Bazil-uddin-khan 24k-0559. BSCS 2H. Date 18th Feb 2024

ASSIGNMENT:-1

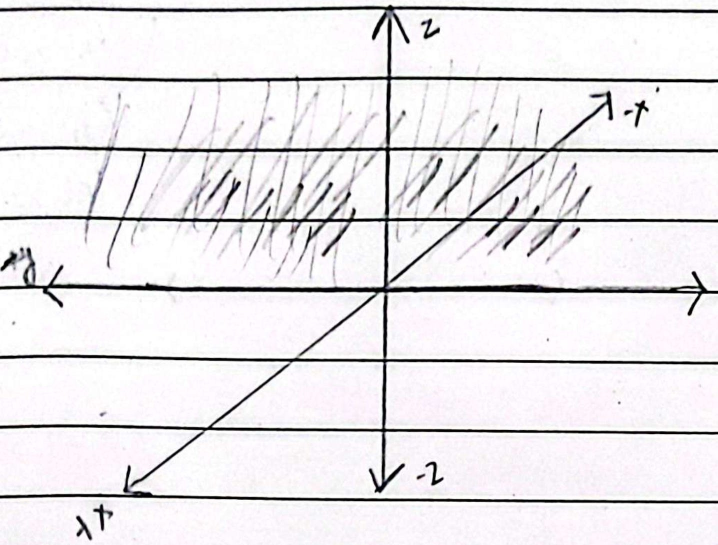
Problem 1:-

Q1). Ans:-

$f(x,y,z) = xy\ln z$

$z > 0$

Domain $\{f(x,y,z) \mid z > 0\}$

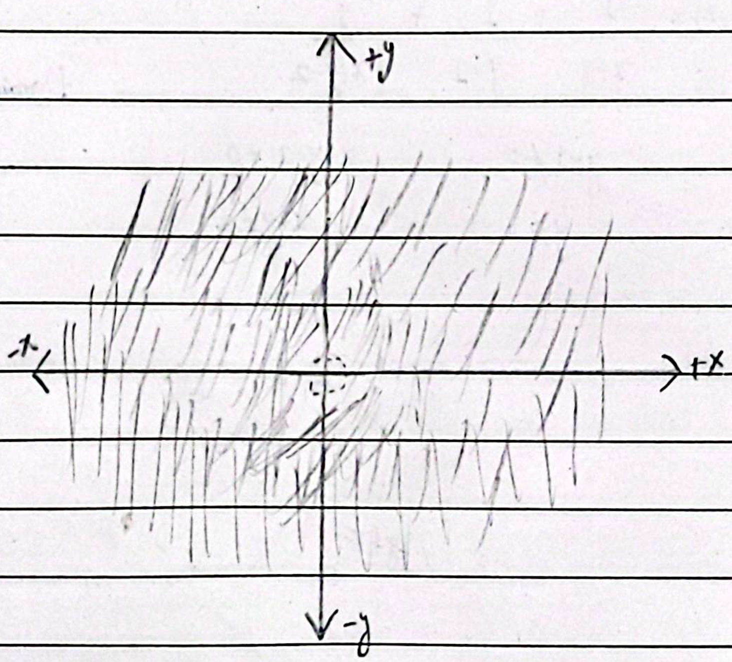


Q2). Ans:- $f(x,y) = \ln(x^2+y^2)$

$x^2+y^2 > 0.$

so Domain $\{f(x,y) \mid x^2+y^2 > 0\}$

All x, y values except 0.



