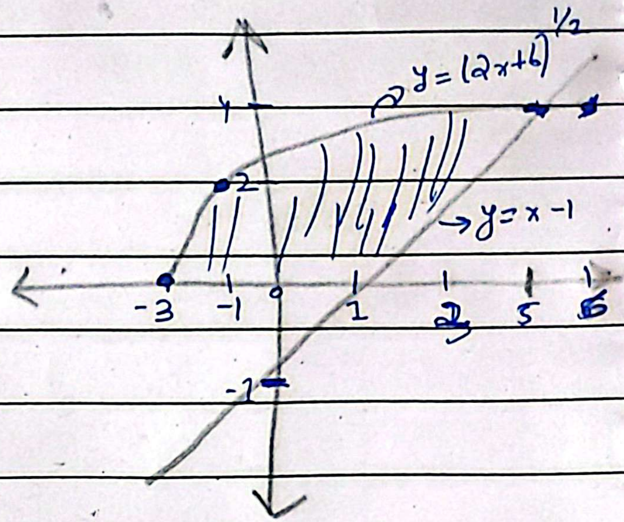
Bazil-uddin-khan 246-0559 BSCS 24. "Assignment - 3"

Q1).

Ans:- $\iint_D xy \, dA$. $y = x-1$, $y^2 = 2x+6$



$$\int_{x=-1}^{x=5} \int_{y=x-1}^{y=(2x+6)^{1/2}} xy \, dy \, dx$$

$$\int_{-1}^{5} \int_{y=x-1}^{y=(2x+6)^{1/2}} \frac{xy^2}{2} \, dy \, dx$$

$$\int_{-1}^{5} \left| \frac{x((2x+6)^{1/2})^2}{2} - \frac{x(x-1)^2}{2} \right| dx$$

$$\int_{-1}^{5} \left| \frac{2x^2+6x}{2} - \frac{x(x^2-2x+1)}{2} \right| dx$$

$$\int_{-1}^{5} x^2 + 3x - \frac{x^3}{2} + x^2 - \frac{x}{2} \, dx$$

$$\int_{-1}^{5} \frac{x^3}{3} + \frac{3x^2}{2} - \frac{x^4}{8} + \frac{x^3}{3} - \frac{x^2}{4}$$

$$= \left| \frac{5^3}{3} + \frac{3(5)^2}{2} - \frac{5^4}{8} + \frac{5^3}{3} - \frac{5^2}{4} \right| - \left| \frac{(-1)^3}{3} + \frac{3(-1)^2}{2} - \frac{(-1)^4}{8} + \frac{(-1)^3}{3} - \frac{(-1)^2}{4} \right| = 36$$

Ans

$\because (x-1)^2 = 2x+6$

$x^2 - 2x + 1 = 2x + 6$

$x^2 - 4x - 5 = 0$

$x^2 - 5x + x - 5 = 0$

$x(x-5) + (x-5) = 0$

$\therefore x = 5, x = -1$

Date:

Bazil-uddin-khan 24k-0559 BSCS-2H.

Q2).

Ans:- $z = 2x + y^2$

$x = y^2,\ x = y^3 \longrightarrow y = \sqrt{x},\ y = x^{1/3}$

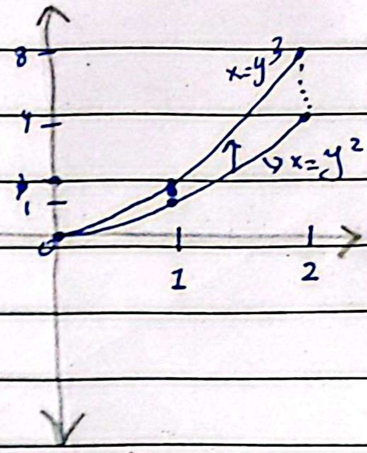
$\because y^2 = y^3$

$y^3 - y^2 = 0$

$y^2(y-1) = 0$

$\because \boxed{y=0, y=1}$

$\boxed{x=0, x=1}$



$$\int_{x=0}^{x=1}\int_{y=x^{1/2}}^{y=x^{1/3}} 2x + y^2 \, dy\,dx$$

$$\int_{x=0}^{x=1}\int_{y=x^{1/2}}^{y=x^{1/3}} 2xy + \frac{y^3}{3} \, dy\,dx$$

$$\int_{x=0}^{x=1} \left| \left(2x(x)^{1/3} + \frac{(x^{1/3})^3}{3}\right) - \left(2x(x)^{1/2} + \frac{(x^{1/2})^3}{3}\right) \right| dx$$

