Bazil-uddin-Umar $3x^2+C^{x}$ 24k-0559 Date

Ex 13.2:

Q1). $\lim_{(x,y)\to(1,3)} [4(1)(9)-1] \to 35$

Q2). $\lim_{(x,y)\to(0,0)} \frac{4(0)-0}{-1} = 0$

Q3). $\lim_{(x,y)\to(-1,2)} \frac{xy^3}{x+y} = \frac{(-1)(8)}{-1+2} = -8$

Q4). $\lim_{(x,y)\to(1,-3)} e^{2x-y^2} = e^{2-9} \to e^{-7}$

Q5). $\lim_{(x,y)\to(0,0)} \ln(1+0) \to 0$

Q6). $\lim_{(x,y)\to(4,-2)} 4\sqrt[3]{8+2(4)} \to 0$

Q7) (a). $\lim_{(x,y)\to(0,0)} \frac{3}{x^2+2y^2}$

Along $\to y=0$ $\lim_{x\to 0} \frac{3}{x^2} = \infty$, ∞

$x=t$

$\frac{3}{t^2+2t^2} \to \frac{3}{3t^2} = \infty$ } so DNE as goes in infinity

$\frac{3(x^2-2y^2)}{x^2+2y^2(x^2-2y^2)} \to \frac{3x^2-6y^2}{x^4-4y^4} \to$

Along $y=x$

$\frac{\cancel{3}}{\cancel{3}x^2} \to \frac{1}{x^2}$

(b). $\lim_{(x,y)\to(0,0)} \frac{x+y}{2x^2+y^2} = \frac{x+y}{2x^2+y^2} = \infty$

Through different paths

Q8) (a). $\lim_{(x,y)\to(0,0)} \frac{x-y}{r^2}$

$\lim_{r\to 0} \frac{r\cos\theta - r\sin\theta}{r^2} = \frac{\cos\theta-\sin\theta}{r}$

Along $x=0$

$\lim_{y\to 0} \frac{-y}{y^2} = -\frac{1}{y}$ $y=x \to$

$= -\infty$

$\lim_{x\to 0} \frac{x}{x^2} = \frac{1}{x} = \infty$ limit donot exist

Along $y=x$ $\Rightarrow$ $\frac{x-x}{2x^2} = 0$ $0 \neq \infty$

(b). $\lim_{(x,y)\to(0,0)} \frac{\cos xy}{x^2+y^2}$

Along $x=0$ ∞, Along $y=0 \to \infty$ } $\to$ limit DNE

$\cancel{y=1/2x} \to \cos($

Q9) $\lim_{z\to 0^+} \frac{\sin(z^2)}{z^2} = 9$

Q10). $\frac{1-\cos(z)}{z} = 0 \to \frac{1-\cos(x^2+y^2)}{x^2+y^2}$

Q11). $\lim_{(z\to 0^+)} e^{-1/z^2} \to \frac{1}{e^{\infty}} \to 0$

$\hookrightarrow e^{\infty}$

e^{-1/x^2+y^2}

Q12). $\frac{e^{-1/z^2}}{z} = \frac{\frac{1}{e^{\infty}}}{z} \to \frac{\frac{1}{1}}{0} = 0$

$\frac{e^{-1/\sqrt{x^2+y^2}}}{\sqrt{x^2+y^2}}$ 0

Bazil-uddin-lehan 244-0559

Date

13-22:-

Q13). $\lim_{(x,y)\to(0,0)} \frac{x^4-y^4}{x^2+y^2}$

Along $y=0$ $\lim_{x\to 0} \frac{x^4}{x^2} = 0$

$\lim x=0$ $\lim_{y=0} \to 0$

$\frac{(x^2-y^2)\cancel{(x^2+y^2)}}{\cancel{(x^2+y^2)}} + 0$

$y^2 = x \to$

$\frac{x^4-y^4}{r^2} \to \frac{(r\cos\theta)^4-(r\sin\theta)^4}{r^2} = \frac{r^4\cos^4\theta - r^4\sin^4\theta}{r^2} \to r^2(\cos^4\theta - \sin^4\theta) \to 0$

so limit $\to 0$

Q14). $\lim_{(x,y)\to(0,0)}$

Along $x=0$ $\downarrow$ 0

Along $y=0$ $\downarrow$ 0

$\frac{r^4\cos^4\theta - 16r^4\sin^4\theta}{r^2\cos^2\theta + 4(r^2\sin^2\theta)} \to \frac{\cancel{r^4}(\cos^4\theta - 16\sin^4\theta)}{\cancel{r^2}(\cos^2\theta + 4\sin^2\theta)}$

$(\cos^2\theta - 4\sin^2\theta)(\cos^2\theta + 4\sin^2\theta)$ $\quad \frac{r^2(\cos^4\theta - 16\sin^4\theta)}{\cos^2\theta + 4\sin^2\theta}$

$r^2 \frac{(\cos^2\theta - 4\sin^2\theta)\cancel{(\cos^2\theta+4\sin^2\theta)}}{\cancel{\cos^2\theta+4\sin^2\theta}}$

when $r \to 0 \to$ 0 $\to$ limit exists

Q15). $\lim_{(x,y)\to(0,0)} \frac{xy}{3x^2+2y^2}$

limit do not exist at (0,0).

