation (4)). We defined a minimum living unit for a household in the model to prevent a household settling on an unrealistic small living area (Table 1). Households compare the potential utilities in candidate locations and their current location and make decisions either to relocate to the new location or to stay at the current location. After all households finish their decisions and movements, the rent of the patch will be updated by a

Table 1. Default values in the model.

Variable name	Value
Number of households	500
Preference to other goods (α)	0.5
Preference to area (β)	0.5
Living units per patch	100
Minimum living unit for a household	10
Transport costs per unit	10
Initial and minimum rent per living unit	10

simple rule explained in section "Update of land rents", and the household's current utility will be adjusted accordingly.

$$OLA = (1 - \alpha) \times \frac{income - transport\ cost\ per\ unit * distance\ to\ CBD}{rent} \tag{3}$$

$$living\ area = \begin{cases} OLA, & OLA \leq ALA \\ ALA, & OLA > ALA \end{cases} \tag{4}$$

In all consecutive time steps, the residential mobility decision of households follows a two-step approach (Brown and Moore, 1970). This means that the households first evaluate the stress of moving and then start their search and relocation process. The stress stems from the individual evaluation of the current utility and income, which we call satisfaction level and is compared to the rest of the city. It is calculated as the ratio of the difference of their current utility and income and the difference between the minimum utility and income in the city (equation (5)). Compared to simply calculating the ratio of utility and income, this approach also accounts for the overall range of satisfaction levels in the city and depicts a higher sensitivity of satisfaction at the lower end of the income spectrum, which aims to reflect the often more urgently required change of housing for such households. The households compare their individual satisfaction level with the overall satisfaction level in the city (equation (6)). The idea behind this comparison is to measure whether households acquire proper utility based on their income. If a household's individual level is lower than the average, the household does not optimally use its income. Therefore, the household is under stress and starts searching for a new location to maximize its satisfaction level.

$$\text{Individual satisfaction level} = \frac{\text{individual utility} - \text{minimum utility of households}}{\text{individual income} - \text{minimum income of households}} \tag{5}$$

$$\text{Overall satisfaction level} = \frac{\text{median utility of households} - \text{minimum utlity of households}}{\text{median income of households} - \text{minimum income of households}} \tag{6}$$

As a consequence, households do not compete directly in the sense of bidding for specific patches or OLA. However, as the rent changes based on the occupation rate of the patch and its surrounding patches, households compete indirectly, as they can spend different units of income.

Update of land rents. A series of simple rules mimics the market dynamics and the basic relationship between supply and demand. The rent of a patch is influenced by its own and its Moore neighbourhood patches' occupancy rates, which are summarized in the rent index (equation (7)). The rent of the patch changes according to the value of the rent index of the patch. If a patch and its neighbouring patches have a high occupancy rate, the rent index of this patch will be high. In this case, we assume that this patch is desirable among households, and the rent of this patch will increase in the next step. When the rent index of a patch is larger than 0.9, the rent of this patch will increase by 10%. When the rent index is smaller than 0.5, the rent will decrease by 10% (but is not allowed to be below the initial and minimum rent). These values are chosen to allow the model to behave in line with basic market mechanisms.

$$Rent\ index = 0.3 \times \frac{occupied\ area\ in\ neighborhood}{total\ area\ in\ neighborhood} + 0.7 \times \frac{occupied\ area\ in\ the\ patch}{total\ area\ in\ the\ patch} \tag{7}$$

$$Rent\ of\ patch\ (t+1) = \begin{cases} 110\% * rent\ per\ living\ unit\ (t), rent\ index > 0.9 \\ rent\ per\ living\ unit\ (t), 0.5 \leq rent\ index \leq 0.9 \\ 90\% * rent\ per\ living\ unit\ (t), rent\ index < 0.5 \end{cases} \tag{8}$$

In equation (8), t represents a time step.

Output evaluation

Urban sprawl

Urban sprawl as an emergent property of the system is measured using two indicators, namely residential area and the population density gradient. These two indicators could respectively indicate urban sprawl from the physical structure and the socio-economic aspect.

Residential area. The residential area is the most direct indicator of urban sprawl. A high value of residential area indicates serious urban sprawl. Because we do not change total population number in the simulation, the residential area is also equivalent to the net density, which is calculated by the population divided by the residential area.

