Date:

Convex optimization::

"If hessian matrix is positive definite then f is concave and stationary point is absolute minimum point."

$z=f(x,y)$ Hessian matrix:: it consists of all second order partial derivatives of f.

$z=f(x,y) \to$

$$H_1 = \begin{array}{c|cc} & f_x & f_y \\ f_x & f_{xx} & f_{xy} \\ f_y & f_{yx} & f_{yy} \end{array}$$

$w=f(x,y,z)$

$$H_2 = \begin{array}{c|ccc} & f_x & f_y & f_z \\ f_x & f_{xx} & f_{yx} & f_{zx} \\ f_y & f_{yx} & f_{yy} & f_{yz} \\ f_z & f_{zx} & f_{zy} & f_{zz} \end{array}$$

$f_{xy}=f_{yx}$

$f_{xy}=f_{yx}$

$f_{xz}=f_{zx}$

$f_{yz}=f_{zy}$

$\boxed{H=H \text{ symmetric matrix}}$

(convex) (concave).

$z=f(x,y)$.

Positive Definite:-	Negative definite
• $D_1 = f_{xx} > 0$, $D_2 = \lvert H_1\rvert > 0$	• $D_1 = f_{xx} < 0$, $D_2 = \lvert H_2\rvert > 0$
• $D_1 = f_{xx} > 0$, $D_2 = \begin{vmatrix} f_{xx} & f_{xy} \\ f_{yy} & f_{yx} \end{vmatrix} > 0$	• $D_1 = f_{xx} < 0$, $D_2 = \lvert H_2\rvert$ ↓ $\begin{vmatrix} f_{xx} & f_{xy} \\ f_{yy} & f_{yx} \end{vmatrix} > 0$
• $D_3 = \lvert H_3\rvert > 0$, ↳ whole matrix	• $D_3 \lvert H_3\rvert < 0$

Q1) $f(x,y,z) = x^4+y^4+z^4+x^2+y^2+z^2$

$f_x = 4x^3+2x$, $f_y = 4y^3+2y$, $f_z = 4z^3+2z$

$f_{xx} = 12x^2+2$ $f_{yy} = 12y^2+2$, $f_{zz} = 12z^2+2$

$f_{xy} = 0$ $f_{xz}=0$, $f_{yz}=0$

$$H_2 = \begin{bmatrix} 12x^2+2 & 0 & 0 \\ 0 & 12y^2+2 & 0 \\ 0 & 0 & 12z^2+2 \end{bmatrix}$$

find stationary by putting $=0$

$4x^3+2x=0$

$2x(2x^2+1)=0$

$x=0$, $y=0$, $z=0$

CP $(0,0,0) \to$ Min. $\boxed{0}$

$$\underset{(0,0,0)}{H_2} = \begin{bmatrix} 2 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 2 \end{bmatrix}$$

$D_1 = 2>0$, $D_2 = 4>0$, $D_3 = 8>0$

↳ As diagonal convex positive / check on this matrix is positive definite