UNIT-I

COMBINATORICS: Permutation and Combination, Repetition and Constrained Repetition, Binomial Coefficients, Binomial Theorem.

PROBABILITY: Definition of Probability, Conditional Probability, Baye's Theorem.

[No. of Hrs: 11]

UNIT – II

PROBABILITY DISTRIBUTIONS: Review of Mean & Standard Deviation, Mathematical Expectation, Moments, Moment Generating Functions, Binomial, Poisson and Normal Distributions. **[No. of Hrs: 10]**

UNIT-III

INTERPOLATION: Operators: Shift, Forward Difference, Backward Difference Operators and their Inter-relation, Interpolation Formulae-Newton's Forward, Backward and Divided Difference Formulae: Lagrange's Formula.

SOLUTION OF NON LINEAR EQUATION: Bisection Method, False Position Method, Newton – Raphson Method for Solving Equation Involving One Variable only.

[No. of Hrs: 12]

UNIT – IV

SOLUTION OF LINEAR SIMULTANEOUS EQUATIONS: Gaussian Elimination Method with and without Row Interchange: LU Decomposition: Gauss - Jacobi and Gauss-Seidel Method; Gauss – Jordan Method and to find Inverse of a Matrix by this Method.

NUMERICAL DIFFERENTIATION- First and Second Order Derivatives at Tabular and Non-Tabular Points, Numerical Integration, Trapezoidal Rule, Simpsons 1/3 Rule: Error in Each Formula (without proof).

[No. of Hrs: 11]

Factorial Notation

For any positive integer n, **n!** means:

$n! = n\,(n-1)\,(n-2)\quad .\quad .\quad .\quad (3)\,(2)\,(1)$

0! will be defined as equal to **one**.

Examples:
$4! = 4 \bullet 3 \bullet 2 \bullet 1 = 24$

The factorial symbol only affects the number it follows unless grouping symbols are used.

$3 \bullet 5! = 3 \bullet 5 \bullet 4 \bullet 3 \bullet 2 \bullet 1 = 360$

$(3 \bullet 5)! = 15! =$ big number

1! = 1
2! = 2.1! = 2.1
3! = 3.2! = 3.2.1! = 3.2.1 = 6
4! = 4.3! = 4.3.2! = 4.3.2.1! = 4.3.2.1 = 24
5! = 5.4! = 5.4.3! = 5.4.3.2! = 5.4.3.2.1 = 120
...
n! = n.(n-1)! = n.(n-1).(n-2)! =...= n.(n-1).(n-2)...3.2.1

Suppose $\frac{15!}{12!} = \frac{15.14.13.12!}{12!} = 15.14.13 = 2730$

Permutation and Combination

Fundamental Counting Principle

If there are m ways to choose a first item and n ways to choose a second item after the first item has been chosen, then there are $m \cdot n$ ways to choose both items.

A lunch special includes one main item, one side, and one drink.

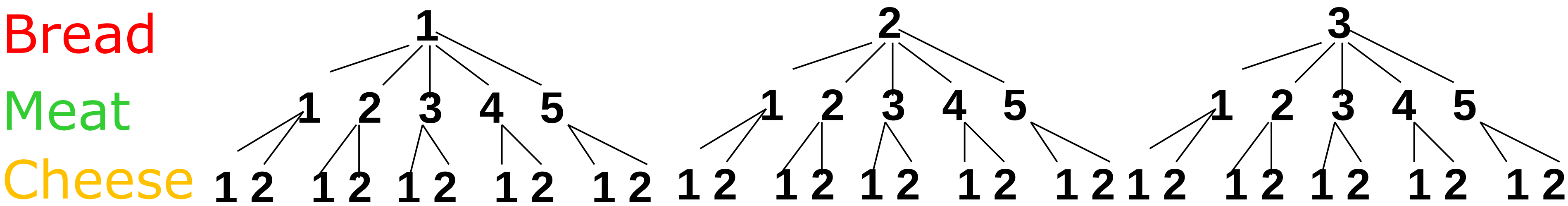
Main Item	Side	Drink
Hamburger	Chips	Juice
Hot dog	Apple	Water
Pizza	Crackers	Milk
Salad		

How many different meals can you choose if you pick one main item, one side, and one drink?