Population density gradient. The other common indicator of urban sprawl in the literature is the population density gradient (Ewing et al., 2002; Torrens, 2008). It describes the spatial distribution of the population within an urban area, often with a population density decreases from the city centre to the outer regions. The most commonly used model to describe the density gradient is the negative exponential function (equation (9)) introduced by Clark (1951). Gradient values are commonly calculated by fitting a linear model of the logarithm of density against distance to the CBD using ordinary least-squares techniques (Mills and Tan, 1980).

$$Population\ density\ (distance\ to\ CBD) = Ae^{-b \times distance\ to\ CBD} \tag{9}$$

where A is the hypothetical density in the city centre, and b is the density gradient.

Income segregation. Income segregation needs to be measured along a continuous dimension for income. Therefore, indicators for racial segregation, such as the index of dissimilarity, cannot be applied simply (Watson, 2009). There are some indicators specifically designed to measure the income segregation. Here, the outputs of our ABM are evaluated using the neighbourhood sorting index (Jargowsky, 1996) and the centile gap index (Watson, 2009).

By applying both indicators, we are able to compare the results to previous findings and estimate each indicator's usability.

Neighbourhood Sorting Index (NSI). The Neighbourhood Sorting Index (NSI), a measure of overall income segregation across districts, was developed by Jargowsky (1996). The NSI is the ratio of the square root of the between-district income variance to the total income variance (equation (10)). Jargowsky (1996) mentioned that the method "*implicitly controls for both the overall income level because it is based on deviations from the mean household income, and overall income inequality, since it is expressed as a percentage of total income variance*." A limitation of the NSI in regard to empirical data is that it requires estimating the total variance of income in a city, which is generally not available. NSI ranges from 0 to 1, where 0 means perfect integration and 1 means perfect segregation.

$$NSI = \frac{\sigma_N}{\sigma_H} = \frac{\sqrt{\frac{\sum_{n=1}^{N} h_n \times \left(\overline{income_n} - \overline{income}\right)^2}{H}}}{\sqrt{\frac{\sum_{i=1}^{H} \left(income_i - \overline{income}\right)^2}{H}}} \tag{10}$$

where i is the household's index, n is the index of neighbourhood, h_n is the number of households in neighborhood n, and H and N are the total number of households and neighbourhoods, respectively.

Centile Gap Index (CGI). The Centile Gap Index (CGI) was developed by Watson (2009) and is based on the spatial distribution of households by income percentile. It represents the deviation of average income from the median for each district compared to that which would be achieved under perfect integration. Under perfect integration, the CGI equals 0. In contrast, a completely segregated city would consist of homogenous neighbourhoods, which would lead to a CGI of 1. Similarly to the NSI, the CGI is not related to the shape of the income distribution.

$$CGI = \left(0.25 - \frac{1}{H}\sum_{i} |P_i - P_{medti}|\right) \Big/ 0.25 \tag{11}$$

where i is the household's index, H is the number of households in the metropolitan area, P_i is the estimated percentile in the metropolitan area m of the income distribution of household i, and P_{medti} is the estimated income percentile of median households in the tract of household i.

Simulation experiments

We varied income level and income inequality in 275 simulations. These simulations include combinations of different mean income (from 700 to 1700, with intervals of 100) and the Gini coefficient (from 0.1 to 0.5, with intervals of 0.1) with five replicas for each combination. The range of the Gini coefficient here represents the range found in empirical studies. The lower boundary of the hypothetical income was set to 700 to avoid societies without a middle class for a high inequality level (Gini equal to 0.5) and low mean income (e.g. 600). The upper boundary of 1700 was set because higher mean incomes always lead to the pattern that all households occupy an individual patch. Each simulation was run for 500 households. We chose the number of households because of the balance of computation time and meaningful outputs. A city with too many households in the artificial city model will often have the pattern that households occupied all the patches under many circumstances. This effect will blur the effect of income to urban-form patterns.

To investigate the sensitivity of the model to parameters settings including city size, we conducted a sensitivity analysis (results are summarised in section ‘‘Sensitivity analysis’’). First, we tested simulations with proportionally increased population and the size of the landscape. We expanded the length of landscape in the model by factor of 1.3, 1.8 and 2.2. Therefore, the same analysis was done for cities with 1.96, 3.24 and 4.84 times the size as the original model. Furthermore, we conducted an extensive sensitivity analysis for the parameters preference to other goods, transport cost per unit and initial and minimum rent per living unit.

