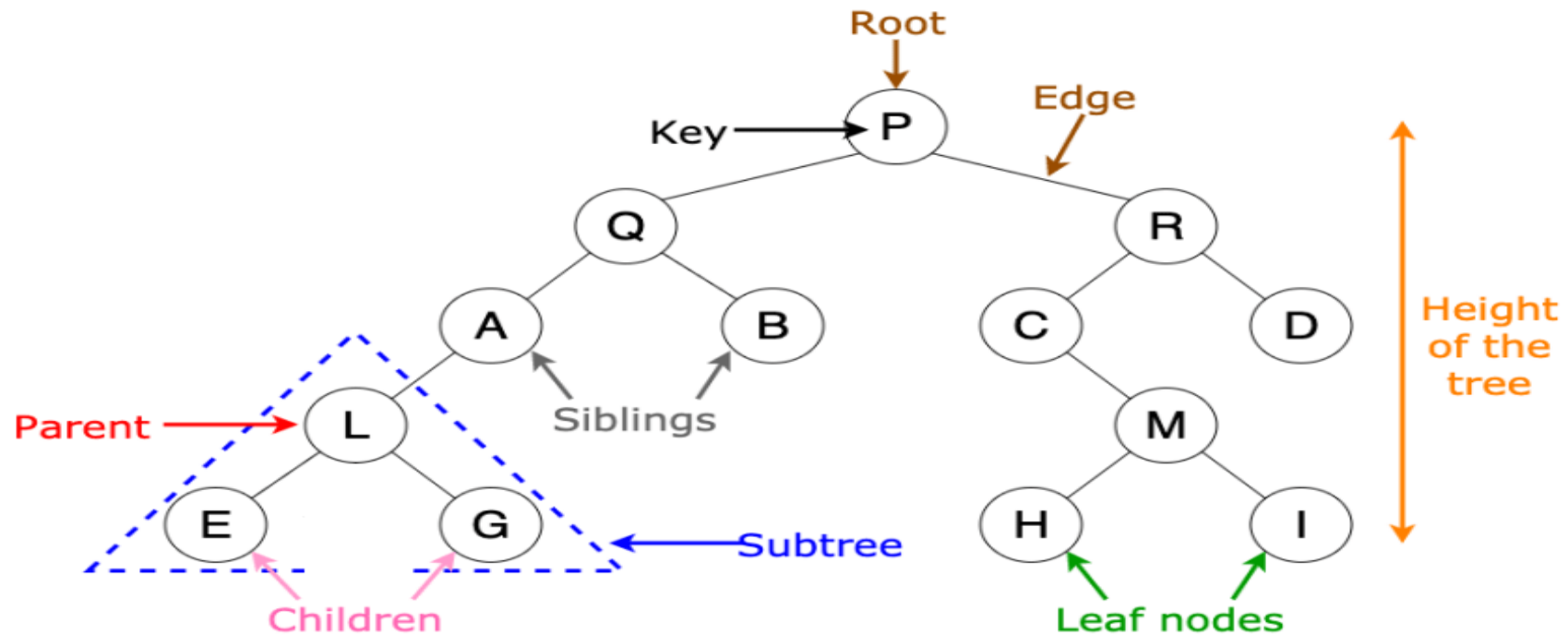




BINARY TREES

Binary Trees

- ▶ A **binary tree** is a hierarchical data structure in which each node has at most two children.