$4 \times 3 \times 3 = 36$

Benefits of the Fundamental Counting Principle

A sandwich can be made with 3 different types of bread, 5 different meats, and 2 types of cheese. How many types of sandwiches can be made if each sandwich consists of one bread, one meat, and one cheese.



There are 30 possible types of sandwiches (cumbersome)

Benefits of the Fundamental Counting Principle

A sandwich can be made with 3 different types of bread, 5 different meats, and 2 types of cheese. How many types of sandwiches can be made if each sandwich consists of one bread, one meat, and one cheese.

$$3 \times 5 \times 2 = 30$$

There are 30 possible types of sandwiches.

Examples

1. Suppose a questionnaire contains 5 questions in which 3 questions have 2 possible answers and the remaining 2 questions have 3 possible answers. Then in how many ways can questionnaire be answered?

Solution:

For 3 questions: since each of the three question can be answered in 2 ways => all 3 questions can be answered in 2*2*2 = 8 ways

Similarly, the rest 2 questions can be answered in 3*3 = 9 ways

Thus the total ways to answer the questionnaire = 8*9 = 72

Examples

2. A computer program consist of one letter followed by three digits. In how many ways different label identifiers are possible if (i) repetition is not allowed (ii) repetition is allowed

Solution: Total letters = 26, total digits = 10
(i) Without repetition:

26	10	9	8	= 18720

(i) With repetition:

26	10	10	10	= 26000

Example

How many 8-digit telephone numbers are possible, if (i) only even digits may used? (ii) The number must be a multiple of 100?

Solution:

(i) Possible even digits : 2, 4, 6, 8

total possible numbers = 4*4*4*… *4 (8 times) = 4^8

(ii) Since we need multiple of 100 => last two places should have digit 0

First place can be filled with digits between 1 – 9 = 9 ways

Rest 5 places can be filled with one of the 10 digits in 10^5 ways.

Total possible phone numbers = 9*10^5

(1-9)9	**10**	**10**	**10**	**10**	**10**	**(0)1**	**(0)1**

Definition

A **combination** is a grouping of outcomes in which the order does not matter.

A **permutation** is an arrangement of outcomes in which the order does matter.

Introduction

- Let a, b and c be the three objects. How many selections are possible taking two objects at a time.

$\Rightarrow$ab, ac, bc

If all three have to be selected => abc

How many possible arrangements are there taking two objects

$\Rightarrow$ab, ba, ac, ca, bc, cb = 6 arrangements

$\Rightarrow$All three taken together => abc, acb, bac, bca, cab, cba = 3! = 3.2.1 = 6

3	**2**	**1=3.2.1=6 =3!**

There are n! ways to arrange n objects all together.

Permutation Rule……

$$_nP_r = \frac{n!}{(n-r)!}$$

where n = total number of objects and r = how many you need.

"n objects taken r at a time"

Also denoted as P(n, r). If r = n, P(n, n) = n!

1	2	3		.....	r
n	(n-1)	(n-2)			(n-(r-1))

n balls to be placed in r – boxes

Total no. of ways = $n.(n-1).(n-2)\ldots.(n-r+1)$

$=(n.(n-1).(n-2)\ldots.(n-r+1).(n-r).(n-r-1)\ldots 3.2.1)/(n-r).(n-r-1)\ldots 3.2.1$

=n!/(n-r)! = P(n, r)

Examples

Evaluate:

(i) P(10, 2)
(ii) P(50, 49)
(iii) P(m+n, 2)
(iv) P(6, 1) + P(9, 2)

Solution:

(i) $P(10,2) = \frac{10!}{(10-2)!} = \frac{10.9.8!}{8!} = 90$

(ii) 50!

(iii) $P(m+n,2) = \frac{(m+n)!}{\left((m+n)-2\right)!} = \frac{(m+n).(m+n-1).(m+n-2)!}{(m+n-2)!} =$

(m+n).(m+n-1)

(iv) $P(6,1) = \frac{6!}{(6-1)!} = \frac{6!}{5!} = \frac{6.5!}{5!} = 6, P(9,2) = 72 \Rightarrow 6 + 72 =$ 78

Examples

Find r if

(i) P(10, r) = 2.P(9,r)   (ii) 4.P(6, r) = P(6, r+1)

Solution:

(ii) $4.\left(\frac{6!}{(6-r)!}\right) = \left(\frac{6!}{\left(6-(r+1)\right)!}\right) \Rightarrow 4 = \frac{(6-r)!}{(6-r-1)!} \Rightarrow 4 = \frac{(6-r).(6-r-1)!}{(6-r-1)!} \Rightarrow 4 = 6 - r \Rightarrow r = 6 - 4 = 2$

Examples

If 2.P(5, 3) = P(n, 4), find n.