Results

Overview of simulation runs

Our results show that after 50 time steps, households rarely relocated anymore so that equilibrium was reached. To be on the safe side, we report simulation results after 100 time steps. The overall patterns from the simulations were calculated by four indicators. We used residential area and density gradient to measure the urban sprawl, and NSI and CGI to measure the income segregation. The indicators’ range over all simulations and the indicators’ coefficient of variation from each simulation replication are shown in Table 2. The small values for the mean of the coefficient of variation indicate that results from different simulations in the same income structure are similar.

Figure 3 shows strong correlations between the two indicators for urban sprawl as well as the two for income segregation. Spearman’s rank correlation between residential area and density gradient is $r_S = -0.94$ ($p < 0.001$) and $r_S = 0.79$ ($p < 0.001$) between the CGI and the NSI.

Validation 1: Income level and sprawl

We aimed to analyse whether the empirical finding of a positive correlation between income level and urban sprawl can be reproduced in the model. The results of simulations with varying mean income and income inequalities (Figure 4) clearly indicate that mean income has a strong effect on both indicators of urban sprawl. The correlation coefficient between mean income and density gradient is $r_S = -0.82$ ($p < 0.001$), while the correlation coefficient between mean income and residential area is $r_S = 0.86$ ($p < 0.001$). Interestingly, Figure 4 also shows that higher income inequality (high Gini coefficient) is related to higher density gradients and less residential area, indicating lower sprawl levels. The correlation coefficients

Table 2. Indicator range and coefficient of variation.

Indicators	Min	Max	Mean of coefficient of variation
Residential area	183	500	0.02
Density gradient	0.01	0.50	0.11
NSI	0.16	0.64	0.10
CGI	0.03	0.42	0.11

between the Gini coefficient of income and urban sprawl indicators (density gradient and residential area) are $r_S = 0.51$ ($p < 0.001$) and $r_S = -0.42$ ($p < 0.001$), respectively.

Validation 2: Income inequality and income segregation

The second empirical finding we analysed to validate the model is whether income inequality is positively correlated with income segregation. To test this correlation in the model, we examined the effect of income inequality (measured by the Gini coefficient) on income segregation at different income levels.

The relationship between the Gini coefficient and the two segregation indicators (NSI and CGI) is shown in Figure 5. The correlation coefficient between the Gini coefficient of income and the segregation indicators is $r_S = 0.43$ (NSI, $p < 0.001$) and 0.64 (CGI, $p < 0.001$), respectively. The income segregation at a very high income inequality level (Gini coefficient = 0.5) may be lower than for a smaller Gini coefficient (Gini coefficient = 0.4). The variability between replicates of income segregation at very high income inequality is also large.

There is an interesting but not very obvious negative correlation between income level and income segregation. The correlation coefficient between mean income and the NSI is

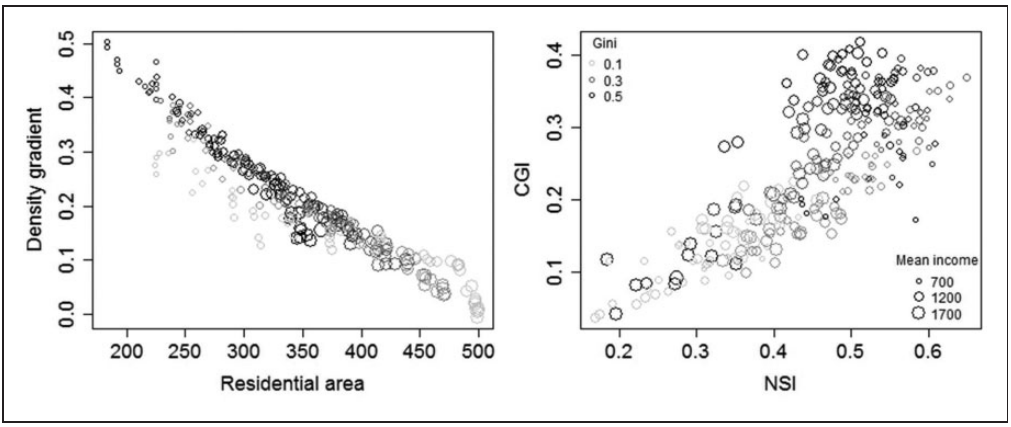

Figure 3. Relationship among urban sprawl indicators (left) and segregation indicators (right).

