Ex 15.2:- (7-14, 19-30, 37-40).

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Q7). $\int_C F \cdot dr$

Ans:- $F(x,y) = 8\hat{i} + 8\hat{j}$

$P(-4,4), Q(-4,5)$

$x = -4 + k_1 t_1 \qquad x = -4$

$y = 4 + k_2 t_1 \qquad y = 4 + t$

At t=1 $\quad -4 = -4 + k_1 \rightarrow k_1 = 0$

At t=1 $\quad 5 = 4 + k_1 \rightarrow k_1 = 1$

$\vec{r}(t) = -4\hat{i} + (4+t)\hat{j}$

$\vec{r}'(t) = 0\hat{i} + 1\hat{j} \rightarrow \|\vec{r}'\| = 1$

$\int_0^1 (8\hat{i} + 8\hat{j}) \cdot (0\hat{i} + 1\hat{j}) = 8$

Q8). same as Q7.

Q9). $F(x,y) = 2x\hat{j} \quad P(-2,4), Q(-2,11)$

Ans:- $x = -2 + k_1 t_1 \quad \underline{t=1} \quad x = -2$

$y = 4 + k_1 t_1 \quad \underline{t=1} \quad y = 4 + 7t$

$\vec{r}'(t) = 0\hat{i} + 7\hat{j}$

$\int_0^1 (0\hat{i} + 2x\hat{j}) \cdot (0\hat{i} + 7\hat{j})$

$\int_0^1 14x \, dt$

$\int_0^1 -28t \rightarrow -28$

Q10). $F(x,y) = -8x\hat{i} + 3y\hat{j} \quad P(-1,0), Q(6,0)$

Ans:- $x = -1 + k_2 t_1, \; y = 0 + k_2 t_2$

At t=1:- $x = 7, \; y = 0$

$\vec{r}'(t) = 7\hat{i} + 0\hat{j}$

$\int_0^1 (-8x\hat{i} + 3y\hat{j}) \cdot (7\hat{i} + 0\hat{j})$

$\cancel{\int_0^1 -56x \rightarrow -112}$

$-56(-1+7t)$

$\int_0^1 56 - 392t = -140$

Q11). (a). $x = 2t, \; y = t^2$

Ans:- $\int_0^1 \quad \vec{r}(t) = 2t\hat{i} + t^2\hat{j}$

$\vec{r}'(t) = 2\hat{i} + 2t\hat{j}$

$|\vec{r}'(t)| = \sqrt{4 + 4t^2}$

$\int_0^1 (2t - t)\, 2(1+t^2)^{1/2} \, dt.$

$\int_0^1 2t(1+t^2)^{1/2} \, dt$

$= 1.22.$

(b). $\int_C (x - \sqrt{y}) \, dx. \quad \because \boxed{dx = 2dt}$

$\int_0^1 2t = 1$

(c). $\int_C (x - \sqrt{y}) \, dy \qquad dy = 2t \, dt$

$\int_0^1 \frac{2t^2}{3} \rightarrow \frac{2}{3}$

Q12). (a). $x = t, \; y = 3t^2, \; z = 6t^3$

Ans:- $\vec{r}(t) = t\hat{i} + 3t^2\hat{j} + 6t^3\hat{k}$

$\vec{r}(t)' = \hat{i} + 6t\hat{j} + 18t^2\hat{k}$

$\int_0^1 (t)(3t^2)(6t^3)^2 \times \sqrt{1 + 36t^2 + 324t^4}$

$\int_0^1 3t^3 \times 36t^6 (1 + 36t^2 + 324t^4)^{1/2} \, dt$

(b). $\int_0^1 3t^3 \times 36t^6 \, dt.$

$108t^9 \rightarrow \frac{108}{10}$

(c)., (d) cosy.

Q13). (a) $\int_C (3x + 2y)\,dx + (2x - y)\,dy.$

Ans:- $x = 0 + k_1 t_1 \quad x = t$

$y = 0 + k_2 t_2 \quad y = t. \quad ds = \sqrt{2} \, dt$

$\int_0^1 \left[(5t) + (t) \right] \sqrt{2} \, dt$

$\sqrt{2} \int_0^1 \frac{6t^2}{2} = 3\sqrt{2}$

Q13). (b). $y = x^2, \; x = t, \; y = t$

$x = \sqrt{t}$

See 13, 14

[19-30]

Q19).

