Multivariable Calculus (MT1008)

Solution Sessional-II Exam

Time Allowed: 1 Hour Max. Marks: 30

Total Questions: 03

Date: April 10, 2025

Time: 11:00 am - 12:00 pm

Rubrics:

- Full marks require correct steps and calculations.
- Partial credit for incorrect computations but correct setup.
- Clear notation and justification are encouraged for full marks. Deduct one mark for wrong notation in a question.

CLO #1: Understand the basic concepts and know the basic techniques of differential & integral calculus of functions of several variables.

Q.1 [13 Marks]

a) Evaluate the double integral over the rectangular region R by first identifying it as a volume of a solid. [4 Marks]

$$\iint_{R} xy e^{y^{2}x} dA; R = \{(x,y) | 1 \le x \le 4, 0 \le y \le 1\}$$

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$$\lim_{R \to \infty} xy e^{y^{2}x} dA = -(61 \text{ Mark})$$

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b) Find an equation of tangent plane and a vector equation for the normal line to the surface at the given point P. [5 Marks]

$$z = 3x^2y^{-1} + x^{-1}y^2$$
; $P(1, -1 - 2)$.

Solution:

Find partial derivatives:
$$f_x=6xy^{-1}-x^{-2}y^2$$
 $f_y=-3x^2y^{-2}+2x^{-1}y$ At $(1,-1)$: $f_x(1,-1)=-6-1=-7$, $f_y(1,-1)=-3-2=-5$ (2 Marks)

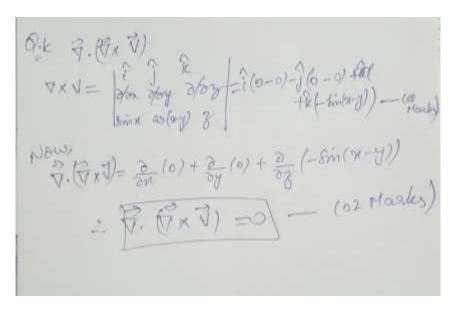
Equation of tangent plane:
$$z + 2 = -7(x - 1) - 5(y + 1)$$
 $z = -7x - 5y + 10 - 2$ $z = -7x - 5y + 8$ (2 Marks)

Equation of normal line:
$$x=1+7t, \quad y=-1+5t, \quad z=-2+t$$

c) Given a vector field. Find
$$div(curl V)$$
.

[4 Marks]

$$\mathbf{V}(x, y, z) = \sin x \ \mathbf{i} + \cos(x - y)\mathbf{j} + z\mathbf{k}$$



CLO #3: Solve problems involving maxima and minima, line integral and surface integral, and vector calculus.

Q.2

[11 Marks]

a) Given a function
$$f(x, y, z) = 3x^2 + 6xz + y^2 - 4yz + 8z^2$$
. Then,

[3+3=6 Marks]

- i) Compute the Hessian matrix of the function given.
- ii) Is *f* convex or concave?

Solution:

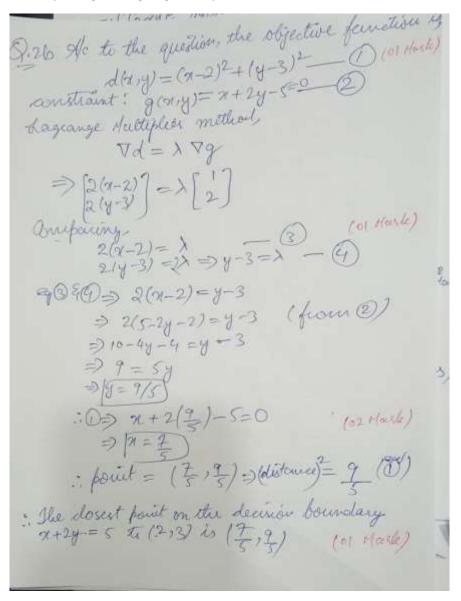
(02 Marks)

Hessian Matrix:
$$H=egin{bmatrix} 6&0&6\0&2&-4\6&-4&16 \end{bmatrix}$$

Step 1: Compute Leading Principal Minors $D_1 = 6$, $D_2 = \begin{vmatrix} 6 & 0 \\ 0 & 2 \end{vmatrix} = 12$, $D_3 = \begin{vmatrix} 6 & 0 & 6 \\ 0 & 2 & -4 \\ 6 & -4 & 16 \end{vmatrix} = 56$ (03 Marks)

Since all minors are positive, it is convex.

b) In a data clustering problem, we want to find the closest point on a given decision boundary to a reference point. Suppose a data point (2,3) represents a new observation, and we need to find the closest point (x,y) on the decision boundary, x+2y=5. Find the point (x,y) on the decision boundary that minimizes the distance between the observation (2,3) and any point (x,y) on the decision boundary using the Lagrange multiplier method. [5 Marks]



CLO #4: Apply gradient and derivative for solving various problems arising in sciences.

Q.3 [6 Marks]

A neural network model has two hyperparameters, *learning rate* (x) and *regularization parameter* (y), which affect the model's performance. The loss function is given by:

$$L(x, y) = (x - 4)^2 + 2(y + 3)^2$$

Perform two iterations of gradient-descent algorithm with learning rate, $\alpha=0.1$ and starting point (8,-1). Solution:

Gradient:

$$\nabla L = \begin{bmatrix} 2(x-4) \\ 4(y+3) \end{bmatrix} \underline{\text{At (8, -1): }} \nabla L = \begin{bmatrix} 8 \\ 8 \end{bmatrix}$$
 (2 Marks)

First Iteration:
$$(x, y) = (8, -1) - 0.1 \cdot (8, 8) = (7.2, -1.8)$$
 (2 Marks)

Second Iteration:
$$\nabla L(7.2, -1.8) = \begin{bmatrix} 6.4 \\ 4.8 \end{bmatrix} (x, y) = (7.2, -1.8) - 0.1 \cdot (6.4, 4.8) = (6.56, -2.28)$$
 (2 Marks)

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