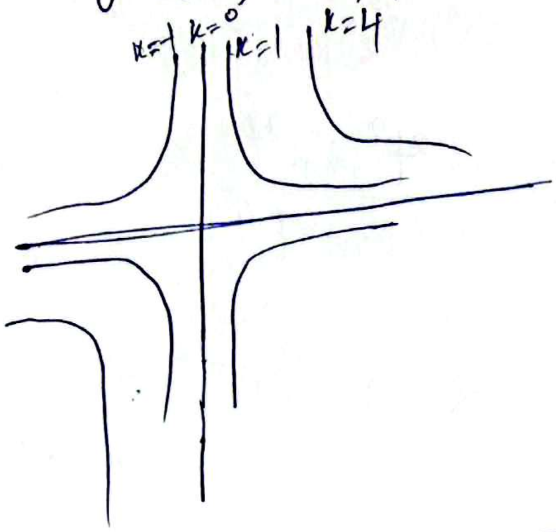
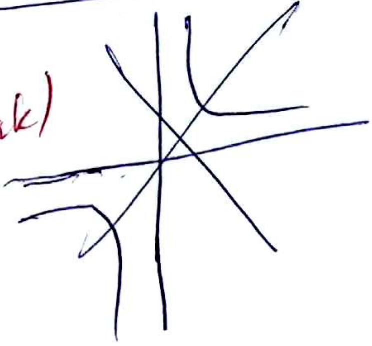
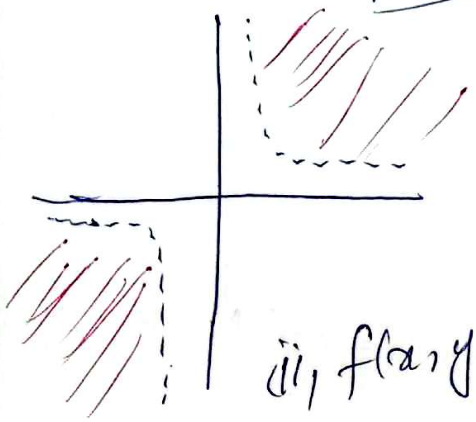
Q.1a $z = xy$; $k = 0, 1, -1, 4$
for level curves put $z = k$
$\therefore xy = k$; $k = 0, 1, -1, 4$



b) (i) $f(x,y) = \ln(xy - 1)$
Domain : $xy - 1 > 0$ (1 Mark)
$\Rightarrow \boxed{xy > 1}$ domain



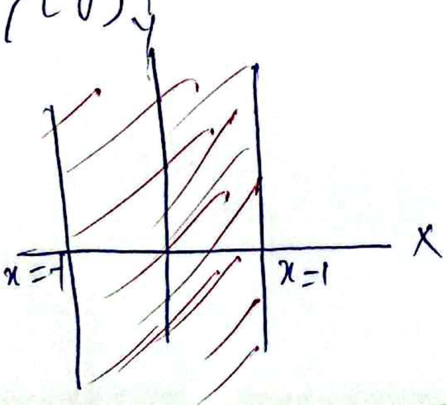


(1 Mark)

(ii) $f(x,y) = \dfrac{\sin^{-1}x}{e^y}$

for numerator, $-1 \le x \le 1$
denominator, $e^y > 0 \quad \forall y \in \mathbb{R}$

$\therefore$ Domain $= \left\{ \begin{bmatrix} x \\ y \end{bmatrix} \middle| -1 \le x \le 1,\ y \in \mathbb{R} \right\}$ (1 Mark)



(1 Mark)

Q.1C

(i) $\lim\limits_{(x,y)\to(0,0)} \dfrac{y^2\sin^2 x}{x^4+y^4}$

(1 Mark for path)

Along $y = mx$

$x = t,\ y = mt$

$t \to 0$ as $x \to 0$

$\therefore \lim\limits_{(x,y)\to(0,0)} \dfrac{y^2\sin^2 x}{x^4+y^4} = \lim\limits_{t\to 0} \dfrac{m^2t^2\sin^2 t}{t^4+m^4t^4}$

$= \lim\limits_{t\to 0} \dfrac{m^2\cancel{t^2}}{\cancel{t^2}(1+m^4)} \dfrac{\sin^2 t}{t^2}$

(1 Mark)

$= \dfrac{m^2}{1+m^4}$

$\therefore$ limit is depending on value of m

$\therefore$ function approaches different limit along different curves

(1 Mark)

$\therefore$ limit does not exist

(ii) $\lim\limits_{(x,y)\to(0,0)} \dfrac{x^2+y^2}{\sqrt{x^2+y^2+1}-1} \times \dfrac{\sqrt{x^2+y^2+1}+1}{\sqrt{x^2+y^2+1}+1}$ (1 Mark)

$= \lim\limits_{(x,y)\to(0,0)} \dfrac{(x^2+y^2)(\sqrt{x^2+y^2+1}+1)}{(x^2+y^2+1-1)}$ (1 Mark)