Ans:- $\int_C \frac{1}{1+x} \, ds \qquad r(t) = t\hat{i} + \frac{2}{3}t^{3/2}\hat{j}$

$\int_0^3 \frac{1}{1+t} (1+t)^{1/2} \, dt.$

$\vec{r}(t) = \sqrt{1 + (t^{1/2})^2} = \sqrt{1+t}$

$\int_0^3 (1+t)^{-1/2} \, dt.$

$\int_0^3 2(1+t)^{1/2} = 4 - 2 = 2$

Q20) $\int_C \frac{x}{1+y^2} \, ds$

$\int_0^1 \frac{1+2t}{1+t^2} \sqrt{5} \, dt.$

$\vec{r}t = \sqrt{(2)^2 + (1)^2} \rightarrow \sqrt{5}$

$\sqrt{5} \left[\int_0^1 \frac{1}{1+t^2} dt + \int_0^1 \frac{2t}{1+t^2} dt \right]$

$\sqrt{5} \int_0^1 \tan^{-1}(t) + \ln(1+t^2)$

$\cancel{6.26} \quad \cancel{4.2}$

$\cancel{6.57} \quad 3.306$

21). $\int_0^1 3t^2 (t^2) \left(\frac{2t^3}{3}\right) \sqrt{1 + 4t^2 + 4t^4} \, dt$

$\int_0^1 t^7 (1 + 4t^2 + 4t^4)^{1/2} \, dt.$

$\int_0^1 t^7 (1 + 2t) \, dt$

$\int_0^1 t^7 + 2t^8 \, dt$

$-2\sin t + 2\cos t + 1$ Bazil-uddin Khan 24k-0559 4.

Q22) $\int_C \frac{e^{-z}}{x^2+y^2}\, ds$

$\int_0^{2\pi} \frac{e^{-t}}{4\cos^2 t + 4\sin^2 t} \times \left[\sqrt{4\cos^2 t + 4\sin^2 t}\ \sqrt{5}\right] dt$

$-\frac{\sqrt{5}}{4}\int_0^{2\pi} + e^{-t} dt = -\frac{\sqrt{5}}{4} e^{-2\pi} + \frac{\sqrt{5}}{4} e^{0}$

$\boxed{\frac{\sqrt{5}}{4}\left(1 - e^{-2\pi}\right)}$

Q23). $\int_C (x+2y)\,dx + (x-y)\,dy.$

$x = 2\cos t,\ y = 4\sin t,\ 0 \le t \le \pi/4$

Ans:- $\int_0^{\pi/4}$

$\vec{r}'(t) = -2\sin t\,\hat{i} + 4\cos t\,\hat{j}$

$|\vec{r}'(t)|^2 = \sqrt{4\sin^2 t + 16\cos^2 t}$ ×

$dx = -2\sin t\,dt,\ dy = 4\cos t\,dt.$

$\int_0^{\pi/4} (2\cos t + 8\sin t)(-2\sin t\,dt)$

$+$

$(2\cos t - 4\sin t)(4\cos t\,dt)$

$-4\sin t\cos t - 16\sin^2 t\,dt$

$+$

$\cancel{4\cos t}\ 8\cos^2 t\,dt$

$-16\sin t\cos t$

$= \int_0^{\pi/4} 8\cos^2 t - 16\sin^2 t - 20\sin t\cos t\,dt$

$= 1-\pi$

Q24). $(x^2-y^2)\,dx + x\,dy$

Ans:- $x = t^{2/3},\ y = t$

$\int_{-1}^{1}$

$\vec{r}'(t) = \frac{2}{3}t^{-1/3} + 1$

$\vec{r}'(t) = \left(\sqrt{\frac{4}{9}t^{-2/3} + 1}\right) dt.$

$\int_{-1}^{1} \left[(t^{4/3} - t^2)\left(\frac{2}{3}t^{-1/3}\,dt\right) + (t^{2/3})(1)\right] \times \left[\frac{4}{9}t^{-2/3} + 1\right]^{1/2} dt.$

