ACTL3141 Assignment: Mortality of Chilean Pensioners

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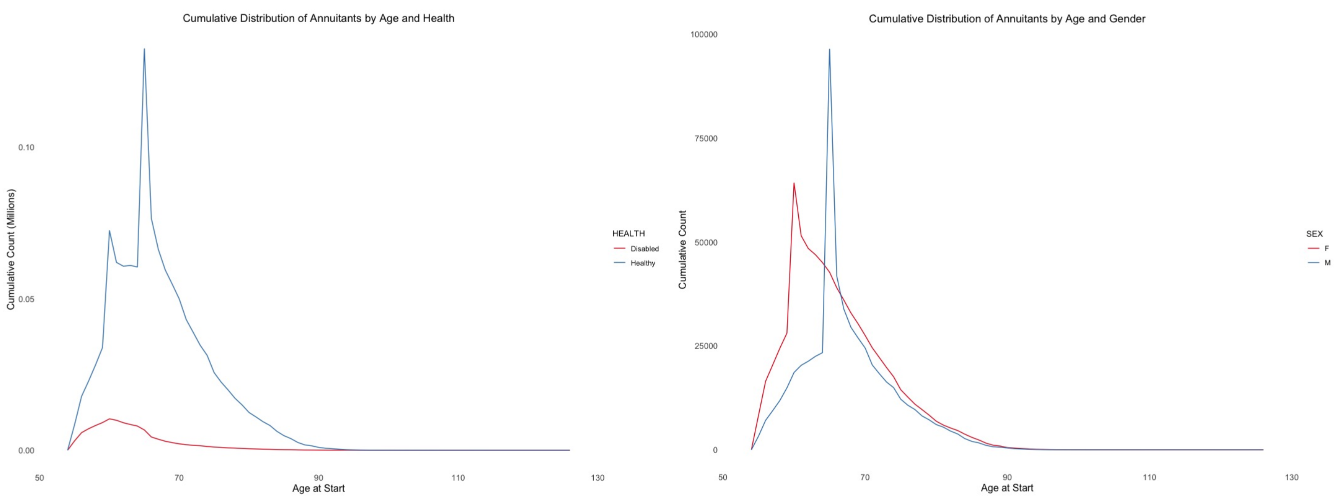
1 Executive Summary

This report aims to examine a set of Chilean mortality data by providing descriptive analysis of the mortality dataset of Chilean annuitants. Furthermore, the report aims to utilise methods such as Kaplan Meier estimation, Cox Regression and the Log-Rank test to study differences in mortality by sex, disability status and type of annuitant. This report uses both univariate and multivariate analysis to provide insights into which factors are significant when considered alone and how the effects of variables may be confounded by the effects of another. The report graduates a unisex period life table for the years 2014-2018 for healthy annuitants between the age range of 60-100 using graduation approaches such as the Gompertz model, the Makeham model and splines. This report also discusses from an ethical perspective whether insurers should use gender as a rating factor for setting their annuity prices.

2 Descriptive analysis of the annuitants



(a) Cumulative Frequency of Annuitants by Health (b) Cumulative Frequency of Annuitants by Gender

Figure 1: Distribution of Annuitants by Gender

The cumulative distribution of annuitants, as depicted in Figure 1, unveils critical insights into the demographic and health composition of Chilean pensioners. The cumulative frequency distribution of annuitants by health in Figure 1a reveals a significant discrepancy between healthy and disabled annuitants at retirement ages with a clear majority being healthy. This dominance of healthy annuitants underscores the critical role that health status plays in survival analysis and as shown in Appendix A, the relatively lower proportion of disabled annuitants, especially amongst females, suggests areas for investigation. The cumulative distribution of annuitants by gender reveals a skew towards younger ages with a high concentration of annuitants initiating their annuities at earlier retirement ages. This skewness is particularly notable among female annuitants with the cumulative distribution frequency for females peaking earlier than the frequency for males which could be reflective of broader socio-economic dynamics that influence retirement planning among genders in Chile.

3 Survival Analysis

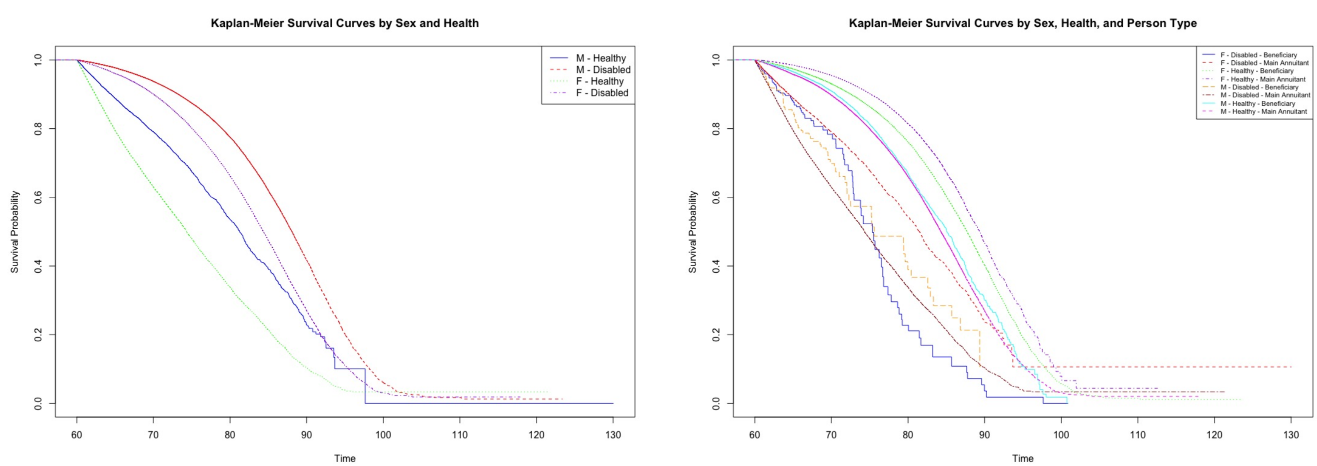
3.1 Kaplan-Meier Survival Curves

Kaplan-Meier estimation offers an intuitive visualisation of survival probabilities over time, allowing for an initial, straightforward comparison across different annuitant subgroups based on single variables such as gender, health status, and annuitant type.

