Probability

$$P(E) = \frac{\#\text{ of times } E \text{ occurs}}{\text{Total } \#\text{ of times all events occur}}$$

$$P(E) = \lim_{n\to\infty} \frac{\text{frequency}_E}{n}$$

Coin (Un-biased)

$\{H, T\}$

$P(C=H) = \frac{1}{2}$

$P(C=T) = \frac{1}{2}$

Experiment

Trial$_1$	Trail$_2$	Trail$_3$	----
H	H	T	---- ---- ..

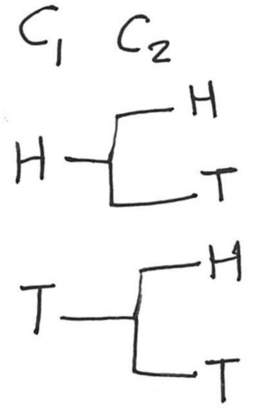
$P(C=H) = \frac{2}{3}$ | $P(C=T) = \frac{1}{3}$

$P(E) \geq 0$

$P(E) \leq 1$

$P(C=H) \approx \frac{1}{2}$ $P(C=T) \approx \frac{1}{2}$

Joint Probability



$$E = \{HH, TT, HT, TH\}$$

$$P(HH) = \frac{1}{4} = P(TT) = P(HT) = P(TH)$$

Conditional Probability

$$P(A|B) = \frac{P(A,B)}{P(B)} \qquad P(A,B) = P(B,A)$$

$$P(A \cup B) = P(A) + P(B) - P(A \cap B)$$

$C_1 \quad C_2$

$$P(H,H) = P(TT) = P(HT) = P(TH) = \frac{1}{4}\}$$

$$P(C_1 = H | C_2 = H) = \frac{P(C_1 = H, C_2 = H)}{P(C_2 = H)}$$

$C_1 \perp C_2$

(Independent)

$$= \frac{\frac{1}{4}}{\frac{1}{2}} = \frac{1}{2}$$

$$P(C_1 = H | C_2 = H) = \frac{1}{2} = P(C_1 = H)$$

C_1 C_2

Coin (mirror reflection of C_1)

$E = \{HH, TT\}$

$P(HH) = \frac{1}{2} = P(TT)$

H H

T T

$$P(C_1 = H \mid C_2 = H) = \frac{P(C_1 = H, C_2 = H)}{P(C_2 = H)}$$

$$= \frac{1}{2} \Big/ \frac{1}{2} = 1$$

$P(C_1 = H \mid C_2 = H) = 1$ $P(C_1 = H) = \frac{1}{2}$

C_2 is dependent on C_1 (fully)

BP = High
Normal

$P(BP = High \mid Age \geq 70) = \uparrow$

$$P(BP = High \mid \underline{\underline{\text{Sky Color} = \text{Light blue}}}) = ?$$

$P(BP = High)$

Bayes Theorem

1 — $$P(A|B) = \frac{P(A,B)}{P(B)}$$

2 — $$P(B|A) = \frac{P(B,A)}{P(A)}$$

$$\underline{P(A,B) = P(B,A)}$$

$$P(C_1=H, C_2=T)$$
$$= P(C_2=T, C_1=H)$$

$$P(A,B) = P(A|B) \cdot P(B) \text{ — } 1$$

$$P(B,A) = P(B|A) \cdot P(A) \text{ — } 2$$

$$P(A|B) \cdot P(B) = P(B|A) \cdot P(A)$$

$$\boxed{P(A|B) = \frac{P(B|A) \cdot P(A)}{P(B)}}$$

Bayes Theorem

$C_1 \quad C_2 \qquad E = \{HH, HT, TH, TT\}$

$$P(C_1 = H) = P(C_1 = H, C_2 = H) + P(C_1 = H, C_2 = T)$$

marginal prob.

joint prob.

$$= \frac{1}{4} + \frac{1}{4}$$

$$= \frac{1}{2}$$

Marginalization