

8TH Oct Friday @ 10:30 AM (FLA Lecture-3)

Topics Covered

- **String**
 - **Operations on String**
- **Finite Automata**

→ strings "An alphabet is a non-empty finite set of symbols.
denoted by Σ .

eg. $\Sigma = \{a, b, c\}$ is an alphabet.

→ A string is a finite sequence of symbols. (U or w)

eg. $U = abcab$ is a string on $\Sigma = \{a, b, c\}$.

The empty string (no symbol at all) denoted by λ or ϵ .

- A part of string is a substring.

bca is a substring of $abcab$.

Note:- A beginning of a string (up to any symbol) is a prefix & an ending is a suffix.

a b c a b
prefixes.

a b c a b
suffixes.

*** "A string is a prefix & suffix of itself. λ or ϵ is a prefix & suffix of any string."

⇒ operations on string:-

1) Finding the length $\Sigma = \{a, b\}$

$w = abba$

$|w| = 4$

2) concatenation

$w = abc, v = ab$

$wv = \underbrace{abc}_w \underbrace{ab}_v$

* → $\lambda w = w\lambda = w$.

3) Power:- $w^0 = \lambda$ (null string)

$$w^1 = w \Rightarrow w^2 = ww \Rightarrow w^3 = w.w^2 = w.w.w$$

$$w^n = w.w^{n-1} = w.w. \dots w \text{ (n-times)}$$

4) Reverse:- $w^R \rightarrow w$ in reverse order

$$w = abc$$

$$w^R \rightarrow cba$$

5) Palindrome: $|w| = |w^R|$

Word & its reversal have same value.

$$\left\langle \begin{array}{l} w = aba \\ w^R = aba \end{array} \right\rangle$$

Even Palindrome:

i) $w = w^R$

ii) $|w|$ is even.

Odd Palindrome:

i) $w = w^R$

ii) $|w|$ is odd.

eg $\lambda^R = \lambda$ (null) $\rightarrow 0$ is even palindrome.

$a^R = a$ (odd) \rightarrow odd Palindrome.

eg No. of palindrome of length 8 over $\Sigma = \{0,1\}$,

$$\rightarrow 2^4 = 16.$$

* No. of palindromes of length n over $\Sigma = k$ is

$$\boxed{k^{\lceil n/2 \rceil}},$$

6) Kleen star / Kleen's closure Σ^*

If $\Sigma = \{a, b\}$

Σ^* = the set of all strings which can be constructed by using the symbols from Σ including λ .

eg. $\Sigma = \{a, b\}$

$\hookrightarrow \Sigma^* = \{\lambda, a, b, aa, bb, ba, ab, aaa, aba, \dots\}$

it is a universal language.

eg. $\Sigma = \{a\}$

$a^* = \{a^*\} = \Sigma^* = \{\lambda, a, a^2, a^3, \dots\}$

7.) Kleene plus / +ve closure (Σ^+) is the set of all strings which can be constructed using the symbols of Σ excluding λ .

$\Sigma^+ = \{a, b, aa, bb, aaa, \dots\}$

$\therefore \boxed{\Sigma^* - \Sigma^+ = \{\lambda\}}$

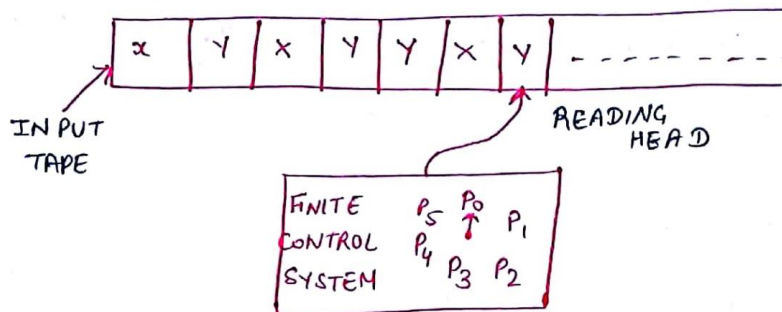
We can say,

$\Rightarrow [\Sigma^+ \cup \{\lambda\} = \Sigma^*] \Rightarrow \{\Sigma^* \cup \{\lambda\}\} = \Sigma^*$

FINITE AUTOMATA

→ Finite Automaton is called "finite" because no. of possible states and no. of letter in the alphabet are both finite and "automation" because the change of the state is totally governed by the input.

It is deterministic, what state is next is automatic not will-full, just as the motion of the hands of clock is automatic, while the motion of hands of a human is presumably the result of desire and thought.



Here, $P_0, P_1, P_2, P_3, P_4, P_5$ are states in Finite Control System

x and y are input symbols.

- At regular interval the automation reads one symbol from the input tape and then enters in a new state that depends only on the current state and the symbol just read.
- After reading an input symbol, reading head moves one square to the right on the input tape, so that on the next move, it will read the symbol in next tape square. Repeat it again and again.

The automation then indicates approval or disapproval.

→ If it winds up in one of a set of final states the input strings is considered to be accepted.

The language accepted by the machine is the set of strings, it accepts.

Definition:-

DETERMINISTIC FINITE AUTOMATA (DFA):

A deterministic Finite Automata is a quintuple

$$M = (Q, q_0, F, \Sigma, \delta)$$

where, Q : is a non-empty finite set of states present in finite control. (q_0, q_1, q_2, \dots)

Σ : is a non-empty finite set of input symbols which can be passed to finite state machine. (a, b, c, \dots)

q_0 : is a starting state, one of the state in Q .

F : is a non-empty set of final states or accepting states, set of final states belongs to Q .

δ : is a function called transition function that takes two arguments a state and a input symbol, it returns a single state. $\boxed{\delta: Q \times \Sigma \rightarrow Q}$

Let 'q' is the state and 'a' be input symbol passed to the transition function as:

$$\delta(q, a) = q'$$

q' is output of the function.

A single state q' may be q . It can be:

$$\boxed{\delta(q, a)} \longrightarrow q' \quad (\checkmark)$$

but q' may be same as q . ($q' = q$)

$$\boxed{\delta(q, a)} \begin{cases} \longrightarrow q' \\ \longrightarrow q'' \end{cases} \quad (X)$$

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