5.6.2. Liang-Barsky Clipping Algorithm

rectangular boundary

The Liang-Barsky clipping algorithm is another clipping algorithm that can be applied to regular rectangular clipping windows only. This algorithm deals with a line in its parametric form.

The parametric equation of a line between points (x_1, y_1) and (x_2, y_2) in terms of parameters t, can be expressed as

$$x = x_1 + \Delta x \,.\, t \qquad \ldots(5.14)$$
$$y = y_1 + \Delta y \,.\, t$$

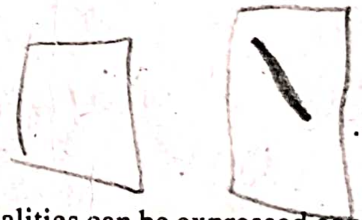
where $\Delta x = x_2 - x_1$ and $\Delta y = y_2 - y_1$. The parametric line is defined in the range of $0 \le t \le 1$. For extended line this range is $-\infty \le t \le +\infty$. The extended parametricl line cuts the extended clipping window boudaries at parameter values t_L, t_R, t_B and t_T, where the suffixes indicate left, right, bottom and top boundaries respectively.

Let $t_{\min}$ and $t_{\max}$ are the extreme parameter values for the visible portion of the line where $t_{\min} \le t_{\max}$. Thus we can define

$$t_{\min} = \max(0, t_L, t_B)$$
$$t_{\max} = \min(1, t_R, t_T) \qquad \ldots(5.15)$$

Now, for any point (x, y) inside the clipping window $[(x, y)$ is a point on the line between points (x_1, y_1) and $(x_2, y_2)]$, the following inequalities hold.

$$x_{\min} \le x_1 + \Delta x \,.\, t$$
$$x_{\max} \ge x1 + \Delta x \,.\, t$$
$$y_{\min} \le y1 + \Delta y \,.\, t \qquad \ldots(5.16)$$
$$y_{\max} \ge y1 + \Delta y \,.\, t$$



where t is the value of the parameter at point (x, y). The above four inequalities can be expressed commonly as

$$p_i t \le q_i \,; \quad \text{for } i = 1, 2, 3, 4 \qquad \ldots(5.17)$$

where

$$p_1 = -\Delta x\,; \quad q_1 = x_1 - x_{\min}$$
$$p_2 = \Delta x\,; \quad q_2 = x_{\max} - x_1$$
$$p_3 = -\Delta y\,; \quad q_3 = y_1 - y_{\min}$$
$$p_4 = \Delta y\,; \quad q_4 = y_{\max} - y_1$$

In the above equations, the indices $i = 1, 2, 3$ and 4 stand for four window boundaries, i.e. left, right, bottom and top boundaries respectively.

Now, we can observe the following facts :

1. $p_i = 0 \Rightarrow$ the line is parallel to the ith boundary

   $-q_i < 0 \Rightarrow$ the line is completely on the invisible side

   $-q_i \ge 0 \Rightarrow$ the line is completely on the visible side.
2. $p_i < 0 \Rightarrow$ the line comes from outside to inside the window intersecting the ith boundary.
3. $p_i > 0 \Rightarrow$ the line goes from inside to outside the window intersecting the ith boundary.
4. $p_i \ne 0 \Rightarrow$ the value of the parameter t at the intersection with the ith boundary is found to be $t = \dfrac{q_i}{p_i}$.

Liang-Barsky Clipping Algorithm

begin

$p_1 = x_1 - x_2$; $q_1 = x_1 - x_{min}$;

$p_2 = x_2 - x_1$; $q_2 = q_2 = x_{max} - x_1$;

$p_3 = y_1 - y_2$; $q_3 = y_1 - y_{min}$;

$p_4 = y_2 - y_1$; $q_4 = y_{max} - y_1$;

$t_{min} = 1$; $t_{max} = 0$;

for $t = 1$ to 4 do

begin

if $((p_i = 0)\ (q_i = 0))$ exclude line;

if $(p_i < 0)$

begin

$t = \frac{q_i}{p_i}$;

if $(t > t_{max})\ t_{max} = t$;

end

if $(p_i > 0)$

begin

$t = \frac{q_i}{p_i}$;

if $(t < t_{min})\ t_{min} = t$;

end

end

if $(t_{max} < t_{min})$ exclude line;

else use t_{max} and t_{min}

to compute the visible part of the line.

end

Example 5.10

Use the Liang-Barsky algorithm to clip the lines given in figure.