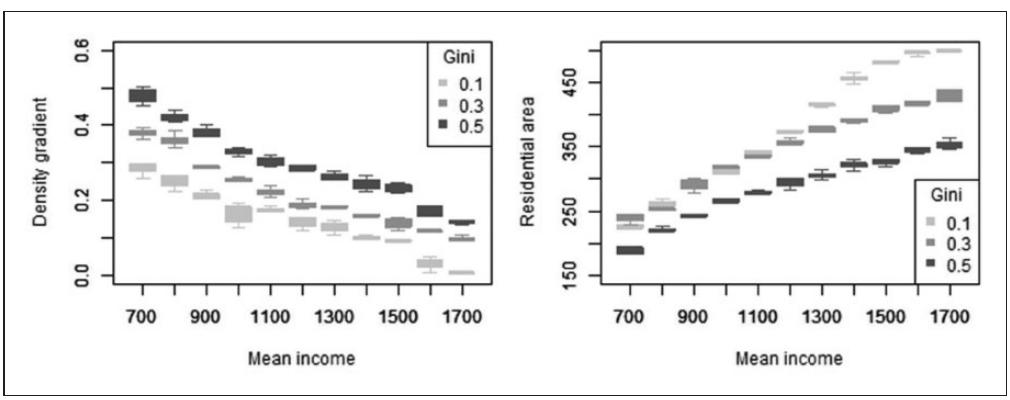

Figure 4. Relationship between mean income and density gradient (left) and residential area (right).

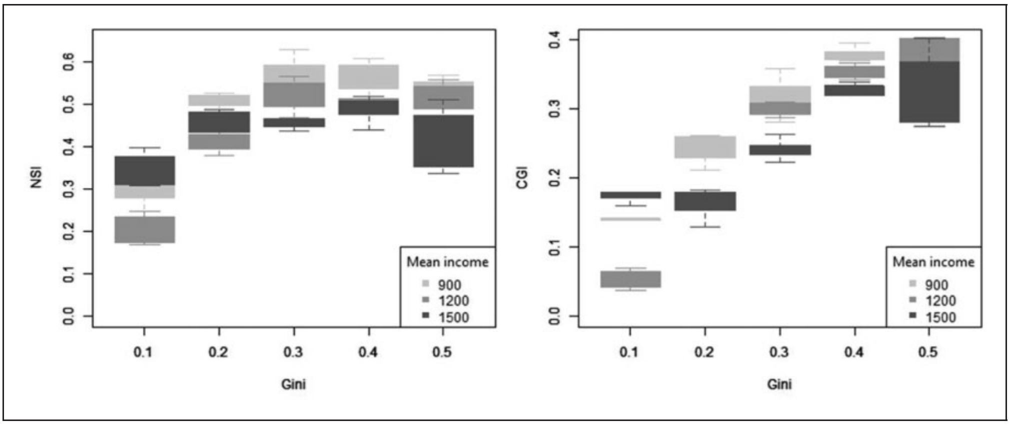

Figure 5. Relationship between Gini's coefficient and income segregation indicators.

Table 3. Spearman's rank correlation coefficients between urban sprawl and segregation indicators. All coefficients are significant ($p < 0.001$).

	Density gradient	Residential area
NSI	0.56	−0.42
CGI	0.46	−0.35

$r_S = -0.41$ ($p < 0.001$), while the correlation coefficient between mean income and the CGI is $r_S = -0.19$ ($p < 0.05$).

The relationship between sprawl and income segregation

To answer our research question, "what is the relationship between urban sprawl and income segregation?", we analysed simulations with all combinations of five levels of income inequality (Gini coefficient) and 11 levels of mean income. Figure 6 shows that both segregation indicators are positively correlated with the density gradient and negatively correlated with residential area, indicating that high levels of segregation are associated with low urban sprawl and vice versa. The correlation coefficients among the four indicators are given in Table 3. Furthermore, Figure 6 shows that no simulations produce results that fall into the range of the lowest (expect for density gradient is the highest) 10% of values for both types of indicators, avoiding both sprawl and segregation at the same time (grey shading in Figure 6). We also calculated the correlation coefficients between indicators on the three increased scales. The correlation coefficients are very close to what we present here.

Sensitivity analysis

Furthermore, a sensitivity analysis was conducted to investigate whether these patterns also hold for other parameter settings. Detailed results are given in the supplementary material S2.