$$\int_{x=0}^{x=1} 2x^{4/3} + \frac{x}{3} - 2x^{3/2} - \frac{x^{3/2}}{3} \, dx.$$

$$\int_{x=0}^{x=1} \frac{6x^{7/3}}{7} + \frac{x^2}{6} - \frac{4x^{5/2}}{5} - \frac{2x^{5/2}}{15}$$

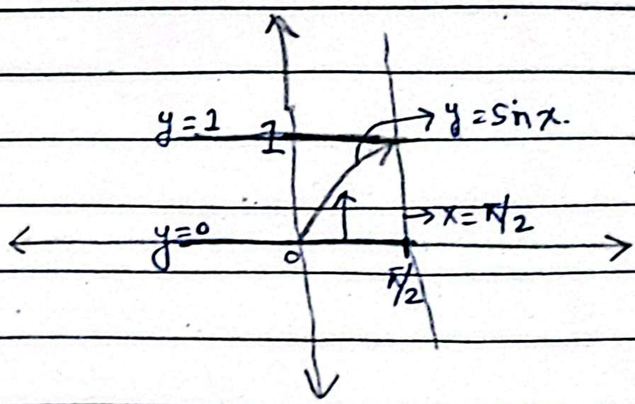
$$= \frac{6(1)^{7/3}}{7} + \frac{1^2}{6} - \frac{4(1)^{5/2}}{5} - \frac{2(1)^{5/2}}{15} - 0$$

$$= \boxed{\frac{19}{210}}$$ **Ans:-**

Bazil-Uddin-Khan 244-0559 BSCS-2H **Date:**

Q3).

Ans:-
$$\int_0^1 \int_{\arcsin y}^{\pi/2} \cos x \sqrt{1+\cos^2 x}\, dx\, dy.$$



$x=\pi/2,\ x=\sin^{-1}(y) \rightarrow y=\sin x$

$y=1,\ y=0 \rightarrow x=0.$

$$\int_{x=0}^{x=\pi/2} \int_{y=0}^{y=\sin x} \cos x \sqrt{1+\cos^2 x}\; dy\, dx$$

$$\int_{x=0}^{x=\pi/2} \left| y \cos x (1+\cos^2 x)^{1/2} \right|_{y=0}^{y=\sin x} dx$$

$$\int_{x=0}^{x=\pi/2} \sin x \cos x (1+\cos^2 x)^{1/2} - 0 \quad dx$$

$$\int_{x=0}^{x=\pi/2} \sin x \cos x (1+\cos^2 x)^{1/2} - 0 \quad dx$$

$$\int_{x=0}^{x=\pi/2} \frac{1}{2} \frac{-2\sin x \cos x\,(1+\cos^2 x)^{1/2+1}}{1/2+1}\, dx$$

$$\int_{x=0}^{x=\pi/2} -\frac{1}{\cancel{2}} \times \cancel{2}\, \frac{(1+\cos^2 x)^{3/2}}{3}$$

$$\int_{x=0}^{x=\pi/2} -\frac{(1+\cos^2 x)^{3/2}}{3}$$

$$= -\frac{(1+\cos^2(\pi/2))^{3/2}}{3} - \left(-\frac{(1+\cos^2(0))^{3/2}}{3}\right)$$

$$= -\frac{1}{3} + \frac{1}{3}(2)^{3/2} = \frac{1}{3}\left(2^{3/2}-1\right) = \boxed{0.609}\ \text{Ans:}$$

Bazil-Uddin-khan 246-0559 BSCS-2H. Date:

Q4).

