13.1 :- Bazil-ud-Din Ieran Date

## Intro to multivariable Functions :-

- To graph a function you must have 1 Dimension more than the # of independent variables.

$f(x) = x+1 \rightarrow y = x+1$ (2-D)

$g(x,y) = x^2+y^2 \rightarrow z = x^2+y^2$ (3-D)

$h(x,y,z) = \dfrac{x^2+y^2}{z-3} = w = \dfrac{x^2+y^2}{z-3}$ (4-D)

Q1) 1.84

Q2) 0.14

Q3). $\sqrt{5}$

## To Graph of Domain of Function :-

- Must have same Dimension as # of Independent variables.

Q). $f(x)$, D: 1-D

$g(x,y)$, D: 2-D

$h(x,y,z)$, D: 3-D

Q). $f(x,y,z) = \sqrt{x^2+2y^2+3z^2}$ 4-D Graph, 3D-Domain

Q). $f(x,y) = \dfrac{xy}{x-y}$ $D\{(x,y) \mid x \neq y\}$ Range → The output "z"

$R\{z \mid -\infty < z < \infty\}$

Q). $g(x,y) = \sqrt{4-x^2-y^2} \rightarrow 4-x^2-y^2 \geq 0$

$D\{(x,y) \mid x^2+y^2 \leq 4\}$ $4 \leq x^2+y^2$

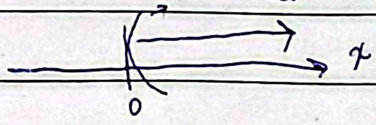
Note :- " $y > x^2$, $y = -1$ here $x^2$ parabola will be doted"

$R\{z \mid 0 \leq z \leq 2\}$

## GRAPhing :- Domain :-

not included so or could use O → circle on 0.

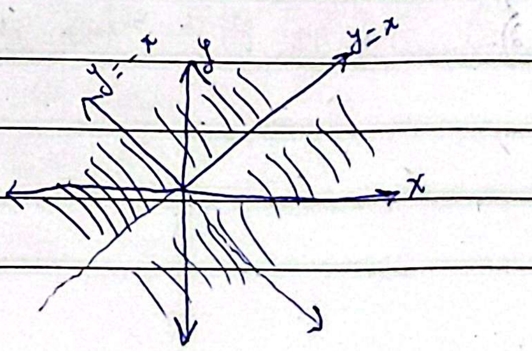
Q). $f(x) = \dfrac{1}{\sqrt{x}}$ D :-



Q). $f(x,y) = \dfrac{xy}{x^2-y^2}$ D :- $\{x,y \mid y \neq x, y \neq -x\}$

$x^2 \neq y^2$

$x = \pm y$



$y-x$

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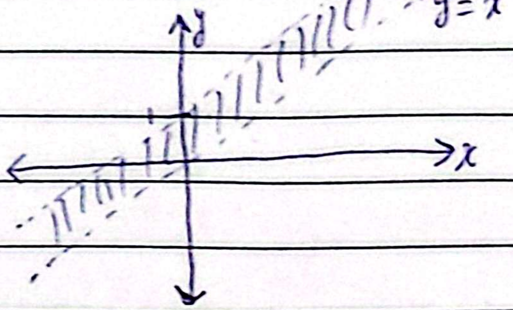
Q). $f(x,y) = \dfrac{\ln(y-x)}{\sqrt{x-y+1}}$




**Ans:-** $y - x > 0$ , $x - y + 1 \geq 0$

$y > x$ $\quad$ $\cancel{x-y}$ $1 \geq y - x$

$y < x+1$

↳ shade below that.

