Objectives

- Multiplexer and Demultiplexer
- Encoder and Decoder
- BCD, and Gray Code Representation

Multiplexer and Demultiplexer

Multiplexer

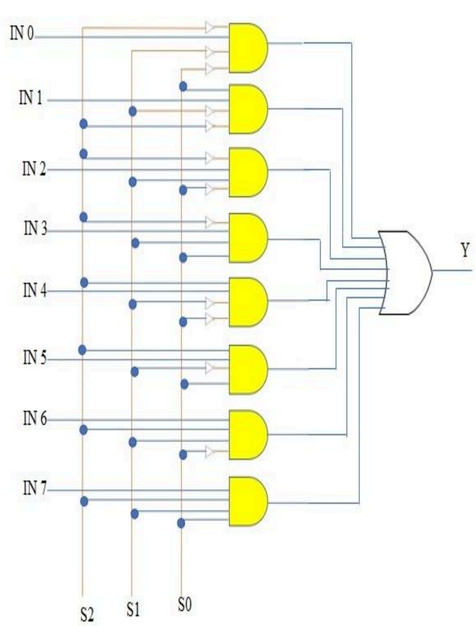
A multiplexer is a logical circuit made up of three main parts:

1. **Input lines**
2. **Address (select) lines**
3. **Output line**

A multiplexer has `n` address (select) inputs and 2^n data inputs. The address lines are used to select which one of the data inputs is routed to the output.
For example, a **8:1 multiplexer** has:

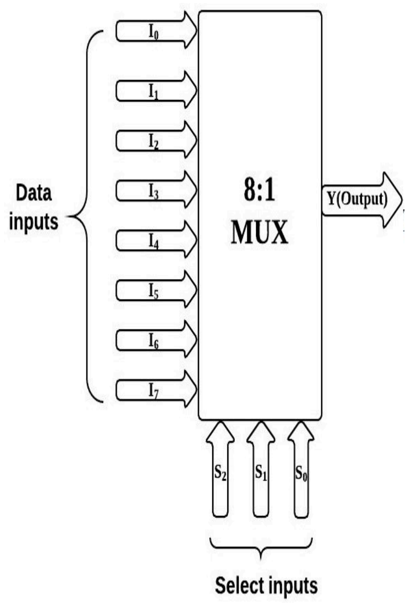
- 3 address (select) lines
- 8 input lines
  The address configuration determines which input is connected to the output. For instance:
- Address `000` selects the **first** input pin
- Address `111` selects the **last** input pin

The structure of a multiplexer is illustrated below:



- The IN_x are the input pins
- The S_x are the address (select) pins
- Y is the output

The circuit can be simplified as shown in the following diagram:



A multiplexer is useful for building and simplifying logic circuits.

Using a Multiplexer in Logic Design

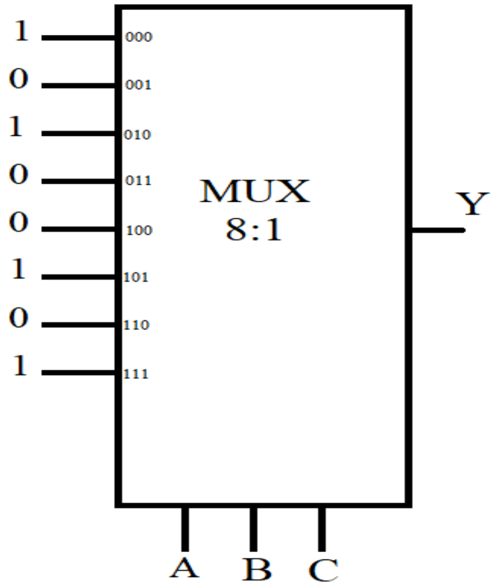
Let's suppose we are given the following truth table:

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

We can implement this truth table using an 8:1 multiplexer, where:

- Inputs A, B, and C serve as the address (select) lines.
- The values in the F column represent the data inputs to the multiplexer.

The corresponding circuit is shown below:



We can simplify the circuit by using a 4:1 multiplexer instead. In this case:

- B and C will be used as address lines.
- **A** will be used to control the output based on logic.