Ans:-

$r = 2 + 2\sin\theta$

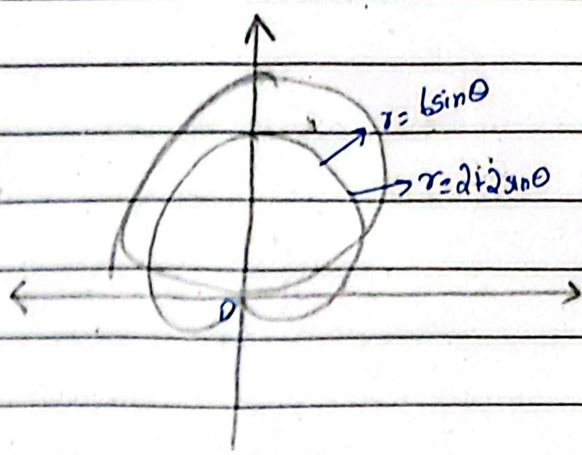
$r = 6\sin\theta$

$2 + 2\sin\theta = 6\sin\theta$

$4\sin\theta = 2$

$\sin\theta = \frac{1}{2}$

$\theta = \frac{\pi}{6}$, and $\theta = \frac{5\pi}{6}$



$$\int_{\theta=\pi/6}^{\theta=5\pi/6} \int_{r=2+2\sin\theta}^{r=6\sin\theta} 1 \; r\,dr\,d\theta$$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} \int_{2+2\sin\theta}^{r=6\sin\theta} \frac{r^2}{2}\,dr\,d\theta$$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} \left| \frac{36\sin^2\theta}{2} - \frac{(2+2\sin\theta)^2}{2} \right| d\theta$$

R.W :-

$\therefore (2\sin\theta + 2)^2$

$\therefore 4\sin^2\theta + 8\sin\theta + 4$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} 18\sin^2\theta - \left(\frac{4\sin^2\theta + 8\sin\theta + 4}{2}\right) d\theta$$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} 18\sin^2\theta - 2\sin^2\theta - 4\sin\theta - 2 \; d\theta$$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} 18\left(\frac{1-\cos2\theta}{2}\right) - 2\left(\frac{1-\cos2\theta}{2}\right) - 4\sin\theta - 2$$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} 9(1-\cos2\theta) - 1 + \cos2\theta - 4\sin\theta - 2$$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} 9 - 9\cos2\theta - 1 + \cos2\theta - 4\sin\theta - 2$$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} 6 - 8\cos2\theta - 4\sin\theta \; d\theta$$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} 6\theta - \frac{8}{2}\sin2\theta + 4\cos\theta$$

$$\int_{\theta=\pi/6}^{\theta=5\pi/6} 6\theta - 4\sin2\theta + 4\cos\theta$$

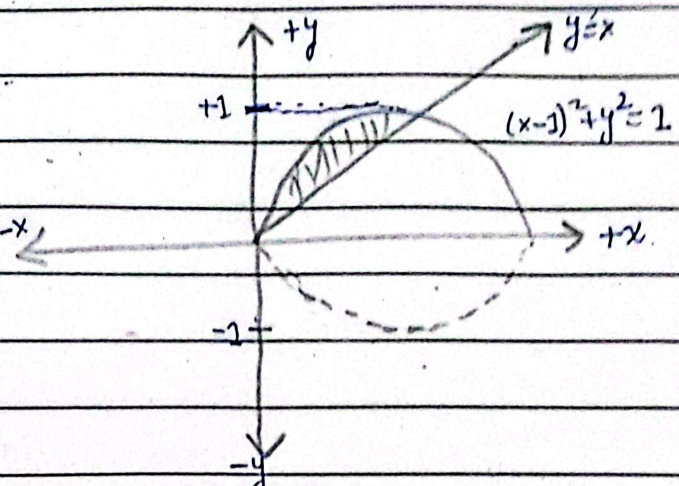
$$= \left|6\left(\frac{5\pi}{6}\right) - 4\sin\left(2\times\frac{5\pi}{6}\right) + 4\cos\left(\frac{5\pi}{6}\right)\right| - \left|6\left(\frac{\pi}{6}\right) - 4\sin\left(\frac{\pi}{3}\right) + 4\cos\left(\frac{\pi}{6}\right)\right|$$

$$= 4\pi$$

Ans:-

Bazil-uddin-khan 24k-0559 BSCS 2M.

