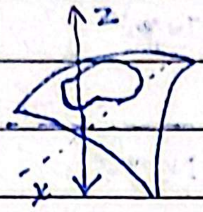
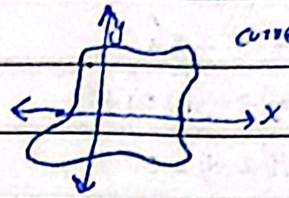
Bezil-ullin-Lehan Date

13.7: TANGENT PLANES AND NORMAL LINES:-

$f(x,y) = z$ and it will have a level curve $f(x,y) = c$.



consider surface $f(x,y)=z$ and will have a level curve of $f(x,y)=c$



curve (2-D).
(1-Dimensionless).

let the curve be represented by a vector function:- $F(t) = \underbrace{x(t)\,\hat{i} + y(t)\,\hat{j}}$ (level curve at c).

so function for level curve is

$$f(x(t), y(t)) = c$$

"it's a vector function so that we can create a level curve"

Take Derivative with Respect to "t" (independent variable).

$$f'(x(t), y(t)) = 0$$

$$= \frac{df}{dx} \times \frac{dx}{dt} + \frac{df}{dy} \cdot \frac{dy}{dt}$$

"it is dot Product" → $\left(\frac{df}{dx}\hat{i} + \frac{df}{dy}\hat{j}\right) \cdot \left(\frac{dx}{dt}\hat{i} + \frac{dy}{dt}\hat{j}\right) = 0$

$$(f_x\hat{i} + f_y\hat{j})(dx/dt\,\hat{i} + dy/dt\,\hat{j}) = 0$$

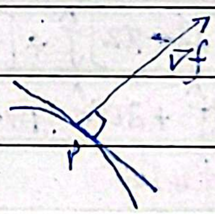
$$\boxed{\nabla f(x,y) \cdot \vec{r}\,(t) = 0}$$

What this means:-

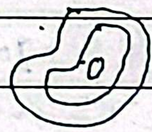
→ $\vec{r}(t)$ Gives a Tangent vector of level curve.

→ when a Dot Product $= 0$ The two vectors are ortagonal.

so ∇f Gives us the slope of tangent Normal to a level curve at a Point.



→ The Fastest way up a Hill is Along the Path ⊥ To a level curve ∇f.



$\nabla f(x,y)$ is the Normal to a level curve $f(x,y) = c$

$\nabla f(x,y,z)$ Is the Normal To A level Surface $f(x,y,z) = c$

Date

Goal:- Tangent planes and Normal lines both needed:- (i) Normal vector, (ii) ∇f.

Q) For the tangent line and Normal line to $x^2-y^2=16$ @ $P(5,3)$.
2-D

:- create a function that is 1 dimension higher than this than find gradient.
and that higher function normal line will tell function of level curve.

So $f(x,y)=x^2-y^2$ [DONT right constant, only variables].

So 3-D

$x^2-y^2=16$ is a level curve
To $f(x,y)$, means $\nabla f(x,y)$.
Gives Normal to $x^2-y^2=16$

2D

$\nabla f(x,y) = 2x\hat{i}-2y\hat{J}$ ← Normal vector any level curve $x^2-y^2=c$.

$(5,3) = \boxed{10\hat{i}-6\hat{J}}$ The $P(5,3)$ Gives a specific Normal To a specific curve $(x^2-y^2=16)$.

$m_n = \frac{-6}{10} = \frac{-3}{5}$

$y-(3)=\left(\frac{-3}{5}\right)(x-5)$ → Normal line eq on curve

$\boxed{y=\frac{-3}{5}x+8}$

$m_T = 5/3$ $y-3=\frac{5}{3}(x-3)$ ⟹ $\boxed{y=\frac{5}{3}x-\frac{16}{3}}$ → Tangent.

Q). Find Normal vector To $-x^2+y^2-z^2=4$ @ $P(1,3,2)$.

$F(x,y,z) = -x^2+y^2-z^2$ → 4-D or $f(x,y,z)=4$ → Represent level curves.

$\nabla f = (-2x)\hat{i}+2y\hat{J}-2z\hat{k}$

$(1,3,2)$ $\nabla f = -2\hat{i}+6\hat{J}-4\hat{k}$ [Normal vector to a family of Normal vector that's you created]
$f(x,y,z)$

The specific Normal to $-x^2+y^2-z^2=4$ @ $P(1,3,2)$.