Q3). Ans:- $f(x,y) = \sin^{-1}(y-x) \rightarrow \dfrac{1}{\sqrt{1-(y-x)^2}}$

$1-(y-x)^2 > 0 \rightarrow y-x < 1$

$(y-x)^2 < 1$ and $y-x > -1$

so Domain $\{f(x,y) \mid (y-x < 1) \,\&\&\, (y-x) > -1\}$

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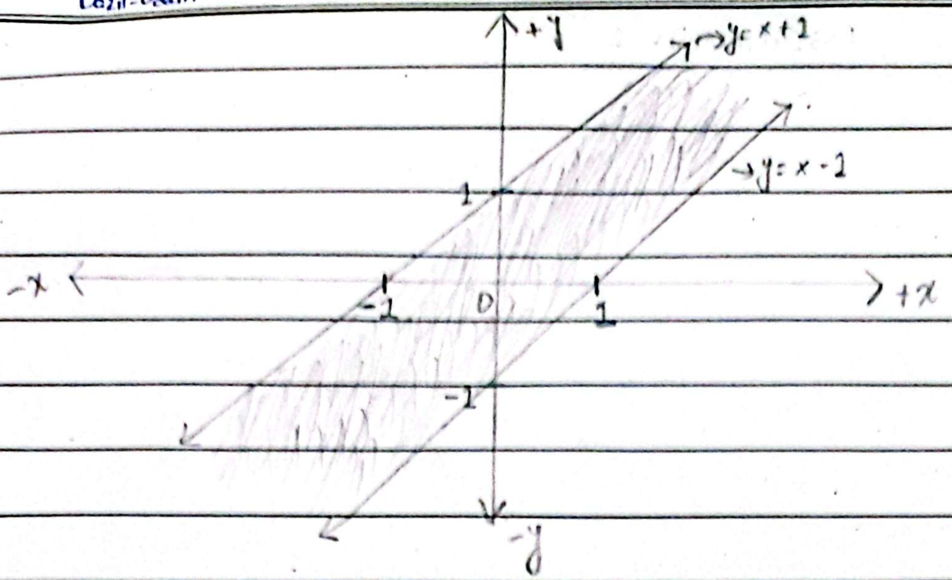


$y - x = 1 \rightarrow y = x + 1$

$y - x = -1 \rightarrow y = x - 1$

At $y=0, x=-1$,

At $x=0, y=1$

At $y=0, x=1$

At $x=0, y=-1$

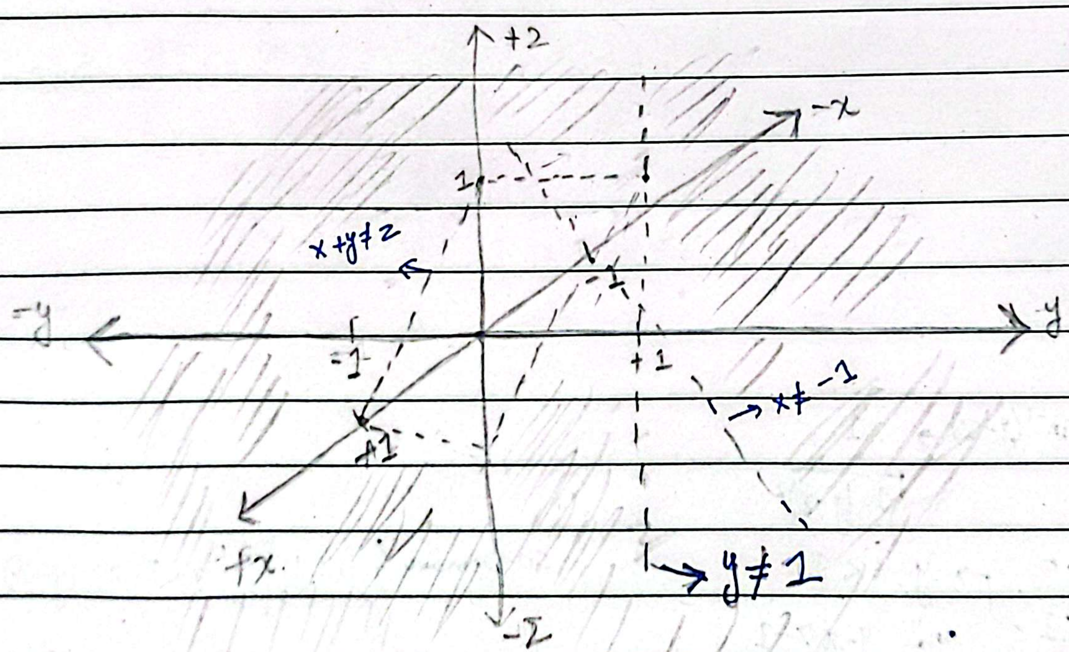
Q4).