$D\{(x,y) \mid y > x \text{ and } y < x+1\}$

Q). $f(x,y,z) = \sqrt{9-x^2-y^2-z^2}$ $\quad$ 4-D GRAPH

**Ans:-** $9-x^2-y^2-z^2 \geq 0$ $\quad$ 3-D Domain

$9-(x^2+y^2+z^2) \geq 0$ $\quad$ "Inside of a sphere Y Radius 3 centre ORIGIN"

$D\{(x,y,z) \mid 9 \leq (x^2+y^2+z^2)\}$

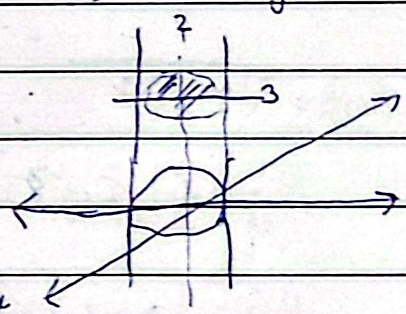
Q) $f(x,y,z) = \dfrac{\sqrt{4-x^2-y^2}}{z-3}$

$D\{(x,y,z) \mid x^2+y^2 \leq 4,\ z \neq 3\}$

**Ans:-** $z-3 \neq 0$ $\quad$ $4-x^2-y^2 \geq 0$ $\quad$ ↳ it's a cylinder

$\boxed{z \neq 3}$ $\quad$ $4-(x^2+y^2) \geq 0$

$4 \leq (x^2+y^2) \rightarrow \boxed{(x^2+y^2) \leq 4}$




How To GRAPH a function:

1). set $f(x,y) = z$

2). Try to get a surface you know.

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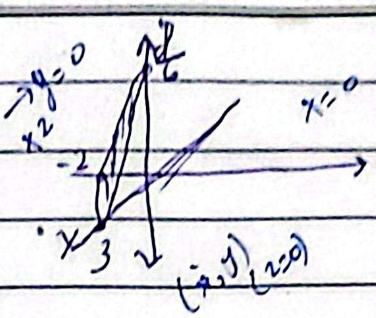
**Ex:-** $f(x,y) = 6 - 2x + 3y.$




$xz, yz, xy.$

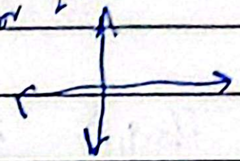
**Ans:-** $z = 6 - 2x + 3y$

$2x - 3y + z = 6$

$2x = 6$ $\quad$ $-3y = 6$ $\quad$ $z = 6$

$x = 3$ $\quad$ $y = -2$

for 2 variable



**Ex:-** $f(x,y) = 9 - x^2 - y^2$ ( $\because$ if $+x, y$ coefficients are -ve so downward else if positive so upward)

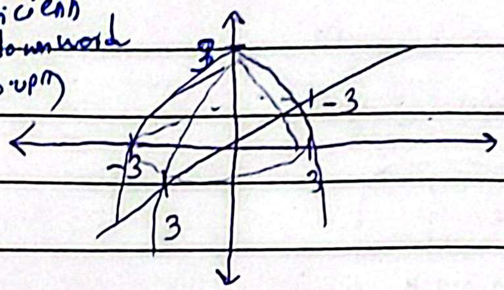
$z = 9 - x^2 - y^2$

$z = 9 - x^2 - y^2$

$x^2 + y^2 + z = 9$

Parabola Alone 'z'

Opening Toward $\rightarrow -z$




for 2D on y-axis

$x = 0, z = 0$

if $z = 3$ so $(z, (x,y))$ → z parallel to x-y plane

**Ex:-** $g(x,y) = \frac{1}{2}\sqrt{36 - 9x^2 - 36y^2}$ $\rightarrow$ $36 - 9x^2 - 36y^2 \geq 0$

$9(4 - x^2 - 9y^2 \geq 0$

Take square on both side

$36 \leq 9x^2 + 36y^2 \rightarrow 9x^2 + 36y^2 \leq 36$

$x^2 + 4y^2 \leq 4$

$z = \frac{1}{2}\sqrt{36 - 9x^2 - 36y^2}$

in yz

$4z^2 = 36 - 9x^2 - 36y^2$

$9x^2 + 36y^2 + 4z^2 = 36 \rightarrow \frac{x^2}{4} + \frac{y^2}{1} + \frac{z^2}{9} = 1$ $\rightarrow$ (Ellipsoid)

contour maps:- ↳ A set of all level cu[rves]

**Level Curves:-** Curve when a plane intersects one surface at different levels along the axes of different

Set $f = k$

· for 3 dimensions $\rightarrow w = f(x,y,z)$

" if for hyperbola $k > 0$ (left, right) if $k < 0$ (up down)

