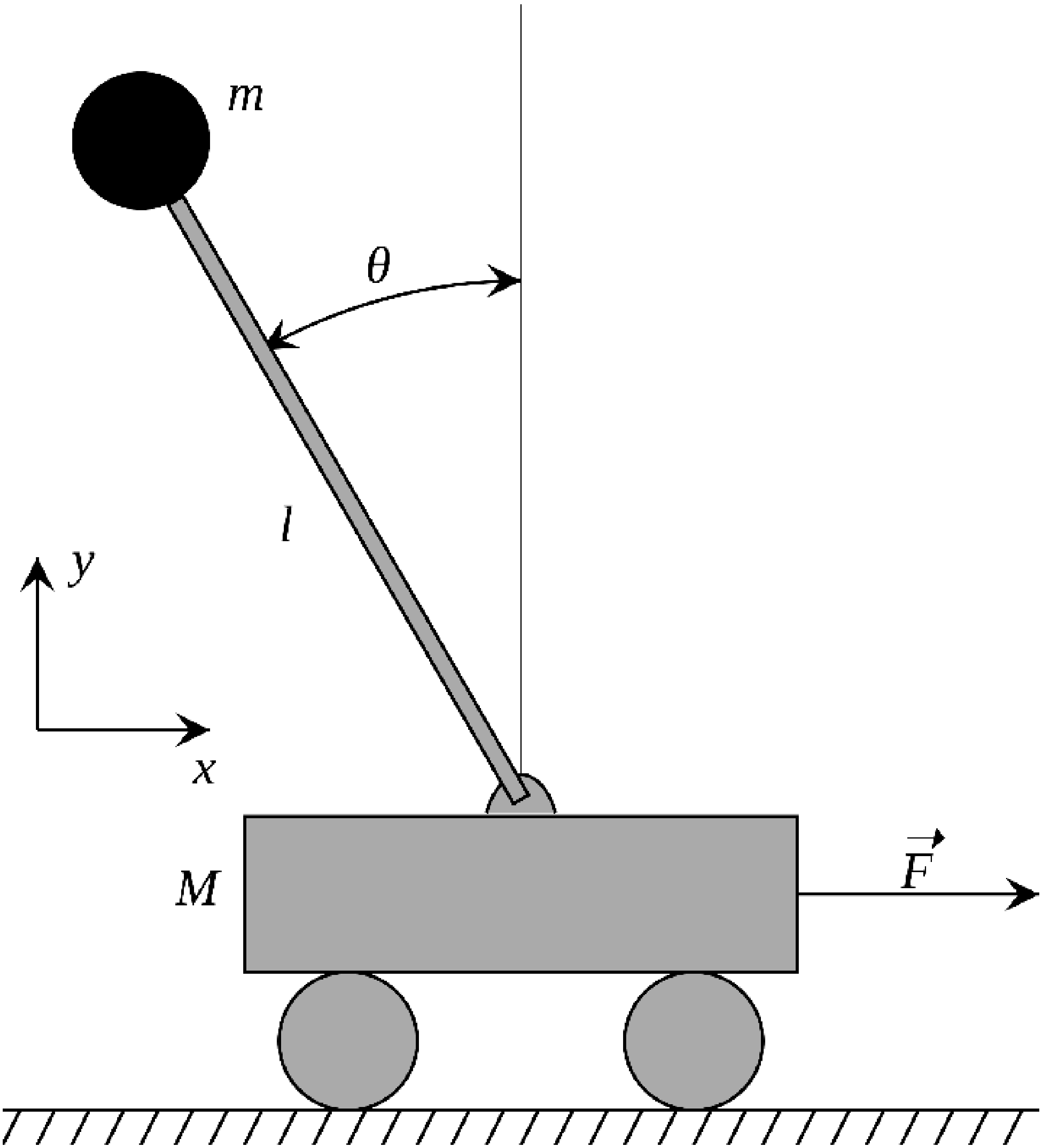
Homework 1

Problem 1-5

Question: Derive the equations of motion of a pendulum balancing on a cart, and linearize them about the steady state $\boldsymbol{\theta} = \mathbf{0}$ and $\mathbf{x} = \mathbf{0}$. Express the equations in state space form as $\dot{\mathbf{x}} = \mathbf{A}\mathbf{x} + \mathbf{B}\mathbf{u}$ and $\mathbf{y} = \mathbf{C}\mathbf{x}$



```
clc
close all
clear all

addpath Screws
addpath fcn_support
% Defining symbols
syms i q di dq ddi ddq t force tau  C real
M = 0.5;
m = .2;
l = .3;
g = 9.8;
syms i0 q0 di0 dq0 ddi0 ddq0






P1 = [  i;
    0];

P2 = [  i - l*sin(q);
     l * cos(q)];

p_v = [i;q];
dp_v = [di;dq];
%

% Taking derivative to compute velocities
V1 = get_vel(P1 ,p_v,dp_v);
V2 =get_vel(P2,p_v,dp_v);

% Computing Kinetic energy and potential energy
KE1 =simplify(1/2*M*V1'*V1);
KE2 =simplify(1/2*m*V2'*V2);

PE1 = M*g*P1(2);
PE2 = m*g*P2(2);

% Define Lagrangian
KE_total = KE1 + KE2;
PE_total = PE1 + PE2;



L = KE_total - PE_total;

[D,C,G] = get_mat(KE_total, PE_total, p_v,dp_v);
D = simplify(D);
C = simplify(C);
G = simplify(G);


