Exercise Questions of Double Integr

(10) $\int_0^1 \int_x^{\sqrt{x}} (x^2+y^2)\, dy\, dx$

$$= \int_0^1 \left(x^2 y + \frac{y^3}{3} \right)_x^{\sqrt{x}} dx$$

$$= \int_0^1 \left(x^{5/2} + \frac{x^{3/2}}{3} - x^3 - \frac{x^3}{3} \right) dx$$

$$= \int_0^1 \left(x^{5/2} + \frac{x^{3/2}}{3} - \frac{4x^3}{3} \right) dx$$

$$= \left(\frac{x^{7/2}}{7/2} + \frac{x^{5/2}}{\frac{5}{2} \cdot 3} - \frac{4}{3} \cdot \frac{x^4}{4} \right)_0^1$$

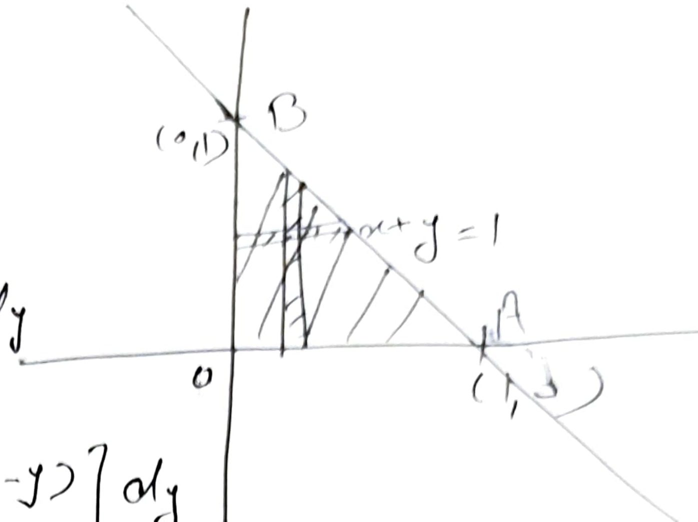
$$= \frac{2}{7} + \frac{2}{15} - \frac{1}{3}$$

$$= \frac{30+14-35}{105} = \frac{44-35}{105} = \frac{9}{105} = \frac{3}{35}$$

(11) Evaluate $\iint (x^2+y^2)\, dx\, dy$ over the region in the positive quadrant for which $x+y \le 1$.

Solu:- Consider $\iint_{OAB} (x^2+y^2)\, dx\, dy$

$$= \int_0^1 \int_0^{1-y} (x^2+y^2)\,dx\,dy$$



$$= \int_0^1 \left(\frac{x^3}{3} + xy^2\right)_0^{1-y} dy$$

$$= \int_0^1 \left[\frac{(1-y)^3}{3} + y^2(1-y)\right] dy$$

$$= \int_0^1 \left[\frac{(1-y)^3}{3} + y^2 - y^3\right] dy$$

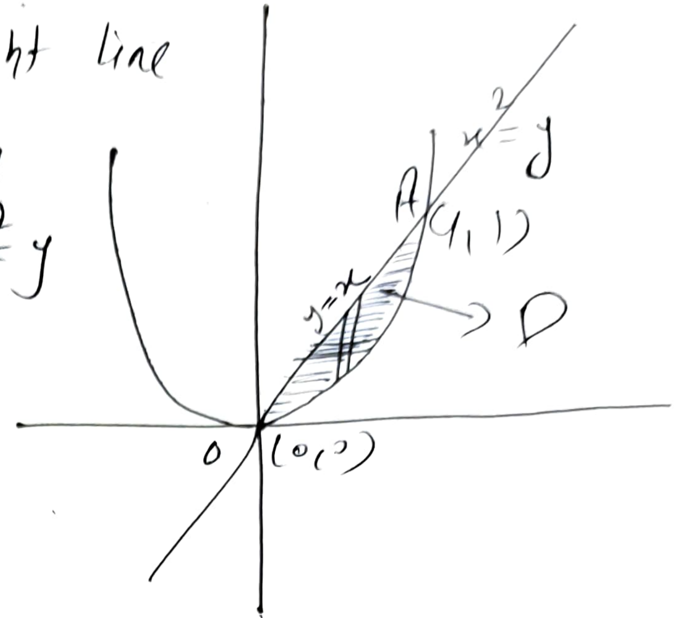
$$= \left(\frac{(1-y)^4}{(-4)(3)} + \frac{y^3}{3} - \frac{y^4}{4}\right)_0^1$$