Sol.: $\frac{n!}{(n-4)!} = 2.\left(\frac{5!}{(5-3)!}\right) = 5! \Rightarrow n(n-1).(n-2).(n-3) = 5! \Rightarrow n.(n-1).(n-2).(n-3) = 5.(5-1).(5-2).(5-3)$

On comparing both sides, n = 5

If P(n-1, 3):P(n, 4) = 1:9, find n.

R = 9 $$\frac{\frac{(n-1)!}{(n-4)!}}{\left(\frac{n!}{(n-4)!}\right)} = \frac{1}{9} \Rightarrow \frac{(n-1)!}{n.(n-1)!} = \frac{1}{9} \Rightarrow \frac{1}{n} = \frac{1}{9} \Rightarrow n = 9$$

Examples

Prove that:

(i) $P(n, n) = P(n, n-1) = n!$

(ii) $P(n, n) = 2.P(n, n-2)$

Examples

In how many ways can the letters of word EDUCATION be arranged such that
(i) vowels should be only in odd places.
(ii) Beginning and ending with vowels.
(iii) Beginning and ending with consonants.

C	L	L	L	L	L	L	L	C

Solution:
Total number of letters = 9
Vowels = 5, consonants = 4
(i) There are 5 odd places and 5 vowels need to be placed there => total ways = $5! = 120$
Remaining 4 places have to filled by 4 consonants in 4! Ways = 24
Total number of ways = $120 * 24 = 2880$

(ii) The first and last place should be occupied with vowel. No. of ways = $P(5, 2) = 5!/(5-2)! = 5!/3! = (5.4.3!)/3! = 20 = 5.4$
Remaining 7 place have to be filled with 7 letters in 7! Ways
Total arrangements = $7!*20 = 5040*20 = 100800$
(iii) Total arrangements = $P(4, 2)*7! = 12*5040 = 60480$

Examples

In how many ways can 5 boys be arranged in a queue such that

(i) Two of them are always together.
(ii) Two of them are never together.

Solution:

(i) Let A and B are always together. Consider them as one unit. Now 4 boys can be arranged in a queue in 4! Ways. A and B can arrange themselves in 2! Ways. Hence total queues = 4!*2! = 48
(ii) Two of them are never together = Total ways – no. of ways in which two are together

Total ways of arranging 5 boys in a queue = 5!

When two of them are never together = 5! – 48 = 120 – 48 = 72

Example

How many passwords with alphabets and numbers of length 6 can be made if

(i) First three places are alphabets followed by numbers
(ii) First and last places are to filled with alphabets
(iii) Last two place will be filled with alphabets

Solution: Number of alphabets = 26, digits = 10 (including 0)
(i) First 3 places will be filled by 3 alphabets in P(26, 3) ways and last 3 places to be filled with 3 digits in P(10, 3)
Total ways = P(26, 3)* P(10, 3) = 26P3 * 10P3 = $\left(\frac{26!}{(26-3)!}\right) * \left(\frac{10!}{(10-3)!}\right) = (26.25.24) * (10.9.8)$

(ii) Total ways =26*P(10, 4)*25 = P(26, 2)*P(10, 4)
(iii) Same as 2nd

(i) No. of ways for having alpha at first and last place = P(26, 2)
(ii) For middle 4 places
Case 1: all alpha, no digit => P(24, 4)*P(10, 0)
Case 2: 3 alpha, 1 digit => P(24, 3)*P(10, 1)
Case 3: 2 alpha, 2 digits => P(24, 2)*P(10, 2)
Case 4: 1 alpha, 3 digits => P(24, 1)*P(10, 3)
Case 5: no alpha, all digits => P(24, 0)*P(10, 4)

Total ways = (i)*[sum of all cases]

Short cut:
Remaining alpha = 24, digits = 10
Alpha numeric series of length 4 => P(24+10, 4)
Total passwords = P(26, 2)*P(34, 4)

Permutation rule when some things repeat……

$$ {}_{n}P_{r} = \frac{n!}{k_1!k_2!k_3!\ldots k_p!} $$

- It reads: the no. of permutations of n objects in which k1 are alike, k2 are alike, etc.
- k1 +k2+k3+…+kp = n