Ans:- $f(x,y,z) = \dfrac{1}{x+1} + \dfrac{1}{y-1} + \dfrac{1}{x+y-2}$

Domain $\{ f(x,y,z) \mid x \neq -1, y \neq 1, x+y \neq 2 \}$

$x+1 \neq 0$ $\quad$ $y-1 \neq 0$ $\quad$ $x+y-2 \neq 0$

$x \neq -1$ $\quad$ $y \neq 1$ $\quad$ $x+y \neq 2$



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Problem 2 :-

Q1). $f(x,y) = 4x^2+y^2+1$

$k = 4x^2+y^2+1$

$k-1 = 4x^2+y^2$

At k=2 : $4x^2+y^2 = 1$

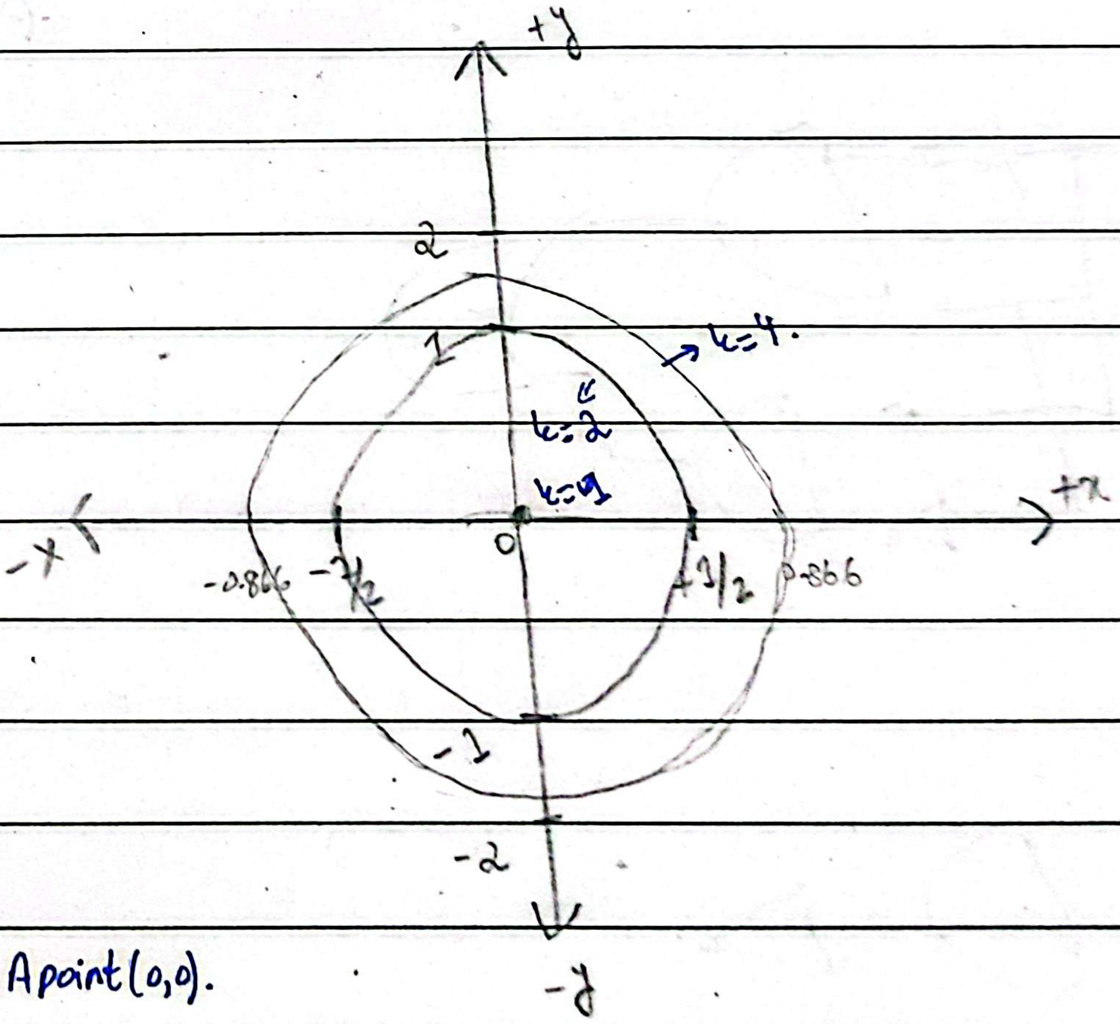
$$\frac{x^2}{1/4}+\frac{y^2}{1} = 1.$$

At k=4 :- $4x^2+y^2 = 4-1$

$$\frac{x^2}{3/4}+\frac{y^2}{3} = 1.$$

At k=1 :- $1-1 = 4x^2+y^2$

$4x^2+y^2 = 0 \rightarrow$ A point (0,0).



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Problem 2:-

Q2).

