

50. Gaussian copula: hazard rate of A and B are 1% and 2%. A contract pay you \$ 1 if A defaults earlier than B. The price of the contract has lowest price when the correlation is (0, 75%, 100%, ...)?
51. Monte Carlo method: to find the mean of A, find a variable B, such that  $\rho(A, B) = c$ , and simulate  $A + B(\mathbb{E}(B) - B)$  instead of A. What is B?  $[\frac{\rho(a,b)}{\sigma_a}, \frac{\text{cov}(a,b)}{\sigma_b}, 1, -1]$
52. Monte Carlo method: we need to simulate some rare event, so u need to be simulated in a different measure. What is the name of this technique? [sequential resampling?]
53. Compute the variance of

$$\int_{t_1}^{t_2} W_t^2 dW_t$$

where  $W_t$  is a Brownian Motion.

54. Given a dividend-paying stock (discrete dividend), what is the difference between the price obtained by modeling the stock with a Geometric Brownian Motion and with a jump-diffusion process?
55. A coupon bond has 5% coupon, 10 year maturity, 1000 face value, 6% yield, compute the price.
56. A coupon bond has 3% coupon, 10 year maturity and the bond trades at par (100), what is the price if rate goes up 1bps: [99.91, 99.99, 100.01, ...]
57. Which contract has a higher value: FRA vs. Eurodollar future?
58. In 2009 a trader believes that the dividend in 2011 will be lower than expectation: which strategy should s/he follow? long/short 2010 forward and long/short 2011 forward?
59. (unclear) A trader observes that the implied volatility of OTM calls and puts are higher than that of ATM options. What is the best strategy to follow: calendar spread, bull spread, bear spread, butterfly spread? [butterfly?]
60. If an asset price is positively correlated with interest rate, the future price is higher/lower than the forward price? (higher)
61. If interest rate is deterministic, future price is higher/lower than forward price? (equal)

38. What is the expected time for a Brownian motion  $W_t$  to hit any of the barriers  $\pm 1$ ?
39. Given that a zero bond price is  $P(0, T) = \frac{1}{1+T^2}$ , what is the forward rate between  $(T_1, T_2)$ ?
40. An asset (uncertain) following a Geometric Brownian Motion has a value of \$0 dollar now and of \$1 in a year: what is the price at  $t=0.5$ ?
41. Express the Radon-Nikodym derivative to go from the real probability measure to the forward measure with respect to a bond price  $P(0, t)$
42. Given two independent Brownian Motions  $B_t$  and  $W_t$ , both starting at point  $(1, 1)$ , what is the probability of the curve  $(B_t, W_t)$  of hitting the positive x-axis before hitting the negative x-axis?
43. Given a Geometric Brownian Motion with drift  $r$  and volatility  $\sigma$  representing the exchange rate USD/GBP in the USD risk neutral measure, what is the drift term for the exchange rate GBP/USD in the GBP risk neutral measure?  $[-r, -r - \sigma^2, -r + \sigma^2]$
44. Given the ODE
- $$\frac{du}{dt} - 5u = 0$$
- and using the backward Euler-method, what choices of  $dt$  can make  $u$  unstable?  $[3, 5, 8, 11]$
45. Given the Brownian Motion  $W_t$  and  $X = e^{W_t}$ , what is  $E[X]$  at  $t = 2$ ?  $[e]$
46. A stock price follows a Geometric Brownian Motion with  $r = 0.05$ ,  $\sigma = 0.3$ ,  $spot = 100$ , what is the probability of price being less than 50 in one year from now?
47. If a European binary call option is worth \$0.1, how is the equivalent American binary option worth: \$0.1, \$0.2, \$0.3, \$0.4
48. given two equivalent measure,  $\mathbb{P}$  and  $\mathbb{P}'$ , knowing  $\mathbb{P}(A) = 0.5$ , what is  $\mathbb{P}'(A)$ ?  $[0, 1, \text{undetermined}]$
49. Today the spot price of the underlying stock is \$10, if you think that one-month later the stock price will be \$8 and \$10 with equal probability and the risk-free interest rate is 5%, what is the price of call option with strike \$10?  $[\text{between } 5/1.05 \text{ and } 10/1.05 \text{ because of risk-free probability}]$

