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MVC Assignment #2:-

Q1).

Ans:- $\cancel{y}$ $c(x,y) = e^x + \sin y - 4x - y.$

$f_x = e^x - 4$ $\quad c_x = 0 \rightarrow 0 = e^x - 4$

$f_y = \cos y - 1$ $\quad 4 = e^x$

$0 = \cos y - 1$ $\quad \ln 4 = x \cancel{\ln e} \rightarrow \boxed{x = \ln 4}$

$1 = \cos y$ $\quad C.P\ (\ln 4, 0).$

$y = \cos^{-1}(1) \rightarrow \boxed{y = 0}$

Second-derevative test:-

$C_{xx} = e^x \quad , C_{xy} = 0$

$C_{yy} = -\sin y$

$D = f_{xx(x,y)} \times C_{yy(x,y)} - (C_{xy})^2_{(x,y)}$

$D = (e^x)_{(x,y)} \times (-\sin y)_{(x,y)} - (0)^2$

$D = (e^{\ln 4}) \times (-\sin 0) - (0)^2$

$D = 0 \rightarrow$ so At the critical Point $(\ln 4, 0)$ it is inconclusive.

Q2). Ans:- $m(a,b) = 100a + 150b - 2a^2 - 3b^2 - ab.$

$m_a = 100 - 4a - b \quad \rightarrow 100 - 4a - b = 0$

$m_b = 150 - 6b - a \quad \rightarrow 150 - 6b - a = 0$

$4a + b = 100$

$a + 6b = 150$

$a = \frac{100 - b}{4} = 150 - 6b$

$= 100 - b = 600 - 24b$

$23b = 500$

$b = \frac{500}{23} \rightarrow$ so $a = \frac{100 - \frac{500}{23}}{4} = \frac{450}{23}$

$CP = \left(\frac{450}{23}, \frac{500}{23}\right)$

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Q2) Ans:-

Second derivative test:-

C.P $\left(\frac{450}{23}, \frac{500}{23}\right)$

$M_{aa} = -4$

$M_{bb} = -6$

$M_{ab} = -1$

So. $M_{aa(a,b)} \times M_{bb(a,b)} - M_{ab(a,b)}^2$

$= -4 \times -6 - (-1)^2$

$= 24 - 1 = 23 \rightarrow > 0$

but $M_{aa} = -4 < 0$

so local Max at critical point $\left(\frac{450}{23}, \frac{500}{23}\right)$.

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Q3). $C(m,n) = 3m^2 + 2mn + 4n^2$

$f = m^2 + n^2 + nm - 400 = 0$
↳ constrain.

Ans:- $\nabla C = \lambda \nabla f$

$(6m+2n)\hat{i} + (8n+2m)\hat{j} = \lambda\left[(2m+n)\hat{i} + (2n+m)\hat{j}\right]$

$\lambda = \dfrac{6m+2n}{2m+n}$, $\lambda = \dfrac{8n+2m}{2n+m}$

$\dfrac{6m+2n}{2m+n} = \dfrac{8n+2m}{2n+m}$

$(6m+2n)(2n+m) = (8n+2m)(2m+n)$

$\cancel{2}(3m+n)(2n+m) = \cancel{2}(4n+m)(2m+n)$

$6mn + 3m^2 + 2n^2 + mn = 8nm + 4n^2 + 2m^2 + mn$

$3m^2 + 2n^2 + 7mn = 4n^2 + 2m^2 + 9nm$

$m^2 - 2n^2 - 2mn = 0$, $m^2 + n^2 + nm = 400$

Now solve both eqs. we get different answers $(\pm 16.33, \pm 5.98)$

→ $(+16.33, +5.98)$, $(16.33, -5.98)$
, $(-16.33, -5.98)$
, $(-16.33, 5.98)$

But we will take only m,n positive integers so

Ans:- $\boxed{\text{optimum values are } (16.33, 5.98) \approx (16, 6).}$

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Q4).

