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3034

B. Tech 3rd Semester (CSE) Examination – December, 2019

MATHEMATICS - III

(Multivariable Calculus and Differential Equations)

Paper : BSC-Math-203-G

Time : Three Hours] *[Maximum Marks : 75*

Before answering the questions, candidates should ensure that they have been supplied the correct and complete question paper. No complaint in this regard, will be entertained after examination.

Note : Attempt ***five*** questions in total, first being ***compulsory*** and select ***one*** from each unit.

1. (a) Investigate the continuity of the function

$$f(x,y) = xy/(x^2+y^2),\ (x,y) \neq (0,0)$$

$$= 0,\ (x,y) = (0,0)$$

at the origin.

$(D(D-1))[D^2-5D+6+8]+16$

$[D^2-5D+14]+16$

D^2

P. T. O.

(b) If $u = \sin^{-1}(x-y), x = 3t$ and $y = 4t^3$. find $\frac{du}{dt}$.

(c) Evaluate $\iint_R r^2 \sin\theta \, dr \, d\theta$ where, R is the semi-circle $r = 2a\cos\theta$ above the initial line

(d) Solve $\cos^2 x \frac{dy}{dx} + y = \tan x$

(e) Solve $\frac{d^4y}{dx^4} + 8\frac{d^2y}{dx^2} + 16y = 0$

(f) Using method of variation of parameter solve,

$$\frac{d^2y}{dx^2} - 2\frac{dy}{dx} + y = e^x / x.$$ 15

UNIT – I

2. (a) Let $r^2 = x^2 + y^2 + z^2$ and $v = r^m$, prove that

$$V_{xx} + V_{yy} + V_{zz} = m(m+1)\, r^{m-2}$$

(b) If $u = \sin^{-1}\frac{x+y}{\sqrt{x}+\sqrt{y}}$, find the value of

$$x^2\frac{\partial^2 u}{\partial x^2} + 2xy\frac{\partial^2 u}{\partial x \partial y} + y^2\frac{\partial^2 u}{\partial y^2}$$ 15

3. (a) In a plane triangle, find the maximum value of $\cos A \cos B \cos C$.

(b) Find the points on the surface $z^2 = xy + 1$, nearest to the origin. 15

UNIT – II

4. (a) By changing the order of integration, evaluate the following double integral $\int_0^1 \int_x^{\sqrt{2-x^2}} \frac{x\,dy\,dx}{\sqrt{x^2+y^2}}$.

(b) Evaluate $\iint_D xy\sqrt{1-x-y}\;dx\,dy$ where D is the region bounded by $x = 0$, $y = 0$, and $x + y = 1$ using the transformation $x + y = u$, $y = uv$ 15

5. (a) Find the area between the parabolas $y^2 = 4ax$ and $x^2 = 4ay$.

(b) Evaluate : 15

$$\int_{-c}^{e} \int_{-b}^{b} \int_{-a}^{a} (x^2 + y^2 + z^2)\,dx\,dy\,dz$$

UNIT – III

6. (a) Solve $\sec^2 y \frac{dy}{dx} + x \tan y = x^3$

(b) Solve $(x^4 e^x - 2mxy^2)\,dx + 2mx^2\,y\,dy = 0$ 15

7. (a) If the temp. of air is $30^\circ C$ and the substance cools trom $100^\circ C$ to $70^\circ C$ in 15 minutes. Find when the temperature will be $40^\circ C$.

(b) Find the orthogonal trisection of the family of curves $s^2 = a^2 \cos 2\theta$. 15

UNIT – IV

8. (a) Solve $\frac{d^2y}{dx^2} + 2\frac{dy}{dx} + y = e^{2x} - \cos^2 x$.

(b) Solve $(2x+3)^2 \frac{d^2y}{dx^2} - (2x+3)\frac{dy}{dx} - 12y = 6x$ 15

9. (a) Solve the following simultaneous equations $\frac{dx}{dt} + 2y = e^t$, $\frac{dy}{dt} - 2x = e^{-t}$.

(b) An alternative E. M. F E sinpt is applied to a circuit at $t = 0$. Given the equation for the current i as $L\frac{d^2i}{dt^2} + R\frac{di}{dt} + \frac{i}{c} = pE\,\text{cospt}$, find the current i when : 15

(i) $CR^2 > 4L$

(ii) $CR^2 < 4L$
