Applied Physics NS (1001)

Linear Motion

- Motion
- Position and displacement
- Average velocity and average speed
- Instantaneous velocity and speed
- Acceleration
- Constant acceleration: A special case
- Free fall acceleration

Motion

- **Motion** – an object's change in position relative to a reference point

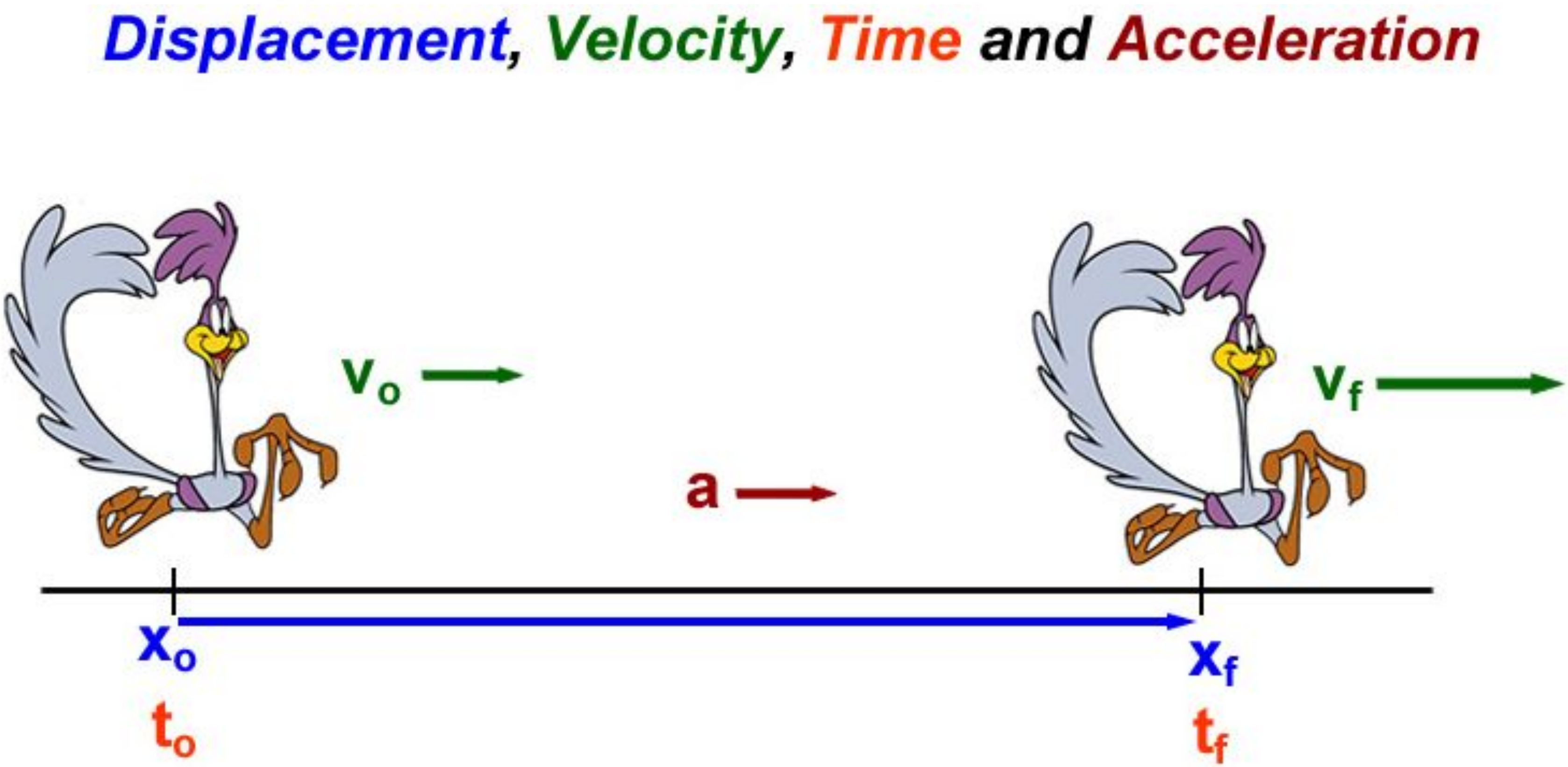
Kinematics – describing motion

Object is treated as a **particle** (a point-like concentration of matter that has no size, no shape and no internal structure).

Questions to ask:

- Where is the particle?
- How fast is it moving?
- How rapidly is it speeding up or slowing down?

Basic Quantities in Kinematics



Displacement

Displacement is a change of position in time.

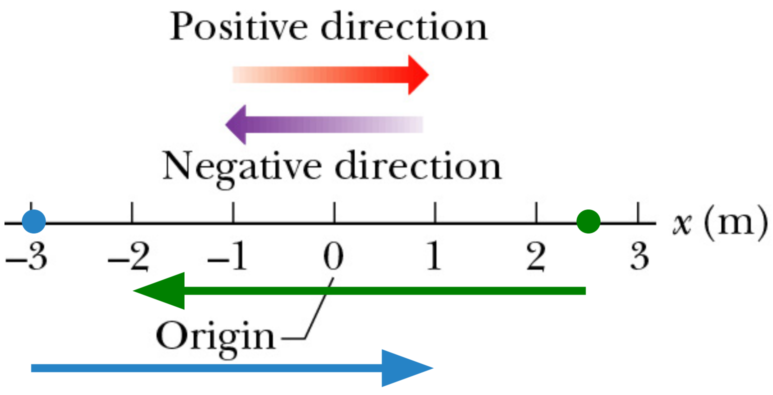
Displacement: $\boxed{\Delta x = x_f(t_f) - x_i(t_i)}$

- f stands for final and i stands for initial.

It is a vector quantity.

It has both magnitude and direction: + or − sign

It has units of [length]: meters.



$x_1(t_1) = +2.5\ m$
$x_2(t_2) = -2.0\ m$
$\Delta x = -2.0\ m - 2.5\ m = -4.5\ m$

$x_1(t_1) = -3.0\ m$
$x_2(t_2) = +1.0\ m$
$\Delta x = +1.0\ m + 3.0\ m = +4.0\ m$

Check Point

Here are three pairs of initial and final positions, respectively, along an x axis. Which pairs give a negative displacement: (a) -3 m, $+5$ m; (b) -3 m, -7 m; (c) 7 m, -3 m?

Distance and Position-time graph

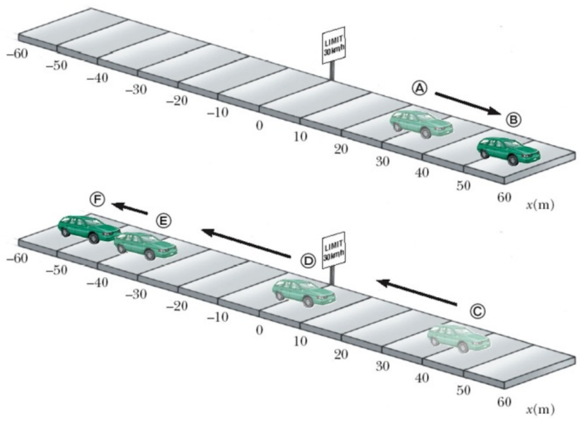
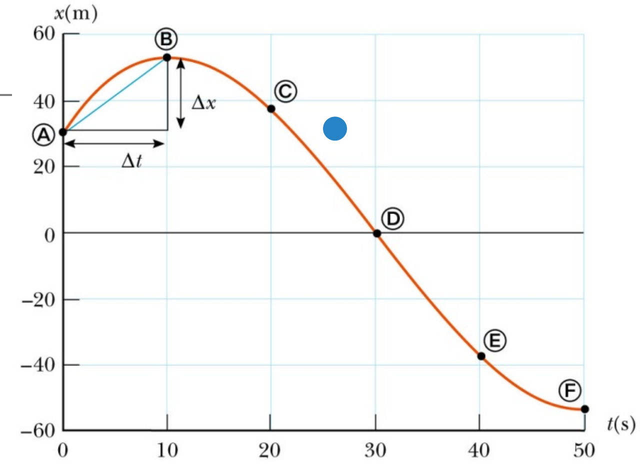


