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Ans 1a) $x\,dy - y\,dx = (x^2+y^2)\,dx$

Divide by (x^2+y^2) on both side.

$$\Rightarrow \frac{x\,dy - y\,dx}{(x^2+y^2)} = dx$$

$$\Rightarrow \frac{x\,dy - y\,dx}{x^2\left(1+\left(\frac{y}{x}\right)^2\right)} = dx$$

$$\Rightarrow \frac{\frac{y\,dx - x\,dy}{x^2}}{\left(1+\left(\frac{y}{x}\right)^2\right)} = -dx$$

$$\Rightarrow \frac{d\left(\frac{x}{y}\right)}{\left(1+\left(\frac{y}{x}\right)^2\right)} = -dx \qquad \left[\because \frac{y\,dx - x\,dy}{x^2} = d\left(\frac{y}{y}\right)\right.$$

Integrate both side

$$\Rightarrow \int \frac{d\left(\frac{x}{y}\right)}{\left(1+\left(\frac{y}{x}\right)^2\right)} = -\int dx$$

$$\Rightarrow \tan^{-1}\left(\frac{y}{x}\right) = -x + c$$

Ans 1c) $y'' + 4y = 4\sec^2 2x$

Sol^n The symbolic form of given differential eq^n is

$$(D^2+4)y = 4\sec^2 2x$$

Now, AE is $D^2+4=0$

$\Rightarrow D = \pm 2i$

So, CF is $y = e^{0x}(c_1\cos 2x + c_2 \sin 2x)$

$\Rightarrow$ cf is $y = c_1 \cos 2x + c_2 \sin 2x$

So, here $y_1 = \cos 2x$ & $y_2 = \sin 2x$

$$W = \begin{vmatrix} y_1 & y_2 \\ y_1' & y_2' \end{vmatrix}$$

$$= \begin{vmatrix} \cos 2x & \sin 2x \\ -2\sin 2x & 2\cos 2x \end{vmatrix}$$

$= 2\cos^2 2x + 2\sin^2 2x$

$= 2(1)$

$= 2$

Now, PI $= u_1 y_1 + u_2 y_2$; $\begin{cases} u_1 = -\int \frac{y_2 X}{W} dx \\ u_2 = \int \frac{y_1 X}{W} dx \end{cases}$

where $X = 4\sec^2 2x$

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$$\Rightarrow u_1 = -\int \frac{\sin 2x \left(4\sec^2 2x\right)}{2}\, dx$$

$$= -2\int \tan 2x \cdot \sec 2x \, dx$$

$$= -\sec 2x$$

$$\Rightarrow u_2 = \int \frac{\cos 2x \cdot \left(4\sec^2 2x\right)}{2}\, dx$$

$$= 2\int \sec 2x \, dx$$

$$= 2 \cdot \frac{1}{2} \log\left|\sec 2x + \tan 2x\right|$$

$$= \log\left|\sec 2x + \tan 2x\right|$$

$$\therefore PI = -\sec 2x \cdot \cos 2x + \log\left|\sec 2x + \tan 2x\right| \sin 2x$$

$$= -1 + \log\left|\sec 2x + \tan 2x\right| \sin 2x$$

So, complete solution (CS) is

$$\boxed{y = c_1 \cos 2x + c_2 \sin 2x + \log\left|\sec 2x + \tan 2x\right| \sin 2x - 1}$$

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Ans 2a) $(D-2)^2 y = 8(e^{2x} + \sin 2x + x^2)$

A.E is ~~(D-2)(D-2)=0~~ $(D-2)(D-2)=0$

$\therefore D = 2, 2$

Now, CF is $y = (c_1 + c_2 x)e^{2x}$

So, $PI = \frac{1}{(D-2)^2}\left[8(e^{2x} + \sin 2x + x^2)\right]$

$\Rightarrow PI = 8\left[\frac{e^{2x}}{(D-2)^2} + \frac{\sin 2x}{(D-2)^2} + \frac{x^2}{(D-2)^2}\right]$

