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Line Drawing Algorithms

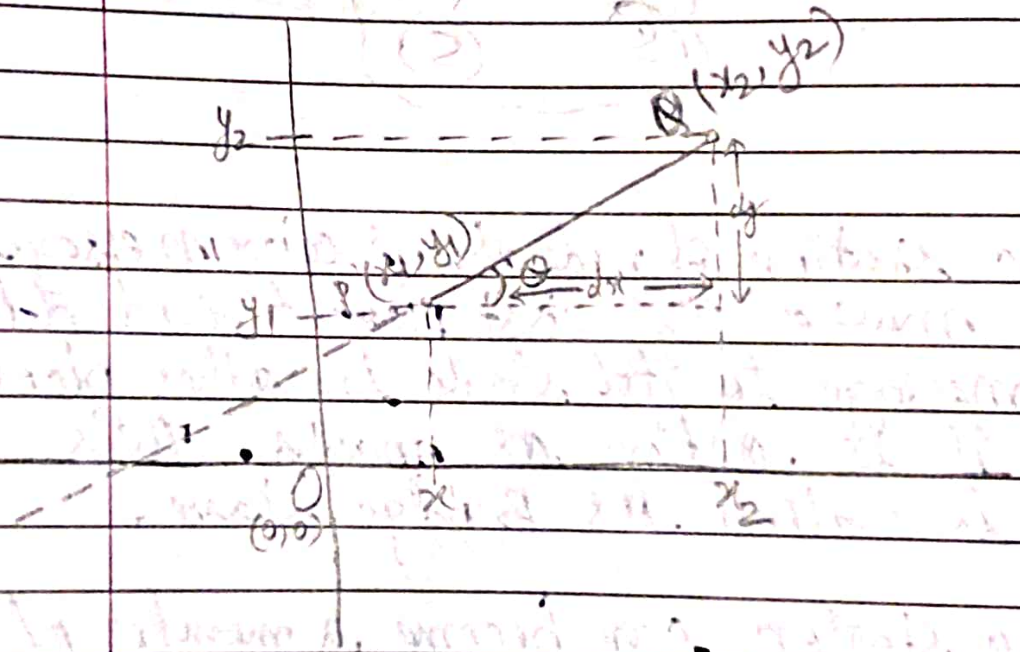
→ DDA (Digital Differential Algo)
→ Brusenham's Algo

Line Equations

1) $y = mx + c$

2) $y - y_1 = \frac{y_2 - y_1}{x_2 - x_1}(x - x_1)$

3) $\frac{x}{a} + \frac{y}{b} = 1$



$m = \tan\theta$

$\tan\theta = \frac{dy}{dx}$

$m = \frac{y_2 - y_1}{x_2 - x_1}$

DDA Algorithm

DDA (x_1, y_1, x_2, y_2)

$dx = x_2 - x_1$; $dy = y_2 - y_1$

if abs(dx) > abs(dy)
    step = abs(dx)
else
    step = abs(dy)

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$$x_{inc} = \frac{dx}{step}$$

$$y_{inc} = \frac{dy}{step}$$

```
for (i=1; i<=step; i++)
{   putpixel (x1, y1)
     x1 = x1 + xinc ;
     y1 = y1 + yinc ;
 }
}
```