TABLE 2.1

Position of the Car at Various Times

Position	t (s)	x (m)
A	0	30
B	10	52
C	20	38
D	30	0
E	40	− 37
F	50	− 53



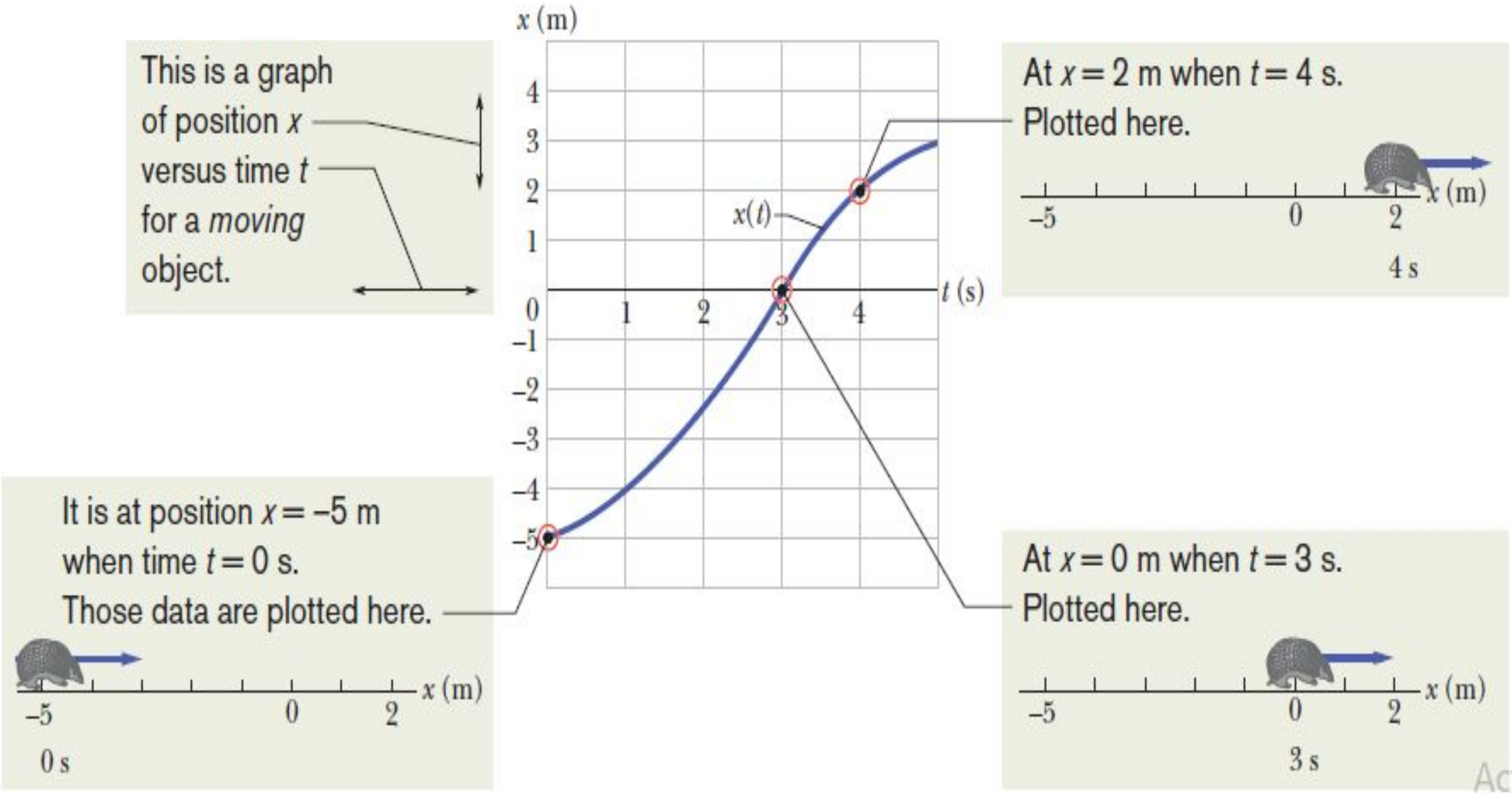
Displacement in space

- From A to B: $\Delta x = x_B - x_A = 52\text{ m} - 30\text{ m} = 22\text{ m}$
- From A to C: $\Delta x = x_c - x_A = 38\text{ m} - 30\text{ m} = 8\text{ m}$

Distance is the length of a path followed by a particle

- from A to B: $d = |x_B - x_A| = |52\text{ m} - 30\text{ m}| = 22\text{ m}$
- from A to C: $d = |x_B - x_A| + |x_C - x_B| = 22\text{ m} + |38\text{ m} - 52\text{ m}| = 36\text{ m}$

Displacement is not Distance.


This is a graph of position x versus time t for a moving object.
x (m)
t (s)
x(t)
At x = 2 m when t = 4 s. Plotted here.
At x = 0 m when t = 3 s. Plotted here.
It is at position x = –5 m when time t = 0 s. Those data are plotted here.
0 s
3 s
4 s

Speed

Speed is a ***scalar*** quantity which refers to "how fast an object is moving.“

Speed can be thought of as the ***rate*** at which an object covers distance, or distance per time.

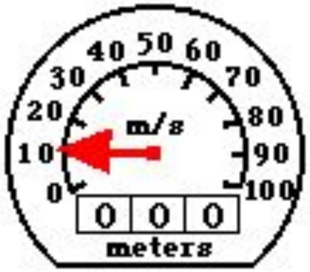
A fast-moving object has a high speed; a slow-moving object has a low speed.

An object with no movement at all has a zero speed.

Normally, objects do not travel at a constant speed

$$\text{Average Speed} = \frac{\text{total distance}}{\text{total time}}$$

Instantaneous Speed and Average Speed



Instantaneous speed is the speed at any given instant in time.

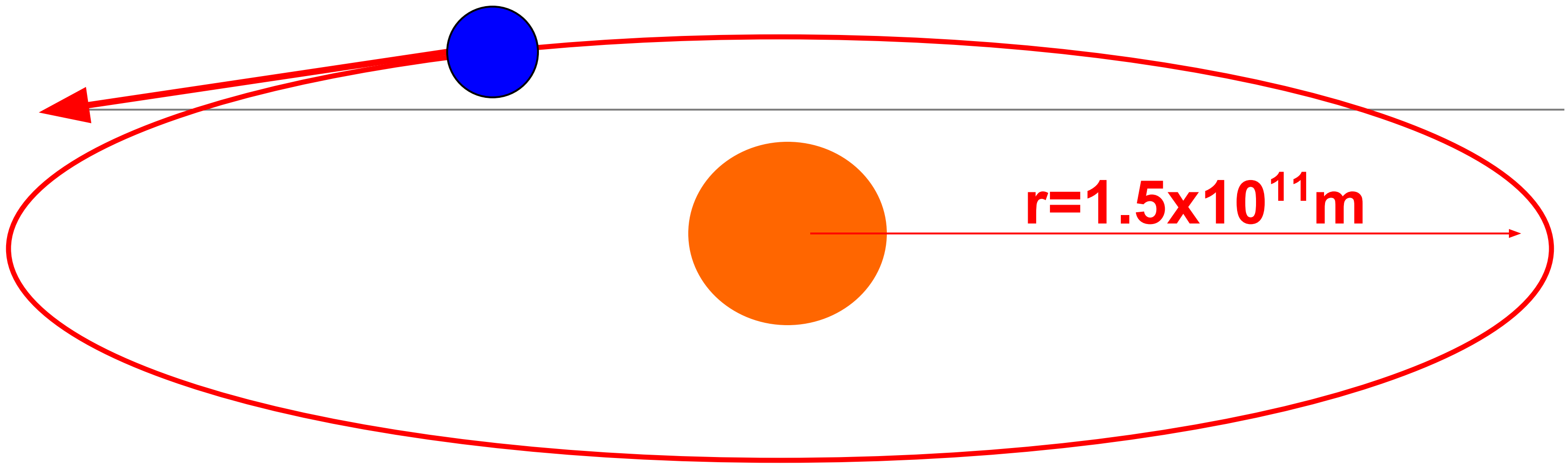
- What the speedometer reads when you look at it

Average speed is the average of all instantaneous speeds

- The average of an infinite number of speedometer readings during a trip
- Found simply by a total distance/total time ratio

$$\text{Average Speed} = \frac{\text{Distance Traveled}}{\text{Time of Travel}}$$

Earth's motion around the Sun



$$V = \frac{\text{distance}}{\text{elapsed time}} = \frac{2\,\pi\,r}{\text{1year}} = \frac{2 \times 3.14 \times 1.5 \times 10^{11}\text{m}}{365\ \cancel{\text{days}} \times 24\ \text{hr}/\cancel{\text{day}}}$$

$$= \frac{9.4 \times 10^{11}\ \text{m}}{8760\ \text{hr}} = \frac{9.4 \times 10^{11}\ \text{m}}{8.76\ \times 10^{3}\ \text{hr}} = \frac{9.4}{8.7} \times 10^{11-3}\ \text{m/hr}$$

$=10^{-3}$km

$$= 1.1\times10^{8}\,\text{m/hr} = 1.1\times10^{5}\,\text{km/hr}$$

$\approx$ 110,000 km/hr

Tip of a watch’s minute hand

$$V = \frac{\text{distance}}{\text{elapsed time}} = \frac{2\,\pi\,r}{1\text{hr}} = \frac{2 \times 3.14 \times 1\text{cm}}{60\ \cancel{\text{min}} \times 60\ \text{s}/\cancel{\text{min}}}$$

$$= \frac{6.28\ \text{cm}}{3600\ \text{s}} = \frac{6.28\ \text{cm}}{3.6 \times 10^{3}\text{s}} = 1.7\text{x}10^{-3}\ \text{cm/s}$$

$=10^{-2}\text{m}$

$$= 1.7\text{x}10^{-5}\ \text{m/s}$$

Velocity

Velocity is the rate of change of position.

Velocity is a vector quantity.

Velocity has both magnitude and direction.

Velocity has a unit of [length/time]: meter/second.

We will be concerned with three quantities, defined as:

displacement

distance

- Average velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

- Average speed

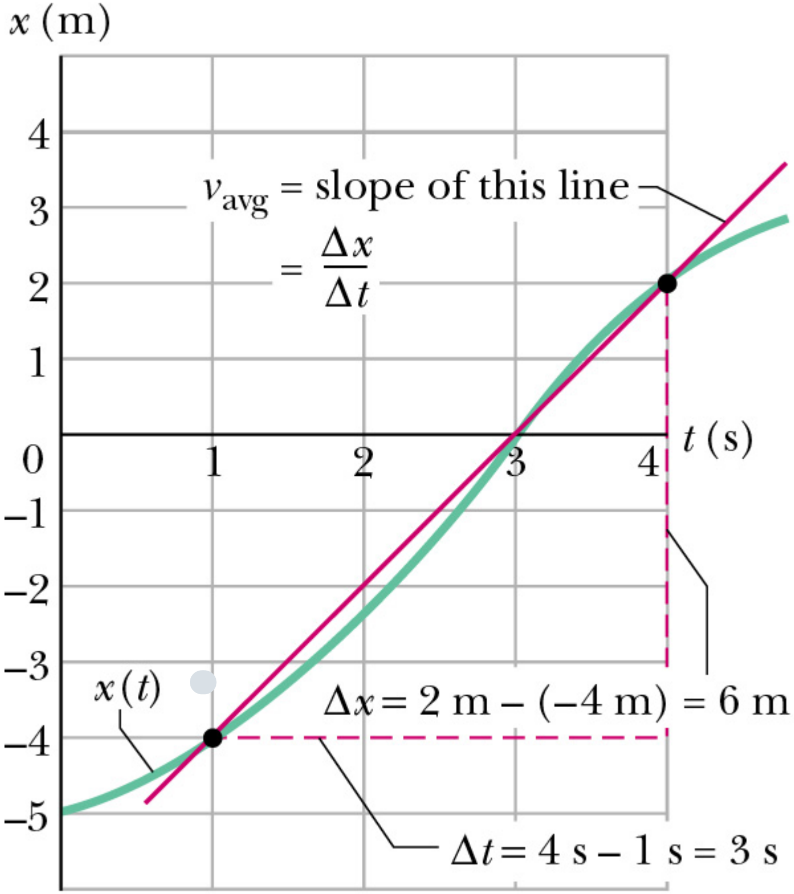
$$\boxed{s_{\text{avg}} = \frac{\text{total distance}}{\Delta t}}$$

- Instantaneous velocity

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}}$$

displacement

Average Velocity

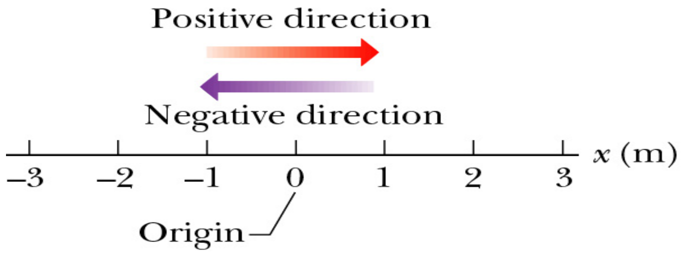


Average velocity

$$v_{avg} = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

is the slope of the line segment between end points on a graph.