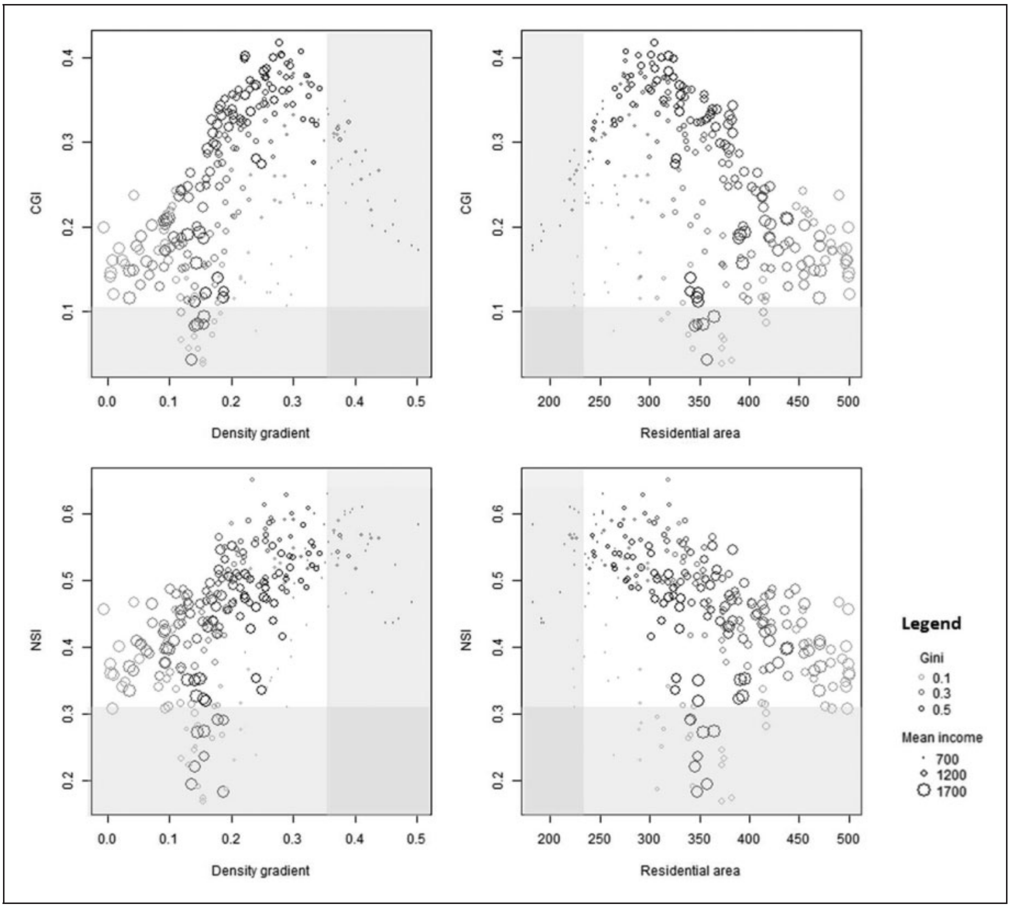

Figure 6. Relationship between urban sprawl indicators (density gradient and residential area) and income segregation indicators (CGI and NSI). Top left: density gradient – CGI; top right: residential area – CGI; bottom left: density gradient – NSI; top right: residential area – NSI. The grey areas encompass the smallest 10% of values (largest 10% for density gradient), indicating a potentially preferable "non-sprawl" and "non-segregation" situation.

In a nutshell, the sensitivity analyses show that the size of the city, the preference for other goods, the transport costs as well as initial/minimum rent per living unit have effects on the investigated patterns. Once we narrowed down the parameter space to exclude implausible simulation setups (such as large numbers of households leaving the system due to too high costs compared to the available income), we found qualitatively similar positive relationships for income level and sprawl (validation 1) and income inequality and segregation (validation 2). For our research question on the relationship of urban sprawl and income segregation, we also found consistent negative correlations in this parameter space.

Discussion

Usability of indicators

In this study, we used two indicators each to quantify urban sprawl and income segregation. Urban sprawl was measured by total residential area and the density gradient of population density, which were shown to reveal different aspects of the urban form, namely the physical and the socio-economic urban form, in empirical studies, e.g. for Europe (Guo et al., 2017). However, our results show that these two urban sprawl indicators are so strongly correlated that they were exchangeable in the context of this model. The high correlation between the two sprawl indicator results is mainly due to the fixed population and city area in the model, which lead to the same gross population density in the city. The correlation between residential area and density gradient would be weaker if the model complexity increased by varying landscape size and population. Nevertheless, considering comparability to findings from empirical studies and possible future work, whether empirical or theoretical, we report both urban sprawl indicators to quantify the model results.