% Now express this in the form of dx/dt = f(x,u)
Z = [i;q;di;dq]; % Vector of state space
ddZ0 = [0;0]; % Vector of SS accelerations [ddi0;ddq0]
Z0 = [0;0;0;0]; % Vector of SS  [i0;q0;di0;dq0]
force_tau = [force;0]; % Vector of force and torque
```

```
% Function to calculate Linearized representation
[A_lin,B_lin] = linearize_DCG(D,C,G,Z,force_tau,Z0,ddZ0);
A_lin = simplify(A_lin)
```

A_lin =

$$\begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{98}{25} & 0 & 0 \\ 0 & \frac{686}{15} & 0 & 0 \end{pmatrix}$$

```
B_lin = simplify(B_lin)
```

B_lin =

$$\begin{pmatrix} 0 & 0 \\ 0 & 0 \\ 2 & 0 \\ \frac{20}{3} & 0 \end{pmatrix}$$

So we have

$$\dot{Z} = \begin{pmatrix} 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & \frac{gm}{M} & 0 & 0 \\ 0 & \frac{g(M+m)}{Ml} & 0 & 0 \end{pmatrix} Z + \begin{pmatrix} 0 \\ 0 \\ \frac{1}{M} \\ \frac{1}{Ml} \end{pmatrix} u$$

$$Y = \begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \end{pmatrix} Z$$

```
A_lin = double(A_lin);
A_int = [0 1 0 0 0 0;
    0 0 0 1 0 0;
    0 0 0 0 1 0;
    0 0 3.92 0 0 0;
    0 0 686/15 0 0 0;
    0 0 0 0 0 -5];


B_lin = double(B_lin(:,1));
```