It is a vector (i.e. is signed), and displacement direction sets its sign.

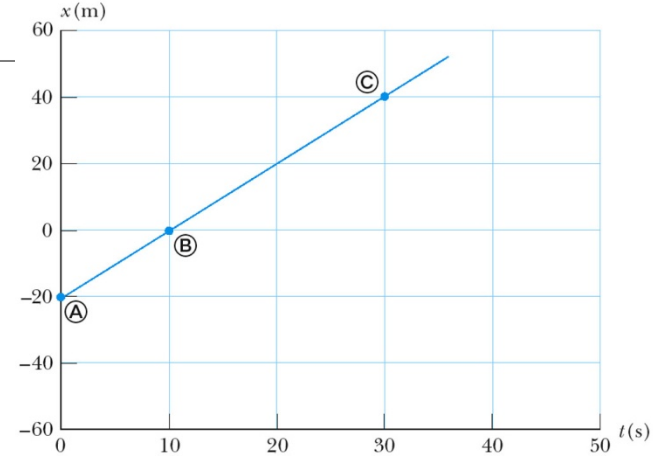


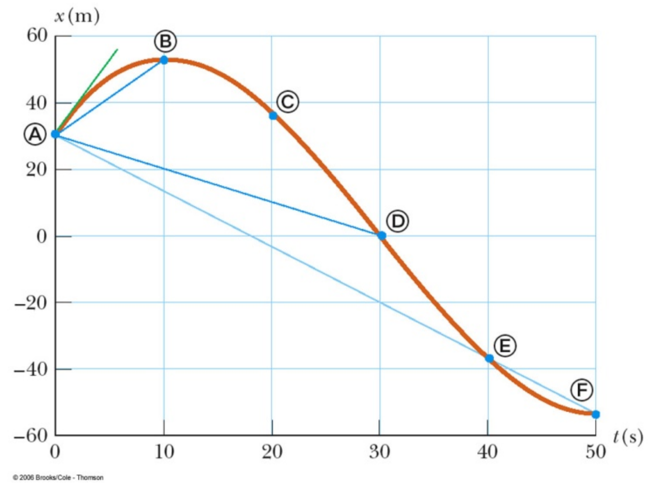
Graphical Interpretation of Velocity

Velocity can be determined from a position-time graph

Average velocity equals the slope of the line joining the initial and final positions. It is a vector quantity.

An object moving with a constant velocity will have a graph that is a straight line.



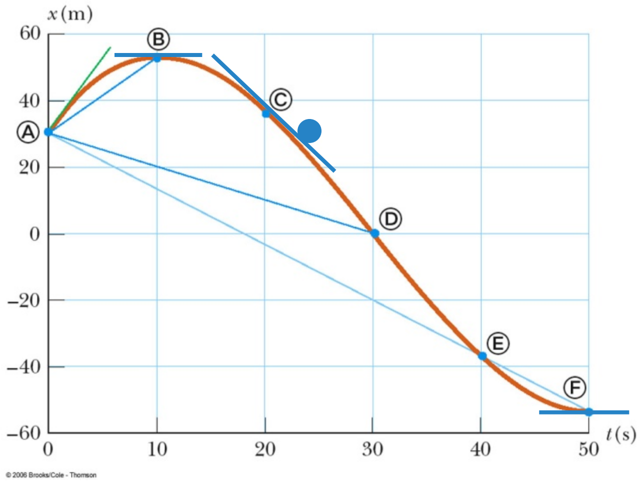


Instantaneous Velocity

Instantaneous means "at some given instant". The instantaneous velocity indicates what is happening at every point of time.

Limiting process:

- Chords approach the tangent as $\Delta t \Rightarrow 0$
- Slope measure rate of change of position



Instantaneous velocity:

$$v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}$$

It is a vector quantity.

Dimension: length/time (L/T), [m/s].

It is the slope of the tangent line to x(t).

Instantaneous velocity v(t) is a function of time.

Uniform Velocity

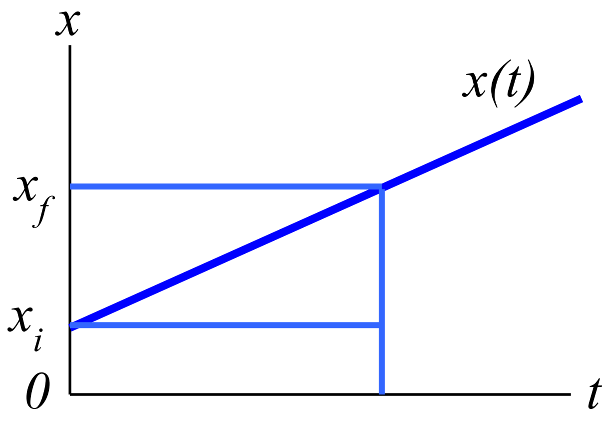
Uniform velocity is the special case of constant velocity

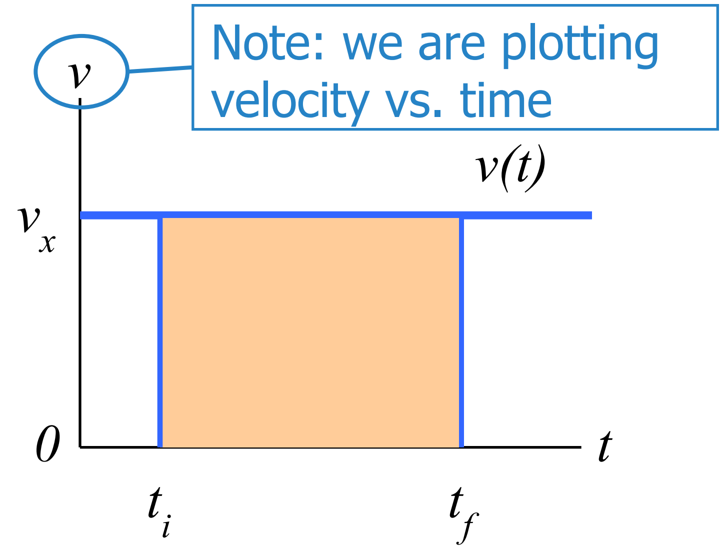
In this case, instantaneous velocities are always the same, all the instantaneous velocities will also equal the average velocity

Begin with then

$$v_x = \frac{\Delta x}{\Delta t} = \frac{x_f - x_i}{\Delta t}$$

$$x_f = x_i + v_x \Delta t$$



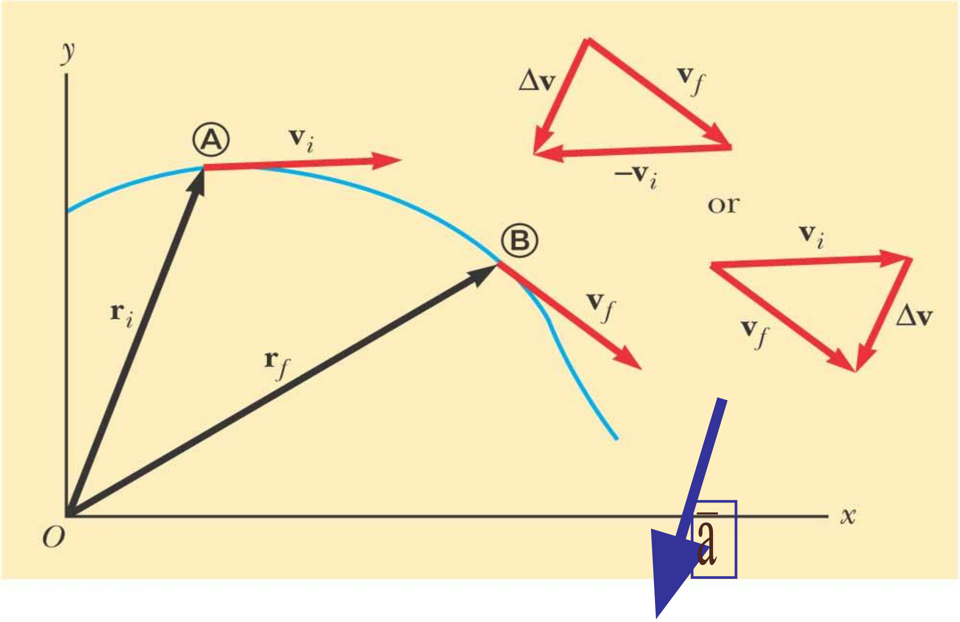


ACCELERATION

The average acceleration of the particle is defined as the change in velocity Δ vx divided by the time interval Δ t during which that change occurred:

$$\bar{a}_x \equiv \frac{\Delta v_x}{\Delta t} = \frac{v_{xf} - v_{xi}}{t_f - t_i}$$

The average acceleration is a vector quantity directed along Δv .



Instantaneous Acceleration

The instantaneous acceleration equals the derivative of the velocity with respect to time, which by definition is the slope of the velocity–time graph

$$a_x \equiv \lim_{\Delta t \to 0} \frac{\Delta v_x}{\Delta t} = \frac{dv_x}{dt}$$

Average Acceleration

Changing velocity (non-uniform) means an acceleration is present.

