1. Evaluate $\iint_R (3x+4y^2)\,dA$, where R is [3 Marks]

the region in the upper half-plane bounded by circles $x^2+y^2=1$ and $x^2+y^2=4$.

$$\iint_R (3x+4y^2)\,dA = \int_0^{\pi}\int_1^2 (3r\cos\theta + 4r^2\sin^2\theta)\,r\,dr\,d\theta$$

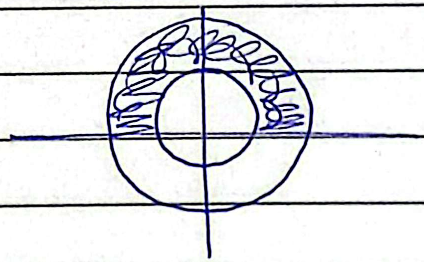
$$= \int_0^{\pi}\int_1^2 (3r^2\cos\theta + 4r^3\sin^2\theta)\,dr\,d\theta$$

$$= \int_0^{\pi}\Big[r^3\cos\theta + r^4\sin^2\theta \Big]_1^2 d\theta$$

$$\int_0^{\pi}\left(8\cos\theta + 16\sin^2\theta - \cos\theta - \sin^2\theta \right)d\theta$$

$$= \int_0^{1}(7\cos\theta + 15\sin^2\theta)\,d\theta = \int_0^{1}\left(7\cos\theta + \frac{15}{2}(1-\cos 2\theta)\right)d\theta$$

$$= \left[7\sin\theta + \frac{15\theta}{2} - \frac{15}{4}\sin 2\theta \right]_0^{\pi} = \frac{15\pi}{2}$$



② Evaluate $\int_C (x^3-y)\,dx + (x+y^3)\,dy$, where C is the triangle with vertices $(0,0)$, $(1,0)$ and $(1,1)$. [3 Marks]

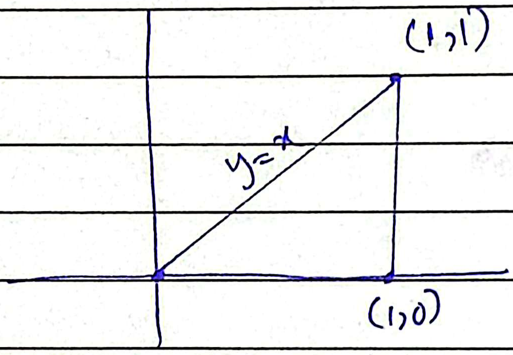
Using Green's theorem

$f(x,y) = x^3 - y$, $g(x,y) = x + y^3$

$f_y(x,y) = -1$, $g_x(x,y) = 1$

$$\iint_R (g_x - f_y)\,dA = \iint_R (1-(-1))\,dA$$

$$= \iint_R 2\,dA$$



Type I.

$$\int_0^1 \int_0^x 2\,dy\,dx$$

$$= \int_0^1 [2y]_0^x\,dx = \int_0^1 2x\,dx$$

$$= [x^2]_0^1 = 1$$

③ Evaluate $\int_C y\,dx + z\,dy + x\,dz$, C is the line segment from $(2,0,0)$ to $(3,4,5)$.

[4 marks]

$r_0 : (2,0,0)$, $r_1 : (3,4,5)$

$$r(t) = (1-t)r_0 + t r_1 = (1-t)\langle 2,0,0\rangle + t\langle 3,4,5\rangle$$
$$= \langle 2-2t, 0, 0\rangle + \langle 3t, 4t, 5t\rangle$$
$$= \langle 2-2t+3t, 4t, 5t\rangle$$
$$= \langle 2+t, 4t, 5t\rangle$$

$x = 2+t$, $y = 4t$, $z = 5t$, $0 \le t \le 1$

$dx = dt$, $dy = 4dt$, $dz = 5dt$

$$\int_C y\,dx + z\,dy + x\,dz = \int_0^1 4t\,dt + 5t(4dt) + (2+t)5dt$$
$$= \int_0^1 (4t + 20t + 10 + 5t)\,dt$$
$$= \int_0^1 (29t + 10)\,dt = \left[\frac{29t^2}{2} + 10t\right]_0^1$$
$$= \frac{29}{2} + 10 = \frac{49}{2} = 24.5$$