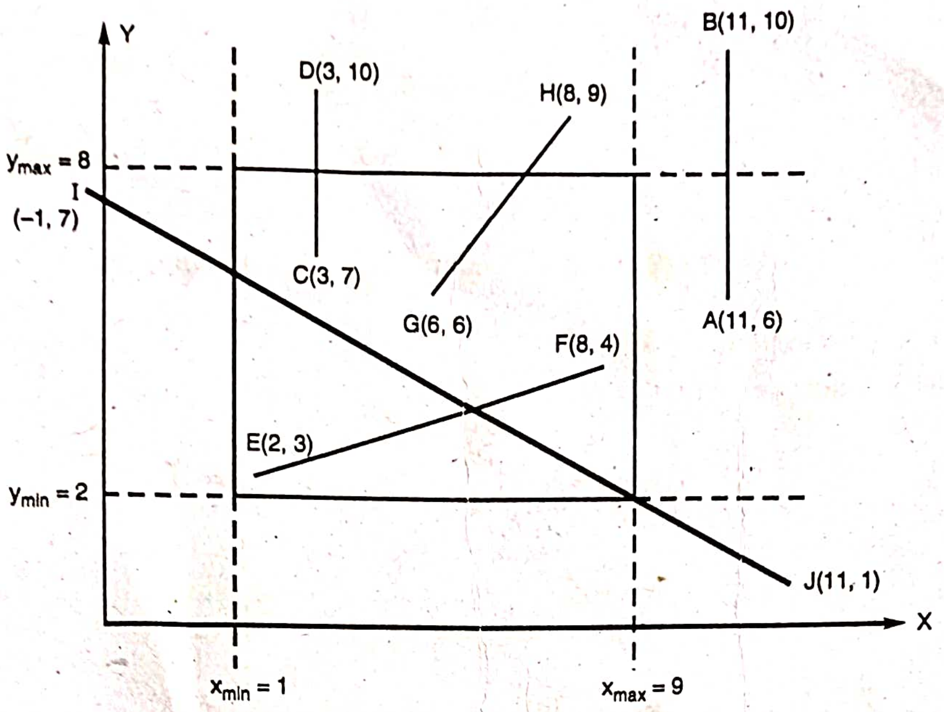


Fig. 5.11

Solution. For line *AB*, we have

$$p_1 = 0 \qquad q_1 = 10$$
$$p_2 = 0 \qquad q_2 = -2$$
$$p_3 = -4 \qquad q_3 = 4$$
$$p_4 = 4 \qquad q_4 = 2$$

Since $p_2 = 0$ and $q_2 < -2$, *AB* is completely outside the right boundary.

For line *CD*, we have

$$p_1 = 0 \qquad q_1 = 2$$
$$p_2 = 0 \qquad q_2 = 6$$
$$p_3 = -3 \qquad q_3 = 5 \qquad r_3 = -\frac{5}{3}$$
$$p_4 = 3 \qquad q_4 = 1 \qquad r_4 = \frac{1}{3}$$

Thus, $u_1 = \max\left(0, -\frac{5}{3}\right) = 0$ and $u_2 = \min\left(1, \frac{1}{3}\right) = \frac{1}{3}$. Since $u_1 < u_2$, the two endpoints of the clipped line are (3, 7) and $\left(3, 7 + 3\left(\frac{1}{3}\right)\right) = (3, 8)$.

For line *EF*, we have

$$p_1 = -6 \qquad q_1 = 1 \qquad r_1 = -\frac{1}{6}$$
$$p_2 = 6 \qquad q_2 = 7 \qquad r_2 = \frac{7}{6}$$

$$p_3 = -1 \qquad q_3 = 1 \qquad r_3 = -\frac{1}{1}$$

$$p_4 = 1 \qquad q_4 = 5 \qquad r_4 = \frac{5}{1}$$

Thus, $u_1 = \max\left(0, -\frac{1}{6}, -1\right) = 0$ and $u_2 = \min\left(1, \frac{7}{6}, 5\right) = 1$. Since $u_1 = 0$ and $u_2 = 1$, line EF is completely inside the clipping window.

For line GH, we have

$$p_1 = -2 \qquad q_1 = 5 \qquad r_1 = -\frac{5}{2}$$

$$p_2 = 2 \qquad q_2 = 3 \qquad r_2 = \frac{3}{2}$$

$$p_3 = -3 \qquad q_3 = 4 \qquad r_3 = -\frac{4}{3}$$

$$p_4 = 3 \qquad q_4 = 2 \qquad r_4 = \frac{2}{3}$$

Thus, $u_1 = \max\left(0, -\frac{5}{2}, -\frac{4}{3}\right) = 0$ and $u_2 = \min\left(1, \frac{3}{2}, \frac{2}{3}\right) = \frac{2}{3}$. Since, $u_1 < u_2$, the two endpoints of the clipped line are (6, 6) and $\left(6 + 2\left(\frac{2}{3}\right), 6 + 3\left(\frac{2}{3}\right)\right) = \left(7\frac{1}{3}, 8\right)$.

For line IJ, we have

$$p_1 = -12 \qquad q_1 = -2 \qquad r_1 = \frac{1}{6}$$

$$p_2 = 12 \qquad q_2 = 10 \qquad r_2 = \frac{5}{6}$$

$$p_3 = 6 \qquad q_3 = 5 \qquad r_3 = \frac{5}{6}$$

$$p_4 = -6 \qquad q_4 = 1 \qquad r_4 = -\frac{1}{6}$$

Thus, $u_1 = \max\left(0, \frac{1}{6}, -\frac{1}{6}\right) = \frac{1}{6}$ and $u_2 = \min\left(1, \frac{5}{6}, \frac{5}{6}\right) = \frac{5}{6}$. Since $u_1 < u_2$, the two endpoints of the clipped line are $\left(-1 + 12\left(\frac{1}{6}\right), 7 + (-6)\left(\frac{1}{6}\right)\right) = (1, 6)$ and $\left(-1 + 12\left(\frac{5}{6}\right), 7 + (-6)\left(\frac{5}{6}\right)\right) = (9, 2)$.