Single Snapshot Joint Estimation of Angles and Times of Arrival: A 2D Matrix Pencil Approach

A. BAZZI[1,2] **D.T.M. SLOCK**[1] L. MEILHAC[2]

[1]EURECOM-Mobile Communications Department

[2]CEVA-RivieraWaves

ICC, 2016

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3. 2D-Matrix Pencil Approach
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   - AoA Estimation via 2D-Matrix Pencils
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Intro

- Indoor Localisation:
  1. **Radio-Based: Online estimation of AoA (Angle of Arrival) and/or ToA (Time of Arrival) , etc.**

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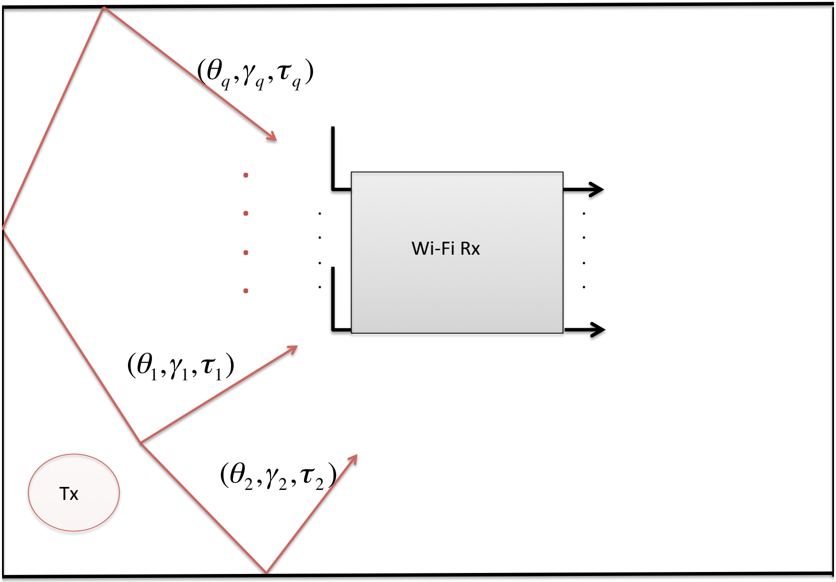
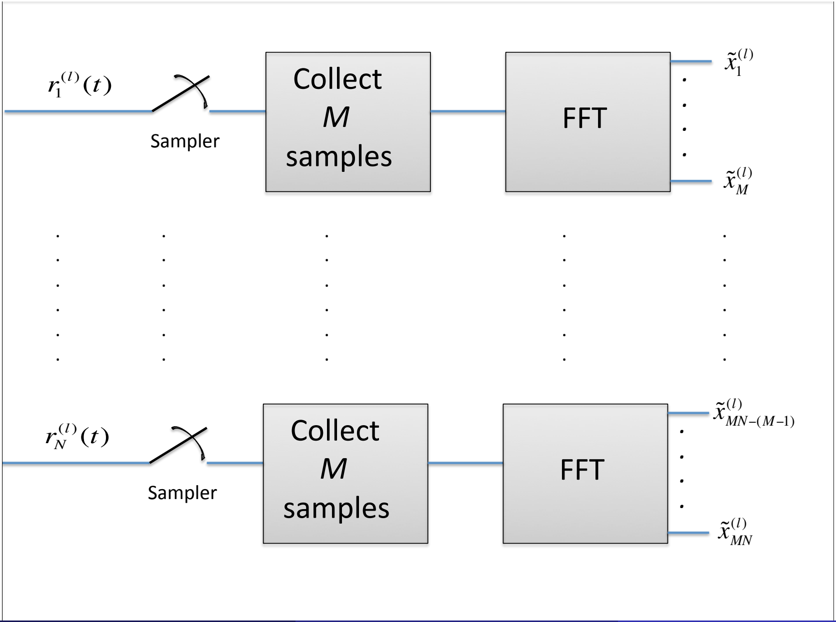
- We focus on **Radio-Based**, and in particular **JADE**.
- Assume single user SIMO transmitting known OFDM symbols in a multipath channel.

