see this last Ex if needed

Ch # 13.5 :- Bazil-uddin-khan

chain Rule FOR MVC :- $\boxed{\frac{dw}{dt} = \frac{dy}{dx} \cdot dy/dt}$

"Intervariables give Path b/w dependent and independent" "x" works as "The INTERMEDIATE" variable → 3 variables but still only 1 independent variable that is "t"

Ex:. $w = x^2 - y^2$, $x = t^2 + 1$, $y = t^3 + t$: W is dependent, T → independent so x, y Im.

I want $\boxed{dw/dy}$ Two paths from "w to t"

As "t" changes (ΔT), "x" and "y" Respond By changing Also ($\Delta x, \Delta y$) and both $\Delta x, \Delta y$ cause a change in 'w' (Δw)

$\Delta w =$ The amount "x" changes "w" + The amount "y" changes "w"

$m = \frac{Rise}{Run} \Rightarrow Run\,m = Rise$ Rate of change in x-direction : Δx + R. of. C in y-Dir Δy

$\Delta w =$ (Partial derivatives) $\frac{dw}{dx} \cdot \Delta x + \frac{dw}{dy} \cdot \Delta y \div \Delta t$

$\frac{\Delta w}{\Delta t} = \frac{dw}{dx} \cdot \frac{\Delta x}{\Delta t} + \frac{dw}{dy} \frac{\Delta y}{\Delta t}$ AND make $\Delta t = 0$ $\lim_{\Delta b \to 0} \frac{\Delta a}{\Delta b} = \frac{da}{db}$

$\lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \lim_{\Delta t \to 0} \frac{dw}{dx} \cdot \frac{\Delta x}{\Delta t} + \lim_{\Delta t \to 0} \frac{dw}{dy} \frac{\Delta y}{\Delta t} \rightarrow \lim_{\Delta t \to 0} \frac{\Delta w}{\Delta t} = \frac{dw}{dx} \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} + \frac{dw}{dy} \lim_{\Delta t \to 0} \frac{\Delta y}{\Delta t}$

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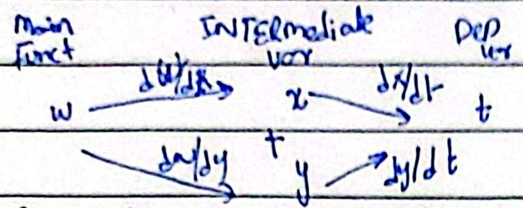
Formula ← $\frac{dw}{dt} = \frac{dw}{dx} \cdot \frac{dx}{dt} + \frac{dw}{dy} \cdot \frac{dy}{dt}$

derivated Normal deriv.

Make a this →



0.2   -0.2

$$\frac{dz}{dx} = 2x + \left(z\cos(xyz)(yz)\frac{dz}{dx}\right) = 0$$

$$\frac{dz}{dy} = \frac{dw}{dx} = (2w)\frac{dw}{dx}$$

$w^2 + w\sin(xyz) = 1$

$2w\frac{dw}{dx}$:

Q). $w = x^2y - xy^3$ $x = \cos t$, $y = e^t$ find $dw/dt|_{t=0}$

$$\frac{dw}{dt} = (2xy - y^3)(-\sin t) + (x^2 - 3y^2x)(e^t)$$

At $t=0$

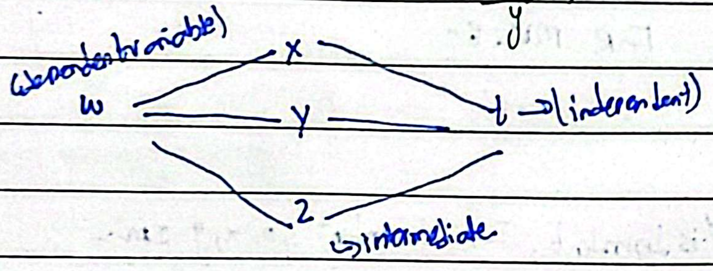
$x = \cos(0) = 1$, $y = e^0 = 1 \Rightarrow y = 1$ $\left.\frac{dw}{dt}\right|_{t=0,\ y=1,\ x=1}$ $\left(2(1)(1) - (1)^3\right)(-\sin 0) + \left(1^2 - 3(1)^2(1)\right)(e^0)$

