$$= \begin{bmatrix} \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ -\frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} \frac{1}{\sqrt{2}} & -\frac{1}{\sqrt{2}} & 0 \\ \frac{1}{\sqrt{2}} & \frac{1}{\sqrt{2}} & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Now let us apply the resultant matrix over (x, y) :

$$\begin{bmatrix} 0 & 1 & 0 \\ 1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix} \times \begin{bmatrix} x \\ y \\ 1 \end{bmatrix} = \begin{bmatrix} y \\ x \\ 1 \end{bmatrix}$$

3.3.3. Shearing

Shearing is a kind of transformations which produces distortion in the shape of an object. The effect of shear is that if the object is composed of layers, they are caused to slide over each other in the direction of applied force. The amount of slide varies inversely to the distance of a layer from the layer on which force is applied with zero effect on the base layer. Very commonly shear operations are performed to shift coordinates along either the *X*- or *Y*-axis.

For Shear along *x*-axis

$$\frac{\Delta x}{y} = \tan\theta$$

$$\Rightarrow \qquad \Delta x = y \tan\theta \qquad \text{...}(i)$$

If after shearing x' and y' are new values.

From. Eq. (*i*)

$$x' = x + y\tan\theta \qquad [Sh_x = \tan\theta)$$

$$y' = y$$

$$\left.\begin{aligned} x' &= x + \Delta x \\ y' &= y \end{aligned}\right\}$$

this value is taken from matrix.

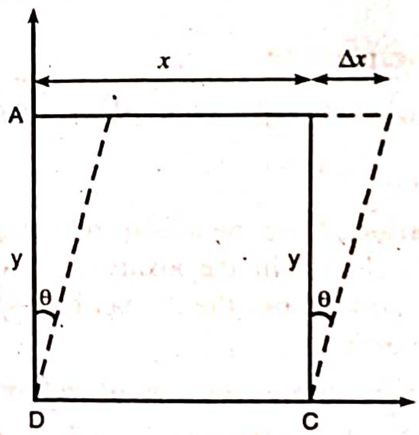


Fig. 3.19

For shear along the *x*-direction, we can use the following matrix:

$$T_{\text{shear}_x} = \begin{bmatrix} 1 & sh_x & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{...(3.33)}$$

The effect of this transformation is as follows :

$$x' = x + sh_x \times y, \; y' = y \qquad \text{...(3.34)}$$

The shear factor sh_x may assume any real value. Positive or negative values of the shear factor move coordinates along positive or negative directions of the *X*-axis respectively.

The y-direction shear can be similarly defined as :

$$T_{\text{shear}_y} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{...(3.35)}$$

The above shearing effects have been achieved by using the origin as the default reference point. If we wish to use another reference point (x_{ref}, y_{ref}), we may use the following relations:

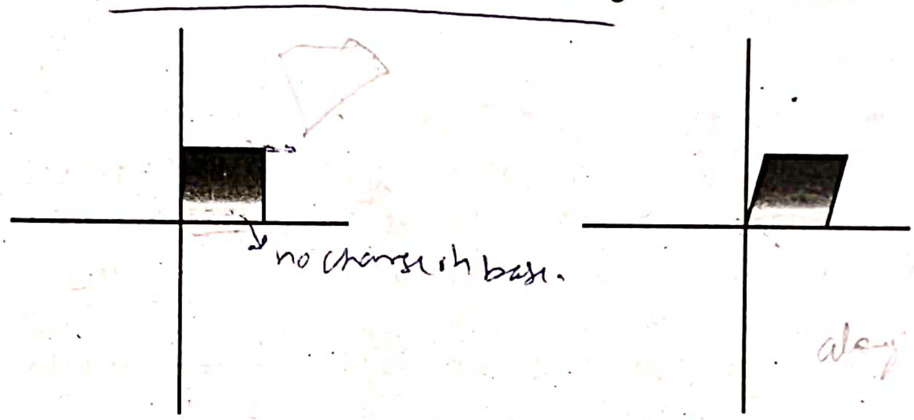
$$T_{\text{shear}_x} = \begin{bmatrix} 1 & sh_x & sh_x \times y_{ref} \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{...(3.36)}$$

column ~~row~~ wise representation

and

$$T_{\text{shear}_y} = \begin{bmatrix} 1 & 0 & 0 \\ sh_y & 1 & sh_y \times x_{ref} \\ 0 & 0 & 1 \end{bmatrix} \qquad \text{...(3.37)}$$

It is notable that the shearing transformation can be implemented using a sequence of rotation and scaling operations. Different kinds of shearing are shown in Figures 3.20 to 3.22.