Now, $\frac{e^{2x}}{(D-2)^2} = x \cdot \frac{e^{2x}}{2(D-2)}$ [case of failure]

$= x^2 \cdot \frac{e^{2x}}{2}$

Now, $\frac{1}{(D-2)^2}\sin 2x = \frac{\sin 2x}{D^2-4D+4}$

$= \frac{1}{-2^2-4D+4}\sin 2x$ [Putting $D^2=-2^2$]

$= \frac{-1}{4D}\sin 2x$

~~Integrate the above eq^n~~

~~$= -\int \frac{1}{4D}\sin 2x\, dx$~~

~~$= \frac{1}{4}\int \sin 2x\, dx$~~

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$$= -\frac{1}{4}\int \sin 2x\,dx$$

$$= -\frac{1}{4}\left(-\frac{\cos 2x}{2}\right) = \frac{1}{8}\cos 2x$$

Now,

$$\frac{1}{(D-2)^2}x^2 = \frac{1}{(2-D)^2}x^2$$

$$= \frac{1}{4\left(1-\frac{D}{2}\right)^2}x^2.$$

$$= \frac{1}{4}\left(1-\frac{D}{2}\right)^{2}x^2$$

$$= \frac{1}{4}\left[1-2\left(-\frac{D}{2}\right)+\frac{(-2)(-3)}{2}\left(\frac{D}{2}\right)^2 - - -\right]x^2$$

$$= \frac{1}{4}\left[1+D+\frac{3}{4}D^2+ - - -\right]x^2$$

$$= \frac{1}{4}\left[x^2+D(x^2)+\frac{3}{4}D^2(x^2)\right]$$

$$PI = 8\left[\frac{x^2}{2}e^{2x}+\frac{1}{8}\cos 2x+\frac{1}{4}\left(x^2+2x+\frac{3}{2}\right)\right]$$

$$PI = 4x^2e^{2x}+\cos 2x+2x^2+4x+3$$

Hence, complete solution (CS) is

$$y = (C_1+C_2x)e^{2x}+4x^2e^{2x}+\cos 2x+2x^2+4x+3$$

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Ans 3a) $\frac{dy}{dx} + \frac{1}{x} = \frac{e^y}{x^2}$

This eq^n is Bernoulli's Eq^n
So divide by e^y on both side

$\Rightarrow e^{-y}\frac{dy}{dx} + \frac{e^{-y}}{x} = \frac{1}{x^2}$ — (1)

Now, Put $e^{-y} = z$

$\Rightarrow -e^{-y}\frac{dy}{dx} = \frac{dz}{dx}$

$\Rightarrow e^{-y}\frac{dy}{dx} = \frac{-dz}{dx}$

Put this value in eq^n (1)

$\Rightarrow \frac{-dz}{dx} + \frac{z}{x} = \frac{1}{x^2}$

Multiply by (-1) on both sides

$\Rightarrow \frac{dz}{dx} - \frac{z}{x} = -\frac{1}{x^2}$

Now, it is in the form of $\left(\frac{dz}{dx} + Pz = Q\right)$

$\therefore P = \frac{-1}{x}, \quad Q = \frac{-1}{x^2}$

$\Rightarrow IF = e^{\int -\frac{1}{x}} = e^{-\log x}$

$IF = \frac{1}{x}$

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So, complete solution is

$z\,(IF) = \int Q.(IF)$

$$\Rightarrow z \cdot \frac{1}{x} = \int \left(\frac{-1}{x^2} \cdot \frac{1}{x} \right) dx$$

$$\frac{z}{x} = -\int \frac{1}{x^3}\, dx$$

$$\frac{z}{x} = -\left[\frac{-1}{2x^2} \right] + C$$

$$\Rightarrow \frac{z}{x} = \frac{1}{2x^2} + C \quad \text{—— (2)}$$

Now, Put the value of z i.e., e^{-y} in eqn (2)

$$\boxed{\frac{e^{-y}}{x} = \frac{1}{2x^2} + C}$$

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