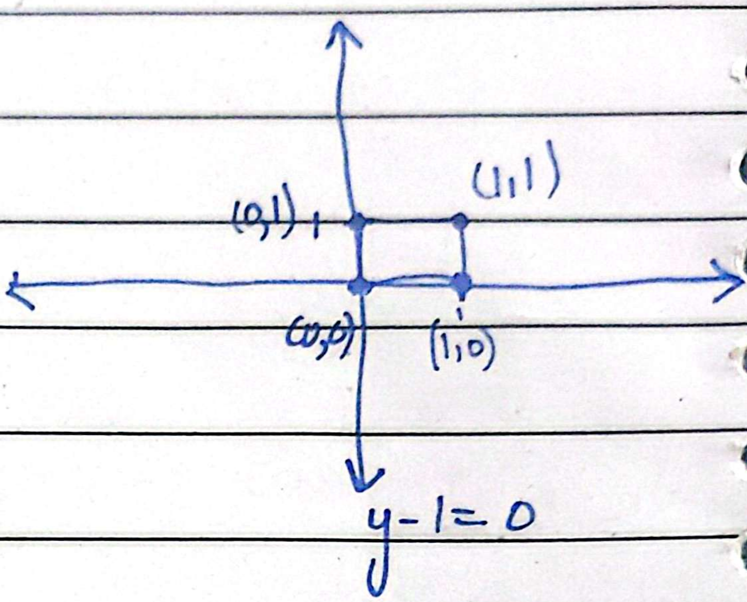
Ex 15.4 :- (1-14). (Do Q6, Q2). ✓ (Done). Bazil-uddin-khan 24k-0559

Q1).

Ans:-

$$\int y^2\,dx + x^2\,dy$$

$\;\;\;\;\;\;\;\;\;\;P \qquad\qquad Q$



$$\int_{x=0}^{x=1}\int_{y=0}^{y=1} 2x - 2y \; dy\,dx$$

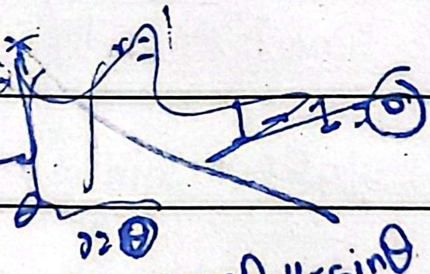
$$\int_0^1\int_0^1 2xy - y^2 \; dx$$

$$\int_0^1 [2yx - 1]\,dx \rightarrow \int_0^1 x^2 - x \rightarrow \textcircled{0}$$

Q2). unit circle radius = 1

Ans:-

$$\int_0^{2\pi}\int_0^{\sqrt{1-x^2}} ~~y\,dx + x\,dy~~ \; dy\,dx$$



"we have to evaluate linearly"

so = $x = \cos\theta,\ y = \sin\theta$

$$\rightarrow \int_0^{2\pi} 1 - 1 = \textcircled{0}$$

"No need to use radius as line"

Q3).

Ans:

$$\int_C 3xy\,dx + 2xy\,dy$$

$\;\;\;\;\;\;\;P \qquad\qquad Q$

$$\int_{x=-2}^{x=4}\int_{y=1}^{y=2} 2y - 3x \; dy\,dx = \textcircled{0}$$

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04).

$\iint_R$ $y = 3\sin\theta$

$\int_0^{2\pi}\int_0^3 1 + 2y \, dz$

$\int_0^{2\pi}\int_0^3 (1 + 2(3\sin\theta)r \, dr\,d\theta$

$\int_0^{2\pi}\int_0^3 r + 6\sin\theta r \, dr\,d\theta$

$\int_0^{2\pi} \left| \frac{r^2}{2} + 3r^2\sin\theta \right|_0^3 d\theta$

$\int_0^{2\pi} \left| \frac{9}{2} + 27\sin\theta \right| = 9\pi$

05).

Ans:- $\int_{x=0}^{\pi/2}\int_{y=0}^{y=\pi/2} -y\cos x + x\sin y \, dy\,dx$

$x^2 + y^2 + 2y + 1 = 0$

$r^2 = 2y + 1$

$x^2 + y^2 + 2y + 1 = 1$

$r^2 = -2y$

$r = -2\sin\theta$

06).

Ans:-

$\int_{\theta=0}^{\pi}\int_{r=0}^{r=-2\sin\theta} \sec^2 x - \tan^2 x \, r \, dr\,d\theta$

1 As it is $\iint 1 \, dA$

exact circle formula $\pi r^2 = \pi(1)^2 = \pi$

07).

$\int_0^{2\pi}\int_{r=0}^{r=2} (1+1)\, r \, dr\,d\theta$

$\int_0^{2\pi} \frac{r^2}{2} \, d\theta \to \int_0^{2\pi} 2 \to 8\pi$

08).

$\int_{x=0}^{x=1}\int_{y=x^2}^{y=x} 2x - 2y \, dy\,dx$

$x^2 - x = 0$

$x(x-1) = 0$

$x = 0, x = 1$

09).

Ans:-

(0,4), (2,0)

$-4 = -2$

$y - 4 = -2(x - 0)$

$y - 4 = -2x$

$x = -$

$\int_{x=0}^{x=2}\int_{y=0}^{y=4-2x} \frac{-y}{1+y} - \frac{1}{1+y} \, dy\,dx$

$\int_{y=0}^{y=4}\int_{x=0}^{\frac{y-4}{-2}} \frac{-y}{1+y} - \frac{1}{1+y}$

$\frac{-(1+y)}{1+y} \to \int_{y=0}^{y=4-2x} -y$

$\int_{x=0}^{x=2} -4 + 2x \, dx$

$-4x + x^2 = -4$

10).

Ans:-

$\int_{\theta=0}^{\theta=\pi/2}\int_{r=0}^{r=4} (-y^2 - x^2)\, r \, dr\,d\theta$

$-\int_{\theta=0}^{\theta=\pi/2}\int_{r=0}^{r=4} r^3 \, dr\,d\theta$

11).

Ans:-

$\int_{x=0}^{x=2}\int_{y=0}^{y=1} \left(\frac{-y^2}{1+y^2} + \frac{1}{1+y^2}\right) dy\,dx$

12).

Ans:-

$\int_{x=0}^{x=3}\int_{y=}^{y=3} \left(\cos^2 x - \cos x\cos y\right) = 0$

$\cos x \cos y$

13).

$y = x^2 \to x^4 - x = 0$

$x(x^3 - 1) = 0$

$x = 0, x = 1$

$\int_{x=0}^{x=1}\int_{y=x^2}^{y=\sqrt{x}} y^2 - x^2 \, dy\,dx = 0$

14).

$x^2 - 2x = 0$

(a). $x = 0, x = 2$

$y = 0, y = 4$

$\int_{x=0}^{x=2}\int_{y=x^2}^{y=2x} -6x + 2y \, dy\,dx$