Examples of Binary Trees

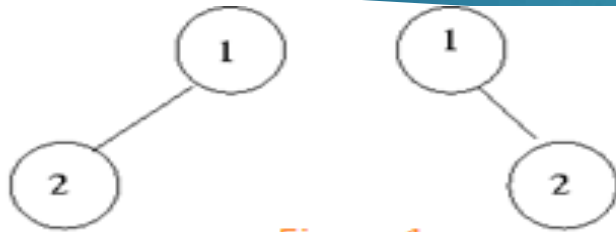


Figure 1

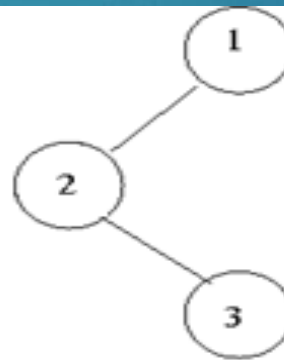


Figure 2

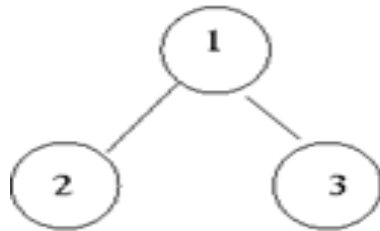
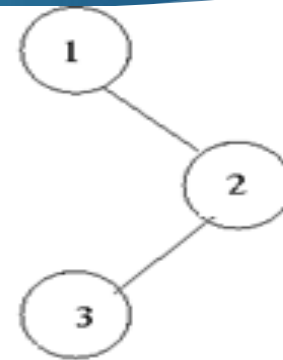


Figure 3

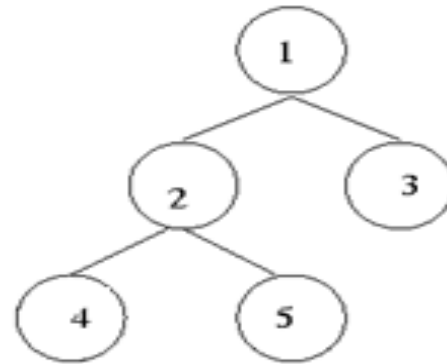


Figure 4

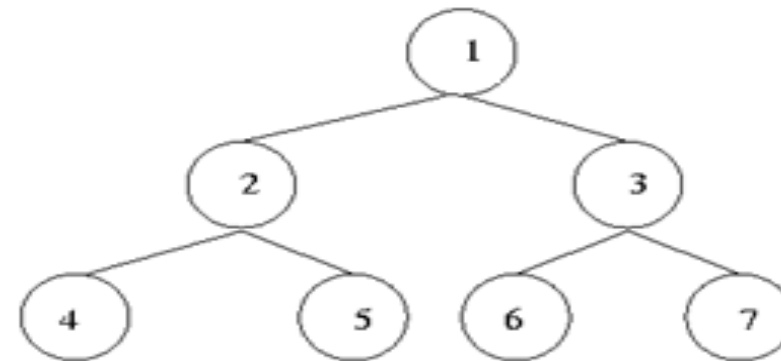


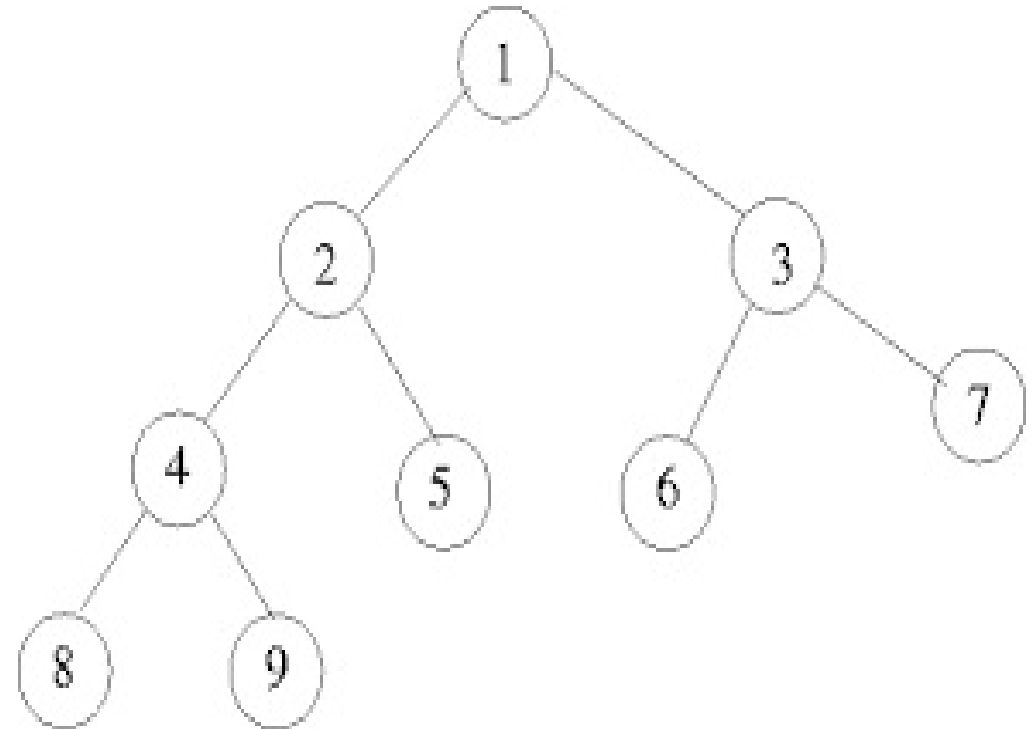
Figure 5

Properties of Binary Trees

- Number of internal nodes = (Leaf node-1)