Single Variable Analysis

The Kaplan-Meier survival curves for single variables (refer to Appendix B Figure 6) - gender, health status, and type of annuitant - reveals key findings. As shown in Figure 6a, gender significantly impacts survival probabilities with females consistently exhibiting higher survival probabilities than males, a trend consistent with global mortality observations. The survival curves in Figure 6b reveals healthy annuitants exhibiting markedly better survival probabilities than their disabled counterparts, the distinction underscores the need to incorporate health status into life tables. The survival probabilities between Main Annuitants and Beneficiaries are compared in Figure 6c, Main Annuitants show a lesser survival probability, potentially reflecting age or health status at policy initiation, considering beneficiaries are typically younger or in better health.

Multiple Variable Analysis



(a) KM Survival Curves by Sex and Health

(b) KM Survival Curves by Sex, Health and Type

Figure 2: Multiple Variable Kaplan-Meier Survival Curves

The multiple variable Kaplan-Meier curves demonstrate a clear differentiation in survival probabilities based on health status within each sex category. The different survival rates among healthy and disabled beneficiaries, across both males and females in Figure 2a, underscores the necessity for distinct life tables. This differentiation is crucial as it reflects not just biological or demographic disparities but also the socio- economic dimensions of mortality risk. The survival probabilities for disabled beneficiaries being markedly lower than those for other groups illuminate the substantial impact of health status on mortality, which varies significantly with gender and annuitant type as shown in Figure 2b. The differences between Main Annuitants and Beneficiaries, adds another layer of complexity. These findings argue for a nuanced approach to life table, supporting the use of five distinct life tables for the Chilean pension system as currently prescribed.

3.2 Log-Rank and Peto-Peto Tests

The choice of Log-Rank and Peto-Peto tests in the analysis stems from their statistical capabilities in comparing survival distributions. The Log-Rank test accounts for differences in survival probabilities over the entire study period and particularly suited for the Chilean mortality dataset where mortality risks evolve with age. Conversely, the Peto-Peto test offers robustness against varying hazard ratios, making it invaluable for assessing the consistency of mortality differences across subgroups. The utilisation of these tests revealed statistically significant differences in survival probabilities among the annuitants, reaffirming the observations from the Kaplan-Meier survival analysis. The variation in mortality risk by the co-variates highlights the limitations of a unisex appraoch to life table construction in Chile. The Log-Rank test yielded a χ^2 statistic of 11,142.93 for gender ($p < 0.001$), underscoring significant survival differences between male and female annuitants. Similarly, health status revealed a chi-squared value of 7,127.93 ($p < 0.001$), emphasising the profound impact of health on mortality risk. These significant p-values indicate that the observed differences in survival rates are highly unlikely to be due to chance (Appendix C). Parallel findings from the Peto-Peto test, such as a χ^2 statistic of 11,162.40 for gender ($p < 0.001$), corroborate the Log-Rank test results, reinforcing the validity of the survival disparities identified (Appendix C). Multiple variable tests (Appendix D) reveal compounded disparities in survival probabilities that single variable analyses alone could not fully uncover. For instance, when analysing the combined effects of gender and health status, the Log-Rank test showed a significantly increased χ^2 statistic compared to the analysis of gender alone. This indicates not only a difference in survival rates between genders but also how health status exacerbates these differences. Similar trends were observed with the addition of annuitant type. These results provide a compelling justification for the use of 5 distinct life tables in Chile and suggests a unisex approach would be inadequate in capturing intricate mortality patterns.

3.3 Cox Regression Model

The Cox regression model was chosen due to its capability to estimate the hazard ratios associated with multiple covariates. The use of multiple Cox models enables a layered exploration into how various factors interact to influence mortality. As evidenced in Appendix E, each model incrementally introduces additional co-variates allowing the discernment of isolated effects of individual co-variates and how these effects adjust in the presence of other factors. The single-variable Cox models showed that being male (indicated by 'SEXM') is associated with an increased hazard ratio of 1.788, suggesting higher mortality risk compared to females. Similarly, the 'Health' variable indicated that healthy individuals have a lower mortality risk with a hazard ratio of 0.302 compared to their disabled counterparts. Main annuitants had an increased risk of mortality. In the multivariate models, each predictor remained statistically significant however the magnitude of their coefficiencies changed highlighting interdepencies amongst the variables. The interaction of sex with health status indicates a differentiation in mortality risks that validates the separation of life tables for male and female annuitants, both healthy and disabled. This distinction aligns with the nuanced approach currently embodied by the "CB-H-2020" and "MI-H-2020" tables for males and the "RV-M-2020", "B-M-2020", and "MI-M-2020" tables for females. Furthermore, the significant hazard ratios for main annuitants and beneficiaries in the Cox models indicate the need for separate life tables based on the type of annuitant. This insight affirms the use of the "RV-M-2020" table specifically for main female annuitants and the "B-M-2020" for female beneficiaries. The baseline hazard function depicted in Figure 7 (Appendix E) illustrates the non-constant risk of death over time, with the hazard rate increasing noticeably at higher ages indicating the mortality risk accelerates when individuals age, in line with mortality trends. The results from the Cox Regression models supports the utilisation of 5 different life tables in Chile, reflecting the diverse risk profiles based on sex, health status and annuitant type.

4 Graduation of unisex life table for healthy annuitants

4.1 Graduation Methods

The graduation of mortality rates for healthy Chilean annuitants aged 60 to 100 was accomplished through various established actuarial models. This summary discusses the key findings from the application of the Gompertz model, the Makeham model, and spline methods, highlighting their capabilities in capturing the mortality trends in the dataset. Before graduating the data the central exposed to risk (E_x^c) and deaths (D_x) were calculated (Refer to Appendix F for calculations) to obtain the mortality rates ($\mu_x = \frac{D_x}{E_x^c}$) under a uniform distribution of deaths assumption.

The Gompertz model, with its assumption of exponentially increasing mortality, yielded a consistent fit across the age spectrum, affirming its long-standing application in demographic studies. The Makeham model, which supplements the Gompertz law with a constant age - independent mortality component, provided an enhanced fit, particularly in the lower and upper tails of the age range. Meanwhile, spline methods — both cubic and smoothing — offered a nuanced and flexible approach, allowing for localised adjustments to the mortality curve.

4.2 Results and Choice of Graduation

Analytical Summary (Refer to Appendices G and H for Results):

Upon evaluating the performance of various mortality models, we observe distinct variations in fit quality as evidenced by statistical testing. The Gompertz model diverges notably from the observed data, registering a χ^2 statistic of 195.5704 against a critical value of 54.57 and yielding a p-value of < 0.001. These figures indicate significant under performance in capturing the dataset's complexity. The Makeham model, which adjusts for age-independent factors, provides an enhanced fit with a χ^2 statistic of 121.2184. Although this value still considerably exceeds the critical threshold of 53.38, the associated p-value of 1.27e-10 denotes a better alignment with the data compared to the Gompertz model.

