$\frac{dy}{dH} = 0.007184\,(w)^{0.425}\,H^{0.725-1}\,(0.725) = 0.0076\,m^2/kg$

13.4:- DIFFERENTIALS:-

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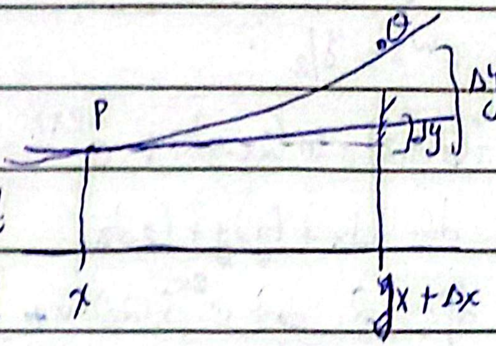
Δy gives the change in height from the point P to point Q.

OR

$(x$ to $x+\Delta x)$



Δy = Incremental

dy = Differential

The dx is the distance travelled in horizontal along x-axis

Δx is curve from P to Q.

$dx = \Delta x$, $\Delta y \neq dy$

But As Δx gets really small $(\Delta x \to 0)$ Then $\Delta y \approx dy$

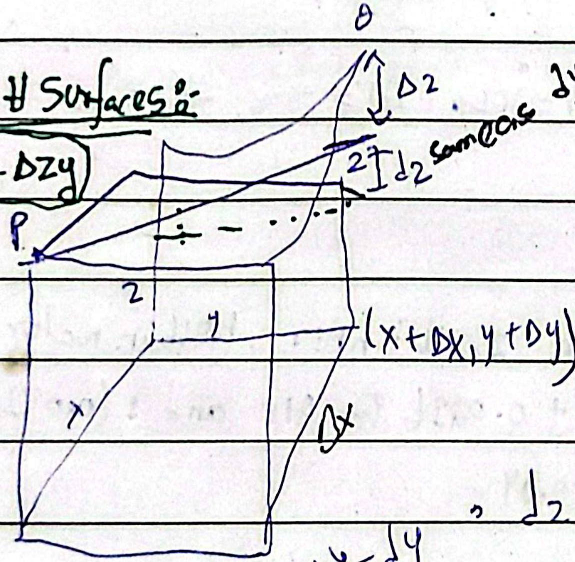
$\therefore\ m = \frac{Rise}{Run} = \frac{Rise}{(\Delta x + x) - (x)} = \frac{Rise}{\Delta x} \to \frac{dy}{dx}$

$f'(x) = dy/dx$ $\quad m = \frac{dy}{dx}$

$$dy = f'(x)\,dx$$

FOR R-3 # Surfaces:-

$\boxed{\Delta z = \Delta z_x + \Delta z_y}$



$\Delta x = dx$, $\Delta y = dy$

$x \neq y$ are

Given a surface from P to Q & A

Tangent Plane of P'

Δz is the actual chang from P to Q (x, y) to $(x+\Delta x, y+\Delta y)$

, dz is the change in height from

P To a point on the tangent plane $(x+\Delta x, y+\Delta x)$

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IF we are in the "x-Direction" we are ∥ xz-Parallel-Plane and Y constant.

$\Delta z_x \approx dz_x = f_x\,dx \rightarrow \Delta z \approx dz_x + \Delta z_y = f_x\,dx + f_y\,dy$

$\cancel{\Delta z \approx} f_x\,dx + f_y\,dy$

Q). Find diff of $z = ye^{x^2-y^2}$ $\Delta z \approx dz = f_x dx + f_y dy$

$dz = f_x dx + f_y dy$

$z_x = ye^{x^2-y^2} \times (2x)$

$z_y = (y)(e^{x^2-y^2})(-2y) + (e^{x^2-y^2})(1)$

$\boxed{dz = 2xye^{x^2-y^2}dx + (1-2y^2)e^{x^2-y^2}dy}$

↳ Approximation of change in height b/w 2 points.

Ex:- $w = x^2e^y + y\ln z$, Find dw? $dw = w_x dx + w_y dy + w_z dz$

$w_x = 2xe^y$

$w_y = x^2e^y + \ln z$

$w_z = y/z$

$dw = 2xe^y dx + (x^2ye^y) + \ln z + y/z$

Just Put in in df eq

$\boxed{x=2, y=3, z=2, dx=0.01, dy=-0.03, dz=0.004}$

Approx change from $(2,3,0) \rightarrow (2.01, 2.97, 0.004)$

Ex:- $f(x,y,z) = \ln(2x-y) + e^{2xz}$ find df? $f_x = \frac{1}{(2x-y)}(2) + e^{2xz} \times 2z$ $(\Delta f \approx df)$

$df = f_x dx + f_y dy + f_z dz$ $df = 0.21, \Delta f \approx 0.21$

$df = \frac{2}{(2x-y)}dx + e^{2xz}x(2z)dx + \frac{1}{2x-y}dy + e^{2xz}(2x)dy$ $= \frac{1}{2x-y}(-1)$, $f_z = e^{2xz}(2x)$

Ex:- Pressure of a certain Gas can be ~~described~~ calculated by $P = \frac{8.314T}{V}$ Find dP. If volume Goes from $20L \rightarrow 20.2L$ and Temp from $300K \rightarrow 295K$. Find Approx change in Pressure.

$f_T = \frac{8.314}{V}$, $f_V = \frac{-8.314T}{V^2}$

$dP = f_T dT + f_V dT$

$dP \approx \left(\frac{8.314}{V}\right)dT + \left(\frac{-8.314T}{V^2}\right)dV$ $T = 300K, V = 20L, \Delta T = -5K, \Delta V = 0.2$

$\Delta P \approx dP \rightarrow -3.3256\,Pa$ net drop in Pressure

Q). $S = \frac{A}{A-W}$ find Max Error if the weights were 2.2lb in Air, 1.8lb in water & you know your scales are accurate $\pm 0.02lb$ for Air and $\pm(0.0423)$ for H_2O.