Extending the concepts

Q1). more intermediate variables :-

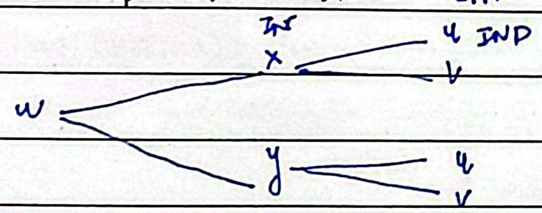
case 1 :-

Q). $w = \tan^{-1}(xz) + \frac{z}{y}$, $x = t$, $y = t^2$, $z = \sin ht$



$$\frac{dw}{dt} = \frac{1}{1+(xz)^2}\,x(2)(1) + \left(-\frac{z}{y^2}\right)(2t) + \left(\frac{1}{1+(xz)^2}\times x + \frac{1}{y}\right)(\cosh t)$$

case 2 :- more independent variables :- CAN'T use dw/dz ... Back to Partials.

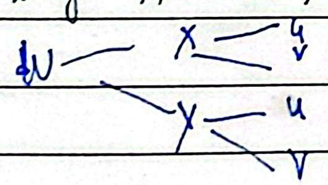


we can only derivate $\frac{dw}{du}$ or $\frac{dw}{dv}$

x, y } are only middleman

$$\frac{dx}{du} = \frac{dw}{dx} \times \frac{dx}{du} + \frac{dw}{dy} \times \frac{dy}{du}$$ OR $$\frac{dx}{dv} = \frac{dw}{dy} \times \frac{dy}{dv} + \frac{dw}{dx} \times \frac{dx}{dv}$$ only 1 independent var

Q) $w = x^3 + y^3$, $x = u^2 + v^2$, $Y = 2uv$



$$\frac{dw}{du} = (3x^2)(2u) + (3y^2)(2v), \quad \frac{dw}{dv} = (3x^2)(2v) + (3y^2)(2u)$$

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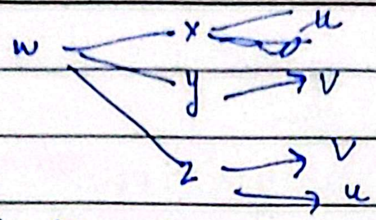
Q). $w = x\tan^{-1}(yz)$, $x = u^{3/2}$, $y = e^{-2v}$ $z = v\cos u$ Date

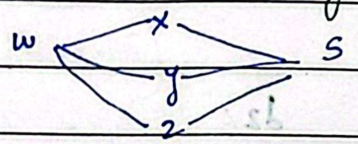
Ans:-



$$\frac{dw}{du} = \tan^{-1}(yz)\left(\frac{1}{2}u^{-1/2}\right) + \left(\frac{1}{1+(yz)^2}x(y)\right)\left(-\cancel{2e}^{-2v} \; -v\sin u\right)$$

$$\frac{dw}{dy} = \frac{dw}{dy}\frac{dy}{dv} + \frac{dw}{dz} \times \frac{dz}{dv} \rightarrow \left(\frac{xz}{1+(yz)^2}\right)\left(-2e^{-2u}\right) + \left(\frac{xy}{(1+(yz))^2}\right) \times \left(\cancel{-v\sin u}\ \cos u\right)$$

Q). $w = x^2y + y^2z^3$, $X = r\cos(s)$, $y = r\sin(s)$, $z = re^s$ $\left. \frac{ds}{dw} \right|_{r=1,\, s=0}$



$$\frac{dw}{ds} = (2xy)(-r\sin s) + (x^2 + 2yz^3)(r\cos s) + (\cancel{3z^2}\ y^2)(re^s)$$

$x = 1\cos 0 = 1$

$y = 1\sin 0 = 0$

$z = 1$

$$\frac{dw}{ds} = 2(1 \times 1)(-1\sin 0) + (1^2 + 2(0)(1^3))(1\cos 0) + (3 \times 1e^0)$$

$\cancel{3} = 1$

• let $w = f(x, y)$ where $f(x,y)$ is a function such that $y = g(x)$ implicit and x is the only independent Variable.

$$\frac{dw}{dx} = \frac{df}{dx} + \frac{df}{dy} \cdot \frac{dy}{dx}$$

$$\boxed{\frac{dy}{dx} = -\frac{f_x}{f_y}} \rightarrow \text{To do implicit differentiation}$$