- How many different words can be made out of letters of the word ALLAHABAD?
- Solution: Total 9 letters out of which 4 are alike (A), 2 are of second kind (L), 1 is of third kind (H), 1 is of fourth kind (B) and 1 is of fifth kind (D) => k1 = 4, k2= 2, k3 = k4= k5 = 1
- Total possible words = $\frac{9!}{4!*2!*1!*1!*1!} = \frac{9!}{4!*2!} = 7560$

Example

- In how many ways 3 red, 4 black and 2 yellow balls be arranged in a row?
- Solution: k1 = 3, k2 = 4, k3 = 2, Total balls = k1 +k2+k3 = 3+4+2 = 9
- Total permutations to arrange in a row =

$$\frac{9!}{3!*4!*2!} = \frac{9.8.7.6.5.4!}{3.2.4!.2} = 9.4.7.5 = 36 * 35 = 1260$$

Example

- There are 3 books of Maths, 4 of English and 4 of Science. (i) In how many ways these can be arranged in a row? (ii) In how many ways these can be arranged in a shelf of library?
- Solution: (i) Total ways = $\frac{11!}{3!.4!.4!} = \frac{11.10.9.8.7.6.5}{6*24} = 11.10.3.7.5 = 11.1050 = 11550$
- (ii) For library
- 3 books of maths should be together and hence can be arranged in 3! Ways
- 4 books of English can be arranged in 4! Ways and 4 books of science can be arranged in 4! Ways.
- The three groups can arrange themselves in 3! ways
- Total possible arrangements = 3!*(4!*4!*3!)
- MES, MSE, EMS, ESM, SEM, SME = 3!

Example

- Arrange the letters of the word MODIRAM so that D and I are always together.

Solution:

Since M is repeating twice, k1 = 2. rest all letters are appearing once. Considering D and I as one unit, total letters = 6

Total permutations = $\frac{6!}{2!} = 360$

Since D and I can arrange themselves in 2! Ways, so total no. of words in which D and I are together, = 360*2! = 720

MO**DI**RAM, MO**ID**RAM

Dictionary Ranking

- The letters of the word MOHAN are written in all possible orders and these words are written out as in a dictionary. Find the rank of the word MOHAN.
- Solution: Total distinct letters = 5 => total possible words = 5! = 120
- According to dictionary order,
- Count the words starting with A
- The first letter is A => First place is fixed for A. Rest 4 places can be filled with 4 letters in 4! Ways => total words starting with A = 4! = 24
- Total Words starting with H => 24
- Rank of MOHAN will lie between 48 and (120-48 = 72)

Count the words starting with M

First we will count the words starting with MA => first two positions are fixed for MA

Rest 3 position can be filled with 3 letters in 3! Ways

No. of words starting with MA = 3! = 6

No. of words starting with MH = 3! = 6

No. of words starting with MN = 3! = 6

No. of words starting with MO = 3! = 6 out of which one word is MOHAN => rank of MOHAN lies between (48+18 = 66, 72)

Words starting with MOA = (first three places fixed, rest 2 places can be filled in 2! Ways) = 2