Ans:- $-2 = 2x - 6y + 2$

$\boxed{z=4}$ $z = 6y - 2x - 2$

$k + 2 = 6y - 2x$

k=1:- $6y - 2x = 3$

At $y=0$, $x = -3/2$

At $x=0$, $y = 1/2$

k=2:- $4 = 6y - 2x$

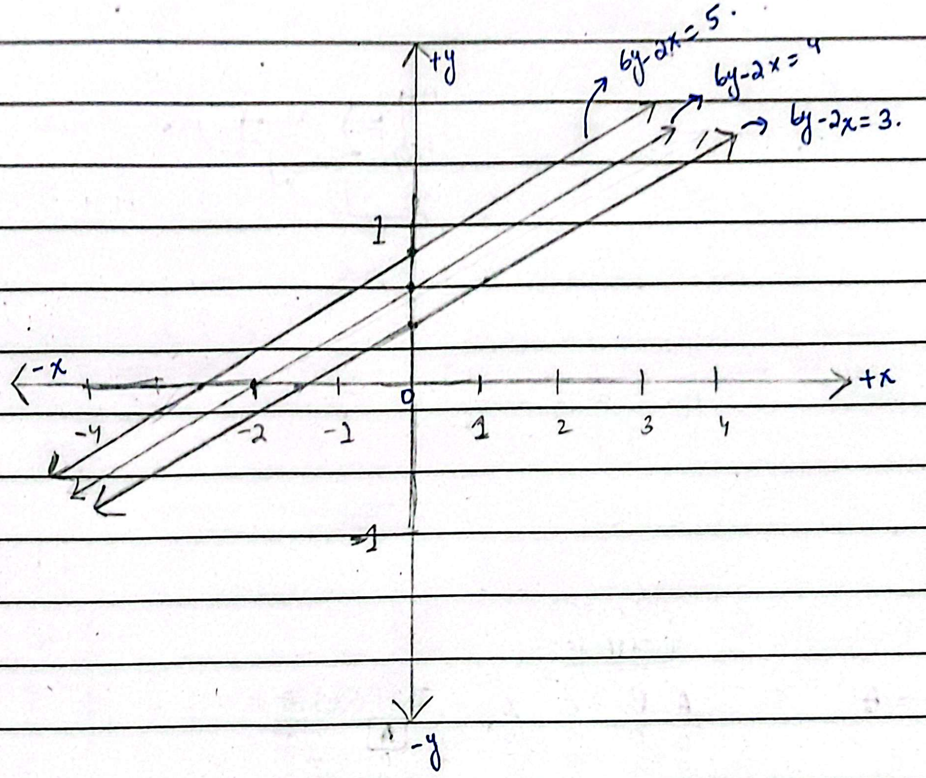
At $y=0$, $x = -2$

At $x=0$, $y = 2/3$

k=3:- $6y - 2x = 5$

At $x=0 \rightarrow y = 5/6$

At $y=0 \rightarrow y = -5/2$.



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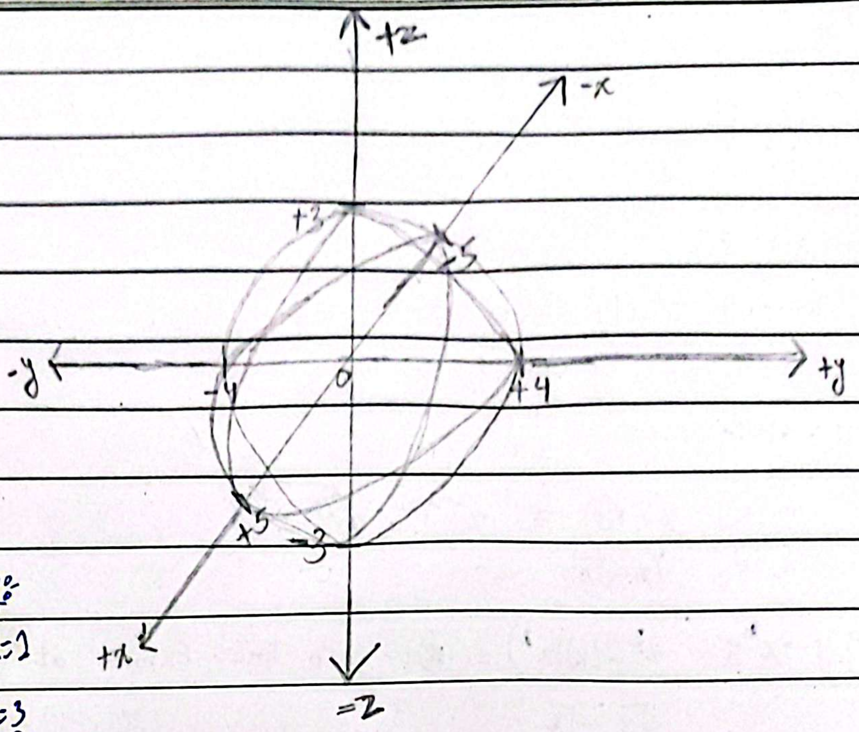
Problem 3.8:

Q1). $\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} = k.$

Ans:-



At $k=1$:- $\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} = 1$

$x^2 = 25 \to x = 5$ or $x = -5$

$y^2 = 16 \to y = +4, y = -4.$

$z^2 = 9 \to z = +3, z = -3.$

xy Plane:

$\frac{x^2}{25} + \frac{y^2}{16} = 1$

$a=5, b=4$

$x=5, y=4$

yz-Plane:

$\frac{y^2}{16} + \frac{z^2}{9} = 1$

$a=4, y=3$

$y=4, z=3$

xz-Plane:

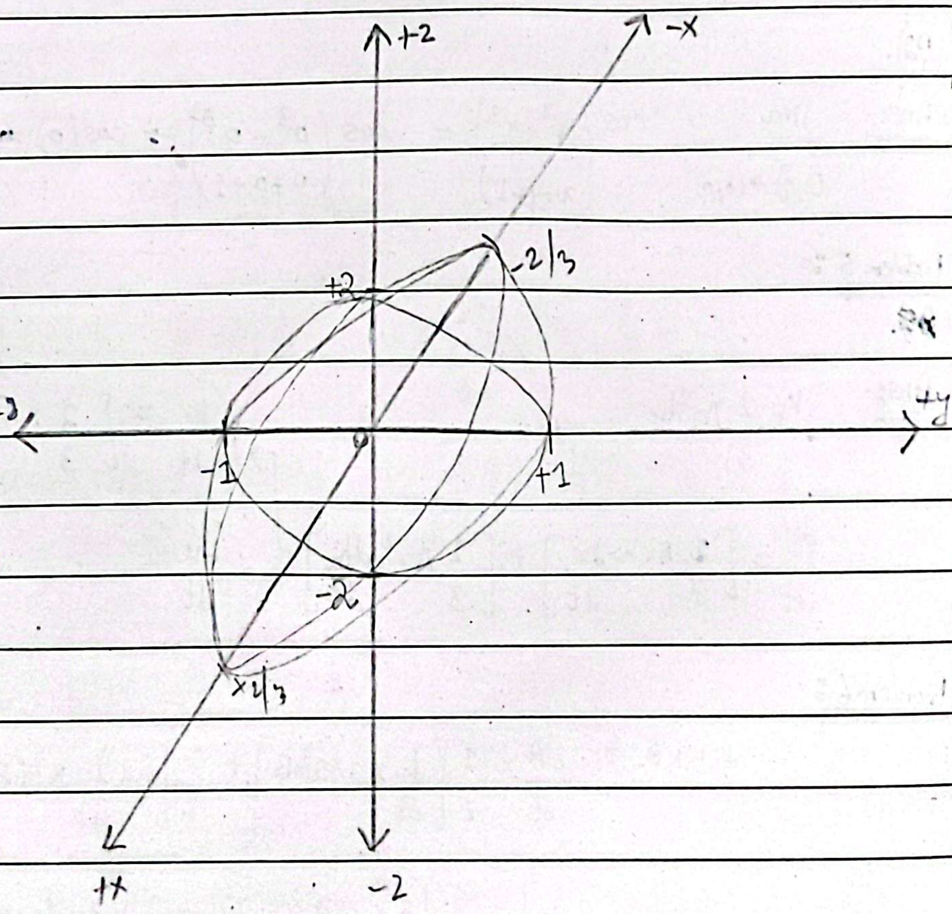
$\frac{x^2}{25} + \frac{z^2}{9} = 1$

$a=5, b=3$

$x=5, z=3$

Q2).