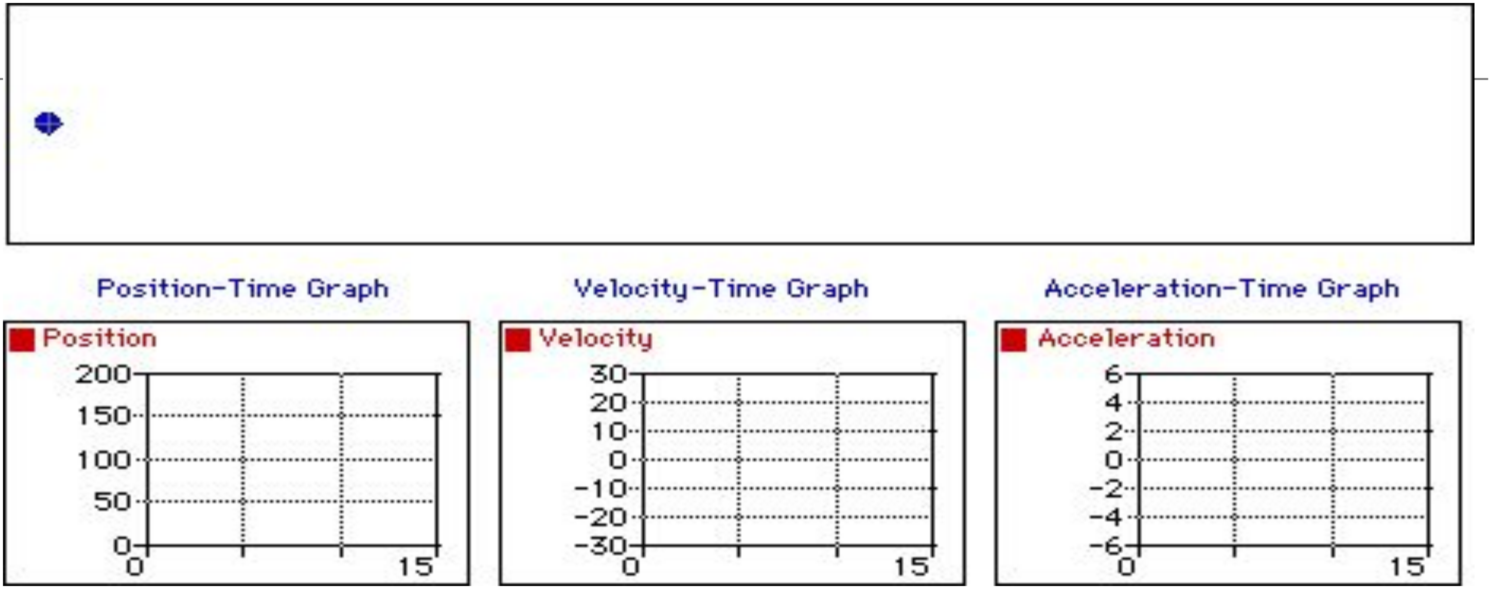
Acceleration is the rate of change of velocity.

Acceleration is a vector quantity.

Acceleration has both magnitude and direction.

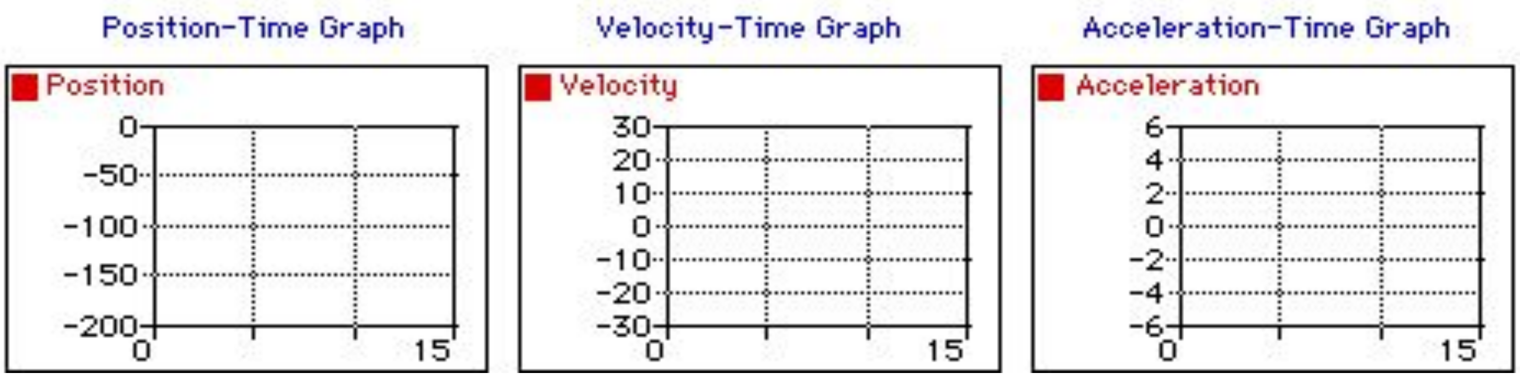
Acceleration has a dimensions of length/time^2: [m/s^2].

Constant velocity and acceleration



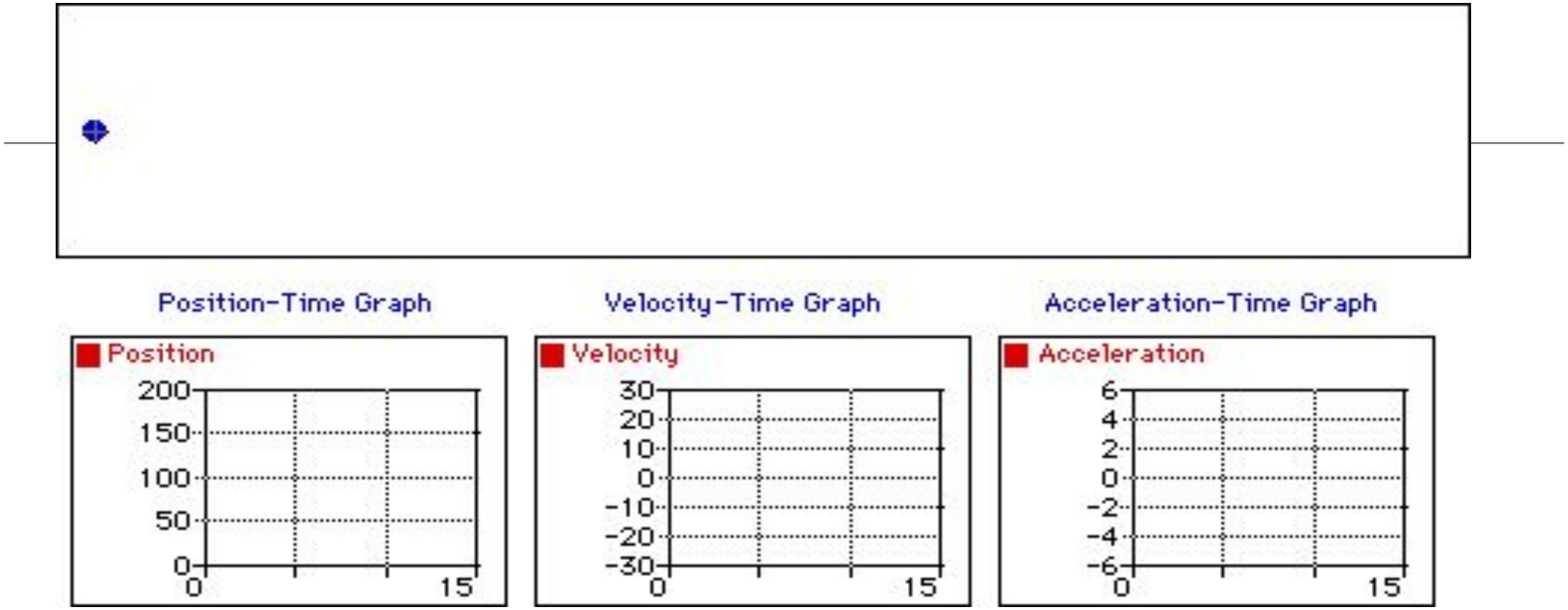
Observe that the object above moves with a constant velocity in the positive direction. The dot diagram shows that each consecutive dot is the same distance apart (i.e., a constant velocity). The position-time graph shows that the slope is both constant (meaning a constant velocity) and positive (meaning a positive velocity). The velocity-time graph shows a horizontal line with zero slope (meaning that there is zero acceleration); the line is located in the positive region of the graph (corresponding to a positive velocity). The acceleration-time graph shows a horizontal line at the zero mark (meaning zero acceleration).

Constant negative velocity and acceleration



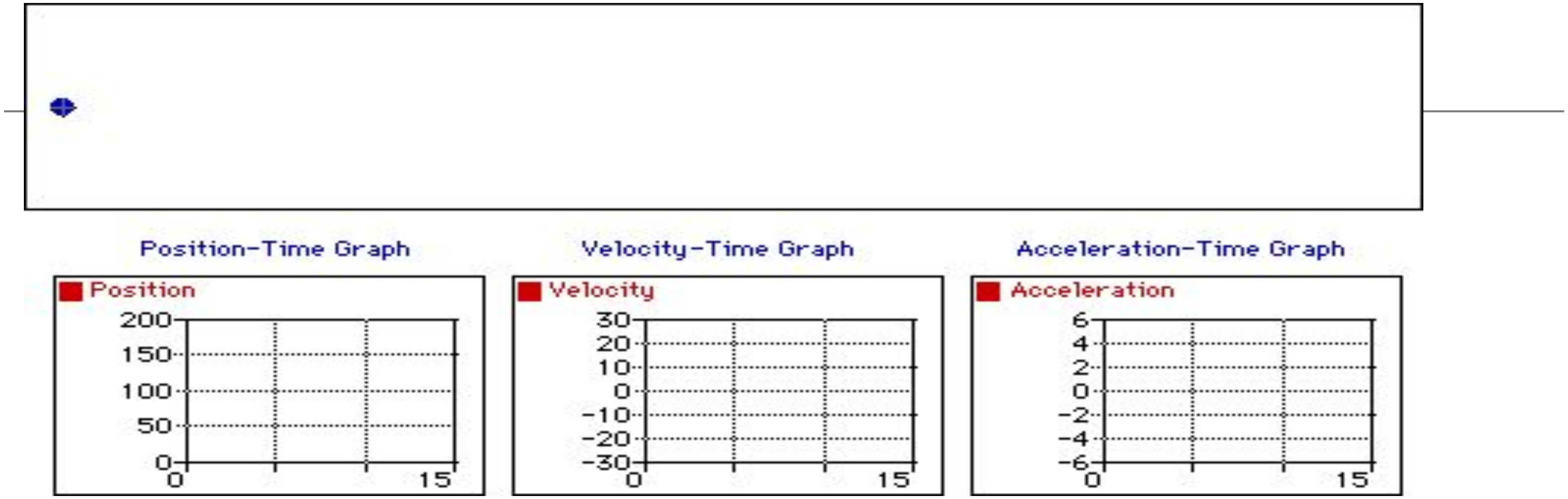
Observe that the object above moves with a constant velocity in the negative direction. The dot diagram shows that each consecutive dot is the same distance apart (i.e., a constant velocity). The position-time graph shows that the slope is both constant (meaning a constant velocity) and negative (meaning a negative velocity). The velocity-time graph shows a horizontal line with zero slope (meaning that there is zero acceleration); the line is located in the negative region of the graph (corresponding to a negative velocity). The acceleration-time graph shows a horizontal line at the zero mark (meaning zero acceleration).