In this figure: Leaf node = 5

So, Internal node = (Leaf node - 1)
= (5-1)
= 4



- Total number of nodes = $(2 * \text{Internal node}) + 1$

Example: If the leaf node is 5 then how many nodes exist in the tree ?

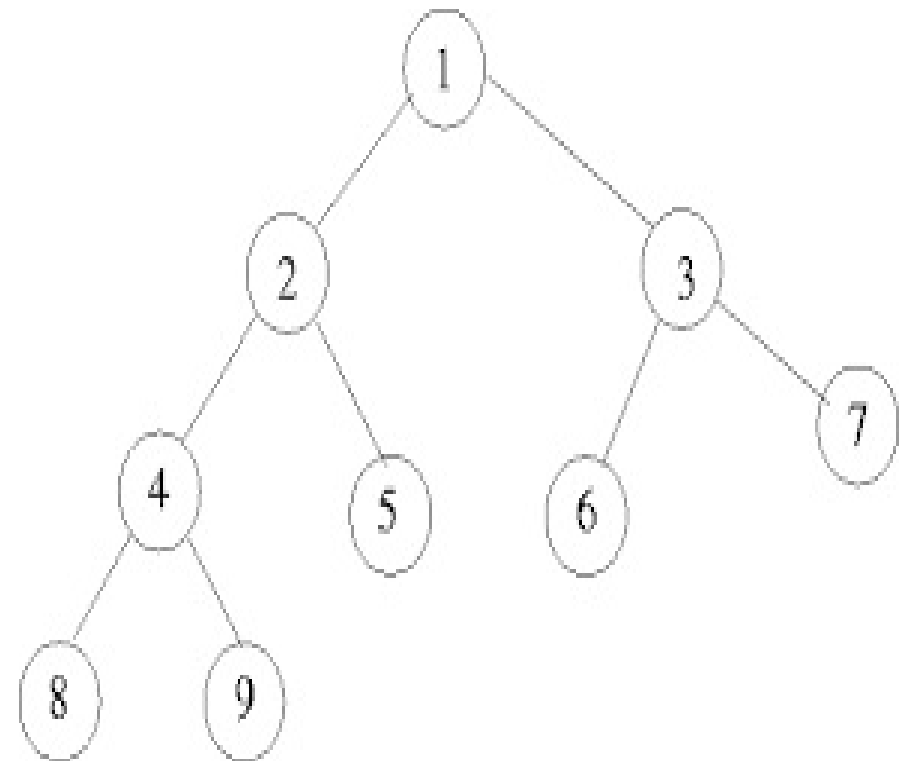
Leaf node = 5

Internal nodes = (Leaf node - 1)
= 4

So , Total number of nodes = $(2 * \text{Internal nodes}) + 1$
= $(2 * 4) + 1$
= 9