Q). $f(x,y) = \sqrt{36 - x^2 - y^2}$

$k = \sqrt{36 - x^2 - y^2}$ $\rightarrow$ $x^2 + y^2 = 36 - k^2$ $\rightarrow$ substitute some k values

So circles many · Radius of 4 getting smaller until $k = 4$

Q). $f(x,y) = y^2 - x^2$

$k = y^2 - x^2 \rightarrow \frac{y^2}{k} - \frac{x^2}{k} = 1$ $\rightarrow$ Hyperbola's "x" or "y" depending on "k"

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Ex:- $f(x,y) = \ln(x+y)$ Bazil-ud-din-khan

$k = \ln(x+y)$

$e^k = x+y$

$y = -x + e^k$ → line slope $(-1)$ $y$ In → $e^k$.

$4x^2+y^2 = 3$

$\frac{x^2}{3/4} + \frac{y^2}{3} = 1 \rightarrow \left(\pm\sqrt{\frac{3}{4}}, \pm\sqrt{3}\right)$

$\frac{x^2}{1} + \frac{y^2}{1/4} = 1$

For $k<0$ NO level

Ex:- $f(x,y,z) = 2x+4y-3z+1$

$k = 2x+4y-3z+1$

$k-1 = 2x+4y-3z$ Planes $n/e = \langle 2,4,-3\rangle$

if $k = x^2+y^2-z^2$

if $k>0$ → one sheet Hyperbola

$k=0$ → cone

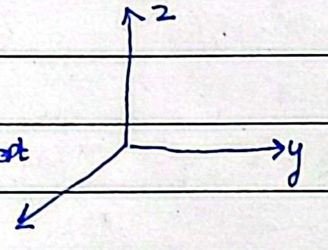
$k<0$ → 2 sheet Hyperbola

$\boxed{\text{if written as level curves of } z = x^2+y^2 : k = 0,1,2,3,4 \text{ Take k values and put in z.}}$

Sketching 3D GRAPHS:-

(1). Plane:- $x + y + z = 1$

$a$ ↓ x intercept, $b$ ↓ y-intercept, $c$ → z intercept



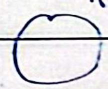
$z - x^2 - y^2 + 4 = k$   $k = z - x^2 - y^2 + 4$

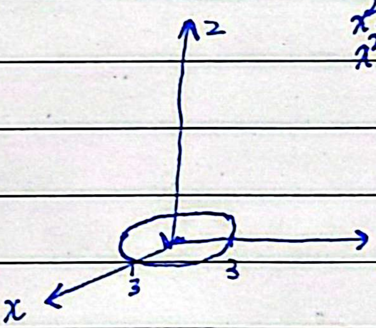
$4 - k = x^2+y^2-z$   $x^2+y^2-z = 4-k$

$x^2+y^2-z = -3$

$x^2+y^2 = -3+z$

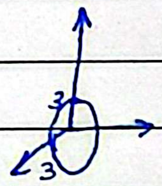
if $z=4$





(2).(i) cylinder:- $x^2+y^2 = a^2$ ↓ Circle   $x^2+y^2+z^2 = a^2$ so

if $x^2+z^2 = a^2$



(ii) Sphere:- $x^2+y^2+z^2 = a^2$

when $z=0$ → $x^2+y^2 = 3^2$

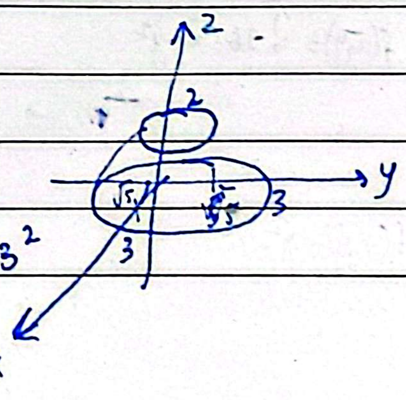
upper hemisphere → $z = \sqrt{3^2-(x^2+y^2)}$