Position varies with positive velocity and acceleration



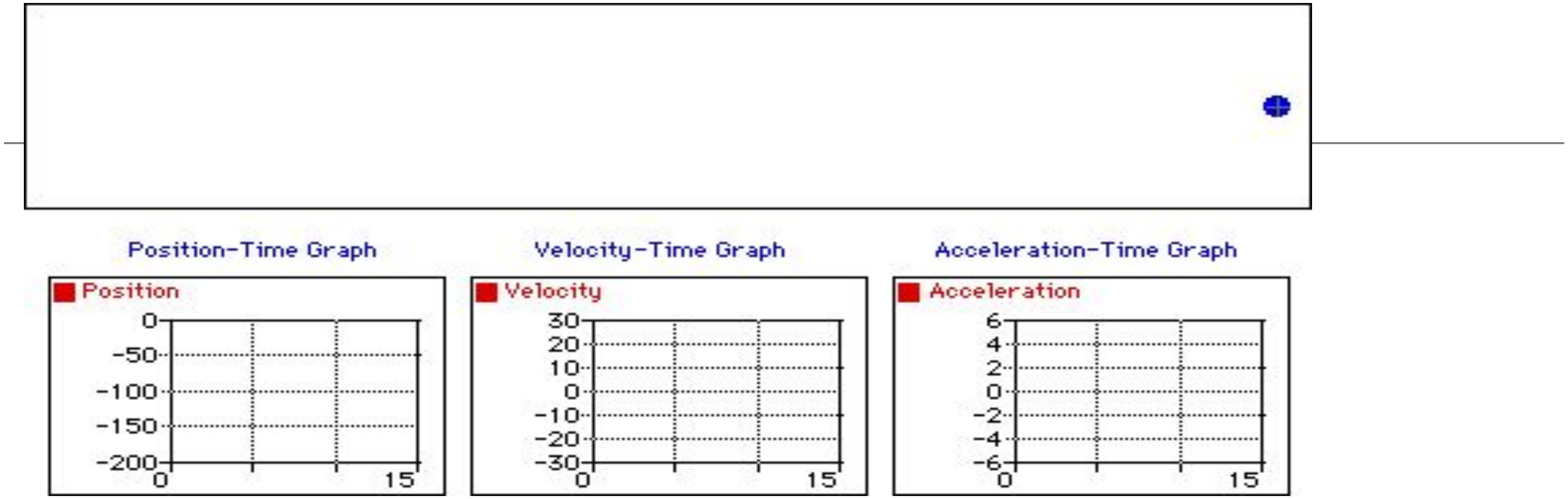
Observe that the object above moves in the positive direction with a changing velocity. An object which moves in the positive direction has a positive velocity. If the object is speeding up, then its acceleration vector is directed in the same direction as its motion (in this case, a positive acceleration). The dot diagram shows that each consecutive dot is not the same distance apart (i.e., a changing velocity). The position-time graph shows that the slope is changing (meaning a changing velocity) and positive (meaning a positive velocity). The velocity-time graph shows a line with a positive (upward) slope (meaning that there is a positive acceleration); the line is located in the positive region of the graph (corresponding to a positive velocity). The acceleration-time graph shows a horizontal line in the positive region of the graph (meaning a positive acceleration).

Position varies with positive velocity and negative acceleration



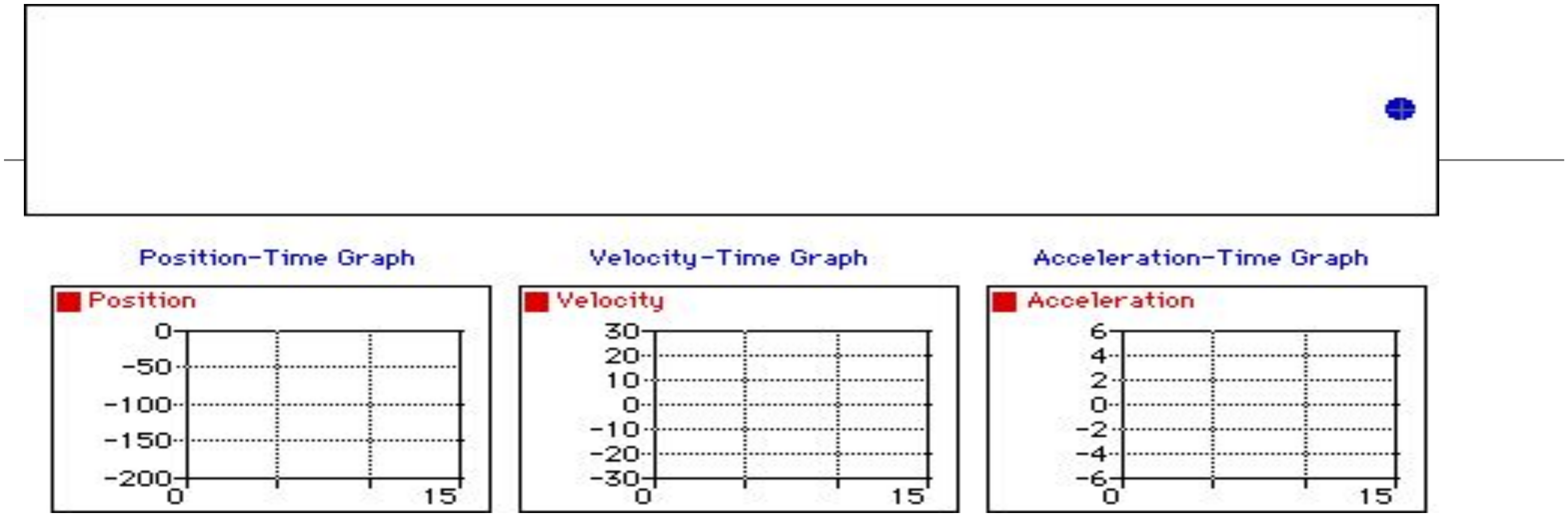
Observe that the object above moves in the positive direction with a changing velocity. An object which moves in the positive direction has a positive velocity. If the object is slowing down then its acceleration vector is directed in the opposite direction as its motion (in this case, a negative acceleration). The dot diagram shows that each consecutive dot is not the same distance apart (i.e., a changing velocity). The position-time graph shows that the slope is changing (meaning a changing velocity) and positive (meaning a positive velocity). The velocity-time graph shows a line with a negative (downward) slope (meaning that there is a negative acceleration); the line is located in the positive region of the graph (corresponding to a positive velocity). The acceleration-time graph shows a horizontal line in the negative region of the graph (meaning a negative acceleration).

Decrease position with velocity and acceleration



Observe that the object above moves in the negative direction with a changing velocity. An object which moves in the negative direction has a negative velocity. If the object is speeding up then its acceleration vector is directed in the same direction as its motion (in this case, a negative acceleration). The dot diagram shows that each consecutive dot is not the same distance apart (i.e., a changing velocity). The position-time graph shows that the slope is changing (meaning a changing velocity) and negative (meaning a negative velocity). The velocity-time graph shows a line with a negative (downward) slope (meaning that there is a negative acceleration); the line is located in the negative region of the graph (corresponding to a negative velocity). The acceleration-time graph shows a horizontal line in the negative region of the graph (meaning a negative acceleration).

Decrease position with negative velocity and positive acceleration

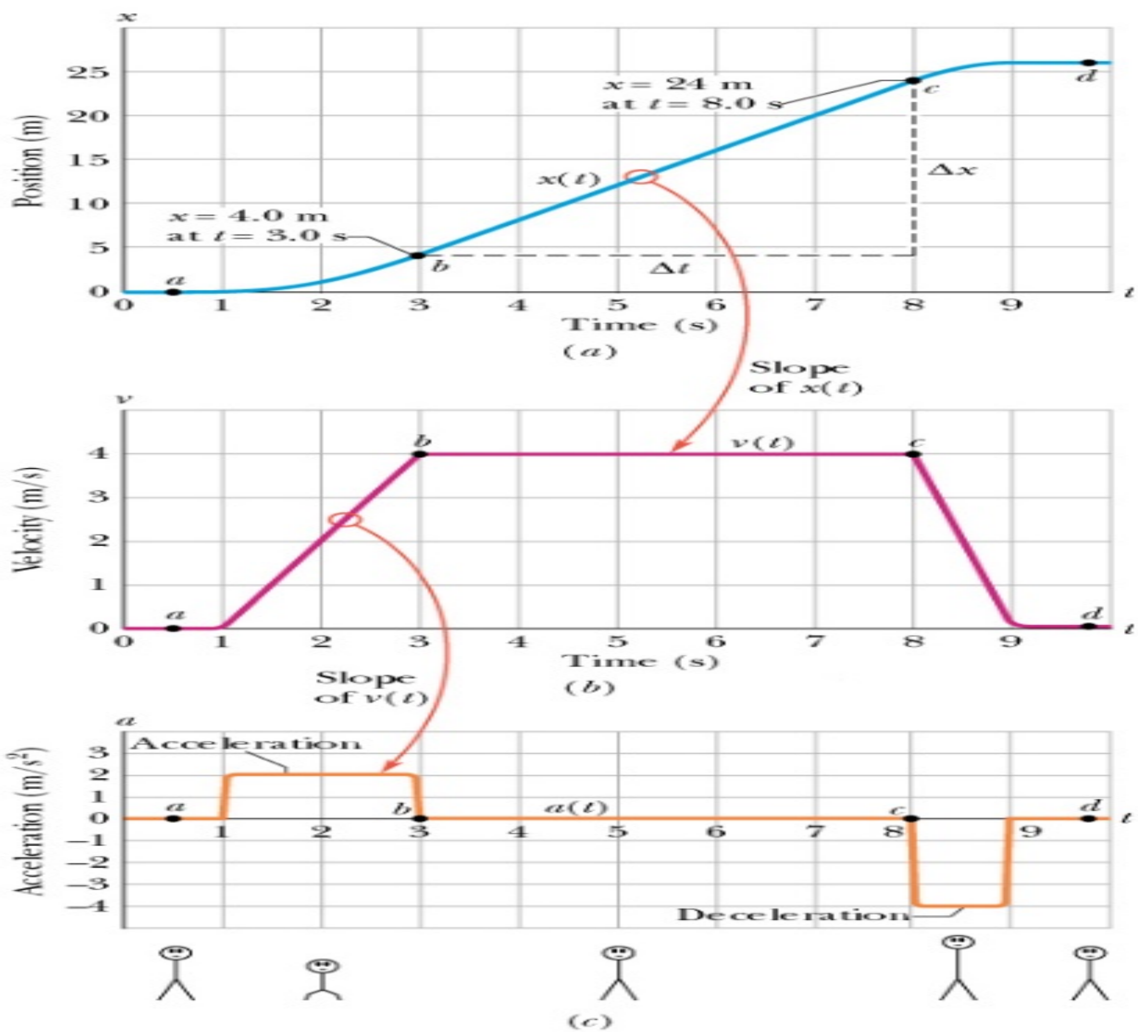


Observe that the object above moves in the negative direction with a changing velocity. An object which moves in the negative direction has a negative velocity. If the object is slowing down then its acceleration vector is directed in the opposite direction as its motion (in this case, a positive acceleration). The dot diagram shows that each consecutive dot is not the same distance apart (i.e., a changing velocity). The position-time graph shows that the slope is changing (meaning a changing velocity) and negative (meaning a negative velocity). The velocity-time graph shows a line with a positive (upward) slope (meaning that there is a positive acceleration); the line is located in the negative region of the graph (corresponding to a negative velocity). The acceleration-time graph shows a horizontal line in the positive region of the graph (meaning a positive acceleration).

