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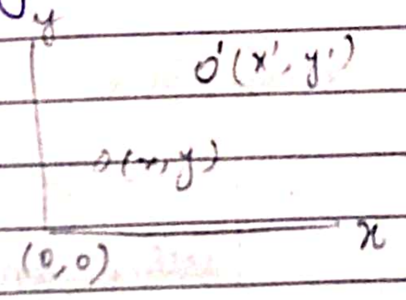
Transformation

i) Geometric
ii) Co-ordinate

Any change in shape of an object w.r.t translation, rotation, scaling, etc.



Column Representation of matrix

$$x' = 2x + 3y$$

$$\begin{array}{cc} x & y \\ \hline \end{array}$$
$$\begin{bmatrix} 2 & 3 \\ 3 & 5 \end{bmatrix}$$

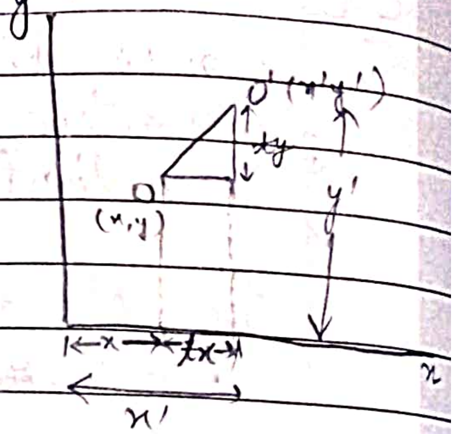
Operations for transformation

Translation
Rotation
Scaling
Shearing
Reflection

Translation (about the origin)

coefficient of :

$$\begin{array}{c|ccc} & x & y & const \\ x' & \multicolumn{3}{c}{\left[\begin{array}{ccc} 1 & 0 & t_x \\ 0 & 1 & t_y \end{array}\right]} \\ y' & & & \end{array}$$



$x' = x + t_x$

$y' = y + t_y$

~~When an operation come~~, To make an operation matrix homogeneous by place ~~adding~~ one at diagonal in the next row and place zero at other position.

As per requirement, add (1, 1, ...) rows and (1, 1, 1, ...) columns in a object matrix to make it homogeneous.

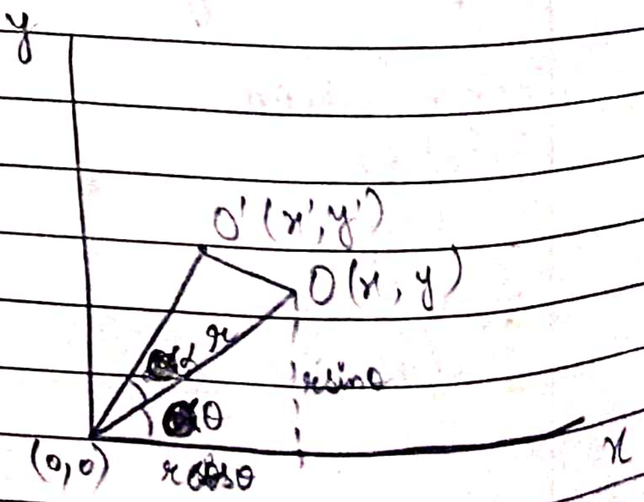
Rotation about origin

$x = r\cos\theta$

$y = r\sin\theta$

$x' = r\cos(\theta + \alpha)$

$y' = r\sin(\theta + \alpha)$



$\sin(A+B) = \sin A \cos B + \cos A \sin B$

$\cos(A+B) = \cos A \cos B - \sin A \sin B$

$x' = r\cos(\theta+\alpha) = r(\cos\theta . \cos\alpha + \sin\theta \sin\alpha)$

$= r\cos\theta \cos\alpha - r\sin\theta \sin\alpha$

$x' = x\cos\alpha - y\sin\alpha$

$y' = r\sin(\theta+\alpha) = r\sin\theta\cos\alpha + r\cos\theta \sin\alpha$

$= y\cos\alpha + x\sin\alpha$

$y' = x\sin\alpha + y\cos\alpha$

$$\begin{array}{c|cc} & x & y \\ \hline x' & \cos\alpha & -\sin\alpha \\ y' & \sin\alpha & \cos\alpha \end{array}$$

