Date 2 feb 2025

Bazil-uddin-uon

ch 13.6 :- → Directional Derivatives :-

Concept :-

To give direction we must have a vector (unit vector on the plane of the independent VAR)

$\frac{df}{dx/dy}$ Are (Dir. Derivatives $\hat{i}$ or $\hat{j}$)

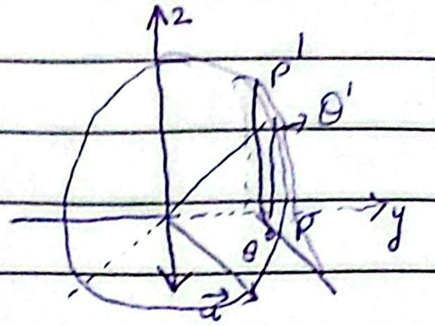
IDEA :- Things we need ; (1) Need $\hat{u}$ (on x-y plane)

(2) Need (Point) for specific R.o.C

lets take a line through "P" $\parallel$ to $\vec{u}$

create a plane parallel to the u-z plane.

that is a curve on the surface in plane



"P', P, Q, Q' are on the same surface"

$P'Q'$ is a $\boxed{\text{Secant}}$ line in the plane.

let As $Q \xrightarrow{\text{approach}} P$ the secant approaches the tangent at P'.

So, Both Δx and Δy should reach to → 0.

Now $\vec{PQ} \parallel \hat{u}$ = $\vec{PQ} = \Delta x \hat{i} + \Delta y \hat{j}$

If parallel $\vec{PQ} = h \cdot \hat{u} \rightarrow h(u_1 \hat{i} + u_2 \hat{j})$ $\Delta x = hu_1$, $\Delta y = hu_2$

So it's a scalar multiple

$\Delta x \hat{i} + \Delta y \hat{j} = h(u_1 \hat{i}) + (hu_2 \hat{j})$ vectors are equal.

$$\lim_{\substack{\Delta x \\ \Delta y \to 0}} \rightarrow \frac{f(x+\Delta x, y+\Delta y) - f(x,y)}{\sqrt{(\Delta x)^2 + (\Delta y)^2}}$$

distance formula ← $\sqrt{(\Delta x)^2 + (\Delta y)^2}$

$\frac{\text{change in Run}}{\text{change in Rise}}$ = $\boxed{\text{it just slope}}$

Directional Dr $\xrightarrow{\lim}_{h \to 0} \frac{f'(x+h \cdot u_1, y+hu_2) - f(x,y)}{h}$

$\rightarrow h = \sqrt{h^2 u_1^2 + h^2 u_2^2}$

$h \cdot \sqrt{u_1^2 + u_2^2}$

$h \cdot 1$

How to do :-

$D_{\hat{u}} f(x,y) = f_x \cdot u_1 + f_y \cdot u_2$

For $\bar{u}$ → A unit vector. u = unit vector

4). ISL
5). DLD
6). CDP

1) Calc
2) CCE
3) EW

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For A function $f(x,y)$ and A unit vector $\to \hat{u} = u_1\hat{i} + u_2\hat{j}$

$D_u f(x,y) = f_x u_1 + f_y u_2$

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Ex:- Find the Derivative of $f(x,y) = x^3 - 2x^2 + y^3$ @ $P(1,2)$ in the direction of the vector that makes AN Angle of $\theta = \pi/6$, w/ 'X'-Axes?