Q). $2x^2 + 3(xy)^{1/2} - 2y - 4 = 0$ $y = gx$

Find $\frac{dy}{dx} = \frac{-f_x}{f_y} = -\frac{\left(4x + \frac{3y}{2}(xy)^{-1/2}\right)}{\frac{3x}{2}(xy)^{-1/2} - 2}$

$f_x = 4x + 3\left(\frac{1}{2}(xy)^{-1/2} \times y\right)$

$f_y = 3\left(\frac{1}{2}(xy)^{-1/2} \times x - 2\right)$

For 3 variable :-

$w = f(x, y, z) = 0$ for $z = g(x, y)$

$$\frac{dz}{dx} = -\frac{f_x}{f_z}$$

$$\frac{dz}{dy} = -\frac{f_y}{f_z}$$

(b). Actual = ← $-1.96 \rightarrow -1.92$

$\ln(2.02 + (0.97 \times 1.01)) = 0.0375091309$

EXB.5:

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Q1). $z = 3x^2y^3$

$$\frac{dz}{dt} = (6xy^3)(4t^3) + (9x^2y^2)(2t)$$

Q2). $z = \ln(2x^2+y)$

$$\frac{dz}{dt} = \left(\frac{1}{2x^2+y}\right)(4x)\left(\frac{1}{2\sqrt{t}}\right) + \left(\frac{1}{2x^2+y} \times 1\right)\left(\frac{2}{3}t^{-1/3}\right)$$

Q3). $$\frac{dz}{dt} = (-3\sin x - \cos xy\, xy)\left(\frac{-1}{t^2}\right) + (-\cos xy\, x\, x)(3)$$

Q4). $$\frac{dz}{dt} = \left(\frac{1}{2\sqrt{1+x-2xy^4}} \times (1-2y^4)\right)\left(\frac{1}{t}\right) + \left(\frac{1}{2\sqrt{1+x-2xy^4}} \times (-8xy^3)\right)(1)$$

Q5). $$\frac{dz}{dt} = (e^{1-xy})(-y)\left(\frac{1}{3}t^{-2/3}\right) + (e^{1-xy})(-x)(3t^2)$$

Q6). $$\frac{dz}{dt} = (-2\sinh(xy)x\, y)\left(\frac{1}{2}\right) + (2\sinh(xy)xx)e^t$$

Q7). $$\frac{dw}{dt} = (10xy^3z^4)(2t) + (15x^2y^2z^4)(3t^2) + (20x^2y^3z^3)(5t^4)$$

Q8).

$$\frac{dw}{dt} = \left(\frac{1}{3x^2-2y+4z^3} \times 6x\right)\left(\frac{1}{2}t^{-1/2}\right) + \left(\frac{1}{3x^2-2y+4z^3} \times -2\right)\left(\frac{2}{3}t^{-1/3}\right) + \left(\frac{1}{3x^2-2y+4z^3} \times 12z^2\right)(-2t^{-3})$$

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$$\frac{dw}{dt} = \left(-5\sin xy \times y - \cos xz(z)\right)\left(\frac{-1}{t^2}\right) + \left(-5\sin(xy)\times x\right)(2) + \left(\cancel{-5\sin(xy)} - \cos(xz)(x)\right)(3t^2)$$

10). $$\frac{dw}{dt} = \left(\frac{1}{2\sqrt{1+x-2yz^4}} \times 1 - 2yz^4\right)\left(\frac{+1}{t^2}\right) + \left(\frac{1}{2\sqrt{1+x-2yz^4}} \times (-2z^4 x)\right)(1) + \left(\frac{1}{2\sqrt{1+x-2yz^4}} \times 8yz^3 x\right)(4)$$

11). $$\frac{dw}{dt} = \left(3x^2y^2z^4\right)(2t) + \left(2x^3yz^4\right)(1) + \left(4z^3y^2x^3\right)(8t^3) \rightarrow \boxed{}$$ $\rightarrow t=1$, $x=1, y=3, z=2$

12). $$\frac{dw}{dt} = \left(x\sin yz^2\right)(-\sin t) + \left(x\cos(yz^2)(z^2)\right)(2t) + \left(x\cos(yz^2)\times 2zy\right)(e^t) \rightarrow \boxed{}$$ $t=0, x=1, y=0, z=1$

