The Small Open Economy (SOE) RBC Model

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In this notebook, I replicate the numerical results and simulations of the External Debt-Elastic Interest Rate (EDEIR) model developed by Uribe and Schmitt-Grohé (2003, 2017). Although Uribe and Schmitt-Grohé (2003, 2017) provide Matlab-based implementations of several small open economy real business cycle (SOE-RBC) models—including versions with endogenous interest-rate premia—I focus here exclusively on the specific EDEIR configuration used in their quantitative exercises.

The goal of this notebook is to produce a transparent Dynare-based implementation of the EDEIR model and to reproduce its steady state, impulse responses, and stochastic simulations. Unlike the Matlab implementations by Uribe and Schmitt-Grohé (2003, 2017)—which specify the equilibrium conditions in nonlinear levels and then obtain the linear approximation numerically—my Dynare code inputs the analytically linearized model directly. I also compare my Dynare implementation with the influential Dynare code developed by Pfeifer, who writes the SOE-RBC model in nonlinear form and relies on Dynare's internal linearization routines.

Despite methodological differences—analytical versus automatic linearization—both approaches generate results consistent with the benchmark findings in Uribe and Schmitt-Grohé (2003, 2017). This confirms the robustness of the EDEIR framework and highlights Dynare's reliability for replicating canonical models in international macroeconomics.

Note 01.

For reference, the broader collection of small open economy (SOE–RBC) models and replication files by Uribe and Schmitt-Grohé (2003, 2017) is available at the following link:

https://www.columbia.edu/~mu2166/closing.htm

Additionally, Pfeifer's Dynare implementation of the small open economy model of Uribe and Schmitt-Grohé (2003) can be downloaded from:

https://github.com/JohannesPfeifer/DSGE_mod/blob/master/SGU_2003/SGU_2003.mod

Model Assumptions

Centralized solution

The economy:

- The economy is inhabited by an infinite number of identical households that face fluctuations in their income. It is a small open economy with no transfers, taxes, or government spending. It is an exchange economy—there is no money, and therefore no issues related to exchange rates.
- There are capital adjustment costs, and the interest rate at which domestic agents borrow from the rest of the world is constant. Domestic households have access to a single risk-free international bond ("incomplete international asset markets"). Access to the international asset market allows the economy to smooth consumption in the face of uncertainty in domestic income. This contrasts with a framework in which agents can trade a full set of state-contingent assets ("complete markets") (Lubik, 2007). Agents do not face frictions when adjusting the size of their asset portfolios.

The households:

- In this economy, the representative household consumes and produces BB & SS, invests in physical capital, and borrows from the rest of the world through the international asset market.
- Production, employment, and the use of physical capital all take place within the representative household.
- The household's utility function is assumed to be of the GHH form (Greenwood, Hercowitz, and Huffman, 1988) because, as will be shown later, it makes labor supply (the marginal rate of substitution between consumption and leisure) independent of the consumption level, and because this class of utility functions is appealing from an empirical standpoint.
- The representative household's preferences, denoted by U—that is, the expected value of the sum of current utility and discounted future utility flows—are given by:

$$U = E_0 \sum_{t=0}^{\infty} \beta^t u\left(c_t, h_t\right) = E_0 \sum_{t=0}^{\infty} \beta^t \left[\frac{\left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{1-\sigma} - 1}{1-\sigma} \right]; \sigma > 0,\ \omega > 1, \beta \in (0,1) \tag{1}$$

Where $E_t(\cdot)$ denotes the mathematical expectations conditioned on the information available at period t, and β is the constant subjective intertemporal discount factor. The utility $u(c_t, h_t)$ of the representative household depends on consumption c_t and labor supply h_t in period t, with the following properties: $u_c > 0$, $u_h < 0$, $u_{cc} < 0$, $u_{hh} < 0$, and $u_{cc}u_{hh} > u_{ch}^2$. The utility function of the representative household, of the GHH type, exhibits relative risk aversion with respect to the consumption-labor composite index $G(c_t, h_t) = c_t - \frac{h_t^{\omega}}{\omega}$, where σ measures the degree of relative risk aversion, $1/\sigma$ represents the intertemporal elasticity of substitution, and $1/(\omega - 1)$ corresponds to the elasticity of labor supply with respect to the real wage. The parameter σ also captures the degree of relative risk aversion.

The period-by-period budget constraint of the representative household takes the following form:

$$d_t = (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \Phi\left(k_{t+1} - k_t\right) = (1 + r_{t-1})d_{t-1} - y_t + c_t + i_t + \frac{\phi}{2}\left(k_{t+1} - k_t\right)^2 \tag{2}$$

Where d_t represents the household debt at the end of period t, c_t is consumption, i_t is gross investment, and k_t is the stock of physical capital. The interest rate at which domestic households can borrow from the rest of the world is denoted by r_t. y_t denotes domestic output. $\Phi\left(k_{t+1} - k_t\right) = \frac{\phi}{2}\left(k_{t+1} - k_t\right)^2$, and the function $\Phi(\cdot)$ captures capital adjustment costs and satisfies: $\Phi(0) = \Phi'(0) = 0 \quad \text{and} \quad \Phi''(0) > 0$. These conditions ensure that, in the steady state, capital adjustment costs are zero and that the relative price of capital goods in terms of consumption goods is unity. Capital adjustment costs are included to prevent excessive volatility in i_t in response to changes in the marginal productivity of capital. Finally, $1/\sigma$ represents a measure of intertemporal substitution elasticity, while $1/(\omega - 1)$ measures The law of movement of the capital stock (with depreciation rate "δ") is given by:

$$k_{t+1} = (1 - \delta)\, k_t + i_t, \quad \delta \in (0, 1) \tag{3}$$

Product y_t is produced by a production function that uses physical capital and labor as inputs. The technology used by the representative household to produce a quantity y_t of goods and services in period t is of the Cobb-Douglas type:

$$y_t = A_t F\left(k_t, h_t\right) = A_t k_t^{\alpha} h_t^{1-\alpha}, \quad \alpha \in (0, 1) \tag{4}$$

Where A_t is the total factor productivity (TFP) that represents the aggregate level of efficiency in production, α is the partial elasticity of production with respect to capital, $1 - \alpha$ is the partial elasticity of production with respect to labor.

It is also assumed that the production function satisfies the following properties: $F_k > 0$, $F_h > 0$, $F_{kk} < 0$, $F_{hh} < 0$, $F(k_t, h_t) = by_t = F(bk_t, bh_t)$ with $b > 0$, $\lim_{k \to 0} F_k \to +\infty$, $\lim_{k \to \infty} F_k = 0$, $\lim_{h \to 0} F_h \to +\infty$, $\lim_{h \to \infty} F_h = 0$.

To rule out the possibility that the representative household could borrow indefinitely and roll over its debt forever (i.e., engage in Ponzi schemes), I assume that the representative household faces a natural debt limit. Households are subject to the following sequence of borrowing constraints, which prevent participation in Ponzi games (No-Ponzi Game Condition):

$$\lim_{j \to \infty} E_t \left[\frac{d_{t+j}}{\prod_{s=0}^{j} (1 + r_s)} \right] = 0 \tag{5}$$

Households choose the sequences $\{c_t, i_t, y_t, h_t, k_{t+1}, d_t\}_{t=0}^{\infty}$ to maximize the utility function U given by (1), subject to constraints (2), (3), (4), and (5).

Substituting formulas (3) and (4) into equation (2):

$$d_t = (1 + r_{t-1})d_{t-1} - A_t k_t^{\alpha} h_t^{1-\alpha} + c_t + k_{t+1} - (1 - \delta)k_t + \frac{\phi}{2}(k_{t+1} - k_t)^2 \tag{6}$$

The dynamic Lagrangian corresponding to the representative household optimization problem is:

$$\begin{aligned} \max_{\{d_t, c_t, h_t, k_{t+1}, \mu_t\}} \quad \mathbb{E}_0 \sum_{t=0}^{\infty} \beta^t \Bigg[& \frac{\left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{1-\sigma} - 1}{1 - \sigma} \\ & + \mu_t \Big(d_t - (1 + r_{t-1})d_{t-1} + A_t k_t^{\alpha} h_t^{1-\alpha} - c_t - k_{t+1} + (1 - \delta)k_t - \frac{\phi}{2}(k_{t+1} - k_t)^2 \Big) \Bigg] \end{aligned} \tag{7}$$

Where $\beta^t \mu_t$ is the Lagrange multiplier [present value of the marginal utility of consumption according to formula (9)] associated with the budget constraint of period t.

The first-order conditions (FOCs) are given by:

$$\frac{\partial \mathcal{L}}{\partial d_t} = \beta^t \left\{ \mu_t - \beta(1 + r_t)E_t\left[\mu_{t+1}\right] \right\} = 0 \leftrightarrow \mu_t = \beta(1 + r_t)E_t\left[\mu_{t+1}\right] \tag{8}$$

$$\frac{\partial \mathcal{L}}{\partial c_t} = \beta^t \left[u_c(c_t, h_t) - \mu_t\right] = \beta^t \left[\left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{-\sigma} - \mu_t \right] = 0 \leftrightarrow \mu_t = u_c(c_t, h_t) = \left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{-\sigma} \tag{9}$$

$$\begin{aligned} & \frac{\partial \mathcal{L}}{\partial h_t} = \beta^t \left[u_h(c_t, h_t) + \mu_t A_t F_h(k_t, h_t)\right] = \beta^t \left[-\left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{-\sigma} h_t^{\omega-1} + (1 - \alpha)\, \mu_t A_t k_t^{\alpha} h_t^{-\alpha} \right] = 0 \\ & \leftrightarrow \left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{-\sigma} h_t^{\omega-1} = (1 - \alpha)\, \mu_t A_t k_t^{\alpha} h_t^{-\alpha} \end{aligned} \tag{10}$$

$$\frac{\partial \mathcal{L}}{\partial k_{t+1}} = -\beta^t \mu_t[1 + \Phi'(k_{t+1} - k_t)] + \beta^{t+1} E_t\Big[\mu_{t+1}[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})]\Big] = 0$$

$$\mu_t[1 + \Phi'(k_{t+1} - k_t)] = \beta E_t\Big[\mu_{t+1}[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2} - k_{t+1})]\Big]$$

From equation (2), taking into account that $\Phi\left(k_{t+1} - k_t\right) = \frac{\phi}{2}\left(k_{t+1} - k_t\right)^2$:

$$\mu_t[1 + \phi(k_{t+1} - k_t)] = \beta E_t\Big[\mu_{t+1}[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \phi(k_{t+2} - k_{t+1})]\Big] \tag{11}$$

$$\frac{\partial \mathcal{L}}{\partial \mu_t} = \beta^t \left[d_t - (1 + r_{t-1})d_{t-1} + A_t k_t^{\alpha} h_t^{1-\alpha} - c_t - k_{t+1} + (1 - \delta)\, k_t - \frac{\phi}{2}\left(k_{t+1} - k_t\right)^2 \right] = 0$$

$$A_t k_t^{\alpha} h_t^{1-\alpha} + (1 - \delta)\, k_t + d_t = c_t + (1 + r_{t-1})d_{t-1} + k_{t+1} + \frac{\phi}{2}\left(k_{t+1} - k_t\right)^2 \tag{12}$$

Replacing formula (9) in equation (10) yields the intratemporal optimality condition (the labor supply):

$$-\frac{u_h(c_t,h_t)}{u_c(c_t,h_t)} = A_t F_h(k_t,h_t) \rightarrow \frac{\left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{-\sigma} h_t^{\omega-1}}{\left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{-\sigma}} = A_t(1-\alpha)k_t^{\alpha}h_t^{-\alpha} \rightarrow h_t^{\omega-1} = A_t(1-\alpha)k_t^{\alpha}h_t^{-\alpha} \tag{13}$$

The representative household's labor–supply condition states that the (absolute) marginal rate of substitution between leisure/labor and consumption —which rises with hours worked when consumption is held constant— must equal the marginal product of labor (i.e., labor demand, the real wage in the decentralized equilibrium), which falls as hours worked increase while the capital stock is kept constant.

*

The variable A_t is taken to be the sole driver of aggregate fluctuations in the model. Its long-run growth component is ignored, and it is assumed to follow a first-order autoregressive process:

$$ln(A_t) = \rho ln(A_{t-1}) + \eta\varepsilon_t, \quad \rho \in (-1,1) \tag{14}$$

Where ρ governs the serial autocorrelation of A_t, and ε_t are white-noise innovations (exogenous and stochastic technology shocks), Gaussian, independent, and identically distributed such that:

$$\mathbb{E}_t(\varepsilon_t) = 0, \qquad \mathbb{E}_t(\varepsilon_t,\, \varepsilon_m) = \begin{cases} \sigma_\varepsilon^2, & \text{if } t = m, \\ 0, & \text{if } t \neq m. \end{cases}$$

According to equation (14), the conditional expectation of next period's productivity shock—given the information available at time t—is equal to a fraction ρ of the current productivity disturbance:

$$E_t\left[ln\left(A_{t+1}\right)\right] = \rho ln(A_t) \tag{15}$$

More broadly, under the assumed AR(1) structure of the productivity shock, its expected value j periods ahead, conditional on information available today, equals a fraction ρ^j of its current value.

$$E_t\left[ln\left(A_{t+j}\right)\right] = \rho^j ln(A_t) \tag{16}$$

*

It is assumed that the interest rate faced by domestic agents, r_t, rises with the economy's cross-sectional average level of external debt, denoted by $\widetilde{d}_t$. Formally, the interest rate is given by:

$$r_t = r^o + p\left(\widetilde{d}_t\right) \tag{17}$$

Where r^o is the constant world interest rate and $p(\cdot)$ represents a country-specific interest rate premium. Households take the path of $\widetilde{d}_t$ as exogenously determined. For simplicity, the world interest rate r^o is assumed to be constant, and the premium function $p(\cdot)$ is taken to be strictly increasing. As will be shown shortly, assuming a debt-elastic interest rate premium guarantees that the model possesses a steady state independent of initial conditions. Moreover, this assumption ensures that a first-order approximation of the equilibrium dynamics converges to the exact (nonlinear) dynamics as the magnitude of the underlying shocks becomes small.

- Following to Schmitt-Grohé and Uribe (2003), the country's interest-rate premium is given by:

$$p\left(\widetilde{d}_t\right) = \psi_1\left(e^{\widetilde{d}_t - \bar{d}} - 1\right) \tag{18}$$

Where $\psi_1 > 0$ and $\bar{d}$ are parameters. Under this specification, the country risk premium is an increasing and convex function of net external debt.

Rational Expectations Equilibrium

Because all agents are assumed to be identical, in equilibrium the average level of debt across agents must coincide with each agent's individual debt level, that is:

$$\widetilde{d}_t = d_t \tag{19}$$

Substituting formula (18) into equation (17) and taking into account equation (19):

$$r_t = r^o + p\left(d_t\right) = r^o + \psi_1\left(e^{d_t - \bar{d}} - 1\right) \tag{20}$$

Substituting formula (20) into equation (6):

$$d_t = \left[1 + r^o + \psi_1\left(e^{d_{t-1} - \bar{d}} - 1\right)\right] d_{t-1} - A_t k_t^{\alpha} h_t^{1-\alpha} + c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi}{2}(k_{t+1} - k_t)^2 \tag{21}$$

Replacing equation (9) into (8) yields the Euler equation for the optimal intertemporal allocation of consumption:

$$u_c(c_t,h_t) = \beta(1+r_t)E_t\Big[u_c(c_{t+1},h_{t+1})\Big] \rightarrow \left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{-\sigma} = \beta(1+r_t)E_t\left[\left(c_{t+1} - \frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\right] \tag{22}$$

The Euler equation states that, at the margin, the representative household is willing to give up one unit of current consumption—valued at its marginal utility—if it is compensated with an additional unit of consumption in the next period, adjusted by the relevant (discounted) interest rate and evaluated according to its uncertain contribution to utility (Lubik, 2007).