1). Point $(1,2)$ $\quad \hat{u} = \cos\pi/6\,\hat{i} + \sin\pi/6\,\hat{j} \to \hat{u} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$

$\hat{u} = \cos\theta\hat{i} + \sin\theta\hat{j}$ $\quad f_x = 3x^2 - 4x \quad f_y = 3y^2$

$$\boxed{D_{\hat{u}} f(x,y) = (3x^2 - 4x)\left(\frac{\sqrt{3}}{2}\right) + (3y^2)(1/2)}$$ $\to$ slope of tangent line so this is a scalar function

eq. of slope of tangent line to the surface but only for $\vec{u}$ direction

$$D\hat{u}(1,2) = \left[3(1)^2 - 4(1)\right]\frac{\sqrt{3}}{2} + 3(2)^2\left(\frac{1}{2}\right) = \frac{6-\sqrt{3}}{2}$$

if we change a qs a little bit:-

Ex:- Find the derivative of $f(x,y) = x^3 - 2x^2 + y^3$ @ $P(1,2)$ in the Direction of vector $\vec{v} = 2\sqrt{3}\hat{i} + 2\hat{j}$ Not the Then is the unit vector

$$\hat{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{2\sqrt{3}\hat{i} + 2\hat{j}}{\sqrt{(2\sqrt{3})^2 + (2)^2}} = \frac{2\sqrt{3}}{4}\hat{i} + \frac{2}{4}\hat{j}$$

Remaing answer is same as above.

$$D_u f(x,y) = f_x u_1 + f_y u_2 = \underbrace{(f_x\hat{i} + f_y\hat{j})}_{\text{weird vector}}(\hat{u})$$

we call weird vector The GRADIENT

gradient of $f \to \quad \nabla f(x,y) = f_x\hat{i} + f_y\hat{j} \to D_u f(x,y) = \nabla f(x,y) \cdot \vec{u}$

Notes:- $D\hat{u}$ is a slope, A scalar NOT A vector

∇f is a vector. (part of $D\hat{u}$)

— Relates to the "grade" ("climb") of the surface [FOR A specific Grade must have a point and $\hat{u}$]

Properties of ∇f:-

#1 If $\nabla f = \vec{0}$ THEN $D\hat{u}f = 0$ for any $\hat{u}$

#2. $f(x,y)$ it has max value of $\|\nabla f(x,y)\|$ and this will happen only when $\hat{u} = c\nabla f$; 'c' is a scalar. (in some direction)

$$D_u f(x,y) = \nabla f \cdot \hat{u} = \|\Delta f\| \cdot \|\Delta \hat{u}\| \cos\theta$$

$$D_u f(x,y) = \|\nabla f\| \cdot \|\hat{u}\| \cos\theta$$ Max value of $\cos\theta = 1$ when $\theta = 0°$

$$D_u f(x,y) = \|\nabla f\| \cdot \cos\theta$$ GENIUS so angle is b/w ∇f and $\vec{u}$ is "0" $\to \|$

when your unitvector is in same direction as gradient vector.

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- Any other θ Gives A $\cancel{x} < 1$ and Takes A fraction of ∇f.... $D\vec{u}$ Becomes less.

3). $D\hat{u} f(x,y)$ has Its min value of $-\|\nabla f(x,y)\|$ $c\theta = \pi$

So ∇f Gives vector for the STEEPEST GRADE of a surface at a point.

If $\hat{u}$ is not $\parallel$ To ∇f

Think of $\hat{u}$ As Turning

$D\vec{u} f$ from Direction of steepest climb, ∇f.

Q). Find $\nabla f(x,y)$ for $f(x,y) = \dfrac{1}{x^2+y^2}$ @ $(1,2)$.

$f_x = 1(x^2+y^2)^{-1} \rightarrow \dfrac{-1}{(x^2+y^2)^2}(2x)$ $\quad f_y = \dfrac{-2y}{(x^2+y^2)^2}$

"The vector for max slope of surface $\in$ At Anypoint"

$$\nabla f(x,y) = \frac{-2x}{(x^2+y^2)^2}\hat{i} - \frac{2y}{(x^2+y^2)^2}\hat{j}$$

$$\nabla f(1,2) = \frac{-2(1)}{(1^2+2^2)^2}\hat{i} - \frac{2(2)}{(1^2+2^2)^2}\hat{j} = \frac{-2}{25}\hat{i} - \frac{4}{25}\hat{j}$$

$\rightarrow$ This is just vector that gives direction at tangent line.