We also used two indicators to measure the residential income segregation level, namely the NSI, which is widely used, and the CGI, which is supposed to overcome the high data demand of the NSI. Overall, we found a high correlation between the two indicators, but with considerable variation between the two for high CGI and NSI values (i.e. high segregation). In empirical studies, the range of segregation levels is small: 0.3–0.5 for NSI (Jargowsky, 1996) and 0.1–0.2 for CGI (Watson, 2009). Our results (Figure 3) show that for this range, the correlation between the two is very good. The CGI has the advantage of requiring no assumptions about the income distribution because the distribution of income is normally unknown in reality. However, in this modelling study, we have perfect information on household income, so the NSI has the advantage of easier implementation and less calculation time.

Pattern validation

The empirical findings from the literature and our model findings about the relationship between urban sprawl and mean income as well as between income inequality and segregation are conceptually summarized in Figure 7.

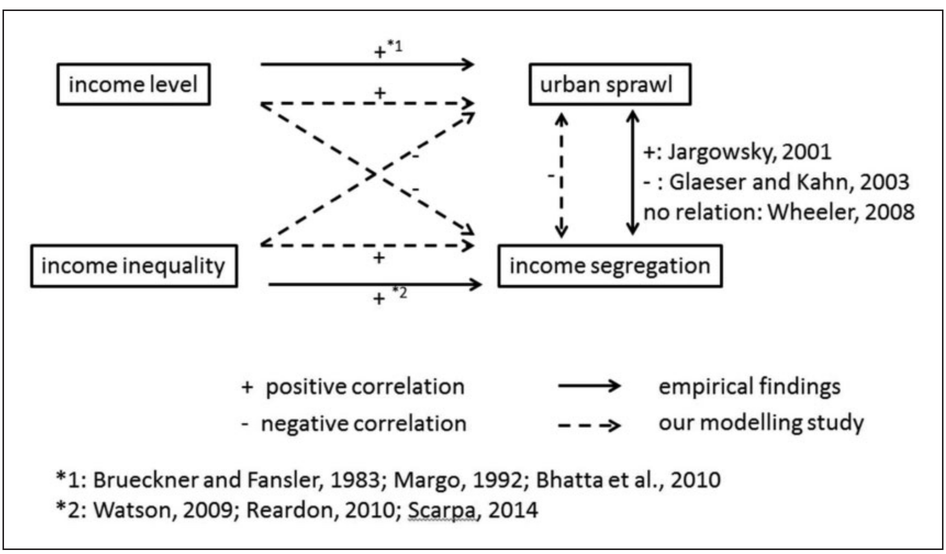

Figure 7. A conceptual diagram representing modelling results of this paper and the empirical results found in the literature.

The positive correlation between urban sprawl and mean income is clearly demonstrated in Figure 4 and is supported by previous empirical work (Bhatta et al., 2010; Brueckner and Fansler, 1983; Margo, 1992). When the income level is high, households have more income to spend on transportation and they are able to afford a greater living area. Therefore, the city expands when the overall income level increases. Our model results also confirm the positive correlation between income inequality and segregation (Figure 5). The results match the empirical findings from Watson (2009) and Reardon and Bischoff (2011). In the model, the districts far away from the city centre have a higher mean income than the districts close to the city centre. The fact that the model was able to simulate both empirical patterns raised our confidence in model performance when further elaborating on the relationship between urban sprawl and income segregation. It is worthwhile note that the segregation pattern is an emergent pattern from the model without explicit rules for segregation, e.g. the tolerant rate or different qualities of amenities. The segregation pattern would most likely be even clearer if we accounted for the preference for the specific environment (amenities) and neighbourhood characteristics in the model.

Interestingly, we also see relatively lower income segregation at a Gini coefficient of 0.5 than 0.4 (i.e. very high income inequality) in the model. The likely reason for this pattern is the large economic advantage the relatively rich households have in a very unequal situation. The affordability of transportation and utility derived from a large living area leads to many relatively rich households choosing to live in the city outskirts. This is the major reason for income segregation in the model. However, in a model scenario with very high inequality (Gini coefficient = 0.5), relatively rich households have so many economic advantages that some of them can even choose to live close to the city centre with large living areas, even though the rent close to the city is relatively high, which can be observed in the model and reality. Another potential explanation may be the distribution of income behind the high Gini coefficient; in this case, the vast majority of households have very low incomes, while only very few are very rich. Distributing these households into the landscape might lead to a drop in the NSI again.