Substituting equation (20) into formula (22):

$$u_c(c_t,h_t)=\beta\left[1+r^o+p\left(d_t\right)\right]E_t\Big[u_c(c_{t+1},h_{t+1})\Big]$$

$$\left(c_t-\frac{h_t^{\omega}}{\omega}\right)^{-\sigma}=\beta\left[1+r^o+\psi_1\left(e^{d_t-\bar{d}}-1\right)\right]E_t\left[\left(c_{t+1}-\frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\right] \tag{23}$$

Replacing formula (9) in ecuation (11) yields the following expression:

$$\left(c_t-\frac{h_t^{\omega}}{\omega}\right)^{-\sigma}\left[1+\phi\left(k_{t+1}-k_t\right)\right]=\beta E_t\left\{\left(c_{t+1}-\frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\left[A_{t+1}\alpha k_{t+1}^{\alpha-1}h_{t+1}^{1-\alpha}+1-\delta+\phi\left(k_{t+2}-k_{t+1}\right)\right]\right\} \tag{24}$$

Substituting equation (20) into formula (5):

$$\lim_{j\to\infty}E_t\left[\frac{d_{t+j}}{\prod_{s=0}^{j}\left[1+r^o+p\left(d_s\right)\right]}\right]=\lim_{j\to\infty}E_t\left[\frac{d_{t+j}}{\prod_{s=0}^{j}\left[r^o+\psi_1\left(e^{d_s-\bar{d}}-1\right)\right]}\right]=0 \tag{25}$$

A rational expectations equilibrium in this centralized economy is defined as a sequence of processes $\{d_t,c_t,h_t,k_{t+1},A_t\}_{t=0}^{\infty}$ that satisfy equations (13), (14), (21), (23), (24), and (25), given the initial condition d_{-1}, A_0, k_0, and the exogenous sequence $\{\varepsilon_t\}_{t=0}^{\infty}$.

Given the equilibrium paths of consumption, labor, capital, and debt, investment can be obtained from equation (3), output from formula (4), and the interest rate from equation (20). With these components, one can then construct the equilibrium path of the trade balance tb_t using the market-clearing condition for the domestic goods and services market under zero government spending:

$$y_t=c_t+i_t+\frac{\phi}{2}\left(k_{t+1}-k_t\right)^2+tb_t$$

$$tb_t=y_t-c_t-i_t-\frac{\phi}{2}\left(k_{t+1}-k_t\right)^2 \tag{26}$$

Additionally, the current account ca_t can be computed as the difference between the trade balance and the interest payments on the previous period's external debt, or equivalently as the (absolute) change in the country's external debt:

$$ca_t=tb_t-r_{t-1}d_{t-1}=d_{t-1}-d_t \tag{27}$$

Note 02:
The non-linear model can be summarised as follows:

$$\begin{cases}
h_t^{\omega-1}=A_t(1-\alpha)k_t^{\alpha}h_t^{-\alpha}\\
d_t=\left[1+r^o+\psi_1\left(e^{d_{t-1}-\bar{d}}-1\right)\right]d_{t-1}-A_tk_t^{\alpha}h_t^{1-\alpha}+c_t+k_{t+1}-(1-\delta)k_t+\frac{\phi}{2}(k_{t+1}-k_t)^2\\
\left(c_t-\frac{h_t^{\omega}}{\omega}\right)^{-\sigma}=\beta\left[1+r^o+\psi_1\left(e^{d_t-\bar{d}}-1\right)\right]E_t\left[\left(c_{t+1}-\frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\right]\\
\left(c_t-\frac{h_t^{\omega}}{\omega}\right)^{-\sigma}\left[1+\phi\left(k_{t+1}-k_t\right)\right]=\beta E_t\left\{\left(c_{t+1}-\frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\left[A_{t+1}\alpha k_{t+1}^{\alpha-1}h_{t+1}^{1-\alpha}+1-\delta+\phi\left(k_{t+2}-k_{t+1}\right)\right]\right\}\\
\ln\mathrm{A}_t=\rho lnA_{t-1}+\eta\varepsilon_t\\
y_t=A_tk_t^{\alpha}h_t^{1-\alpha}\\
i_t=k_{t+1}-(1-\delta)\,k_t\\
ca_t=tb_t-r_{t-1}d_{t-1}=d_{t-1}-d_t\\
cay_t=\frac{ca_t}{y_t}\\
r_t=r^o+p\left(d_t\right)=r^o+\psi_1\left(e^{d_t-\bar{d}}-1\right)\\
tb_t=y_t-c_t-i_t-\frac{\phi}{2}\left(k_{t+1}-k_t\right)^2\\
tby_t=\frac{tb_t}{y_t}\\
\lim_{j\to\infty}E_t\left[\frac{d_{t+j}}{\prod_{s=0}^{j}\left[r^o+\psi_1\left(e^{d_s-\bar{d}}-1\right)\right]}\right]=0
\end{cases} \tag{*}$$

Non-stochastic Steady-State

The deterministic (non-stochastic) steady state corresponds to the model's equilibrium under two conditions: (i) all exogenous shocks are set to zero in the current period and throughout the entire past, and (ii) uncertainty is absent, meaning that agents perfectly foresee that no shocks will occur in the future. Variables written without a time subscript and with the subscript SS represent their steady-state values: $A_{SS},\ y_{SS},\ k_{SS},\ h_{SS},\ c_{SS},\ d_{SS},\ ca_{SS},\ tb_{SS}$.

From equation (2):

$$d_{SS}=(1+r_{SS})d_{SS}-y_{SS}+c_{SS}+i_{SS}+\frac{\phi}{2}\left(k_{SS}-k_{SS}\right)^2$$

$$\boxed{r_{SS}d_{SS}-y_{SS}+c_{SS}+i_{SS}=0} \tag{28}$$

From formula (3):

$$k_{SS} = (1-\delta)\, k_{SS} + i_{SS}$$

$$\boxed{i_{SS} = \delta k_{SS}} \tag{29}$$

From equation (4):

$$\boxed{y_{SS} = A_{SS}F(k_{SS}, h_{SS}) = A_{SS}k_{SS}^{\alpha}h_{SS}^{1-\alpha}} \tag{30}$$

From formula (13):

$$h_{SS}^{\omega-1} = A_{SS}(1-\alpha)k_{SS}^{\alpha}h_{SS}^{-\alpha} \tag{31}$$

From equation (14), taking into account that in the steady state $\varepsilon_{SS} = 0$:

$$\mathrm{lnA}_{SS} = \rho lnA_{SS} + \eta\varepsilon_{SS} = \rho lnA_{SS}$$

$$(1-\rho)\, lnA_{SS} = 0$$

$$\boxed{A_{SS} = 1} \tag{32}$$

From formula (21):

$$d_{SS} = \left[1 + r^{o} + \psi_1\left(e^{d_{SS}-\bar{d}} - 1\right)\right] d_{SS} - A_{SS}k_{SS}^{\alpha}h_{SS}^{1-\alpha} + c_{SS} + k_{SS} - (1-\delta)k_{SS} + \frac{\phi}{2}(k_{SS} - k_{SS})^2$$

$$d_{SS} = \left[1 + r^{o} + \psi_1\left(e^{d_{SS}-\bar{d}} - 1\right)\right] d_{SS} - A_{SS}k_{SS}^{\alpha}h_{SS}^{1-\alpha} + c_{SS} + \delta k_{SS} \tag{33}$$

From equation (23):

$$\left(c_{SS} - \frac{h_{SS}^{\omega}}{\omega}\right)^{-\sigma} = \beta\left[1 + r^{o} + \psi_1\left(e^{d_{SS}-\bar{d}} - 1\right)\right] E_t\left[\left(c_{SS} - \frac{h_{SS}^{\omega}}{\omega}\right)^{-\sigma}\right]$$

$$\left(c_{SS} - \frac{h_{SS}^{\omega}}{\omega}\right)^{-\sigma} = \beta\left[1 + r^{o} + \psi_1\left(e^{d_{SS}-\bar{d}} - 1\right)\right]\left(c_{SS} - \frac{h_{SS}^{\omega}}{\omega}\right)^{-\sigma}$$

$$1 = \beta\left[1 + r^{o} + \psi_1\left(e^{d_{SS}-\bar{d}} - 1\right)\right] \tag{34}$$

For normalization purposes (not to guarantee the stationarity of the model), it is assumed that:

$$\boxed{\beta(1 + r^{o}) = 1} \tag{35}$$

Substituting equation (32) into formula (31):

$$h_{SS}^{\omega-1} = (1-\alpha)k_{SS}^{\alpha}h_{SS}^{-\alpha}$$

$$h_{SS}^{\omega-1} = (1-\alpha)\left(\frac{k_{SS}}{h_{SS}}\right)^{\alpha}$$

$$\boxed{h_{SS} = [(1-\alpha)\kappa^{\alpha}]^{\left(\frac{1}{\omega-1}\right)}} \tag{36}$$

Where $\kappa = k_{SS}/h_{SS}$ is the steady-state capital-labor ratio.

Replacing equation (32) into formula (33) and taking into account that $\kappa = k_{SS}/h_{SS}$:

$$d_{SS} = \left[1 + r^{o} + \psi_1\left(e^{d_{SS}-\bar{d}} - 1\right)\right] d_{SS} - k_{SS}^{\alpha}h_{SS}^{1-\alpha} + c_{SS} + \delta k_{SS}$$

$$0 = \left[r^{o} + \psi_1\left(e^{d_{SS}-\bar{d}} - 1\right)\right] d_{SS} - \kappa^{\alpha}h_{SS} + c_{SS} + \delta k_{SS}$$

$$c_{SS} = -\left[r^{o} + \psi_1\left(e^{d_{SS}-\bar{d}} - 1\right)\right] d_{SS} + \kappa^{\alpha}h_{SS} - \delta k_{SS} \tag{37}$$

Substituting equation (35) into formula (34):

$$0 = \beta\psi_1\left(e^{d_{SS}-\bar{d}} - 1\right)$$

$$\boxed{d_{SS} = \bar{d}} \tag{38}$$

From equation (24), taking into account equation (32) and that $\kappa = k_{SS}/h_{SS}$:

$$\left(c_{SS} - \frac{h_{SS}^{\omega}}{\omega}\right)^{-\sigma} = \beta E_t\left[\left(c_{SS} - \frac{h_{SS}^{\omega}}{\omega}\right)^{-\sigma}\left(\alpha\kappa^{\alpha-1} + 1 - \delta\right)\right]$$

$$\left(c_{SS} - \frac{h_{SS}^{\omega}}{\omega}\right)^{-\sigma} = \beta\left(c_{SS} - \frac{h_{SS}^{\omega}}{\omega}\right)^{-\sigma}\left(\alpha\kappa^{\alpha-1} + 1 - \delta\right)$$

$$1 = \beta\left(\alpha\kappa^{\alpha-1} + 1 - \delta\right)$$

$$\boxed{\kappa = \left(\frac{\beta^{-1} - 1 + \delta}{\alpha}\right)^{\left(\frac{1}{\alpha - 1}\right)}} \tag{39}$$

Considering that $\kappa = k_{SS}/h_{SS}$:

$$\boxed{k_{SS} = \kappa h_{SS}} \tag{40}$$

Replacing equation (38) into formula (37):

$$\boxed{c_{SS} = -r^o \bar{d} + \kappa^\alpha h_{SS} - \delta k_{SS}} \tag{41}$$

From equation (26):

$$tb_{SS} = y_{SS} - c_{SS} - i_{SS} - \frac{\phi}{2}(k_{SS} - k_{SS})^2$$

$$\boxed{tb_{SS} = y_{SS} - c_{SS} - i_{SS}} \tag{42}$$

From formula (27):

$$\boxed{ca_{SS} = tb_{SS} - r_{SS} d_{SS} = \hat{d}_{SS} - \hat{d}_{SS} = 0} \tag{43}$$

From equation (20), taking into account equation (38):

$$\boxed{r_{SS} = r^o} \tag{44}$$

Linearization of the model

Now the system of difference equations of the non-linear SOE RBC model will be linearized around its steady state in terms of log deviations $\hat{x}_t = (x_t - x_{SS})/x_{SS} \approx \ln(x_t) - \ln(x_{SS}) \leftrightarrow x_{SS} \neq 0$, for $x_t = y_t, c_t, h_t, k_t, A_t$, and with respect to deviations in levels $\hat{z}_t = z_t - z_{SS}$, for $z_t = r_t, d_t, ca_t, tb_t$. For a nonlinear function f of several variables, I use one of the following first-order Taylor linearizations around their respective steady states, as appropriate:

$$f(\vec{x}_t) \approx f(\vec{x}^*) + \frac{\partial f(\vec{x}^*)}{\partial x_{1,t}} \hat{x}_{1,t} x_{1,SS} + \cdots + \frac{\partial f(\vec{x}^*)}{\partial x_{n,t}} \hat{x}_{n,t} x_{n,SS} \leftrightarrow x_{i,SS} \neq 0 \;\; (i = 1, 2, \cdots, n) \tag{45}$$

Where $\vec{x}^* = (x_{1,SS}, x_{2,SS}, \ldots, x_{n,SS})$ and $x_{i,t} - x_{i,SS} = \hat{x}_{i,t} x_{i,SS} \leftrightarrow x_{iSS} \neq 0 \;\; (i = 1, 2, \cdots, n)$.

$$f(\vec{z}_t) \approx f(\vec{z}^*) + \frac{\partial f(\vec{z}^*)}{\partial z_{1,t}} \hat{z}_{1,t} + \cdots + \frac{\partial f(\vec{z}^*)}{\partial z_{n,t}} \hat{z}_{n,t} \tag{46}$$

Where $\vec{z}^* = (z_{1,SS}, z_{2,SS}, \ldots, z_{n,SS})$ and $z_{i,t} - z_{i,SS} = \hat{z}_{i,t} \; (i = 1, 2, \cdots, n)$.