Ans:- $f(x,y,z) = 9x^2 + 4y^2 + z^2$



At $k=4$:- $k = 9x^2 + 4y^2 + z^2$

$4 = 9x^2 + 4y^2 + z^2$

$1 = \frac{9x^2}{4} + \frac{y^2}{1} + \frac{z^2}{4}$

$1 = \frac{x^2}{4/9} + \frac{y^2}{1} + \frac{z^2}{4}$

$a^2 = 4/9 \to x = +2/3, x = -2/3$

$b^2 = 1 \to y = +1, y = -1.$

$c^2 = 4. \to z = +2, z = -2.$

xy Plane:

$\frac{x^2}{4/9} + \frac{y^2}{1} = 1$

$a = \pm 2/3, b = \pm 1$

yz Plane:

$y^2 + \frac{z^2}{4} = 1$

$a = \pm 1, b = \pm 2$

xz Plane:

$\frac{x^2}{4/9} + \frac{z^2}{4} = 1$

$a = \pm 2/3, b = \pm 2.$

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Problem 4:-

Q1). $\lim_{(x,y)\to(0,0)} \dfrac{x^2-xy}{\sqrt{x}-\sqrt{y}}$

Ans:- Along $x\to 0$:-

$\lim_{y\to 0} f(x,y) \quad \dfrac{0^2-0(y)}{\sqrt{0}-\sqrt{y}} = 0$

Along $y\to 0$:-

$\lim_{x\to 0} f(x,y) \quad \dfrac{x^2-x(0)}{\sqrt{x}-\sqrt{0}} = x^{2-1/2} = x^{3/2} = 0$

so limit is 0.

Along $y\to x^2$:- $\dfrac{x^2-(x)(x^2)}{\sqrt{x}-\sqrt{x}} = \dfrac{0}{0}$ So limit Exists at 0.

~~Apply l-hopital~~ $\cancel{2x-3y}$ / simplify it →

$\lim_{(y,x)\to(0,0)} \dfrac{(x^2-xy)(\sqrt{x}+\sqrt{y})}{(\sqrt{x}-\sqrt{y})(\sqrt{x}+\sqrt{y})} = \dfrac{x\cancel{(x-y)}(\sqrt{x}+\sqrt{y})}{\cancel{(x-y)}} = x(\sqrt{x}+\sqrt{y}) = 0(0+0) = 0.$

Q2).

Ans:- $\lim_{(x,y)\to(0,0)} \cos\left(\dfrac{x^2-y^3}{x+y+1}\right) = \cos\left(\dfrac{0^3-0^3}{0+0+1}\right) \to \cos(0) = 1$ So limit Exists and it is 1.

Problem 5:-

~~Q4~~.

Ans:- $V = \dfrac{1}{3}\pi r^2 h$

$\dfrac{dV}{dt} = \left[\dfrac{2}{3}\pi r h \dfrac{dr}{dt}\right] + \left[\dfrac{1}{3}\pi r^2 \dfrac{dh}{dt}\right]$

$\dfrac{dV}{dt} = \left[\dfrac{2}{3} \times \pi \times (1.5) \times 3.2 \times \left(\dfrac{-2}{10}\right)\right] + \left[\dfrac{1}{3} \times \pi \times (1.5)^2 \times \left(\dfrac{3}{10}\right)\right]$

$\dfrac{dV}{dt} =$

$\rightarrow \boxed{\dfrac{dV}{dt} = -1.30\,cm^3/s}$

Problem 6:-

Ans:- $A = \dfrac{1}{2}ac\sin B \rightarrow \dfrac{dA}{dt} = \dfrac{1}{2}\left[\dfrac{da}{dt} \times c \times \sin B\right] + \dfrac{1}{2}\left[a \times \dfrac{dc}{dt} \times \sin B\right] + \dfrac{1}{2}\left[a \times c \times \cos B \times \dfrac{dB}{dt}\right]$

$\dfrac{dA}{dt} = \dfrac{1}{2}\left[0.4 \times 4 \times \sin\dfrac{\pi}{6}\right] + \dfrac{1}{2}\left[3 \times -0.8 \times \sin\dfrac{\pi}{6}\right] + \dfrac{1}{2}\left[3 \times 4 \times \cos\left(\dfrac{\pi}{6}\right) \times 0.2\right] = \boxed{\dfrac{dA}{dt} = 0.8392 \text{ unit } s^2/s.}$

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Problem 7:-

Ans:- $PV^{1.4} = k$

$$dk = V^{1.4}dp + 1.4PV^{0.4}dv$$

Now divide whole by eq.

$$dk = \frac{\cancel{V^{1.4}}dp}{P\cancel{V^{1.4}}} + \frac{1.4\cancel{P}V^{0.4}dv}{\cancel{P}V^{1.4}}$$

$$dk = \frac{dp}{P} + 1.4\frac{dv}{V}$$

$dp/p = 0.04$.