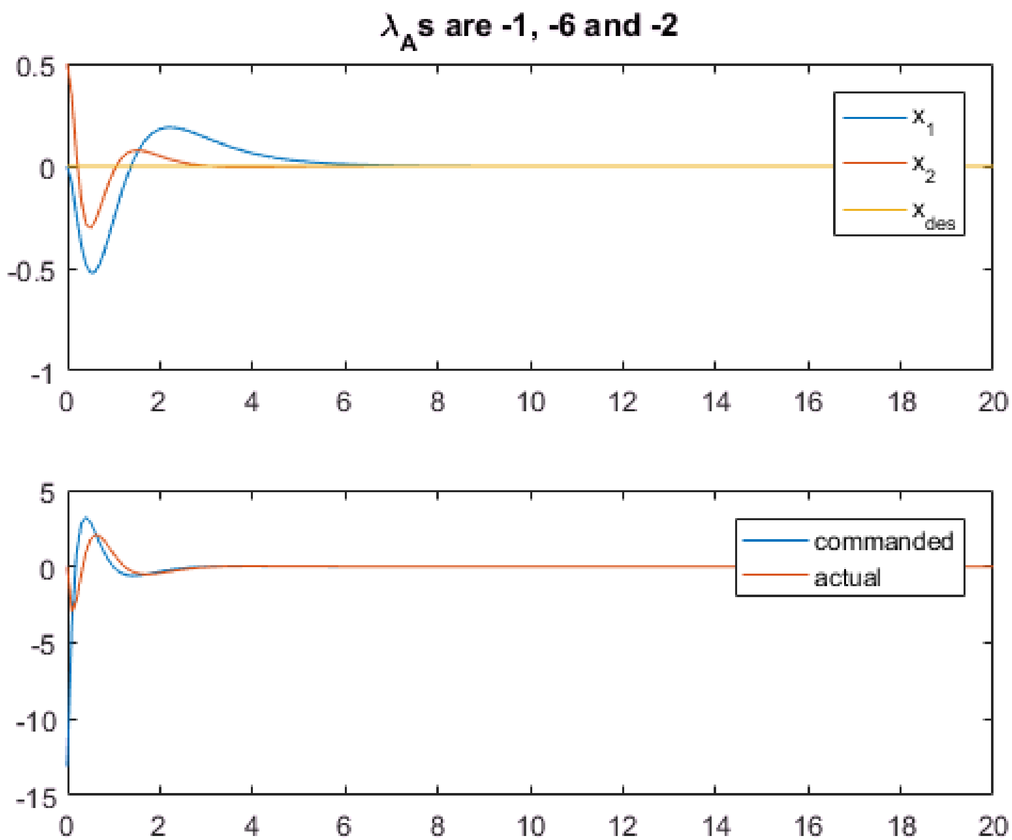
```
B_int = [0;0;0;2;20/3;5];

poles = eig(A_lin);
C = [1 0 0 0;
    0 1 0 0];
D = [0;0];




p = [-1;-2;-3;-4;-5;-6];
%co = ctrb(A_int,B_int);
%rank(co)

K = place(A_int,B_int,p);
%[v,d] = eig(A_int - B_int*K);
control = @(t,x)[-K*x];
%A = @(x)[x(3,:);
 %   x(4,:);
 %   (m*l*sin(x(2,:))*x(4,:)^2 - m*g*cos(x(2,:))*sin(x(2,:)))/(M + m - m*cos(x(2,:))^2);
 %   (-(M+m)*g*sin(x(2,:)) + m*l*cos(x(2,:))*sin(x(2,:))*x(4,:)^2)/(m*l*cos(x(2,:))^2 - (M+m)*
%B = @(x)[0;0;1/(M + m - m*cos(x(2,:))^2);cos(x(2,:))/(m*l*cos(x(2,:))^2 - (M+m)*l)];
sys_dyn = @(t,x)[A_int*x + B_int*control(t,x)];
x0 = [0;0;.5;0;.5;0];
Tspan = 0:0.1:20;
[t,x] = ode45(sys_dyn, Tspan, x0);
figure;
subplot(2,1,1)
plot(t,x(:,2),t,x(:,3),t,zeros(size(t)))
legend('x_1','x_2', 'x_{des}')
title(['\lambda_As are ' num2str(d(1,1)) ', ' num2str(d(2,2)) ' and ' num2str(d(3,3))])
subplot(2,1,2)
plot(t',control(t',x'),t',x(:,6)')
legend('commanded','actual')
```



λAs are -1, -6 and -2
x1
x2
xdes
commanded
actual
0.5
0
-0.5
-1
5
0
-5
-10
-15
0
2
4
6
8
10
12
14
16
18
20