Ans:- $E(p,q,r) = p^2 + 4q^2 + 2r^2$ constraint → $p^2+q^2+r^2 = 250{,}000$

So Min consumption at (500,0,0). $\sigma(p,q,r) =$

C optimal

$(E_p\hat{i} + E_q\hat{j} + E_r\hat{k}) = \lambda(\sigma_p\hat{i} + \sigma_q\hat{j} + \sigma_r\hat{k})$

$= (2p\hat{i} + 8q\hat{j} + 4r\hat{k}) = \lambda(2p\hat{i} + 2q\hat{j} + 2r\hat{k})$

$2p = \lambda 2p = 2p - \lambda 2p = 0$ $8q = \lambda 2q \rightarrow 8q - \lambda 2q = 0$ $4r = 2r\lambda = 4r - 2r\lambda = 0$

$\lambda = 1$ $2p(1-\lambda) = 0$ $\lambda = 4$ $2q(4-\lambda) = 0$ $\lambda = 2$ $2r(2-\lambda) = 0$

$p = 0$ So $q = 0$ $r = 0$

But all p, q, r → cannot be 0 together of 2000

So if $p = 0, q = 500, r = 0$

$p = 500, q = 0, r = 0$ So → $(0,500,0) = 2{,}000{,}000$ → Ans: So for minimum consumption only at (500,0,0)

$p = 0, q = 0, r = 500$ $(500,0,0) = 250{,}000$

$(0,0,500) = 500{,}000$

↳ Max at consumption (0,500,0)

Q5).

Ans:- $J(\theta_1, \theta_2) = \sin(\theta_1) + \theta_2^2 + \theta_1\theta_2$ $\alpha = 0.1$, $\theta_1^0 = 1$, $\theta_2^0 = 1$

$f(\theta_1) =$

$J_1 \rightarrow f(\theta_1,\theta_2) = \sin(1) + (1)^2 + (1)(1) = \boxed{2.84}$ $\nabla J = (\cos(\theta_1) + \theta_2, 2\theta_2 + \theta_1)$

$J_2 = J_1 + \alpha(-\nabla J_1(\theta_1,\theta_2)) = (1,1) + 0.1(-(1.54,3))$ so at $\nabla J_{(1,1)} = (1.54, 3)$

$= (1,1) - 0.1(1.54, 3)$

$J_2 = (1 - (0.1 \times 1.54), 1 - (0.1 \times 3)) \rightarrow \boxed{(0.846, 0.7)}$ → 1st iteration

$J_3 = J_2 + \alpha(-\nabla J_2(\theta_1,\theta_2))$

$J_3 = (0.846, 0.7) + \alpha(-(1.36, 2.246))$ At $\nabla J(0.846, 0.7) = (1.36, 2.246)$.

$J_3 = (0.846, 0.7) - (0.1)(1.36, 2.246) = J_3 = (0.846 - (0.1 \times 1.36), 0.7 - (0.1 \times 2.246))$

Ans: → $J_3 = \boxed{(0.71, 0.4754)}$ → 2nd Iteration

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Q6).

Ans:- $R(f,g,h) = f^2 + 2g^2 + 3h^2 + fg - 5f - 7g - 9h + 30$

$\alpha = 0.05$, $(f_0, g_0, h_0) = (1,1,1)$

$R_1(1,1,1) = 16$

$\nabla R = (2f + g - 5,\ 4g + f - 7,\ 6h - 9)$

$R_2 = R_1 + \cancel{\alpha}(-\nabla R(f,g,h))$

At $\nabla R_1 = (-2, -2, -3)$ at $(1,1,1)$

$R = (1,1,1) + (0.05)(-(-2,-2,-3))$

$R_2 = (1,1,1) - (0.05)(-2,-2,-3) = (1-(0.05\times-2),\ 1-(0.05\times-2),\ 1-(0.05\times-3))$

$R_2 = (1.1,\ 1.1,\ 1.15)$. → 1st iteration

$R_3 = R_2 + \alpha(-\nabla R_2(f,g,h))$.