The cubic spline model had quantile-based knots to capture underlying mortality trends across the entire age spectrum (detailed in Appendix F2) achieved a χ^2 statistic of 75.82554 and a p-value of 5.01e-05. This outcome suggests an improved fit over the parametric models, but it does not conclusively indicate an optimal representation of mortality trends due to the arbitrary nature of knot placement, which can potentially introduce bias. The smoothing spline model exhibits a balanced performance. It records a χ^2 statistic of 141.9967, aligning closely with its critical value of 49.80185, and presents a statistically significant p-value of 7.77e-15. Furthermore, its Generalised Cross-Validation (GCV) score of 0.001537138 validates the model's efficacy in smoothing, demonstrating an equilibrium between data fitting and generalisation capabilities without overfitting.

Recommendation: Given these considerations, the smoothing spline model is strongly suggested

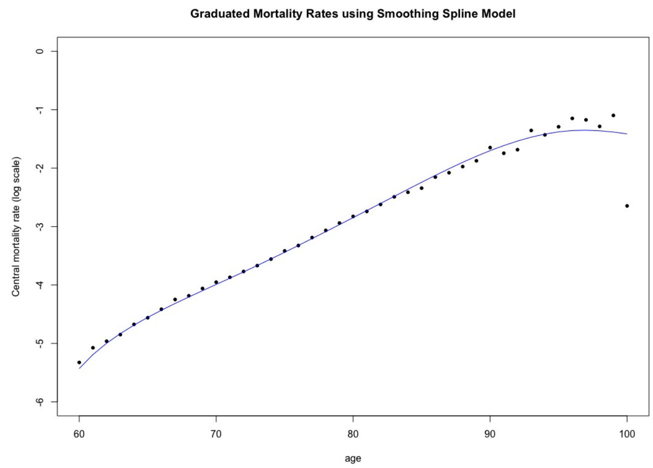
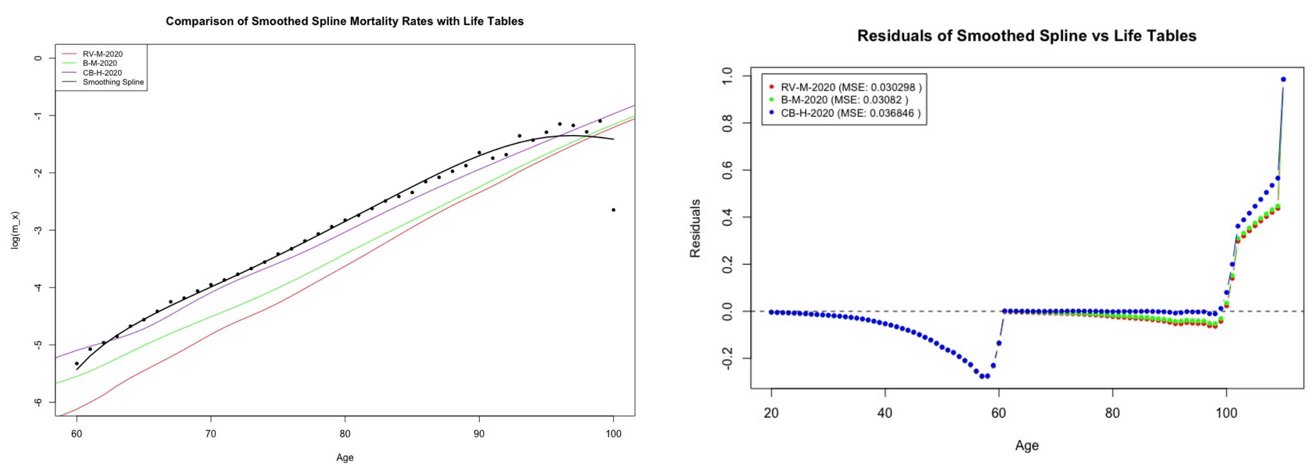


Figure 3: Graduated Mortality Rates using Smoothing Spline with spar = 0.7

for graduating the mortality data. Its statistically robust performance, coupled with the optimal GCV score, implies that it provides a reliable representation of the mortality trend without being susceptible to overfitting.

4.3 Comparison with Current Life Tables



(a) Side-by-Side Mortality Rates

(b) Residuals

Figure 4: Comparison of Graduated Mortality Rates with Life Tables

Figure 4a exhibits the smoothed spline mortality rates alongside the graduated rates from the life tables: "CB-H-2020" for healthy male annuitants, "RV-M-2020" for healthy female main annuitants, and "B-M-2020" for healthy female beneficiary annuitants. The plotted $\log(\mu_x)$ indicates that the spline model aligns closely with the life tables across most ages. This suggests that the model is capturing the general mortality trend accurately for healthy annuitants across both genders and annuitant types. The convergence of the spline with "CB-H-2020" suggests that the healthy male mortality rates are best represented by the smoothing spline. There is a noticeable divergence observed between the graduated rates and the mortality rates of female annuitants, particularly in "RV-M-2020". The residual plot in Figure 4b provides insight into the variance of the spline model from the actual life tables' rates. The residuals for "CB-H-2020" exhibit a slight increase with age which indicates the model may underestimate mortality rates for older male annuitants. The residuals for "RV-M-2020" and "B-M-2020" appear to be contained within a narrow range, suggesting a consistent model performance across the ages 60 to 100. The density plot (Appendix I Figure 11) assesses the deviation of the smoothing spline model from the data. The distributions are somewhat normally distributed but exhibit slight skews which implies that the spline model provides a reasonable estimation of the mortality rates. The regression summaries (Appendix I Table 15, Table 16 and Table 17) provide statistical measures of the fit between the smoothed spline rates and the life tables. The spline model has a statistically significant fit to the life table rates with the coefficient for the spline being noticeably higher for "CB-H-2020" (0.9252), suggesting the strongest relationship between the smoothing spline model and the mortality rates for the healthy male annuitants.