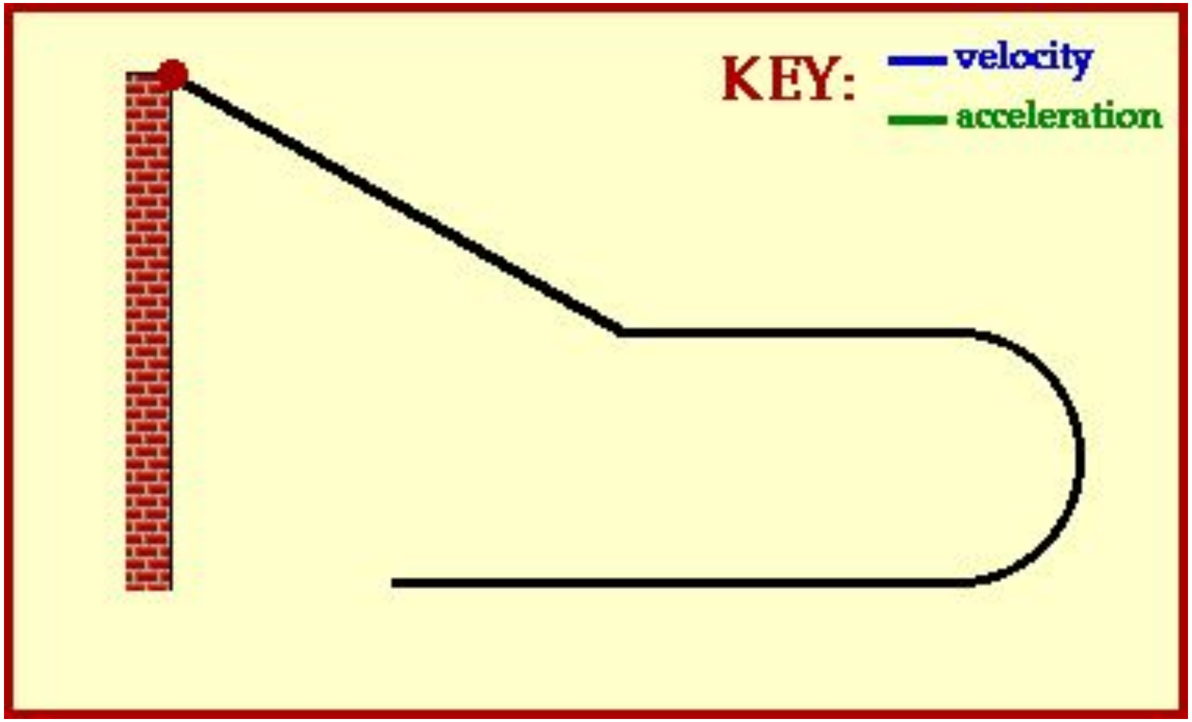
Check Point

A wombat moves along an x axis. What is the sign of its acceleration if it is moving (a) in the positive direction with increasing speed, (b) in the positive direction with decreasing speed, (c) in the negative direction with increasing speed, and (d) in the negative direction with decreasing speed?

Example:
An elevator cab that is initially stationary , then moves upward, and then stops.



Direction of acceleration and velocity



Relationship between Acceleration and Velocity (First Stage)

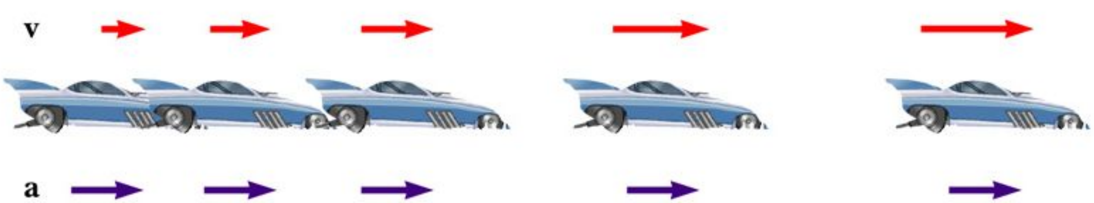
Velocity and acceleration are in the same direction

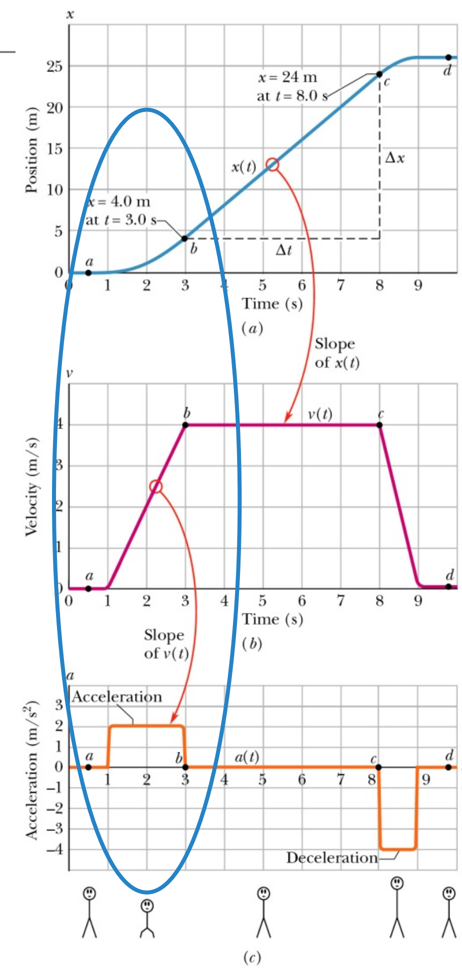
Acceleration is uniform (blue arrows maintain the same length)

Velocity is increasing (red arrows are getting longer)

$$v_f(t) = v_i + at$$

Positive velocity and positive acceleration



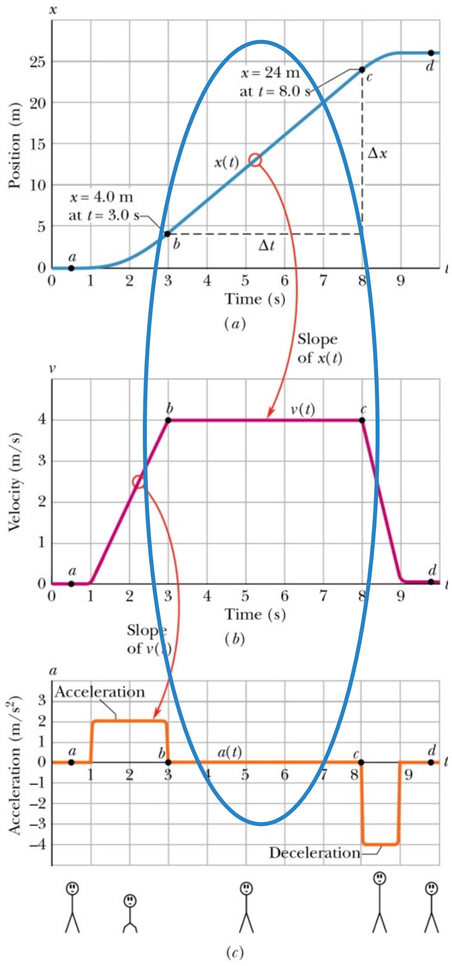


Relationship between Acceleration and Velocity (Second Stage)

Uniform velocity (shown by red arrows maintaining the same size)

Acceleration equals zero

$$v_f(t) = v_i + at$$

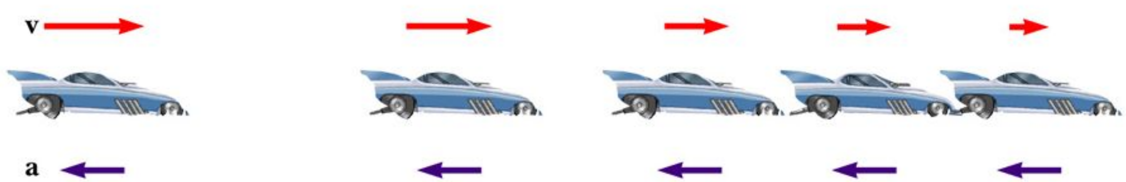


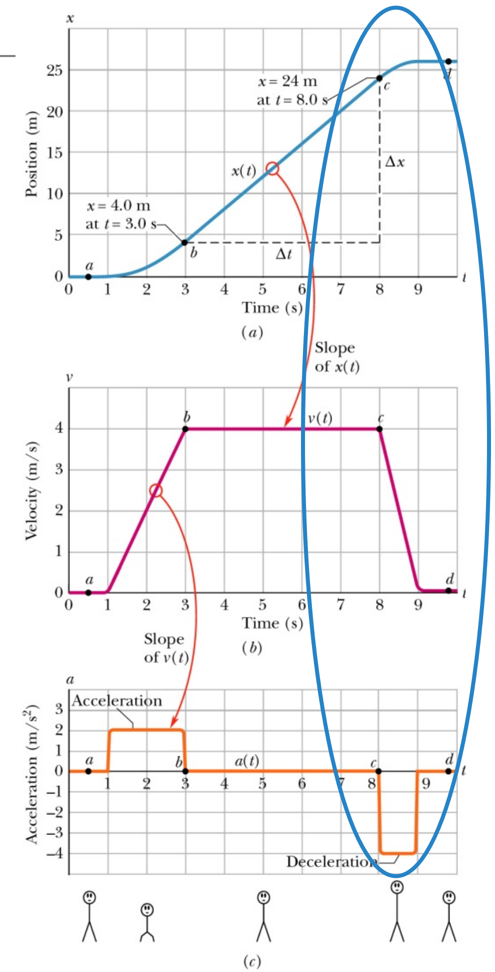
Relationship between Acceleration and Velocity (Third Stage)

- Acceleration and velocity are in opposite directions
- Acceleration is uniform (blue arrows maintain the same length)
- Velocity is decreasing (red arrows are getting shorter)

$$v_f(t) = v_i + at$$

- Velocity is positive and acceleration is negative





Kinematic Variables: x, v, a

Position is a function of time: $x = x(t)$

Velocity is the rate of change of position.