At $\nabla R_2 = (-1.7, -1.5, -2.1)$ at $(1.1, 1.1, 1.15)$

$= (1.1, 1.1, 1.15) + 0.05(-(-1.7, -1.5, -2.1))$

$= (1.1, 1.1, 1.15) - 0.05(-1.7, -1.5, -2.1)$

$= (1.1 - (0.05\times -1.7),\ 1.1 - (0.05\times -1.5),\ 1.15 - (0.05\times -2.1))$

$R_3 = (1.185,\ 1.175,\ 1.255)$. → $\boxed{\text{2nd iteration}}$ Ans:

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Q7).

Ans:- $J(\theta_1, \theta_2, \theta_3) = e^{\theta_1} + \theta_2^2 + 4\theta_3^2 + \theta_1\theta_2$

$J\theta_1 = e^{\theta_1} + \theta_2$, $J\theta_{11} = e^{\theta_1}$, $J\theta_{12} = 1$, $J\theta_{13} = 0$

$J\theta_2 = 2\theta_2 + \theta_1$, $J\theta_{22} = 2$, $J\theta_{21} = 1$, $J\theta_{23} = 0$

$J\theta_3 = 8\theta_3$, $J\theta_{33} = 8$, $J\theta_{31} = 0$, $J\theta_{32} = 0$

But if we see $e^{\theta_1} = 0$ or $2e^{\theta_1} + \theta_1 = 0$

$\frac{\theta_1}{2} = e^{\theta_1}$

"So no direct optimum values but could come from approximation" will have no direct real solutions "

$$\text{Matrix} = \begin{array}{c} \\ \theta_1 \\ \theta_2 \\ \theta_3 \end{array} \begin{array}{c} \begin{array}{ccc} \theta_1 & \theta_2 & \theta_3 \end{array} \\ \begin{bmatrix} J\theta_{11} & J\theta_{12} & J\theta_{13} \\ J\theta_{21} & J\theta_{22} & J\theta_{23} \\ J\theta_{31} & J\theta_{32} & J\theta_{33} \end{bmatrix} \end{array}$$

$$H = \begin{bmatrix} e^{\theta_1} & 1 & 0 \\ 1 & 2 & 0 \\ 0 & 0 & 8 \end{bmatrix}$$

(i) $D_1 = e^{\theta_1} > 0$

(ii) $D_2 = \begin{vmatrix} e^{\theta_1} & 1 \\ 1 & 2 \end{vmatrix} \rightarrow |2e^{\theta_1} - 1| > 0 \rightarrow 2e^{\theta_1} - 1 > 0$

(iii) $D_3 = \left| e^{\theta_1}[16] - 8 \right| > 0 \rightarrow 16e^{\theta_1} - 8 > 0$

$8(2e^{\theta_1} - 1) > 0$

Hessian matrix is +ve definite and has would be local minimum

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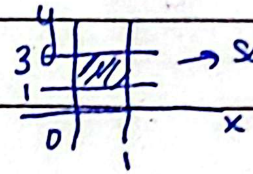
Q8). <u>Ans:-</u> $T(x,y) = xy - x^2$ Region $0 \le x \le 1$, $1 \le y \le 3$



→ so Rectangular.

$$\mathcal{E} = \iint_{(D)} xy - x^2 \; dA$$

$$= \int_1^3 \int_0^1 xy - x^2 \; dx\,dy.$$

$$= \int_1^3 \left[\int_0^1 \left(\frac{x^2 y}{2} - \frac{x^3}{3} \right) \right] dy$$

$$= \int_1^3 \left[\left| \frac{x^2 y}{2} - \frac{x^3}{3} \right|_0^1 \right] dy$$

$$= \int_1^3 \left[\frac{y}{2} - \frac{1}{3} - 0 \right] dy \rightarrow \int_1^3 \frac{y}{2} - \frac{1}{3} dy \rightarrow \int_1^3 \frac{y^2}{4} - \frac{y}{3}$$

$$= \left| \frac{3^2}{4} - \frac{3}{3} \right| - \left| \frac{1^2}{4} - \frac{1}{3} \right|$$

$$= \left| \frac{9}{4} - 1 \right| - \left| \frac{1}{4} - \frac{1}{3} \right| = \boxed{4/3} \text{ Ans:}$$