$z^2 = -(x^2+y^2)+3^2 \rightarrow z^2+y^2+x^2 = 3^2$

$z=0, \rightarrow x^2+y^2 = 3^2$

$z = +2$

$x^2+y^2 = (\sqrt{5})^2$



# PRACTICE:- Q's:-

$4 \le (x^2+y^2)$

$(x^2+y^2) \geq 4$

Ex:- 13.1

Q1). (a). $f(2,1) \rightarrow (2)^2(1)+1 = 5$

(b) $f(1,2) \rightarrow 3$

(c). $f(0,0) \rightarrow 1$

(d). $f(1,-3) \rightarrow -2$

(e). $f(3a,a) \rightarrow 9a^3+1$

(f) $f(ab,a-b) \rightarrow (a^2b^2)(a-b)+1 \longrightarrow \boxed{a^3b^2 - a^2b^3+1}$

Q2). (a). $f(x,y) = x + \sqrt[3]{xy}$

$t+t \rightarrow 2t$

(b). $2x$

(c). $2y^2 + (8y^3)^{1/3} \rightarrow \boxed{2y^2+8y}$

Q3) (a). $(x+y)(x-y)+3$

$\boxed{x^2-y^2+3}$

(b). $(xy)(3x^2y^3)+3$

$\boxed{3x^3y^4+3}$

Q4). (a). $\frac{x}{y} = \frac{x}{y}\sin\left(\frac{x}{y}\right)$ $g(x/y)$

(b). $g(xy) \rightarrow (xy)\sin(xy)$

(c). $g(x-y) \rightarrow (x-y)\sin(x-y)$

Q5). $f(g(x), h(y)) = xe^{xy}$

$xe^{xy} = (x^3)e^{x^3(3y+1)}$

$xy = x^2(x^3(3y+1))$

$y = x^4 3y + x^4$

$\rightarrow \boxed{x^3 e^{x^3(3y+1)}}$

Q6). $y\sin(x^2y) =$

$\pi xy(\sin(x^2y^3)^2 x\pi xy)$

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Q7) $f(x,y) = x + 3x^2y^2$ , $x(t) = t^2$, $y(t) = t^3$

(a).

$f(x(t),y(t)) \to t + 3t^2(t)^2 \to \boxed{t+3t^4}$

(b). 0

(c). $f(x(2), y(2))$

$2^2+3[(2)^2]^2[(2)^2]^3 \to \cancel{1024}$ $24580$

Q8). $x(t) = \ln(t^2+2)$ , $y(t) = t^{3/2}$

$\boxed{(t)^{3/2}(e^{-3x\ln(t^2+1)})}$

17-20:-

17).(a). 19

(b) -9

(c) 3

(d) $a^6+3$

(e) $t(t^4)(-t^3)+3 \to -t^8+3$

(f) $(a+b)(a-b)^2(b)^3+3$

$(a+b)(a^2-2ab+b^2)(b)^3+3$

18). (a). $(x^2)(x+y)(x-y)+x+y$

$(x^3+yx^2)(x-y)+x+y$

$x^4-\cancel{x^3y}+\cancel{yx^3}-y^2x^2+x+y$

$\boxed{x^4-y^2x^2+x+y}$

(b). $(x2)(xy)\left(\frac{y}{x}\right)+x$

$x\,2y^2+x \to \boxed{x(2y^2+1)}$

Q19). $(y+1)\,e^{(x^2)(y+1)(z^2)}$

$\boxed{(y+1)\,e^{x^2z^2y+x^2z^2}}$

20).

$\left(\frac{xy}{2}\right)\sin\left(x^2z^3x\pi xy2\right)$

$\boxed{\left(\frac{xy}{2}\right)\sin\left(x^3z^3x\pi y\right)}$

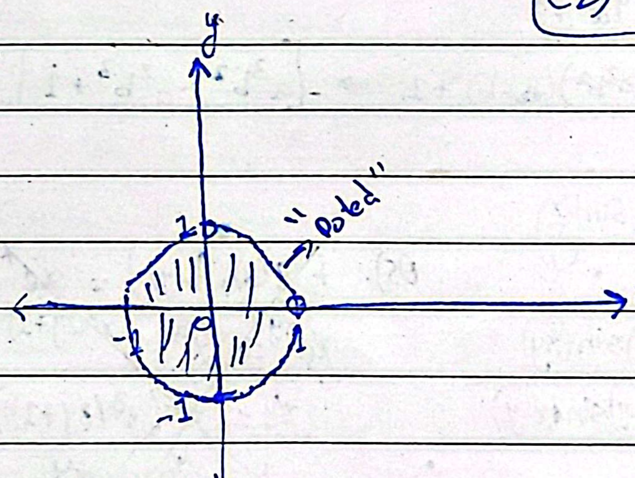
23-28:- (43-44) / (53-64)