Also, for $x_t = y_t, c_t, h_t, k_t, A_t$, I also employ Uhlig's (1999) linearization method, which does not require calculating derivatives and is based on specific approximation techniques. In particular, for $\hat{x}_t \approx \ln x_t - \ln x_{SS}$, the following approximation will be employed:

$$x_t \approx x_{SS} e^{\hat{x}_t} \tag{47}$$

Furthermore, let y_t be a variable such that $\hat{y}_t$ (its logarithmic deviation from its steady state) is a real number close to zero; then, the following approximations are also valid:

$$\hat{x}_t \hat{y}_t \approx 0 \tag{48}$$

$$e^{\hat{x}_t + a\hat{y}_t} \approx 1 + \hat{x}_t + a\hat{y}_t \tag{49}$$

The equation to linearize is (13):

$$h_t^{\omega - 1} = A_t (1 - \alpha) k_t^\alpha h_t^{-\alpha}$$

Considering equation (47):

$$\left[h_{SS} e^{\hat{h}_t}\right]^{w-1} \approx A_{SS} e^{\hat{A}_t} (1 - \alpha) \left[k_{SS} e^{\hat{k}_t}\right]^\alpha \left[h_{SS} e^{\hat{h}_t}\right]^{-\alpha}$$

$$h_{SS}^{w-1} e^{(w-1)\hat{h}_t} \approx \left[(1-\alpha) A_{SS} {k_{SS}}^\alpha {h_{SS}}^{-\alpha}\right] e^{\left(\hat{A}_t + \alpha \hat{k}_t - \alpha \hat{h}_t\right)}$$

$$h_{SS}^{w-1} e^{(w-1)\hat{h}_t} \approx \left[(1-\alpha) A_{SS} \left(\frac{k_{SS}}{h_{SS}}\right)^\alpha\right] e^{\left(\hat{A}_t + \alpha \hat{k}_t - \alpha \hat{h}_t\right)}$$

Taking into account that $\kappa = k_{SS}/h_{SS}$:

$$h_{SS}^{w-1} e^{(w-1)\hat{h}_t} \approx \left[(1-\alpha) A_{SS} \kappa^\alpha\right] e^{\left(\hat{A}_t + \alpha \hat{k}_t - \alpha \hat{h}_t\right)}$$

According to equations (32) and (36):

$$e^{(w-1)\hat{h}_t} \approx e^{\hat{A}_t + \alpha \hat{k}_t - \alpha \hat{h}_t}$$

$$\boxed{(\alpha + w - 1)\hat{h}_t \approx \hat{A}_t + \alpha \hat{k}_t} \tag{50}$$

The equation to linearize is (21):

$$d_t = \left[1 + r^o + \psi_1\left(e^{d_{t-1}-\bar{d}} - 1\right)\right] d_{t-1} - A_t k_t^{\alpha} h_t^{1-\alpha} + c_t + k_{t+1} - (1-\delta)k_t + \frac{\phi}{2}(k_{t+1} - k_t)^2$$

Considering formula (47) and that $\hat{d}_t = d_t - d_{SS} = d_t - \bar{d}$:

$$\hat{d}_t + \bar{d} \;\approx \left[1 + r^o + \psi_1\left(\frac{e^{d_{t-1}}}{e^{\bar{d}}} - 1\right)\right](\hat{d}_{t-1} + \bar{d}) - A_{SS} k_{SS}^{\alpha} h_{SS}^{1-\alpha}\, e^{\left[\hat{A}_t + \alpha \hat{k}_t + (1-\alpha)\hat{h}_t\right]}$$

$$+\; c_{SS} e^{\hat{c}_t} + k_{SS} e^{\hat{k}_{t+1}} - (1-\delta)\, k_{SS} e^{\hat{k}_t} + \frac{\phi}{2}\left(k_{SS} e^{\hat{k}_{t+1}} - k_{SS} e^{\hat{k}_t}\right)^2$$

Taking into account expressions (30), (46) and (49):

$$\hat{d}_t + \bar{d} \;\approx \left[1 + r^o + \psi_1\left(\frac{e^{\bar{d}} + e^{\bar{d}}\hat{d}_{t-1}}{e^{\bar{d}}} - 1\right)\right](\hat{d}_{t-1} + \bar{d}) - y_{SS}\left[1 + \hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t\right]$$

$$+\, c_{SS}(1 + \hat{c}_t) + k_{SS}(1 + \hat{k}_{t+1}) - (1-\delta)k_{SS}(1 + \hat{k}_t) + \frac{\phi k_{SS}^2}{2}\,(\hat{k}_{t+1} - \hat{k}_t)^2$$

$$\hat{d}_t + \bar{d} \;\approx \left(1 + r^o + \psi_1\hat{d}_{t-1}\right)(\hat{d}_{t-1} + \bar{d}) - y_{SS}\left[1 + \hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t\right]$$

$$+\; c_{SS}(1 + \hat{c}_t) + k_{SS}(1 + \hat{k}_{t+1}) - (1-\delta)k_{SS}(1 + \hat{k}_t) + \frac{\phi k_{SS}^2}{2}\,(\hat{k}_{t+1}^2 - 2\hat{k}_{t+1}\hat{k}_t + \hat{k}_t^2)$$

Considering formula (48), the higher-order terms (products of small deviations) are neglecting. Then, $\hat{k}_{t+1}^2 \approx 0$, $\hat{k}_{t+1}\hat{k}_t \approx 0$, $\hat{k}_t^2 \approx 0$, and $\hat{d}_{t-1}^2 \approx 0$:

$$\hat{d}_t \approx (1 + r^o)\,\hat{d}_{t-1} + \bar{d}\,r^o + \psi_1\bar{d}\,\hat{d}_{t-1} - y_{SS} - y_{SS}\left[\hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t\right] + c_{SS} + c_{SS}\hat{c}_t + k_{SS}\hat{k}_{t+1} + \delta k_{SS} - (1-\delta)k_{SS}\hat{k}_t$$

Taking into account formulas (28) and (29):

$$\hat{d}_t \approx \left(1 + r^o + \psi_1\bar{d}\right)\hat{d}_{t-1} - y_{SS}\left[\hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t\right] + c_{SS}\hat{c}_t + k_{SS}\left[\hat{k}_{t+1} - (1-\delta)\hat{k}_t\right]$$

$$\boxed{\left(\frac{1}{y_{SS}}\right)\hat{d}_t \approx \left(\frac{1 + r^o + \psi_1\bar{d}}{y_{SS}}\right)\hat{d}_{t-1} + \left(\frac{c_{ss}}{y_{SS}}\right)\hat{c}_t + \left(\frac{k_{SS}}{y_{SS}}\right)\left[\hat{k}_{t+1} - (1-\delta)\hat{k}_t\right] - \hat{A}_t - \alpha\hat{k}_t - (1-\alpha)\hat{h}_t} \tag{51}$$

The equation to linearize is (23):

$$\left(c_t - \frac{h_t^{\omega}}{\omega}\right)^{-\sigma} = \beta\left[1 + r^o + \psi_1\left(e^{d_t - \bar{d}} - 1\right)\right] E_t\left[\left(c_{t+1} - \frac{h_{t+1}^{\omega}}{\omega}\right)^{-\sigma}\right]$$

Considering formula (45):

$$\left(c_{SS} - \frac{h_{SS}{}^{w}}{w}\right)^{-\sigma} - \sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-\sigma-1}\hat{c}_t + \sigma h_{ss}^{w}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-\sigma-1}\hat{h}_t$$
$$\approx \beta\left[1 + r^o + \psi_1\left(\frac{e^{\bar{d}} + e^{\bar{d}}\hat{d}_t}{e^{\bar{d}}} - 1\right)\right] E_t\left[\left(c_{SS} - \frac{h_{SS}{}^{w}}{w}\right)^{-\sigma} - \sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-\sigma-1}\hat{c}_{t+1} + \sigma h_{ss}^{w}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-\sigma-1}\hat{h}_{t+1}\right]$$

Dividing by $\left(c_{SS} - \frac{h_{SS}{}^{w}}{w}\right)^{-\sigma} > 0$:

$$1 - \sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{c}_t + \sigma h_{ss}^{w}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{h}_t$$
$$\approx \beta\left[1 + r^o + \psi_1\hat{d}_t\right] E_t\left[1 - \sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{c}_{t+1} + \sigma h_{ss}^{w}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{h}_{t+1}\right]$$

Taking into account equation (35):

$$1 - \sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{c}_t + \sigma h_{ss}^{w}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{h}_t$$
$$\approx \left(1 + \beta\psi_1\hat{d}_t\right)\left(1 - \sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1} E_t[\hat{c}_{t+1}] + \sigma h_{ss}^{w}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1} E_t[\hat{h}_{t+1}]\right)$$

Expanding the product on the right-hand side of the approximation symbol:

$$1 - \sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{c}_t + \sigma h_{ss}^{w}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{h}_t$$
$$\approx 1 + \beta\psi_1\hat{d}_t - \sigma\, c_{SS}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1} E_t[\hat{c}_{t+1}] + \sigma\, h_{SS}^{w}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1} E_t[\hat{h}_{t+1}]$$
$$-\; \beta\psi_1\sigma\, c_{SS}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{d}_t\, E_t[\hat{c}_{t+1}] + \beta\psi_1\sigma\, h_{SS}^{w}\left(c_{SS} - \frac{h_{SS}^{w}}{w}\right)^{-1}\hat{d}_t\, E_t[\hat{h}_{t+1}]$$

Neglecting higher-order terms (products of small deviations) such as $\hat{d}_t E_t[\hat{c}_{t+1}]$ and $\hat{d}_t E_t[\hat{h}_{t+1}]$:

$$1-\sigma c_{SS}\Big(c_{SS}-\frac{h_{SS}^w}{w}\Big)^{-1}\hat{c}_t+\sigma h_{ss}^w\Big(c_{SS}-\frac{h_{SS}^w}{w}\Big)^{-1}\hat{h}_t$$
$$\approx 1+\beta\psi_1\hat{d}_t-\sigma\, c_{SS}\Big(c_{SS}-\frac{h_{SS}^w}{w}\Big)^{-1}E_t[\hat{c}_{t+1}]+\sigma\, h_{SS}^w\Big(c_{SS}-\frac{h_{SS}^w}{w}\Big)^{-1}E_t[\hat{h}_{t+1}]$$

$$\boxed{\begin{aligned}&-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t\\&\quad\approx\beta\psi_1\hat{d}_t-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}E_t[\hat{c}_{t+1}]+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}E_t[\hat{h}_{t+1}]\end{aligned}} \tag{52}$$

Note 03:
Since it was assumed that the marginal utility of consumption was strictly positive, $(c_t-h_t^\omega/\omega)^{-\sigma}>0$, in the steady state $c_{ss}-h_{ss}^\omega/\omega>0$ is verified.

The equation to linearize is (24):

$$\Big(c_t-\frac{h_t^\omega}{\omega}\Big)^{-\sigma}\Big[1+\phi(k_{t+1}-k_t)\Big]=\beta E_t\Big\{\Big(c_{t+1}-\frac{h_{t+1}^\omega}{\omega}\Big)^{-\sigma}\Big[A_{t+1}\alpha k_{t+1}^{\alpha-1}h_{t+1}^{1-\alpha}+1-\delta+\phi(k_{t+2}-k_{t+1})\Big]\Big\}$$

Taking into account formulas (45) and (47):

$$\begin{aligned}&\left[\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-\sigma}-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-\sigma-1}\hat{c}_t+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-\sigma-1}\hat{h}_t\right]\left[1+\phi\left(k_{SS}e^{\hat{k}_{t+1}}-k_{SS}e^{\hat{k}_t}\right)\right]\\&\approx\beta E_t\left\{\left[\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-\sigma}-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-\sigma-1}\hat{c}_{t+1}+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-\sigma-1}\hat{h}_{t+1}\right]\right.\\&\quad\left.\times\left[A_{SS}\alpha k_{SS}^{\alpha-1}h_{SS}^{1-\alpha}\,e^{\hat{A}_{t+1}+(\alpha-1)\hat{k}_{t+1}+(1-\alpha)\hat{h}_{t+1}}+1-\delta+\phi k_{SS}\left(e^{\hat{k}_{t+2}}-e^{\hat{k}_{t+1}}\right)\right]\right\}\end{aligned}$$

Dividing by $\left(c_{SS}-\frac{h_{SS}{}^w}{w}\right)^{-\sigma}>0$:

$$\begin{aligned}&\left[1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t\right]\left[1+\phi\left(k_{SS}e^{\hat{k}_{t+1}}-k_{SS}e^{\hat{k}_t}\right)\right]\\&\approx\beta E_t\left\{\left[1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_{t+1}+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_{t+1}\right]\right.\\&\quad\left.\times\left[A_{SS}\alpha k_{SS}^{\alpha-1}h_{SS}^{1-\alpha}\,e^{\hat{A}_{t+1}+(\alpha-1)\hat{k}_{t+1}+(1-\alpha)\hat{h}_{t+1}}+1-\delta+\phi k_{SS}\left(e^{\hat{k}_{t+2}}-e^{\hat{k}_{t+1}}\right)\right]\right\}\end{aligned}$$

Considering (49) and that $A_{SS}=1$ and $\kappa=k_{SS}/h_{SS}$:

$$\begin{aligned}&\left[1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t\right]\left[1+\phi k_{SS}\left(\hat{k}_{t+1}-\hat{k}_t\right)\right]\\&\approx\beta E_t\left\{\left[1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_{t+1}+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_{t+1}\right]\right.\\&\quad\left.\times\left[\alpha\kappa^{\alpha-1}[1+\hat{A}_{t+1}+(\alpha-1)\hat{k}_{t+1}+(1-\alpha)\hat{h}_{t+1}]+1-\delta+\phi k_{SS}\left(\hat{k}_{t+2}-\hat{k}_{t+1}\right)\right]\right\}\end{aligned}$$

Since after substituting (35) into (39) it turns out that $\alpha\kappa^{\alpha-1}=r^o+\delta$:

$$\begin{aligned}&\left[1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t\right]\left[1+\phi k_{SS}\left(\hat{k}_{t+1}-\hat{k}_t\right)\right]\\&\approx\beta E_t\left\{\left[1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_{t+1}+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_{t+1}\right]\right.\\&\quad\left.\times\left[(r^o+\delta)\,[1+\hat{A}_{t+1}+(\alpha-1)\hat{k}_{t+1}+(1-\alpha)\hat{h}_{t+1}]+1-\delta+\phi k_{SS}\left(\hat{k}_{t+2}-\hat{k}_{t+1}\right)\right]\right\}\end{aligned}$$

Expanding the product on the left-hand side of the approximation symbol and neglecting higher-order terms (products of small deviations) such as $\hat{c}_t\hat{k}_t$, $\hat{c}_t\hat{k}_{t+1}$, $\hat{h}_t\hat{k}_{t+1}$, and $\hat{h}_t\hat{k}_t$:

$$\begin{aligned}&1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t\ +\phi k_{SS}\left(\hat{k}_{t+1}-\hat{k}_t\right)\\&\approx\beta E_t\left\{\left[1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_{t+1}+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_{t+1}\right]\right.\\&\quad\left.\times\left[(r^o+\delta)\,[1+\hat{A}_{t+1}+(\alpha-1)\hat{k}_{t+1}+(1-\alpha)\hat{h}_{t+1}]+1-\delta+\phi k_{SS}\left(\hat{k}_{t+2}-\hat{k}_{t+1}\right)\right]\right\}\end{aligned}$$

Expanding the product on the right-hand side of the approximation symbol and neglecting higher-order terms (products of small deviations) such as $\hat{c}_{t+1}\hat{A}_{t+1}$, $\hat{c}_{t+1}\hat{k}_{t+1}$, $\hat{c}_{t+1}\hat{h}_{t+1}$, $\hat{c}_{t+1}\hat{k}_{t+2}$, $\hat{h}_{t+1}\hat{A}_{t+1}$, $\hat{h}_{t+1}\hat{k}_{t+1}$, $\hat{h}_{t+1}\hat{h}_{t+1}$, and $\hat{h}_{t+1}\hat{k}_{t+2}$:

$$\begin{aligned}
&1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t\ +\phi k_{SS}\left(\hat{k}_{t+1}-\hat{k}_t\right)\\
&\approx \beta E_t\Bigg\{(r^o+\delta)\left[1+\hat{A}_{t+1}+(\alpha-1)\hat{k}_{t+1}+(1-\alpha)\hat{h}_{t+1}\right]+1-\delta+\phi k_{SS}\left(\hat{k}_{t+2}-\hat{k}_{t+1}\right)\\
&\quad-(1+r^o)\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_{t+1}+(1+r^o)\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_{t+1}\Bigg\}
\end{aligned}$$