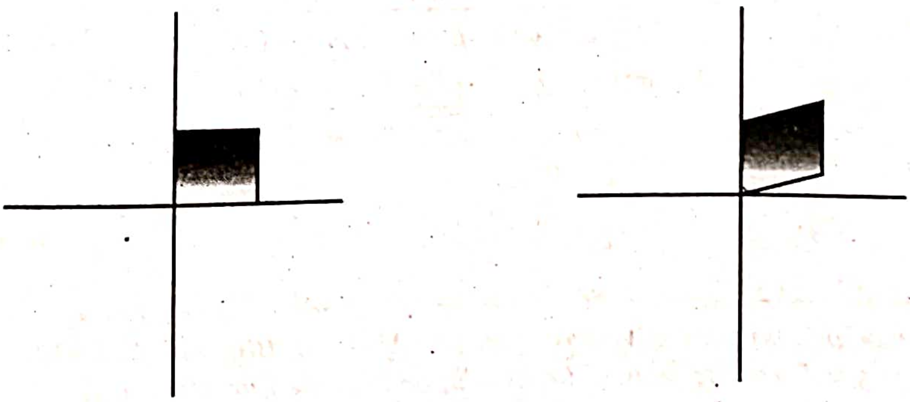


no change in base.

along X-direction

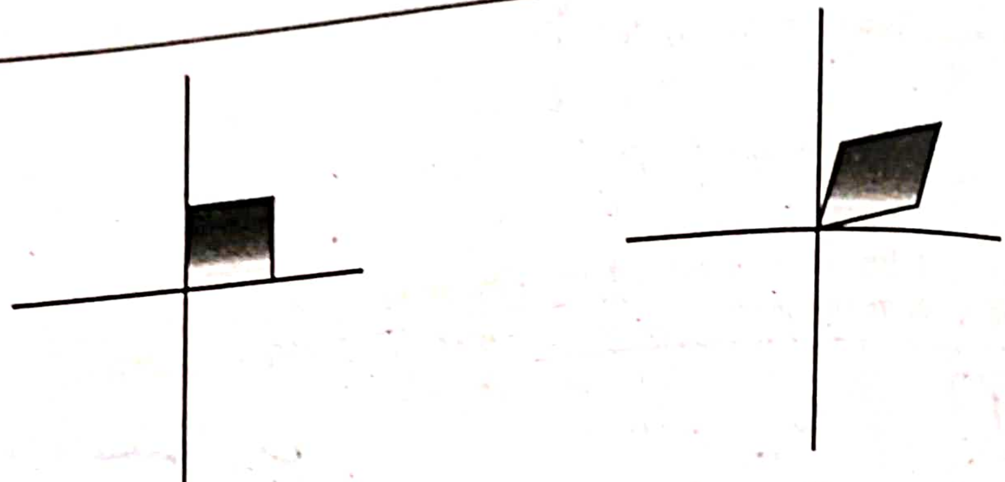
(a) (b)

Fig. 3.20. Shearing along x-direction : (a) before transformation, (b) after transformation.



(a) (b)

Fig. 3.21. Shearing along y-direction : (a) before transformation and (b) after transformation.



(a) (b)

Fig. 3.22. Shearing along x- and y-directions : (a) before transformation and (b) after transformation.

Example 3.11

Find the form of the matrix for reflection about a line L with slope m and y intercept (0, b).

Solution. Applying the fact that the angle of inclination of a line is related to its slope m by the equation $\tan(\theta) = m$, we have with $v = bJ$,

$$M_L = T_v . R_\theta . M_x . R_{-\theta} . T_{-y}$$

$$= \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & b \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta) & -\sin(\theta) & 0 \\ \sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} \cos(\theta) & \sin(\theta) & 0 \\ -\sin(\theta) & \cos(\theta) & 0 \\ 0 & 0 & 1 \end{pmatrix} \begin{pmatrix} 1 & 0 & 0 \\ 0 & 1 & -b \\ 0 & 0 & 1 \end{pmatrix}$$

Now if $\tan(\theta) = m$, standard trigonometry yields $\sin(\theta) = \dfrac{m}{\sqrt{m^2+1}}$ and $\cos(\theta) = \dfrac{1}{\sqrt{m^2+1}}$

Substituting these values for $\sin(\theta)$ and $\cos(\theta)$ after matrix multiplication, we have

$$M_L = \begin{pmatrix} \dfrac{1-m^2}{m^2+1} & \dfrac{2m}{m^2+1} & \dfrac{-2bm}{m^2+1} \\ \dfrac{2m}{m^2+1} & \dfrac{m^2-1}{m^2+1} & \dfrac{2b}{m^2+1} \\ 0 & 0 & 1 \end{pmatrix}$$