Proposition :-

A proposition is a declarative sentence that is either true or false, but not both.

Ex:-1 Washington D.C is the capital of the United States of America.

Ex:-2 Toronto is the capital of India

Ex-3 $1+1=2$

Ex-4 $2+2=3$

Consider the following sentences

(i) What time is it?
(ii) Read this carefully.

] not declarative statements

(iii) $x+1=2$
(iv) $x+y=z$

] they are neither true nor false.

Note:- We use letters to denote propositions. Usually we use p, q, r, s ----

Note:- If we have a proposition p which is true we represent it by giving p the value T
p T, and we call T truth value of p
$||^{ly}$ if p is false we give it truth value F.

Propositional Calculus :

The area of logic that deals with propositions is called the propositional calculus.

Fact : propositional calculus was developed systematically by Greek philosopher Aristotle more than 2300 yrs ago.

Compound propositions

mathematical statements that are constructed by combining one or more propositions are called compound propositions.

Logical operators :

Definition : Let p be a proposition.

The statement "It is not the case that p" is another proposition, called the negation of p.

The negation of p is denoted by $\neg p$

$\neg p$ is read "not p"

Ex Find the negation of the proposition

"Today is Friday"

Sol^n "It is not the case that today is Friday"

Today is not Friday.

Truth table :-

p	$\neg p$
T	F
F	T

Note :- $\neg$ is known as negation operator.

Definition (Conjunction)

Let p and q be propositions. The proposition "p and q", denoted $p \wedge q$, is the proposition that is true when both p and q are true and it is false otherwise.

The proposition $p \wedge q$ is called conjunction of p and q.

Truth table

p	q	$p \wedge q$
T	T	T
F	T	F
T	F	F

Definition →: (Disjunction)

let p and q be propositions. The proposition "p or q", denoted $p \vee q$, is the proposition that is false when p and q are both false and true otherwise. The proposition $p \vee q$ is called disjunction of p and q.

Truth table

p	q	$p \vee q$
T	F	T
F	T	T
F	F	T

Definition →: (Exclusive or)

let p and q be propositions. The exclusive or of p and q, denoted by $p \oplus q$, is the proposition that is true when exactly one of p and q is true and false otherwise.

Truth table

p	q	$p \oplus q$
T	T	F
T	F	T
F	T	T
F	F	F

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Definition (Implication)

Let p and q be propositions. The implication $p \to q$ is the proposition that is false when p is true and q is false, and true otherwise.

Truth table

p	q	$p \to q$
T	T	T
T	F	F
F	F	T
F	T	T

Note:: in $p \to q$, p is called Hypothesis, q is called Conclusion.

Note-: If $p \to q$ is a proposition.

Then
(I) $q \to p$ is called Converse of $p \to q$
(II) $\neg q \to \neg p$ is called Contrapositive of $p \to q$
(III) $\neg p \to \neg q$ is called Inverse of $p \to q$

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Ex: what are the contrapositive, the converse, and the inverse of the implication
" The home team wins whenever it is raining"?

Solution

(I) p : it is raining
q : The home team wins.

(i) $p \to q$
If it is raining, then the home team wins.

(ii) Contrapositive

$\neg q \to \neg p$
If the home team does not win, then it is not raining.

(iii) Converse

$q \to p$
If The home team wins, then it is raining

(iv) Inverse

$\neg p \to \neg q$

If it is not raining, then the home team does not win.

Definition (Biconditional)

let p and q be propositions. The biconditional $p \leftrightarrow q$ is the proposition that is true when p and q have the same truth values, and false otherwise.

p	q	$p \leftrightarrow q$
T	T	T
T	F	F
F	T	F
F	F	T

Propositional Equivalence :-

Definition (Tautology, Contradiction, contingency)

(i) A compound proposition that is always true, no matter the truth value of the propositions that occur in it, is called a tautology

(ii) A compound proposition that is always false is called a Contradiction.

(iii) A proposition that is neither a tautology nor a contradiction is called contingency.

VIII

Ex

p	$\neg p$	$(p \vee \neg p)$	$(p \wedge \neg p)$
T	F	T	F
F	T	T	F

Therefore The proposition $p \vee \neg p$ is tautology and $p \wedge \neg p$ is a contradiction

Logical Equivalence :-

The propositions p and q are called logically Equivalent if $p \leftrightarrow q$ is a tautogogy

The notation $p \equiv q$ denotes that p and q are logically Equivalent.

Rules for logical Equivalence :-

(I) $p \wedge T \equiv p$ $\boxed{\text{(Identity laws)}}$

(II) $p \vee F \equiv p$

(III) $p \vee T \equiv T$ (Domination laws)

(IV) $p \wedge F \equiv F$

(V) $p \vee p \equiv p$ (Idempotent laws)

(VI) $p \wedge p \equiv p$

(VII) $\neg(\neg p) \equiv p$ (Double negation law)

(VIII) $p \vee q \equiv q \vee p$ (Commutative law)

$p \wedge q \equiv q \wedge p$

~~(IX)~~ $\cancel{p \vee q \equiv q \vee p}$ (

$\cancel{p \wedge q \equiv q \wedge p}$

(IX) $(p \vee q) \vee r \equiv p \vee (q \vee r)$ (Associative law)

(X) $(p \wedge q) \wedge r \equiv p \wedge (q \wedge r)$

(XI) $p \vee (q \wedge r) \equiv (p \vee q) \wedge (p \vee r)$ (distributive law)

(XII) $p \wedge (q \vee r) \equiv (p \wedge q) \vee (p \wedge r)$

(XIII) $\neg(p \wedge q) \equiv (\neg p) \vee (\neg q)$ (Demorgen's law)

(XIV) $\neg(p \vee q) \equiv (\neg p) \wedge (\neg q)$

(XV) $p \vee (p \wedge q) \equiv p$
(XVI) $p \wedge (p \vee q) \equiv p$ (Absorption law)

(XVII) $p \vee \neg p \equiv T$
(XVIII) $p \wedge \neg p \equiv F$ (negation law).

Logical Equivalence involving implications.

(I) $p \rightarrow q \equiv \neg p \vee q$
(II) $p \rightarrow q \equiv \neg q \rightarrow \neg p$
(III) $p \vee q \equiv \neg p \rightarrow q$
(IV) $p \wedge q \equiv \neg (p \rightarrow \neg q)$
(V) $\neg (p \rightarrow q) \equiv p \wedge (\neg q)$
(VI) $(p \rightarrow q) \wedge (p \rightarrow r) \equiv p \rightarrow (q \wedge r)$

E.t.c