Taking into account formula (35):

$$\begin{aligned}
&1-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t\ +\phi k_{SS}\left(\hat{k}_{t+1}-\hat{k}_t\right)\\
&\approx E_t\Bigg\{\left(\frac{r^o+\delta}{1+r^o}\right)\left[1+\hat{A}_{t+1}+(\alpha-1)\hat{k}_{t+1}+(1-\alpha)\hat{h}_{t+1}\right]+\left(\frac{1-\delta}{1+r^o}\right)+\left(\frac{\phi k_{SS}}{1+r^o}\right)\left(\hat{k}_{t+2}-\hat{k}_{t+1}\right)\\
&\quad-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_{t+1}+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_{t+1}\Bigg\}
\end{aligned}$$

Simplyfying terms:

$$\begin{aligned}
&-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t\ +\phi k_{SS}\left(\hat{k}_{t+1}-\hat{k}_t\right)\\
&\approx E_t\Bigg\{\left(\frac{r^o+\delta}{1+r^o}\right)\left[\hat{A}_{t+1}+(\alpha-1)\hat{k}_{t+1}+(1-\alpha)\hat{h}_{t+1}\right]+\left(\frac{\phi k_{SS}}{1+r^o}\right)\left(\hat{k}_{t+2}-\hat{k}_{t+1}\right)\\
&\quad-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_{t+1}+\sigma h_{SS}^w\left(c_{SS}-\frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_{t+1}\Bigg\}
\end{aligned}$$

Finally, extracting expectations:

$$\boxed{\begin{aligned}
&-\sigma c_{SS}\Big(c_{SS}-\frac{h_{SS}^w}{w}\Big)^{-1}\hat{c}_t+\sigma h_{ss}^w\Big(c_{SS}-\frac{h_{SS}^w}{w}\Big)^{-1}\hat{h}_t+\phi k_{SS}(\hat{k}_{t+1}-\hat{k}_t)\\
&\approx-\sigma c_{SS}\Big(c_{SS}-\frac{h_{SS}^w}{w}\Big)^{-1}E_t[\hat{c}_{t+1}]+\sigma h_{SS}^w\Big(c_{SS}-\frac{h_{SS}^w}{w}\Big)^{-1}E_t[\hat{h}_{t+1}]\\
&+\left(\frac{r^o+\delta}{1+r^o}\right)E_t[\hat{A}_{t+1}+(\alpha-1)(\hat{k}_{t+1}-\hat{h}_{t+1})]+\left(\frac{\phi k_{SS}}{1+r^o}\right)E_t[\hat{k}_{t+2}-\hat{k}_{t+1}]
\end{aligned}} \tag{53}$$

The equation to linearizate is (14):

$$ln(A_t)=\rho ln(A_{t-1})+\eta\varepsilon_t$$

Let $\hat{A}_t$ denote the log deviation of A_t from its steady state, $\hat{A}_t=ln(A_t)-\ln(A_{SS})$. Since, according to equation (31), $Ass=1$ $[\ln(A_{SS})=0]$:

$$\hat{A}_t=\ln(A_t)$$

Then, in $t-1$:

$$\hat{A}_{t-1}=\ln(A_{t-1})$$

Substituting $ln(A_t)$ and $ln(A_{t-1})$ in equation (14):

$$\boxed{\hat{A}_t=\rho\hat{A}_{t-1}+\eta\varepsilon_t} \tag{54}$$

The equation to linearizate is (4):

$$y_t=A_tk_t^\alpha h_t^{1-\alpha}$$

Considering equation (45):

$$y_{SS}+y_{SS}\hat{y}_t\approx\left(A_{SS}k_{SS}^\alpha h_{SS}^{1-\alpha}\right)+k_{SS}^\alpha h_{SS}^{1-\alpha}A_{SS}\hat{A}_t+A_{SS}\alpha k_{SS}^{\alpha-1}h_{SS}^{1-\alpha}k_{SS}\hat{k}_t+A_{SS}k_{SS}^\alpha(1-\alpha)h_{SS}^{-\alpha}h_{SS}\hat{h}_t$$

$$y_{SS}+y_{SS}\hat{y}_t\approx\left(A_{SS}k_{SS}^\alpha h_{SS}^{1-\alpha}\right)+\left(A_{SS}k_{SS}^\alpha h_{SS}^{1-\alpha}\right)\hat{A}_t+\alpha\left(A_{SS}k_{SS}^\alpha h_{SS}^{1-\alpha}\right)\hat{k}_t+(1-\alpha)\left(A_{SS}k_{SS}^\alpha h_{SS}^{1-\alpha}\right)\hat{h}_t$$

Taking into account equation (30):

$$y_{SS}+y_{SS}\hat{y}_t\approx y_{SS}+y_{SS}\hat{A}_t+\alpha y_{SS}\hat{k}_t+(1-\alpha)y_{SS}\hat{h}_t$$

$$\boxed{\hat{y}_t\approx\hat{A}_t+\alpha\hat{k}_t+(1-\alpha)\hat{h}_t} \tag{55}$$

The equation to linearizate is (3):

$$k_{t+1}=(1-\delta)\,k_t+i_t$$

Considering formula (47):

$$k_{SS}e^{\hat{k}_{t+1}} \approx (1-\delta)\, k_{SS}e^{\hat{k}_t} + i_{SS}e^{\hat{i}_t}$$

Taking into account expression (49):

$$k_{SS}(1+\hat{k}_{t+1}) \approx (1-\delta)\, k_{SS}(1+\hat{k}_t) + i_{SS}(1+\hat{i}_t)$$

$$k_{SS} + k_{SS}\hat{k}_{t+1} \approx (1-\delta)k_{SS} + (1-\delta)\, k_{SS}\hat{k}_t + i_{SS} + i_{SS}\hat{i}_t$$

$$k_{SS}\hat{k}_{t+1} \approx -\delta k_{SS} + (1-\delta)\, k_{SS}\hat{k}_t + i_{SS} + i_{SS}\hat{i}_t$$

Considering formula (29):

$$k_{SS}\hat{k}_{t+1} \approx -\delta k_{SS} + (1-\delta)\, k_{SS}\hat{k}_t + \delta k_{SS} + \delta k_{SS}\hat{i}_t$$

$$k_{SS}\hat{k}_{t+1} \approx (1-\delta)\, k_{SS}\hat{k}_t + \delta k_{SS}\hat{i}_t$$

$$\hat{k}_{t+1} \approx (1-\delta)\, \hat{k}_t + \delta\hat{i}_t$$

$$\boxed{\hat{i}_t \approx \frac{\hat{k}_{t+1} - (1-\delta)\hat{k}_t}{\delta}} \tag{56}$$

The equation to linearizate is (27):

$$ca_t = d_{t-1} - d_t$$

Taking into account that $\hat{ca}_t = ca_t - ca_{SS}$ and that $\hat{d}_t = d_t - d_{SS} = d_t - \bar{d}$:

$$ca_{SS} + \hat{ca}_t \approx d_{SS} + \hat{d}_{t-1} - d_{SS} - \hat{d}_t$$

Considering equation (43), $ca_{SS} = 0$:

$$\boxed{\hat{ca}_t \approx \hat{d}_{t-1} - \hat{d}_t} \tag{57}$$

The equation to linearizate is the current-account-to-output ratio, which is given by:

$$cay_t = \frac{ca_t}{y_t} = ca_t {y_t}^{-1}$$

In the steady state, taking into account that $ca_{SS} = 0$ (eq. 43):

$$cay_{SS} = \frac{ca_{SS}}{y_{SS}} = 0$$

Taking into account that $\hat{cay}_t = cay_t - cay_{SS}$ and the equation (46), the linearization of the current-account-to-output ratio $cay_t = ca_t {y_t}^{-1}$ is given by:

$$cay_{SS} + \hat{cay}_t \approx \frac{ca_{SS}}{y_{SS}} + \left(\frac{1}{y_{SS}}\right)\hat{ca}_t - \left(\frac{ca_{SS}}{y_{SS}^2}\right)\hat{y}_t$$

Considering that $ca_{SS} = cay_{SS} = 0$:

$$\boxed{\hat{cay}_t \approx \frac{\hat{ca}_t}{y_{SS}}} \tag{58}$$

The equation to linearizate is (20):

$$r_t = r^o + p\,(d_t) = r^o + \psi_1\left(e^{d_t - \bar{d}} - 1\right)$$

Taking into account that $\hat{r}_t = r_t - r_{SS} = r_t - r^o$:

$$\hat{r}_t = r_t - r^o = p\,(d_t) = \psi_1\left(e^{d_t-\bar{d}} - 1\right) = \psi_1\left(\frac{e^{d_t}}{e^{\bar{d}}} - 1\right)$$

Considering expression (46) and that $\hat{d}_t = d_t - d_{SS} = d_t - \bar{d}$:

$$\hat{r}_t = p(d_t) \approx \psi_1\left(\frac{e^{\bar{d}} + e^{\bar{d}}\hat{d}_t}{e^{\bar{d}}} - 1\right)$$

$$\boxed{\hat{r}_t = p\,(d_t) \approx \psi_1\hat{d}_t} \tag{59}$$

The equation to linearizate is (27):

$$ca_t = tb_t - r_{t-1}d_{t-1}$$

Taking into account (46) and that $\hat{ca}_t = ca_t - ca_{SS}$ and that $\hat{tb}_t = tb_t - tb_{SS}$:

$$ca_{SS} + \hat{ca}_t \approx tb_{SS} + \hat{tb}_t - \left(r_{SS}d_{SS} + d_{SS}\hat{r}_{t-1} + r_{SS}\hat{d}_{t-1}\right)$$

According to formula (43), $ca_{SS} = 0$ and $tb_{SS} = r_{SS}d_{SS}$:

$$\hat{ca}_t \approx \hat{tb}_t - \left(d_{SS}\hat{r}_{t-1} + r_{SS}\hat{d}_{t-1}\right)$$

Substituting equations (44) and (57):

$$\hat{d}_{t-1} - \hat{d}_t \approx \hat{tb}_t - d_{SS}\hat{r}_{t-1} - r^o\hat{d}_{t-1}$$

$$\boxed{\hat{tb}_t \approx (1+r^o)\hat{d}_{t-1} - \hat{d}_t + d_{SS}\hat{r}_{t-1}} \tag{60}$$

The equation to linearizate is the trade-balance-to-output ratio, which is given by:

$$tby_t = \frac{tb_t}{y_t} = tb_t {y_t}^{-1}$$

In the steady state:

$$tby_{SS} = \frac{tb_{SS}}{y_{SS}}$$

Taking into account that $\hat{tby}_t = tby_t - tby_{SS}$ and the equation (46), the linearization of the trade-balance-to-output ratio $tby_t = tb_t {y_t}^{-1}$ is given by:

$$tby_{ss} + \hat{tby}_t \approx \frac{tb_{SS}}{y_{SS}} + \left(\frac{1}{y_{SS}}\right)\hat{tb}_t - \left(\frac{tb_{SS}}{y_{SS}^2}\right)\hat{y}_t$$

Considering that $tby_{SS} = tb_{SS}/y_{SS}$:

$$\boxed{\hat{tby}_t \approx \frac{\hat{tb}_t - \left(\frac{tb_{SS}}{y_{SS}}\right)\hat{y}_t}{y_{SS}}} \tag{61}$$

Note 04:
The linearized model can be summarised as follows:

$$\begin{cases}
(\alpha + w - 1)\hat{h}_t \approx \hat{A}_t + \alpha\hat{k}_t \\
\left(\frac{1}{y_{SS}}\right)\hat{d}_t \approx \left(\frac{1+r^o+\psi_1\bar{d}}{y_{SS}}\right)\hat{d}_{t-1} + \left(\frac{c_{ss}}{y_{SS}}\right)\hat{c}_t + \left(\frac{k_{SS}}{y_{SS}}\right)\left[\hat{k}_{t+1} - (1-\delta)\hat{k}_t\right] - \hat{A}_t - \alpha\hat{k}_t - (1-\alpha)\hat{h}_t \\
-\sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t + \sigma h_{SS}^w\left(c_{SS} - \frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t \\
\approx \beta\psi_1\hat{d}_t - \sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^w}{w}\right)^{-1}E_t[\hat{c}_{t+1}] + \sigma h_{SS}^w\left(c_{SS} - \frac{h_{SS}^w}{w}\right)^{-1}E_t[\hat{h}_{t+1}] \\
-\sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^w}{w}\right)^{-1}\hat{c}_t + \sigma h_{SS}^w\left(c_{SS} - \frac{h_{SS}^w}{w}\right)^{-1}\hat{h}_t + \phi k_{SS}(\hat{k}_{t+1} - \hat{k}_t) \\
\approx -\sigma c_{SS}\left(c_{SS} - \frac{h_{SS}^w}{w}\right)^{-1}E_t[\hat{c}_{t+1}] + \sigma h_{SS}^w\left(c_{SS} - \frac{h_{SS}^w}{w}\right)^{-1}E_t[\hat{h}_{t+1}] \\
\quad + \left(\frac{r^o+\delta}{1+r^o}\right)E_t[\hat{A}_{t+1} + (\alpha-1)(\hat{k}_{t+1} - \hat{h}_{t+1})] + \left(\frac{\phi k_{SS}}{1+r^o}\right)E_t[\hat{k}_{t+2} - \hat{k}_{t+1}] \\
\hat{A}_t = \rho\hat{A}_{t-1} + \eta\varepsilon_t \\
\hat{y}_t \approx \hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t \\
\hat{i}_t \approx \frac{\hat{k}_{t+1} - (1-\delta)\hat{k}_t}{\delta} \\
\hat{ca}_t \approx \hat{d}_{t-1} - \hat{d}_t \\
\hat{cay}_t \approx \frac{\hat{ca}_t}{y_{SS}} \\
\hat{r}_t = r_t - r^o = p(d_t) \approx \psi_1\hat{d}_t \\
\hat{tb}_t \approx (1+r^o)\hat{d}_{t-1} - \hat{d}_t + d_{SS}\hat{r}_{t-1} \\
\hat{tby}_t \approx \frac{\hat{tb}_t - \left(\frac{tb_{SS}}{y_{SS}}\right)\hat{y}_t}{y_{SS}} \\
\lim_{j\to\infty} E_t\left[\frac{d_{t+j}}{\prod_{s=0}^{j}\left[r^o + \psi_1\hat{d}_s\right]}\right] = 0
\end{cases} \tag{**}$$

Simulating the SOE RBC model with Dynare

Variables

```
mod
var
c    // Logarithmic difference between consumption and its steady-state value
h    // Logarithmic difference between labor and its steady-state value
k    // Logarithmic difference between capital stock and its steady-state value
d    // Logarithmic difference between external debt and its steady-state value
A    // Logarithmic difference between TFP and its steady-state value
y    // Logarithmic difference between GDP and its steady-state value
i    // Logarithmic difference between investment and its steady-state value
ca   // Difference between current account and its steady-state value
cay  // Difference between the current-account-to-output ratio and its steady-state value
```

```
r    // Difference between interest rate and its steady-state value [According to equation (59), the linearized risk premium]
tb   // Difference between the trade balance and its steady-state value
tby; // Difference between the trade-balance-to-output ratio and its steady-state value

varexo epsilon;
```

Parameters

```
mod
parameters Ass alpha beta css delta dss eta hss kappa kss iss phi psi1  ro rho rss sigma w yss tbss cass tbyss cayss riskpremiumss;
```