(1 Mark)

$= 2$ //

Q.1d $u_x = 15\pi \cos(3\pi x)\sin(4\pi y)\cos(10\pi t)$

$u_{xx} = -45\pi^2 \sin(3\pi x)\sin(4\pi y)\cos(10\pi t)$ - [1 Mark]

$u_y = 20\pi \sin(3\pi x)\cos(4\pi y)\cos(10\pi t)$

$u_{yy} = -80\pi^2 \sin(3\pi x)\sin(4\pi y)\cos(10\pi t)$ - [1 Mark]

$u_t = -50\pi \sin(3\pi x)\sin(4\pi y)\sin(10\pi t)$

$u_{tt} = -500\pi^2 \sin(3\pi x)\sin(4\pi y)\cos(10\pi t)$ - [1 Mark]

Now,

$4(u_{xx}+u_{yy}) = -500\pi^2 \sin(3\pi x)\sin(4\pi y)\cos(10\pi t)$ - [1 Mark]

Hence proved.

Q.1e $\frac{\partial w}{\partial x} = \frac{dw}{d\rho}\frac{\partial \rho}{\partial x} = \frac{dw}{d\rho}\cdot\left(\frac{1}{2}(x^2+y^2+z^2)^{-1/2}(2x)\right)$

$= \frac{dw}{\partial\rho}\cdot\frac{x}{\rho}$ — [1 Mark]

Similarly,

$\frac{\partial w}{\partial y} = \frac{dw}{d\rho}\frac{\partial \rho}{\partial y} = \frac{dw}{d\rho}\cdot\frac{y}{\rho}$ - [1 Mark]

$\frac{\partial w}{\partial z} = \frac{dw}{\partial\rho}\frac{\partial \rho}{\partial z} = \frac{dw}{\partial\rho}\cdot\frac{z}{\rho}$ - [1 Mark]

Now,

$\left(\frac{\partial w}{\partial x}\right)^2+\left(\frac{\partial w}{\partial y}\right)^2+\left(\frac{\partial w}{\partial z}\right)^2 = \left(\frac{dw}{d\rho}\right)^2\left(\frac{x^2}{\rho^2}\right)+\left(\frac{dw}{d\rho}\right)^2\left(\frac{y^2}{\rho^2}\right)$
$+\left(\frac{dw}{d\rho}\right)^2\left(\frac{z^2}{\rho^2}\right)$

$= \left[\frac{x^2+y^2+z^2}{\rho^2}\right]\cdot\left(\frac{dw}{d\rho}\right)^2$ - [2 Mark]

$= \frac{\rho^2}{\rho^2}\cdot\left(\frac{dw}{d\rho}\right)^2 = \left(\frac{dw}{d\rho}\right)^2$ - [1 Mark] proved

Q.2a $V=25,\ P=1$

$\therefore T = 16.574\cdot\left(\frac{1}{25}\right) - 0.52754\cdot\frac{1}{25^2} - 0.3879(1) + 12.187(25)(1)$

$\Rightarrow \boxed{T = 304.95\ K}$ — (1 Mark)

$\because T = T(V,P)$

$\therefore dT = T_V\,dV + T_P\,dP$ — (1)

$T_V = -\frac{16.574}{V^2} + \frac{0.52754(2)}{V^3} + 12.187P$

$\Rightarrow \boxed{T_V(25,1) = 12.16055}$ —(1 Mark)

Also, $T_P = -0.3879 + 12.187V$

$\Rightarrow \boxed{T_P(25,1) = 304.29}$ —(1 Mark)

$\therefore (1) \Rightarrow \boxed{dT = 12.1605\,dV + 304.29\,dP}$ — (1 Mark)

Q.2b $\because D_u P\left(\frac{3}{4},\frac{21}{4}\right) = P_x\left(\frac{3}{4},\frac{21}{4}\right)a + P_y\left(\frac{3}{4},\frac{21}{4}\right)b$ — (1)

$\vec{a} = a\hat{i} + b\hat{j} = \cos\theta\,\hat{i} + \sin\theta\,\hat{j} = \frac{\sqrt{3}}{2}\hat{i} + \frac{1}{2}\hat{j}$ — (1 Mark)

$P_x = \frac{\pi}{3}\cos\left(\frac{\pi}{3}x\right)\sin\left(\frac{\pi}{7}y\right) \Rightarrow P_x\left(\frac{3}{4},\frac{21}{4}\right) = \frac{\pi}{6}$ — (1 Mark)

$P_y = \frac{\pi}{7}\sin\left(\frac{\pi}{3}x\right)\cos\left(\frac{\pi}{7}y\right) \Rightarrow P_y\left(\frac{3}{4},\frac{21}{4}\right) = -\frac{\pi}{14}$ — (1 Mark)

$\therefore (1) \Rightarrow \boxed{D_u P\left(\frac{3}{4},\frac{21}{4}\right) = \frac{\pi}{84}(7\sqrt{3}-3) = 0.3413}$

—(1 Mark)

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