Homework 1

Problem 1-3: Positive (semi-)definite matrices

Question: If AA is a matrix whose all elements are real,

1. Show that $\mathbf{A}\mathbf{A}^{\mathbf{T}}$ is positive semi-definite, i.e. eigen values are all greater than or equal to 0.
2. Positive semi-definite matrices have a nice property that for any vector $\mathbf{x}$, $\mathbf{x}^{\mathbf{T}}\mathbf{A}\mathbf{x} \geq \mathbf{0}$. Under what conditions does the equality occur?

In special case where all eigen values are all positive, the matrix $\mathbf{A}$ is called positive definite, and in such cases $\mathbf{x}^{\mathbf{T}}\mathbf{A}\mathbf{x} > \mathbf{0}$

1) Given a matrix $\mathbf{A}$ with all real values let

$$S = A^T A$$

To be considered positive semi-definite we must have for a vector $\mathbf{x}$

$$x^T S x \geq 0$$

Substituting $\mathbf{S}$ we have

$$x^T A^T A x = ||Ax||^2 \geq 0$$

Therefore S must be positive semi-definite

2) The condition where $\mathbf{x}^{\mathbf{T}}\mathbf{S}\mathbf{x} = \mathbf{0}$ occurs only when $\mathbf{x}$ is the zero vector