National University of Computer & Emerging Sciences, Karachi
Spring-2025 FAST School of Computing
MT-2008 Multivariate Calculus

Assignment #2

Q#1 In a machine learning algorithm, the computational cost associated with processing data is modeled by the function:

$$C(x, y) = e^x + \sin y - 4x - y$$

where x represents the complexity of the model and y represents the preprocessing time. Find the critical points and classify them using the second derivative test.

Q#2 A computer system dynamically allocates memory to two processes, P and Q, where the memory allocated (in MB) is represented by:

$$M(a, b) = 100a + 150b - 2a^2 - 3b^2 - ab$$

Find the critical points and determine their nature using the second derivative test.

Q#3 *A machine learning company is optimizing the cost function C(m, n) of training a neural network, given by*

$$C(m, n) = 3m^2 + 2mn + 4n^2$$

where m is the number of hidden layers, and n is the number of epochs. The training system has a constraint:

$$m^2 + n^2 + mn = 400$$

Determine the optimal values of m and n that minimize the training cost.

Q#4A data center wants to minimize its energy consumption E(p, q, r), modeled as:

$$E(p, q, r) = p^2 + 4q^2 + 2r^2$$

where p, q, r represent power consumption in three different server clusters. The total power distribution must satisfy:

$$p^2 + q^2 + r^2 = 250,000$$

Find the optimal power distribution that minimizes total energy consumption.

Q#5: A robotic arm is programmed to follow a trajectory by minimizing an error function given by:

$$J(\theta 1, \theta 2) = \sin(\theta 1) + \theta_2^2 + \theta_1\theta_2$$

*Using **Gradient Descent**, find the update rules θ_1 and θ_2 determine the optimal values iteratively.* (only two iteration)

*Using **Initial values**:* $\theta_1^0 = 1$=*1,* $\theta_2^0 = 1$ *and* $\alpha = 0.1$

Q#6 A cybersecurity model has a risk function:

$$\boldsymbol{R(f,g,h) = f^2 + 2g^2 + 3h^2 + fg - 5f - 7g - 9h + 30}$$

where f, g, and h represent different security configurations. Use gradient descent to minimize risk.

$$\text{Set } \alpha = 0.05 \text{ and start with } (f_0, g_0, h_0) = (1,1,1).$$

Q#7: Consider the cost function:

$$\boldsymbol{J(\theta_1, \theta_2, \theta_3) = e^{\theta_1} + \theta_2^2 + 4\theta_3^2 + \theta_1\theta_2}$$

*Find the **optimal values** using the **Hessian matrix**.*

Q#8 A machine learning model is analyzing the temperature distribution in a region where the heat function is given by:

$$\boldsymbol{T(x,y) = xy - x^2}$$

The region is bounded by the $0 \leq x \leq 1 \; and \; 1 \leq y \leq 3$

Find the total heat energy over this region by evaluating the double integral:

$$\boldsymbol{E = \iint_{[D]} xy - x^2 \, dA}.$$

Q#9: A drone equipped with AI is mapping a mountain ridge where the height is given by:

$$\boldsymbol{z = 10 - x^2 - y^2}$$

The region is a triangular domain with vertices at $(0,0), (2,0), and \; (2,3)$. Find the total volume of the mountain above the xy-plane.

Q#10 a) Find the equation of the tangent plane and the normal vector for the surface:

$$\boldsymbol{f(x,y,z) = x^2 + y^2 - z = 0 \; at \; the \; point \; (1,2,5)}$$

b) Given two surfaces:

$$\boldsymbol{f_1(x,y,z) = x^2 + y^2 + z^2 - 9 = 0 \; and \; f_2(x,y,z) = x + 2y - 3z + 4 = 0}$$

find the angle between their tangent planes at the intersection point $(2, -1, 1)$