Q5)



Ans:-

$$\int_{\theta=\pi/4}^{\theta=\pi/2}\int_{r=0}^{r=2\cos\theta} 2y \; r\,dr\,d\theta.$$

$$\int_{\theta=\pi/4}^{\theta=\pi/2}\int_{r=0}^{r=2\cos\theta} 2y \; r\,dr\,d\theta$$

$$\int_{\theta=\pi/4}^{\theta=\pi/2}\int_{r=0}^{r=2\cos\theta} 2r^2\sin\theta \; dr\,d\theta$$

$$\int_{\theta=\pi/4}^{\theta=\pi/2}\left[\frac{2r^3}{3}\sin\theta\right]_{r=0}^{r=2\cos\theta} d\theta$$

$$\int_{\theta=\pi/4}^{\theta=\pi/2} \frac{2(2\cos\theta)^3}{3}\sin\theta\,d\theta$$

$$-\int_{\theta=\pi/4}^{\theta=\pi/2} -\frac{16}{3}(\cos\theta)^3 \sin\theta\,d\theta$$

$$+\int_{\theta=\pi/4}^{\theta=\pi/2} -\frac{16}{3}\frac{(\cos\theta)^4}{4}$$

$$\int_{\theta=\pi/4}^{\theta=\pi/2} -\frac{4}{3}(\cos\theta)^4$$

$$= -\frac{4}{3}\left(\cos\left(\frac{\pi}{2}\right)\right)^4 - \left[-\frac{4}{3}\left(\cos\left(\frac{\pi}{4}\right)\right)^4\right].$$

$$= \left[-\frac{4}{3}\left(\cos\left(\frac{\pi}{2}\right)\right)^4 + \frac{4}{3}\left(\cos\left(\frac{\pi}{4}\right)\right)^4\right] = \frac{1}{3} \quad \text{Ans:}$$

$\because (x-1)^2+y^2=1$

$\because (x-1)^2+x^2=1$

$\because x^2-2x+1+x^2=1$

$\because 2x^2-2x=0$

$\because 2x(x-1)=0$

$\boxed{x=0,\; x=1}$

$(x^2-2x+1)+y^2=1$

$x^2-2x+y^2=0$

$x^2+y^2=2x$

$r^2=2x$ $\qquad x=r\cos\theta$

$r^{\cancel{2}}=2\cancel{r}\cos\theta$

$r=2\cos\theta$

$r^2=2y$ $\qquad y=r\sin\theta$

$r^{\cancel{2}}=2\cancel{r}\sin\theta$

$r=2\sin\theta$

Bazil-Uddin-khan 244-0559 BSCS 24. Date:

Q6).

Ans:-

$$\int_0^3 \int_0^1 \int_0^{\sqrt{1-z^2}} z e^y \, dx \, dz \, dy$$

$$\int_0^3 \int_0^1 \left| x z e^y \right|_0^{\sqrt{1-z^2}} dz \, dy$$

$$\int_0^3 \int_0^1 \left(\left(\sqrt{1-z^2}\right) z e^y - 0 \right) dz \, dy.$$

$$\int_0^3 \int_0^1 -\frac{1}{2} \left(\sqrt{1-z^2}\right)(-2z) e^y \, dz \, dy$$

$$\int_0^3 \left| -\frac{1}{2} \frac{(1-z^2)^{1/2+1}}{1/2+1} e^y \right|_0^1 dy$$

$$\int_0^3 \left| -\frac{(1-z^2)^{3/2}}{3} e^y \right|_0^1 dy$$

$$\int_0^3 \left(-0 + \left(-\frac{1}{3}\right) e^y \right) dy$$

$$\int_0^3 \frac{1}{3} e^y \, dy = \left|\frac{1}{3} e^y\right|_0^3 \rightarrow \frac{1}{3} e^3 - \frac{1}{3} e^0 = \frac{1}{3} e^3 - \frac{1}{3} = \boxed{\frac{1}{3}\left(e^3 - 1\right)}$$ Ans:

Basil-uddin-khan 24k-0559 BSCS-2H. Date:

Q7).

