Lec - 2
UNIT-3

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Baye's theorem

It is based on conditional probability.

It Describes the probability of an event, based on prior knowledge of conditions that might be related to the event.

It relates conditional probability & marginal probabilities of 2 random events.

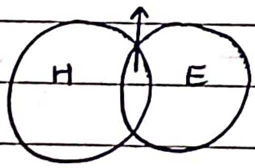
CP → $P(H|E) = \dfrac{\text{no. of times } H \text{ and } E}{\text{no. of times } E}$

Probability of hypothesis H given that evidence E is true.

$$P(H|E) = \frac{P(H \cap E)}{P(E)}$$

eg.

H∩E



If we know P(A|B) we can calculate P(B|A)

from above ~~theorem~~ formula

$$P(A \cap B) = P(A|B) \cdot P(B) \quad \text{—(i)}$$

Also,

$$P(A \cap B) = P(B|A) \cdot P(A) \quad \text{—(ii)}$$

from (i) & (ii)

$$P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

$\therefore$

Baye's theorem.

$$\boxed{P(B|A) = \frac{P(A|B) \cdot P(B)}{P(A)}}$$

P(A|B) → Posterior probability (prob of A when B is true.)

P(B|A) → Likelihood. (prob. of evidence)

P(A) → Prior probability (prob. of hypothesis)

P(B) → Marginal prob (prob. of evidence).

Q. What is the probability that person has disease dengue with neck pain?

given: 80% times dengue causes neck pain.

$$P(\text{dengue}) = \frac{1}{30{,}000}$$

$$P(\text{neckpain}) = .02$$

a → proposition that person has neck pain
b → proposition that person has dengue.

$p(a|b) = .8$

$P(a) = \dfrac{1}{30{,}000}$

$P(b) = .02$

$$p(b|a) = \cancel{\frac{.8 \times .02}{1/30{,}000}} \quad \frac{.8 \times 1/30{,}000}{.02}$$

$= 0.00133.$

Applications:

1. Robot / Automatic M/C: next step is calculated based on previous step.
2. Forecasting weather.
3. Monty hall problem can be solved.

Implementation.

Sort uncertainities into independent items of evidence.

use dempsters rule.

↓

Q. consider the case of car being towed in previous example.

Now, independent event would be

- degree of belief in car being found.
- Mike's reliability
- Mary's reliability.

Combining degrees of belief gets complicated with more evidence & hypothesis.

* Dempster's rule of combination.

$U \rightarrow$ set of all hypothesis. — car stolen — not stolen ⋮

$P(U) \rightarrow$ power set of U

$\phi \rightarrow$ empty set

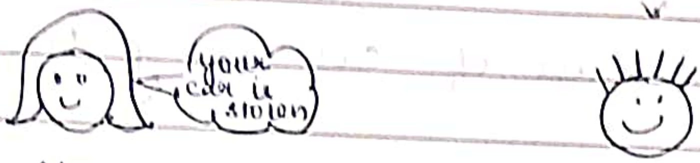
$m(A) \rightarrow$ mass assigned to A [belief function] eg. 0.98 in previous eg.

$$m(\phi) = 0$$

$$\Sigma\, m(A) = 1 \quad , \forall A \in P(U)$$

It is used in obtaining degree of belief for one event from probabilities of related events and combining those evidence beliefs when they are based on independent events.

Part. of definition.

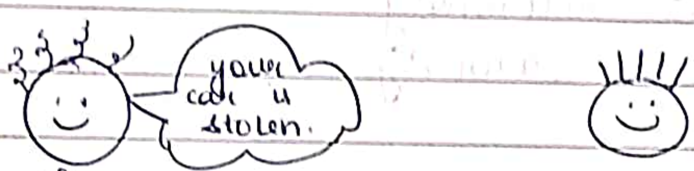


Mary. John.

P(Mary_reliable) = 0·85

degree of belief (car stolen) = 0·85

" (¬ car stolen) = 0



Mike John

P(Mike_reliable) = 0·90

Independent events

(both reliable) Joint probability = 0·85 x 0·90 = 0·765

(neither reliable) = (1 − 0·85) x (1 − ·90) = 0·01

∴ Prob. atleast one is reliable = (1 − 0·015)

= 0·985

This example is easy, now consider that one says car is stolen & other says not stolen, then it will be difficult for John to rely on anyone

→ Dempster - shafer Theory
→ partial order planning.

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Dempster- shafer Theory

It is designed to deal with distinction b/w uncertainity & ignorance.

Rather than calculating probability of a proposition, it computes probability that evidence supports proposition.

usually used in cases where we have insufficient information to estimate 'prior' and 'conditional' probabilities.

Q. An attack on parliament & A B C are possible culprits.

known — $p(C) = .8$

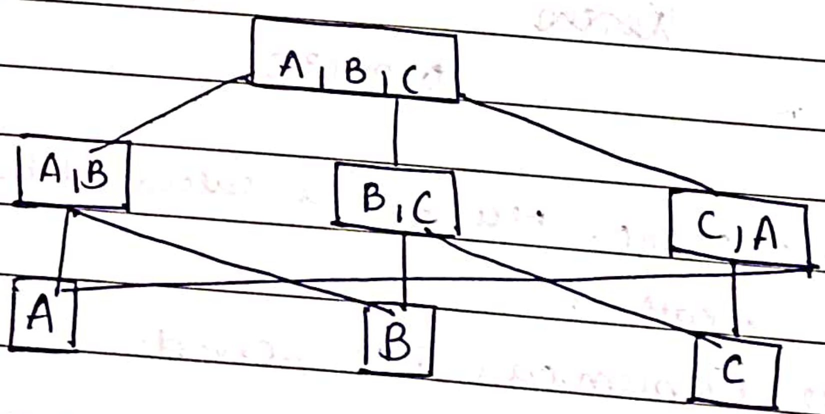
traditional theory says $p(A) + p(B) = .2$

DS doesn't say so.

can say .1

as .1 is uncertainity that neither A, B, C have ~~compa~~ commited the crime

concept of Belief subsets:



Assume m_1 & m_2 to be two belief functions, the way to combine them to form a new belief function is :

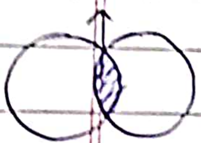
$$m_3(C) = \sum_{A\cap B=C} (m_1(A)\times m_2(B))$$

we have 2 belief functions → m_1 → mary
m_2 → mike

& we need to compute → m_3

normalization factor $= 1 - \underbrace{\sum_{A\cap B=\phi} (m_1(A) \times m_2(A))}_{\text{subsets that do not have intersection}}$

intersection



$$\text{DST} = \frac{\sum_{A\cap B=C} (m_1(A)\times m_2(B))}{1-\sum_{A\cap B=\phi}(m_1(A)\times m_2(B))}$$

$$= \frac{.85 \times .90}{1-(\cancel{.85\times .90})\ .015}$$

$$= \frac{.765}{\cancel{.235}\ .015}$$

$$= 3.255$$