$$\nabla f(x,y,z) = f_x\hat{i} + f_y\hat{j} + f_z\hat{k}$$

Q). For $(x,y,z) = \dfrac{x+y}{x+z}$ find $\nabla f(1,2,3)$?

$$f_x = \frac{z-y}{(x+z)^2} \quad f_y = \frac{1}{(x+z)} \quad f_z = \frac{-(x+y)}{(x+z)^2}$$

$$\nabla f(x,y,z) = \left(\frac{z-y}{(x+z)^2}\right)\hat{i} + \frac{1}{x+z}\hat{j} - \frac{x+y}{(x+z)^2}\hat{k} \quad \nabla f(1,2,3) = \boxed{\frac{1}{16}\hat{i} + \frac{1}{4}\hat{j} - \frac{3}{16}\hat{k}}$$

Q). Find $D_u f(x,y)$ for $f(x,y) = (x^2+y^2+1)^{1/2}$ @ $P(2,2)$ In the direction of $\vec{v} = 3i+4j$?

step 1: find ∇f @ point $\nabla f(2,2) = f_x = \dfrac{x}{\sqrt{x^2+y^2+1}}$, $f_y = \dfrac{y}{\sqrt{x^2+y^2+1}} = \dfrac{x}{\sqrt{x^2+y^2+1}}\hat{i} + \dfrac{y}{\sqrt{x^2+y^2+1}}\hat{j}$

$(2,2) \rightarrow \dfrac{2}{3}\hat{i} + \dfrac{2}{3}\hat{j}$ just take out the magnitude

"if we want to ask steepest slope"

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step 2:- Find $\vec{u}$

$$\vec{u} = \frac{\vec{v}}{|v|} = \frac{3\hat{i}+4\hat{j}}{\sqrt{9+16}} = \frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}$$

$$D_{\hat{u}} f(2,2) = \nabla f(2,2)\,\hat{u}$$

$$= \left(\frac{2\hat{i}}{3} + \frac{2\hat{j}}{3}\right)\left(\frac{3\hat{i}}{5} + \frac{4\hat{j}}{5}\right) = \frac{14}{15}$$ → slope of tangent line at that point only in direction of vector

Q). $f(x,y) = xe^{-y}$ Find $D_{\vec{u}} f(x,y)$ @ $P(2,0)$ In direction of line from $P(2,0)$ To $Q(-1,2)$.

$f_x = e^{-y}$

$f_y = -xe^{-y}$ $\qquad f_x\hat{i} + f_y\hat{j} \rightarrow e^{-y}\hat{i} - xe^{-y}\hat{j}$

$(2,0)$ $\qquad e^{-0}\hat{i} - (2)e^{-0}\hat{j}$

$\boxed{1\hat{i} - 2\hat{j}}$ $\qquad \vec{u} = \frac{1\hat{i}-2\hat{j}}{\sqrt{5}} = \boxed{\frac{\hat{i}}{\sqrt{5}} - \frac{2\hat{j}}{\sqrt{5}}}$

$$\vec{v} = \frac{PQ = (-1-2)\hat{i} + (2-0)\hat{j}}{\sqrt{13}}$$

$= \cancel{(1\hat{i}-2\hat{j})\left(\frac{\hat{i}}{\sqrt{5}} - \frac{2\hat{j}}{\sqrt{5}}\right)} =$

$v = \frac{-3\hat{i}}{\sqrt{13}} + \frac{2\hat{j}}{\sqrt{13}}$

$$(\hat{i}-2\hat{j})\left(\frac{-3\hat{i}}{\sqrt{13}} + \frac{2\hat{j}}{\sqrt{13}}\right) = \frac{-7}{\sqrt{13}}$$

Q). $f(x,y,z)$ find $D_{\vec{u}} f(x,y,z)$ in direction of line from $P(2,2,2)$ & $Q(6,-2,6)$ c $(4,2,2)$