To do this, we restructure the truth table by grouping rows according to values of B and C, and observing how F depends on A:

A	B	C	F
0	0	0	1
1	0	0	0
0	0	1	0
1	0	1	1
0	1	0	1
1	1	0	0
0	1	1	0
1	1	1	1

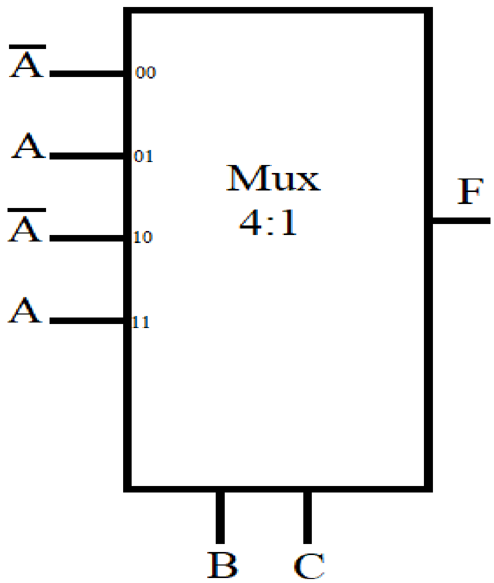
Now we analyze the behavior for each combination of B and C:

- Address 00 (B=0, C=0):
  $F = \overline{A}$
- Address 01 (B=0, C=1):
  $F = A$
- Address 10 (B=1, C=0):
  $F = \overline{A}$
- Address 11 (B=1, C=1):
  $F = A$

Based on this, the inputs of the 4:1 multiplexer will be connected as follows:

Select (B,C)	Input to MUX	Value
00	I_0	$\overline{A}$
01	I_1	A
10	I_2	$\overline{A}$
11	I_3	A

We can now draw the simplified logic circuit using a 4:1 multiplexer and the control signal **A** as input to each data line based on the configuration above.



Demultiplexer

A demultiplexer (or demux) is the opposite of a multiplexer. Instead of selecting one input from many to send to a single output, a demultiplexer routes a single input to one of many outputs, based on the values of its address (select) lines.
A demultiplexer consists of three main parts:

1. **One input line**
2. **Address (select) lines**
3. **Multiple output lines**

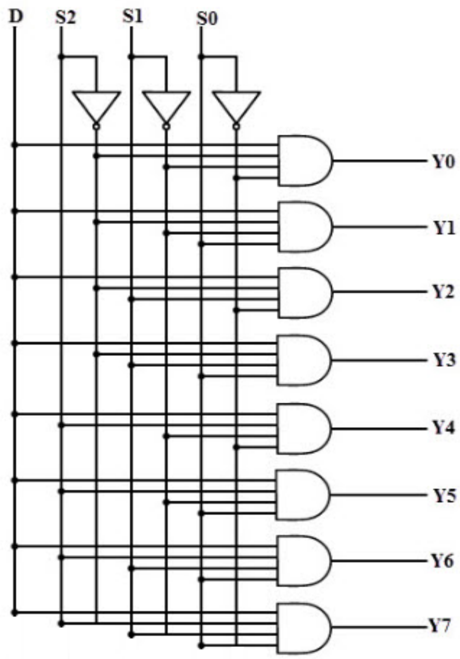
A demultiplexer has `n` address (select) inputs and 2^n output lines. The address inputs determine which output line will carry the input signal, while the others will be set to 0 (or remain inactive).
For example, a 1:8 demultiplexer has:

- 1 input line
- 3 address (select) lines
- 8 output lines

The address configuration determines which output will receive the input signal:

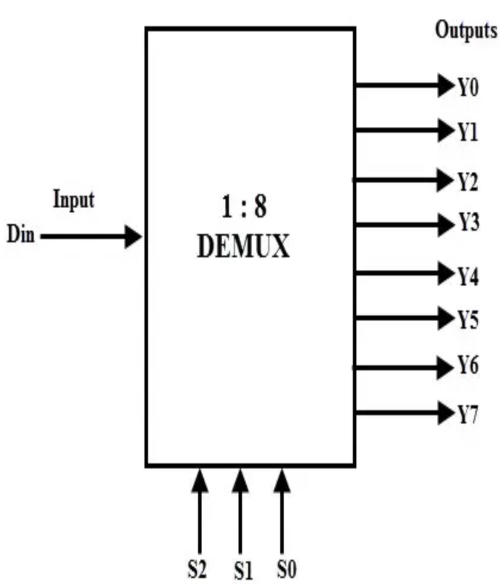
- Address `000` activates **output 0**
- Address `111` activates **output 7**

The structure of a demultiplexer is shown below:



- D is the input signal
- S_x are the address (select) pins
- Y_x are the output lines
  Only one output will be active at a time, based on the address.

