National University of Computer & Emerging Sciences

Multivariable Calculus (MT-2008)
Date: March 1st, 2024
Time: 8:30 am - 9:30 am
Course Instructor(s)

Dr. Fahad Riaz, Dr. Nazish Kanwal
Ms. Fareeha Sultan, MS. Alishba, & Ms. Uzma.

Sessional-I Exam
Total Time: 1 Hour
Total Marks: 30
Total Questions: 04
Semester: SP-2024
Campus: Karachi
Dept: CS, CY, SE & AI.

Student Name ________ **Roll No** ________ **Section** ________ **Student Signature** ________

Instructions:

1. Answer all questions on answer script . Credit will be awarded for correct content and clarity of presentation.
2. All the answers must be solved according to the sequence given in the question paper.
3. There are **4 questions and 2 pages**.

CLO#2: Apply the theory to calculate the gradients, directional derivatives, arc length of curves, area of surfaces, and volume of solids. [**6 marks**]

Question 1

(a) $\boxed{\text{3 points}}$ Find and sketch the domain of $f(x,y) = \sqrt{4-x^2-y^2} + \sqrt{1-x^2}$.

(b) $\boxed{\text{3 points}}$ Identify and sketch the level curves for specified values of k.

$$z = \sqrt{36 - 9x^2 - 4y^2}, \quad k = -1, 0, 6.$$

CLO#2: Apply the theory to calculate the gradients, directional derivatives, arc length of curves, area of surfaces, and volume of solids. [**5 marks**]

Question 2

(a) $\boxed{\text{2 points}}$ Determine whether the limit $\mathbf{\lim_{(x,y)\to(0,0)}} \frac{\mathbf{xy-y^2}}{\mathbf{y^2+x}}$ exists. If it does find the limit, if not, explain why not?

(b) $\boxed{\text{3 points}}$ Determine whether the function $f(x,y)$ **is continuous at** $(0,0)$. Justify your answer.

$$f(x,y) = \begin{cases} \frac{2xy}{x^2+y^2} & \text{if } (x,y) \neq (0,0) \\ 0 & \text{if } (x,y) = (0,0) \end{cases}$$

CLO#2: Apply the theory to calculate the gradients, directional derivatives, arc length of curves, area of surfaces, and volume of solids. [**10 marks**]

Question 3

(a) $\boxed{\text{3 points}}$ Let $z = \ln(e^x + e^y)$ be the function whose all second order partial derivatives are exist. Show that

$$\frac{\partial^2 z}{\partial x^2} + \frac{\partial^2 z}{\partial x \partial y} = 0.$$

(b) $\boxed{\text{3 points}}$ Let $f_x(3,2) = 12.2$, $f_x(3,2.2) = 16.8$ & $f_x(3,1.8) = 7.5$. Estimate the value of $f_{xy}(3,2)$.

(c) $\boxed{\text{4 points}}$ The length a of a side of a triangle is increasing at a rate of 3 $inch/sec$, the length b of another side is decreasing at a rate of 2 $inch/sec$, and the contained angle θ is increasing at a rate of 0.05 $radian/sec$. How fast is the area $A = \frac{1}{2}ab\sin\theta$ of the triangle changing with time when $a = 40$ $inch$, $y = 50$ $inch$, and $\theta = \pi/6$.

CLO#2: Apply the theory to calculate the gradients, directional derivatives, arc length of curves, area of surfaces, and volume of solids. [**9 marks**]

Question 4

(a) $\boxed{\text{5 points}}$ Find the local linear approximation of the function $f(x, y, z) = \sqrt{x^2 + y^2 + z^2}$ at the point $P(3, 2, 6)$ and use it to approximate at point $Q(3.26, 1.97, 5.99)$. Compare your result with the actual value of the distance between the given points.

(b) $\boxed{\text{3 points}}$ Use implicit differentiation to find $\frac{\partial \mathbf{z}}{\partial \mathbf{x}}$ at the given point (where $f(x, y, z) = 0$ defines z implicitly).

$$xe^y + ye^z + 2\ln x - 2 - 3\ln 2 = 0; \quad (1, \ln 2, \ln 3).$$

(c) $\boxed{\text{1 point}}$ Determine whether the statement is true or false. Explain your answer.
If the graph of $z = f(x, y)$ is a plane in 3-space, then both f_x and f_y are constant functions.