Hence,

Total number of nodes = 9



- Maximum number of nodes at any level 'L' = 2^L

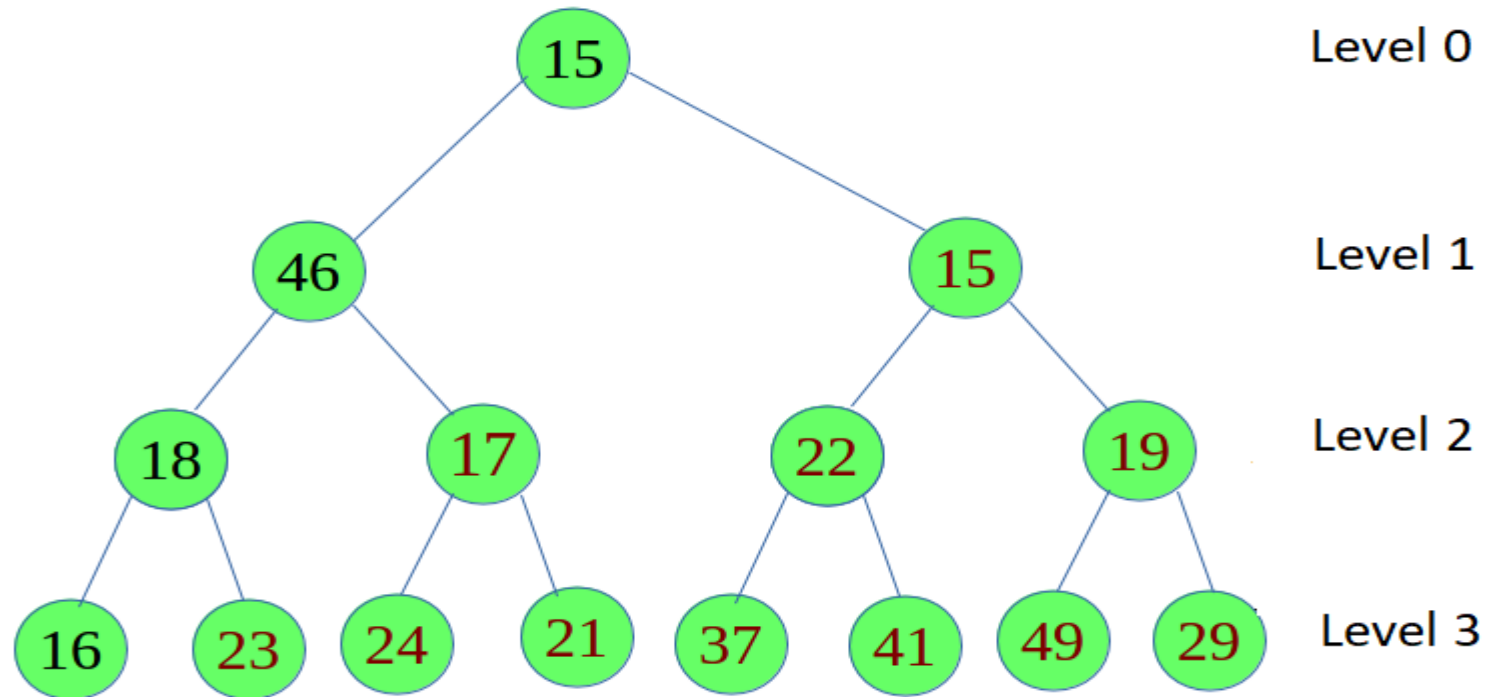
- At Level 0 , Nodes = 2^0

At Level 1, Nodes = 2^1

At Level 2, Nodes = 2^2

At Level 3, Nodes = 2^3

At Level L, Nodes = 2^L



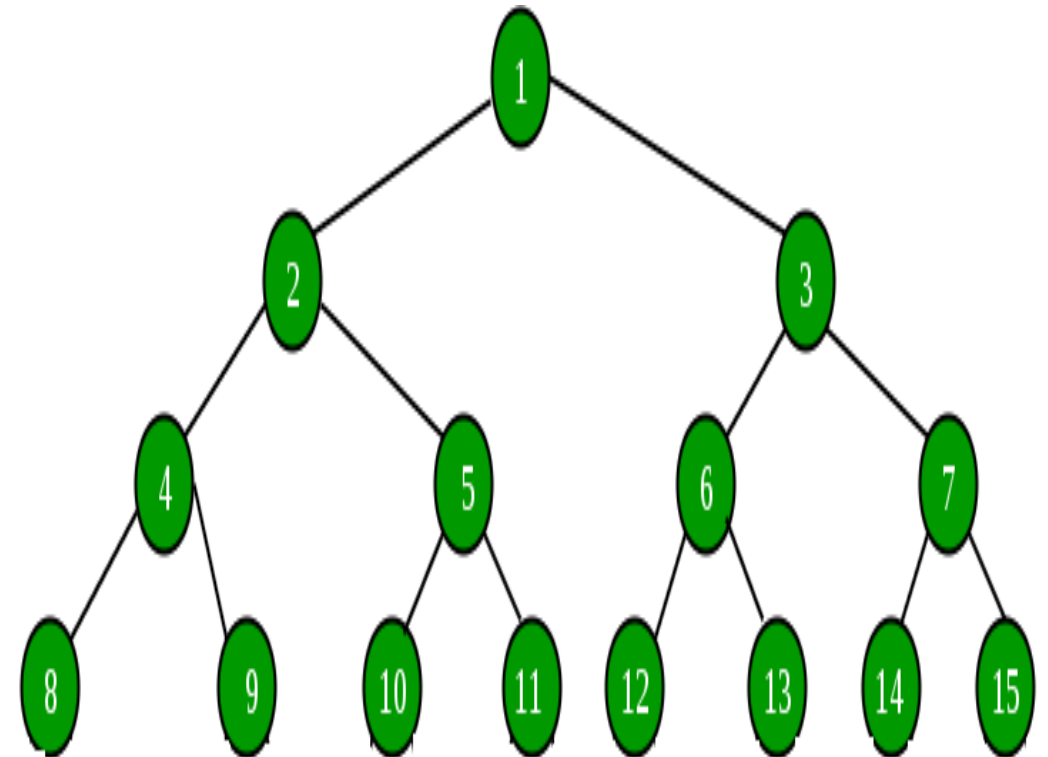
► Maximum number of nodes = $2^{H+1} - 1$