The simplified version of the circuit is shown here:

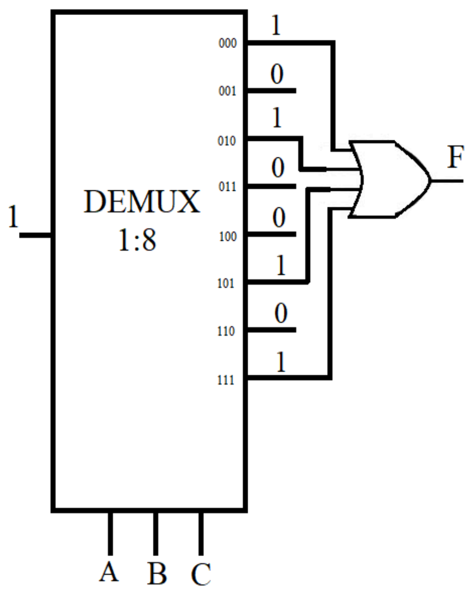


A	B	C	F
0	0	0	1

A	B	C	F
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

We can use B, C, and A as the select lines for a 1:8 demux and activate the input only when F = 1.
To implement the function F, we can OR together the demux outputs that correspond to the adresses where F = 1:

- Adress 000 select the line 0
- Adress 010 select the 2
- Adress 101 select the 5
- Adress 111 select the 7



Encoder and Decoder

Decoder

A decoder is a combinational logic circuit that converts binary input into a one-hot output only one output line is active (HIGH) for each binary input combination.
A decoder has:

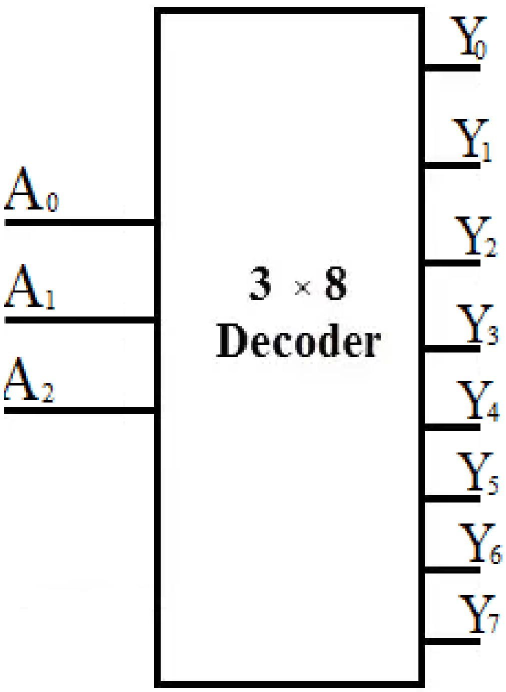
1. n input lines
2. 2^n output lines

Each output line corresponds to one of the possible binary input combinations. For example, a 3-to-8 decoder takes 3 input bits and activates one of the 8 output lines.

Structure of a Decoder

The inputs are interpreted as a binary number, and the output line corresponding to that number is set to 1. For example, in a 3-to-8 decoder:

- Input `000` activates output Y_0
- Input `111` activates output Y_7



- A_0, A_1, A_2 are the input lines
- Y_0 to Y_7 are the output lines

Truth Table of a 3-to-8 Decoder:

A_2	A_1	A_0	Y_0	Y_1	Y_2	Y_3	Y_4	Y_5	Y_6	Y_7
0	0	0	1	0	0	0	0	0	0	0
0	0	1	0	1	0	0	0	0	0	0
0	1	0	0	0	1	0	0	0	0	0
0	1	1	0	0	0	1	0	0	0	0
1	0	0	0	0	0	0	1	0	0	0
1	0	1	0	0	0	0	0	1	0	0
1	1	0	0	0	0	0	0	0	1	0
1	1	1	0	0	0	0	0	0	0	1

Only one output is HIGH at a time the rest are LOW.

