8/10/2020

# Assignment - I

1. Investigate the continuity of the function

$$f(x,y) = \begin{cases} \dfrac{xy}{x^2+y^2} \; ; & (x,y) \neq (0,0) \\ 0 \; ; & (x,y) = (0,0) \end{cases}$$

at the origin.

2. If $V = r^m$ where $r^2 = x^2+y^2+z^2$

show that $V_{xx} + V_{yy} + V_{zz} = m(m+1) r^{m-2}$.

3. If $V = f(r)$ and $r^2 = x^2+y^2+z^2$

Prove that $V_{xx} + V_{yy} + V_{zz} = f''(r) + \frac{2}{r} f'(r)$.

4. If $u = \sin^{-1}\left(\dfrac{x+y}{\sqrt{x}+\sqrt{y}}\right)$

Prove that $x^2 \dfrac{\partial^2 u}{\partial x^2} + 2xy \dfrac{\partial^2 u}{\partial x \partial y} + y^2 \dfrac{\partial^2 u}{\partial y^2}$

$$= \frac{-\sin u \cos 2u}{4 \cos^3 u}$$

5. If $u = \sin^{-1}(x-y)$, $x = 3t$, $y = 4t^3$,

find $\dfrac{du}{dt}$.

(6) Find $\frac{dy}{dx}$, when $x^y + y^x = c$

(7) If $x = e^r \cos\theta$, $y = e^r \sin\theta$

Show that $\frac{\partial^2 u}{\partial x^2} + \frac{\partial^2 u}{\partial y^2} = e^{-2r}\left(\frac{\partial^2 u}{\partial r^2} + \frac{\partial^2 u}{\partial \theta^2}\right)$

(8) In a plane triangle ABC, find the maximum value of $\cos A . \cos B . \cos C$.

(9) Find the points on the surface $z^2 = xy + 1$, nearest to the origin.

(10) Find the dimensions of the rectangular box, open at the top, of maximum capacity whose surface is 432 sq. cm.