$\int_{-1}^{1} \frac{2}{3}t\,dt - \frac{2}{3}t^{5/3}\,dt + t^{2/3}\,dt$

Just solve this

Q25). $\int -y\,dx + x\,dy$

$y^2 = 3x$ $\quad x = 3 + k_1 t_1$

$\quad Y = 3 + k_2 t_2$

$x = 3 - 3t \rightarrow dx = -3dt$

$y = 3 - 3t \rightarrow dy = -3dt$

$\int_0^1 (-3+3t)(-3dt) + (3+3t)(-3dt)$

$\int_0^1 9dt - 9t\,dt - 9dt - 9t\,dt$

$\int_0^1 -18t\,dt$

$\rightarrow \int_0^1 -9t^2\,dt = -9$

$y^2 = 3x$

$(3-3t)^2 = 3(3-3t)$

$\cancel{3-3t=3}$

(1,3-1)

for anticlockwise (counterclockwise)

Remaining 15-20s:- Bazil-uddin khan 244-0559 H.

Circle so do Parametrization $x = t\cos t$, $y = \sin t$ $\to \theta, 0$ to $\pi/2$

(25-30)

Q25). $\int_C -y\,dx + x\,dy$

$y^2 = 3x$ from (3,3) to (0,0)

"as function is given so no need to use r etc." or "t=0 or t=1"

$y^2 = 3x \to x = \frac{y^2}{3}$ Substitution

let y = t

$y = t$. $x = \frac{t^2}{3}$ $dy = dt$, $dx = 2t/3$

$$\int_3^0 \left[-t + \frac{2t}{3} + \frac{t^2}{3}\cdot 1\right] dt = 3$$

Q26). $(y-x)\,dx + x^2y\,dy$

$y^2 = x^3$ (1,-1) to (1,1)

let $y = t$, $x = (t^2)^{1/3}$

$y(t) = t$ $\boxed{x(t) = t^{2/3}}$

$y - \frac{1}{3}$

$$(t - t^{2/3})\times\left(\frac{2}{3}t^{-1/3}\right) + (t^{4/3})(t)\,dt$$

$$(t - t^{2/3})\left(\frac{2}{3}t^{-1/3}\right) + (t^{7/3})\,dt$$

$$\frac{2t^{2/3}}{3} - \frac{2}{3}t^{1/3} + t^{7/3}\,dt$$

$$\frac{6t^{5/3}}{5} - \frac{2t^{4/3}}{3}\cdot\frac{3}{4} + \frac{3t^{10/3}}{10}$$

$$\int_{t=-1}^{t=1} \frac{6t^{5/3}}{5} - \frac{t^{4/3}}{2} + \frac{3t^{10/3}}{10}$$

$= 12/5$

Q27). $(x^2+y^2)\,dx - x\,dy$ Imp.

Ans:- (t,t)

$$\frac{-t}{(1-t^2)^{1/2}} + \frac{t}{(1-t^2)^{1/2}}$$

$x^2+y^2 = 1$, $dy = dt$.

$y = t \to 0$

$x = (1-t^2)^{1/2}$

$\frac{dx}{dt} = \frac{1}{2}(1-t^2)^{-1/2}(-2t)$

$dx = (1-t^2)^{-1/2}(-t)$

$$\int_0^{\pi/2} (\cos t^2 + \sin t^2)(-\sin t) - (\cos t)(\cos t)\,dx$$

$= 1 - \pi/4$

28).

Ans:- $x = 3 - t \Rightarrow dx = -1\,dt$

$y = 4 - 3t \to dy = -3\,dt$

$$\int_0^1 (4-3t)(-1)\,dt + (3-t)(4-3t)(-3\,dt)\,dt$$

29). $dx = e^t\,dt$, $dy = 3e^{3t}\,dt$, $dz = -e^{-t}\,dt$

Just put and solve

30). $dx = \cos t\,dt$, $dy = -\sin t\,dt$, $dz = 2t$

R.W

Face 1:- $x \to [1,3]$, $y \to [0,3]$, $z \to [3,-3]$

$x \to [2,-5]$, $y \to [-\sqrt{2},3]$, $z \to [-3,3]$.