5 Ethical implications of unisex annuity pricing

Drawing from Dobrin's ethical reasoning framework, in the context of unisex pricing in Chile, the ethical dilemma of unisex annuity pricing can be approached systematically and the implications can be studied under the lens of both deontology and utilitarianism. The key stakeholders include the individual policyholders, potential policyholders, and the insurance company. The relevant core values at play are fairness and equality where unisex pricing embodies equal treatment irrespective of gender. Conversely, gender-based pricing aligns with actuarial fairness where individuals pay premiums and receive annuities proportional to their risk. Instituting unisex pricing may homogenise premiums and lead to cross-subsidisation between genders, simplifying insurance structure; it could potentially dilute the accuracy of risk-based pricing and affect the insurer's solvency. Through a deontological lens unisex pricing can be perceived as an obligation to uphold equality, whilst utilitarian ethics justifies gender-based pricing to guarantee statistically sound pricing. To explain unisex pricing, insurers must articulate ethical and actuarial rationale to ensure stakeholders understand the balance between risk assessment, an insurer's solvency and social equity. The pros of using gender as a rating criterion from a utilitarian perspective include: aligning with actuarial fairness because premiums and annuitiues reflect true mortality risks (1) and benefitting groups with lower mortality risks financially with gender-specific tables (2). From a deontological perspective, using gender-specific tables promotes actuarial responsibility (3). Conversely, the cons of using gender as a rating criterion under a utilitarian perspective includes disproportionately affecting one gender, leading to financial inequality and potential hardship (1) and perpetuating social stereotypes and biases (2). Under a deontological lends using gender could be considered discriminatory violating principles of treating individuals equally (3) and disregards the imperative to ensure non-discrimination in business practices (4).

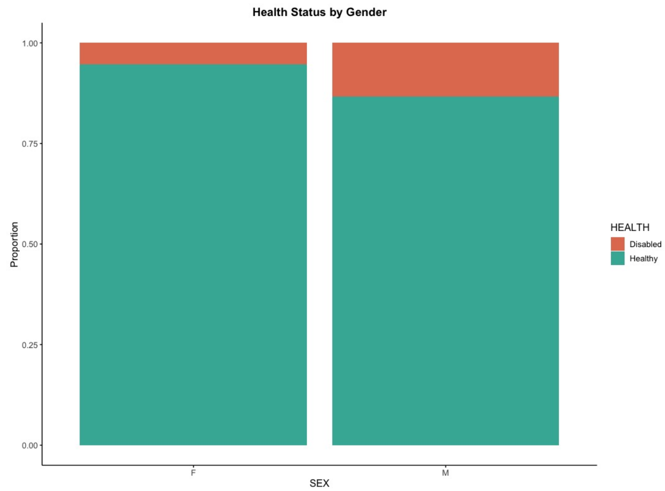
Recommendation: Given the findings of this report, a balanced approach is recommended, where a unisex pricing model is adopted while also considering individual risk factors such as health status and annuitant type. This hybrid approach could satisfy the utilitarian aim of societal welfare and align with the deotological aim of avoiding gender discrimination and treating all policyholders equally.

6 Appendix

6.1 Appendix A. Distribution of annuitants' health profiles by gender

Gender	Disabled (%)	Healthy (%)
F	5.37	94.63
M	13.38	86.62

(a) Proportion of Gender Healthy or Disabled

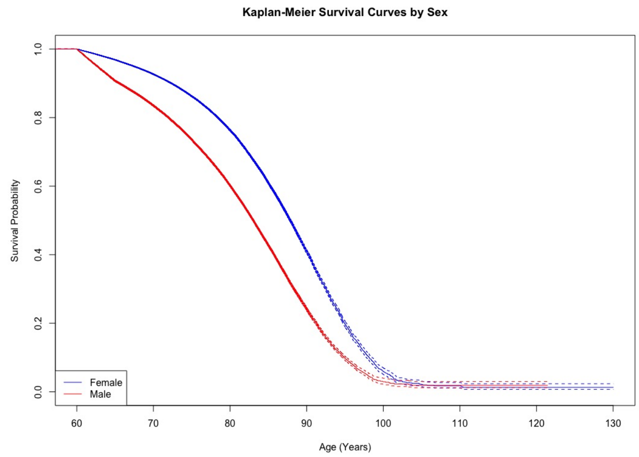


(b) Health Status by Gender

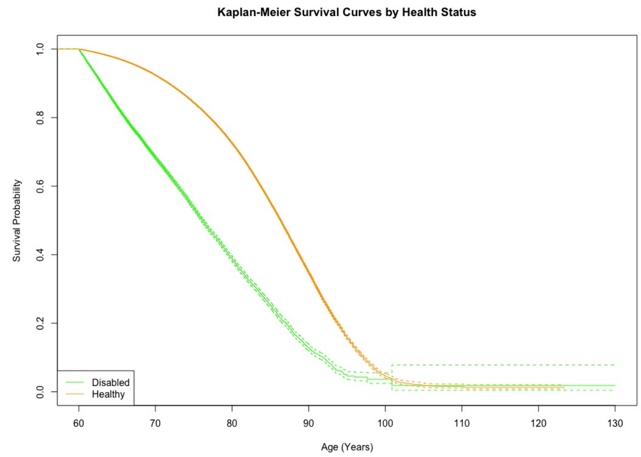
Figure 5: Analysis of Annuitants' Health Profiles by Gender

Figure 3 displays the health status of annuitants by gender, with the bars representing the proportion of individuals who are classified as 'Healthy' and 'Disabled'. For both females (F) and males (M), the plot shows that a majority of the annuitants are in the 'Healthy' category, as indicated by the larger teal section. The 'Disabled' category is represented by the smaller orange-coloured section. The proportion of males classified as 'Disabled' is more than twice the proportion of females classified as 'Disabled'.

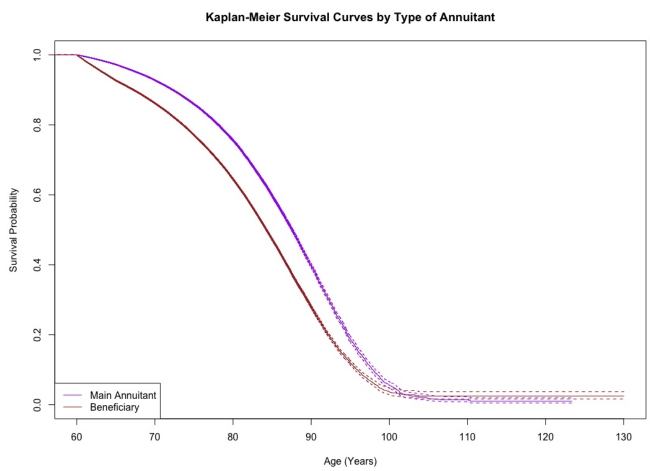
6.2 Appendix B. Single Variable Kaplan-Meier Survival Curves