22. In the Cartesian plane, a circle of radius 3 is centered in (4,0) and a circle of radius 4 is centered in (6,0). Draw the common tangent to both circles touching the two circles in the first quadrant. What is the length of the segment joining the two tangency points? [the lines joining centers and corresponding tangency points are orthogonal to the tangent and hence parallel to each others, drawing the segment from (4,0) orthogonal to the segment joining (6,0) to the corresponding tangency point solves the problem]
23. In C++ if you explicitly declare in a class the assignment operator, what should you also declare: copy constructor, destructor, initialising constructor, comparison operator? [the correct answer is unclear because by the rule of three you should declare both the copy constructor *and* the (virtual) destructor but it was not given as an option]
24. In C++ once you define class A, when is the copy constructor invoked? Several cases given among which this one (copy constructor used both in line 2 and 3):

```
A a;
A b(a);
A b = a;
```

[other case is when an object is passed *by value* to a function]

25. What is the output of this C++ code snippet?

```
int d = 3;
int main()
{
    int a, b = 5, c = 5, d = 5;
    d = a++ - ++b * c / ::d;
    cout << d << endl;
}
```

26. Identify among four long C++ code snippets the one describing BubbleSort. Two areas of focus:

```
for (int i = 0; i < size; ++i)
```

[it is important to have a strictly minor sign]

```
void swap(int& x, int& y)
{
    int temp = x;
    x = y;
    y = temp;
}
```

143. Given a Brownian Motion  $W_t$  compute the expectation of the following integral:

$$\int_0^T W_s^n dW_s$$

144. How a static member data `s` is called within a class `A`? [`A::s`]
145. The spot price of an asset is \$100. A trader think that in one year there is a 0.5 probability that the asset climbs to \$110 and a 0.5 probability that the asset drops to \$90, risk free rate = 0.05. What is the price of a call option strike = 100? [between 5/1.05 10/1.05]
146. Put-Call parity for American options.
147.  $X, Y$  are uniformly distributed in the interval  $[0,1]$ . What is  $\mathbb{P}(XY < 0.5)$ ?
148. Which bond has the greatest duration: 1) 4 years bond with 5% coupon 2) 4 years bond with 1% coupon 3) 1 year bond with 5% coupon 4) 1 year bond with 1% coupon
149. Compute the eigenvectors of:

$$\begin{pmatrix} 3 & -2 \\ 4 & -1 \end{pmatrix}$$

150. Given a linear transformation  $T$  in an  $n$  dimensional linear space and two bases  $\vec{u}$  and  $\vec{v}$  such that  $\vec{u} = M\vec{v}$ . If  $T$  is represented by matrix  $A$  in basis  $\vec{u}$  and by matrix  $B$  in basis  $\vec{v}$ , what is the relationship between  $A$  and  $B$ :  $M^tAM$ ,  $MAM^t$ ,  $MA$ ,  $MM^t$
151. `D` is a derived class from `B`. `f()` is a virtual function in `D` and `B`. The constructor of `D` and `B` calls `f()`. When instantiating an instance of `D`, what is the calling order of `B()`, `B::f()`, `D()`, `D::f()`
152. For a trivial distribution, what is larger: variance or standard deviation?
153. Compute:

$$\int_0^T W_s^2 dW_s$$

where  $W_s$  is a Brownian Motion. Solution:

$$\int_0^T W_s^2 dW_s = \frac{1}{3}W_T^3 - \frac{2}{3} \int_0^T W_s dW_s$$

12. Find the direction of fastest growth of  $f(x) = 1/(1 + ax^2 + bx^2 + cz^2)$  at  $(1,1,-2)$

13.

$$\int_0^\infty e^{\frac{1}{2}x^2}$$

14.

$$\begin{aligned}x'(t) &= ax + by \\y'(t) &= cx + dy \\x(0) &= 4 \quad y(0) = 3\end{aligned}$$

Compute  $x(1)$

15. Under which condition does a linear system not have solutions?
16. There are 3 equations in 6 variables. How many dimensions does the solution have?
17. A symmetric positive definite matrix does not necessarily have which property: 1) diagonal elements (or eigenvalues) strictly positive 2) all positive entries 3) all strictly positive diagonal entries in its Cholesky decomposition 4) invertible
18. There are two regular coins and a fake coin with heads on both sides. Pick a coin, toss it and it comes up head. What is the probability to toss the same coin and get head?
19. There is a regular coin and a fake coin. Pick a coin, toss it up twice and it comes up head both times. What is the probability to have picked up the fake coin?
20. The 52 poker cards are split in 4 equal piles. One pile has two aces, two other piles have one ace each and the remaining pile has none. What is the probability of this happening?
21. Three random variables  $X, Y, Z$  have equal correlation coefficients:  $\rho(X, Y) = \rho(X, Z) = \rho(Y, Z)$ . What is the lower bound on the value of the correlation coefficient?  $[-0.5]$
22. Three random variables  $X, Y, Z$  are such that  $\rho(X, Y) = \rho(Y, Z) = c > 0$ . What is the minimum correlation between  $X$  and  $Z$ ?  $[2c^2 - 1]$
23. Given some data and some statistics, what is the regression function?