Planes:- $\vec{A} = a\hat{i}+b\hat{J}+c\hat{k}$ @ $P(x_0,y_0,z_0)$.

$a(x-x_0)+b(y-y_0)+c(z-z_0)=0$

Normal lines:- $\vec{m} = a\hat{i}+b\hat{J}+c\hat{k}$, $P(x_0,y_0,z_0)$

$\frac{x-x_0}{a} = \frac{y-y_0}{b} = \frac{z-z_0}{c}$

$2x\hat{i} + 8y\hat{j} + 2z\hat{k}$ $2(x-1)+16(y-2)+2(z-2)$

$: 2\hat{i}+16\hat{j}+2\hat{k}$ $2x-2+16y-32+2z-4=0$

$x = x_0 + ab$ $y = y_0 + ab$ $z = z_0 + cb$ Date $2x+16y+2z=38$

$\boxed{x+8y+z=19}$

Q59). Q) $xyz=-4$ σ $P(2,-1,2)$

$f(x,y,z) = xyz$

$\nabla f(x,y,z) = (yz)\hat{i} + (xz)\hat{j} + (xy)\hat{k}$

$\boxed{\nabla f(2,-1,2) = -2\hat{i} + 4\hat{j} - 2\hat{k}}$ → For Tangent/Normal vector only ya hm

→ Divide by 2 short.

Planes:-

$-1(x-2) + 2(y+1) - 1(z-2) = 0$

$\boxed{x-2y+z=6}$ → Tangent plane of mat. xyz.

Normal line $= \frac{x-2}{-1} = \frac{(y+1)}{2} = \frac{z-2}{-1}$ → that can orthogonal.

Q). $xz^2 + yx^2 + y^2 - 2x + 3y + 6 = 0$ σ $P(-2,1,3)$

$f(x,y,z) = xz^2+yx^2+y^2-2x+3y$

$\nabla f(x,y,z) = [(z^2) + (y2x) - 2]\hat{i} + [x^2+2y+3]\hat{j} + [2zx]\hat{k}$

$f(x,y,z) =$

$[9 + (-4) - 2]\hat{i} + [4+2+3]\hat{j} + (-12)\hat{k}$

$\boxed{3\hat{i} + 9\hat{j} - 12\hat{k}}$ Plane:- $\vec{R} = \hat{i} + 3\hat{j} - 4\hat{k}$ Tangent plane

$\div 3$

$1(x+2) + 3(y-1) - 4(z-3) = 0$

$\boxed{x+3y-4z=-11}$

Normal line ← $\frac{x+2}{1} = \frac{y-1}{3} = \frac{z-3}{-4}$

if written like this then do this:

Q). $z = \tan^{-1}(y/x)$. σ $P(1,1,\pi/4)$.

~~z = tan~~ $\tan^{-1}(y/x) - z = 0$

$f(x,y,z) = \tan^{-1}(y/x) - z$

$\nabla f(x,y,z) = \left(\frac{1}{1+(y/x)^2} \times y\right)\hat{i} + \left(\frac{1}{1+(y/x)^2} \times \frac{1}{x}\right)\hat{j} - 1\hat{k}$

$\nabla f(x,y,z) = -\frac{i}{2} + \frac{j}{2} - k$

put values

GENIUS

$\ln(u) \to \frac{1}{u}\left[\frac{1}{\right.$

Ex 13.7:- Date 8 Feb 2025

Q3). $x^2+y^2+z^2=25 \quad P(-3,0,4)$.