Q23). $\ln(1-y^2-x^2)$

$\hookrightarrow 1-y^2-x^2>0$

$1-(y^2+x^2)>0$

$y^2+x^2<1$



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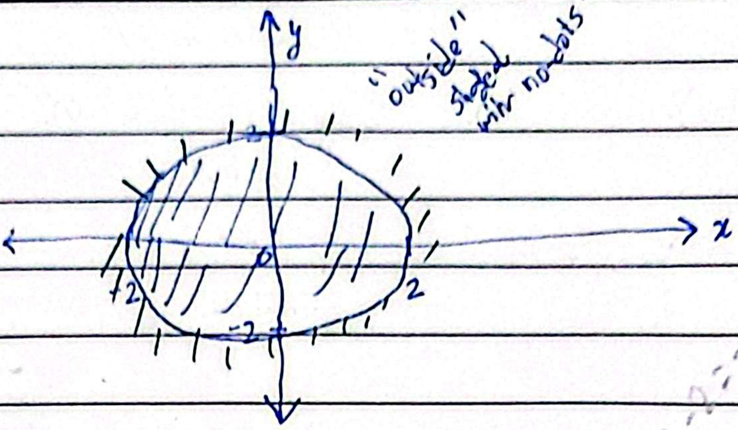
24). $x^2+y^2-4 \geq 0$

$x^2+y^2 \geq 4$

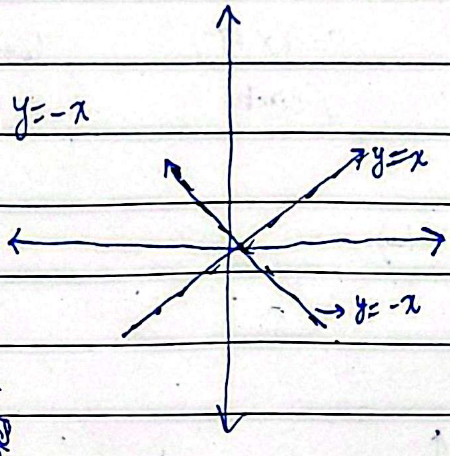
$x^2+y^2 \geq 2^2$




"outside" shaded with no dots"

25). $\dfrac{1}{x-y^2}$ $\quad x \neq y^2$

$y = x, \ y = -x$

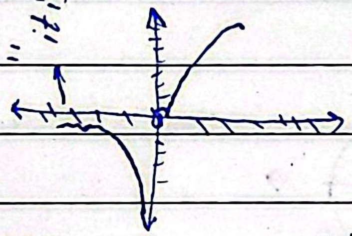



26). $\ln xy$

$xy > 0$ $\quad x<0, y<0$

$x>0, y>0$

" it is dotted "



$x^2+y^2=2^2$

$z = \sqrt{x^2+y^2}$

$-2 = x^2+y$

$x^2+y=-2$

$x^2=-2-y$

27). (a). $xe^{-\sqrt{y+2}} \rightarrow y+2 \geq 0$

y values need to be $\rightarrow$ $y \geq -2$ $\rightarrow$ Domain

(b). $25-x^2-y^2-z^2 \geq 0$

$25-(x^2+y^2+z^2) \geq 0$

$25 \geq (x^2+y^2+z^2) \rightarrow$ Max 25.