(a) Kaplan Meier Survival Curves by Sex



(b) Kaplan Meier Survival Curves by Health Status



(c) Kaplan Meier Survival Curves by Type of Annuitant

Figure 6: Single Variable Kaplan Meier Survival Curves

6.3 Appendix C. Single Variable Log-Rank and Peto-Peto Tests

Group Comparison	Log-Rank χ^2 Statistic	Log-Rank p-Value
Sex	11 142.93	0
Health	7127.93	0
Annuitant Type	5236.54	0

Table 1: Summary of statistical tests results

Group Comparison	Peto-Peto χ^2 Statistic	Peto-Peto p-Value
Sex	11 162.40	0
Health	7201.20	0
Annuitant Type	5243.03	0

Table 2: Summary of statistical tests results

6.4 Appendix D. Multiple Variable Log-Rank and Peto-Peto Tests

Group Comparison	Log-Rank χ^2 Statistic	Log-Rank p-Value
Sex	11 142.93	0
Sex and Health	17 057.36	0
Sex, Health and Annuitant Type	19 668.65	0

Table 3: Summary of statistical tests results

Group Comparison	Peto-Peto χ^2 Statistic	Peto-Peto p-Value
Sex	11 162.40	0
Sex and Health	17 149.63	0
Sex, Health and Annuitant Type	19 764.47	0

Table 4: Summary of statistical tests results

6.5 Appendix E. Results from Cox Regression

Model	Term	Estimate	Exp(Coef)	Std. Error	Statistic	p-value
cox_model.1	SEXM	0.581	1.79	0.00632	92.0	$< 2 \times 10^{-16}$
cox_model.2	Healthy	-1.20	0.302	0.00868	-138.0	$< 2 \times 10^{-16}$
cox_model.3	Main Annuitant	0.417	1.52	0.00658	63.3	$< 2 \times 10^{-16}$
cox_model.4	SEXM	0.467	1.60	0.00644	72.6	$< 2 \times 10^{-16}$
cox_model.4	Healthy	-1.06	0.348	0.00887	-119.0	$< 2 \times 10^{-16}$
cox_model.5	SEXM	0.561	1.75	0.00867	64.7	$< 2 \times 10^{-16}$
cox_model.5	Healthy	-1.09	0.336	0.00915	-119.0	$< 2 \times 10^{-16}$
cox_model.5	Main Annuitant	-0.153	0.858	0.00932	-16.4	4.95×10^{-61}

Table 5: Cox model summary

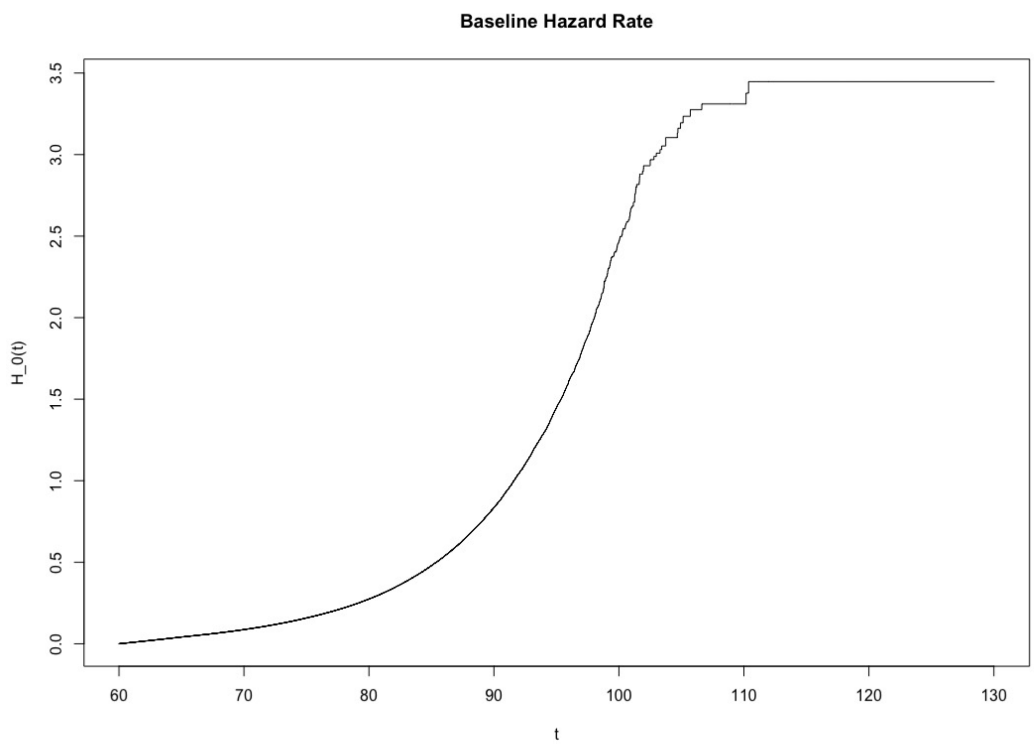


Figure 7: Baseline Hazard Rate

6.6 Appendix F. Mortality Rate Calculations and Model Fitting Process

6.6.1 Appendix F1. Mortality Rate Calculations

```
# Select healthy annuitants within the age range 60 to 100.
healthy_data <- subset(mortality_data, HEALTH == "Healthy" &
                        age_at_start >= 60 & age_at_start <= 100)

# Calculate the days each annuitant was in the study.
healthy_data$days_in_study <- pmax(as.numeric(as.Date(healthy_data$
   DATE_END) -
                                   as.Date(healthy_data$DATE_START)),
                                       0)

# Adjust the days in study for those who died, assuming deaths are
   uniformly distributed.
healthy_data$adjusted_days_in_study <- ifelse(healthy_data$DEATH,
                                              healthy_data$days_in_
                                                 study - 0.5 *
                                                 365.25,
                                              healthy_data$days_in_
                                                 study)

# Aggregate deaths by age at entry.
Dx <- tapply(healthy_data$delta, healthy_data$AGE_AT_ENTRY_floor, sum
   , na.rm = TRUE)

# Calculate the total years lived by age at entry.
Ex <- tapply(healthy_data$adjusted_days_in_study / 365.25,
             healthy_data$AGE_AT_ENTRY_floor, sum, na.rm = TRUE)

# Calculate the central mortality rate for each age.
mx <- Dx / Ex

# Define the age range of interest.
x <- 60:100

# Ensure that the Ex, Dx, and mx vectors are numeric and correspond
   to the defined age range.
Ex <- as.numeric(Ex[as.character(x)])
Dx <- as.numeric(Dx[as.character(x)])
mx <- as.numeric(mx[as.character(x)])
```