The relationship between urban sprawl and segregation

The relationship between urban sprawl and income segregation has rarely been investigated in empirical studies. Scholars have presented different results regarding the relationship between these two undesired phenomena in urban development. For example, Wheeler (2006) found that they are not related, Jargowsky (2002) stated that urban sprawl has a positive effect on urban segregation while Glaeser and Kahn (2004) found a negative effect. Finally, the modelling study within the Schelling framework by Spielman and Harrison (2014) also found a positive effect. Thus, our main interest in this modelling study is to shed light on this disputed issue. The model results show that the two phenomena are negatively correlated, which means that more urban sprawl is associated with less income segregation (Figure 6). This contradicts those results from the studies mentioned above that found either a positive or no relationship (Figure 7).

The interactions between mean income and segregation as well as income inequality and urban sprawl explain the negative correlation. First, high income inequality has a negative effect on urban sprawl. This pattern can be explained by the assumptions made in the Alonso, Muth, and Mills model. The high Gini coefficient with a fixed mean income and a right-skewed distribution means that there are a large number of households with income lower than the mean income. This condition is similar to the case with a low mean income but more equality. Therefore, these comparably poor households, which are the majority in the city, are restricted to living closer to the centre to avoid high transportation costs, meaning that this city is more compact than a city with the same mean income but a lower Gini coefficient. Second, the income level also has a negative effect on income segregation. One explanation is that when the mean income is high in the model city, the comparably poor households also have the ability to live in the city outskirts in small living units. This is not achievable for the comparably poor households in a society with lower mean income in the model. Therefore, in the simulation results, the higher potential for mobility leads to a negative correlation of income level and segregation. Thus, urban sprawl and segregation respond inversely to changes in both income level and distribution. These two reasons lead to the observed negative correlation between sprawl and segregation. Glaeser and Kahn (2004) argue that sprawling cities are normally car-oriented and are less segregated compared to compact cities with multiple transport modes. In a city with multiple transport modes, the rich will live in the car zones while the poor prefer to live close to public transport. Therefore, compared to compact cities, sprawling cities have less segregation. Contrary to our findings here, Spielman and Harrison (2014) found a positive relationship between urban sprawl and segregation within a Schelling framework. In their simulation experiments, the built environment is a property of the landscape and is varied when initialising the simulation experiments. The agents in the model only have preferences to live close to specific other population groups, and the simulations show that these preferences can be realised more effectively within a larger space. In our approach, both the built environment and segregation are emergent properties of the system, as the agents consider costs and available living units in their location choice. Due to the processes of price formation and occupation of living units summarised above, contradictory findings emerge when using market-oriented versus neighbourhood-similarity assumptions for the location choice.

Figure 6 shows that no simulation runs within the Alonso framework ended within a range that has small values (smallest 10% in the simulation) for all of the indicators of urban sprawl and income segregation, at the same time (as shown with the grey areas in Figure 6). We conclude from this result that, under the free market conditions implemented in our model, the optimal outcome for urban planners, namely low levels of urban sprawl and income segregation, is not likely to happen. Examining Figure 6 more closely, we can see that in a poor (small dots) and equal society (lighter dots), the goal of little sprawl and weak income segregation is more likely to be achieved. However, that would not maximize overall welfare. To conclude, avoiding urban sprawl and income segregation in a city might indeed be a "wicked problem" (Schönwandt et al., 2013), and solutions to this problem should be tested systematically. Mechanistic models, such as the one we present here considering possible extensions, appear to offer a good potential to further deepen our knowledge of the complex interplay of processes governing complex systems like urban areas.

Limitations and future research

As mentioned in the previous sections, the city model presented here is a stylized one and does not – on top of policy instruments such as planning – consider physical properties of the landscape, such as rivers and hills, among others. Furthermore, we did not capture different transport choices in the model and believe that further differentiation of transport modes would further enrich the explanatory power of the model. While the monocentric structure is a clear simplification of a real city, we considered it a good starting point for our study, exploring basic patterns and principles of population distribution. Also, our stylized approach assumes that a housing market exists and that floor space and transportation are the most important factors in residential mobility. We believe that next to North America, this also holds for many European cities, for example. Another difference between the above-mentioned empirical studies and the model is that the data in empirical studies consist of time-series with changes of population size. In this model, we chose to simulate different combinations of income level and income distribution with fixed population numbers to minimize the complexity of the model and the set of simulations. The rent development mechanism described in section "Update of land rents" applies a simple approach to mimic market dynamics. A more sophisticated market model will not only increase credibility of the model, but might also influence sprawl and segregation (Huang et al., 2013; Sun et al., 2014), so that it is worthwhile to further develop the model in this respect. However, the results indicate that the model stemming from urban theory can explain the emergence of empirical patterns. This in turn demonstrates that a virtual city in a simulation model can be used to explore urban theory and thus understand patterns in the real world.