$\mathcal{E} = \frac{4}{3}$ Total heat energy over reigion

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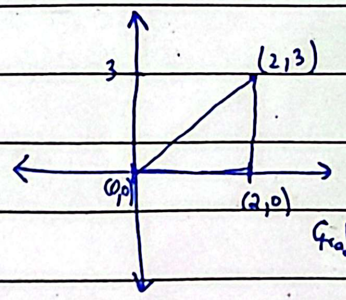
Q9).



Gradient $= \frac{3-0}{2-0} = \frac{3}{2}$

$y - 0 = \frac{3}{2}(x-0)$

$2y = 3x$

$y = 3x/2$

Ans:- $z = 10 - x^2 - y^2$

$$\text{Volume} = \int_0^2 \int_0^{3x/2} (10 - x^2 - y^2)\, dy\, dx$$

$$\text{volume} = \int_0^2 \left[\int_0^{3x/2} \left[10 - x^2 - y^2\right] dy \right] dx$$

$$\text{volume} = \int_0^2 \left| 10y - x^2y - \frac{y^3}{3} \right|_0^{3x/2} dx$$

$$V = \int_0^2 \left| \frac{30x}{2} - \frac{3x^3}{2} - \frac{27x^3}{24} \right| dx$$

$$V = \int_0^2 \frac{30x^2}{4} - \frac{3x^4}{8} - \frac{27x^4}{96} \rightarrow V = \frac{30(2)^2}{4} - \frac{3(2)^4}{8} - \frac{27(2)^4}{96} - 0$$

$$\boxed{\text{Volume} = 19.5\ m^3}$$

Q10)

(a).

Ans:- $f(x,y,z) = x^2+y^2-z=0$ at point $(1,2,5)$.

$f(x,y,z) = x^2+y^2-z$

$= f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$

$= 2x\hat{i} + 2y\hat{j} - 1\hat{k}$ put point $(1,2,5)$

$= 2\hat{i} + 4\hat{j} - 1\hat{k}$

so tangent plane eq $= 2(x-1)+4(y-2)-1(z-5)$

$= 2x-2+4y-8-z+5$

$\boxed{\text{Tangent eq plane} = 2x+4y-z=5}$

equation

$2i+4j-1k$

Normal $= \langle 2,4,-1\rangle$

vector to surface

(b).

Ans:- ~~$f(x,y,z) = x^2 \ldots = 0$ at d~~

$f_1(x,y,z) = x^2+y^2+z^2-9=0$

$f_2(x,y,z) = x+2y-3z+4=0$

Point $(2,-1,1)$

$f_1 = f_x i + f_y j + f_z k$

$f_2 = f_x i + f_y j + f_z k$

$f_1 = 2x\hat{i} + 2y\hat{j} + 2z\hat{k}$

$f_2 = 1\hat{i} + 2\hat{j} - 3\hat{k}$

Point at $f_1 = (4\hat{i} - 2\hat{j} + 2\hat{k})$.

$f_2 = (1\hat{i} + 2\hat{j} - 3\hat{k})$

For Angle $F_1 \cdot F_2 = |F_1||F_2|\cos\theta$

$(4i-2j+2k)\cdot(1i+2j-3k) = |2\sqrt{6}| \times \sqrt{14}\cos\theta$

$4-4-6 = 2\sqrt{6}\times\sqrt{14}\cos\theta$

$\cos\theta = \dfrac{-6}{2\sqrt{6}\times\sqrt{14}}$ $= \boxed{\theta = 109.1^\circ \text{ degrees or } 1.90 \text{ radians.}}$