LOGIC AND DISCRETE MATH 240

PROFESSOR PLESS

Homework 1

Peiyun(Seed) Zeng

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1. Let M be the proposition “you cause a memory overflow error”
   Let P be the proposition “you use pointers incorrectly”

   Express each of the following statements using M, P, and logical connectives, for example, in an expression that might look like: $(M \wedge P) \longrightarrow \neg P$.

   a) You cause a memory overflow error only if you use pointers incorrectly
   b) You use pointers correctly but you cause a memory overflow error.
   c) If you used pointers incorrectly but didn't cause memory overflow error then you used pointers correctly.

Solutions:

a) The key word here is only if. It implies that"you use pointers correctly"(P) is the **necessary** condition for "you cause a memory flow"(M)

   So the final answer becomes: $(M \rightarrow P)$

b) first, English word but has the same logic meaning as $\wedge$ (the logic **and**). Also, "you use pointers correctly" is an negation of P

   then obviously we can rewrite the sentecne as: $(\neg P \wedge M)$

c) Let's walk through this sentence. "if you use d pointers incoreccctly" is P. "but" as we discussed is logic "and" ($\wedge$). you did not cause memory overflow is the negation of M ($\neg M$) .

The "if then" is equivalent to $\rightarrow$

Then the final answer **becomes:** $(P \wedge \neg M) \rightarrow \neg P$

2. Consider the expression: $(Q \wedge P) \vee \neg(\neg Q \longrightarrow \neg P)$. In any way that you like, find an equivalent expression that is as short as possible. Prove that your expression is equivalent.

Solutions:

1. **The first approach**- using boolean algebra rules:

   a) Identity: $P \hat{} T \equiv P$
   b) Identity: $P \vee F \equiv P$
   c) Domination: $P \hat{} F \equiv F$
   d) Domination: $P \vee T \equiv \mathrm{T}$
   e) Negation : $P \wedge \neg P \equiv F$
   f) Negation : $P \vee \neg P \equiv T$
   g) Idempotent : $P \wedge P \equiv P$
   h) Idempotent : $P \vee P \equiv P$
   i) Double Negation : $\neg\neg P \equiv P$
   j) Commutative Law : $P \wedge Q \equiv Q \wedge P$
   k) Commutative Law : $P \vee Q \equiv Q \vee P$
   l) DeMorgans Law : $\neg(P \vee Q) \equiv (\neg P \wedge \neg Q)$
   m) DeMorgans Law: $\neg(P \wedge Q) <==> (\neg P \vee \neg Q)$
   n) Implies Rule : $P \rightarrow Q \equiv (\neg P \vee Q)$
   o) Contrapositive: $P \rightarrow Q \equiv \neg Q \rightarrow \neg P$
   p) Absorption : $P \wedge (P \vee Q) \equiv P$
   q) Absorption : $P \vee (P \wedge Q) \equiv P$
   r) Associative Law : $P \vee (Q \vee R) \equiv (P \vee Q) \vee R$
   s) Associative Law : $P \wedge (Q \wedge R) \equiv (P \wedge Q) \wedge R$
   t) Distributive Law: $P \wedge (Q \vee R) \equiv (P \wedge Q) \vee (P \wedge R)$
   u) Distributive Law : $P \vee (Q \wedge R) \equiv (P \vee Q) \wedge (P \vee R)$

Now let's simply the Expression by applying the rules above

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\begin{aligned}
&(Q \wedge P) \vee \neg(\neg Q \longrightarrow \neg P) && \\
&= (Q \wedge p) \quad \vee \quad \neg(P \rightarrow Q) && \text{(Applying rule [o])} \\
&= (Q \wedge p) \quad \vee \quad \neg(\neg P \vee Q) && \text{(applying rule [n])} \\
&= (Q \wedge p) \quad \vee \quad (P \wedge \neg Q) && \text{(Applying rule [l]- DeMorgan's Theoreom)} \\
&= P \wedge (Q \vee \neg Q) && \text{(Applying rule [t] and rule [j])} \\
&= P \wedge T && \text{(Applying rule [f])} \\
&= P && \text{(OMG! I love this result)}
\end{aligned}
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2. The second approach, which is essentially a better and more reliable approach is by using **Truth Table** and **K-Map**