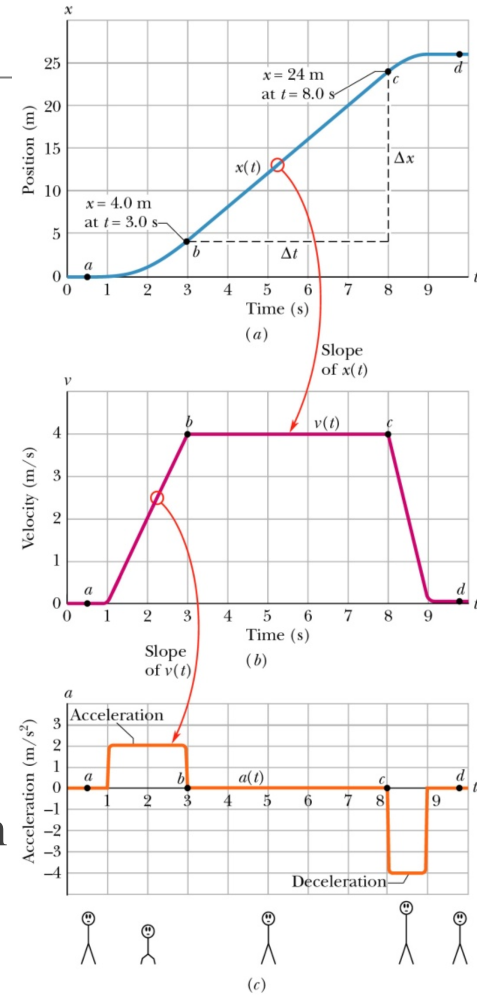
Acceleration is the rate of change of velocity.

$$\boxed{v = \lim_{\Delta t \to 0} \frac{\Delta x}{\Delta t} = \frac{dx}{dt}} \qquad a = \lim_{\Delta t \to 0} \frac{\Delta v}{\Delta t} = \frac{dv}{dt}$$

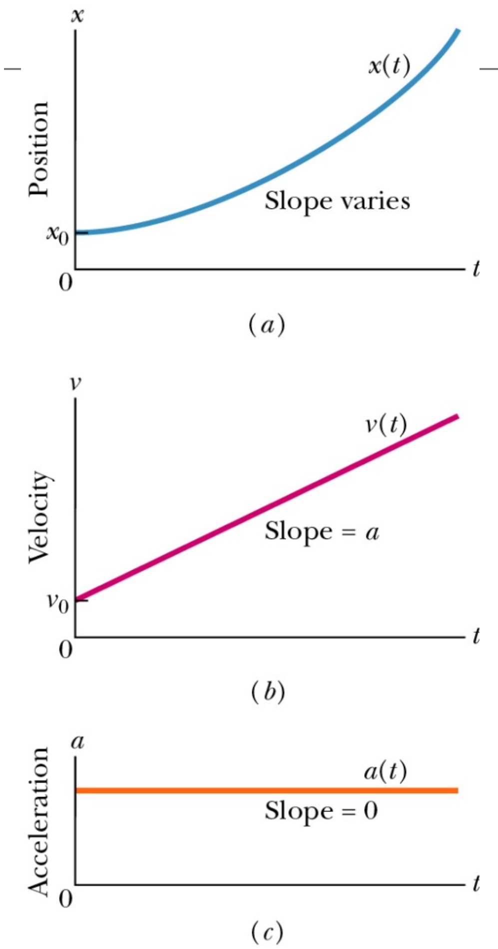
Position Velocity Acceleration

Graphical relationship between x, v, and a

This same plot can apply to an elevator that is initially stationary, then moves upward, and then stops. Plot v and a as a function of time.



Special Case: Motion with Uniform Acceleration (our typical case)



Acceleration is a constant

Kinematic Equations (which we will derive in a moment)

$$v = v_0 + at$$

$$\Delta x = \bar{v}t = \frac{1}{2}(v_0 + v)t$$

$$\Delta x = v_0 t + \tfrac{1}{2}at^2$$

$$v^2 = v_0{}^2 + 2a\Delta x$$

Derivation of the Equation (1)

- Given initial conditions:
  a(t) = constant = a, v(t = 0) = v_0, x(t = 0) = x_0

  Start with definition of average acceleration:

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t - t_0} = \frac{v - v_0}{t - 0} = \frac{v - v_0}{t} = a$$

  We immediately get the first equation

$$\boxed{v = v_0 + at}$$

  Shows velocity as a function of acceleration and time

  Use when you don't know and aren't asked to find the displacement

Derivation of the Equation (2)

Given initial conditions:

- $a(t) = \text{constant} = a$, $v(t = 0) = v_0$, $x(t = 0) = x_0$

Start with definition of average velocity:

$$v_{avg} = \frac{x - x_0}{t} = \frac{\Delta x}{t}$$

Since velocity changes at a constant rate, we have

$$\boxed{\Delta x = v_{avg} t = \frac{1}{2}(v_0 + v)t}$$

Gives displacement as a function of velocity and time

Use when you don’t know and aren’t asked for the acceleration

Derivation of the Equation (3)

- Given initial conditions:
  $a(t) = \text{constant} = a,\ v(t=0) = v_0,\ x(t=0) = x_0$

Start with the two just-derived equations:

$$v = v_0 + at \qquad \Delta x = v_{avg} t = \frac{1}{2}(v_0 + v)t$$

We have $\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2}(v_0 + v_0 + at)t$

$$\boxed{\Delta x = x - x_0 = v_0 t + \frac{1}{2}at^2}$$

Gives displacement as a function of all three quantities: time, initial velocity and acceleration

Use when you don't know and aren't asked to find the final velocity

Derivation of the Equation (4)

Given initial conditions:

- $a(t) = \text{constant} = a$, $v(t = 0) = v_0$, $x(t = 0) = x_0$

Rearrange the definition of average acceleration

, to find the time

$$a_{avg} = \frac{\Delta v}{\Delta t} = \frac{v - v_0}{t} = a \qquad t = \frac{v - v_0}{a}$$

Use it to eliminate *t* in the second equation:

, rearrange to get

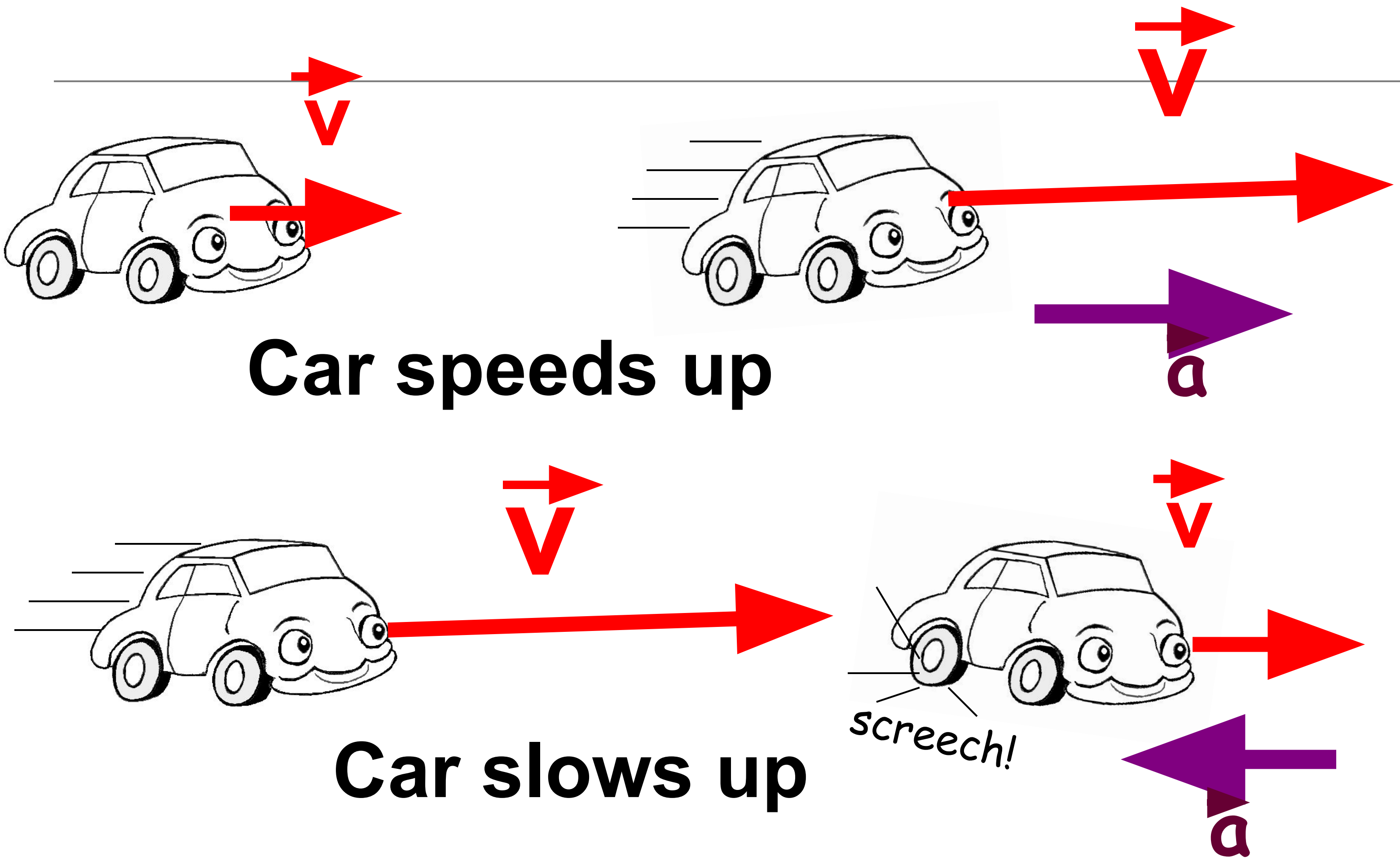
$$\Delta x = \frac{1}{2}(v_0 + v)t = \frac{1}{2a}(v + v_0)(v - v_0) = \frac{v^2 - {v_0}^2}{2a}$$