Words starting with MOH = 2 out of which one word is MOHAN

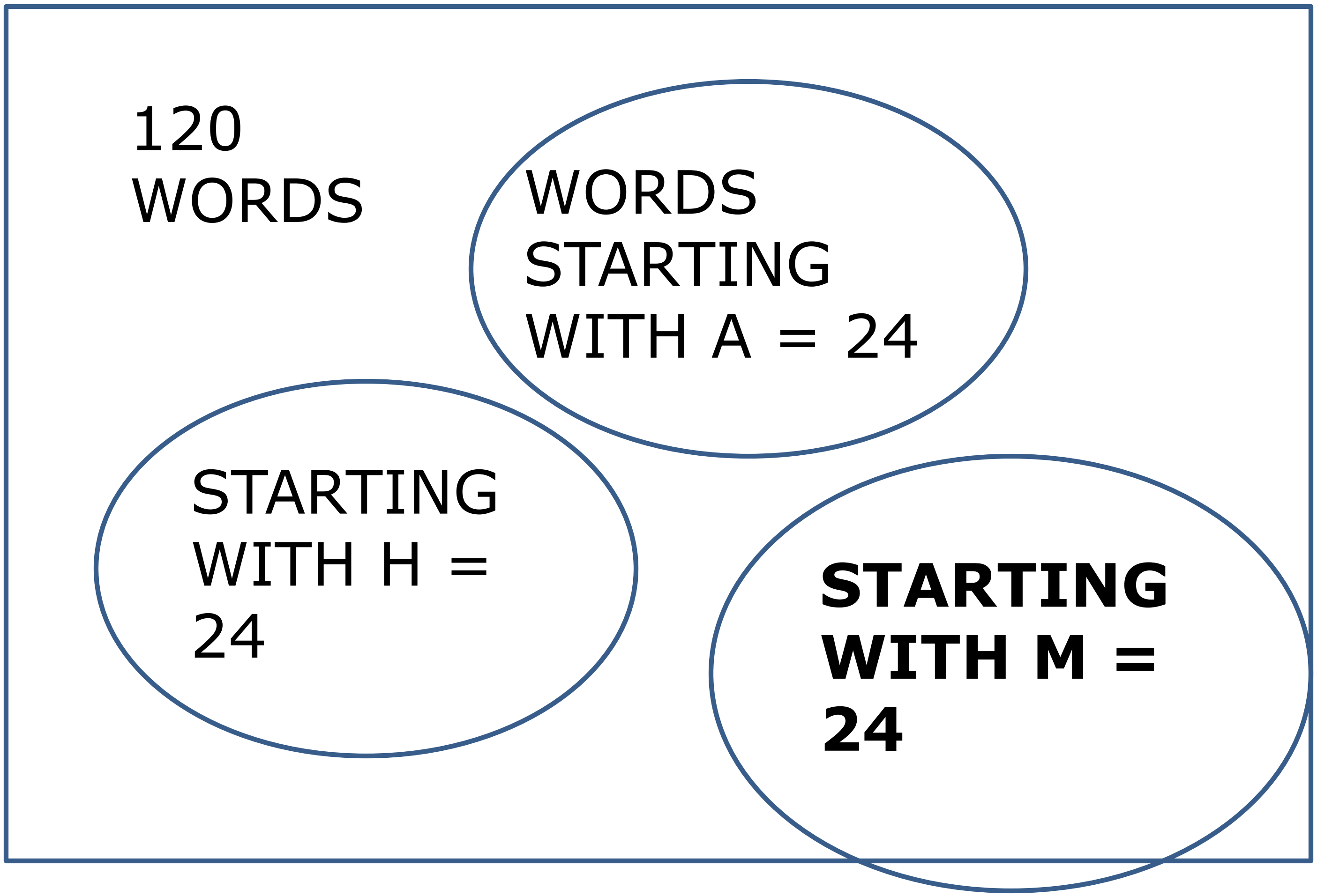
$\Rightarrow$ Rank of MOHAN will be between (66+2=68, 72)

69^{th} and 70^{th} word starting with MOH

MOHAN, MOHNA (according to dictionary)

=> Rank of MOHAN = 24+24+6+6+6=2+1 = 69

TOTAL LENTH OF WORD = 5



MOHAN

TOTAL WORDS = 24
MA = 3! = 6
MH = 6
MN = 6
MO = 6 OUT OF WHICH ONE WORD IS MOHAN

MOA = 2! = 2
MOH = 2
MOHAN = 1

RANK IS 24 + 24 +6+6+6+2+1 = 69

Example

- Question: Find the rank of the word ZENITH, if arranged in a dictionary.