Ans: $\int_C F \cdot dr$ where $F(x,y,z) = x\hat{i} - z\hat{j} + y\hat{k}$

$r(t) = 2t\,i + 3t\,j - t^2 k$ $(-1 \le t \le 1)$.

$r'(t) = 2\hat{i} + 3\hat{j} - 2t\hat{k}$

$x = 2t;\ y = 3t,\ z = -t^2$

$$\int_{-1}^{1} (x\hat{i} - z\hat{j} + y\hat{k}) \cdot (2\hat{i} + 3\hat{j} - 2t\hat{k})\, dt$$

$$\int_{-1}^{1} 2x - 3z - 2ty \; dt$$

$$\int_{-1}^{1} 2(2t) - 3(-t^2) - 2t(3t)\, dt$$

$$\int_{-1}^{1} 4t + 3t^2 - 6t^2 \; dt$$

$$\int_{-1}^{1} 4t - 3t^2 \; dt$$

$$\int_{-1}^{1} t^2 - t^3 \longrightarrow (1-1) - (1-(-1)^3) = 0 - 2 = -2$$

Ans = -

Bazil-Uddin-khan 24k-0559 BSCS 2H. Date:

Q8).

$$\int_C xyz^2\, ds \qquad (-1,5,0) \text{ to } (1,6,4)$$

$t=0 \qquad t=1$

Ans:-

$x = c_1 + k_1t_1 \rightarrow x = -1 + k_1t_1 \quad = 1 = -1 + k_1(1) \rightarrow k_1 = 2$

$y = c_2 + k_2t_2 \rightarrow y = 5 + k_2t_2 \quad = 6 = 5 + k_2(1) \rightarrow k_2 = 1$

$z = c_3 + k_3t_3 \rightarrow z = 0 + k_3t_3 \quad = 4 = 0 + k_3(1) \rightarrow k_3 = 4.$

$x = -1 + 2t,$

$y = 5 + t$

$z = 4t.$

$ds = |\vec{r(t)}|\, dt$

$(\vec{r})_t = (1+2t)\hat{i} + (5+t)\hat{j} + (4t)\hat{k}$

$|(\vec{r})'_t| = 2\hat{i} + 1\hat{j} + 4\hat{k}$

$||\vec{r(t)}|| = \sqrt{2^2+1^2+4^2} = \boxed{\sqrt{21}}$

$$\int_{t=0}^{t=1} (-1+2t)(5+t)16t^2 \times \sqrt{21}\, dt$$

$\therefore (2t-1)(t+5)$

$\therefore (2t^2+10t-t-5)$

$\therefore 2t^2+9t-5$

$$\sqrt{21}\int_{t=0}^{t=1} (2t^2+9t-5).16t^2\, dt$$

$$\sqrt{21}\int_{t=0}^{t=1} 32t^4 + 144t^3 - 80t^2\, dt.$$

$$\sqrt{21}\int_{t=0}^{t=1} \frac{32t^5}{5} + \frac{144t^4}{4} - \frac{80t^3}{3}$$

$$\sqrt{21}\left(\frac{32(1)^5}{5} + \frac{144(1)^4}{4} - \frac{80(1)^3}{3}\right) - \sqrt{21}(0)$$

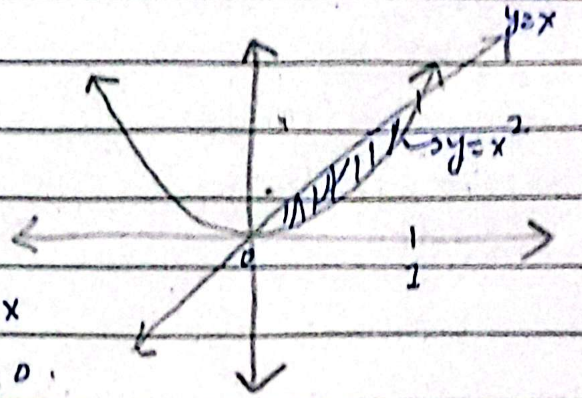
$= 72.099 - 0$

$= \boxed{72.1}$ $72.099 \approx$ Ans:- or $\boxed{\frac{236\sqrt{21}}{15}}$ Ans:

Bazil-uddin-khan 244-0559 BSCS-2H

Q9).