Parameter values

```
mod
sigma=2;
delta=0.1;
ro=0.04;
beta=1/(1+ro);
alpha=0.32;
w=1.455;
phi=0.028;
psi1=0.000742;
rho=0.42;
eta=0.0129;
kappa=((beta^(-1)-1+delta)/alpha)^(1/(alpha-1));
Ass=1;                                          // steady-state TFP expressed in levels
dss=0.7442;                                     // steady-state external debt expressed in levels
rss=ro;                                         // steady-state interest rate expressed as a decimal
hss=((1-alpha)*kappa^(alpha))^(1/(w-1));        // steady-state labor supply expressed in levels
kss=kappa*hss;                                  // steady-state capital stock expressed in levels
yss=kappa^(alpha)*hss;                          // steady-state GDP expressed in levels
css= yss-delta*kss-ro*dss;                      // steady-state consumption expressed in levels
iss=delta*kss;                                  // steady-state investment expressed in levels
tbss=yss-css-iss;                               // steady-state trade balance expressed in levels
cass=tbss-rss*dss;                              // steady-state current account expressed in levels
tbyss=tbss/yss;                                 // steady-state trade-balance-to-output ratio expressed as a decimal
cayss=cass/yss;                                 // steady-state current-account-to-output ratio expressed as a decimal
riskpremiumss=psi1*(exp(dss-dss)-1);            // steady-state risk premium expressed in levels [riskpremiumss =p(dss)=rss-ro=0]
```

Model

```
mod

model (linear);

// Labor supply / production block

alpha*k(-1)+A=(alpha+w-1)*h;

// Debt accumulation

(1/yss)*d=(1/yss)*(psi1*dss+1+ro)*d(-1)+(css/yss)*c+(kss/yss)*(k-(1-delta)*k(-1))-A-alpha*k(-1)-(1-alpha)*h;

// Euler equation

sigma*hss^w*(css-(1/w)*hss^w)^(-1)*h-sigma*css*(css-(1/w)*hss^w)^(-1)*c=beta*psi1*d+sigma*hss^w*(css-(1/w)*hss^w)^(-1)*h(+1)-sigma*css*(css-(1/w)*hss^w)^(-1)*c(+1);

// Capital accumulation FOC

-sigma*css*(css-(1/w)*hss^w)^(-1)*c+sigma*hss^w*(css-(1/w)*hss^w)^(-1)*h+phi*kss*(k-k(-1))=-sigma*css*(css-(1/w)*hss^w)^(-1)*c(+1)+sigma*hss^w*(css-(1/w)*hss^w)^(-1)*h(+1)+((ro+delta)/(1+ro))*(A(+1)+(alpha-1)*(k-h(+1)))+(phi*kss/(1+ro))*(k(+1)-k);

// TFP shock

A=rho*A(-1)+eta*epsilon;

// National accounts

y=A+alpha*k(-1)+(1-alpha)*h;
i=(k-(1-delta)*k(-1))/delta;
ca=d(-1)-d;
cay=ca/yss;
r=psi1*d;
tb=(1+ro)*d(-1)-d+dss*r(-1);
tby=(tb-(tbss/yss)*y)/yss;

end;
```

Steady State

```
mod
initval;
c = 0;
h = 0;
k = 0;
d = 0;
A = 0;
y = 0;
i = 0;
ca = 0;
cay= 0;
r= 0;
tb = 0;
tby= 0;
end;

steady;
```

Displaying steady state in *levels*

```
mod
disp('========= STEADY STATE IN LEVELS =========');
disp(['A_ss   = ', num2str(Ass)]);
disp(['d_ss   = ', num2str(dss)]);
disp(['h_ss   = ', num2str(hss)]);
disp(['k_ss   = ', num2str(kss)]);
disp(['y_ss   = ', num2str(yss)]);
disp(['c_ss   = ', num2str(css)]);
disp(['i_ss   = ', num2str(iss)]);
disp(['tb_ss  = ', num2str(tbss)]);
disp(['ca_ss  = ', num2str(cass)]);
disp(['r_ss   = ', num2str(rss)]);
disp(['tby_ss   = ', num2str(tbyss)]);
disp(['cay_ss   = ', num2str(cayss)]);
disp(['riskpremium_ss   = ', num2str(riskpremiumss)]);
disp('==========================================');
```

Checking the Blanchard-Kahn conditions

```
mod
check;
```

TPF Schock

```
mod
shocks;
var epsilon; stderr 1/eta; //Normalize to a shock of unit magnitude
end;
```

Simulation of impulse-response functions

Normalized Impulse response functions (IRFs)

```
mod
disp("============== Normalized IRFs (stderr = 1/eta) ==============");
stoch_simul(irf=11, order=1, nograph);


figure(1);


% Detect actual length generated by Dynare
n_irf = length(y_epsilon);

% Horizon according to Dynare
horizon = 0:(n_irf-1);

subplot(4,3,1);  plot(horizon, y_epsilon, 'b', 'LineWidth', 2);   grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{y_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);
title('GDP','FontSize',12);
subplot(4,3,2);  plot(horizon, c_epsilon, 'b', 'LineWidth', 2);   grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{c_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);
title('Consume','FontSize',12);
subplot(4,3,3);  plot(horizon, i_epsilon, 'b', 'LineWidth', 2);   grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{i_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);
title('Investment','FontSize',12);

subplot(4,3,4);  plot(horizon, h_epsilon, 'b', 'LineWidth', 2);   grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{h_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);
title('Labor','FontSize',12);
```

```
subplot(4,3,5);  plot(horizon, tby_epsilon, 'b', 'LineWidth', 2); grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{tby_t}$','Interpreter','latex','FontSize',13);  set(get(gca,'ylabel'),'Rotation',0);
title('Trade balance / GDP','FontSize',12);
subplot(4,3,6);  plot(horizon, A_epsilon, 'r', 'LineWidth', 2);   grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{A_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);       title('TFP
shock','FontSize',12);

subplot(4,3,7);  plot(horizon, d_epsilon, 'b', 'LineWidth', 2);   grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{d_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);
title('External Debt','FontSize',12 );
subplot(4,3,8);  plot(horizon, cay_epsilon, 'b', 'LineWidth', 2); grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{cay_t}$','Interpreter','latex','FontSize',13);  set(get(gca,'ylabel'),'Rotation',0);
title('Current account / GDP','FontSize',12);
subplot(4,3,9);  plot(horizon, k_epsilon, 'b', 'LineWidth', 2);   grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{k_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);
title('Capital stock','FontSize',12);

subplot(4,3,10); plot(horizon, r_epsilon, 'b', 'LineWidth', 2);   grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{r_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);       title('Risk
premium','FontSize',12);
subplot(4,3,11); plot(horizon, tb_epsilon, 'b', 'LineWidth', 2);  grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{tb_t}$','Interpreter','latex','FontSize',13);   set(get(gca,'ylabel'),'Rotation',0);
title('Trade balance','FontSize',12);
subplot(4,3,12); plot(horizon, ca_epsilon, 'b', 'LineWidth', 2);  grid on; xlabel('Time','FontSize',11);
ylabel('$\hat{ca_t}$','Interpreter','latex','FontSize',13);   set(get(gca,'ylabel'),'Rotation',0);
title('Current account','FontSize',12);

% Adjust the design to prevent subplots from overlapping.
set(gcf,'Position',[100 100 1200 800]);
```

Stochastic simulations

```
mod

disp("============== Stochastic Moments (stderr = 1) ==============");
shocks;
var epsilon = 1;
end;

stoch_simul(irf=0, order=1, periods = 150000, drop = 50000);


figure(2);

subplot(4,3,1); plot(y); title('GDP','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{y_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,2); plot(c); title('Consume','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{c_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,3); plot(i); title('Investment','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{i_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,4); plot(h); title('Labor','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{h_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,5); plot(tby); title('Trade balance / GDP','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{tby_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,6); plot(A, 'r'); title('TFP shock','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{A_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,7); plot(d); title('External Debt','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{d_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,8); plot(cay); title('Current account/ GDP','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{cay_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,9); plot(k); title('Capital stock','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{k_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,10); plot(r); title('Risk premium','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{r_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,11); plot(tb); title('Trade balance','FontSize',12); xlabel('Time','FontSize',11); ylabel("tb");
set(get(gca,'ylabel'),'Rotation',0);

subplot(4,3,12); plot(ca); title('Current account','FontSize',12); xlabel('Time','FontSize',11);
ylabel('$\hat{ca_t}$','Interpreter','latex','FontSize',13);    set(get(gca,'ylabel'),'Rotation',0);

% Adjust the design to prevent subplots from overlapping.
set(gcf,'Position',[100 100 1200 800]);
```

Outcomes from Dynare numerical simulations

Outcomes generated by the simulation of a unit, unexpected, one-period TFP disturbance

```
STEADY-STATE RESULTS:

c             0
h             0
k             0
d             0
A             0
y             0
i             0
ca            0
cay           0
r             0
tb            0
tby           0

========= STEADY STATE IN LEVELS =========
A_ss   = 1
d_ss   = 0.7442
h_ss   = 1.0074
k_ss   = 3.3977
y_ss   = 1.4865
c_ss   = 1.117
i_ss   = 0.33977
tb_ss  = 0.029768
ca_ss  = 7.2858e-17
r_ss   = 0.04
tby_ss    = 0.020026
cay_ss    = 4.9014e-17
riskpremium_ss    = 0
==========================================


EIGENVALUES:
         Modulus             Real         Imaginary

       1.288e-16       -1.288e-16                 0
            0.42             0.42                 0
          0.4779           0.4779                 0
          0.9967           0.9967                 0
           1.044            1.044                 0
           2.176            2.176                 0
       5.732e+14        5.732e+14                 0
             Inf              Inf                 0


There are 4 eigenvalue(s) larger than 1 in modulus
for 4 forward-looking variable(s)

The rank condition is verified.
```

```
============== Normalized IRFs (stderr = 1/eta) ==============

MODEL SUMMARY

  Number of variables:          12
  Number of stochastic shocks:  1
  Number of state variables:    4
  Number of jumpers:            4
  Number of static variables:   6


MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS
Variables        epsilon
epsilon      6009.254252
```

POLICY AND TRANSITION FUNCTIONS

Variable	c	h	k	d	A	y	i	ca	cay	r	tb	tby
d(-1)	-0.039227	0.000000	-0.006592	0.974341	0.000000	0.000000	-0.065918	0.025659	0.017262	0.000723	0.065659	0.044171
r(-1)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.744200	0.500643
k(-1)	0.506431	0.412903	0.500311	-1.685401	0.000000	0.600774	-3.996887	1.685401	1.133815	-0.001251	1.685401	1.125721
A(-1)	0.529381	0.541935	0.282285	0.378288	0.420000	0.788516	2.822848	-0.378288	-0.254484	0.000281	-0.378288	-0.265107
epsilon	0.016260	0.016645	0.008670	0.011619	0.012900	0.024219	0.086702	-0.011619	-0.007816	0.000009	-0.011619	-0.008143

THEORETICAL MOMENTS

VARIABLE	MEAN	STD. DEV.	VARIANCE
c	0.0000	2.0981	4.4020
h	0.0000	1.6423	2.6973
k	0.0000	1.1213	1.2572
d	0.0000	27.8431	775.2404
A	0.0000	1.1019	1.2142
y	0.0000	2.3896	5.7102
i	0.0000	7.0071	49.0990
ca	0.0000	1.6743	2.8031
cay	0.0000	1.1263	1.2686
r	0.0000	0.0207	0.0004
tb	0.0000	2.0474	4.1917
tby	0.0000	1.3780	1.8988

MATRIX OF CORRELATIONS

Variables	c	h	k	d	A	y	i	ca	cay	r	tb	tby
c	1.0000	0.8440	0.9385	-0.6128	0.7524	0.8440	0.5177	0.0654	0.0654	-0.6128	-0.2823	-0.3019
h	0.8440	1.0000	0.9447	-0.0935	0.9645	1.0000	0.6688	0.0503	0.0503	-0.0935	-0.0088	-0.0322
k	0.9385	0.9447	1.0000	-0.3470	0.8345	0.9447	0.4645	0.2539	0.2539	-0.3470	0.0247	0.0026
d	-0.6128	-0.0935	-0.3470	1.0000	0.0201	-0.0935	0.0114	-0.0301	-0.0301	1.0000	0.5259	0.5278
A	0.7524	0.9645	0.8345	0.0201	1.0000	0.9645	0.8306	-0.1936	-0.1936	0.0201	-0.1537	-0.1761
y	0.8440	1.0000	0.9447	-0.0935	0.9645	1.0000	0.6688	0.0503	0.0503	-0.0935	-0.0088	-0.0322
i	0.5177	0.6688	0.4645	0.0114	0.8306	0.6688	1.0000	-0.7068	-0.7068	0.0114	-0.5952	-0.6105
ca	0.0654	0.0503	0.2539	-0.0301	-0.1936	0.0503	-0.7068	1.0000	1.0000	-0.0301	0.8343	0.8328
cay	0.0654	0.0503	0.2539	-0.0301	-0.1936	0.0503	-0.7068	1.0000	1.0000	-0.0301	0.8343	0.8328
r	-0.6128	-0.0935	-0.3470	1.0000	0.0201	-0.0935	0.0114	-0.0301	-0.0301	1.0000	0.5259	0.5278
tb	-0.2823	-0.0088	0.0247	0.5259	-0.1537	-0.0088	-0.5952	0.8343	0.8343	0.5259	1.0000	0.9997
tby	-0.3019	-0.0322	0.0026	0.5278	-0.1761	-0.0322	-0.6105	0.8328	0.8328	0.5278	0.9997	1.0000

COEFFICIENTS OF AUTOCORRELATION

Order	1	2	3	4	5
c	0.7822	0.6367	0.5493	0.4996	0.4721
h	0.6170	0.3603	0.2066	0.1201	0.0733
k	0.7886	0.5562	0.3897	0.2868	0.2276
d	0.9982	0.9952	0.9919	0.9886	0.9853
A	0.4200	0.1764	0.0741	0.0311	0.0131
y	0.6170	0.3603	0.2066	0.1201	0.0733
i	0.0686	-0.1379	-0.1363	-0.0935	-0.0553
ca	0.3220	0.0875	0.0130	-0.0067	-0.0096
cay	0.3220	0.0875	0.0130	-0.0067	-0.0096
r	0.9982	0.9952	0.9919	0.9886	0.9853
tb	0.5276	0.3636	0.3108	0.2960	0.2930
tby	0.5148	0.3533	0.3054	0.2945	0.2941