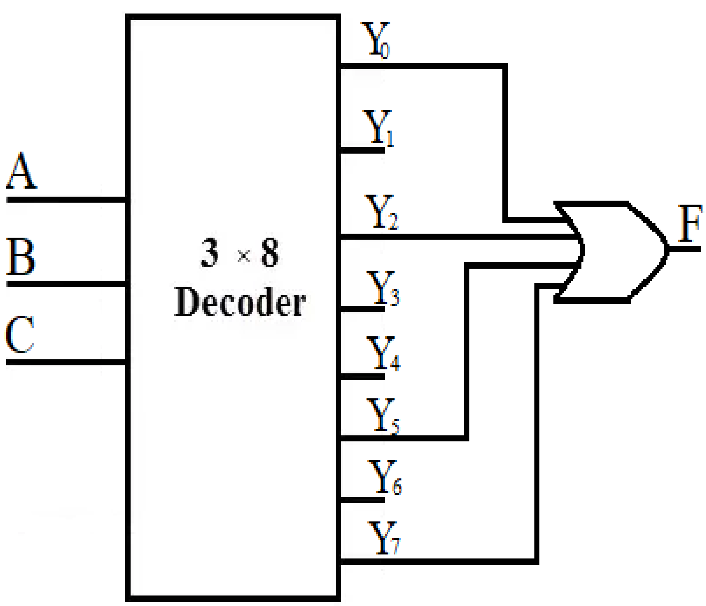
Using a Decoder in Logic Design

Suppose we are given the following truth table:

A	B	C	F
0	0	0	1
0	0	1	0
0	1	0	1
0	1	1	0
1	0	0	0
1	0	1	1
1	1	0	0
1	1	1	1

We can implement this using a 3-to-8 decoder:

- Inputs A, B, and C go into the decoder.
- Each decoder output corresponds to one row in the truth table.
- The function F is defined by OR'ing the outputs where F = 1:
  - F = $Y_0 + Y_2 + Y_5 + Y_7$



This approach helps simplify Boolean functions and implement them efficiently using decoder logic.

Encoder

An encoder is a combinational logic circuit that converts multiple input lines into a smaller number of output lines. It performs the reverse operation of a decoder.

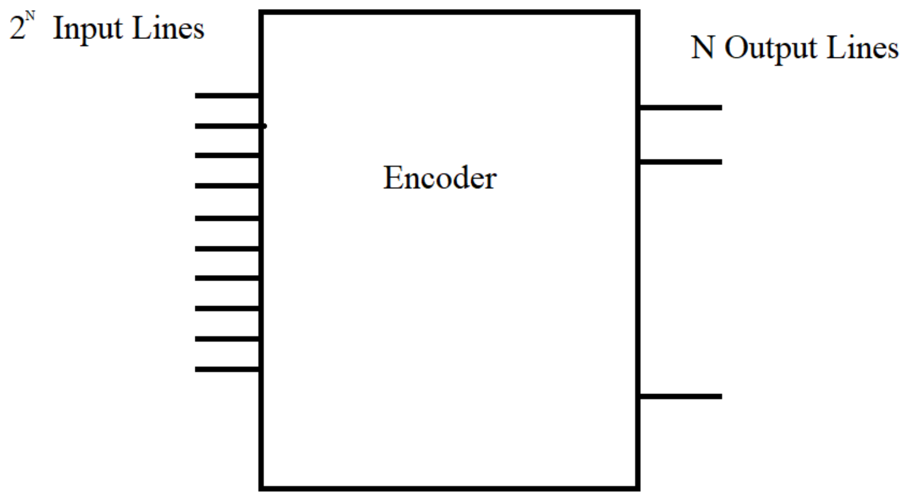
An encoder has:

1. 2^n input lines
2. n output lines

Only one input should be active (i.e., HIGH or 1) at a time. The encoder identifies the active input line and outputs the corresponding binary code.
For example, a 8-to-3 encoder has:

- 8 input lines (I_0 to I_7)
- 3 output lines (Y_0, Y_1, Y_2)

If $I_5 = 1$ and all other inputs are 0, the output will be the binary representation of 5: `101`.
Here is the symbol and structure of a typical encoder:



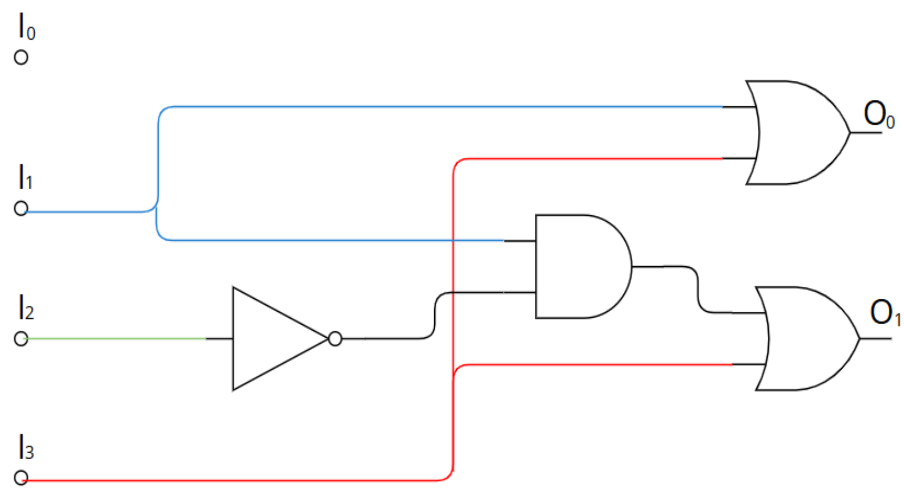
- The I_x are the input lines
- The Y_x are the encoded output lines

The output is determined by the position of the active input line:

Input Line Active	Output ($Y_2Y_1Y_0$)
I_0	000
I_1	001
I_2	010
I_3	011
I_4	100
I_5	101
I_6	110
I_7	111

Priority Encoder

In practical applications, sometimes more than one input line might be active simultaneously. A priority encoder resolves this by assigning priority to the highest-numbered input.
For instance, if both $I_2 = 1$ and $I_6 = 1$, the encoder will output the binary code for input 6: `110`.
A typical priority encoder also includes a valid output signal that indicates whether at least one input is active.
Here is a simplified circuit diagram of a 4-to-2 priority encoder:



BCD, and Gray Code Representation

Introduction

In digital systems, numbers are typically represented in binary form. However, in certain applications, alternative coding schemes are used to improve reliability, compatibility, or ease of interpretation. Two widely known representations are Binary-Coded Decimal (BCD) and Gray code.

BCD Representation

Binary-Coded Decimal (BCD) is a method of encoding decimal numbers where each digit is represented separately using its 4-bit binary equivalent. This representation is particularly useful in systems that interface with human-readable decimal input or output, such as digital clocks, calculators, and electronic meters.
In BCD, only the binary combinations from **0000** to **1001** are valid, corresponding to the decimal digits 0 through 9. The table below shows the basic BCD representation for each digit:

Digit	BCD
0	0000
1	0001
2	0010
3	0011
4	0100
5	0101
6	0110
7	0111

Digit	BCD
8	1000
9	1001

Example:
To represent the decimal number **17** in BCD:

- First, write the BCD for 1: **0001**
- Then, write the BCD for 7: **0111**

So the BCD representation of 17 is:
$17 = (00010111)_{BCD}$

Gray Representation

Gray code is a binary numeral system where two successive values differ by only one bit. This unique property makes Gray code highly valuable in digital systems where minimizing errors during transitions is important such as in analog-to-digital converters, rotary encoders, and error correction in digital communication.
Unlike standard binary, where multiple bits may change between consecutive numbers, Gray code ensures that only one bit changes at a time. This reduces the risk of misinterpretation caused by timing issues or signal noise during transitions.
Below is the basic 3-bit Gray code sequence:

Decimal	Binary	Gray Code
0	000	000
1	001	001
2	010	011
3	011	010
4	100	110
5	101	111
6	110	101
7	111	100

Gray code representation is especially useful when constructing Karnaugh maps, as it ensures that only one variable changes between adjacent cells

Converting between BCD and Gray

Converting BCD to Gray

We convert BCD to Gray by firstly convert BCD decimal then we convert the decimal result to binary after that we convert the binary result to Gray using the following operation

- Keep the first (most significant) bit the same.
- Each next bit = current binary bit XOR previous binary bit.