Q13). $$\frac{dz}{dt} = (3)(2t) + (-1)(3t^2) \rightarrow t=2 \rightarrow \boxed{}$$

Q14). $$\frac{dw}{dt} = (1)(1) + (2)(\cos\pi t)(\pi) + (3)(2t) \quad t=1 \rightarrow \boxed{}$$

Q17). $\frac{dz}{d}$

$z \to x \to u, v$; $z \to y \to u, v$

$$\frac{dz}{du} = (16xy-2)(v) + (8x^2+3)(1)$$
$$\frac{dz}{dv} = (16xy-2)(u) + (8x^2+3)(-1)$$

Q18).

$z \to x \to u, v$; $z \to y \to u, v$

$$\frac{dz}{du} = (2x - y\sec^2 x)(1/v) + (-\tan x)(2uv^2)$$
$$\frac{dz}{dv} = (-\tan x)(2uv^2) + (2x - y\sec^2 x)(-u/v^2)$$

Q19).

$z \to x \to u$; $z \to y \to v$

$$\frac{dz}{du} = (1/y)(-2\sin u), \quad \frac{dz}{dv} = (-x/y^2)\left[\cancel{u \times \frac{1}{u}} + \cancel{(\ln u)}\right]$$ $(3\cos v)$

Q20).

$z \to x \to u, v$; $z \to y \to u, v$

$$\frac{dz}{du} = (3)\left(1 + \frac{v}{u}\right) + (-2)(2u)$$
$$\frac{dz}{dv} = (3)(\ln u) + (-2)\left(-\left[v \times \frac{1}{v} + \ln v\right]\right)$$

Q21).

$z \to x \to u, v$; $z \to y \to v$

$$\frac{dz}{du} = (e^{x^2y})(2xy)\left(\frac{1}{2\sqrt{uv}} \times v\right) + (0), \quad \frac{dz}{dv} = \left(e^{\frac{x^2}{y}}\right)(2xy)\left(\frac{1 \times u}{2\sqrt{uv}}\right) + (e^{x^2y})(x^2)\left(\frac{-1}{v^2}\right)$$

Rectangle $= lw \to P = 2(l+w)$

$200 = 2\left(1.5 + \frac{dw}{dt}\right) \to (66)$

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22). $z = \cos x \sin y$

$z \to x \to u, v$; $z \to y \to u, v$

$$\frac{dz}{du} = (-\sin x \sin y)(1) + (\cos x \cos y)(2u)$$

$$\frac{dz}{dv} = (-\sin x \sin y)(-1) + (\cos x \cos y)(2v)$$

23). $T \to x \to r, \theta$; $T \to y \to r, \theta$

$$\frac{dT}{dr} = (2xy - y^3)(\cos\theta) + (x^2 - x)(\sin\theta)$$

$$\frac{dT}{d\theta} = (2xy - y^3)(-r\sin\theta) + (x^2 - x)(r\cos\theta)$$

24). $R \to S \to \phi$; $R \to T \to \phi$

$$\frac{dR}{d\theta} = \left(e^{2s-t^2}\right)(2)(3) + \left(e^{2s-t^2}\right)(-2t)\left(\frac{1}{2}\phi^{-1/2}\right)$$

25) $T \to u \to x, y$; $T \to v \to x, y$

$$\frac{dT}{dx} = \left(\frac{1}{v}\right)(2x) + \left(-u/v^2\right)(4y^3) \qquad \frac{dT}{dy} = \left(\frac{1}{v}\right)(-2y) + \left(\frac{-u}{v^2}\right)(12xy^2)$$

26). $w \to r \to u, v$; $w \to s \to u, v$

$$\frac{dw}{du} = \left[\frac{(r^2+s^2)(s) - (rs)(2r)}{(r^2+s^2)^2}\right][(v)] + \left[\frac{(r^2+s^2)(r) - (rs)(2s)}{(r^2+s^2)^2}\right][1]$$

$$\frac{dw}{dv} = \left[\frac{(r^2+s^2)(s) - (2r^2 s)}{(r^2+s^2)^2}\right][u] + \left[\frac{(r^2+s^2)(r) - 2rs^2}{(r^2+s^2)^2}\right][-2]$$