$P(x_1, y_1) = (2, 2)$; $Q(x_2, y_2) = (9, 2)$

$dx = x_2 - x_1 = 9 - 2 = 7$

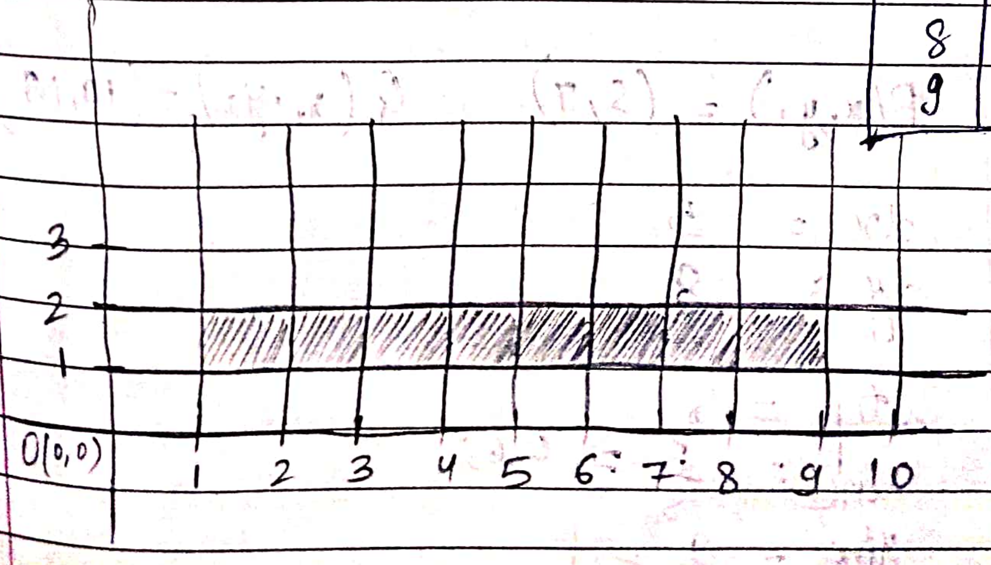
$dy = y_2 - y_1 = 2 - 2 = 0$

$step = 7$

$x_{inc} = \frac{7}{7} = 1$

$y_{inc} = \frac{0}{7} = 0$

x	y
2	2
3	2
4	2
5	2
6	2
7	2
8	2
9	2



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Q. $P(x_1, y_1) = (5, 4)$ $Q(x_2, y_2) = (12, 7)$