$$\boxed{v^2 = {v_0}^2 + 2a\Delta x = {v_0}^2 + 2a(x - x_0)}$$

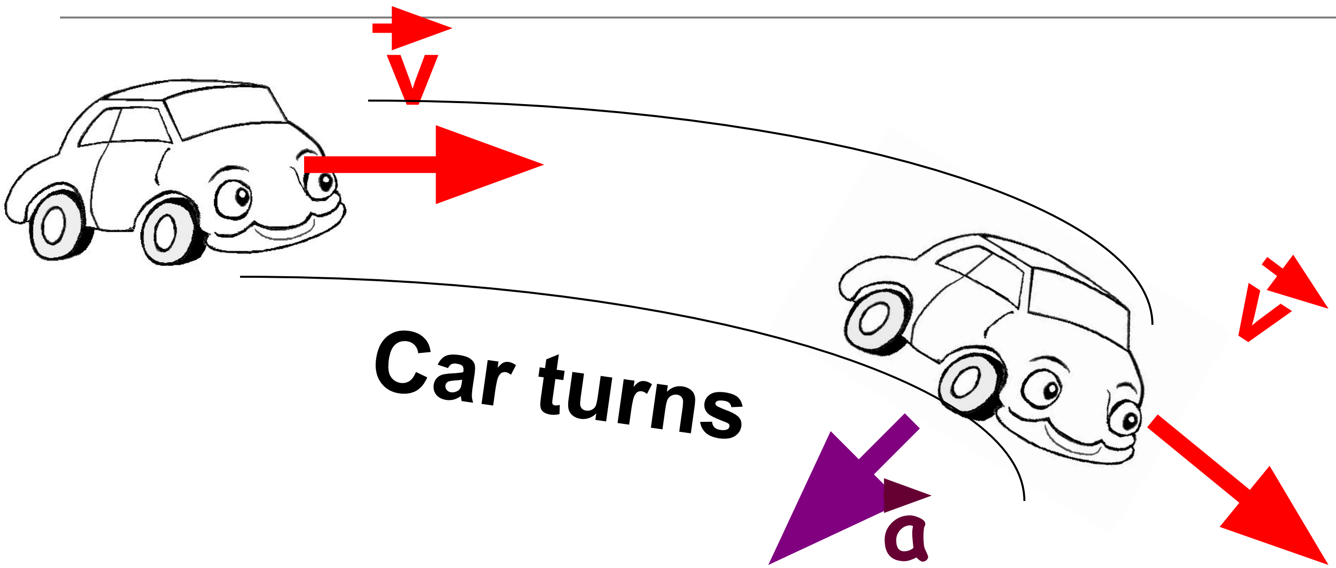
Gives velocity as a function of acceleration and displacement

Use when you don't know and aren't asked for the time

Different ways to change $\vec{V}$



Accelerations



In all three cases, $\vec{V}$ changes.
Therefore these are all examples of accelerations

1. The position of a particle moving in a straight line is given by $X = 5+2t +4t^2-t^3$, where x is in meter. (a) Find an expression for the velocity as a function of time. (b) Find the position of the particle at t=0 , 1, 0.1 , and 0.01sec. (c) What is the average velocity between t=0sec and t=1 sec , between t=0 t=0.1sec , and between t=0 and t=0.01sec ? (d) What is the instantaneous velocity at t=0sec.(e) What conclusion do you draw from the answers in (c) and (d)?
2. A car starts from rest and accelerates uniformly to a speed of 25 m/s in 8 sec. (a) What is the acceleration? (b) How far did it travel in 8 sec? (Ans: 3.13 m/s^2 , 100m)
3. A boy stands on the edge of a building 10m above the ground and throws a ball upward with an intial velocity of 12 m/s. It misses the roof on the way down and falls to the ground. Find how long the ball was in the air and its velocity just before it strikes the ground. (Hint: take y=0 at t=0 and y final as -10m) (Ans: 3.11 sec , 18.44 m/s)

Q1: what we want to achieve by studying physics?

Q 2. Three ropes are tied to a stake and the following forces are exerted. Using rectangular component method to find the resultant force

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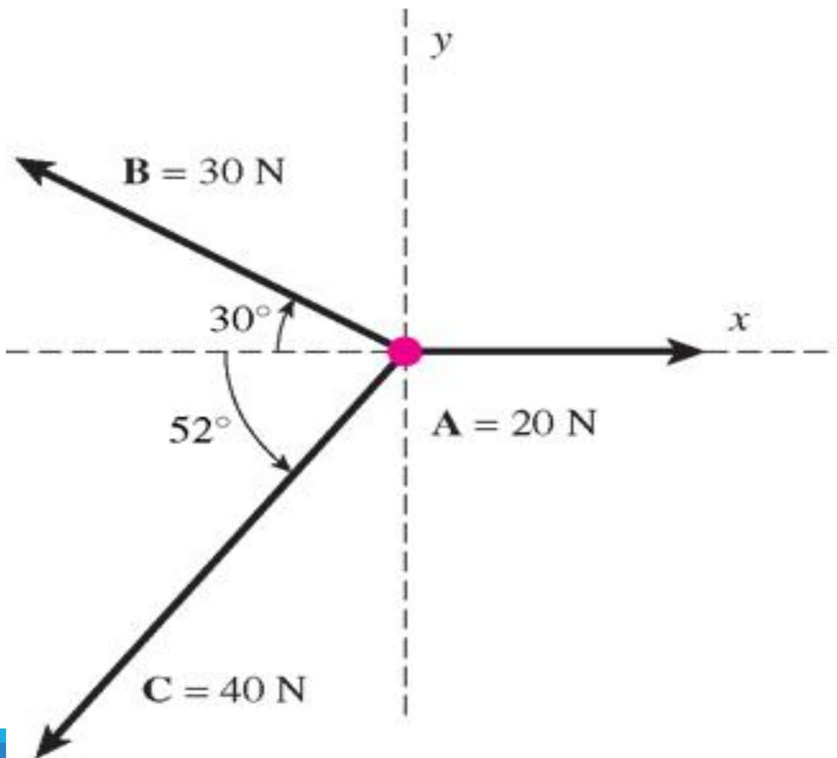
Q3. Find (i) dot product (ii) the cross product (iii) angle between of the following vectors (iv) A+B

$$\vec{A} = 2\hat{i} + 3\hat{j} + 2\hat{k}$$

$$\vec{B} = -4\hat{i} + 5\hat{j} - 3\hat{k}$$



(a) Rough sketch

Applied Phys Quiz 1 Section 1F

Quiz 1A

•34 Two vectors are presented as $\vec{a} = 3.0\hat{i} + 5.0\hat{j}$ and $\vec{b} = 2.0\hat{i} + 4.0\hat{j}$. Find (a) $\vec{a} \times \vec{b}$, (b) $\vec{a} \cdot \vec{b}$, (c) $(\vec{a} + \vec{b}) \cdot \vec{b}$, and (d) the component of $\vec{a}$ along the direction of $\vec{b}$. (*Hint:* For (d), consider Eq. 3-20 and Fig. 3-18.)

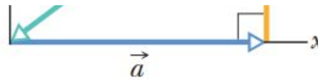


Figure 3-32 Problems 33 and 54.

•35 Two vectors, $\vec{r}$ and $\vec{s}$, lie in the xy plane. Their magnitudes are 4.50 and 7.30 units, respectively, and their directions are 320° and 85.0°, respectively, as measured counterclockwise from the positive x axis. What are the values of (a) $\vec{r} \cdot \vec{s}$ and (b) $\vec{r} \times \vec{s}$?

•36 If $\vec{d}_1 = 3\hat{i} - 2\hat{j} + 4\hat{k}$ and $\vec{d}_2 = -5\hat{i} + 2\hat{j} - \hat{k}$, then what is $(\vec{d}_1 + \vec{d}_2) \cdot (\vec{d}_1 \times 4\vec{d}_2)$?

•37 Three vectors are given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} - 2.0\hat{k}$, $\vec{b} = -1.0\hat{i} - 4.0\hat{j} + 2.0\hat{k}$, and $\vec{c} = 2.0\hat{i} + 2.0\hat{j} + 1.0\hat{k}$. Find (a) $\vec{a} \cdot (\vec{b} \times \vec{c})$, (b) $\vec{a} \cdot (\vec{b} + \vec{c})$, and (c) $\vec{a} \times (\vec{b} + \vec{c})$.

Quiz 1B

••39 Vector $\vec{A}$ has a magnitude of 6.00 units, vector $\vec{B}$ has a magnitude of 7.00 units, and $\vec{A} \cdot \vec{B}$ has a value of 14.0. What is the angle between the directions of $\vec{A}$ and $\vec{B}$?

••40 GO Displacement $\vec{d}_1$ is in the yz plane 63.0° from the positive direction of the y axis, has a positive z component, and has a magnitude of 4.50 m. Displacement $\vec{d}_2$ is in the xz plane 30.0° from the positive direction of the x axis, has a positive z component, and has magnitude 1.40 m. What are (a) $\vec{d}_1 \cdot \vec{d}_2$, (b) $\vec{d}_1 \times \vec{d}_2$, and (c) the angle between $\vec{d}_1$ and $\vec{d}_2$?

••41 SSM ILW WWW Use the definition of scalar product, $\vec{a} \cdot \vec{b} = ab \cos\theta$, and the fact that $\vec{a} \cdot \vec{b} = a_x b_x + a_y b_y + a_z b_z$ to calculate the angle between the two vectors given by $\vec{a} = 3.0\hat{i} + 3.0\hat{j} + 3.0\hat{k}$ and $\vec{b} = 2.0\hat{i} + 1.0\hat{j} + 3.0\hat{k}$.

••42 In a meeting of mimes, mime 1 goes through a displacement $\vec{d}_1 = (4.0\text{ m})\hat{i} + (5.0\text{ m})\hat{j}$ and mime 2 goes through a displacement $\vec{d}_2 = (-3.0\text{ m})\hat{i} + (4.0\text{ m})\hat{j}$. What are (a) $\vec{d}_1 \times \vec{d}_2$, (b) $\vec{d}_1 \cdot \vec{d}_2$, (c) $(\vec{d}_1 + \vec{d}_2) \cdot \vec{d}_2$, and (d) the component of $\vec{d}_1$ along the direction of $\vec{d}_2$? (*Hint:* For (d), see Eq. 3-20 and