24. Give a uniform distribution  $U[0, a]$ , what is the maximum likelihood estimator (MLE) of  $a$ ?  $[\frac{n}{n+1} \max(x_i)]$
25. Flip a fair coin 10000 times, what is the probability of more than 4950 heads?
26. Flip a coin 100 times, what is the probability of more than 60 heads?
27. Take  $n$  iid random variables, what is

$$\lim_{n \rightarrow \infty} \frac{\sum_{i=1}^n X_i}{\sqrt{n}}$$

28. In a drawer, there are 18 socks, 12 red and 6 black. Pick two socks randomly, what is the probability of drawing a pair of the same colour?
29. Pick 2 cards from a 52 cards deck. What is the probability to have a pair?
30.  $X_1$  and  $X_2$  are iid random variables, if for any  $a, b$  we can find  $c, d$  so that  $aX_1 + bX_2$  has same distribution of  $cX + d$ , then the distribution is: normal, Poisson, stable?
31. The random variable  $X$  has mean 0 and variance 1. Compute

$$\mathbb{E}[(x+2)^2]$$

32. The random variables  $X$  and  $Y$  have mean 0 and variance 1. Is the following expectation undetermined? If not, compute it.

$$\mathbb{E}[X|X+Y=1]$$

33. Two people are arriving at a time which is uniformly distributed between 12pm-1pm. What is the probability of one waiting the other for less than 15 minutes?
34. What is the distribution of sample volatility divided by sample mean?  
 $\chi^2, F, t, \Gamma$
35. Compute the mode of (11,11,29,41,41,41)
36. Compute the standard deviation of (11,11,29,41,41,41)
37. 25 people (numbered from 1 to 25) sit randomly in 25 chairs (numbered from 1 to 25). What is the expected number of people sitting in a chair with a number matching their own?

75. (unsure) On a 32-bit computer, what is the output of the following non-optimised code:

```
uint -32 a = 16;  
c = 32;  
printf("\%d") of unsigned int(a) - unsigned int(c);
```

0,8,16,32?

76. How many bits are required to store the number 1,000,000: 17,18,19,20?  
[20?]

77. What is the time complexity of merge sort? [ $O(n \log(n))$ ]

78. Time complexity of binary tree searching (in term of  $o()$  notation)?

79. How many comparisons are needed to sort 5 elements? [4]

80. Searching an element among 10,000 elements costs 1ns. How much time does it cost to search an element among 100,000 elements: 10ns , 35ns , 100ns , 350ns?

81. In which instance [ ] cannot be used: string, vector, list, ...? [list]

82. Which of the following does not store multiple elements contiguously: list, deque, vector, string(list)?

83. What is void\* in C++? (null pointer/pointer to nothing?)

84. A 3-year duration coupon bond which has yield 5% has a price of 94, what is par yield? [use the equation  $dP/P = -D \cdot dy$ , 3%]

85. The underlying asset is modeled with a brownian motion starting at 0, a contract on this asset will pay 0 if in 4 years the asset price ever below than -2, pay 1 otherwise, what is the price of this contract? [using reflection principle]

86. Give the definition of swap rate.

87. A set has 10 numbers, how many steps do you need to find both the maximum and minimum number in this set? [ $1.5 \cdot 10 - 2 = 13$ ]

88. You have a view that the realized volatility on S&P is much higher than implied vol, what will you do: buy out-of-the-money call/put option or buy/sell at-the-money straddle?

62. You are long a long-term ATM call and short a short-term ATM call, adjust the ratio to make the total vega of the portfolio zero. If before expiry of the short-term option, spot = strike again, vega: positive, negative, zero? (positive?)
63. The trader wants to do a delta-hedge strategy, but he plugs wrong implied volatility  $\sigma$ , assume the portfolio is  $-C + \Delta S$ , what will the portfolio value change? when the stock increase/decrease, the value increase/decrease..
64. You have a binary option, increase of delta: (positive, negative, it depends, ...)
65. Theta is the partial derivative of? [time]
66. Which of the following is not an assumption of the Black Scholes model: free borrow and lending at risk-free rate, no transaction cost, same tax rate...?
67. Volatility surface: on the strike dimension is flat, on the other dimension, the volatility shape is: 1) an increasing function of strike 2) a decreasing function of strike 3) frown 4) smile
68. (unclear) condition for an American call option to being exercised. [dividend=0? or not?]
69. A default protection seller is long or short the credit risk?
70. In order to hedge the interest rate risk of a 7 year bond worth 100MM, you need to short a 2 year bond of what value? [350MM?]
71. What is the the output type of the operator "<<" called in the construct "cout <<t <<..."? ["ostream& operator <<(ostream& out, T c)"]
72. How are static member data initialisation? [choose the one initialized out of the class]
73. Static member initialisation in a template class: how many instances of initialisation can there be? 1) cannot have static member in template class 2) only one instance 3) as many as instances of classes 4) as many as instantiations of classes
74. Static member initialisation in the multi-thread case.