$dv/v = -0.015$ $\quad dk = 0.04 + 1.4(-0.015) = \boxed{0.019}$ % error in k

Problem 8:-

Ans:- $f_T(-15,30) = 1.3 \ (13/10)$

$$\Delta T = -5 \rightarrow \frac{f(-15-5,30) - f(-15,30)}{-5} = \frac{-33-(-26)}{-5} = \boxed{\frac{7}{5}}$$

$$\text{So Average is} = \frac{\frac{6}{5}+\frac{7}{5}}{2} = \cancel{1.3}\ 1.3$$

$$\Delta T = +5 \rightarrow \frac{f(-15+5,30) - f(-15,30)}{+5} = \frac{-20-(-26)}{5} = \boxed{\frac{6}{5}}$$

$f_v(-15,30) = -0.15 \ (-3/20)$.

$$\Delta v = -10 \rightarrow \frac{f(-15,20) - f(-15,30)}{-10} = \frac{-24-(-26)}{-10} = \boxed{\frac{2}{-10}}$$

AND

$$\Delta v = +10 \rightarrow \frac{f(-15,30+10) - f(-15,30)}{10} = \frac{-27+(-26)}{10} = \frac{-1}{10}$$

$$\text{So Average} = \frac{-\frac{2}{10} - \frac{1}{10}}{2} = -0.15$$

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Q9)

(a).

Ans: $\frac{dh}{dv}$ represents the rate at which the height is changing with respect to v increasing while keeping t as constant.

$\frac{dh}{dt}$ = represents the rate at which the height with respect to duration is changing while keeping windspeed (v) as constant.

(b).

Ans: $f_v(40,15) = 1$

Now Take Average $= \frac{\frac{11}{10} + \frac{9}{10}}{2} = 1$

$\nabla v = +10 \rightarrow \frac{f(40+10, 15) - f(40,15)}{+10} = \frac{36 - 25}{10} = \frac{11}{10}$

$\nabla v = -10 \rightarrow \frac{f(40-10, 15) - f(40,15)}{-10} = \frac{16 - 25}{-10} = \frac{-9}{-10} = \frac{9}{10}$

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Q9b) (continue).

Ans:- $f_t(40,15) = 0.7$.

Nowtake Average $= \dfrac{\frac{3}{5}+\frac{4}{5}}{2} = 0.7$

$\nabla t = +5 \rightarrow \dfrac{f_t(40,15+5) - f(40,15)}{+5} = \dfrac{28-25}{5} = \dfrac{3}{5}$

$\nabla t = -5 \rightarrow \dfrac{f(40,15-5) - f(40,15)}{-5} = \dfrac{21-25}{-5} = \dfrac{+4}{+5} = \dfrac{4}{5}$

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Problem 10:-

Q1). Ans:- $f(x,y,z) = xe^y + ye^z + ze^x$ at Point $(0,0,0)$, vector $(\vec{v}) = 5\hat{i} + 1\hat{j} - 2\hat{k}$.

$\nabla f = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$

(gradient) $\nabla f = (e^y + ze^x)\hat{i} + (xe^y + e^z)\hat{j} + (ye^z + e^x)\hat{k}$

$\nabla f = (e^0 + 0(e^0))\hat{i} + (0.e^0 + e^0)\hat{j} + (0e^0 + e^0)\hat{k} \rightarrow \boxed{\nabla f_{(0,0,0)} = (1\hat{i} + 1\hat{j} + 1\hat{k})}$ Gradient.