Soln (0,1),(0,1),(0,1)

if $x = 2$:- $(y z)$

$$\int_{-\sqrt{2}}^{3}\int_{-3}^{3} 2yz\,dy\,dz = 9z - [+9z] = 0$$

if $x = -5$:-

$$\int\int_0$$

if $y = 2$:-

$$\int_{-5}^{2}\int_{-3}^{3} -\sqrt{2}xz\,dz\,dx = 0$$

if $y = 3$

$$\int_{-5}^{2}\int_{-3}^{3} 3xz\,dz\,dx$$

$-\frac{53y^2}{}$ =

if $z = 3$:-

$$\int_{-\sqrt{2}}^{3}\int_{-5}^{2} 3xy\,dx\,dy$$

$$\int_{-\sqrt{2}}^{3}\int_{-5}^{2} \frac{3x^2}{2}y\,dy$$

$$\int_{-\sqrt{2}}^{3} 6y - \frac{75y}{2}\,dy$$

$$\int^3 -\frac{63y}{2}\,dy$$

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37-40

(Q37). $f(x,y) = x^2\hat{i} + xy\hat{j}$

$r(t) = 2\cos t\,\hat{i} + 2\sin t\,\hat{j}$

$$\int_0^{\pi} (x^2\hat{i} + xy\hat{j}) \cdot ($$

$r'(t) = -2\sin t\,\hat{i} + 2\cos t\,\hat{j}$

$||r'(t)|| = \sqrt{4\sin^2 t + 4\cos^2 t} = 2$

$$\int_0^{\pi} (4\cos^2 t\,\hat{i} + 4\cos t\sin t\,\hat{j}) \cdot (-2\sin t\,\hat{i} + 2\cos t\,\hat{j})$$

$$\int_0^{\pi} -16\cos^2 t\sin t + 16\cos^2 t\sin t$$

$$\int_0^{\pi} -16 + 16 = 0$$

(Q38). $F(x,y) = \underbrace{x^2 y}_{f}\,\hat{i} + \underbrace{4}_{g}\,\hat{j}$

$\vec{r}(t) = e^t\hat{i} + e^{-t}\hat{j}$

$$\int_0^1 \left[(e^t)^2 (e^{-t})\hat{i} + 4\hat{j}\right] \cdot (e^t\hat{i} - e^{-t}\hat{j})$$

$x(dt$

$\sqrt{(e^{t^2}) - (e^{-t^2})}$

$e^{t^2} - \frac{1}{e^{t^2}}$

$\frac{e^{t^4} - 1}{e^{t^2}}$

$\frac{(e^{t^2} - 2)(e^{t^2} + 1)}{e^t}$

$(e^{t2} - 4e^{-t}) \left(\frac{(e^{t^2} - 1)(e^{t^2} + 1)}{e^t}\right)$

40). $F(x,y,z) = z\hat{i} + x\hat{j} + y\hat{k}$

$c: r(t) = \sin t\,\hat{i} + 3\sin t\,\hat{j} + \sin^2 t\,k \qquad x = \sin t,\ y = 3\sin t,\ z = \sin^2 t.$

$\int_0^{\pi/2} \left(\sin^2 t\,\hat{i} + \sin t\,\hat{j} + 3\sin t\,\hat{k}\right)\cdot\left(\cos t\,\hat{i} + 3\cos t\,\hat{j} + 2\sin t\cos t\,\hat{k}\right) \quad \vec{r}'(t) = \cos t\,\hat{i} + 3\cos t\,\hat{j} + 2\sin t\cos t\,\hat{k}$

$\int_0^{\pi/2} \cos t\sin^2 t + 3\cos t\sin t + 6\sin^2 t\cos t$

$\int_0^{\pi/2} 7\cos t\sin^2 t + 3\cos t\sin t\,dt$

$\int_0^{\pi/2} \frac{7\sin^3 t}{3} + \frac{3\sin^2 t}{2}\,dt \rightarrow \frac{7}{3} + \frac{3}{2} = \frac{23}{6}$

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