where H is the height of the tree

Example: If height of the tree is three then find the maximum number of nodes.

Height, $H=3$

$$\begin{aligned}\text{Maximum number of nodes} &= 2^{H+1} - 1 \\ &= 2^{3+1} - 1 \\ &= 15\end{aligned}$$

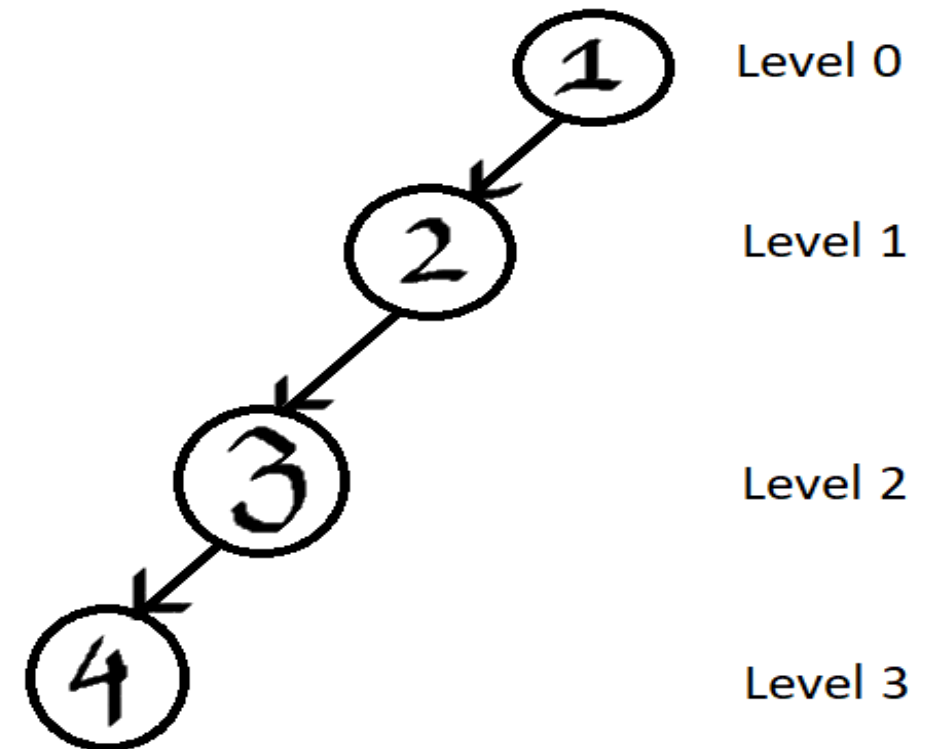


► Minimum number of nodes = $H+1$

Example: If height of the tree is three then find the minimum number of nodes.

Height, $H = 3$

$$\begin{aligned}\text{Minimum number of nodes} &= H+1 \\ &= 3+1 \\ &= 4\end{aligned}$$



Full Binary Tree

- ▶ A binary tree is a full binary tree if every node has zero or two children.