"circle has $x^2+y^2=r^2$" if we add one $z=r^2$ with $x^2+y^2+z^2=r^2$ then it is a sphere"

(c). All real No. with Points at radius 5, on sphere

28) (a). $4-x^2 \geq 0$

$-x^2+4 \geq 0$

$(-x+2)(+x+2) \geq 0$

$-x+2 \geq 0 \quad x+2 \geq 0$

$x \leq 2 \quad x \geq -2 \rightarrow \boxed{-2 \leq x \leq 2}$

$x^2+y^2=z-3$

$z > 3$

$x^2+y^2=4$

(b). $y-2x>0$

$y>2x \rightarrow \dfrac{y}{x} > 2$ or $y>2x$ Not 0.

All Real No. where x is always have value that will Result in all there where y is greater than 2x.

y>2x

(c) $\dfrac{xyz}{x+y+z}$ $\quad x+y+z \neq 0$

$x+y \neq -z$

↳ Denominator should not be 0

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43-44:- Bozil Uddin Lehon

(a). $k = 5x^2 - 5y^2$ (line).

$k=0 \to 0 = 5x^2 - 5y^2$

$5x^2 - 5y^2 = k$ $x^2 = y^2 \to y = x, y = -x$

hyperbola

(b). $k = y - 4x^2$

$k=0 \to 0 = y - 4x^2$

$y = 4x^2$

(Parabola)

(c). $k = x^2 + 3y^2 \to$ (Ellipse)

(d). $k = 3x^2$ (line)

44) (a). $k = x^2 - 2xy + y^2$

$k=0 \to 0 = x^2 - 2xy + y^2$

(lines) $0 = (x-y)^2$

(b) $k = 2x^2 + 2y^2$

$0 = 2(x^2+y^2)$

(circle)

(c). $k = x^2 - 2x - y^2$

$k + 2x = x^2 - y^2$ (hyperbola)

as $x^2 \neq y^2$

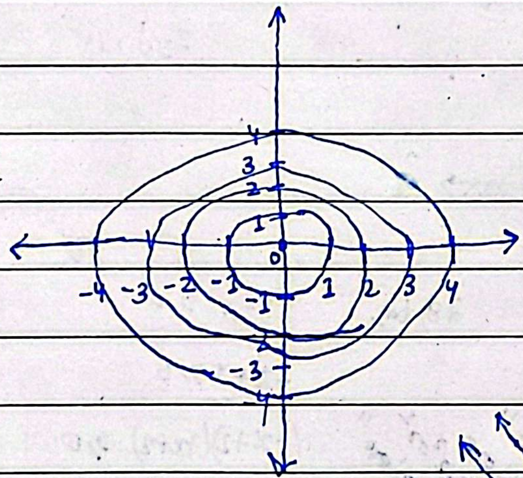
(d). $k=0 \to$ $x = 2y^2$ (Parabola).

(51-56)

51). $z = x^2 + y^2$

$k = x^2 + y^2$

it is having radius $\sqrt{k}$



52). $k = y/x$

$y = -2x$

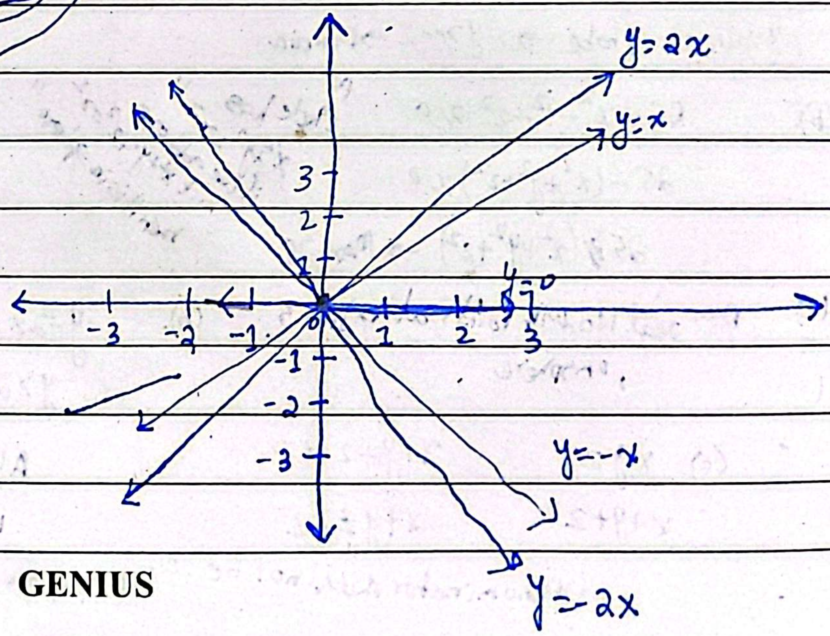
$y = -x$

$y = 0$

$y = x$

$y = 2x$

Not at origin



53). $z = x^2 + y$ (Parabola).