$$= \frac{(1-1)^4}{(-4)(3)} + \frac{1}{3} - \frac{1}{4} - \left[\frac{(1-0)}{(-4)(3)} + \frac{0}{3} - \frac{0}{4}\right]$$

$$= \frac{1}{3} - \frac{1}{4} + \frac{1}{12} = \frac{4-3+1}{12} = \frac{2}{12} = \frac{1}{6}$$

(15) Evaluate $\iint xy(x+y)\,dx\,dy$ over the area between $y = x^2$ and $y = x$.

Soln. The straight line $y=x$ intersects the parabola ~~at~~ $x^2=y$



when $x^2=x$

$\Rightarrow x(x-1)=0$

$\Rightarrow x=0,1$

if $x=0, y=0$

and if $x=1, y=1$

So, intersection points are $(0,0)$ and $(1,1)$

Now $\iint_D xy(x+y)\,dx\,dy$

$$= \int_0^1 \int_{x^2}^{x} xy(x+y)\,dy\,dx$$

$$= \int_0^1 \int_{x^2}^{x} (x^2y+xy^2)\,dy\,dx$$

$$= \int_0^1 \left(\frac{x^2y^2}{2}+\frac{xy^3}{3}\right)_{x^2}^{x} dx$$

$$= \int_0^1 \left(\frac{x^2.x^2}{2}+\frac{x.x^3}{3}-\frac{x^2.x^4}{2}-\frac{x.x^6}{3}\right)dx$$

$$= \int_0^1 \left(\frac{x^4}{2} + \frac{x^4}{3} - \frac{x^6}{2} - \frac{x^7}{3} \right) dx$$

$$= \left(\frac{x^5}{5(2)} + \frac{x^5}{5(3)} - \frac{x^7}{2(7)} - \frac{x^8}{8(3)} \right)_0^1$$

$$= \frac{1}{10} + \frac{1}{15} - \frac{1}{14} - \frac{1}{24}$$

$$= \frac{84 + 56 - 60 - 35}{840}$$

2	10	15	14	24
3	5	15	7	12
5	5	5	7	4
	1	1	7	4

$$= \frac{140 - 95}{840}$$

$$= \frac{45}{840} = \frac{3}{56}$$

Some Important Results

(1) $\int_0^{\pi/2} \sin^n \theta \, d\theta = \dfrac{(n-1) \times \text{go on deminishing by } 2}{n \times \text{go on " " } 2} \times \dfrac{\pi}{2}$

only if n is even

Otherwise $= \dfrac{(n-1) \times \text{go on deminishing by } 2}{n \times \text{go on " " } 2}$

eg. $\int_0^{\pi/2} \sin^4\theta \, d\theta$

$$= \frac{3.1}{4.2} \times \frac{\pi}{2} = \frac{3\pi}{16}$$

and $\int_0^{\pi/2} \sin^5\theta \, d\theta = \frac{4.2.}{5.3.1} = \frac{8}{15}.$

similarly

(2) $\int_0^{\pi/2} \cos^n\theta \, d\theta =$

$$= \begin{cases} \dfrac{(n-1) \times \text{so on deminishing by } 2}{n \times \text{'' '' '' by } 2} \times \dfrac{\pi}{2} & \text{if } n \text{ is even} \\ \text{otherwise } \dfrac{(n-1) \times \text{so on deminishing by } 2}{n \times \text{'' '' '' by } 2.} \end{cases}$$