$\hookrightarrow = \sqrt{xyz}$

Ans:- $f_x = \frac{1}{2\sqrt{xyz}} \times yz$, $f_y = \frac{1}{2\sqrt{xyz}} \times xz$ $\quad f_z = \frac{1}{2\sqrt{xyz}} \times xy$

$$f_x = \frac{yz}{2\sqrt{xyz}}\hat{i} + \frac{xz}{2\sqrt{xyz}}\hat{j} + \frac{xy}{2\sqrt{xyz}}\hat{k}$$ put $(4,2,2) \rightarrow$ so $\boxed{\frac{\hat{i}}{2} + \hat{j} + \hat{k}}$

$$\vec{u} = \frac{\vec{v}}{|\vec{v}|} = \frac{(4\hat{i} - 4\hat{j} + 4\hat{k})}{(4^2+4^2+4^2)^{1/2}} = \boxed{\frac{1}{3} - \frac{2\hat{j}}{3} + \frac{2\hat{k}}{3}}$$ ← DOT Product

$$\nabla f(4,2,2)\cdot\hat{u}$$

$$\left(\frac{\hat{i}}{2} + \hat{j} + \hat{k}\right)\left(\frac{\hat{i}}{3} - \frac{2}{3} + \frac{2\hat{k}}{3}\right) = 1/2$$

$\boxed{(45-48)\ \&\ (35,36).}$

Ex 12.6 :- (1-45, 53-66). Bezil-udlin-chan.

1). : $D_u = f_x(i) + f_x(j)$

$$\underset{(1,1)}{D_u} = \frac{3}{2}(1+xy)^{1/2}(y)\left(\frac{1}{\sqrt{2}}\right) + \frac{3}{2}(1+xy)^{1/2}(x)\left(\frac{1}{\sqrt{2}}\right) \quad 6\sqrt{2}$$

2). $\underset{(3,5)}{D_u} = 5\cos(5x-3y)\left(\frac{3}{5}\right) + 3\cos(5x-3y)\left(\frac{-4}{5}\right) \rightarrow 27/5$

3). $\underset{(0,0)}{D_u} = \left[\frac{1}{1+x^2+y} \cdot x(2x)\right]\left[\frac{-1}{\sqrt{10}}\right] + \frac{2y}{1+x^2+y^2}\,\frac{x-3}{\sqrt{10}} = 0$

4). $\underset{(3,4)}{D_u} = \left[\frac{(x-y)(c)-(cx+dy)(1)}{(x-y)^2}\right]\left(\frac{4}{5}\right) + \left[\frac{(x-y)(d)-(cx+dy)(-1)}{(x-y)^2}\right]\left(\frac{3}{5}\right)$

5). $D_u = 4x^5y^2z^3$

$(2,-1,1)$ $u = 1/3 i + 2/3 j - 2/3 k$

$$D_u = (20x^4y^2z^3)\left(\frac{1}{3}\right) + (8x^5yz^3)(2/3) - (12x^5y^2z^2)(2/3)$$

$\boxed{D_u = -320}$

6) $\underset{(0,2,3)}{D_u} = (ye^{xz}(z))\left(\frac{2}{7}\right) + (ye^{xz})\left(\frac{-3}{7}\right) + (yxe^{xz}+2z)\left(\frac{6}{7}\right)$

$\boxed{D_u = 87/7}$

7). $\underset{(-1,2,4)}{D_u} = \left(\frac{2x}{x^2+2y^2+3z^2}\right)\left(\frac{-3}{13}\right) + \left(\frac{4y}{x^2+2y^2+3z^2}\right)\left(\frac{-4}{13}\right) + \left(\frac{6z}{x^2+2y^2+3z^2}\right)\left(\frac{12}{13}\right)$

$\boxed{D_u = -314/741}$

8). $\underset{(1/2,\,1/3,\,\pi)}{D_u} = (\cos xyz)(yz)\left(\frac{2}{\sqrt{3}}\right) + \cos(xyz)(xz)\left(\frac{-1}{\sqrt{3}}\right) + \cos(xyz)(xy)\left(\frac{1}{\sqrt{3}}\right)$