$\text{vector}(\vec{v}) = \dfrac{5\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{5^2+1^2+2^2}} = \dfrac{5\hat{i} + \hat{j} - 2\hat{k}}{\sqrt{30}} \rightarrow \boxed{\dfrac{5\hat{i}}{\sqrt{30}} + \dfrac{\hat{j}}{\sqrt{30}} - \dfrac{2\hat{k}}{\sqrt{30}}}$

Now dotProduct of both $\rightarrow$ $\vec{V} \cdot \vec{\nabla f}$ $\left(\dfrac{5\hat{i}}{\sqrt{30}} + \dfrac{\hat{j}}{\sqrt{30}} - \dfrac{2\hat{k}}{\sqrt{30}}\right) \cdot \left(1\hat{i} + 1\hat{j} + 1\hat{k}\right)$

$= \dfrac{5}{\sqrt{30}} + \dfrac{1}{\sqrt{30}} - \dfrac{2}{\sqrt{30}} \rightarrow \boxed{\dfrac{4}{\sqrt{30}}}$ Ans:-

Q2). Ans:- $f(x,y,z) = \sqrt{xyz}$ at Point $(3,2,6)$, vector $(\vec{v} = -1\hat{i} - 2\hat{j} + 2\hat{k})$

∇f (gradient) $= f_x\hat{i} + f_y\hat{j} + f_z\hat{k} = \left(\dfrac{1}{2\sqrt{xyz}} \times yz\right)\hat{i} + \left(\dfrac{1}{2\sqrt{xyz}} \times xz\right)\hat{j} + \left(\dfrac{1}{2\sqrt{xyz}} \times xy\right)\hat{k}$

$f(3,2,6) = \left(\dfrac{1}{2\sqrt{3\times2\times6}} \times 2\times6\right)\hat{i} + \left(\dfrac{1}{2\sqrt{3\times2\times6}} \times 3\times6\right)\hat{j} + \left(\dfrac{1}{2\sqrt{3\times2\times6}} \times 3\times2\right)\hat{k} = \boxed{\nabla f(3,2,6) = 1\hat{i} + \dfrac{3}{2}\hat{j} + \dfrac{1}{2}\hat{k}}$

$\text{unit vector } (\vec{v}) = \dfrac{-1\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{(-1)^2+(-2)^2+(2)^2}} = \dfrac{-1\hat{i} - 2\hat{j} + 2\hat{k}}{\sqrt{1+4+4}} \rightarrow \boxed{\vec{v} = \dfrac{-1}{3}\hat{i} - \dfrac{2}{3}\hat{j} + \dfrac{2}{3}\hat{k}}$

Now dot Product of $\nabla f_{(3,2,6)} \cdot \vec{v} = \left(1\hat{i} + \dfrac{3}{2}\hat{j} + \dfrac{1}{2}\hat{k}\right) \cdot \left(\dfrac{-1}{3}\hat{i} - \dfrac{2}{3}\hat{j} + \dfrac{2}{3}\hat{k}\right)$

$= \dfrac{-1}{3} - 1 + \dfrac{1}{3} = \boxed{-1}$ Ans:-

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Problem 10:-

Q3) $f(x,y) = x - \frac{y^2}{x} + \sqrt{3}\sec^{-1}(2xy)$ at Point (1,1) Vector $\vec{v} = (12\hat{i} + 5\hat{j})$.

Ans:-

$$\nabla f_{(x,y)} = f_x\hat{i} + f_y\hat{j} = \left(1 + \frac{y^2}{x^2} + \sqrt{3}\left[\frac{1}{\cancel{2xy}\sqrt{(2xy)^2-1}} \times \cancel{2y}\right]\right)\hat{i} + \left(-\frac{2y}{x} + \sqrt{3}\left[\frac{1}{\cancel{2xy}\sqrt{(2xy)^2-1}} \times \cancel{2x}\right]\right)\hat{j}$$

$$\nabla f_{(1,1)} = \left(1 + \frac{y^2}{x^2} + \frac{\sqrt{3}}{x\sqrt{(2xy)^2-1}}\right)\hat{i} + \left(\frac{-2y}{x} + \frac{\sqrt{3}}{y\sqrt{(2xy)^2-1}}\right)\hat{j} = \boxed{3\hat{i} - 1\hat{j}}$$

$$\text{unit vector } (\vec{v}) = \frac{12\hat{i} + 5\hat{j}}{\sqrt{12^2+5^2}} = \boxed{\frac{12}{13}\hat{i} + \frac{5}{13}\hat{j}}$$

$$\text{Dot Product} = \nabla f_{(x,y)} \cdot (\vec{v})$$

$$\text{Now, Dot Product} = (3\hat{i} - 1\hat{j}) \cdot \left(\frac{12}{13}\hat{i} + \frac{5}{13}\hat{j}\right) = \frac{36}{13} - \frac{5}{13} = \boxed{\frac{31}{13}} \text{ Ans:-}$$