Figure 01: Impulse-response functions

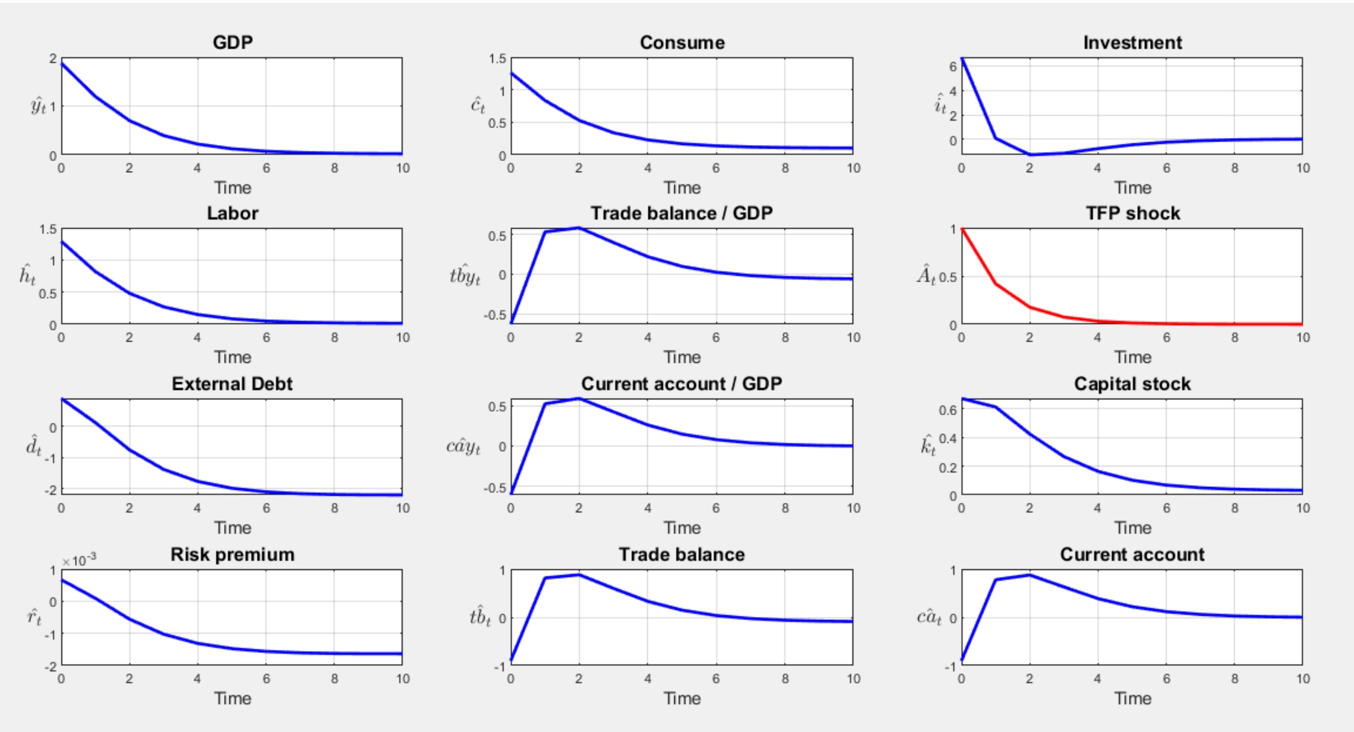

Figure 01 presents the impulse response functions of key macroeconomic variables following a one-percent technology shock in period 0 (The red curve in the Figure 01). In this small open economy RBC framework, the positive total factor productivity (TFP) shock triggers an immediate expansion in output, consumption, investment, and hours worked. Investment reacts particularly strongly—several times larger than the output response—underscoring its central role in generating the initial deterioration of the trade-balance-to-output ratio. This deterioration occurs because the surge in domestic absorption, (i.e., the sum $c_0 + i_0$), more than offsets the contemporaneous increase in production. After its sharp jump, investment temporarily overshoots and later falls slightly below steady state, reflecting intertemporal substitution in capital accumulation. As the TFP shock gradually fades, all real variables converge back to their long-run levels, Uribe & Schmitt-Grohé (2017).

The responses of external sector variables reveal the adjustment process characteristic of small open economies. The trade balance and the current account drop on impact as the economy temporarily increases borrowing from abroad to finance higher domestic absorption. Both series then reverse and rise above baseline before slowly returning to steady state, capturing the gradual correction of external imbalances. This pattern is mirrored in the trajectory of external debt: it initially becomes more negative (higher borrowing), followed by a gradual repayment phase consistent with the improving trade and current account positions.

The capital stock increases smoothly and persistently due to higher investment, displaying the expected hump-shaped path typical of capital accumulation dynamics. Hours worked rise on impact and gradually fall back as the incentives created by higher marginal productivity weaken over time. The real interest rate exhibits a small and short-lived decline, reflecting intertemporal smoothing motives and the temporary nature of the shock.

Altogether, the full set of IRFs illustrates the canonical propagation mechanisms of the SOE RBC model: procyclical behavior of output, consumption, investment, and labor; countercyclical responses of the trade balance and current account; and a gradual external adjustment supported by borrowing and subsequent repayment. The dynamics align with the patterns emphasized in Uribe & Schmitt-Grohé (2017).

Outcomes from stochastic simulations under successive i.i.d. technology shocks drawn from a normal distribution

```
============== Stochastic Moments (stderr = 1) ==============

MODEL SUMMARY

  Number of variables:          12
  Number of stochastic shocks: 1
  Number of state variables:    4
  Number of jumpers:            4
  Number of static variables:   6


MATRIX OF COVARIANCE OF EXOGENOUS SHOCKS
Variables      epsilon
epsilon       1.000000
```

POLICY AND TRANSITION FUNCTIONS

Variable	c	h	k	d	A	y	i	ca	cay	r	tb	tby
d(-1)	-0.039227	0.000000	-0.006592	0.974341	0.000000	0.000000	-0.065918	0.025659	0.017262	0.000723	0.065659	0.044171
r(-1)	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.000000	0.744200	0.500643
k(-1)	0.506431	0.412903	0.500311	-1.685401	0.000000	0.600774	-3.996887	1.685401	1.133815	-0.001251	1.685401	1.125721
A(-1)	0.529381	0.541935	0.282285	0.378288	0.420000	0.788516	2.822848	-0.378288	-0.254484	0.000281	-0.378288	-0.265107
epsilon	0.016260	0.016645	0.008670	0.011619	0.012900	0.024219	0.086702	-0.011619	-0.007816	0.000009	-0.011619	-0.008143

Unconditional Second moments

MOMENTS OF SIMULATED VARIABLES

VARIABLE	MEAN	STD. DEV.	VARIANCE	SKEWNESS	KURTOSIS
c	-0.000119	0.026951	0.000726	0.009001	0.021661
h	-0.000020	0.021188	0.000449	-0.001643	0.016584
k	-0.000037	0.014440	0.000209	-0.000986	0.024033
d	0.002435	0.355655	0.126490	-0.049656	0.059753
A	-0.000004	0.014227	0.000202	-0.004559	0.001448
y	-0.000029	0.030828	0.000950	-0.001643	0.016584
i	-0.000040	0.090558	0.008201	-0.013780	-0.015835
ca	0.000004	0.021623	0.000468	0.007372	0.013445
cay	0.000003	0.014547	0.000212	0.007372	0.013445
r	0.000002	0.000264	0.000000	-0.049656	0.059753
tb	0.000103	0.026354	0.000695	-0.001134	0.002076
tby	0.000070	0.017738	0.000315	-0.001376	0.003221

CORRELATION OF SIMULATED VARIABLES

VARIABLE	c	h	k	d	A	y	i	ca	cay	r	tb	tby
c	1.0000	0.8458	0.9388	-0.6082	0.7551	0.8458	0.5196	0.0640	0.0640	-0.6082	-0.2782	-0.2978
h	0.8458	1.0000	0.9451	-0.0911	0.9646	1.0000	0.6693	0.0482	0.0482	-0.0911	-0.0087	-0.0321
k	0.9388	0.9451	1.0000	-0.3429	0.8352	0.9451	0.4650	0.2527	0.2527	-0.3429	0.0281	0.0059
d	-0.6082	-0.0911	-0.3429	1.0000	0.0215	-0.0911	0.0129	-0.0304	-0.0304	1.0000	0.5213	0.5232
A	0.7551	0.9646	0.8352	0.0215	1.0000	0.9646	0.8310	-0.1958	-0.1958	0.0215	-0.1553	-0.1779
y	0.8458	1.0000	0.9451	-0.0911	0.9646	1.0000	0.6693	0.0482	0.0482	-0.0911	-0.0087	-0.0321
i	0.5196	0.6693	0.4650	0.0129	0.8310	0.6693	1.0000	-0.7079	-0.7079	0.0129	-0.5973	-0.6127
ca	0.0640	0.0482	0.2527	-0.0304	-0.1958	0.0482	-0.7079	1.0000	1.0000	-0.0304	0.8371	0.8356
cay	0.0640	0.0482	0.2527	-0.0304	-0.1958	0.0482	-0.7079	1.0000	1.0000	-0.0304	0.8371	0.8356
r	-0.6082	-0.0911	-0.3429	1.0000	0.0215	-0.0911	0.0129	-0.0304	-0.0304	1.0000	0.5213	0.5232
tb	-0.2782	-0.0087	0.0281	0.5213	-0.1553	-0.0087	-0.5973	0.8371	0.8371	0.5213	1.0000	0.9997
tby	-0.2978	-0.0321	0.0059	0.5232	-0.1779	-0.0321	-0.6127	0.8356	0.8356	0.5232	0.9997	1.0000

POLICY AND TRANSITION FUNCTIONS

Variable	c	h	k	d	A	y	i	ca	cay	r	tb	tby
d(-1)	-0.039227	0	-0.006592	0.974341	0	0	-0.065918	0.025659	0.017262	0.000723	0.065659	0.044171
r(-1)	0	0	0	0	0	0	0	0	0	0	0.744200	0.500643
k(-1)	0.506431	0.412903	0.500311	-1.685401	0	0.600774	-3.996887	1.685401	1.133815	-0.001251	1.685401	1.125721
A(-1)	0.529381	0.541935	0.282285	0.378288	0.420000	0.788516	2.822848	-0.378288	-0.254484	0.000281	-0.378288	-0.265107
epsilon	0.016260	0.016645	0.008670	0.011619	0.012900	0.024219	0.086702	-0.011619	-0.007816	0.000009	-0.011619	-0.008143

AUTOCORRELATION OF SIMULATED VARIABLES

VARIABLE	1	2	3	4	5
c	0.7796	0.6321	0.5431	0.4931	0.4643
h	0.6158	0.3576	0.2026	0.1162	0.0676
k	0.7871	0.5526	0.3844	0.2805	0.2198
d	0.9981	0.9951	0.9918	0.9884	0.9850
A	0.4192	0.1746	0.0710	0.0294	0.0093
y	0.6158	0.3576	0.2026	0.1162	0.0676
i	0.0693	-0.1376	-0.1375	-0.0915	-0.0561
ca	0.3215	0.0862	0.0105	-0.0074	-0.0125
cay	0.3215	0.0862	0.0105	-0.0074	-0.0125
r	0.9981	0.9951	0.9918	0.9884	0.9850
tb	0.5241	0.3584	0.3044	0.2908	0.2863
tby	0.5112	0.3481	0.2990	0.2894	0.2874

Figure 02: Stochastic Simulations

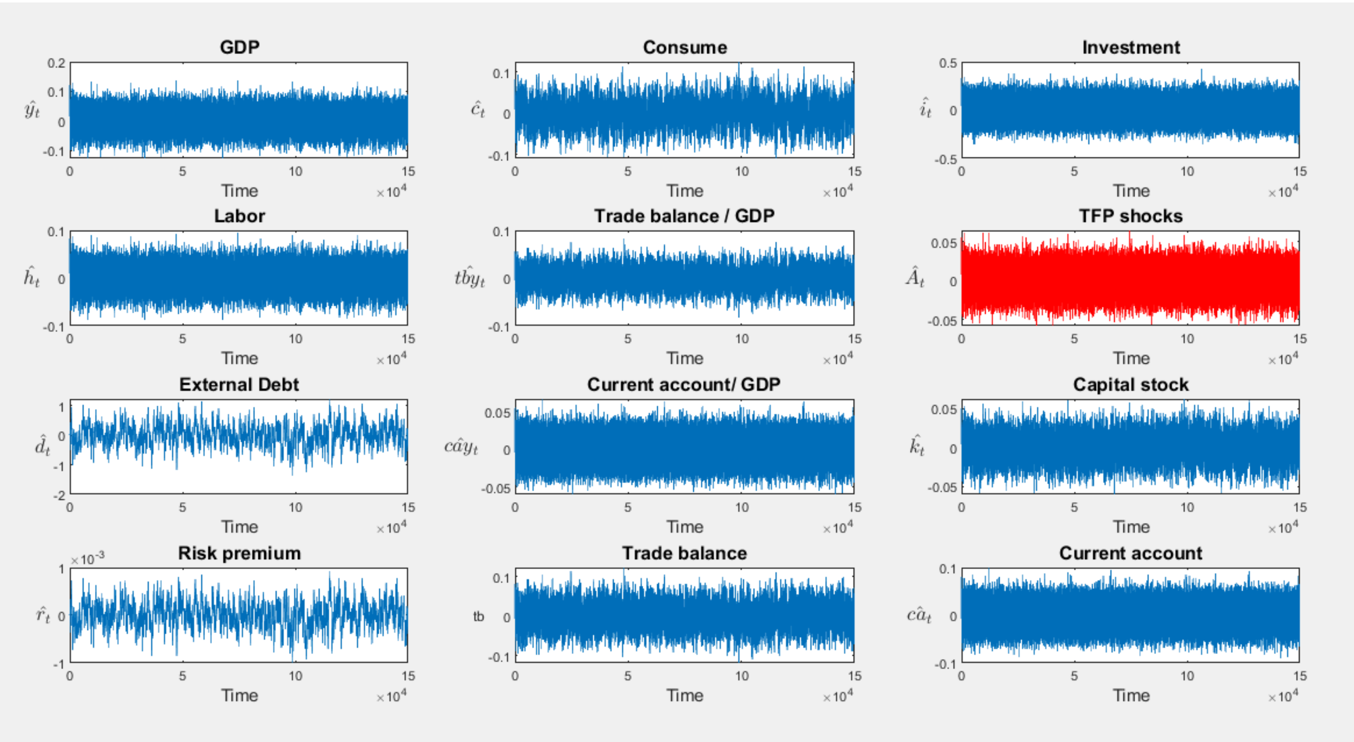

Figure 02 displays long-horizon simulated trajectories of the main macroeconomic variables of the small open economy RBC model under successive i.i.d. normally distributed technology shocks (red trajectory in the Figure 02). Unlike impulse-response functions—which trace the economy's reaction to a single, isolated shock—these plots show the unconditional dynamics of each variable when the economy is continually hit by random disturbances every period.

Overall, these long-run simulated paths illustrate how the economy behaves under continuous random disturbances: all variables remain stationary and mean-reverting, yet display volatilities and comovement patterns characteristic of the small open economy RBC framework. The figure effectively complements the impulse-response analysis by showing the time-series properties the model implies in the presence of persistent stochastic shocks.

Tble 01: Empirical and Theoretical Second Moments

Variable	Canadian data 1946-1985			Canadian data 1960-2011			Matlab-based simulations (Uribe & Schmitt-Grohé)			Dynare-based simulations (own)			Dynare-based simulations (Pfeifer)		
	σx_t	$\rho x_t, x_{t-1}$	$\rho x_t, GDP_t$	σx_t	$\rho x_t, x_{t-1}$	$\rho x_t, GDP_t$	σx_t	$\rho x_t, x_{t-1}$	$\rho x_t, GDP_t$	σx_t	$\rho x_t, x_{t-1}$	$\rho x_t, GDP_t$	σx_t	$\rho x_t, x_{t-1}$	$\rho x_t, GDP_t$
y	2.81	0.62	1	3.71	0.86	1	3.08	0.62	1	3.08	0.62	1	3.08	0.62	1
c	2.46	0.7	0.59	2.19	0.7	0.62	2.71	0.78	0.84	2.70	0.78	0.85	2.71	0.78	0.84
i	9.82	0.31	0.64	10.31	0.69	0.8	9.04	0.07	0.67	9.06	0.07	0.67	9.04	0.07	0.67
h	2.02	0.54	0.8	3.68	0.75	0.78	2.12	0.62	1	2.12	0.62	1	2.12	0.62	1
tb/y	1.87	0.66	-0.13	1.72	0.76	0.12	1.78	0.51	-0.04	1.77	0.51	-0.03	1.78	0.51	-0.04
ca/y							1.45	0.32	0.05	1.45	0.32	0.05	1.45	0.32	0.05
d										35.57	0.998	-0.09	35.92	0.998	-0.09
k										1.44	0.79	0.95	1.45	0.79	0.94
A										1.42	0.42	0.96	1.42	0.42	0.96
risk premium										0.03	0.998	-0.091	0.03	0.998	-0.094
tb										2.64	0.52	-0.01			
ca										2.16	0.32	0.05			

The last table compares the empirical second moments of the Canadian economy with the predictions of the real business cycle (RBC) model of a small open economy (SOE)—with an elastic interest premium on debt—by Uribe and Schmitt-Grohé (2003, 2017) and with two sets of simulated results: the Dynare implementation by Pfeifer and my own Dynare simulations.