$= -1.28$

GENIUS

$\hat{u} = \frac{4i - 3j}{\sqrt{4^2+3^2}}$

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9). $\underset{(2,1)}{Df} = (12x^2y^2)\left(\frac{4}{5}\right) - (8x^3y)\left(\frac{3}{5}\right)$

$\boxed{=}$

10) $\underset{(1,0)}{Df} = (27x^2)\left(\frac{1}{\sqrt{2}}\right) + (6y^2)\left(\frac{-1}{\sqrt{2}}\right)$

$\boxed{}$

11). $\underset{(1,4)}{Df} = \left(\frac{y^2}{x}\right)\left(\frac{-3}{\sqrt{18}}\right) + (2y\ln x)\left(\frac{+3j}{\sqrt{18}}\right) = \boxed{}$

$\hat{u} = \frac{-3i + 3j}{\sqrt{9+9}}$

12). $\underset{(0,\pi/4)}{Df} = (e^x\cos y)\left(\frac{5}{\sqrt{29}}\right) + (e^x - \sin y)\left(\frac{-2j}{\sqrt{29}}\right) = \boxed{}$

$u = \frac{5i - 2j}{\sqrt{29}}$

13). $\underset{(-2,2)}{Df} = \left(\frac{1}{1+(y/x)^2} \times \frac{-y}{x^2}\right)\left(\frac{-1}{\sqrt{2}}\right) + \left(\frac{1}{1+(y/x)^2} \times \frac{1}{x}\right)\left(\frac{-1}{\sqrt{2}}\right)$

$= \boxed{}$

14). $\underset{(0,0)}{Df} = \cancel{(e^y + ye^x)}$

$(e^y - ye^x)\left(\frac{5}{\sqrt{29}}\right) - \left(\frac{2}{\sqrt{29}}\right)(xe^y - e^x) = \boxed{}$

15). $\underset{(-3,0,4)}{Df} = (y)\left(\frac{1}{\sqrt{3}}\right) + (x)\left(\frac{1}{\sqrt{3}}\right) + (2z)\left(\frac{1}{\sqrt{3}}\right) = \boxed{}$

16). $\underset{(-3,1,4)}{Df} = \left(-\frac{1}{2\sqrt{x^2+z^2}} \times 2x\right)\left(\frac{2}{3}\right) + \left(1\right)\left(\frac{-2}{3}\right) + \left(\frac{-1}{2\sqrt{x^2+z^2}} \times 2z\right)\left(\frac{-1}{3}\right)$

$u = \frac{2i - 2j - k}{\sqrt{4+4+1}}$ $= \boxed{}$

17). $f(x,y,z) = \frac{2-x}{2+y} = \left(\frac{-1}{2+y}\right)\left(\frac{-6}{7}\right) + \left(\frac{-2}{(2+y)^2}\right)\left(\frac{3}{7}\right) + \left(\frac{(2+y)(1) - (2-x)(\ldots}{(2+y)^2}\right.$

$\hookrightarrow \times \left(-2/7\right)$

$u = \frac{-6i + 3j - 2k}{\sqrt{36+9+4} = \sqrt{49}}$ $= \boxed{}$

18). $\underset{(-2,2,-1)}{df} = \left(e^{x+y+3z}\right)\left(\frac{20}{\sqrt{441}}\right) + \left(e^{x+y+3z}\right)\left(\frac{-4}{\sqrt{441}}\right) + \left(e^{x+y+3z}\right)(3)\left(\frac{5}{\sqrt{441}}\right)$

