LOGIC AND DISCRETE MATH 240

PROFESSOR PLESS

Homework 2

Peiyun(Seed) Zeng

Due September 12, 2014

Useful Logical equivalences gonna be cited in this Homework

1. Identity: $P \wedge T \equiv P$
2. Identity: $P \vee F \equiv P$
3. Domination: $P \wedge F \equiv F$
4. Domination: $P \vee T \equiv \mathrm{T}$
5. Negation : $P \wedge \neg P \equiv F$
6. Negation : $P \vee \neg P \equiv T$
7. Idempotent : $P \wedge P \equiv P$
8. Idempotent : $P \vee P \equiv P$
9. Double Negation : $\neg\neg P \equiv P$
10. Commutative Law : $P \wedge Q \equiv Q \wedge P$
11. Commutative Law : $P \vee Q \equiv Q \vee P$
12. DeMorgans Law : $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
13. DeMorgans Law: $\neg(P \wedge Q) <==> (\neg P \vee \neg Q)$
14. Implies Rule : $P \rightarrow Q \equiv (\neg P \vee Q)$

15. Contrapositive: $P \rightarrow Q \equiv \neg Q \rightarrow \neg P$

16. Absorption : $P \wedge (P \vee Q) \equiv P$

17. Absorption : $P \vee (P \wedge Q) \equiv P$

18. Associative Law : $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$

19. Associative Law : $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$

20. Distributive Law: $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$

21. Distributive Law : $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

Problems to Turn in

1. Using (and citing) rules of inference and logical equivalence, Prove the following form of argument is valid:

$$
\begin{array}{l}
P \longrightarrow (Q \vee R) \\
R \longrightarrow S \\
\neg S \longrightarrow \neg Q \\
P \\
\hline
S
\end{array}
$$

Solution:

1.P	(Given)
2.$P \longrightarrow (Q \vee R)$	(Given)
3.$R \longrightarrow S$	(Given)
4.$\neg S \longrightarrow \neg Q$	(Given)
5.$Q \vee R$	(From 1 and 2)
6.$Q \vee R$	(From 5 and apply rule(i)–double negation)
7.$\neg Q \longrightarrow R$	(Apply rule(n)–Implies rule)
8.$\neg Q \longrightarrow S$	(from 7 and 3; Infer From Hypothetical Syllogism)
9.$Q \longrightarrow S$	(from 4 and apply rule(o) –Contrapositive)
10.$Q \vee S$	(From 9 and apply rule (n)–implies rule)
11.$\neg Q \vee S$	(From 10 and apply rule (n)–implies rule)
12.S	(S must be True from 10 and 11)

Extra Note on step 12:

a) either "$\neg Q$" or "Q" must be False

b) $\neg Q \vee S$ is True

So S must be True
So the inference logic is a tautology (if we assume all premises are true).
Thus, the argument is Valid

2. Using (and citing) rules of inference and logical equivalence, Prove the following form of argument is valid:

$$\neg P(a) \longrightarrow Q(a)$$
$$P(a) \longrightarrow Q(a)$$
$$\forall x, Q(x) \longrightarrow S(x)$$
$$\overline{}$$
$$S(a)$$

Solution:

1.$\forall x, Q(x) \longrightarrow S(x)$ (Given)

2.$\neg P(a) \longrightarrow Q(a)$ (Given)

3.$P(a) \longrightarrow Q(a)$ (Given)

4.From 1, every x in domain,including a,will have to Satisfy the function $\rightarrow Q(a) \longrightarrow S(a)$

5.$Q(a) \longrightarrow S(a)$ (from 4)

6.$\neg P(a) \longrightarrow S(a)$ (From 2 and 5; Infer From Hypothetical Syllogism)

7.$P(a) \longrightarrow S(a)$ (From 3 and 5; Infer From Hypothetical Syllogism)

8.$P(a) \vee S(a)$ (From 6 and apply rule (n)–implies rule)

9.$\neg P(a) \vee S(a)$ (From 7 and apply rule (n)–implies rule)

10.$S(a)$ (From 9)

Extra Note on step 10:

a) either "$\neg P(a)$"or"$P(a)$" must be False

b) $\neg P(a) \vee S(a)$ is True

So S(a) must be True
So the inference logic is a tautology (if we assume all premises are true).
Thus, the argument is Valid

3. Write the logical negation of the following quantified statement, and express your answer in a form where the negation sign is inside the scope of all the quantifiers. Use logical equivalences to prove that your formula is equivalent

$$\forall x P(x) \longrightarrow \forall y (Q(y) \wedge R(y))$$

Solution:

Ok let's quote the General DeMorgan Theorem here (introduced by Dr.pless in lecture and proof won't be shown here)

$$\neg[\forall \mathbf{x} \mathbf{P}(\mathbf{x})] \equiv \exists \mathbf{x} \neg \mathbf{P}(\mathbf{x})$$

We state here first **the final result of the negation** Shall be

$$\forall x P(x) \wedge \exists y (\neg Q(y) \vee \neg R(y))$$

Now Let's **Prove it**

$$\begin{aligned}
\text{The negation of} \quad & \forall x P(x) \longrightarrow \forall y (Q(y) \wedge R(y)) \\
& \equiv \neg[\forall x P(x) \longrightarrow \forall y (Q(y) \wedge R(y))] \\
& \equiv \neg[\neg \forall x P(x) \vee \forall y (Q(y) \wedge R(y))] && \text{(Apply rule[n]–Implies Rule)} \\
& \equiv \forall x P(x) \wedge \neg \forall y (Q(y) \wedge R(y)) && \text{(Apply rule[l]–DeMorgan's Rule)} \\
& \equiv \forall x P(x) \wedge \exists y (\neg Q(y) \vee \neg R(y)) \\
& && \text{(Apply rule[m]–DeMorgan's Rule and Generalized DeMorgan's Theorem)}
\end{aligned}$$

By using logical equivalences, our final result is **proved**