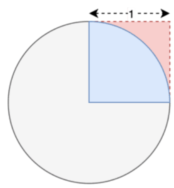
Compulsory Project 1 for STAT 250

Deadline: Fredag 18. Mars, kl. 23:59

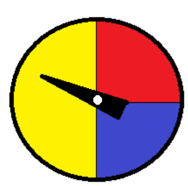
Task 1: Cut a unit circle in four equal segments. The blue quarter circle has a radius of 1 and it's area is defined by the following formula:



$$
\begin{aligned}
A_{quarter} &= \frac{\pi \cdot r^2}{4} \\
&= \frac{\pi \cdot 1^2}{4} \quad with\ r = 1 \\
&= \frac{\pi}{4}
\end{aligned}
$$

The quarter circle fits perfectly well into the red unit square with an edge length of 1.
Based on the above picture and formula, please design a Monte Carlo experiment to approximate π. Please use your imagination and simulation tools from STAT250. You have quite a lot freedom in designing here, as long as your result converge to pi. Please give out also argument of your experiment design, not just submit code.

Task 2: We spin the following spinner, if the black arrow is landing on 'yellow' you gain 1 point, 'red' you lose 1 point and 'blue' you gain 2 points. Again, use your imagination and design a Monte Carlo method to approximate the probability of the event "after 10 spins, you'll have less than 0 points".



Task 3: A discrete random variable X has probability mass function

x	0	1	2	3	4
$p(x)$	0.1	0.2	0.2	0.2	0.3

Use the inverse transform method to generate a random sample of size 1000 from the distribution of X. Construct a relative frequency table and compare the empirical with the theoretical probabilities.

Task 4: Generate a random sample of size 1000 from the Beta(3,2) distribution by acceptance-rejection method. Graph the histogram of the sample and compare it with the theoretical Beta(3,2) distribution. You can choose your own $g(x)$ function.

Task 5: Use acceptance-rejection method to generate data standard normal distribution $f(x) = \frac{1}{\sqrt{2\pi}} e^{-\frac{x^2}{2}}$, $-\infty < x < \infty$ from $g(x) = \frac{1}{2} e^{-|x|}$, $-\infty < x < \infty$.

Task 6: Generate a random sample of size 1000 from a normal location mixture:

$$p_1 N(0, 1) + (1- p_1) N(3, 1)$$

Graph the histogram of the sample with $p_1 = 0.75$. Choose different values of p_1 and observe whether the empirical distribution of the mixture appears to be bimodal.

Task 7: A *compound Poisson process* is a stochastic process $\{X(t), t \geq 0\}$, $X(t) = \sum_{i=1}^{N(t)} Y_i, t \geq 0$, where $\{N(t), t \geq 0\}$ is a Poisson(λ) process and Y_1, Y_2 ,... are i.i.d. from F_Y and independent of $\{N(t), t \geq 0\}$. Then $E[X(t)]=t\lambda E[Y_1]$ and $Var[X(t)]=t\lambda E[Y_1^2]$.

Write a program to simulate a compound Poisson(λ)-Gamma process (where Y has a Gamma distribution). Estimate the mean and the variance of $X(10)$ for several choices of the parameters and compare with the theoretical values.

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Task 8: Find two importance functions that are supported on $(1, \infty)$ and are 'close' to :

$$g(x) = \frac{x^2}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right); x > 1$$

Which of your two importance functions should produce the smaller variance in estimating:

$$\int_1^\infty \frac{x^2}{\sqrt{2\pi}} \exp\left(-\frac{x^2}{2}\right) dx$$

You can plot the shape of $g(x)$ first when choosing importance functions.

Task 9: $X_1, X_2, \ldots, X_n$ are independent random variable from distribution $N(\mu, \sigma^2)$. Suppose we do not know σ^2, write a function to get the 95% confidence interval for μ, and this function should be function of μ and n.

Task 10: Suppose that $X_1, X_2, \ldots, X_n \sim N(\mu, \sigma^2)$. Test $H_0: \mu = 100$ against $H_1: \mu \neq 100$. Set $\alpha = 0.05$, plot the power curves for the t-test with sample sizes 10, 20, 30, 40, and 50. Plot the curves on the same graph, each in a different color or different line type.