$u = \frac{20i - 4j + 5k}{\sqrt{441}}$ $\boxed{df =}$

19). $z = \cos\pi/3\, i + \sin\theta j$

$z = 1/2\, i + \sqrt{3}/2\, j$

$\frac{y}{2\sqrt{xy}}\left(\frac{1}{2}\right) + \frac{x}{2\sqrt{xy}}\left(\frac{\sqrt{3}}{2}\right) \rightarrow \boxed{\frac{y+\sqrt{3}}{8}}$

20). $z = 0i + 1j$

$\left[\frac{(x+y)(1) - (x-y)}{(x+y)^2}\right](0) + \left[\frac{(x+y)(1) - (x-y)(-1)}{(x+y)^2}\right]$

$= 2$

21). $z = \frac{\sqrt{3}}{2} i + \frac{1}{2} j$

$2(\sec^2(2x+y))\left(\frac{\sqrt{3}}{2}\right)$

22). $-i + 0j$

$(\cosh x \cosh y)(-1) = -1$

23). $\left(\frac{(x+y)(1) - (x)(1)}{(x+y)^2}\right)\left(\frac{-1}{\sqrt{2}}\right) + \left(\frac{(x+y)(0) - (x)(1)}{(x+y)^2}\right)\left(\frac{-1}{\sqrt{2}}\right)$

$= \sqrt{2}/8$

(1,1) (or 1)
(-1, -2)

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24). $\left(-e^{-x}\sec y\right)(\hat{i}) + \left(e^{-x}\sec y \tan y\right)(\cancel{\pi}\cdot \hat{j})$

$x=0, y=\pi/4$

$\cos\theta\, \hat{i} + \sin \pi/4\, \hat{j}$ $\left(-\sqrt{2}\hat{i} + \sqrt{2}\hat{j}\right)$

$(0-0), (0-\pi/4) \rightarrow \dfrac{0i - \pi/4 J}{\sqrt{0^2 + (\pi/4)^2}} = \dfrac{-\pi/4\, J + 0i}{\pi/4}$

$(\sqrt{2})(-1) \rightarrow \boxed{-\sqrt{2}}$ $\boxed{-1J + 0i}$

25). $(1,1)$

$f_x = \dfrac{1}{2\sqrt{xy}}\, y e^y\, i = 1.36i \rightarrow 1.36i + 2.72J$

$f_y = \left(\sqrt{xy}\right)(e^y) + \left(\dfrac{1}{2\sqrt{xy}} \times x\right)(e^y) J = 2.72J$

$\left((0-1)i, (-1-1)J\right) \rightarrow \dfrac{-1i - 2J}{\sqrt{5}} \rightarrow \dfrac{-1i}{\sqrt{5}} - \dfrac{2J}{\sqrt{5}}$

$\left(1.36i + 2.72J\right)\left(\dfrac{-1i}{\sqrt{5}} - \dfrac{2J}{\sqrt{5}}\right) = \boxed{-3.04}$

26). $\left[\dfrac{-y}{(x+y)^2}\right] i + \left[\dfrac{(x+y)(1) - (y)(1)}{(x+y)^2}\right] J \rightarrow \dfrac{-y}{(x+y)^2} i + \dfrac{x}{(x+y)^2} J$

$\boxed{\text{Unit} = 0i + 0J \text{ vector}}$ $\dfrac{-3i}{25} + \dfrac{2J}{25}$

27). $\left[-y(x+2)^2\right][i] + \left(\dfrac{1}{x+2}\right)(J) \div \left[\dfrac{y}{(x+2)^2}\right] \times J \rightarrow \boxed{-1i + 1J - 1k}$

$(\cancel{0+1})i\ (-1-2)i + (1)J + (1)k$

$\dfrac{(-3i + 1J + 1k)}{\sqrt{11}} = \left(-1i + 1J - 1k\right)\left(\dfrac{-3i}{\sqrt{11}} + \dfrac{1J}{\sqrt{11}} + \dfrac{1k}{\sqrt{11}}\right)$

$\dfrac{3}{\sqrt{11}} + \dfrac{1}{\sqrt{11}} - \dfrac{1}{\sqrt{11}} = \boxed{\dfrac{3}{\sqrt{11}}}$

