Homework 1

Problem 1-3a: Similarity Transforms:

Question: Two square-matrices $\mathbf{A}$ and $\mathbf{C}$ are called similar if there exists an invertible square matrix $\mathbf{P}$ such that,

$$\mathbf{C} = \mathbf{P}^{-1}\mathbf{AP}$$

The transformation $\mathbf{P}^{-1}\mathbf{AP}$ is also called similarity or conjugation of $\mathbf{A}$. We will encounter this form of equation when we do transformation of variables in a linear system.

1. Show that $\mathbf{A}$ and $\mathbf{C}$ have same eigen values.
2. Do $\mathbf{A}$ and $\mathbf{C}$ have the same eigen vectors?
3. Consider the dynamic system given by $\dot{\mathbf{x}} = \mathbf{Ax} + \mathbf{Bu}.$ Apply a transformation $\mathbf{P}$ such that the transformed variable $\mathbf{x} = \mathbf{P}\hat{\mathbf{x}}$.
4. What will happen in the special case where $\mathbf{P}$ is the vector of eigen values? What can you say about the stability and controlability of the tranformed system by looking at the new dynamics equations?
5. Consider the system given below, and transform it such that $\mathbf{P}$ is the vector of eigen values. Confirm your predictions from part 4.

$$\mathbf{A} = \begin{pmatrix} 0 & 1 & .5 \\ -2 & -3 & 4 \\ 1 & 8 & 2 \end{pmatrix}$$

$$\mathbf{B} = \begin{pmatrix} 0 \\ 1 \\ 1 \end{pmatrix}$$

Problem 4

1) Given

$$\mathbf{C} = \mathbf{P}^{-1}\mathbf{AP}$$

$$\Rightarrow \mathbf{PCP}^{-1} = \mathbf{A}$$

Since

$$\mathbf{A\nu} = \lambda\mathbf{\nu}$$

$$\Longrightarrow \mathbf{PCP^{-1}\nu = \lambda\nu}$$

Therefore

$$\Longrightarrow \mathbf{CP^{-1}\nu = \lambda P^{-1}\nu}$$

by letting $\mathbf{P^{-1}\nu = \mu}$ we have

$$\mathbf{C\mu = \lambda\mu}$$

Proving that eigenvalues of $\mathbf{A}$ are also eigen values of $\mathbf{C}$

2) $\mathbf{A}$ and $\mathbf{C}$ do not have the same eigenvectors since $\boldsymbol{\nu}$ is not necessarily equal to $\mathbf{P^{-1}\nu}$

3) With $\mathbf{x = P\hat{x}}$. we apply our transformation and have

$$P^{-1}\dot{x} = P^{-1}AP\hat{x} + P^{-1}B$$

4) In the special case where P is the vecctor of eigen values then **A** will transform into the matrix of eigenvalues along the diagnal since

$$AP = P\Lambda$$

$$\longrightarrow P^{-1}AP = \Lambda$$

5)

```
A = [0 1 .5;
    -2 -3 4;
    1 8 2];

B = [0;
    1;
    1]
```

```
B = 3x1 double

     0
     1
     1
```

```
[V, D] = eig(A)
```

```
V = 3x3 double

   -0.0600    0.9274    0.1509
```

```
    0.7321   -0.1970    0.3884
   -0.6785    0.3179    0.9090
```

```
D = 3x3 double

   -6.5432         0         0
         0   -0.0411         0
         0         0    5.5843
```

```
inv(V)*A*V
```

```
ans = 3x3 double

   -6.5432    0.0000    0.0000
    0.0000   -0.0411   -0.0000
   -0.0000    0.0000    5.5843
```

```
inv(V)*B
```

```
ans = 3x1 double

    0.4889
   -0.2193
    1.5417
```

Since one of the eigenvalues of **A** has a positive real part, we know that this system is unstable, as **A** will eventually drive the output to infinity along the third eigenvector.