Homework 1

Problem 1-4

Question: Equations of motion of a simple point mass moving along a straight line are given by $\mathbf{M\ddot{x} = F}$ with measurement of position only. Express the equation in state space form as $\mathbf{\dot{x} = Ax + Bu}$ and $\mathbf{y = Cx}$.

```
clc
close all
clear all

addpath Screws
addpath fcn_support

syms m X0 dX0 ddX0 x10 x20 x30 dx10 dx20 dx30

X = sym('x', [3 1]); % 3 dimension position vector
d_X = sym('dx', [3 1]); % velocity vector
V = get_vel(X,X,d_X);
KE = 1/2*m*V'*V;
[D,C,G] = get_mat(KE, 0, X,d_X);
D = simplify(D);
C = simplify(C);
G = simplify(G);

Z = [X(1); X(2); X(3); d_X(1); d_X(2); d_X(3)]; %vector in state-space
ddX0 = [0; 0; 0];
X0 = [x10; x20; x30; dx10; dx20; dx30];
F = sym('f', [3 1]);
[A_lin,B_lin] = linearize_DCG(D,C,G,Z,F,X0,ddX0);
A_lin = simplify(A_lin)
```

```
A_lin =
```

$$\begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$$

```
B_lin = simplify(B_lin)
```

```
B_lin =
```

$$\begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & \frac{1}{m} & 0 \\ 0 & 0 & \frac{1}{m} \end{pmatrix}$$

So in state space we have:

$$\dot{Z} = \begin{pmatrix} 0 & 0 & 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix} Z + \begin{pmatrix} 0 & 0 & 0 \\ 0 & 0 & 0 \\ 0 & 0 & 0 \\ \frac{1}{m} & 0 & 0 \\ 0 & \frac{1}{m} & 0 \\ 0 & 0 & \frac{1}{m} \end{pmatrix} u$$

$$Y = \begin{pmatrix} 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 & 0 & 0 \end{pmatrix} Z$$