Assignment 1

Problem 1: **Find and sketch the domain of the following functions**

1. $f(x,y,z) = xylnz$
2. $f(x,y) = ln(x^2 + y^2)$
3. $f(x,y) = sin^{-1}(y - x)$
4. $f(x,y,z) = \frac{1}{x+1} + \frac{1}{y-1} + \frac{1}{x+y-z}$

Problem 2: Draw an assortment of level curves (contour plot) in the functions domain

1. $f(x,y) = 4x^2 + y^2 + 1$
2. $-2 = 2x - 6y + z$

Problem 3: Sketch the level surface for the given function at the specific value of *k*.

1. $\frac{x^2}{25} + \frac{y^2}{16} + \frac{z^2}{9} = k$
2. $f(x,y,z) = 9x^2 + 4y^2 + z^2, k = 4$

Problem 4: Find

1. $\lim_{(x,y)\to(0,0)} \frac{x^2-xy}{\sqrt{x}-\sqrt{y}}$
2. $\lim_{(x,y)\to(0,0)} cos\frac{x^2-y^3}{x+y+1}$

Problem 5: The height of a right circular cone is increasing at 3mm/s and its radius is decreasing at 2mm/s. Determine, correct to 3 significant figures, the rate at which the volume is changing (in) when the height is 3.2 cm and the radius is 1.5 cm.

Problem 6: The area A of a triangle is given by $A = \frac{1}{2}acsinB$, where *B* is the angle between sides a and c. If a is increasing at 0.4 units/s, c is decreasing at 0.8 units/s and B is increasing at 0.2 units/s, find the rate of change of the area of the triangle, correct to 3 significant figures, when a is 3 units, c is 4units and B is $\pi/6$ radians.

Problem 7: Pressure p and volume V of a gas are connected by the equation $pV^{1.4} = k$ Determine the approximate percentage error in k when the pressure is increased by 4% and the volume is decreased by 1.5%.

Problem 8: The wind-chill index W is the perceived temperature when the actual temperature is T and the wind speed is v, so we can write $W = f(T, v)$. The following table of values is an excerpt from Table

Wind speed (km/h)

Actual temperature (°C) T \ v	20	30	40	50	60	70
−10	−18	−20	−21	−22	−23	−23
−15	−24	−26	−27	−29	−30	−30
−20	−30	−33	−34	−35	−36	−37
−25	−37	−39	−41	−42	−43	−44

Estimate the values of $f_T(-15, 30)$ and $f_v(-15, 30)$. What are the practical interpretations of these values?

Problem 9: The wave heights h in the open sea depend on the speed v of the wind and the length of time that the wind has been blowing at that speed. Values of the function $h = f(v, t)$. are recorded in feet in the following table.

Duration (hours)

Wind speed (knots) v \ t	5	10	15	20	30	40	50
10	2	2	2	2	2	2	2
15	4	4	5	5	5	5	5
20	5	7	8	8	9	9	9
30	9	13	16	17	18	19	19
40	14	21	25	28	31	33	33
50	19	29	36	40	45	48	50
60	24	37	47	54	62	67	69

(a) What are the meanings of the partial derivatives $\partial h/\partial v$ and $\partial h/\partial t$?

(b) Estimate the values of $f_v(40, 15)$ and $f_t(40, 15)$. What are the practical interpretations of these values?

Problem 10: Find the directional derivative of the function at the given point in the direction of the vector *v*.

1. $f(x,y,z) = xe^y + ye^z + ze^x$, $(0,0,0)$, $v = \langle 5,1,-2 \rangle$
2. $f(x,y,z) = \sqrt{xyz}$, $(3,2,6)$, $v = \langle -1,-2,2 \rangle$
3. $f(x,y) = x - \frac{y^2}{x} + \sqrt{3}sec^{-1}(2xy)$, $(1,1)$, $v = 12i + 5j$