$dx = x_2 - x_1 = 12 - 5 = 7$

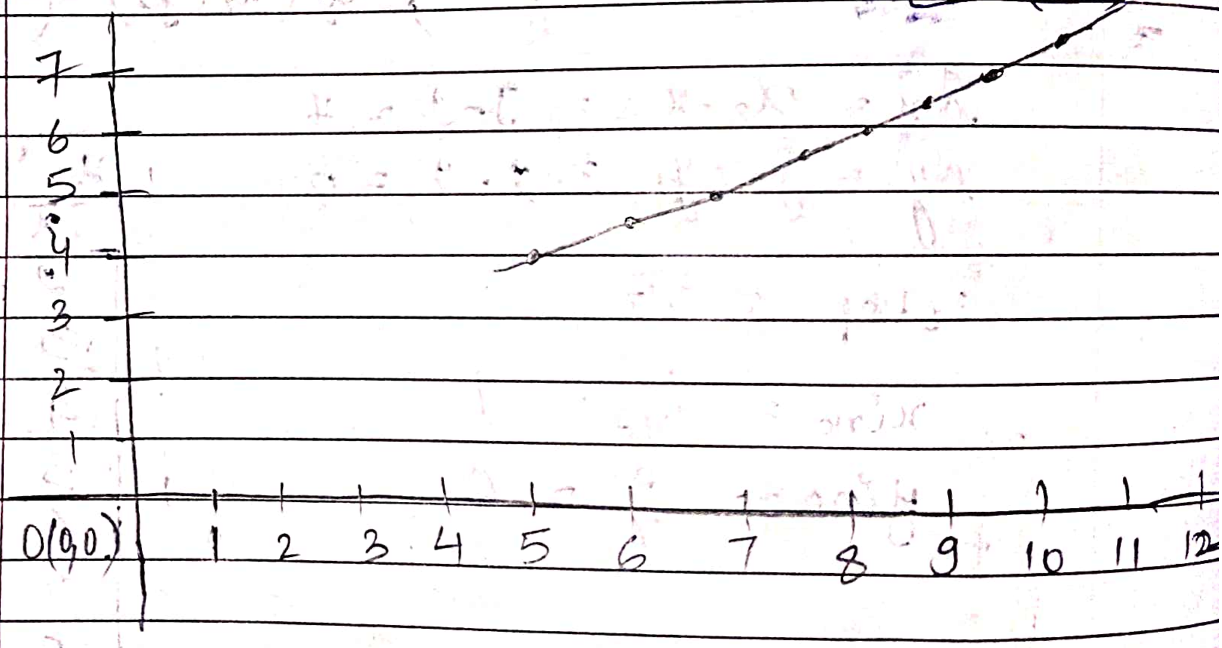
$dy = y_2 - y_1 = 7 - 4 = 3$

step $= 7$

$x_{inc} = \frac{7}{7} = 1$

$y_{inc} = \frac{3}{7} = 0.42$

x	y
5	4
6	4.42
7	4.84
8	5.26
9	5.68
10	6.10
11	6.52
12	6.94



Q. $P(x_1, y_1) = (5, 7)$ $Q(x_2, y_2) = (10, 15)$

$dx = 5$

$dy = 8$

Step $= 8$

$x_{inc} = \frac{5}{8} = 0.62$

$y_{inc} = \frac{8}{8} = 1$

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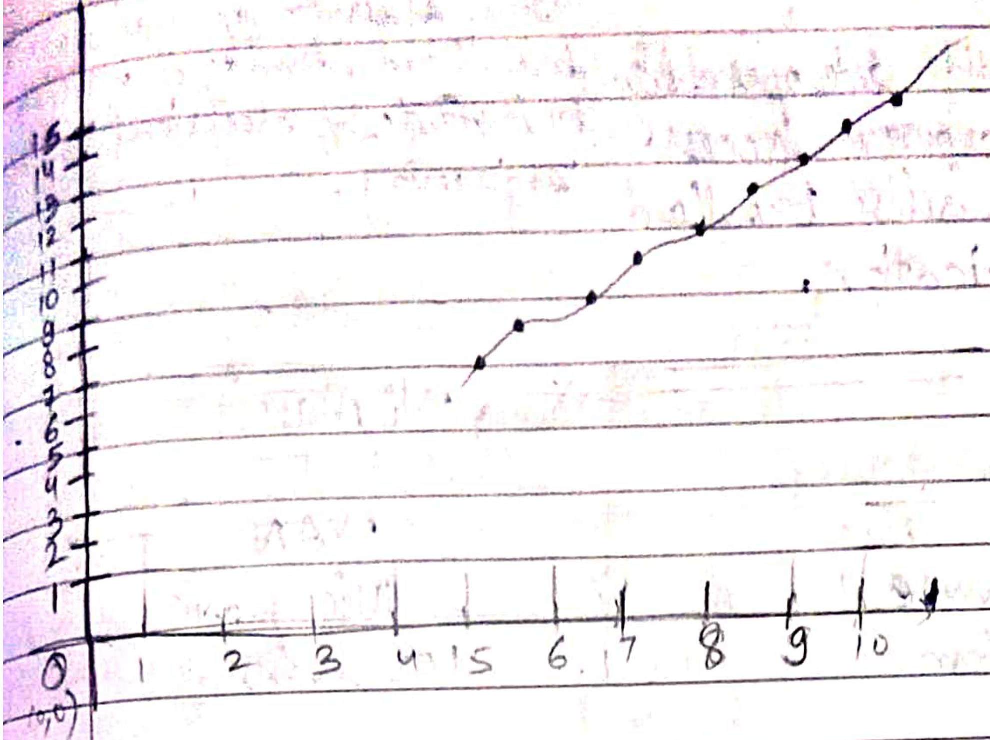