Scenario



$(\theta_q, \gamma_q, \tau_q)$
Wi-Fi Rx
$(\theta_1, \gamma_1, \tau_1)$
$(\theta_2, \gamma_2, \tau_2)$
Tx

Processing OFDM analog symbol



Formulation and Problem Statement

System Model

$$\mathbf{x} = \mathbf{H}\boldsymbol{\gamma} + \mathbf{n}$$

where

- $\mathbf{H} = [\mathbf{a}(\theta_1) \otimes \mathbf{c}(\tau_1), \ldots, \mathbf{a}(\theta_q) \otimes \mathbf{c}(\tau_q)]$ assumed to be full rank.
  - $\mathbf{a}_N(\theta) = [1, z_\theta, \ldots, z_\theta^{N-1}]^\mathsf{T}$ with $z_\theta = e^{-j2\pi \frac{d}{\lambda}\sin(\theta)}$
  - $\mathbf{c}_M(\tau) = [z_\tau^{-\frac{M-1}{2}} \ldots z_\tau^{\frac{M-1}{2}}]^\mathsf{T}$ with $z_\tau = e^{-j2\pi\tau\Delta_f}$
- $\boldsymbol{\gamma} = [\gamma_1 \ldots \gamma_q]^T \in \mathbb{C}^{q\times 1}$
- $\mathbf{n} \in \mathbb{C}^{MN\times 1} \backsim \mathcal{N}(\mathbf{0}, \sigma^2\mathbf{I})$ background noise vector.

Problem Statement

Given one snapshot $\mathbf{x}$ *and the number of multipath components* q*, jointly estimate the AoAs/ToAs of each path, i.e.* $\{(\theta_i, \tau_i)\}_{i=1}^q$.

Outline

We start by forming a matrix from the data vector $\mathbf{x}$. Let $\mathbf{X} \in \mathbb{C}^{M_p \times K_M}$ Hankel block matrix defined as follows

$$\mathbf{X} = \begin{pmatrix} \mathbf{X}_1 & \mathbf{X}_2 & \cdots & \mathbf{X}_{K_M} \\ \mathbf{X}_2 & \mathbf{X}_3 & \cdots & \mathbf{X}_{K_M+1} \\ \vdots & \vdots & \ddots & \vdots \\ \mathbf{X}_{M_p} & \mathbf{X}_{M_p+1} & \cdots & \mathbf{X}_M \end{pmatrix}$$

where

$$\mathbf{X}_i = \begin{pmatrix} x_{i,1} & x_{i,2} & \cdots & x_{i,K_N} \\ x_{i,2} & x_{i,3} & \cdots & x_{i,K_N+1} \\ \vdots & \vdots & \ddots & \vdots \\ x_{i,N_p} & x_{i,N_p+1} & \cdots & x_{i,N} \end{pmatrix} \in \mathbb{C}^{N_p \times K_N}$$

with

$$K_M = M - M_p + 1 \qquad K_N = N - N_p + 1$$

Reformulation (cont'd)

The matrix $\mathbf{X}$ could be written as

$$\mathbf{X} = \mathbf{L}\mathbf{\Gamma}\mathbf{R}^T + \mathbf{N}$$

where

- $\mathbf{L}^T = \left[\begin{array}{c|c|c|c} \mathbf{A}_{N_p} & \mathbf{A}_{N_p}\mathbf{D}_\tau & \dots & \mathbf{A}_{N_p}\mathbf{D}_\tau^{M_p-1} \end{array} \right] \in \mathbb{C}^{q \times M_p N_p}$
- $\mathbf{R}^T = \left[\begin{array}{c|c|c|c} \mathbf{A}_{K_N} & \mathbf{A}_{K_N}\mathbf{D}_\tau & \dots & \mathbf{A}_{K_M}\mathbf{D}_\tau^{M_p-1} \end{array} \right] \in \mathbb{C}^{q \times K_M K_N}$
- $\mathbf{D}_\tau = \text{diag}\ [z_{\tau_1} \dots z_{\tau_q}] \in \mathbb{C}^{q \times q}$
- $\mathbf{A}_N = [\mathbf{a}_N(\theta_1) \dots \mathbf{a}_N(\theta_q)] \in \mathbb{C}^{N \times q}$
- $\mathbf{\Gamma} = \text{diag}\ [\gamma_1 \dots \gamma_q] \in \mathbb{C}^{q \times q}$

ToA Estimation

Let

$$\mathbf{X}_{left} \triangleq \mathbf{X}_l = \mathbf{X}^{\langle :,1:K_N(K_M-1)\rangle} \qquad \mathbf{X}_{right} \triangleq \mathbf{X}_r = \mathbf{X}^{\langle :,(K_N+1):K_N K_M\rangle}$$