28). $f(x,y,z) = x^3 y^2 z^5 - 2xz + yz + 3x$

$\left(3\cancel{x^2 y^2 z^5}\right)(i)$ $\left(3x^2y^2z^5 - 2z + 3\right)i + \left(2yx^3z^5 + z\right)J + \left(5z^4y^2x^3 - 2x + y\right)k$

$\cancel{\left(3x^2 y^2 z^5\right)}$

$\nabla = 13i + 5J - 20k$

$\left(0-(-1)\right)i + \left(0-(-2)\right)J + \left(-1-1\right)k$

$\dfrac{1i + 2J + (-2)k}{\sqrt{9}}$

$(13)\left(\dfrac{1}{3}\right) + (5)\left(\dfrac{2}{3}\right) - 20\left(\dfrac{-2}{3}\right) = 21$

29). $D_u f(1,2) = -5$, $D_v f(1,2) = 10$

$u = 3/5 i - 4/5 J$, $v = 4/5 i + 3/5 J$

$-5 = (f_x)(+3/5) - f_y(4/5)$

$10 = f(x)(4/5) \cdot f_y(3/5)$

$-25 = 3f_x - 4f_y$, $50 = 4f_x + 3f_y$

(a) $f_x = 5$, (b) $f_y = 10$

(c). $(5)(-5) + (10)(10) \rightarrow 75$

30). $f_x(5,1) = -3$, $f_y = 2$

$f_x i + f_y J$ $(-4,3)(-5,1)$

$\boxed{-3i + 2J}$ $\left(\dfrac{1i}{\sqrt{5}} + \dfrac{2J}{\sqrt{5}}\right)$ $(-4+5)i + (3-1)J$

$\dfrac{1i + 2J}{\sqrt{5}}$

$\dfrac{-3}{\sqrt{5}} + \dfrac{4}{\sqrt{5}} \rightarrow \dfrac{1}{\sqrt{5}}$

31). ~~Ex 32~~ (II)

32). P.

$\boxed{\rightarrow 2x + 5x}$

$\rightarrow 2x^2 - 5x + 6$

$\rightarrow \dfrac{2x^{3\times 2}}{3\times 2} - \dfrac{5x^{2}}{2\times 5} + \dfrac{6x}{1}$

(33.40) $\rightarrow$

Q33). $\nabla z = \cos(7y^2 - 7xy)(-7y)\hat{i} + \cos(7y^2 - 7xy)(14y - 7x)J$

Q34). $\nabla z = \left[7\cos\left(\frac{6x}{y}\right)\times\frac{6}{y}\right]\hat{i} + \left[7\cos\left(\frac{6x}{y}\right)\left(\frac{-6x}{y^2}\right)\right]J$

Q35). $\nabla z = \left(\dfrac{(6x-7y)(6) - (6x+7y)(6)}{(6x-7y)^2}\right)\hat{i} + \left(\dfrac{(6x-7y)(7) - (6x+7y)(-7)}{(6x-7y)^2}\right)J$

Q36). $\nabla z = \left(\dfrac{(x+8y)(6e^{3y}) - (6xe^{3y})(1)}{(x+8y)^2}\right)\hat{i} + \left(\dfrac{(x+8y)(18xe^{3y}) - (6xe^{3y})(8)}{(x+8y)^2}\right)J$

Q37). $\nabla w = (-9x^8)\hat{i} + (-3y^2)J + (12z^{11})k.$

Q38). $\nabla w = (e^{8y}\sin 6z)\hat{i} + (8xe^{8y}\sin 6z)J + (6xe^{8y}\cos 6z)k.$

Q39). $\nabla w = \left(\dfrac{1}{\sqrt{x^2+y^2+z^2}} \times \dfrac{1\times 2x}{2\sqrt{x^2+y^2+z^2}}\right) \rightarrow \dfrac{1}{2}\left(\dfrac{2x}{x^2+y^2+z^2}\hat{i} + \dfrac{2y}{x^2+y^2+z^2}J + \dfrac{2z}{x^2+y^2+z^2}\right)J.$