x	y
5	7
5.62	8
6.24	9
6.86	10
7.48	11
8.10	12
8.72	13
9.32	14
9.94	15

Q. $P(17, 14)$ $Q(12, 9)$

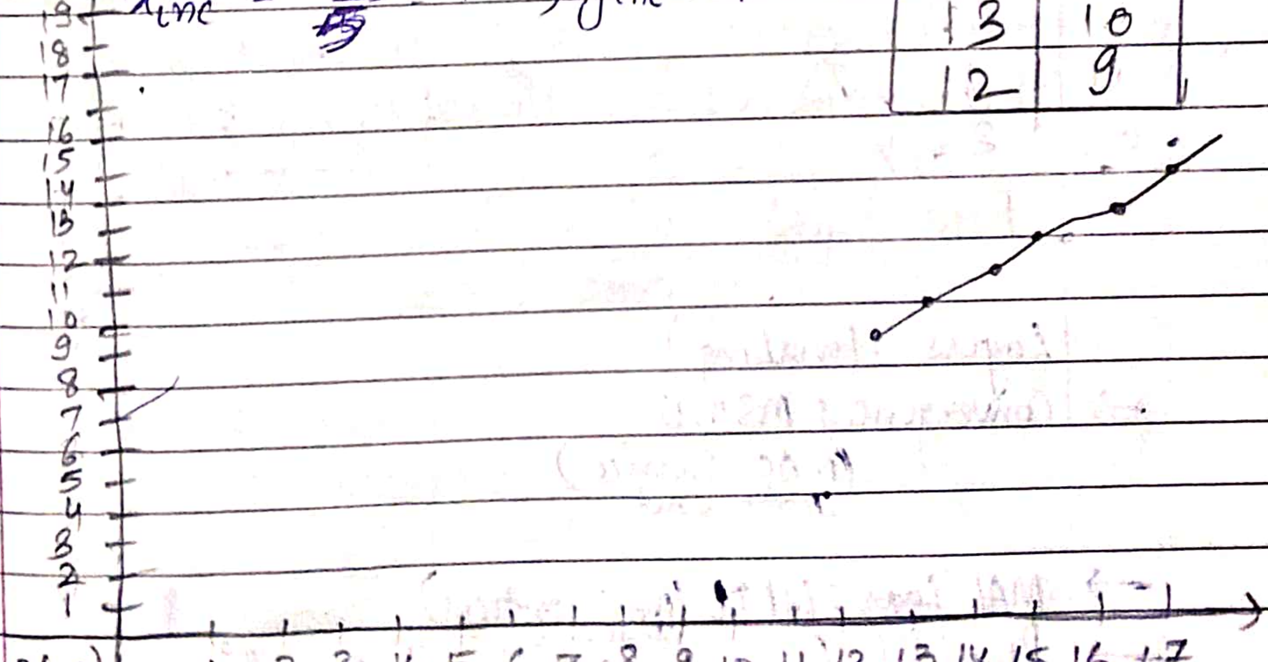
$dx = -5$

$dy = -5$

$step = +5$

$x_{inc} = \frac{-5}{\cancel{5}} = -1$, $y_{inc} = -1$

x	y
17	14
16	13
15	12
14	11
13	10
12	9



Q. $P(12, 9)$ $Q(17, 14)$

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Q. P $(14, 6)$ and Q $(19, 11)$

($x_1 = 14$, $y_1 = 6$; $x_2 = 19$, $y_2 = 11$)

$dx = x_2 - x_1 = 5$

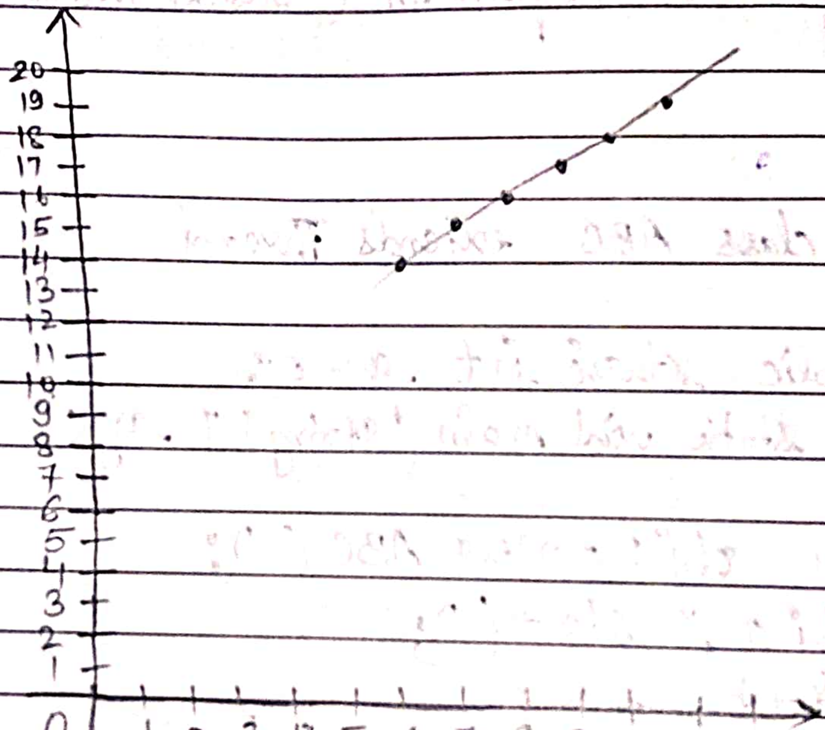
$dy = y_2 - y_1 = 5$

step $= 5$

$x_{inc} = \frac{5}{5} = 1$

$y_{inc} = \frac{5}{5} = 1$

x	y
14	6
15	7
16	8
17	9
18	10
19	11



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Bresenham's Algorithm (x_1, y_1, x_2, y_2)

```
{   x = x1 ;
    y = y1 ;

    dx = x2 - x1 ;
    dy = y2 - y1 ;

    P = 2dy - dx;

    while (x <= x2)
    {
      putpixel (x, y);
        x++;
      if (P<0)
      {
          P = P + 2dy ;
      }

      else {
        P = P + 2dy - 2dx ;
        y++;
      }
    }
}
```