Q. Translate a point $P(-2,3)$ about the origin.

$t_x = -2$, $t_y = 3$

$$T_v = \begin{bmatrix} 1 & 0 & -2 \\ 0 & 1 & 3 \end{bmatrix}$$

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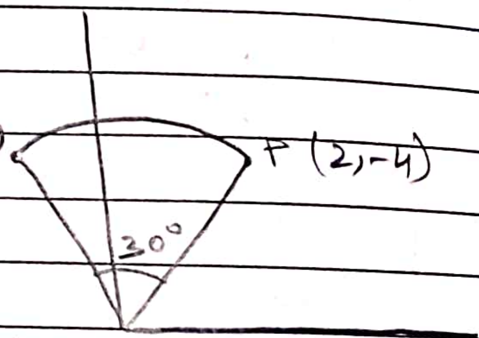
Pg 78 Q.4.2 a) Rotation of an object by 30° about the origin

$$R_\alpha = \begin{bmatrix} \cos 30^\circ & -\sin 30^\circ \\ \sin 30^\circ & \cos 30^\circ \end{bmatrix}$$

$$= \begin{bmatrix} \frac{\sqrt{3}}{2} & -\frac{1}{2} \\ \frac{1}{2} & \sqrt{3}/2 \end{bmatrix}$$

b) Object matrix $= \begin{bmatrix} 2 \\ -4 \end{bmatrix}$



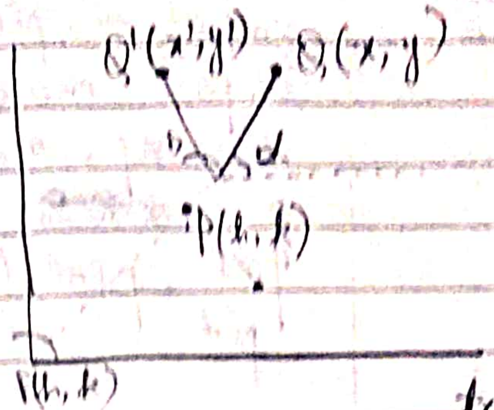
$$P'(x', y') = \text{operation . object}$$

$$= \begin{bmatrix} \sqrt{3}/2 & -1/2 \\ 1/2 & \sqrt{3}/2 \end{bmatrix} \begin{bmatrix} 2 \\ -4 \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3} - 2\sqrt{3} & -1+2 \\ 1-2 & \sqrt{3} - 2\sqrt{3} \end{bmatrix}$$

$$= \begin{bmatrix} \sqrt{3}(1-2) & 1 \\ -1 & \sqrt{3}(1-2) \end{bmatrix}$$

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$



$$R_{\theta,P} = T_v R_\theta T_{-v}$$

$t_x = h$
$t_y = k$

$$R_{\theta,P} = \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & -\sin\theta & 0 \\ \sin\theta & \cos\theta & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}$$

Q. Perform a 45° rotation of triangle A(0,0) B(1,1), C(5,2).
a) about the origin
b) about the point P(−1, −1)

Sol^n a) $R_\theta = \begin{bmatrix} \cos 45^\circ & -\sin 45^\circ \\ \sin 45^\circ & \cos 45^\circ \end{bmatrix}$

$$= \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} \\ 1/\sqrt{2} & 1/\sqrt{2} \end{bmatrix}$$

$$R = \begin{bmatrix} 1/\sqrt{2} & -1/\sqrt{2} & 0 \\ 1/\sqrt{2} & 1/\sqrt{2} & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 0 & 1 & 5 \\ 0 & 1 & 2 \\ 1 & 1 & 1 \end{bmatrix}$$

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Scaling (w.r.t origin)

$$S_{Sx,Sy} = \begin{bmatrix} S_x & 0 & 0 \\ 0 & S_y & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Translate, then scaling, then reverse the translation in scaling w.r.t fixed point.

$$S_{a,b,P} = T_V \cdot S_{a,b} \cdot T_{-V}$$

$$= \begin{bmatrix} 1 & 0 & h \\ 0 & 1 & k \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} a & 0 & 0 \\ 0 & b & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & -h \\ 0 & 1 & -k \\ 0 & 0 & 1 \end{bmatrix}$$

ex 4.8 A (0,0), B(1,1) and C(5,2)
C is fixed point.

$$T_V = \begin{bmatrix} 1 & 0 & 5 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \qquad (\because h = 5,\ k = 2)$$

$$T_{-V} = \begin{bmatrix} 1 & 0 & -5 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$S_{2,2} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