In a noiseless case, it is easy to see that

$$\mathbf{X}_l = \mathbf{L}\boldsymbol{\Gamma}\mathbf{R}_o^{\mathsf{T}} \qquad \mathbf{X}_r = \mathbf{L}\boldsymbol{\Gamma}\mathbf{D}_\tau\mathbf{R}_o^{\mathsf{T}} \qquad \mathbf{R}_o = \mathbf{R}^{\langle 1:K_N(K_M-1),:\rangle}$$

Using the fact that the matrix pencil $\mathbf{X}_r - \lambda\mathbf{X}_l$ drops rank at $\lambda = z_{\tau_i}$.
The estimates $\{\hat{z}_{\tau_i}^{\mathsf{MP}}\}_{i=1}^{q}$ are obtained by the eigenvalues of $\mathbf{T}$

$$\mathbf{T} = \boldsymbol{\Lambda}^{-1}\mathbf{U}^H\mathbf{X}_r\mathbf{V} \text{ where } \quad [\mathbf{U}, \boldsymbol{\Lambda}, \mathbf{V}] = \mathsf{svd}(\mathbf{X}_l)$$

ToA Estimation

Conditions for ToA Estimation via 2D-Matrix Pencils

- **A.1**: $q \leq \min \{M_p N_p, K_N(K_M - 1)\}$
- **A.2**: $P \leq \min \{N_p, K_N\}$
- **A.3**: $Q \leq \min \{M_p, K_M - 1\}$

where

- P is the maximum number of paths arriving at the same time but with different angles of arrival.
- Q is the maximum number of paths arriving at the same angle but with different times of arrival.

Outline

AoA Estimation

In a similar manner

AoA Estimation is done seperately, with a shuffled version of $\mathbf{X}$.

Conditions for AoA Estimation via 2D-Matrix Pencils

- **B.1**: $q \leq \text{min}\ \{M_p N_p, K_M(K_N - 1)\}$
- **B.2**: $P \leq \text{min}\ \{N_p, K_N - 1\}$
- **B.3**: $Q \leq \text{min}\ \{M_p, K_M\}$

where

- P is the maximum number of paths arriving at the same time but with different angles of arrival.
- Q is the maximum number of paths arriving at the same angle but with different times of arrival.

Outline

Algorithm 1

- Step 1: Form the Hankel-block-Hankel matrix $\mathbf{X}$.
- Step 2: Obtain $\{\hat{z}_{\tau_i}^{\mathsf{MP}}\}_{i=1}^{q}$.
- Step 3: Estimate the ToAs of the q paths by $\hat{\tau}_i^{\mathsf{MP}} = -\frac{\mathsf{ang}\{\hat{z}_{\tau_i}^{\mathsf{MP}}\}}{2\pi\triangle_f}$.
- Step 4: Form

$$\mathbf{Z} = \begin{pmatrix} X_{1,1} & X_{2,1} & \cdots & X_{M,1} \\ X_{1,2} & X_{2,2} & \cdots & X_{M,2} \\ \vdots & \vdots & \ddots & \vdots \\ X_{1,N} & X_{2,N} & \cdots & X_{M,N} \end{pmatrix} = \mathbf{A}_N \boldsymbol{\Gamma} \mathbf{C}_M^T + \mathbf{W}$$

where $\mathbf{W}$ is the noise part. This step comprises in estimating the term $\mathbf{G} = \mathbf{A}_{\mathsf{N}}\boldsymbol{\Gamma}$ using Least Squares (LS), i.e:

$$\hat{\mathbf{G}} = \arg\min_{\mathbf{G}} \|\mathbf{Z} - \mathbf{G}\tilde{\mathbf{C}}_M^T\|^2 \quad \text{where} \quad \tilde{\mathbf{C}}_M = [\mathbf{c}_M(\hat{\tau}_1^{\mathsf{MP}}) \ldots \mathbf{c}_M(\hat{\tau}_q^{\mathsf{MP}})]$$

The solution of the LS problem is:

$$\hat{\mathbf{G}} = \mathbf{Z}\tilde{\mathbf{C}}_{\mathsf{M}}^{\dagger} \quad \text{where} \quad \tilde{\mathbf{C}}_M^{\dagger} = \tilde{\mathbf{C}}_M^{*}\left(\tilde{\mathbf{C}}_M^T\tilde{\mathbf{C}}_M^{*}\right)^{-1} \quad \text{and}$$