$p = -1$

$-1 + 8 = 7$

Q. $\overset{x_1}{}\ \overset{y_1}{}$ $P(9, 18)$ $\quad$ $Q(14, 22)$ with x_2, y_2 above 14, 22

$dx = x_2 - x_1 = 5$

$dy = y_2 - y_1 = 4$

$P = 2dy - dx = 4 \times 2 - 5$

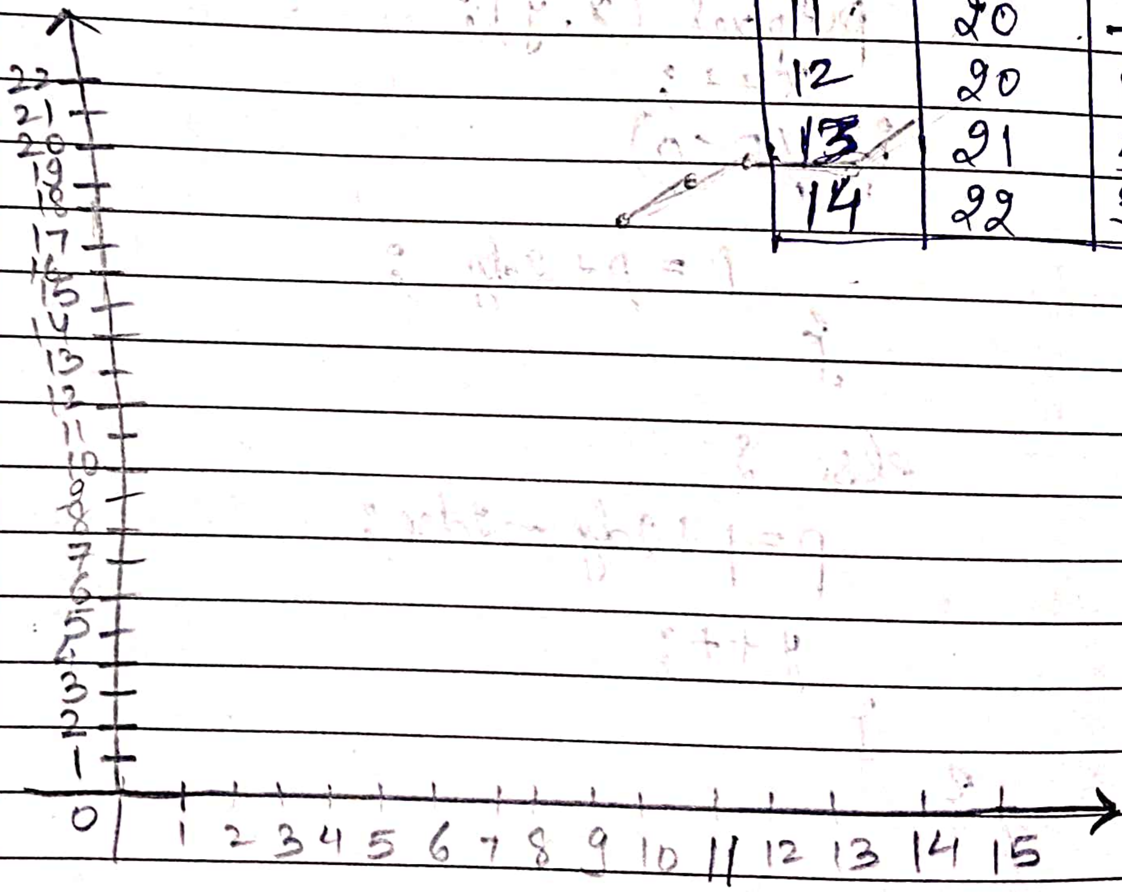
$= \underline{3}$

$P = 3 + 8 - 10$

$P = 11 - 10$

$P = 1$

x	y	p
9	18	3
10	19	1
11	20	-1
12	20	7
13	21	5
14	22	3



P =

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Q. $P(\overset{x_1}{1}, \overset{y_1}{1})$ $Q(\overset{x_2}{5}, \overset{y_2}{3})$