6.6.2 Appendix F2. Model Fitting Process for Splines

```
# Define quantiles to determine the position of knots for cubic
   spline fitting

knots <- quantile(x, probs = c(0.25, 0.5, 0.75))



# Fit the cubic spline model to mortality rates with defined knots

# Weights are used to account for the variance of rates

cubSpline <- lm(mx ~ ns(x, knots = knots), weights = Ex/mx)



# Sequence of ages for plotting smooth curve

x_seq <- seq(min(x), max(x), length.out = 200)

# Calculate fitted mortality rates from the cubic spline model

mx_cubSpline <- predict(cubSpline, newdata = list(x = x_seq))



# Plot the cubic spline fit against the log mortality rates

# 'pch = 20' specifies the type of point.

plot(x, log(mx), pch = 20, xlab = "Age", ylab = "Log Central
   mortality rate",

     main = "Chilean Mortality: Cubic Spline", ylim = range(log(mx),
        log(mx_cubSpline)))

lines(x_seq, log(mx_cubSpline), col = 'blue')
```

Smoothing Splines

```
# Plot raw mortality rates to help choose the smoothing parameter (
   spar)

plot(x, mx, main="Choosing Smoothing Parameter Value", xlab="Age",
```

```
   ylab="Mortality rate")

for(s in seq(0.2, 2, by=0.4)) {

  fit <- smooth.spline(x, mx, spar=s)

  lines(fit, col=rainbow(5)[which(s==seq(0.2, 2, by=0.4))], lwd=2)

}

legend("topleft", legend=seq(0.2, 2, by=0.4), col=rainbow(5), lwd=2,
   title="spar values")



# Fit the smoothing spline using cross-validation to find the optimal
    spar value

fit <- smooth.spline(x, mx, cv=TRUE)

print(fit$spar) # Output the optimal spar value



# Fit the final smoothing spline model using the selected spar value

smSpline <- smooth.spline(x, mx, spar = fit$spar)



# Sequence of ages for plotting smooth curve

x_seq <- seq(min(x), max(x), length.out = 200)

# Calculate fitted mortality rates from the smoothing spline model

mx_smSpline <- predict(smSpline, x = x_seq)



# Plot the smoothing spline fit against the log mortality rates

plot(x, log(mx), pch = 20, xlab = "age", ylab = "Central mortality
   rate (log scale)",

     main = "Graduated Mortality Rates using Smoothing Spline Model",
```

```
    ylim = range(-6, 0))

lines(x_seq, log(mx_smSpline), col = 'blue')
```