Algorithm 1 (cont'd)

- Step 5: Using $\hat{\mathbf{G}}$, we solve the following optimisation problem:

$$\hat{\mathbf{A}}_{\mathsf{N}} = \arg\min_{\mathbf{A}_N} \|\hat{\mathbf{G}} - \mathbf{A}_N \boldsymbol{\Gamma}\|^2 = \sum_{i=1}^{q} \arg\min_{\mathbf{a}_N(\theta_i)} \|\hat{\mathbf{G}}^{\langle :,i \rangle} - \gamma_i \mathbf{a}_N(\theta_i)\|^2$$

which is estimated as $\hat{\mathbf{a}}_N(\theta_i) = \frac{\hat{\mathbf{G}}^{\langle :,i \rangle}}{\|\hat{\mathbf{G}}^{\langle :,i \rangle}\|}$

- Step 6: Estimate the AoAs using LS fit

$$\hat{\mathbf{e}}_i = \arg\min_{\mathbf{e}_i} \|\mathsf{ang}\{\hat{\mathbf{a}}_N(\theta_i)\} - \mathbf{T}\mathbf{e}_i\|^2, \quad i = 1 \dots q$$

$$\mathbf{T} = \begin{pmatrix} 0 & 1 \\ 1 & 1 \\ \vdots & \vdots \\ N-1 & 1 \end{pmatrix} \quad \text{and} \quad \hat{\mathbf{e}}_i = [e_{i,1}, e_{i,2}]^T = \mathbf{T}^{\dagger} \mathsf{ang}\{\hat{\mathbf{a}}_N(\theta_i)\}$$

and the solution is $\hat{\theta}_i = -\sin^{-1}\left(\frac{e_{i,1}\lambda}{2\pi d}\right), \qquad i = 1 \dots q$

Conditions for Algorithm 1

Conditions

- **C.1**: $q \leq \min \{M_p N_p, K_N(K_M - 1), M\}$
- **C.2**: $P = 1$
- **C.3**: $Q \leq \min \{M_p, K_M - 1\}$

where

- P is the maximum number of paths arriving at the same time but with different angles of arrival.
- Q is the maximum number of paths arriving at the same angle but with different times of arrival.

Outline

Algorithm 2

- Step 1: Do *Step 1* till *Step 3* of Algorithm 1.
- Step 4: Form $\mathbf{Y} = \mathbf{X}\mathbf{P}$ where $\mathbf{P}$ is a shuffling matrix.
- Step 5: Obtain $\{\hat{z}_{\theta_i}^{\text{MP}}\}_{i=1}^{q}$ using in a similar manner as ToAs.
- Step 6: Estimate the AoAs of the q paths by the following relation:

$$\hat{\theta}_i^{\text{MP}} = -\sin^{-1}\left(\frac{\text{ang}\{\hat{z}_{\theta_i}^{\text{MP}}\}\lambda}{2\pi d}\right)$$

- Step 7: We have $\{\hat{z}_{\theta_i}^{\text{MP}}\}_{i=1}^{q}$ and $\{\hat{z}_{\tau_i}^{\text{MP}}\}_{i=1}^{q}$, seperately. Our task now "boils down" to pairing.

Algorithm 2 (cont'd)

SubAlgorithm 1: Step 7 of Algorithm 2

INITIALISE Step 7:

$\mathbf{C}_{\mathrm{M}} = [\mathbf{c}_{\mathrm{M}}(\hat{\tau}_1^{\mathrm{MP}}) \ldots \mathbf{c}_{\mathrm{M}}(\hat{\tau}_q^{\mathrm{MP}})]$

$\mathbf{A}_{\mathrm{N}} = [\mathbf{a}_{\mathrm{N}}(\hat{\theta}_1^{\mathrm{MP}}) \ldots \mathbf{a}_{\mathrm{N}}(\hat{\theta}_q^{\mathrm{MP}})]$