$\boxed{0 \neq 1/5}$

$\frac{(r\cos\theta)^2(\cos\theta)^2}{r^2}$

Along $x=0 \to 0$, Along $y=0 \to 0$

Along $y=x$ $\lim_{x\to 0} \frac{x^2}{3x^2+2x^2} \to \frac{\cancel{x^2}}{5\cancel{x^2}} \to \frac{1}{5}$

Q16). $\lim_{(x,y)\to(0,0)} \frac{1-x^2-y^2}{x^2+y^2} \to \frac{1-r^2\cos^2\theta - r^2\sin^2\theta}{r^2} = \frac{1-r^2(\cos^2\theta+\sin^2\theta)}{r^2} = \frac{1-r^2}{r^2}$

$= \infty$

$\lim_{(x,y)\to(0,0)} \to \infty$

Bazil-uddin-khan 244-0559 BSCQ-24. Date

Q17).

$$\lim_{(x,y,z)\to(2,-1,2)} \frac{xz^2}{\sqrt{x^2+y^2+z^2}} = \frac{8}{3}$$

Q18).

$$\lim_{(x,y,z)\to(2,-1,2)} \ln(4+0+1) \to 1.61$$

Q19). $t^2 = x^2+y^2+z^2$

$$\frac{2t\sin(t^2)}{t} \times \frac{1}{2t} \to 0.$$

Q20). $\lim_{(x,y,z)\to(0,0,0)} \frac{\sin\sqrt{x^2+y^2+z^2}}{x^2+y^2+z^2}$ $x=t^2, y=t, z=t$

$$\frac{\sin(\sqrt{3t^2})}{3t^2} \to \frac{\sin(\sqrt{3}\,t)}{3t^2} = 0 \text{ do not exist.}$$

Lhopital → but still it ∞ so limit Not exists

Q21). $\lim_{(x,y,z)\to(0,0,0)} \frac{e^{\sqrt{0^2+0^2+0^2}}}{\sqrt{0}} \to \frac{1}{0} = 0$ (DONOT EXIST)

Q22). $\lim_{(x,y,z)\to(0,0,0)} \tan^{-1}(\infty) \to \frac{\pi}{2}$

Q23). $\lim_{(x,y)\to(0,0)} \sqrt{x^2+y^2}\ \ln(x^2+y^2)$

$\lim_{r\to 0} \cancel{2}\, r\ln(r) \to 0\times -\infty$ (INDERMINATE form)

$$\frac{\ln(r)}{\frac{1}{r}} \to \frac{\frac{1}{r}}{\frac{-1}{r^2}} \to -r \to 0$$

$(x^2-4y^2)(x^2+4y^2)$

x^4-16y^4

$r\sin\theta \ln(r^2)$ $\frac{\ln(r^2)}{\frac{1}{r\sin\theta}} = \frac{\frac{2r}{r^2}}{\frac{2}{r\sin\theta}} = 0$

$(r, 2\sin\theta)$

Q25). $\lim_{(x,y)\to(0,0)} \frac{(r^2\cos^2\theta)(r^2\sin^2\theta)}{r}$

$\to r^3\cos^2\theta\sin^2\theta$

$\to 0$

Q26) $\lim_{(x,y)\to(0,0)} \frac{(r\cos\theta)(r\sin\theta)}{\sqrt{r^2\cos^2\theta+2(r^2\sin^2\theta)}}$

$$\frac{r^2\cos\theta\sin\theta}{r\sqrt{\cos^2\theta+2\sin^2\theta}}$$

$$0 = \frac{r\cos\theta\sin\theta}{\sqrt{\cos^2\theta+2\sin^2\theta}}$$

$\frac{\sin u^2}{u} \to 2u\cos u^2$ ... $(\cos u^2 \times 2u)$

for 2 variables can't apply direct formula so we use polar

Q24). $\lim_{(x,y)\to(0,0)} y\ln(x^2+y^2)$

Ans:- $y\ln(r^2)$

$r\sin\theta \ln(r^2\sin^2\theta + r^2\cos^2\theta)$

$r\sin\theta\ln(r^2)$

$\frac{\frac{1}{r^2}}{1/r\sin\theta}$

$\lim_{r\to 0} 2r\sin\theta\ln(R) = 0$

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Q34). (b).

$$y=x^3$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2} \to \frac{x^6}{2x^6+x^6} \to \frac{1}{3} \qquad \text{so} \to 0 \neq 1/3$$

lim → Along $x=0 \to 0$, Along $y=0 \to 0$.

Not Exists at $(0,0)$.

(a).

Ans:- $f(x,y)\to(0,0)$

$$\lim_{(x,y)\to(0,0)} \frac{x^3y}{2x^6+y^2} = \lim_{m\to0} \frac{mx^4}{2(x^6)+(m^2x^2)} = 0 \qquad \lim \frac{x^2}{2mx^4+m} = 0$$

$$\lim_{(x,y)\to(0,0)} \frac{x^3t}{2x^6+t^2x^4} \to \frac{x^3t}{x^4(2x^2+t^2)} \lim_{t\to0} \frac{t}{2x^2+t^2} = 0$$

Q35) (a).

Ans:-

$x=at$, $y=bt$, $z=ct$

$$\lim_{t\to0} \frac{(at)(bt)(ct)}{a^2t^2+b^2t^2+c^2t^2} = \frac{abct}{a^2+b^2+c^2} = 0$$

Ans:- 5.

(b).

$$\frac{t^2t \times t}{t^4+t^4+t^4} \to \frac{t^4}{3t^4} = \frac{1}{3}$$

Along $x=t^2$, $y=t$, $z=t$

Along $x=0 \to 0$, so $\to 0 \neq 1/3$ limit does not Exist

$$\frac{abct^3}{a^2t^2+b^2t^2+c^2t^2} = \frac{abct}{a^2+b^2+c^2} = 0$$