6	1	4	3	5	2
Z	E	N	I	T	H
5	0	2	1	1	0
5!	4!	3!	2!	1!	0! =1
600	0	12	2	1	0

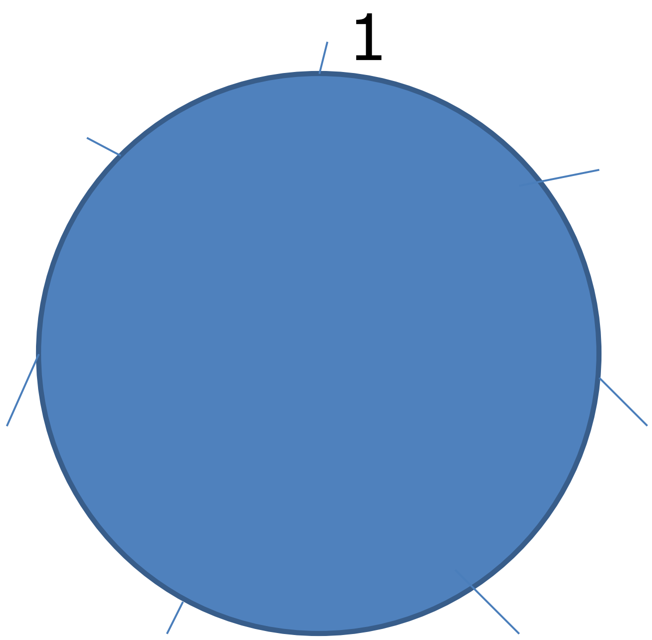
TOTAL = 615 ADD 1 TO GET RANK = 616

Example

- How many 4 digit numbers can be formed using the digits 1, 2, 3, 4, 5, 6, 7. How many of them are greater than 3400?

Circular Permutation

- If n objects are to arranged around a circular table, then possible number of ways are (n-1)!



1 person is arranged to start the order, remaining (n-1) can be arranged in (n-1) places in (n-1)! ways

- If n pearls are to arranged in a necklace, total ways are $(n-1)!/2$
- Example:

In how many ways 5 boys be arranged in a circle?

Solution: $(5-1)! = 4! = 24$

In how many ways 8 different beads can be arranged to form a necklace?

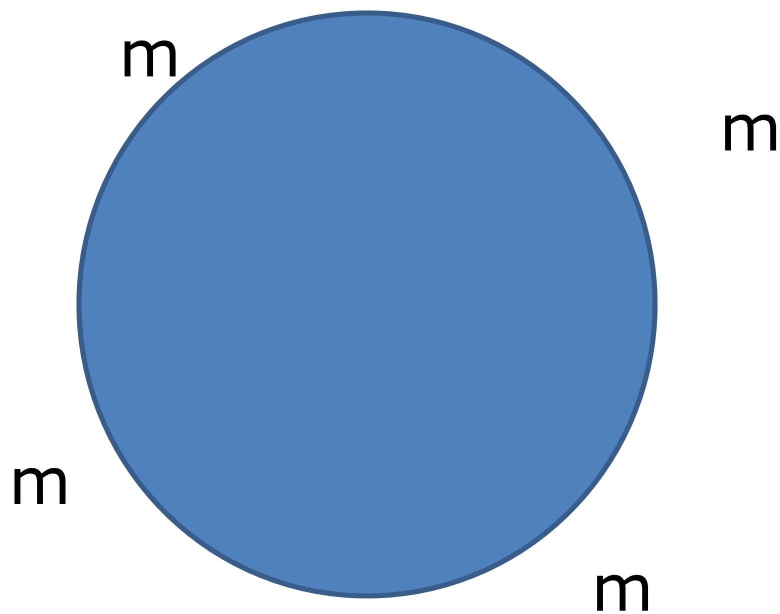
Sol.: $(8-1)!/2 = 7!/2 = 2520$ ways

In how many ways can a party of 4 men and 4 women be seated at a circular table?

Sol.: Total persons = 8, ways = $(8-1)! = 7!$

- In how many ways can a party of 4 men and 4 women be seated at a circular table such that no two men want to sit together?

4 men can be arranged around a circle in 3! Ways.
Creating 4 spaces in between.
Now 4 women can arrange themselves in 4 places in 4! Ways
Total ways to sit = 3!*4! = 24*6 = 144



(4-1)! = 3!

Examples

- In how many ways can 5 girls and 5 boys be arranged in a queue so that no two girl sit together?
- Solution: Arrange 5 boys in 5! ways. This will create 6 gaps in between. So 5 girls can be arranged in 6 gaps by 6P5 ways.
- Total ways = 5!*(6!/(6-5)!) = 5!*6!

	B		B		B		B		B	

Combination Rule……

$$ {}_{n}C_{r} = \frac{n!}{(n-r)!r!} $$

- Read: “n” objects taken “r” at a time.