$\boldsymbol{\Upsilon}_1 = \mathbf{I}_{\mathrm{q}}$

MAIN LOOP of Step 7:

for $l = 1$ *to* $q!$ **do**

- *Step 7.1:* $\mathbf{H} = \mathbf{C}_{\mathrm{M}} \boxtimes (\mathbf{A}_{\mathrm{N}} \boldsymbol{\Upsilon}_l)$
- *Step 7.2:* $\mathscr{P}_{\mathbf{H}} = \mathbf{I}_{\mathrm{MN}} - \mathbf{H}(\mathbf{H}^{\mathrm{H}}\mathbf{H})^{-1}\mathbf{H}^{\mathrm{H}}$
- *Step 7.3:* $\mathbf{b}(l) = \left\|\mathscr{P}_{\mathbf{H}}\mathbf{x}\right\|^2$
- *Step 7.4:* Choose another permutation matrix $\boldsymbol{\Upsilon}_{l+1}$

FIND BEST MATCH:

Step 7.5: Find $\hat{k} = \arg\max_k \mathbf{b}(k)$. This means that all columns of $\mathbf{C}_{\mathrm{M}}$ are matched to columns of $\mathbf{A}_{\mathrm{N}}\boldsymbol{\Upsilon}_{\hat{k}}$ according to the ML criterion in *Step 7.3*.

Conditions for Algorithm 2

Conditions

- **D.1**: $q \leq \min \{M_p N_p, K_N(K_M - 1), K_M(K_N - 1)\}$
- **D.2**: $P \leq \min \{N_p, K_N - 1\}$
- **D.3**: $Q \leq \min \{M_p, K_M - 1\}$

where

- P is the maximum number of paths arriving at the same time but with different angles of arrival.
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Simulation Results

Simulation Parameters

- $N = 3$ antennas.
- $B = 40$ MHz, $M = 64$ subcarriers, $\triangle_f = 0.625$ MHz (subcarrier spacing).
- $M_p = 30$ and $N_p = 2$ (2D-Matrix Pencil parameters).
- $q = 17$ paths, where
  - The first 11 paths arrive with delays $\{\tau_k = 30(k-1)\text{ nsec}\}_{k=1}^{11}$ with corresponding AoAs as $\{\theta_k = -60^\circ\}_{k=1}^{2}$, $\theta_3 = -45^\circ$, $\theta_4 = -20^\circ$, $\{\theta_k = 0^\circ\}_{k=5}^{8}$, $\{\theta_k = 10^\circ\}_{k=9}^{10}$, and $\theta_{11} = 35^\circ$.
  - The 6 other paths arrive with delays $\{\tau_k = 500 + 50(k-12)\text{ nsec}\}_{k=12}^{17}$ with corresponding AoAs as $\{\theta_k = 35^\circ\}_{k=12}^{14}$ and $\{\theta_k = 60^\circ\}_{k=15}^{17}$.
  - The multipath coefficients γ are randomly chosen.
- Vary SNR.

Some Simulations at SNR = 30dB

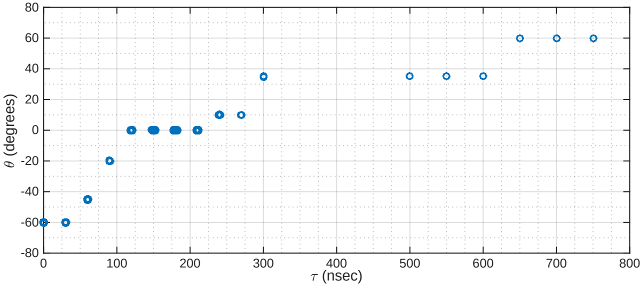

Figure: A scatter plot showing a realisation of Algorithm 1 (with auto pairing) at SNR = 30dB with $q = 17$ multipath components using $N = 3$ antennas and $M = 64$ subcarriers.

Some Simulations at SNR = 20dB

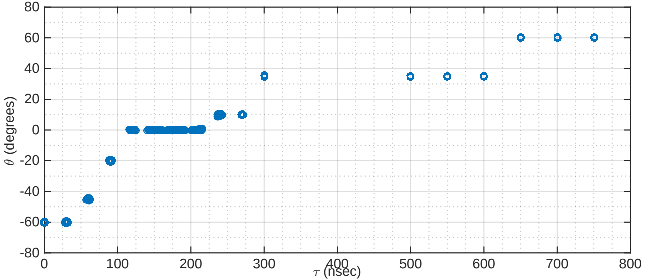

Figure: A scatter plot showing a realisation of Algorithm 1 (with auto pairing) at SNR = 20dB with $q = 17$ multipath components using $N = 3$ antennas and $M = 64$ subcarriers.