<u>Ans:-</u> $\int_C \underbrace{(e^x+y^2)}_{P}\,dx + \underbrace{(e^y+x^2)}_{Q}\,dy$



$\iint_R 2x - 2y$

$\because y=x^2, y=x$

$\because x^2-x=0$

$\because x(x-1)=0$

$\because x=0, x=1$

$\int_{x=0}^{x=1}\int_{y=x^2}^{y=x} (2x-2y)\,dy\,dx$

$\int_0^1 \int_{y=x^2}^{y=x} 2xy - y^2 \,dx$

$\int_0^1 \left| 2xy - y^2 \right|_{y=x^2}^{y=x} dx$

$\int_0^1 \left|2x(x) - x^2\right| - \left|2x(x^2) - (x^2)^2\right| dx$

$\int_0^1 \left|2x^2 - x^2\right| - \left|2x^3 - x^4\right| dx$

$\int_0^1 x^2 - 2x^3 + x^4 \,dx$

$\int_0^1 \frac{x^3}{3} - \frac{x^4}{2} + \frac{x^5}{5} = \left|\frac{(1)^3}{3} - \frac{(1)^4}{2} + \frac{(1)^5}{5}\right| - |0| = \frac{1}{30}$ Ans:

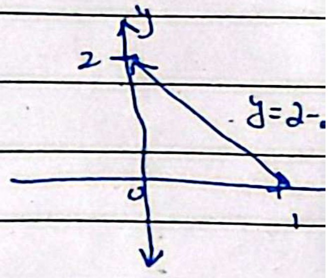
Bazil-Uddin-khan 24k-0559 BSCS-2H.

Q10). $\iint_S xy \, dS$

Ans:- $A = (1,0,0),\ B = (0,2,0),\ C = (0,0,2)$.

$\overrightarrow{AB} = \vec{B} - A = (0,2,0) - (1,0,0) = (-1,2,0)$

$\overrightarrow{AC} = C - A = (0,0,2) - (1,0,0) = (-1,0,2)$



$\vec{n} = \overrightarrow{AB} \times \overrightarrow{AC} = \begin{vmatrix} i & j & k \\ -1 & 2 & 0 \\ -1 & 0 & 2 \end{vmatrix} = i[4-0] - j[-2] + 2\hat{k} = 4\hat{i} + 2\hat{j} + 2\hat{k}$

↳ plane equation.

Use: plane equation $(1,0,0)$.

$4(x-1) + 2y + 2z = 0$

$4x - 4 + 2y + 2z = 0$

$2(2x - 2 + y + z) = 0$

$z = 2 - 2x - y$

$\vec{S}_{(x,y)} = x\hat{i} + y\hat{j} + (2-2x-y)\hat{k}$

$\vec{S_x} = \hat{i} - 2\hat{k}$

$\vec{S_y} = \hat{j} - \hat{k}$

$\begin{matrix} i & j & k \\ 1 & 0 & -2 \\ 0 & 1 & -2 \end{matrix}$

$|\vec{S_x} \times \vec{S_y}| = 2i + j + k = \sqrt{2^2+1^2+1^2} = \sqrt{6}$

$\sqrt{6}\int_0^1 \int_0^{2-2x} xy \, dy\, dx$

$\sqrt{6}\int_0^1 \left[\frac{xy^2}{2}\right]_0^{2-2x} dx \rightarrow \frac{\sqrt{6}}{2}\int_0^1 \frac{x(2-2x)^2}{2} - 0 \cdot dx$

$(-2x+2)(-2x+2)$

$4x^2 - 8x + 4$

$\frac{\sqrt{6}}{2}\int_0^1 \frac{4x^3 - 8x^2 + 4x}{2} \, dx$

$\frac{\sqrt{6}}{2}\int_0^1 2x^3 - 4x^2 + 2x \, dx. \rightarrow \frac{\sqrt{6}}{2}\int_0^1 \frac{x^4}{2} - \frac{4x^3}{3} + x^2$

$= \sqrt{6}\left[\frac{1^4}{2} - \frac{4(1)^3}{3} + 1\right] - [0] = \boxed{\frac{\sqrt{6}}{6}}$ Ans:-