$A = 2.2lb$, $W = 1.8$, $dA = \pm 0.02$, $dW = \pm 0.04$

$$\Delta s \approx ds = f_A dA + f_w dw \qquad f_A = \frac{A}{(A-w)} \rightarrow \frac{(A-w)(1)-(A)(1)}{(A-w)^2} = \boxed{\frac{-w}{(A-w)^2}}$$

$$ds = \left(\frac{-w}{(A-w)^2}\right) dA + \left(\frac{A}{(A-w)^2}\right) dw \qquad f_w = A(A-w)^{-1} \rightarrow A(A-w)^{-1}(-1)(-1)$$

$$\frac{A}{(A-w)^2}$$

Max Error

due to || → -ve ignore

$$|ds| = \left|\left(\frac{-1.8}{2.2-1.8}\right)(\pm 0.02)\right| + \left|\left(\frac{2.2}{(2.2-1.8)^2} \times \pm 0.04\right)\right| = 0.77516$$

$$\text{max \% ERRor} = \left|\frac{\Delta f}{f}\right|$$

local linear Expressions $f(x_0, y_0) + f_x(x_0, y_0)(x - x_0) + f_y(y - y_0)$

$$\left|\frac{\Delta T}{T}\right| \approx \left|\frac{dT}{T}\right|$$

divide dT by function as some values Not given). and we want to find max error to do this

$20(4x-3y+2z)^3(-3)$

$-60(4x-3y+2z)^2(2)$

Ex 13.4:-

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Q9). $\frac{dz}{dx} \approx \frac{dz}{dx}(\Delta x) + \frac{dz}{dy}(\Delta y)$

$\boxed{dz \approx (7\Delta x) - 2(\Delta y)}$

Q10). $dz \approx ye^{xy}\Delta x + xe^{xy}\Delta y$

Q11). $dz \approx 3x^2y^2\Delta x + (2yx^3)\Delta y$

Q12). $dz \approx (10xy^5-2)(\Delta x) + (25y^4x^2+4)\Delta y$

Q13). $dz \approx \left(\frac{1}{(1+(xy)^2} xy\right)(\Delta x) + \left(\frac{1}{1+(xy)^2}\right)(x)\Delta y$

Q14).

$dz \approx \left(-3e^{-3x}\cos 6y\right)dx + \left(e^{-3x} - \sin 6y\, 6\right)dy.$

Q15). $dw \approx (8dx) - (3dy) + (4dz)$

Q16). $dw \approx (e^{xyz})(yz)dx + (e^{xyz} x z)(dy) + (e^{xyz})(yx)dz$

Q17). $dw \approx (3x^2y^2z)dx + (x^3 2yz)dy + (x^3y^2)dz$

Q18). $dw \approx (8xy^3z^7 - 3y)dx + (4x\,x^2 3y^2)(-3x)dy + (4x^2y^3 x 7z^4 + 1)dz$

Q19). $dw \approx \left(\frac{1}{(1+(xyz)^2} x(yz)\right)dx + \left(\frac{1}{(1+(xyz)^2} x(yz)\right)dy + \left(\frac{1}{(1+(xyz)^2} xyx\right)dz$

Q20). $dw \approx \left(\frac{1}{2\sqrt{x}}\right)dx + \left(\frac{1}{2\sqrt{y}}\right)dy + \frac{1}{2\sqrt{z}}dz$

Q21-Q26:-

$x = 1,\ y = 2,\ \Delta x = 0.01,\ \Delta y = 0.04$

Q21). $df = (2x+2y)dx + (2x)dy \rightarrow \boxed{0.10}$

Q22). $df = \left(\frac{1}{3}x^{-2/3}y^{1/2}\right)dx + \left(x^{1/3}\frac{1}{2}y^{-1/2}\right)dy \rightarrow \boxed{-0.045}$

$x = 8,\ y = 9,\ \Delta x = -0.22,\ \Delta y = 0.03$

Q23). $(x+y)(xy)^{-1}$

$df = \left(\frac{xy(1) - (x+y)(y)}{(xy)^2}\right)(dx) + \left(\frac{(xy)(1) - (x+y)}{(xy)^2}\right)dy$

$\Delta x = -0.02$

$\Delta y = -0.04$

$df \approx -0.02$

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Q27). $\ln(1+xy)^{1/2}$ $x=0, y=2, \Delta x=-0.09, \Delta y=-0.02$

$df \approx \frac{1}{(1+xy)^{1/2}} \times \frac{1}{2}(1+xy)^{-1/2}$ $\frac{1}{(1+xy)^{1/2}} xy\, dx + \frac{x}{(1+xy)^{1/2}} dy$

Q29. $x=1, y=-1, z=2, \Delta x=0.01, \Delta y=-0.02, \Delta z=0.02$

$df \approx (2y^2z^3)\Delta x + (4xyz^3)\Delta y + (6z^2xy^2)\Delta z \rightarrow 1.28$

Q26).

$x=-1, y=-2, z=4, \Delta x=-0.04, \Delta y=0.02, \Delta z=-0.03$

$dz \approx \frac{(x+y+z)(yz)-(xyz)(1)}{(x+y+z)^2}dx + \frac{(x+y+z)(xz)-(xyz)(1)}{(x+y+z)^2}dy + \frac{(x+y+z)(xy)-(xyz)}{(x+y+z)^2}$

Q(33-40):-