105. In a multi-thread process, how do you initialise static members? Do you get an error or will it be initialised by first process?
106. The forward rate between  $T_1$  and  $T_2$  is a martingale, what is the corresponding numéraire?
107. Given the sequence 10,29,29,31,33,33,78 can you guess the next number? [it does not match anything in Sloane's sequences dictionary]
108. Which of the following does not store multiple elements contiguously? List, deque, vector, string
109. What is the type of the iterator of the list?
110. The trader wants to implement a delta-hedge strategy, but he uses the wrong  $\sigma$  which is greater than the implied volatility, assume the portfolio is  $-C + \Delta S$  (short 1 unit call option, long  $\Delta$  units of the underlying asset, how will the portfolio value change? When the stock value increases/decreases, the value of the portfolio increases/decreases...
111. Forward rate: given that 1 USD=0.6 GBP, USD rate 5%, GBP rate 2%, what is the 1-year forward rate?
112. Given the random variables  $X, Y, Z$  with  $\rho(X, Y) = \rho(Y, Z) = c > 0$ , what is the minimum correlation between  $X$  and  $Z$ ?
113. Definition of  $R^2$
114. Do the linear regression  $Y = a_1 + b_1 * X$ ,  $X = a_2 + b_2 * Y$ . Is the product  $b_1 b_2$  greater, equal or smaller than 1?
115. A one-year forward contract on a stock has spot price \$100, the stock will pay \$1 dividend at 6-months, what is the price of the contract? [99]
116. Put-Call parity.
117. Give a lower bound for the value of a call option.
118. You roll a fair-dice 100 times, the sum between what interval has probability of 68%? [ $1 \sigma$ ]
119.  $\mathbb{E}[X_i] = 1$ , what of the following is martingale? [ $\frac{(X_1 + X_2 + \dots + X_n)}{n}$ ]
120. Is the stochastic process  $W_t^2 - t$  a martingale?

89. You have a real symmetric matrix  $A$ , you use an algorithm run on your PC to compute  $A^{10}$  and it takes 1 sec, how long does it take to compute  $A^{100}$ ? [2]
90. You roll a coin 100 times, what is the probability of getting at least 65 heads? [ $3\sigma$ ]
91. You put 100 balls into 1000 boxes, what is the expected number of empty boxes?
92. What is  $i^i$  equal to? [ $e^{-\pi/2}$ ]

93.

$$\int_0^1 \log x \, dx$$

94. Under which situation will a linear system has infinite solutions?
95. A barrier option on a stock has strike 80 and barrier 80, the underlying asset has spot price 100 and follows a Geometric Brownian motion, what is the option price?
96. You do a delta hedge strategy on an option, if the underlying suddenly moves a lot, how does the value your portfolio change? [positive]
97. An exotic option pays  $\max(0, (S_2 - S_1) - K)$ . If the correlation between  $S_1$  and  $S_2$  increases, what will the option price do?
98. A random variable  $X$  has mean 0 and variance 1,  $\mathbb{E}[Y|X] = X$  conditioning on  $X$ , what is  $\mathbb{E}[XY]$ ?
99. Given two binary variable such that  $\mathbb{P}(X = 1) = \mathbb{P}(X = 0) = \mathbb{P}(Y = 1) = \mathbb{P}(Y = 0) = 0.5$  and  $\rho(x, y) = 0.5$ , what is  $\mathbb{P}(\max(X, Y) = 1)$ ?
100. How do you initialise static member data: the same as non-static members or using constructor initialisation list?
101. Randomly shuffle 52 poker cards into 4 piles, what is the probability that all four aces are in the first pile?
102. Randomly pick 5 cards out of 52 cards, what is the probability that these 5 cards all have different values?
103. What is the type of access of an Array? [random access]
104. What is the time complexity of removing an element from a vector?