The first two empirical blocks (1946–1985 and 1960–2011) replicate the data analyzed by Uribe and Schmitt-Grohé (2017). In both samples, investment is the most volatile variable, followed by production, consumption, and hours worked, while the trade-balance-to-GDP ratio exhibits the lowest volatility. Serial correlations are positive in all aggregates, with a notable increase in the persistence of investment in the most recent sample. The trade balance is slightly countercyclical in the initial period but becomes slightly procyclical in recent years. In line with Uribe and Schmitt-Grohé (2003, 2017), the Canadian economy experienced an increase in volatility over time, especially in production and hours worked.

The third block reports the second moments implied by the calibrated SOE-RBC model in Uribe and Schmitt-Grohé (2003, 2017). As expected—because several parameters were calibrated to target these empirical moments—the model closely replicates key volatilities and the serial correlation of output. It also performs reasonably well for non-targeted moments: consumption is less volatile than output but more volatile than hours and the trade balance, and the model correctly predicts a countercyclical trade balance. However, the model overstates the comovement between hours and output, generating a correlation close to one, a mechanical implication of the assumed utility specification, Uribe and Schmitt-Grohé (2017). The last two blocks—my Dynare simulations and Pfeifer's Dynare-based simulations—show that both codes reproduce almost exactly the second moments of the SOE-RBC model presented in Uribe and Schmitt-Grohé (2003, 2017).

Finally, although all three implementations solve the same small-open-economy real business cycle (SOE-RBC) framework, they differ in structure, focus, and the way the linear approximation is obtained:

- Uribe and Schmitt-Grohé (2017) Matlab code: defines and analyzes the small open economy model with an External Debt-Elastic Interest Rate (EDEIR) by Uribe and Schmitt-Grohé (2017). The main function, "edeir_model", creates symbolic representations of the model's endogenous variables, exogenous shocks, parameters, and 14 equilibrium conditions that describe debt evolution, output, consumption, labor supply, capital accumulation, investment, trade balance, current account, and TFP dynamics. It then computes first-order derivatives of these conditions with respect to current and future state and control variables (fx, fxp, fy, fyp) around the deterministic steady state, performs log-linearization for numerical stability, and constructs the matrix of shock impacts (ETASHOCK) and its variance-covariance. The function outputs a file (edeir_model_num_eval.m) containing these derivatives for numerical evaluation. Additional scripts, "edeir_run.m" and "edeir_ss.m", compute the steady state, evaluate the derivatives at this steady state, derive first-order policy functions (gx, hx), calculate second moments, correlations, and serial correlations of key variables, and generate impulse-response functions for shocks such as TFP innovations, plotting the dynamic responses of output, consumption, investment, labor, trade balance, and TFP. Overall, the code symbolically linearizes the EDEIR model, prepares it for numerical solution, and computes both theoretical and empirical model statistics.

- Pfeifer's Dynare implementation expresses the model in nonlinear levels, using exponentials and explicit steady-state definitions. Dynare computes the steady state analytically whenever possible and numerically otherwise, and then applies an automatic first-order Taylor linearization via "stoch_simul(order=1)". The code also checks residuals and performs a first-order stochastic simulation to compute standard deviations, autocorrelations, and correlations of key macroeconomic variables, presenting the results in a table format. Finally, it generates impulse response functions (IRFs) to a TFP shock over 10 periods for visualization and analysis. Pfeifer's code is modular and designed to handle multiple model configurations, not just the specific version with an external debt-elastic interest-rate premium.

- My Dynare implementation: focuses exclusively on replicating the SOE-RBC model of Uribe and Schmitt-Grohé with a debt-elastic interest rate premium. The system is supplied already in log-linearized form, with steady-state definitions included. Although it also uses stoch_simul(order=1), the difference is that Dynare works on the analytically linearized system, rather than performing a numerical linearization of a nonlinear model. Likewise, my code performs both IRF analysis and very long stochastic simulations (150,000 periods) under i.i.d. TFP shocks, visualizing trajectories of all variables over time.

Despite these differences, all three implementations produce almost identical second moments, serial correlations, and cyclical correlations, confirming the robustness of the SOE-RBC model's first-order solution and Dynare's reliability in computing it.

Summary of ideas

The notebook develops and replicates the Small Open Economy Real Business Cycle (SOE-RBC) model following the framework of Uribe & Schmitt-Grohé. It begins by presenting the model's structural assumptions—household preferences, firm technology, international financial markets, and the capital-accumulation equation—before deriving the full set of equilibrium conditions. The steady state is computed analytically and checked for consistency, after which the system is log-linearized around this steady state to obtain a reduced-form representation suitable for numerical solution. All parameter values used in the notebook correspond exactly to those reported in Uribe & Schmitt-Grohé (2003) and in their 2017 textbook, ensuring direct comparability to the original quantitative results.

After solving the linear rational-expectations system and obtaining the policy matrices, the notebook shows impulse-response functions to a temporary productivity disturbance, illustrating the dynamic responses of consumption, output, labor, investment, capital, esternal debt, the current account, the risk premium, the trade-balance-to-output ratio, the current-account-to-output ratio, and the trade balance. Stochastic simulations under successive i.i.d. technological shocks with a normal distribution are also presented, showing the raw dynamic behavior of all variables without any Hodrick–Prescott filter. The workflow, from structural formulation to IRFs and stochastic simulations, aims to closely replicate the numerical findings reported by Uribe & Schmitt-Grohé in both their 2003 paper and their 2017 book.

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Supplementary Material

Alternative method of linearizing the first-order conditions (FOCS) of the model

Following to Uribe-Schmitt-Grohé (2017), let's linearize the following multivariable function:

$$N_t = E_t\Big[M\left(x_{1,t}, x_{2,t+1}, z_{1,t}\right)\Big] \tag{A.1}$$

Where N_t, $x_{1,t}$, and $x_{2,t+1}$ are variables expressed in logarithms (such as y_t, c_t, h_t, k_t, k_{t+1}, and k_{t+2}), while $z_{1,t}$ is a variable expressed as a rate (such as r_t), in ratios (such as tb_t/y_t and ca_t/y_t), or that can take negative values (such as d_t, tb_t, and ca_t).

Then:

$$\hat{N}_t = ln(N_t) - ln(N_{SS}) \approx (N_t - N_{SS})/N_{SS} \rightarrow N_t \approx (1+\hat{N}_t)N_{SS} \leftrightarrow N_{SS} \neq 0 \tag{A.2}$$

$$\hat{x}_{1,t} = ln(x_{1,t}) - ln(x_{1,SS}) \approx (x_{1,t} - x_{1,SS})/x_{1,SS} \rightarrow x_{1,t} - x_{1,SS} \approx x_{1,SS}\hat{x}_{1,t} \leftrightarrow x_{1,SS} \neq 0 \tag{A.3}$$

$$\hat{x}_{2,t+1} = ln(x_{2,t+1}) - ln(x_{2,SS}) \approx (x_{2,t+1} - x_{2,SS})/x_{2,SS} \rightarrow x_{2,t+1} - x_{2,SS} \approx x_{2,SS}\hat{x}_{2,t+1} \leftrightarrow x_{2,SS} \neq 0 \tag{A.4}$$

$$\hat{z}_{1,t} = z_{1,t} - z_{1,SS} \tag{A.5}$$

Taking into account (A.1), in the steady state:

$$N_{SS} = E_t\Big[M\left(x_{1,SS}, x_{2,SS}, z_{1,SS}\right)\Big] = M\left(x_{1,SS}, x_{2,SS}, z_{1,SS}\right) = M\left(\vec{x}^*\right) \tag{A.6}$$

Where $\vec{x}^{\,*} = (x_{1,SS}, x_{2,SS}, z_{1,SS})$

To linearize (A.1) the formulas (45), (46), (A.2), (A.3), (A.4) and (A.5) are taken into account:

$$(1+\hat{N}_t)N_{SS} \approx E_t\left[M\left(\vec{x}^*\right) + \frac{\partial M\left(\vec{x}^*\right)}{\partial x_{1,t}}x_{1,SS}\hat{x}_{1,t} + \frac{\partial M\left(\vec{x}^*\right)}{\partial x_{2,t+1}}x_{2,SS}\hat{x}_{2,t+1} + \frac{\partial M\left(\vec{x}^*\right)}{\partial z_{1,t}}\hat{z}_{1,t}\right]$$

$$N_{SS} + N_{SS}\hat{N}_t \approx M\left(\vec{x}^*\right) + \frac{\partial M\left(\vec{x}^*\right)}{\partial x_{1,t}}x_{1,SS}\hat{x}_{1,t} + \frac{\partial M\left(\vec{x}^*\right)}{\partial x_{2,t+1}}E_t\left[\hat{x}_{2,t+1}\right] + \frac{\partial M\left(\vec{x}^*\right)}{\partial z_{1,t}}\hat{z}_{1,t} \tag{A.7}$$

Replacing (A.6) in (A.7):

$$M\left(\vec{x}^*\right) + M\left(\vec{x}^*\right)\hat{N}_t \approx M\left(\vec{x}^*\right) + \frac{\partial M\left(\vec{x}^*\right)}{\partial x_{1,t}}x_{1,SS}\hat{x}_{1,t} + \frac{\partial M\left(\vec{x}^*\right)}{\partial x_{2,t+1}}x_{2,SS}E_t\left[\hat{x}_{2,t+1}\right] + \frac{\partial M\left(\vec{x}^*\right)}{\partial z_{1,t}}\hat{z}_{1,t}$$

$$\hat{N}_t \approx \left[\frac{\partial M\left(\vec{x}^*\right)}{\partial x_{1,t}}\frac{x_{1,SS}}{M\left(\vec{x}^*\right)}\right]\hat{x}_{1,t} + \left[\frac{\partial M\left(\vec{x}^*\right)}{\partial x_{2,t+1}}\frac{x_{2,SS}}{M\left(\vec{x}^*\right)}\right]E_t\left[\hat{x}_{2,t+1}\right] + \left[\frac{\partial M\left(\vec{x}^*\right)}{\partial z_{1,t}}\frac{1}{M\left(\vec{x}^*\right)}\right]\hat{z}_{1,t}$$

$$\boxed{\hat{N}_t \approx \epsilon_{M,x_1}\hat{x}_{1,t} + \epsilon_{M,x_2}E_t\left[\hat{x}_{2,t+1}\right] + \eta_{M,z_1}\hat{z}_{1,t}} \tag{A.8}$$

Where $\epsilon_{M,x_1} = \frac{\partial M(\vec{x}^*)}{\partial x_{1,t}}\frac{x_{1,SS}}{M(\vec{x}^*)}$ is the steady-state partial elasticity of M with respect to $x_{1,t}$, $\epsilon_{M,x_2} = \frac{\partial M(\vec{x}^*)}{\partial x_{2,t+1}}\frac{x_{2,SS}}{M(\vec{x}^*)}$ is the steady-state partial elasticity of M with respect to $x_{2,t+1}$, and $\eta_{M,z_1} = \frac{\partial M(\vec{x}^*)}{\partial z_{1,t}}\frac{1}{M(\vec{x}^*)}$ is the partial semi-elasticity of M with respect to $z_{1,t}$ evaluated at the steady state.

Rewriting the first-order conditions (FOCs) in general terms

Substituting (9) into equation (8):

$$u_c(c_t, h_t) = \beta(1+r_t)E_t\left[u_c(c_{t+1}, h_{t+1})\right] \tag{A.9}$$

From equation (13):

$$u_h(c_t, h_t) = -u_c(c_t, h_t)A_tF_h(k_t, h_t) \tag{A.10}$$

Replacing (9) into equation (11):

$$u_c(c_t, h_t)[1+\Phi'(k_{t+1}-k_t)] = \beta E_t\Big[u_c(c_{t+1}, h_{t+1})[A_{t+1}F_k(k_{t+1}, h_{t+1}) + 1 - \delta + \Phi'(k_{t+2}-k_{t+1})]\Big] \tag{A.11}$$

From equation (12), taking into account (2) and (4):

$$y_t = A_tF\left(k_t, h_t\right) = c_t - d_t - (1-\delta)\,k_t + (1+r_{t-1})d_{t-1} + k_{t+1} + \Phi\left(k_{t+1}-k_t\right) \tag{A.12}$$

Linearization of the first-order conditions (FOCs) of the model

The equation to linearize is (A.9). Taking into account (A.8) and equation (40):

$$\left[\frac{u_{cc}(\vec{x}^*)}{u_c(\vec{x}^*)}c_{SS}\right]\hat{c}_t+\left[\frac{u_{ch}(\vec{x}^*)}{u_c(\vec{x}^*)}h_{SS}\right]\hat{h}_t\approx\left[\frac{\beta u_c(\vec{x}^*)}{\beta(1+r^o)u_c(\vec{x}^*)}\right]\hat{r}_t+\left[\frac{\beta(1+r^o)u_{cc}(\vec{x}^*)c_{SS}}{\beta(1+r^o)u_c(\vec{x}^*)}\right]E_t[\hat{c}_{t+1}]+\left[\frac{\beta(1+r^o)u_{ch}(\vec{x}^*)h_{SS}}{\beta(1+r^o)u_c(\vec{x}^*)}\right]E_t[\hat{h}_{t+1}]$$

Taking into account (35):

$$\left[\frac{u_{cc}(\vec{x}^*)}{u_c(\vec{x}^*)}c_{SS}\right]\hat{c}_t+\left[\frac{u_{ch}(\vec{x}^*)}{u_c(\vec{x}^*)}h_{SS}\right]\hat{h}_t\approx\beta\hat{r}_t+\left[\frac{u_{cc}(\vec{x}^*)c_{SS}}{u_c(\vec{x}^*)}\right]E_t[\hat{c}_{t+1}]+\left[\frac{u_{ch}(\vec{x}^*)h_{SS}}{u_c(\vec{x}^*)}\right]E_t[\hat{h}_{t+1}]$$

Substituting formula (59):

$$\boxed{\epsilon_{cc}\hat{c}_t+\epsilon_{ch}\hat{h}_t\approx\beta\psi_1\hat{d}_t+\epsilon_{cc}E_t[\hat{c}_{t+1}]+\epsilon_{ch}E_t[\hat{h}_{t+1}]}\tag{A.13}$$

Where $\vec{x}^*=(c_{SS},h_{SS}),\varepsilon_{cc}=\frac{u_{hh}(\vec{x}^*)}{u_h(\vec{x}^*)}h_{SS}=-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^{\omega}}{\omega}\right)^{-1}$, and $\epsilon_{ch}=\frac{u_{ch}(\vec{x}^*)}{u_c(\vec{x}^*)}h_{SS}=\sigma h_{SS}^{\omega}\left(c_{SS}-\frac{h_{SS}^{\omega}}{\omega}\right)^{-1}$.