$0 + 4 - 8 = -4$

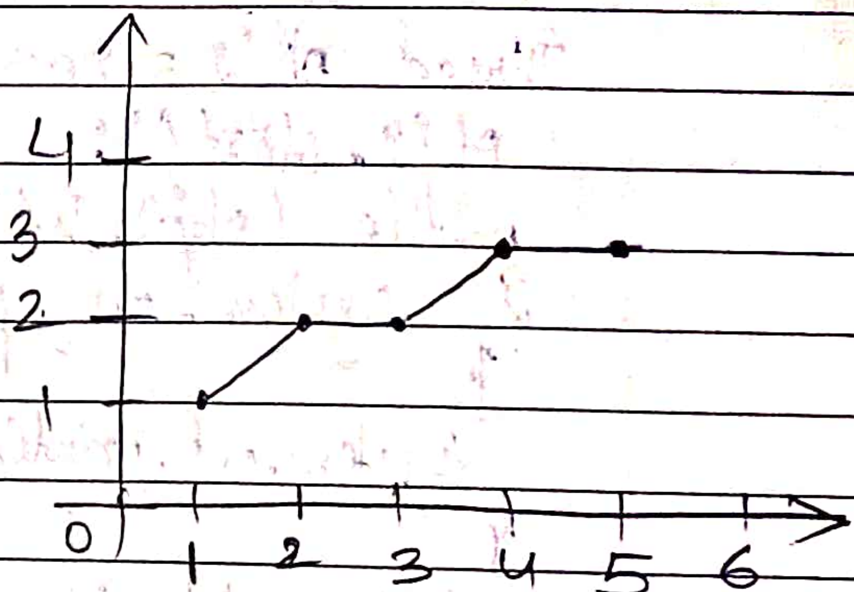
$-4 + 4 = 0$

$dx = x_2 - x_1 = 5 - 1 = 4$

$dy = y_2 - y_1 = 3 - 1 = 2$

$P = 2dy - dx = 4 - 4 = 0$

x	y	P
1	1	0
2	2	-4
3	2	0
4	3	-4
5	3	0



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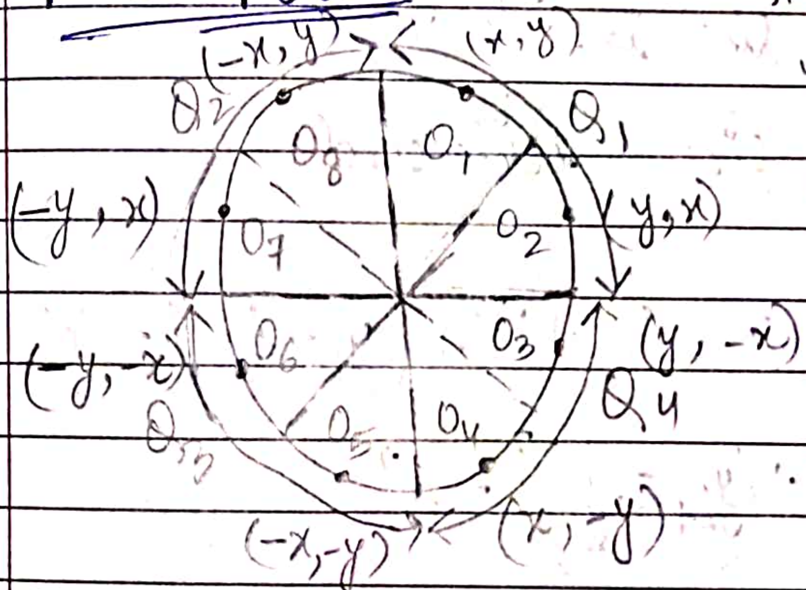
CG