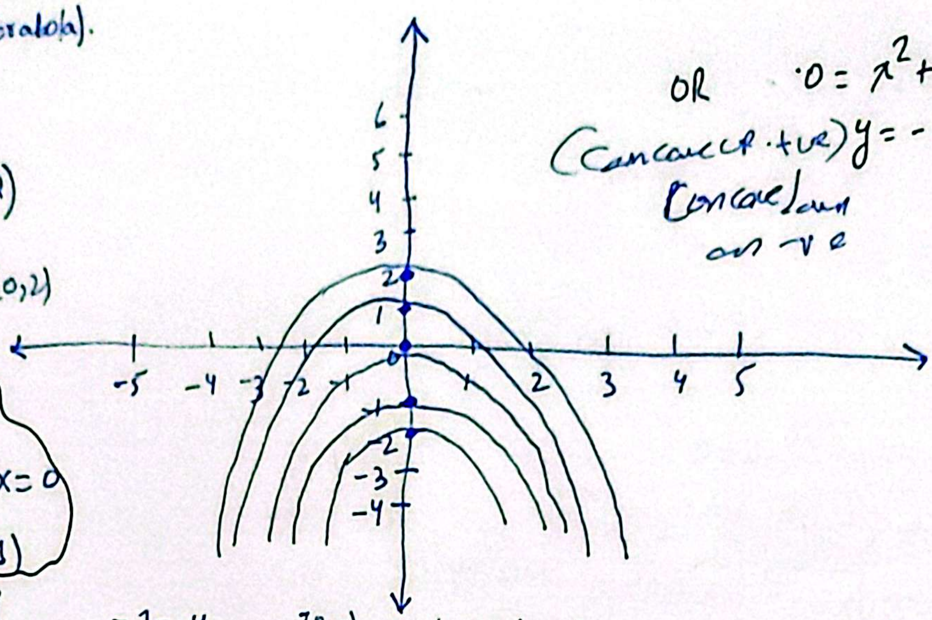
Ans:-

$-2 = x^2 + y$

$x^2 = -(y+2)$

$-y = -2$

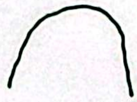
$y = 2$ (0,2)




OR $0 = x^2 + y$

(Concave (-ve)) $y = -x^2$

Concave down on -ve



$y = -x^2$

$\downarrow$

$z_0$

at $y = -3$

if $y = -x^2 - 1$

$\downarrow$

one down

if $y = -x^2 + 1$

$\downarrow$

one up

Note:- (from GTP)

Parabola everytime put $x = 0$

(0, -1)

$2 = x^2 + y$

$-2 = y$ (0,-2)

$-1 = y$

(0,-1)

(0,0)

(0, 1) (0, 2)

54). $z = x^2 + 9y^2 \rightarrow 2 - x^2 = 9y^2 \rightarrow \boxed{y^2 = \frac{2 - x^2}{9}} \rightarrow \boxed{9y^2 = 2 - \cancel{x}^2}$

