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College Name:

DELHI GLOBAL INSTITUTE OF TECHNOLOGY

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Subject: Mathematics-III

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Ans. c) $\int_0^3 \int_0^1 (x^2+3y^2)\, dy\, dx$

$$\int_0^3 \int_0^1 (x^2+3y^2)\, dy\, dx = \int_0^3 \left[x^2 y + \frac{\cancel{3}y^3}{\cancel{3}}\right]_0^1 dx$$

$$= \int_0^3 \left[(x^2+1) - 0\right] dx$$

$$= \int_0^3 (x^2+1)\, dx$$

$$= \left[\frac{x^3}{3} + x\right]_0^3$$

$$= \left[\left(\frac{27}{3} + 3\right) - 0\right]$$

$$= 12$$

Hence, $\int_0^3 \int_0^1 (x^2+3y^2)\, dy\, dx = 12$

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Ans 1(e) $\int_0^a\int_0^a\int_0^a xy+yz+zx \; dx\,dy\,dz$

$$\int_0^a\int_0^a\int_0^a (xy+yz+zx)\,dx\,dy\,dz = \int_0^a\int_0^a \left[\frac{x^2y}{2}+xyz+\frac{x^2z}{2}\right]_0^a dy\,dz$$

$$= \int_0^a\int_0^a \left[\left(\frac{a^2y}{2}+ayz+\frac{a^2z}{2}\right)-0\right]dy\,dz$$

$$= \int_0^a\int_0^a \left(\frac{a^2y}{2}+ayz+\frac{a^2z}{2}\right)dy\,dz$$

$$= \int_0^a \left(\frac{a^2y^2}{4}+\frac{ay^2z}{2}+\frac{a^2yz}{2}\right)_0^a dz$$

$$= \int_0^a \left(\frac{a^4}{4}+\frac{a^3z}{2}+\frac{a^3z}{2}\right)dz$$

$$= \left(\frac{a^4z}{4}+\frac{a^3z^2}{4}+\frac{a^3z^2}{4}\right)_0^a$$

$$= \left[\left(\frac{a^5}{4}+\frac{a^5}{4}+\frac{a^5}{4}\right)-0\right]$$

$$= \frac{3a^5}{4}$$

Hence,

$$\int_0^a\int_0^a\int_0^a (xy+yz+zx)\,dx\,dy\,dz = \frac{3}{4}a^5$$

Ans 2 a) If $u = \dfrac{x^2y^2}{x+y}$, Prove: $x\dfrac{\partial^2 u}{\partial x^2} + y\dfrac{\partial^2 u}{\partial x \partial y} = 2\dfrac{\partial u}{\partial x}$

$$u = \frac{x^2 y^2}{x+y}$$

$$\Rightarrow \quad u = \frac{x^2 \left(y^2/x^2 \right)}{x \left(1 + y/x \right)}$$

$$\Rightarrow \quad u = x^1 f\left(\frac{y}{x}\right)$$

$\therefore$ u is a homogeneous function of degree 1.

By Euler's theorem, we have:

$$x\frac{\partial u}{\partial x} + y\frac{\partial u}{\partial y} = 1u \quad \text{—} (1)$$

differentiate eq^n (1) w.r.t x

$$x\frac{\partial^2 u}{\partial x^2} + (1)\frac{\partial u}{\partial x} + y\frac{\partial^2 u}{\partial x \partial y} = 1\frac{\partial u}{\partial x}$$

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = \frac{\partial u}{\partial x} + \frac{\partial u}{\partial x}$$

$$x\frac{\partial^2 u}{\partial x^2} + y\frac{\partial^2 u}{\partial x \partial y} = 2\frac{\partial u}{\partial x}$$

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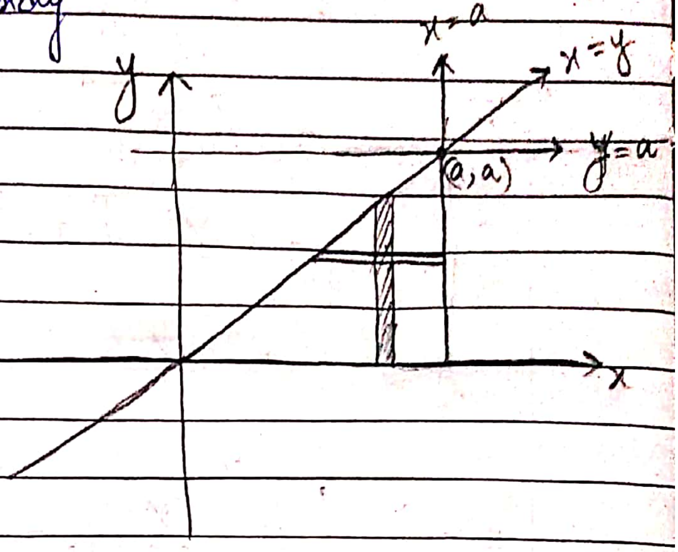
Ans 3 a) $\displaystyle\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dxdy$



The old limits are:

$x = y$ to $x = a$

and

$y = 0$ to $y = a$

Thus the region of integration is bounded by using these limits and using this region we get the new limits for integration.

The new limits are:

$y = 0$ to $y = x$

$x = 0$ to $x = a$

Now, we evaluate after changing the order of integration

$$\int_0^a \int_y^a \frac{x}{x^2+y^2}\, dxdy \; \cancel{dydx} = \int_0^a \int_0^x \frac{x}{x^2+y^2}\, dydx$$

$$= \int_0^a \int_0^x x \cdot \left[\frac{dy}{x^2+y^2}\right] dx$$

$$= \int_0^a x \left[\frac{1}{x}\tan^{-1}\frac{y}{x}\right]_0^x dx$$

$$= \int_0^a x\left[\frac{1}{x}\tan^{-1}\frac{x}{x} - 0\right] dx$$

$$= \int_0^a \tan^{-1}(1) . dx$$

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$$= \int_0^a \frac{\pi}{4} dx$$

$$= \frac{\pi}{4}\left[x\right]_0^a$$

$$= \frac{\pi}{4}\left[a-0\right]$$

$$= \frac{\pi a}{4}$$

$$\Rightarrow \int_0^a \int_0^x \frac{x}{x^2+y^2}\, dy\, dx = \frac{a\pi}{4}$$