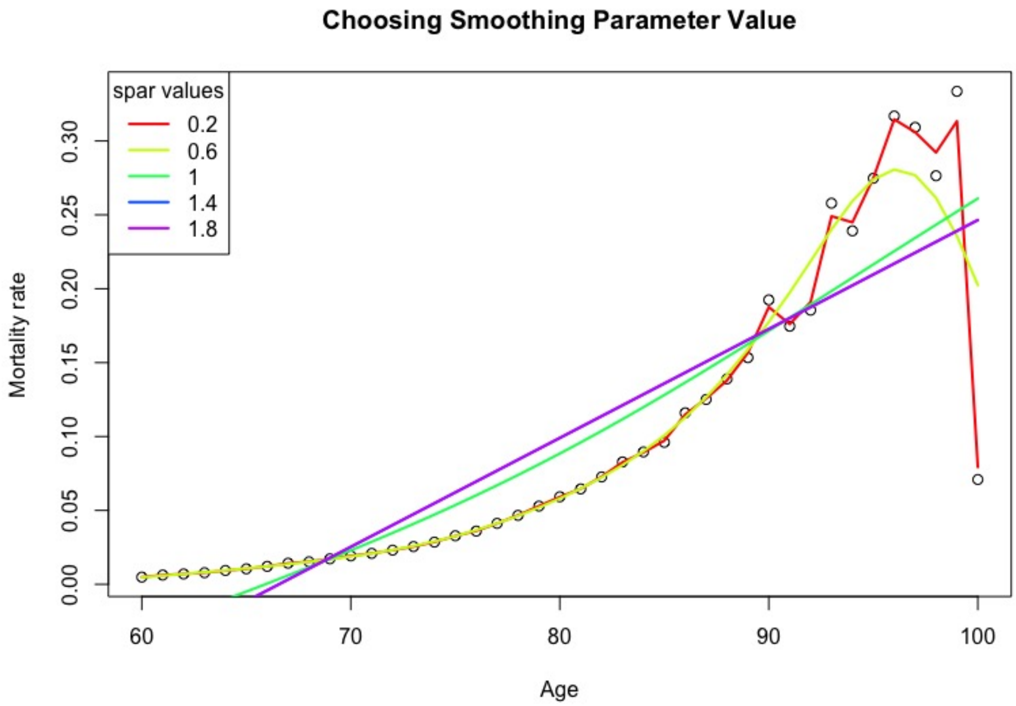


Figure 8: Smoothing Parameter Values Graph

6.7 Appendix G. Graduation Analysis and Results

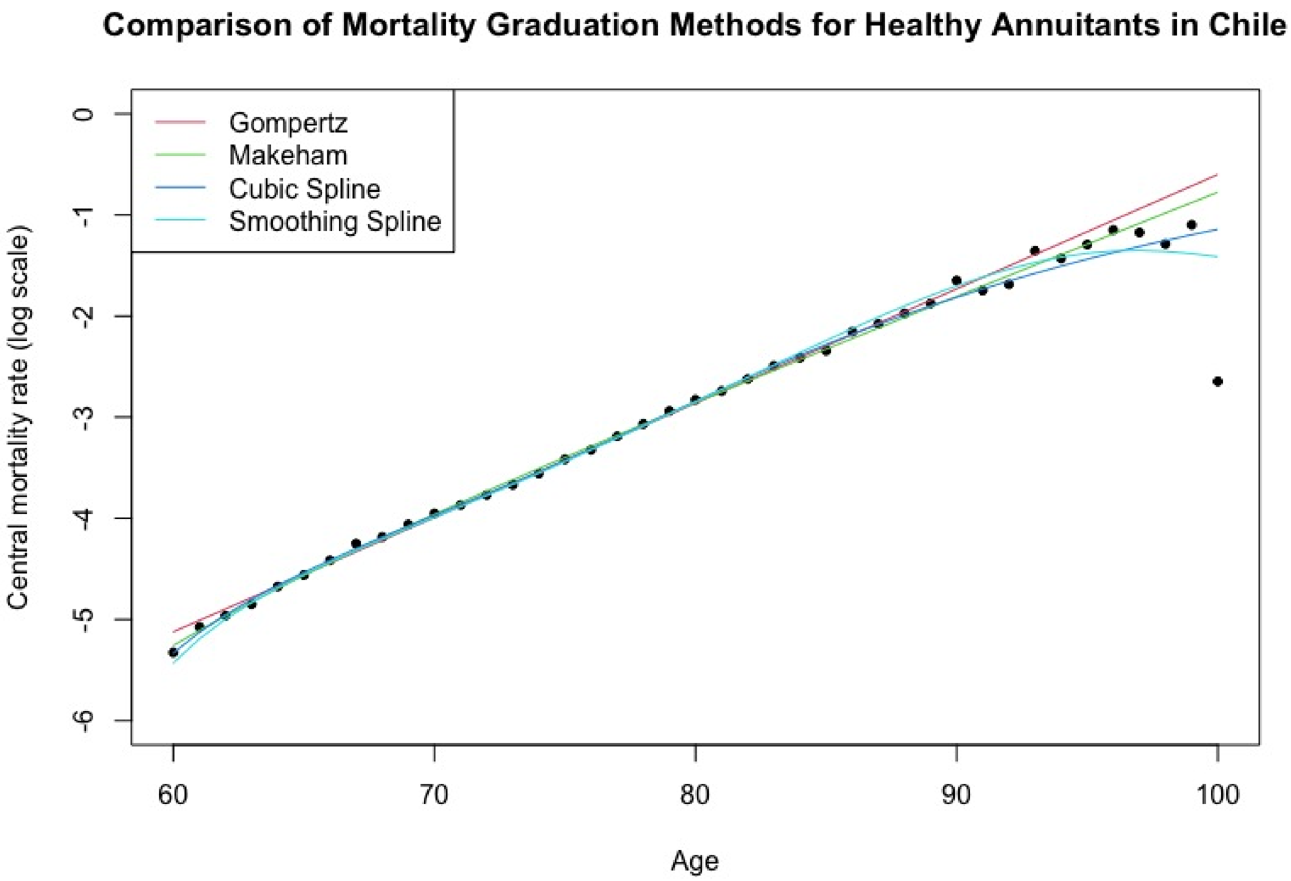


Figure 9: Comparison of Mortality Graduation Methods for Healthy Annuitants in Chile

Parameter	Estimate	Convergence
b0	-11.904	7.38e-07
b1	0.113	-

Table 6: Graduation Results for the Gompertz Model (Iterations to Convergence: 11)

Parameter	Estimate	Convergence
A	-0.002593	8.028e-06
b0	-10.979014	-
b1	0.102092	-

Table 7: Graduation Results for the Makeham Model (Iterations to Convergence: 3)

Parameter	Estimate
Intercept	0.004834
ns(x, knots = knots)1	0.040202
ns(x, knots = knots)2	0.145610
ns(x, knots = knots)3	0.268270
ns(x, knots = knots)4	0.307861

Table 8: Graduation Results for the Cubic Spline Model

Parameter	Value
Smoothing Parameter (spar)	0.7
Lambda	0.001276946
Equivalent Degrees of Freedom (Df)	5.811864
Penalized Criterion (RSS)	0.04642175
GCV	0.001537138

Table 9: Graduation Results for the Smoothing Spline Model

6.8 Appendix H. Statistical Test Analysis and Results of Graduations

Model	χ^2 Statistic	Critical Value	Degrees of Freedom	p-value
Gompertz	195.5704	54.57	39	< 0.001
Makeham	121.2184	53.38	38	1.27e-10
Cubic Spline	75.82554	48.60237	34	5.01e-05
Smoothing Spline	141.9967	49.80185	35	7.77e-15

Table 10: Summary of Chi-Square Test Results for Mortality Models

Model	χ^2 Statistic	Degrees of Freedom	p-value
Gompertz	14.8	3	0.001996
Makeham	14.865	3	0.001936
Cubic Spline	1.6358	3	0.6513
Smoothing Spline	24.241	3	2.225e-05

Table 11: Summary of Standard Deviation Test Results for Mortality Models

Model	Successes	Trials	p-value	Confidence Interval (95%)
Gompertz	17	41	0.3489	[0.2632, 0.5789]
Makeham	23	41	0.5327	[0.3975, 0.7153]
Cubic Spline	23	41	0.5327	[0.3975, 0.7153]
Smoothing Spline	23	41	0.5327	[0.3975, 0.7153]

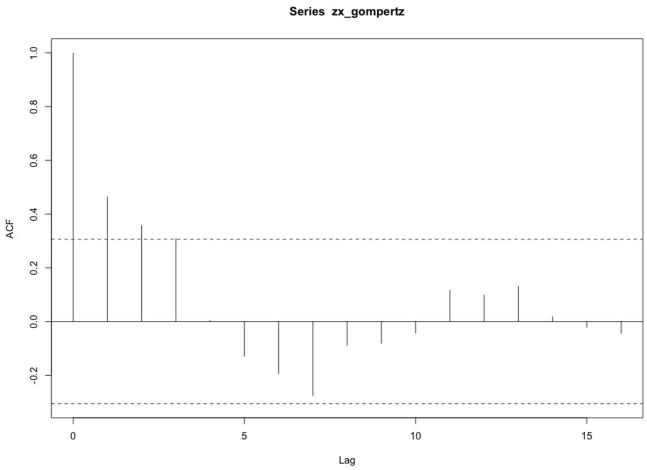
Table 12: Summary of Sign Test Results for Mortality Models

Model	Statistic	Critical Value	p-value
Gompertz	1.137669	1.959964	0.2553
Makeham	0.6730982	1.959964	0.5009
Cubic Spline	0.342862	1.959964	0.7317
Smoothing Spline	2.948643	1.959964	0.0032