Algorithm
- → Line
- → Circle
- → Ellipse
- → Boundary filling
- → Color filling

Circle

Mid Point



Q_1, Q_2, Q_3, Q_4 are quadrants

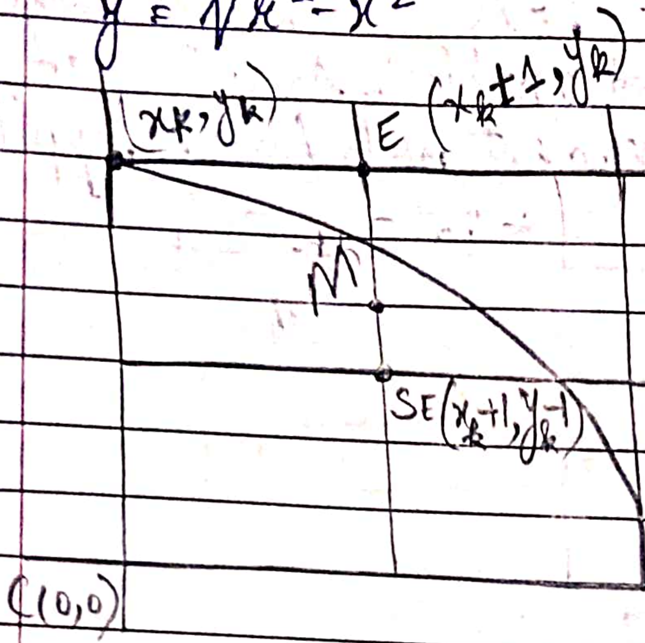
$O_1, O_2, O_3, O_4, O_5, O_6, O_7, O_8$ are octants.

Equation of Circle (c = 0,0)

$$x^2 + y^2 = r^2 \quad \text{— (1)}$$

$$\boxed{x^2 + y^2 - r^2 = 0} \quad \text{— (2)}$$

$$y = \sqrt{r^2 - x^2}$$



$$M\left(\frac{x_k+1+x_k+1}{2}, \frac{y_k+y_k-1}{2}\right)$$

$$M\left(x_k+1, y_k - \frac{1}{2}\right)$$

$$x^2 + y^2 - r^2 = 0$$

Result :

$P = 0$	Point on the circle
$P < 0$	Point lies inside circle
$P > 0$	Point lies outside circle

$$P_k = x_m^2 + y_m^2 - r^2 = (x_k+1)^2 + \left(y_k - \frac{1}{2}\right)^2 - r^2$$

$$P_{k+1} = (x_{k+1}+1)^2 + \left(y_{k+1} - \frac{1}{2}\right)^2 - r^2$$

$\boxed{p_i = \frac{5}{4} - r}$

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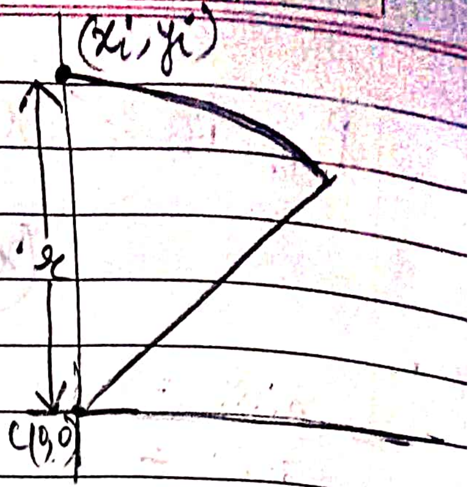
Algorithm



(1) Plot initial point (x_i, y_i)
Such that $x_i = 0$, $y_i = r$.

(2) Find initial decision parameter : $\boxed{p_i = \frac{5}{4} - r}$

(3) If $p_i < 0$, then

$$x_{i+1} = x_i + 1$$
$$y_{i+1} = y_i$$
$$p_{i+1} = p_i + 2x_i + 3$$

(4) If $p_i \geq 0$, then

$$x_{i+1} = x_i + 1$$
$$y_{i+1} = y_i - 1$$
$$p_{i+1} = p_i + 2(x_i - y_i) + 5$$

(5) Repeat step 3, 4 until x becomes greater than or equal to y.