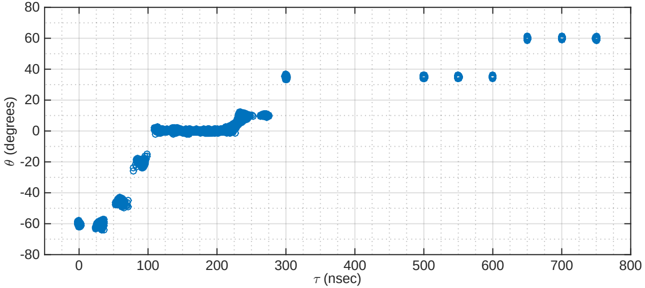

Figure: A scatter plot showing a realisation of Algorithm 1 (with auto pairing) at SNR $= 10$dB with $q = 17$ multipath components using $N = 3$ antennas and $M = 64$ subcarriers.

Some Simulations at SNR = 0dB

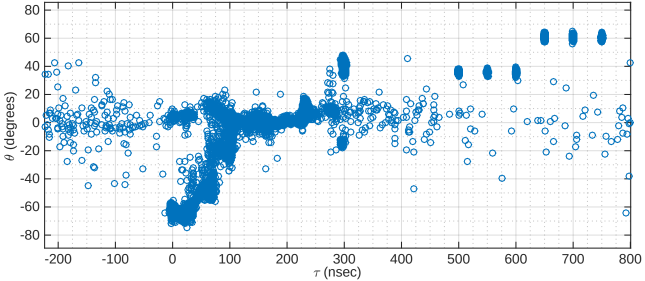

Figure: A scatter plot showing a realisation of Algorithm 1 (with auto pairing) at SNR = 0dB with $q = 17$ multipath components using $N = 3$ antennas and $M = 64$ subcarriers.

MSE Performance

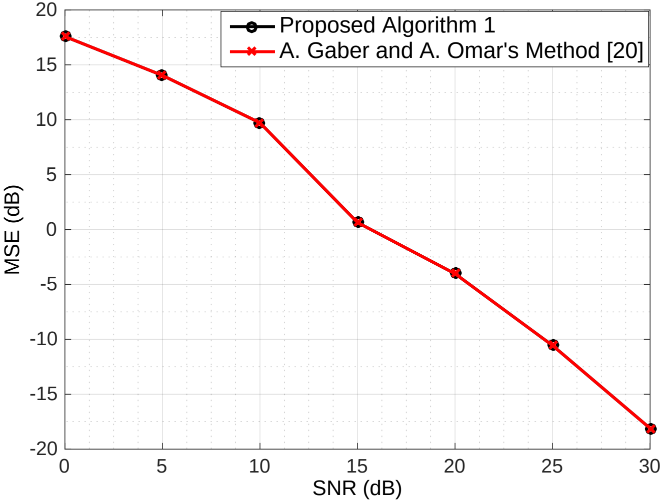

Figure: MSE (dB) of ToAs vs. SNR (dB) with $q = 3$.

MSE Performance (cont'd)

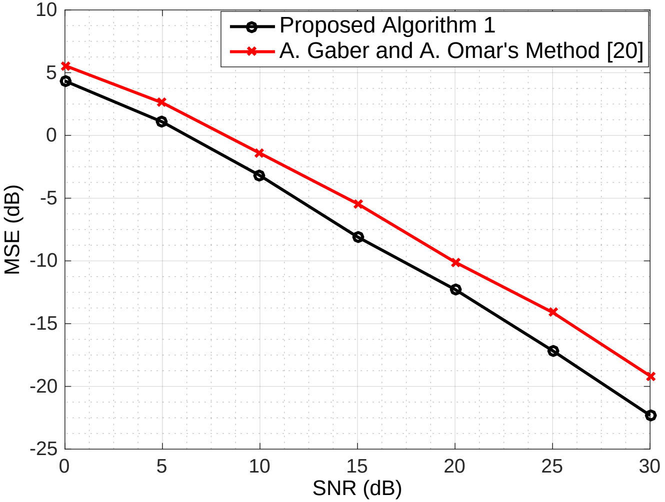

Figure: MSE (dB) of AoAs vs. SNR (dB) with $q = 3$.

QUESTIONS ?