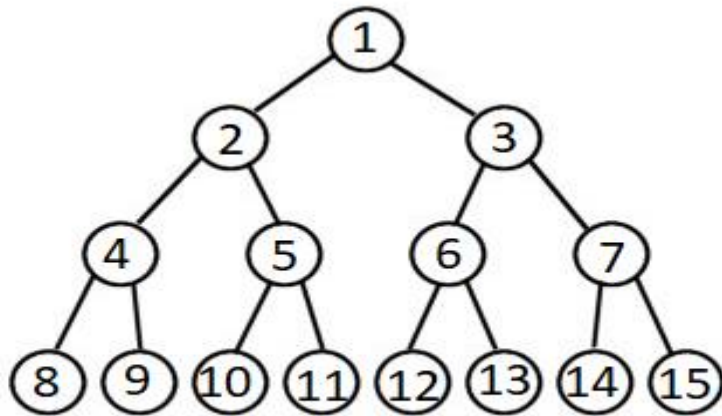


Figure 1

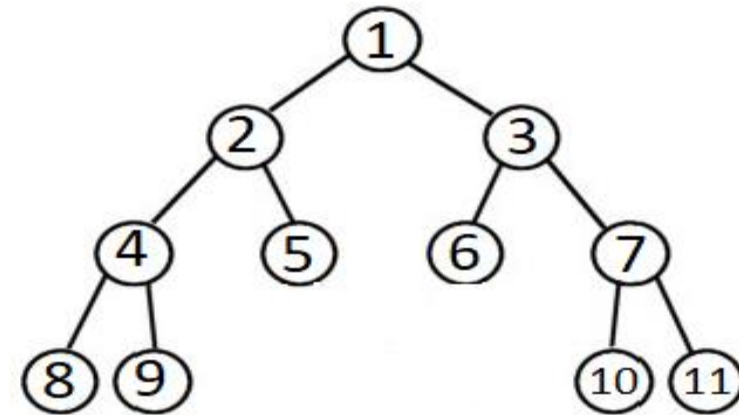


Figure 2

Complete Binary Tree

- ▶ A binary tree is a complete binary tree if all the levels are completely filled except possibly the last level.