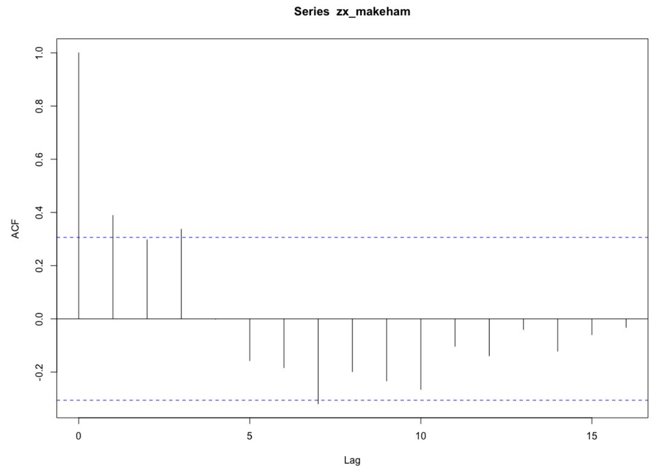
Table 13: Summary of Cumulative Deviation Test Results for Mortality Models

Model	Statistic	Critical Value	p-value
Gompertz	6	7.809555	0.0064
Makeham	6	8.064645	0.0042
Cubic Spline	8	8.064645	0.0855
Smoothing Spline	8	8.064645	0.0855

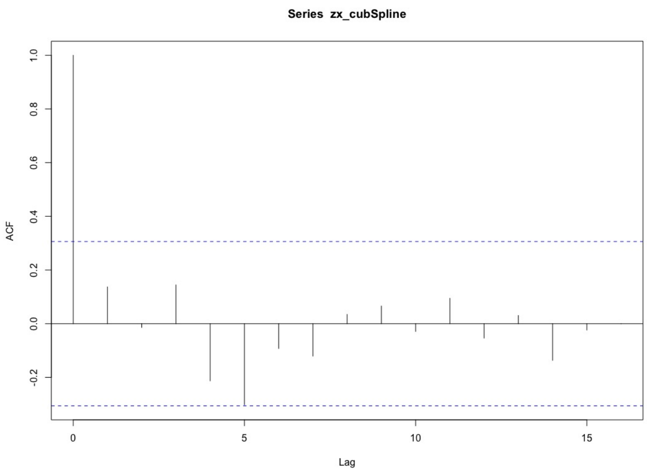
Table 14: Summary of Group Sign Test Results for Mortality Models



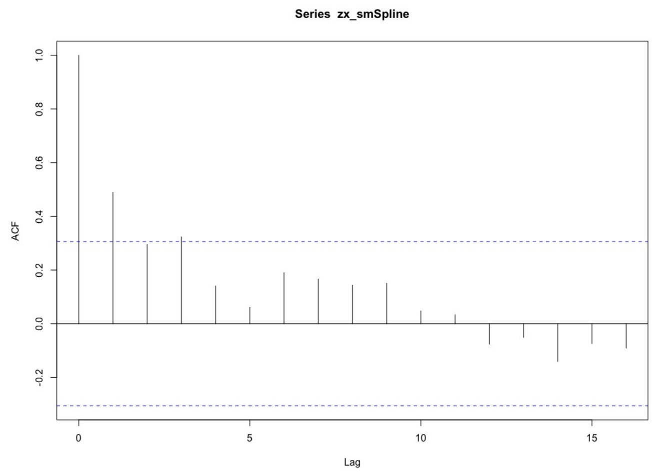
(a) ACF Gompertz



(b) ACF Makeham



(c) ACF Cubic Spline



(d) ACF Smoothing Spline

Figure 10: Serial Correlations Test for Graduation Models

6.9 Appendix I. Comparing Graduated Mortality Rates with Life Table Data

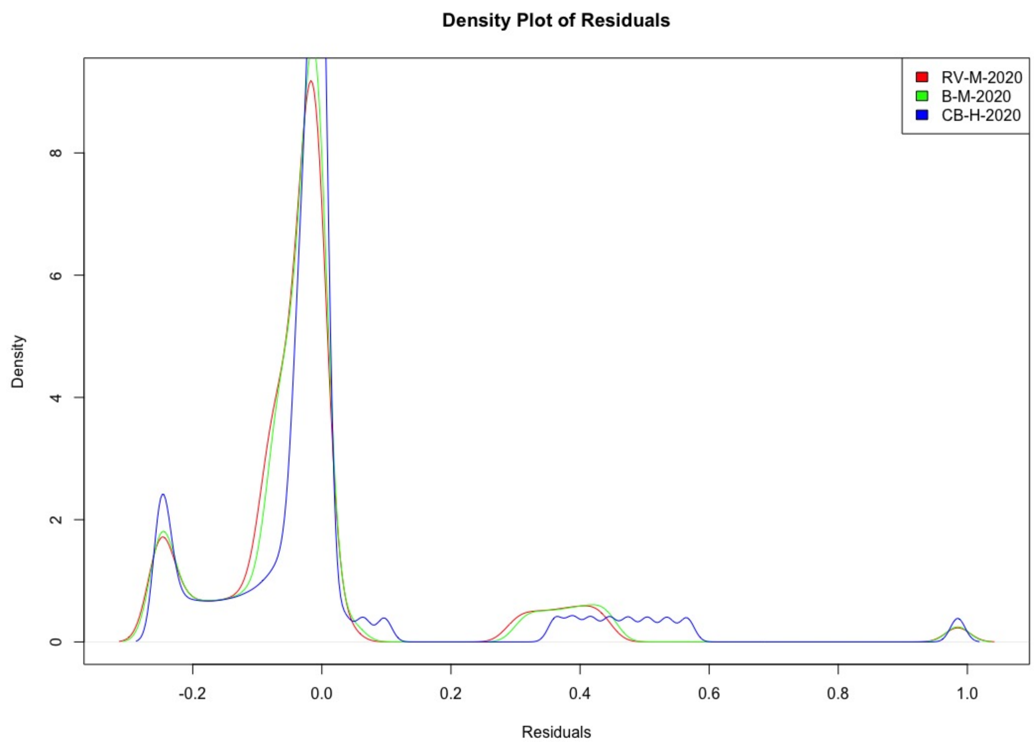


Figure 11: Density Plot of Residuals

Coefficient	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.011662	0.005497	-2.121	0.0403 *
mx_smSpline	0.737495	0.040630	18.151	< 2e-16 ***

Table 15: Regression Summary for RV-M-2020

Coefficient	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.009045	0.005497	-1.645	0.108
mx_smSpline	0.770982	0.040634	18.974	< 2e-16 ***

Table 16: Regression Summary for B-M-2020

Coefficient	Estimate	Std. Error	t value	Pr(>|t|)
(Intercept)	-0.005258	0.005472	-0.961	0.342
mx_smSpline	0.925198	0.040445	22.876	< 2e-16 ***

Table 17: Regression Summary for CB-H-2020