27). $z \to x \to r, \theta$

$$\frac{dz}{dr} = \left(\frac{1}{x^2+1}\right)(2x)(\cos\theta) \qquad \frac{dz}{d\theta} = \left(\frac{1}{x^2+1} \times 2x\right)(-r\sin\theta)$$

28). $u \to r \to x$; $u \to s \to y$; $u \to t \to x, y$

$$\frac{du}{dx} = (s^2 \ln t)(2x) + \left(rs^2 \times \frac{1}{t}\right)(y^3)$$

$$\frac{du}{dy} = (r \ln t\, 2s)(4) + \left(\frac{rs^2}{t}\right)(x\,3y^2)$$

29). $w \to x \to \rho, \phi, \theta$; $w \to y \to \rho, \phi, \theta$; $w \to z \to \phi, \rho$

$$\frac{dw}{d\rho} = (8x)(\sin\phi\cos\theta) + (8y)(\sin\phi\sin\theta) + (2z)(\cos\phi)$$

$$\frac{dw}{d\phi} = (8x)(\rho\cos\phi\cos\theta) + (8y)(\rho\cos\phi\sin\theta) + (2z)(-\rho\sin\phi)$$

$$\frac{dw}{d\theta} = (8x)(\rho\sin\phi\sin\theta) + (8y)(\rho\sin\phi\cos\theta)$$

$w \to r \to s$
$w \to \theta \to s$

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30). $w \to x$, $w \to y \to x$, $w \to z \to x$

$$\frac{dw}{dx} = (6xyz^3)(6x) + (9xy^2z^2)\left(\frac{1}{2\sqrt{x-1}} \times 1\right)$$

31). $\frac{dw}{ds} = ?$ $s = 1/4$

$s = 1/4 \to \pi/4 = \theta,\ r = 1/2$

$w \to r \to s$, $w \to \theta \to s$

$$\frac{dw}{ds} = (2r - \tan\theta)\left(\frac{1}{2\sqrt{s}}\right) + (-r\sec^2\theta)(\pi) \to \boxed{-0.58}$$

32). $x = 1,\ u = 1,\ v = -2$

$y = -8$

$f \to x \to u$, $f \to y \to u, v$

$$\to \frac{df}{du} = (2xy - 1)\left(\frac{1}{2\sqrt{u}}\right) + (2yx^2 + 2)(v^3) \to \boxed{175.5}$$

$$\frac{df}{dv} = (2x^2y + 2)(3v^2u) \to \boxed{-168}$$

33). $z \to x \to r, \theta$; $z \to y \to r, \theta$

$$\frac{dz}{dr} = \left(xye^{x/y} \times \frac{1}{y}\right)(\cancel{\cos\theta})(e^{x/y} x y)(\cos\theta)$$

$$+ \left(xye^{x/y} x - \frac{x}{y^2}\right)(e^{x/y} x x) \times (\sin\theta).$$

$$\frac{dz}{d\theta} = \left(xye^{x/y} \times \frac{1}{y}\right)(-r\sin\theta)$$

34). $t = 3,\ x = 9,\ y = 10$

$z \to x \to t$, $z \to y \to t$

$$\frac{dz}{dt} = (2xy)(2t) + (x^2)(1) \to \boxed{261}$$

35). $A = \frac{1}{2}AB\sin\theta$ $R \to A$, $R \to B$, $R \to \theta$ $\frac{1}{2}AB\cos\theta\ \frac{d\theta}{dt} \Big\} \to (35, 36).$

Q41). $\frac{dy}{dx} = \left(x^2 \times 3y^2\frac{dy}{dx}\right) \cancel{+\sin} + (y^3 \times 2x) - \sin y\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{y^3 2x =}{-(x^2 3y^2 \cancel{-y^3 2x})}\ \sin y \quad -\frac{dx}{dy} \to \frac{2xy^3}{x^2 3y^2 - \sin y}$$

Q42). $3x^2 - (3x)(2y)\frac{dy}{dx} + 3y^2\frac{dy}{dx} = 0$

$$\frac{dy}{dx} = \frac{3x^2}{-(3y^2 - 3x2y)}$$

$$\frac{dx}{dy} = 3x^2 - 3y^2$$