Homework 1

Problem 1: Cayley-Hamilton theorem

Question: Cayley-Hamilton theorem states that a matrix satisfies its own equation, this theorem can be applied to analytically evaluate any function of a matrix. Review the notes on Cayley-Hamilton theorem and given

$$A = \begin{pmatrix} 0 & 1 & .5 \\ -2 & -3 & 4 \\ 1 & 8 & 2 \end{pmatrix}$$

apply Cayley-Hamilton theorem to compute

1. sin(A)
2. e^A

```
clc
close all
clear all

A = [0,1,.5;
    -2, -3, 4;
    1, 8, 2];
%[v,d] = eig(A);


syms l a0 a1 a2
I = eye(3);
```

Characteristic Equation

```
D = det(A - l*I) == 0
```

D $= -l^3 - l^2 + \frac{73\,l}{2} + \frac{3}{2} = 0$

```
P = [-1 -1 73/2 3/2];
roots(P)
```

```
ans = 3x1 double

   -6.5432
    5.5843
   -0.0411
```

Compute : e^A

From Cayley-Hamilton we have:

$$e^{\lambda} = a_0 + a_1\lambda + a_2\lambda^2$$

$$\begin{pmatrix} e^{\lambda_1} \\ e^{\lambda_2} \\ e^{\lambda_3} \end{pmatrix} = \begin{pmatrix} 1 & \lambda_1 & \lambda_1^2 \\ 1 & \lambda_2 & \lambda_2^2 \\ 1 & \lambda_3 & \lambda_3^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

```
s = [1 -6.5432 6.5432^2;
    1 5.5843 5.5843^2;
    1 -0.0411 .0411^2];

a = [exp(-6.5432); exp(5.5843); exp(-.0411)];
```

We the find the Coefficients

```
Coef = inv(s)*a
```

```
Coef = 3x1 double

    2.0081
   25.6678
    3.8759
```

Solution

$$e^A = 2.0081I + 25.6678A + 3.8759A^2$$

Proof

```
S = 2.0081*I + 25.6678*A + 3.8759*(A)^2
```

```
S = 3x3 double

   -3.8058   29.5437   32.2134
```

```
  -12.5766   76.1648   83.2917
  -28.5948  178.2111  194.8141
```

```
B = expm(A)
```

```
B = 3x3 double

   -3.8070   29.5437   32.2136
  -12.5760   76.1656   83.2912
  -28.5955  178.2103  194.8144
```

```
err = (1 - norm(B - S)/norm(B))*100;
fprintf('So our solution is accurate to within %0.4f percent   %. \n', err);
```

```
So our solution is accurate to within 99.9994 percent
```

Compute sin(A)

$$\sin(\lambda) = a_0 + a_1\lambda + a_2\lambda^2$$

$$\begin{pmatrix} sin(\lambda_1) \\ sin(\lambda_2) \\ sin(\lambda_3) \end{pmatrix} = \begin{pmatrix} 1 & \lambda_1 & \lambda_1^2 \\ 1 & \lambda_2 & \lambda_2^2 \\ 1 & \lambda_3 & \lambda_3^2 \end{pmatrix} \begin{pmatrix} a_0 \\ a_1 \\ a_2 \end{pmatrix}$$

```
a2 = [sind(-6.5432); sind(5.5843); sind(-0.0411)];
```

Coefficients

```
Coef2 = inv(s)*a2
```

```
Coef2 = 3x1 double

   -0.0000
    0.0174
    0.0000
```

Solution

$$\sin(A) = 0I + 0.0174\,A - 0\,A^2$$

$$sin(A) = 0.0174\,A$$

Proof

```
S2 = 0.0174*A
```

```
S2 = 3x3 double

         0    0.0174    0.0087
   -0.0348   -0.0522    0.0696
    0.0174    0.1392    0.0348
```

```
B2 = sind(A)
```

```
B2 = 3x3 double

         0    0.0175    0.0087
   -0.0349   -0.0523    0.0698
    0.0175    0.1392    0.0349
```

```
err2 = (1 - norm(B2 - S2)/norm(B2))*100;
fprintf('So our solution is accurate to within %0.4f percent   %. \n', err2);
```

```
So our solution is accurate to within 99.8430 percent
```