Combination

- $C(n, 0) = n!/(n-0)!.0! = n!/n! = 1$
- $C(n, 1) = n!/(n-1)!.1! = n!/(n-1)! = n.(n-1)!/(n-1)! = n$
- $C(n, n) = n!/((n-n)!. n! = n!/n! = 1 = C(n, 0)$
- In general
- $C(n, r) = C(n, n-r)$

Identities to Prove

(i) $C(n,r) = C(n-1,r) + C(n-1,r-1)$

$$Sol.: LHS \Rightarrow C(n,r) = \frac{n!}{(n-r)!.r!} \quad \ldots \ldots (i)$$

$$RHS \Rightarrow C(n-1,r) = \frac{(n-1)!}{(n-1-r)!.r!} \quad \ldots \ldots (1)$$

$$C(n-1,r-1) = \frac{(n-1)!}{\big((n-1)-(r-1)\big)!.(r-1)!}$$

$$= \frac{(n-1)!}{(n-r)!.(r-1)!} \ldots . (2)$$

(n-r)! = (n-r).(n-r-1)! Put this in (2)
And r! = r. (r-1)! Put in (1)

Identity

$$Prove\ that: C(n, r) + C(n, r-1) = C(n+1, r)$$

Solution: LHS

$$\frac{n!}{(n-r)!.r!} + \frac{n!}{\big(n-(r-1)\big)!.(r-1)!}$$

$$= \frac{n!}{(n-r)!.r.(r-1)!} + \frac{n!}{(n-r+1)!.(r-1)!}$$

$$= \frac{n!}{(n-r)!.r(r-1)!} + \frac{n!}{(n-r+1).(n-r)!.(r-1)!}$$

$$= \frac{n!}{(n-r)!.(r-1)!}.\left[\frac{1}{r} + \frac{1}{n-r+1}\right] = \frac{n!}{(n-r)!.(r-1)!}.\left[\frac{n-r+1+r}{r.(n-r+1)}\right]$$

$$= \frac{(n+1).n!}{(n-r+1).(n-r)!.r.(r-1)!} = \frac{(n+1)!}{(n+1-r)!.(r!)} = C(n+1, r)$$

Example

- If $C(2n, 3) = C(2n, 2)$ Find n.

Solution:

$2n!/((2n-3)!.3!) = 2n!/((2n-2)!.2!)$

$= (2n-2).(2n-3)!.2 = (2n-3)!.6$

$\Rightarrow 2(2n-2) = 6 \Rightarrow 4n - 4 = 6$

$\Rightarrow 4n = 10 \Rightarrow n = 10/4 = 5/2$

Note: If $C(n, x) = C(n, y)$ then either $x = y$ or $x+y = n$

Clearly here, $x = 3$ is not equal to $y = 2$ so $x+y = n$, $\Rightarrow 3+2 = 2n \Rightarrow n = 5/2$

Question 1

An English test contains five different essay questions labeled A, B, C, D, and E. You are supposed to choose 2 to answer. How many different ways are there to do this?

The order of outcomes is not important, so this situation involves combinations.

$$ {}^{5}C_{2} = 10 $$

Question 2

A voicemail system password is 1 letter followed by a 3-digit number less than 600. How many different voicemail passwords are possible if all digits are allowed?

The order of outcomes is important, so this situation involves permutations.

$$26 \times 6 \times 10 \times 10 = 15600$$

Question 3

A family of 3 plans to sit in the same row at a movie theater. How many ways can the family be seated in 3 seats?

The order of outcomes is important, so this situation involves permutations.

ABC	BAC	CAB
ACB	BCA	CBA

$3 \times 2 \times 1 = 6$

Question 4

Ingrid is stringing 3 different types of beads on a bracelet. How many ways can she string the next three beads if they must include one bead of each type?

The order of outcomes is important, so this situation involves permutations.

$3 \times 2 \times 1 = 6$

Question 5

Nathan wants to order a sandwich with two of the following ingredients: mushroom, eggplant, tomato, and avocado. How many different sandwiches can Nathan choose?

The order of outcomes is not important, so this situation involves combinations.

$$^{4}C_{2} = 6$$

Question 6

A group of 8 swimmers are swimming in a race. Prizes are given for first, second, and third place. How many different outcomes can there be?

The order of outcomes is important, so this situation involves permutations.

$8 \times 7 \times 6 = 336$

Question 7

How many different ways can 9 people line up for a picture?

The order of outcomes is important, so this situation involves permutations.

$9! = 362{,}880$

Question 8

Four people need to be selected from a class of 15 to help clean up the campus. How many different ways can the 4 people be chosen?

The order of outcomes is not important, so this situation involves combinations.

15 Choose 4 = 1365