Replacing elasticities into (A.13):

$$\boxed{\begin{aligned}&-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^{w}}{w}\right)^{-1}\hat{c}_t+\sigma h_{SS}^{w}\left(c_{SS}-\frac{h_{SS}^{w}}{w}\right)^{-1}\hat{h}_t\\&\approx\beta\psi_1\hat{d}_t-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^{w}}{w}\right)^{-1}E_t[\hat{c}_{t+1}]+\sigma h_{SS}^{w}\left(c_{SS}-\frac{h_{SS}^{w}}{w}\right)^{-1}E_t[\hat{h}_{t+1}]\end{aligned}}\tag{A.14}$$

Expression (A.14) is nothing more than formula (52).

Now, the equation to be linearized is (A.10). Considering (A.8), and formula (40):

$$\begin{aligned}\left[\frac{u_{hc}(\vec{x}^*)}{u_h(\vec{x}^*)}c_{SS}\right]\hat{c}_t+\left[\frac{u_{hh}(\vec{x}^*)}{u_h(\vec{x}^*)}h_{SS}\right]\hat{h}_t&\approx\left[\frac{-u_{cc}(\vec{x}^*)A_{SS}F_h(\vec{y}^*)}{-u_c(\vec{x}^*)A_{SS}F_h(\vec{y}^*)}c_{SS}\right]\hat{c}_t+\left[\frac{-u_{ch}(\vec{x}^*)A_{SS}F_h(\vec{y}^*)}{-u_c(\vec{x}^*)A_{SS}F_h(\vec{y}^*)}h_{SS}\right]\hat{h}_t\\&+\left[\frac{-u_c(\vec{x}^*)A_{SS}F_{hh}(\vec{y}^*)}{-u_c(\vec{x}^*)A_{SS}F_h(\vec{y}^*)}h_{SS}\right]\hat{h}_t+\left[\frac{-u_c(\vec{x}^*)F_h(\vec{y}^*)}{-u_c(\vec{x}^*)A_{SS}F_h(\vec{y}^*)}A_{SS}\right]\hat{A}_t+\left[\frac{-u_c(\vec{x}^*)A_{SS}F_{hk}(\vec{y}^*)}{-u_c(\vec{x}^*)A_{SS}F_h(\vec{y}^*)}k_{SS}\right]\hat{k}_t\end{aligned}$$

Performing some algebraic operations:

$$\left[\frac{u_{hc}(\vec{x}^*)}{u_h(\vec{x}^*)}c_{SS}\right]\hat{c}_t+\left[\frac{u_{hh}(\vec{x}^*)}{u_h(\vec{x}^*)}h_{SS}\right]\hat{h}_t\approx\left[\frac{u_{cc}(\vec{x}^*)}{u_c(\vec{x}^*)}c_{SS}\right]\hat{c}_t+\left[\frac{u_{ch}(\vec{x}^*)}{u_c(c_{SS},h_{SS})}h_{SS}\right]\hat{h}_t+\left[\frac{F_{hh}(\vec{y}^*)}{F_h(\vec{y}^*)}h_{SS}\right]\hat{h}_t+\hat{A}_t+\left[\frac{F_{hk}(\vec{y}^*)}{F_h(\vec{y}^*)}k_{SS}\right]\hat{k}_t$$

Taking into account the definitions of elasticities and the partial derivatives of equation (4):

$$\epsilon_{hc}\hat{c}_t+\epsilon_{hh}\hat{h}_t\approx\epsilon_{cc}\hat{c}_t+\epsilon_{ch}\hat{h}_t-\alpha\hat{h}_t+\hat{A}_t+\alpha\hat{k}_t$$

$$\boxed{(\epsilon_{hc}-\epsilon_{cc})\hat{c}_t+(\epsilon_{hh}-\epsilon_{ch})\hat{h}_t\approx\hat{A}_t+\alpha(\hat{k}_t-\hat{h}_t)}\tag{A.15}$$

Where $\vec{y}^*=(k_{SS},h_{SS})$, $\epsilon_{hc}=\frac{u_{hc}(\vec{x}^*)}{u_h(\vec{x}^*)}c_{SS}=-\sigma c_{SS}\left(c_{SS}-\frac{h_{SS}^{\omega}}{\omega}\right)^{-1}$, and $\epsilon_{hh}=\frac{u_{hh}(\vec{x}^*)}{u_h(\vec{x}^*)}h_{SS}=w-1+\sigma h_{SS}^{\omega}\left(c_{SS}-\frac{h_{SS}^{\omega}}{\omega}\right)^{-1}$.

Substituting the elasticities into (A.15):

$$\boxed{(\alpha+w-1)\hat{h}_t\approx\hat{A}_t+\alpha\hat{k}_t}\tag{A.16}$$

Formula (A.16) is nothing more than expression (50).

The equation to linearize is (A.11). Taking into account (A.8):

$$\begin{aligned}&-\frac{u_{cc}(\vec{x}^*)[1+\Phi'(0)]c_{SS}\hat{c}_t+u_{ch}(\vec{x}^*)[1+\Phi'(0)]h_{SS}\hat{h}_t+u_c(\vec{x}^*)\Phi''(0)k_{SS}(\hat{k}_{t+1}-\hat{k}_t)}{u_c(\vec{x}^*)[1+\Phi'(0)]}\\&\approx\frac{\beta u_{cc}(\vec{x}^*)[A_{SS}F_k(\vec{y}^*)+1-\delta+\Phi'(0)]c_{SS}E_t[\hat{c}_{t+1}]+\Big[\beta u_{ch}(\vec{x}^*)[A_{SS}F_k(\vec{y}^*)+1-\delta+\Phi'(0)]+\beta u_c(\vec{x}^*)A_{SS}F_{kh}(\vec{y}^*)\Big]h_{SS}E_t[\hat{h}_{t+1}]}{\beta u_c(\vec{x}^*)[A_{SS}F_k(\vec{y}^*)+1-\delta+\Phi'(0)]}\\&+\frac{\beta u_c(\vec{x}^*)F_k(\vec{y}^*)A_{SS}E_t[\hat{A}_{t+1}]+\beta u_c(\vec{x}^*)A_{SS}F_{kk}(\vec{y}^*)k_{SS}E_t[\hat{k}_{t+1}]+\beta u_c(\vec{x}^*)\Phi''(0)k_{SS}E_t[\hat{k}_{t+2}-\hat{k}_{t+1}]}{\beta u_c(\vec{x}^*)[A_{SS}F_k(\vec{y}^*)+1-\delta+\Phi'(0)]}\end{aligned}$$

Taking into account equation (32) and that $\Phi'(0)=0$:

$$\begin{aligned}&-\frac{u_{cc}(\vec{x}^*)c_{SS}\hat{c}_t+u_{ch}(\vec{x}^*)h_{SS}\hat{h}_t+u_c(\vec{x}^*)\Phi''(0)k_{SS}(\hat{k}_{t+1}-\hat{k}_t)}{u_c(\vec{x}^*)}\\&\approx\frac{\beta u_{cc}(\vec{x}^*)[F_k(\vec{y}^*)+1-\delta]c_{SS}E_t[\hat{c}_{t+1}]+\Big[\beta u_{ch}(\vec{x}^*)[F_k(\vec{y}^*)+1-\delta]+\beta u_c(\vec{x}^*)F_{kh}(\vec{y}^*)\Big]h_{SS}E_t[\hat{h}_{t+1}]}{\beta u_c(\vec{x}^*)[F_k(\vec{y}^*)+1-\delta]}\\&+\frac{\beta u_c(\vec{x}^*)F_k(\vec{y}^*)E_t[\hat{A}_{t+1}]+\beta u_c(\vec{x}^*)F_{kk}(\vec{y}^*)k_{SS}E_t[\hat{k}_{t+1}]+\beta u_c(\vec{x}^*)\Phi''(0)k_{SS}E_t[\hat{k}_{t+2}-\hat{k}_{t+1}]}{\beta u_c(\vec{x}^*)[F_k(\vec{y}^*)+1-\delta]}\end{aligned}$$

Taking into account the definitions of elasticities:

$$\epsilon_{cc}\hat{c}_t + \epsilon_{ch}\hat{h}_t + \Phi''(0)k_{SS}(\hat{k}_{t+1} - \hat{k}_t) \approx \epsilon_{cc}\hat{E}_t[\hat{c}_{t+1}] + \epsilon_{ch}E_t[\hat{h}_{t+1}]$$
$$+ \frac{F_{kh}(\vec{y}^*)h_{SS}E_t[\hat{h}_{t+1}] + F_k(\vec{y}^*)E_t[\hat{A}_{t+1}] + F_{kk}(\vec{y}^*)k_{SS}E_t[\hat{k}_{t+1}] + \Phi''(0)k_{SS}E_t[\hat{k}_{t+2} - \hat{k}_{t+1}]}{F_k(\vec{y}^*) + 1 - \delta}$$

Considering the partial derivatives of equation (4), equation (32) and that that $\kappa = k_{SS}/h_{SS}$:

$$\epsilon_{cc}\hat{c}_t + \epsilon_{ch}\hat{h}_t + k_{SS}\Phi''(0)(\hat{k}_{t+1} - \hat{k}_t) \approx \epsilon_{cc}\hat{E}_t[\hat{c}_{t+1}] + \epsilon_{ch}E_t[\hat{h}_{t+1}]$$
$$+ \frac{(1-\alpha)\alpha\kappa^{\alpha-1}E_t[\hat{h}_{t+1}] + \alpha\kappa^{\alpha-1}E_t[\hat{A}_{t+1}] + (\alpha-1)\kappa^{\alpha-1}E_t[\hat{k}_{t+1}] + \Phi''(0)k_{SS}E_t[\hat{k}_{t+2} - \hat{k}_{t+1}]}{\alpha\kappa^{\alpha-1} + 1 - \delta}$$

Since after substituting (35) into (39) it turns out that $\alpha\kappa^{\alpha-1} = r^o + \delta$:

$$\begin{aligned}&\epsilon_{cc}\hat{c}_t + \epsilon_{ch}\hat{h}_t + \Phi''(0)k_{SS}(\hat{k}_{t+1} - \hat{k}_t) \approx \epsilon_{cc}\hat{E}_t[\hat{c}_{t+1}] + \epsilon_{ch}E_t[\hat{h}_{t+1}]\\ &+ \left(\frac{r^o+\delta}{1+r^o}\right)\Big[E_t[\hat{A}_{t+1}] + (\alpha-1)\Big[E_t[\hat{k}_{t+1}] - E_t[\hat{h}_{t+1}]\Big]\Big] + \frac{\Phi''(0)k_{SS}}{1+r^o}E_t[\hat{k}_{t+2} - \hat{k}_{t+1}]\end{aligned} \tag{A.17}$$

Substituting the elasticities in (A.17) and taking into account that $\Phi''(0) = \phi$:

$$\boxed{\begin{aligned}&-\sigma c_{SS}\Big(c_{SS} - \frac{h^w_{SS}}{w}\Big)^{-1}\hat{c}_t + \sigma h^w_{ss}\Big(c_{SS} - \frac{h^w_{SS}}{w}\Big)^{-1}\hat{h}_t + \phi k_{SS}(\hat{k}_{t+1} - \hat{k}_t)\\ &\approx -\sigma c_{SS}\Big(c_{SS} - \frac{h^w_{SS}}{w}\Big)^{-1}E_t[\hat{c}_{t+1}] + \sigma h^w_{SS}\Big(c_{SS} - \frac{h^w_{SS}}{w}\Big)^{-1}E_t[\hat{h}_{t+1}]\\ &+ \left(\frac{r^o+\delta}{1+r^o}\right)E_t[\hat{A}_{t+1} + (\alpha-1)(\hat{k}_{t+1} - \hat{h}_{t+1})] + \left(\frac{\phi k_{SS}}{1+r^o}\right)E_t[\hat{k}_{t+2} - \hat{k}_{t+1}]\end{aligned}} \tag{A.18}$$

Formula (A.18) is nothing more than expression (53).

The equation to linearize is (A.12). Taking into account (A.8):

$$\frac{A_{SS}F(\vec{y}^*)\hat{A}_t + A_{SS}F_k(\vec{y}^*)k_{SS}\hat{k}_t + A_{SS}F_h(\vec{y}^*)h_{SS}\hat{h}_t}{A_{SS}F(\vec{y}^*)} \approx \frac{c_{SS}\hat{c}_t - \hat{d}_t - (1-\delta)k_{SS}\hat{k}_t + \bar{d}\,\hat{r}_{t-1} + (1+r^o)\hat{d}_{t-1} + k_{SS}\hat{k}_{t+1} + \Phi'(0)k_{SS}(\hat{k}_{t+1} - \hat{k}_t)}{c_{SS} - \bar{d} - (1-\delta)k_{SS} + (1+r^o)\bar{d} + k_{SS} + \Phi(0)}$$

Considering equation (4), its partial derivatives and that $\Phi(0) = \Phi'(0) = 0$:

$$\frac{A_{SS}F(\vec{y}^*)\hat{A}_t + \alpha A_{SS}F(\vec{y}^*)\hat{k}_t + (1-\alpha)A_{SS}F(\vec{y}^*)\hat{h}_t}{A_{SS}F(\vec{y}^*)} \approx \frac{c_{SS}\hat{c}_t - \hat{d}_t - (1-\delta)k_{SS}\hat{k}_t + \bar{d}\,\hat{r}_{t-1} + (1+r^o)\hat{d}_{t-1} + k_{SS}\hat{k}_{t+1}}{c_{SS} - \delta k_{SS} + r^o\bar{d}}$$

Taking into account equations (28), (29) and (30):

$$\frac{y_{SS}\hat{A}_t + \alpha y_{SS}\hat{k}_t + (1-\alpha)y_{SS}\hat{h}_t}{y_{SS}} \approx \frac{c_{SS}\hat{c}_t - \hat{d}_t - (1-\delta)k_{SS}\hat{k}_t + \bar{d}\,\hat{r}_{t-1} + (1+r^o)\hat{d}_{t-1} + k_{SS}\hat{k}_{t+1}}{y_{SS}}$$

$$\hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t \approx \frac{c_{SS}\hat{c}_t - \hat{d}_t - (1-\delta)k_{SS}\hat{k}_t + \bar{d}\,\hat{r}_{t-1} + (1+r^o)\hat{d}_{t-1} + k_{SS}\hat{k}_{t+1}}{y_{SS}}$$

Considering formula (59):

$$\hat{A}_t + \alpha\hat{k}_t + (1-\alpha)\hat{h}_t \approx \frac{c_{SS}\hat{c}_t - \hat{d}_t - (1-\delta)k_{SS}\hat{k}_t + \bar{d}\,\psi_1\hat{d}_{t-1} + (1+r^o)\hat{d}_{t-1} + k_{SS}\hat{k}_{t+1}}{y_{SS}}$$

$$\boxed{\left(\frac{1}{y_{SS}}\right)\hat{d}_t \approx \left(\frac{1+r^o+\psi_1\bar{d}}{y_{SS}}\right)\hat{d}_{t-1} + \left(\frac{c_{ss}}{y_{SS}}\right)\hat{c}_t + \left(\frac{k_{SS}}{y_{SS}}\right)\Big[\hat{k}_{t+1} - (1-\delta)\hat{k}_t\Big] - \hat{A}_t - \alpha\hat{k}_t - (1-\alpha)\hat{h}_t} \tag{A.19}$$

Expression (A.19) is nothing more than formula (50).