"14.6"

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$\iiint_B f(x,y,z)\,dV$ "• if limits constant do from the pattern 6 Possiblities

• if limits contains functions so then Do as in order given"

Qs (1-8):-

1). $\int_{-1}^{1}\int_{0}^{2}\int_{0}^{1} (x^2+y^2+z^2)\,dx\,dy\,dz$

Ans:- $\int_{-1}^{1}\int_{0}^{2} \left|\left(\frac{x^3}{3} + xy^2 + xz^2\right)\right|_0^1 dy\,dz$

$\int_{-1}^{1}\int_{0}^{2} \frac{1}{3} + y^2 + z^2 \,dy\,dz$

$\int_{-1}^{1} \left|\frac{1}{3}y + \frac{y^3}{3} + yz^2\right|_0^2 dz$

$\int_{-1}^{1} \frac{2}{3} + \frac{8}{3} + 2z^2 \,dz$

$\int_{-1}^{1} \frac{2z}{3} + \frac{8}{3}z + \frac{2z^3}{3} = 8$

2). $\int_{1/3}^{1/2}\int_{0}^{\pi}\int_{0}^{1} 2x\sin xy \,dz\,dy\,dx$

Ans:- $\int_{1/3}^{1/2}\int_{0}^{\pi} \left[\frac{z^2}{2} x\sin xy\right]_0^1 dy\,dx$

$\int_{1/3}^{1/2}\int_{0}^{\pi} \frac{1^2}{2} x\sin xy$

$\int_{1/3}^{1/2}\int_{0}^{\pi} -\frac{1^2}{2}\cos xy \,dx \Rightarrow$

$\int_{1/3}^{1/2} \left| -\frac{1^2}{2}\cos(\pi x) - \left[-\frac{1}{2}\cos(0\times\pi)\right]\right|$

$\int_{1/3}^{1/2} \left| -\frac{\cos(\pi x)}{2} + \frac{1}{2}\right| dx$

$\int_{1/3}^{1/2} -\frac{1}{2\pi}\sin(\pi x) + \frac{1}{2}x \rightarrow 0.0620$

3). $\int_{0}^{2}\int_{-1}^{y^2}\int_{-1}^{z} yz \,dx\,dy\,dz$

Ans:- $\int_{0}^{2}\int_{-1}^{y^2} \left| xyz \right|_{-1}^{2}$

$\int_{0}^{2}\int_{-1}^{y^2} yz^2 + yz \,dy^2\,dy.$

$\int_{0}^{2}\int_{-1}^{y^2} \frac{y^2 z^3}{3} + \frac{y^2 z^2}{2} \,dy$

$\int_{0}^{2} \frac{y^4 z^3}{3} + \frac{y^4 z^2}{2} + \frac{z^2}{2}\,dz$

$\int_{0}^{2} \frac{y^4 z^3}{6} + \frac{y^4 z^2}{4} - \frac{z^3}{6} - \frac{z^2}{4}$

$= \boxed{\frac{8y^4}{6} + y^4 - \frac{8}{6} - 2}$

$\int_{0}^{2}\int_{-1}^{y^2} \frac{yz^3}{3} + \frac{yz^2}{2} dy$

$\int_{0}^{2} \left|\frac{y^7}{3} + \frac{y^5}{2}\right| - \left|-\frac{y}{3} + \frac{y}{2}\right| dy$

$\int_{0}^{2} \frac{y^8}{24} + \frac{y^6}{12} + \frac{y^2}{6} - \frac{y^2}{4} = 47/3$

~~0.062~~

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Q4).

Ans:- $\int_0^{\pi/4}\int_0^1\int_0^{x^2} x\cos y \; dz\, dx\, dy$

$\int_0^{\pi/4}\int_0^1 \left|z\, x\cos y\right|_0^{x^2} dx\,dy$

$\int_0^{\pi/4}\int_0^1 x^3\cos y \; dx\, dy$

$\int_0^{\pi/4}\int_0^1 \frac{x^4}{4}\cos y \; dy$

$\int_0^{\pi/4} \frac{1}{4}\cos y \; dy$

$\int_0^{\pi/4} \frac{1}{4}\sin y \to \frac{\sqrt{2}}{8}$

Q5).

Ans:- $\int_0^3\int_0^{\sqrt{9-z^2}}\int_0^x xy \; dy\,dx\,dz$

$\int_0^3\int_0^{\sqrt{9-z^2}} \left|\frac{xy^2}{2}\right|_0^x dx\,dz$

$\int_0^3\int_0^{\sqrt{9-z^2}} \frac{x^3}{2}\, dx\, dz$

$\int_0^3\int_0^{\sqrt{9-z^2}} \frac{x^4}{8}\, dz$

$\int_0^3 \frac{\left((9-z^2)^{1/2}\right)^4}{8}\, dz$

$\int_0^3 \frac{(9-z^2)^2}{8}\, dz \to \frac{z^4}{8} - \frac{18z^2}{8} + \frac{81}{8}\, dz$

$\int_0^3 \frac{z^5}{40} - \frac{18z^3}{24} + \frac{81}{8}z \to 81/5$

$(9-z^2)^2 : (9-z^2)(9-z^2)$

$z^4 - 18z^2 + 81$

Q6).

Ans:- $\int_1^3\int_x^{x^2}\int_0^{\ln 2} xe^y \; dy\,dz\,dx$

$\int_1^3\int_x^{x^2} \left|xe^y\right|_0^{\ln 2} dz\,dx$

$\int_1^3\int_x^{x^2} xe^{\ln 2} - xe^0 \; dz\,dx$

$\int_1^3\int_x^{x^2} x2 - x \; dz\,dx$

$\int_1^3 \left|\frac{x^2}{2}z - x^2 z\right|_x^{x^2} dx \to \int_1^3 \left[\frac{x^4}{2} - x^3 - \frac{x^3}{2} + x^2\right] dx$

$\int_1^3 \frac{x^5}{10} - \frac{x^4}{4} - \frac{x^4}{8} + \frac{x^3}{3} \to \cancel{-\frac{40}{3}}\ 43/15$

Q7)

Ans:- $\int_0^2\int_0^{\sqrt{4-x^2}}\int_{-5+x^2+y^2}^{3-x^2-y^2} x \; dz\,dy\,dx$

$\int_0^2\int_0^{\sqrt{4-x^2}} xz \; dy\,dz$

$\int_0^2\int_0^{\sqrt{4-x^2}} x(3-x^2-y^2) - x(-5+x^2+y^2)\; dy\,dx$

$\int_0^2\int_0^{\sqrt{4-x^2}} 3x - x^3 - y^2x + 5x - x^3 - xy^2 \; dy\,dx$

$\int_0^2\int_0^{\sqrt{4-x^2}} 8x - 2y^2x - 2x^3\, dy\,dx$

$\cancel{x^3(4-x^2)}$

$\int_0^2 \left|8xy - \frac{2y^3}{3}x - 2x^3y\right|_0^{\sqrt{4-x^2}} dx \to \cancel{64}\ 128/15$

$\int_0^2 8(x)(\sqrt{4-x^2}) - \frac{2(4-x^2)^{3/2}x}{3} - 2x^3(4-x^2)^{1/2}\, dx$

$\int_0^2 -\frac{1}{3}(8\sqrt{4-x^2}) + \frac{2(4-x^2)^{5/2}}{15} + \frac{2x^2(4-x^2)^{3/2}}{3}$

$|0+0+0| - |-8 + \frac{64}{15} + \frac{64}{3}| = \frac{88}{5} \to$