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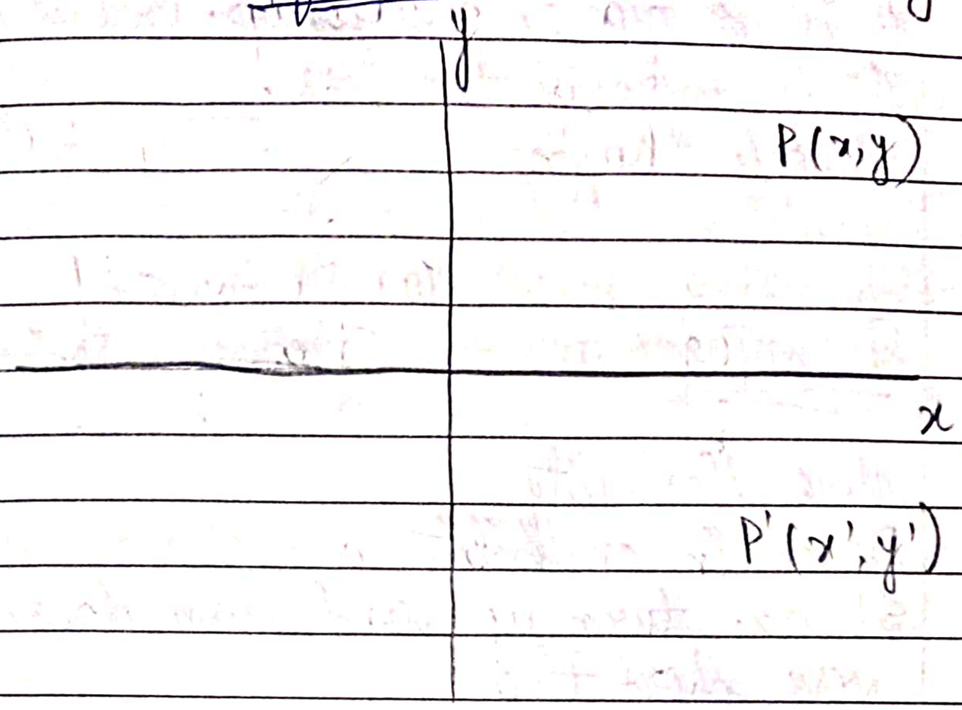
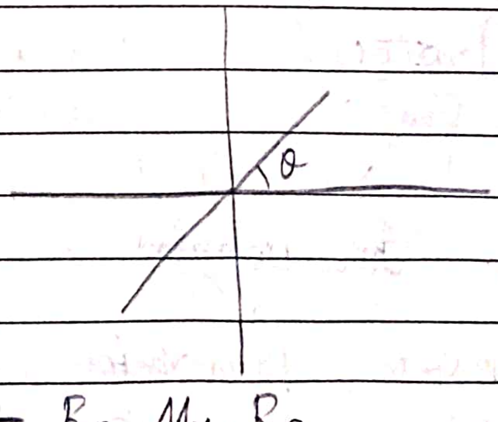
Reflection (about the origin)



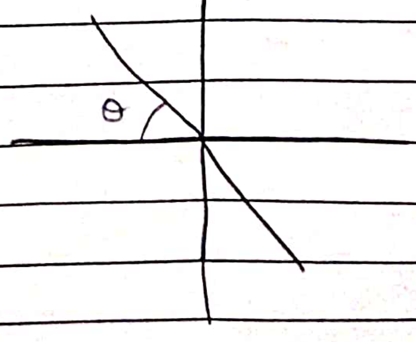
$x' = x$
$y' = -y$

$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}$, $M_y = \begin{bmatrix} -1 & 0 \\ 0 & 1 \end{bmatrix}$



$\tan\theta = m = 1$
$y = x$
$\theta = \tan^{-1} 1$
$\theta = 45^\circ$

$M_{\theta,x} = R_\theta . M_x . R_{-\theta}$



$y = -x$

$M_{\theta,x} = R_\theta . M_x . R_{-\theta}$

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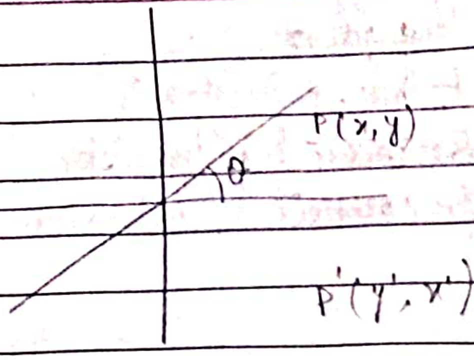


$x' = +y$

$y' = -x$

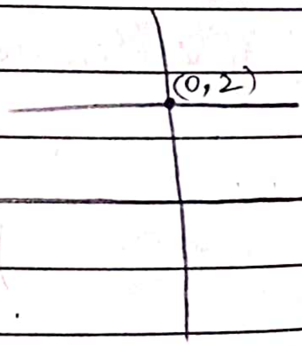
$$M_x = \begin{bmatrix} 0 & 1 & 0 \\ -1 & 0 & 0 \\ 0 & 0 & 1 \end{bmatrix}$$

Example 4.11 $A(-1,0)$ $B(0,-2)$, $C(1,0)$ $D(0,2)$

$$O = \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

a) $y = 2$

$t_x = 0$
$t_y = 2$



$$T_V = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}_{2\times3}$$

$$M_x = \begin{bmatrix} 1 & 0 \\ 0 & -1 \end{bmatrix}_{2\times2}$$

$M_{V,x} = T_V \, M_x \, T_{-V}$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 2 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{bmatrix}$$