The model results show that the desired urban development with low urban sprawl and low income segregation does not happen in the virtual city under free market conditions without any interventions such as urban planning measures. Thus, the model indicates the value of urban planning. In future studies, it would be interesting to systematically investigate which planning polices (zoning, social hosing, etc.) could foster the development of less sprawl and less segregation within the model.

There are some aspects of the model that have not been explored in this study. For example, the preferences for living area and composite goods in the current model are homogeneous. Inputting preferences related to income level may yield different outcomes of the modified model. Following Schelling's approach, the preference for neighbours with similar incomes could be another feature of the model, which could help to further understand the mechanism of segregation. By systematically exploring the additional effects of preferences towards specific income groups, one could more closely investigate the different findings regarding urban sprawl and segregation found within the Alonso (our study) and the Schelling framework (Spielman and Harrison, 2014). Moreover, the population number and income distribution of the city is stable in this study. With changes in population number, we could simulate the growing and shrinking context of a city, which could improve our understanding of urban form development in cities facing the challenge of change. Changing the landscape of a city, for example, different size or setting of geographically heterogeneous landscape (e.g. river, hill) could also advance our understanding of urban sprawl and segregation. Also, the spatial distribution of activities within a city is important. Implementing different centres would require various new assumptions; however, it would be interesting to see with which model assumptions empirical patterns of real, more complex cities could be explained. Such investigations would, however, clearly exceed the focus of our study. Finally, neighbourhood effects also play an important role in reality. In this study, we did not include this on purpose to focus on the relationship between income structure, urban sprawl and income segregation. However, we see the potential of adding this in following studies, for gaining more detailed insights about the effects of social aspects on spatial urban patterns.

Conclusion

Urban sprawl and income segregation are two major urban problems in urban development. Both urban spatial patterns are discussed with respect to income level and income distribution, but are hardly ever analysed together empirically. Empirical studies either found very little evidence for a relationship between sprawl and segregation

(Wheeler, 2006) or a positive relationship (Jargowsky, 2002). To better understand this relationship, we developed a stylized ABM to study this aspect in a virtual laboratory. We utilized Alonso's bid rent theory to model the location choice of households in an unregulated market. We did not specify the explicit rules for segregation in the model. The model provided an emergent pattern of urban sprawl as well as income segregation. We quantified urban sprawl with the total residential area from the physical structure perspective and the population density gradient from a socio-economic perspective. We quantified segregation of income groups with the two indicators, NSI and CGI, which differ regarding their need for data. The results show that the ABM successfully replicates the findings from empirical studies regarding the positive relationships between both income level and urban sprawl and between income distribution and income segregation. Interestingly, the results show a negative correlation between urban sprawl and income segregation, which contradicts other empirical studies.

In the virtual laboratory of our ABM, we can shed some more light on the processes underlying these patterns. In the model, households maximize their utility by choosing their location as well as the size of the living area. In general, relatively rich households have a tendency to move to the city outskirts and have a large living area. At the same time, the relatively poor households stay close to the centre to compensate their disadvantage in mobility. However, this tendency does not apply to all the cases in the model. For example, in a richer society, the relatively poor households have fewer disadvantages in mobility than in a poorer society, which leads to income level having a positive correlation with urban sprawl as well as a negative correlation with income segregation. At the same time, the income inequality has a positive effect on income segregation but a negative correlation with urban sprawl. The combined effects of the negative correlation lead to a wicked problem, i.e. in the simulation we do not create a society with low urban sprawl and low income segregation, which is a state that planners normally wish to achieve. This finding strongly emphasizes the need for planning and unravels some aspects of free-market failure.

This ABM has shown that in an unregulated market, urban sprawl and income segregation are negatively related, contradicting the scarce empirical results currently available. Sound evidence is needed to provide a base for efficient policy instruments that can tackle the complexity of urban sprawl and income segregation. Therefore, this modelling study should be a motivation for both more detailed empirical research to test their relationship empirically in a much broader setting and for further theoretical modelling studies. Moreover, policy implementations, such as zoning or social housing, and their effects should be another focal point of future empirical and modelling studies.

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Note

1. The model is provided at openabm (www.openabm.org/model/5266/). A detailed model description according to the ODD protocol (Grimm et al. 2010) is provided in supplementary material S1.

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