$f(x,y,z) = x^2+y^2+z^2$

$\nabla f(x,y,z) = [2x]\hat{i} + [2y]\hat{j} + 2z[\hat{k}]$

$\nabla f(-3,0,4) = -6\hat{i} + 0\hat{j} + 8\hat{k}$

Plane Tangent:- Divide by 2 $\to -3\hat{i}+0\hat{j}+4\hat{k}$

$-3(x+3)+0(y+0)+4(z-4)=0$

$-3x-9+0+4z-16=0$

$\boxed{-4z+3x=-25}$

Normal = Parametric $\frac{x+3}{-3} = \frac{y}{0} = \frac{z-4}{4}$

Q4). $f(x,y,z) = x^2y-4z^2$

$\nabla f(x,y,z) = (2xy)\hat{i} + (x^2)\hat{j} - (8z)\hat{k}$

$\nabla f(-3,1,-2) = (-6)\hat{i} + 9\hat{j} + 16\hat{k}$

Plane Tangent $= -6(x+3)+9(y-1)+16(z+2)=0$

$-6x-18+9y-9+16z+32=0$

$\boxed{6x-9y-16z=5}$

Normal:- $\frac{x+3}{-6} = \frac{y-1}{9} = \frac{z+2}{16}$

Q5). $f(x,y,z) = x^2-xyz$

$\nabla f(x,y,z) = [2x-yz]\hat{i} + [-xz]\hat{j} + [-xy]\hat{k}$

$\nabla f(-4,5,2) = -18\hat{i} + 8\hat{j} + 20\hat{k}$ Divide by 2

$\to \quad = -9\hat{i} + 4\hat{j} + 10\hat{k}$

$-9(x+4)+4(y-5)+10(z-2)=0$

$\cancel{-9x-36+4x-20+10x-20=0}$

$5x$

Normal:- $\frac{x+4}{-9} = \frac{x-5}{4} = \frac{x-2}{10}$

Q6). $f(x,y,z) = x^2+y^2-z$

$\nabla f(x,y,z) = [2x]\hat{i}+[2y]\hat{j}-[1]\hat{k}$

$\nabla f(2,-3,13) = [4\hat{i}-6\hat{j}-1\hat{k}]$

$4(x-2)-6(y+3)-1(z-13)=0$

$4x-8-6y-18-z+13=0$

Tangent $4x-6y-z=13$

Normal $\frac{x-2}{4} = \frac{y+3}{-6} = \frac{z-13}{-1}$

Q7). $f(x,y,z) = 4x^3y^2+2y-z$

$\nabla f(1,-2,12) = [12x^2y^2]\hat{i}+[8yx^3+2]\hat{j}-[1]\hat{k}$

$\nabla f(1,-2,12) = 48\hat{i}-14\hat{j}-\hat{k}$

See 12 too.

Tangent: $48(x-1)-14(y+2)-1(z-12)=0$

Normal:- $\frac{x-1}{48} = \frac{y+2}{-14} = \frac{z-12}{-1}$

Q8). $f(x,y,z) = \frac{x^7}{2}y^{-2}-z$; P

$\nabla f(2,4,4) = \left[\frac{7x^6}{2}y^{-2}\right]\hat{i}+\left[-x^7y^{-3}\right]\hat{j}-1\hat{k}$

$\nabla f(2,4,4) = 14\hat{i}-2\hat{j}-\hat{k}$

Tangent $14(x-2)-2(y-4)-1(z-4)=0$

Normal $\frac{x-2}{14} = \frac{y-4}{-2} = \frac{z-4}{-1}$

Q9). $f(x,y,z) = xe^{-y}-z$

$\nabla f(x,y,z) = [e^{-y}]\hat{i}+[-xe^{-y}]\hat{j}-1[\hat{k}]$

$\nabla f(1,0,1) = [1\hat{i}-1\hat{j}-1\hat{k}]$

Tangent:- $1(x-1)-1(y-0)-1(z-1)=0$

Normals $\frac{(x-1)}{1} = \frac{y-0}{-1} = \frac{z-1}{-1}$

Q10). $f(x,y,z) = \ln\sqrt{x^2+y^2}-z$

$\nabla f(x,y,z) = \frac{1}{2}\left[\frac{1}{x^2+y^2}\times 2x\right]\hat{i}+\frac{1}{2}\left[\frac{1}{x^2+y^2}\right](2y)\hat{j}-1\hat{k}$

Tangent:- $\left[\frac{x}{x^2+y^2}\right]\hat{i}+\left[\frac{y}{x^2+y^2}\right]\hat{j}-\hat{k}$

$= -\hat{i}+0\hat{j}-1\hat{k}$.

Normal:- ✓

Q11). $e^{3y}\sin 3x - z = 0$

$\nabla f(\pi/6,0,1) = [e^{3y}3\cos 3x]\hat{i}+[3e^{3y}\sin 3x]\hat{j}-1\hat{k}$