

# Homework 1

---

Peiyun(Seed) Zeng  
EN:437801

September 20, 2014

## PROBLEM 1

**Decide whether the following statement is True or False. You must justify your answer to receive full credit (if you think it is true, argue it; if you think it is false, give a counter example).**

(a)  $\mathbf{f(n)} = \mathbf{O(g(n))} \wedge \mathbf{f(n)} = \mathbf{\Omega(h(n))} \longrightarrow \mathbf{g(n)} = \mathbf{\theta(h(n))}$

**Solution:**

This is False. **Counter example:**  $f(n) = n^2; g(n) = n^3; h(n) = n$ . which apparently satisfies our constrain (  $\mathbf{f(n)} = \mathbf{O(g(n))} \wedge \mathbf{f(n)} = \mathbf{\Omega(h(n))} \longrightarrow \mathbf{g(n)} = \mathbf{\theta(h(n))}$  ).

Yet, it **does not** follow that  $g(n) = n^3 = \theta(h(n)) = \theta(n)$

---

# Homework 1

---

Peiyun(Seed) Zeng  
EN:437801

September 20, 2014

## PROBLEM 2

(a)  $T(n) = 7T(n/2) + n^3 * \log n + 10n * \log n$

**Solution:**

Let's apply Master Method one more time

1.  $a = 7, b = 2 \rightarrow x = \log_b a = \log_2 7 \approx 2.8$
2.  $n^3 * \log n$  grows much faster  $\rightarrow f(n) = n^3 * \log n + 10n * \log n = \Theta(n^3 * \log n) \rightarrow y = 3, k = 1$
3.  $y > x \rightarrow$  This Corresponds to Case 3 – The root Dominates since  $f(n)$  grows faster than  $x$
4. **So we can conclude:**  $T(n) = \Theta(n^x) = \Theta(n^{\log_b a}) = \Theta(n^{\log_2 7})$

**Final Result:**  $T(n) = \Theta(n^x) = \Theta(n^{\log_b a}) = \Theta(n^{\log_2 7})$

(b)  $T(n) = 2T(n/8) + \sqrt[3]{n} + 10 \lg n$

**Solution:**

Let's apply Master Method for this one

1.  $a = 2, b = 8 \rightarrow x = \log_b a = \log_8 2 = (1/3)$
2.  $\sqrt[3]{n}$  grows significantly faster than  $10 \lg n \rightarrow f(n) = \sqrt[3]{n} + 10 \lg n = \Theta(\sqrt[3]{n}) \rightarrow y = 1/3, k = 0$
3. This corresponds to Case 2– Roots has equal wight to the leaf since  $x$  grows roughly the rate as the leaf
4. **So we can conclude:**  $T(n) = \Theta(n^x (\log n)^{(k+1)}) = n^{(1/3)} * \log n$

**Final Result:**  $T(n) = \Theta(n^x (\log n)^{(k+1)}) = n^{(1/3)} * \log n$

(c)  $T(n) = 2T(\sqrt{n}) + \lg \lg n$

**Solution:**

Unfortunately, this cannot be directly solved by Master Method. Let's start from substitution r

1. Let  $m = \lg n \equiv n = 2^m \rightarrow$  now substitute into the original equation
2.  $T(2^m) = 2 * T(2^{m/2}) + \lg m$  (equation [1])
3. Now let  $S(m) = 2 * S(m/2) + \lg m$  (equation [2])
4. equation [2] can be solved using **Master theorem**
5.  $a = 2, b = 2, x = \log_2 2 = 1$
6.  $f(n) = \lg m = \Theta(n^0 * (\log n)^1, y = 0, k = 0$   
(Since Log function grows really slow, so log and lg used interchangeably here)
7.  $x > y \rightarrow$  This corresponds to Case 1
8. So we can conclude from master theorem :  $S(m) = \Theta(m^1) = \Theta(m) = \Theta(\lg n)$

**Final Result:**  $T(n) = \lg n$

INTRO TO ALGORITHM, CSE 241

PROFESSOR ZHANG

---

# Homework 1

---

Peiyun(Seed) Zeng  
EN:437801

September 20, 2014

Since I realized there is no extra credit for using latex. and i donot want to draw a tree using latex. Solution will be presented in handwritten (Next Page)  
Welcome to the messy part of the story :) Good Luck