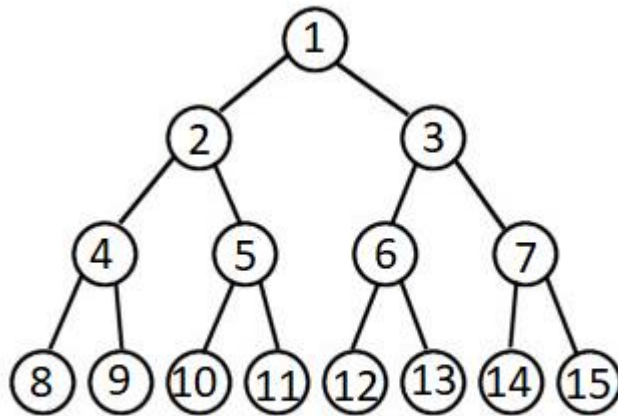


Figure 1

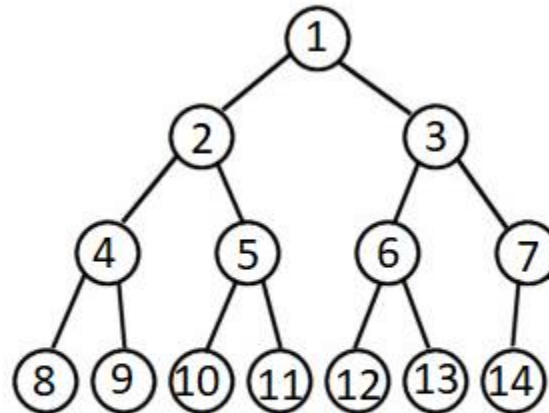


Figure 2

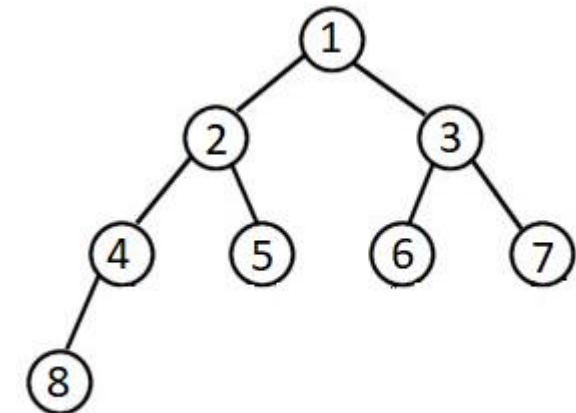
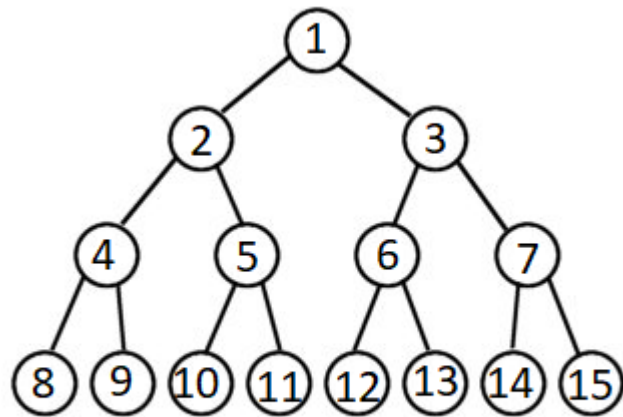


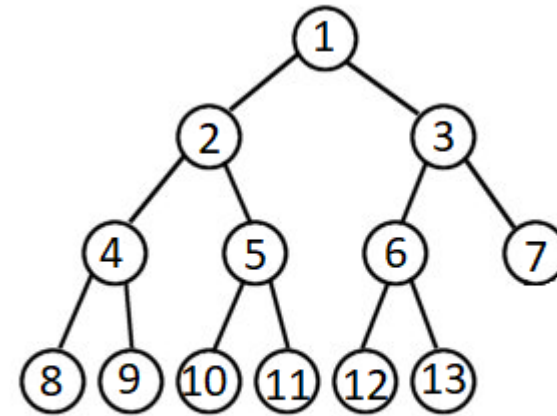
Figure 3

Perfect Binary Tree

- ▶ A binary tree is a perfect binary tree in which all the internal nodes have two children and all leaf nodes are at same level.



Perfect Binary Tree



Not Perfect Binary Tree

Degenerate Binary Tree

- ▶ A binary tree is a degenerate binary tree where for each parent node, there is only one associated child node.

Left Skewed Tree

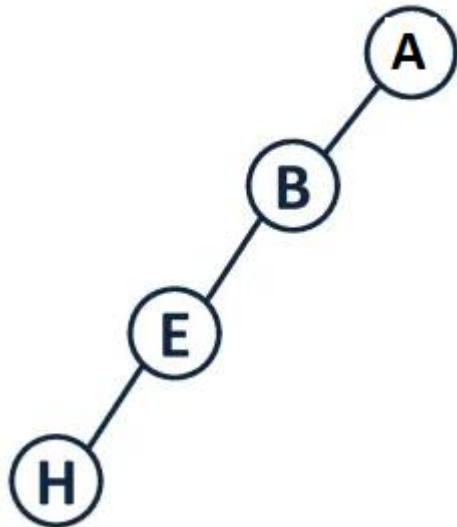


Figure 1

Right Skewed Tree

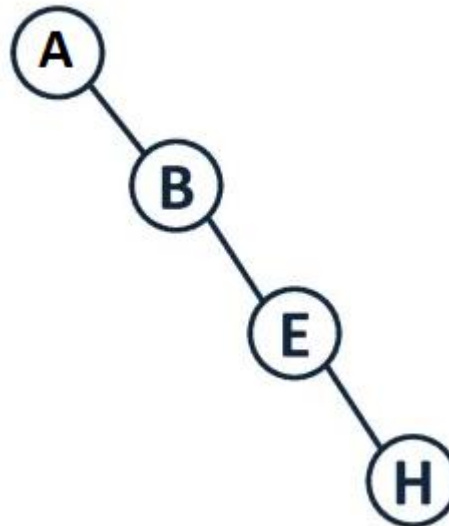


Figure 2

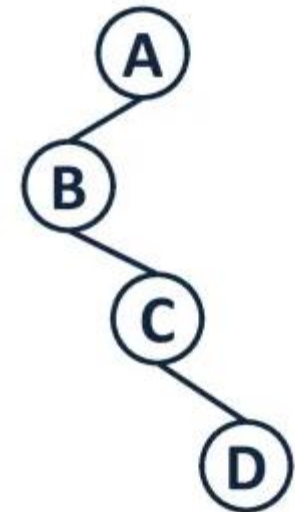


Figure 3