Example convert $(00100010)_{BCD}$ to Gray
The BCD number is split into two 4-bit groups:

- **0010** represents the digit **2**
- **0010** represents the digit **2**

So, the decimal number is:

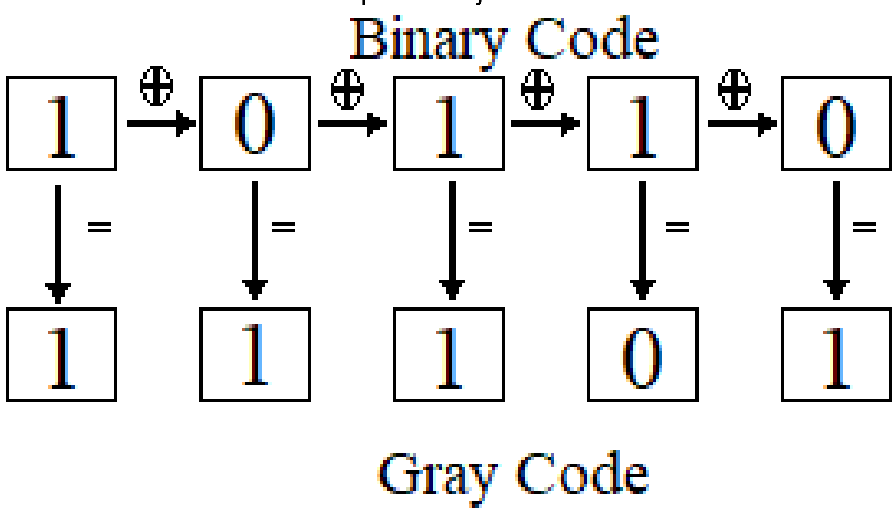
$$(00100010)_{BCD} = 22_{10}$$

Now we Convert 22 to binary:

$$22 = (10110)_2$$

Finally we Convert Binary to Gray Code, to convert binary $(10110)_2$ to Gray code:

- Keep the first bit the same: `1`
- Perform XOR between each pair of adjacent bits:



  The final Gray code is: $(11101)_{Gray}$

Converting Gray to BCD

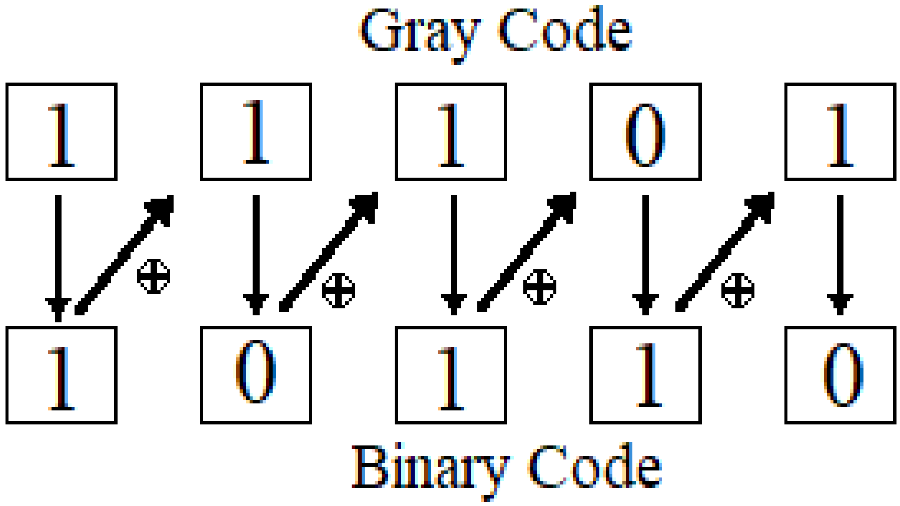
We convert Gray to BCD by first converting it to binary then convert the binary number to decimal after that we convert the resut to BCD, we convert Gray to binary we follow this rule

- Keep the first (most significant) bit the same.
- Each next binary bit = previous binary bit XOR current Gray bit.

Example: Convert $(11101)_{\text{Gray}}$ to BCD
We Convert Gray to Binary Code following this rules:

- Keep the first (most significant) bit the same.

- Each next binary bit = previous binary bit XOR current Gray bit.



We get as result $(10110)_2$, now we convert Binary to Decimal

$$(10110)_2 = (22)_{10}$$

Finally we split the decimal number 22 into its digits:

- First digit: 2 = 0010_{BCD}
- Second digit: 2 = 0010_{BCD}

Final Result:

$$(11101)_{\text{Gray}} = (00100010)_{BCD}$$