Q. Plot the first octant of a circle centered a origin having radius 10 unit.

$x_i = 0$, $y_i = 10$

$$p_i = \frac{5}{4} - 10 = \frac{5-40}{4} = \frac{-35}{4} = -8.75$$

$\Downarrow$

$1.25 \cong 1 \Rightarrow p_i = 1 - 10 = -9$

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x_i	y_i	P_i
0	10	-9
1	10	-6
2	10	-1
3	10	6
4	9	-3
5	9	8
6	8	5
7	7	6

$x_{i+1} = 0+1$

$y_{i+1} = y_i = 10$

$p_{i+1} = -8.75 + 3$

$= -5.75$

$x_{i+1} = 3+1 = 4$

$y_{i+1} = 10-1 = 9$

$p_4 = 6 + 2(3-10) + 5$

$= 6 - 14 + 5$

$= -3$

$P_5 = 8 + 2(-4) + 5$

$P_6 = 5 + 2(-2) + 5$

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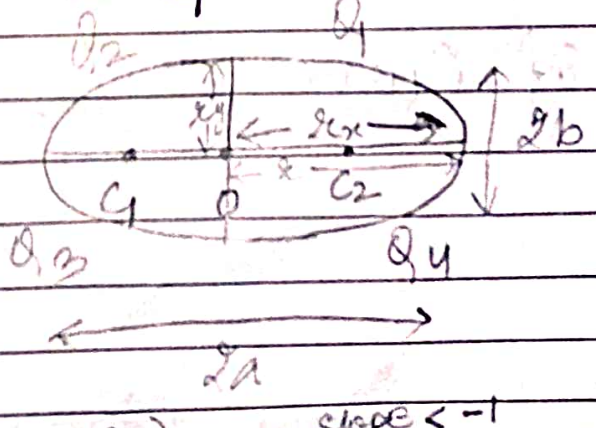
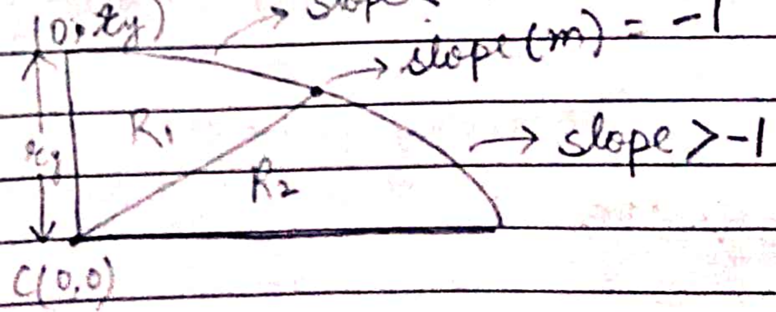
CG

Ellipse



C_1, C_2 are focii



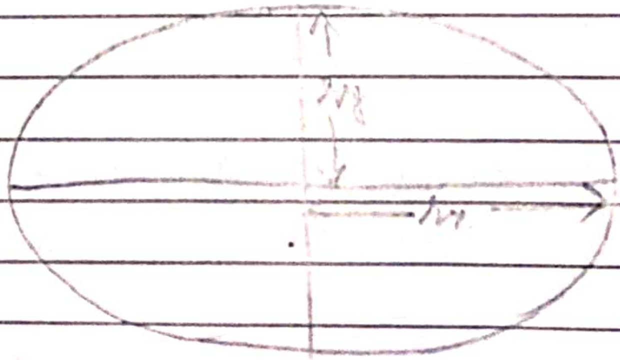
Quadrant-1 : Region-1 (Q1R1) :

Start point $(0, r_y)$
Slope of curve < -1
Take unit steps in positive x direction till boundary between two region is reached.

Quadrant-1 : Region-2 (Q1R2) :

slope > -1
take unit step in negative 'y' direction till the end of quadrant.
On the boundary between two regions (R_1 & R_2) the slope of curve is -1.

Slope of Curve (Ellipse)



Equation :

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} = 1$$

$$b^2x^2 + a^2y^2 = a^2b^2$$

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$$b^2x^2 + a^2y^2 - a^2b^2 = 0$$

$$2a = 2r_x \Rightarrow a = r_x$$
$$2b = 2r_y \Rightarrow b = r_y$$

$$r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2 = 0 \quad \text{— } (1)$$

$$\frac{dy}{dx}\left\{ r_y^2 x^2 + r_x^2 y^2 - r_x^2 r_y^2 \right\}$$

$$\Rightarrow \frac{dy}{dx}\left\{ y^2 \right\} = \frac{dy}{dx}\left(\frac{r_x^2 r_y^2 - r_y^2 x^2}{r_x^2} \right)$$
$$= \frac{dy}{dx}\left(r_y^2 - \frac{r_y^2 x^2}{r_x^2} \right)$$