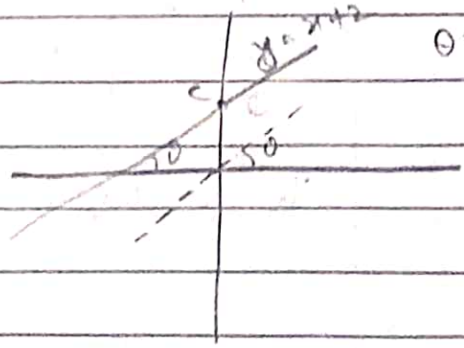
$$= \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix}$$

$$M_{v,x} \cdot O = \begin{bmatrix} 1 & 0 & 0 \\ 0 & -1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & -1 & 4 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} -1 & 0 & 1 & 0 \\ 0 & -2 & 0 & 2 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 1 & 1 \end{bmatrix}$$

$$= \begin{bmatrix} \quad \end{bmatrix}$$

c) $y = x + 2$



$\theta = 45°$

$$T_v = \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 2 \end{bmatrix}$$

$R_\theta = [$

$M_{v,\theta,x} = T_v \cdot R_\theta \cdot M_x R_\theta T_v$

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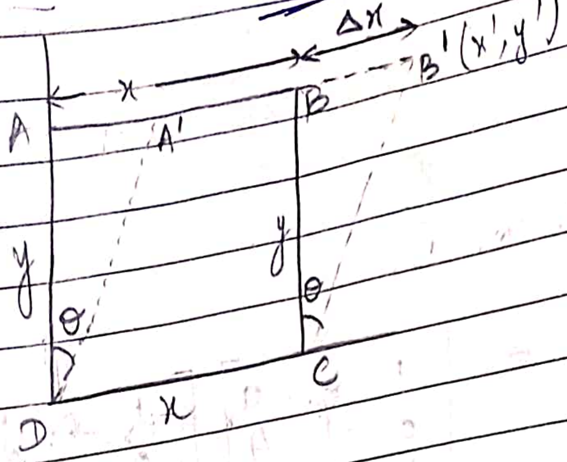
Shearing



$$\tan\theta = \frac{\Delta x}{y}$$

$$x' = x + \Delta x$$
$$y' = y$$

Sh_x $\begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix}$

Inverse Of Geometric Transformation

Inverse of geometric transformation for each transformation matrix = coordinate transformation respectively.

$$T_v = \begin{bmatrix} 1 & 0 & t_x \\ 0 & 1 & t_y \end{bmatrix}$$

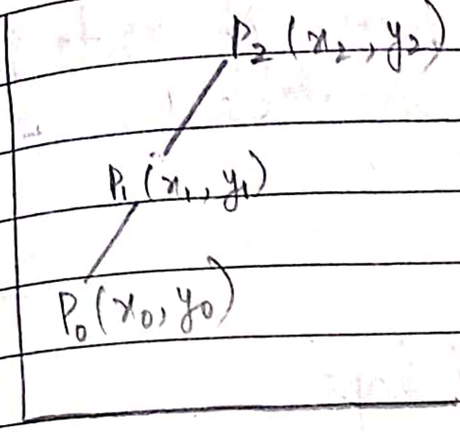
$$T_v^{-1} = T_{-v} = \begin{bmatrix} 1 & 0 & -t_x \\ 0 & 1 & -t_y \end{bmatrix}$$

$$S_{x,y} = \begin{bmatrix} s_x & 0 \\ 0 & s_y \end{bmatrix}$$

$$S^{-1}_{sx,sy} = S_{\frac{1}{sx},\frac{1}{sy}} = \begin{bmatrix} 1/s_x & 0 \\ 0 & 1/s_y \end{bmatrix}$$

$$R_\theta = \begin{bmatrix} \cos\theta & -\sin\theta \\ \sin\theta & \cos\theta \end{bmatrix}$$

$$R_\theta^{-1} = R_{-\theta} = \begin{bmatrix} \cos\theta & \sin\theta \\ -\sin\theta & \cos\theta \end{bmatrix}$$

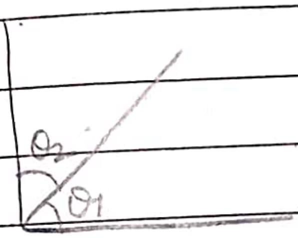


$t_x = t_{x_0} + t_{x_1} + t_{x_2}$

t_{o_0}, t_1, t_3

$t = t_{o_0}, t_1 + t_3$

$T_V = T_{V_0 + V_1 + V_2}$



$\theta = \theta_1 + \theta_2$