$\ln(\sqrt{A}) \rightarrow \frac{1}{2}\ln(A)$

Q40). $\nabla w = \left((e^{-5x})(\sec(x^2yz)\tan(x^2yz)\times 2xyz) + (\sec x^2yz)(-5e^{-5x})\right)\hat{i} + (e^{-5x}\sec x^2yz \times x^2z)J + (e^{-5x}\sec x^2yz)(x^2y)k.$

Q41). $\rightarrow \nabla f = (10x)\hat{i} + (4y^3)J \rightarrow (40\hat{i} + 32J).$

Q42). $\rightarrow \nabla f = (5\cos x^2)(2x)(\hat{i}) + (-\sin 3y)(3)J \rightarrow (627\hat{i} + 0J)$

Q43). $\rightarrow \nabla f = 3(x^2+xy)^2(2x+y)\hat{i} + 3(x^2+xy)^2(x)J \rightarrow (-9\hat{i} - 3J).$

Q44). $\rightarrow \nabla f = \left(\dfrac{y}{x+y+z}\right)\hat{i} + \left[gx\dfrac{1}{x+y+z} + \ln(x+y+z)\right]J + \left[\dfrac{y}{x+y+z}\right]k \rightarrow \boxed{4\hat{i} + 4J + 4k}$

Q45). $\rightarrow \nabla f = (3\tan^2 x\sec^2 x)(2y^2)\hat{i} + (2yz\tan x^3)J + (y^2\tan^3 x)k \rightarrow \boxed{108\hat{i} - 6J + 9k}$

Q53). $(12x^2y)\hat{i} + (8x^3y)J \rightarrow (12\hat{i} - 8J)$

$\dfrac{12\hat{i} - 8J}{4\sqrt{13}}$, rate of change: $4\sqrt{13}$

Q54). $(3)\hat{i} - (1/y)J \rightarrow (3\hat{i} - 1/4J)$

$\dfrac{3\hat{i} - 1/4J}{\sqrt{145}/4}$, rate of change $= \sqrt{145}/4$

Q55). $\left(\dfrac{1\times 2x}{2\sqrt{x^2+y^2}}\right)\hat{i} + \left(\dfrac{2y}{2\sqrt{x^2+y^2}}\right)J \rightarrow \boxed{\frac{4}{5}\hat{i} + \frac{3}{5}J}$ rate of change $= 1$

Q56). $\left(\dfrac{(x+y)(1) - (x)}{(x+y)^2}\right)\hat{i} + \left(\dfrac{-(x)}{(x+y)^2}\right)J \rightarrow \dfrac{\hat{i}}{2} \rightarrow$

$\dfrac{\frac{\hat{i}}{2}}{1/2} \rightarrow$ rate of change $1/2$

Q57). $(3x^2z^2)\hat{i} + (3y^2z)J + (2zx^3 + y^3 + 1)k$

$3\hat{i} - 3J + 0k.$

$\dfrac{3\hat{i} - 3J}{\sqrt{18}} \rightarrow \sqrt{18} \rightarrow$ rate of ch

Q58). $\left(\dfrac{1}{2\sqrt{x-3y+z^2}}\right)\hat{i} + \left(\dfrac{-3}{2\sqrt{x-3y+z^2}}\right)J + \left(\dfrac{2z}{\sqrt{x-3y+z^2}}\right)k$

$\dfrac{\frac{1}{6}\hat{i} - \frac{1}{2}k + \frac{2}{3}k}{7} \rightarrow$ rate of change:

Q49). $\left(-\dfrac{1}{2}(x^2+y^2)^{1/2}\times 2x\right)\hat{i} + \left(-\dfrac{1}{2}(x^2+y^2)^{1/2}(2y)\right)J$

$\boxed{-15\hat{i} - 20J}$