$0 = x^2 + 9y^2$

$\hookrightarrow$ at $x = 0$

$1 = x^2 + 9y^2 \rightarrow 1 = \frac{x^2}{(1)^2} + \frac{y^2}{(1/3)^2}$

$a^2$ $b^2$

$a^2 = 1$

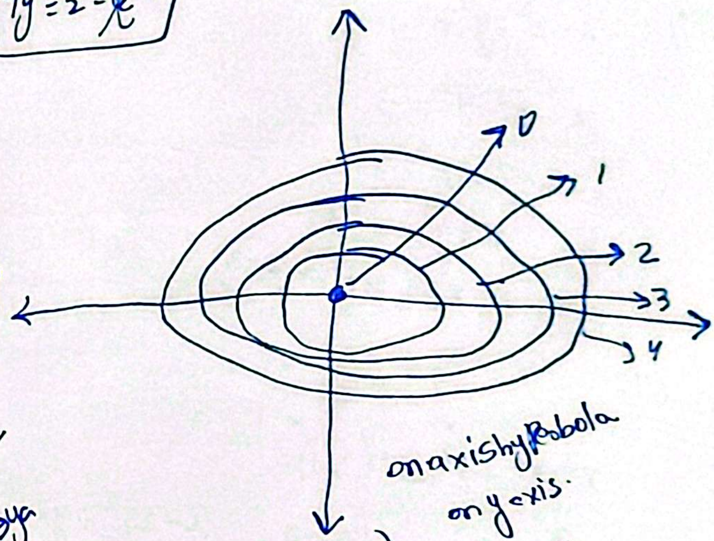
$a = 1, a = -1$

$b^2 = \sqrt{1/3}$

$b = \sqrt{1/3}$

$a \rightarrow$ in $x$

$b \rightarrow$ in $y$




on axis hyperbola on y-axis.

$2 = x^2 + 9y^2$

$\frac{x^2}{(\sqrt{2})^2} + \frac{y^2}{\left(\frac{\sqrt{2}}{3}\right)^2}$

$\downarrow$ $\downarrow$

a b

$\rightarrow y_0$

(56) $y = \sin x$

$y = 2\sin x$

55). $z = x^2 - y^2$, $k = -2, -1, 0, 1, 2$

$0 = x^2 - y^2$

$y = x, y = -x$

$x^2 - y^2 = 0$

point

$y =$

$x^2 - y^2 = 1$

$\frac{x^2}{1} - \frac{y^2}{1}$

$z = k \rightarrow x^2 - y^2 = -2$

$\frac{x^2}{-2} + \frac{y^2}{2} \rightarrow \frac{y^2}{2} - \frac{x^2}{2} = 1$

$\hookrightarrow$ on y axis so

if $k > 0 \rightarrow$ left-right

if $k < 0 \rightarrow$ up-down.

57-60:

57). $f(x,y,z) = 4x^2 + y^2 + 4z^2$; $k = 16$

$4x^2 = 16$ $\quad$ $y^2 = 16$ $\quad$ $4z^2 = 16$

$x^2 = 4$ $\quad$ $y = \pm 4$ $\quad$ $z^2 = 4$

$x = \pm 2$ $\quad$ $(0, \pm 4, 0)$ $\quad$ $(z = \pm 2)$

$(\pm 2, 0, 0)$ $\quad$ $(0, 0, \pm 2)$

58). $f(x,y,z) = x^2 + y^2 - z^2$ $\quad k = 0$

$z^2 = x^2 + y^2$ $\boxed{\text{cone}}$ $\quad + \sqrt{x^2+y^2}$

$z = \pm\sqrt{x^2+y^2}$

just make cone

59). $z - x^2 - y^2 + 4 = 7$

$z - x^2 - y^2 = 3$

$z - 3 = x^2 + y^2$

$x^2 + y^2 = z - 3$

$x^2 = 0, y = 0^2 \rightarrow z = 3$

60). $4x - 2y + z = 1$

$4x - 2y = 1 - z$

if $x = 0, y = 0$

$z = 1$

$(0, 0, 1)$

$x = 0, y = \quad , z = 0$

$0 - 2y + 0 = 1$

$y = -1/2$ $\quad (0, -1/2, 0)$

$x = \quad , y = 0, z = 0$

$4x = 1$

$x = (1/4, 0, 0)$

Just draw on graph.

61-64:

(61) $x^2 + y^2 + z^2 = f(x, y, z)$

$\downarrow$

$(0, 0, 0)$ $\quad$ for $\rightarrow$ $\quad (x-2)^2 + y^2 + z^2$ $\quad$ $x - 2 = 0$

$x = 2$

$(2, 0, 0)$

(62). $(3i - j + 2k) \rightarrow$ parallel planes.