Modeling, Simulation and Control Study for The Quad-Copter UAV

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***Abstract*—In the recent years UAV (Unmanned Aerial Vehicles) having quad-copter helicopter i.e. quad-copter configuration have been receiving increasing attention amongst the global researchers due to its wide-range of applications such as surveillance in military, civilian and disaster management applications. This paper presents our investigations on the modeling, simulation, altitude model validation and comparison of some popular quad-copter control schemes. Quad-copter consists of two pairs of counter rotating rotors situated at the ends of a cross frame, symmetric about the centre of gravity, which coincides with the origin of the reference systems used. The model used in our work is based on the Euler Lagrange method used to derive the defining equations of motions of the six degree of freedom system with minor changes to consider variable air-density. This modified model is then simulated in matlab-simulink framework to dynamically compute the quad-copter altitude and attitude. Also, some commonly used control algorithms like PD, PID and back-stepping control are compared using our modified model simulation framework computing certain well-known performance metrics, like ISE, IAE, for the closed loop altitude and attitude control.**

Keywords—UAV, Quad-copter, Quad-rotor, Modeling, Simulation

I. Introduction

Unmanned Aerial Vehicles (UAV) have been receiving an increasing attention amongst the global researchers, particularly due to its wide range of applications such as surveillance in military, civilian and disaster management [1]. The UAV can be broadly classified in two category i.e. autonomous aero-plane and autonomous helicopter. The helicopters have clear advantages over the aeroplanes due to their specific capabilities like better hovering operation, ability of landing/take-off in limited space, etc. Hence, they are more suitable for a class of surveillance applications, such as, inspection inside factory/residential buildings, reconnaissance within an urban environment, observation of a structurally unsafe building, etc.

A quad-copter is a four propeller helicopter. As shown in Fig.1, the quad-copter consists of two pairs of counter rotating rotors situated at the ends of a cross-frame that is symmetric about the centre of gravity coinciding with the origin of the reference system used. The basic motions of a quad-copter a regenerated by varying the rotational speed of all four rotors, and thereby changing the absolute and differential lift (or thrust) forces. The helicopter tilts towards the direction of low lift rotor, which enables acceleration along that direction. Spinning directions of the rotors are set to balance the moments, therefore eliminating the need for a tail rotor. This flying robot presents the main advantage of having quite simple dynamic features. One of the advantages of quad-rotors is the payload augmentation. They have more lift thrusts than conventional helicopters therefore they offer better payload capacity. Moreover, they are potentially simpler-to-build and highly maneuverable. Quad rotor aerial robot can generate 6-DOF movement in the inertial frame through changing rotational speed of the motors. The 6-Degree of Freedom (DOF) motion include three translational motions along three coordinate axes (i.e. surge, sway and heave) and three rotational motion around three rotary axes (i.e. roll, pitch and yaw).

II. Quad-copter Modeling

There is a significant number of publications describing the mathematical modelling, simulation and control of quad-copter UAV [2], [3], [4], [5] and [6]. They modelled a quad-copter by incorporating the airframe and motor dynamics as well as aerodynamics and gyroscopic effects. In order to derive a complete dynamic model, the rigid body dynamics and the effects of aerodynamics are studied in this section based on the reference [7]. This literature model is modified and discussed in our previous work [8] to account for variations in air-density with change in the altitude. In this paper, we present extension of our previous work [8] by performing altitude model validation for this modified model and investigate several control schemes for this modified model simulation comparing popular control performance metric like ISE, IAE.

The body fixed coordinates for the quad-copter are defined as explained below. As shown in Fig.1, the lever marked with black strip is chosen as the **x** axis, and the perpendicular lever is considered as the **y** axis. The **z** axis is then defined by the right hand rule. For inertial coordinates, the point where the quad rotor starts its flight is set as the origin, and an east-north-up orthogonal coordinate system is established by the right hand rule. With attitude angles defined as in Fig.2.

The transformation matrix $\mathcal{R}$ relating inertial coordinates to body fixed coordinates as in [7] is given by equation (1),

$$\mathcal{R}(\varphi, \theta, \psi) = \begin{bmatrix} c(\psi)c(\theta) & c(\psi)s(\theta)s(\varphi) - s(\psi)c(\varphi) & c(\psi)s(\theta)c(\varphi) + s(\psi)s(\varphi) \\ s(\psi)c(\theta) & s(\psi)s(\theta)s(\varphi) + c(\psi)c(\varphi) & s(\psi)s(\theta)s(\varphi) - s(\varphi)c(\psi) \\ -s(\theta) & c(\theta)s(\varphi) & c(\theta)c(\varphi) \end{bmatrix} \quad (1)$$

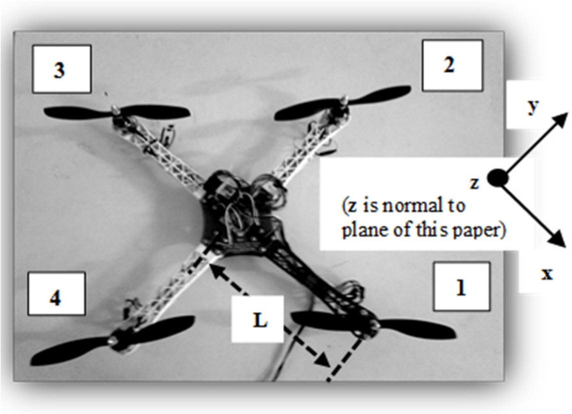

Fig. 1. Our quad-copter machine with reference axis

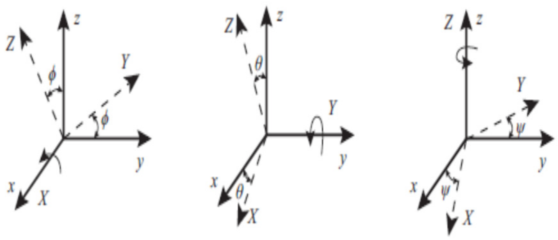
Fig. 2. Quad-copter attitude rotational angles

where, *s and c* stands for trigonometric operators '*sin*' and '*cos*' respectively. Whereas, the operands φ, θ, ψ respectively represent the roll, pitch and yaw attitude angles.

The thrust F produced by each motor is calculated as –

$$F=\rho\,(H)\,C_T\,A\omega_m^2R^2 \qquad (2)$$

Where,

C_T: thrust coefficient $\qquad$ ρ: air density (kg/m^3)
A: rotor disk area (m^2) $\qquad$ R: blade radius (m)
H: Altitude or Height (m) $\qquad$ ω_m: Motor speed (rad/sec)

The consideration of variable air-density in (2) is the key contribution of our work. In the body fixed coordinates, the direct inputs are RPM (revolutions per minute) commands for the motors. The resultant outputs are **z** directional thrusts in body fixed coordinates. However, the concerned outputs are altitude (i.e. position) and attitude. To eliminate this gap, four control variables are defined as [8],

$$\left.\begin{aligned} U_H &= F_1 + F_2 + F_3 + F_4 \\ U_\phi &= (F_2 - F_4)\,L \\ U_\theta &= (F_3 - F_1)\,L \\ U_\psi &= F_1 + F_3 - F_2 - F_4 \end{aligned}\right\} \qquad (3)$$

Where F_1, F_2, F_3 and F_4 are thrusts and L is the lever length. The subscripts correspond to the ordinal numbers in Fig. 1. Thus U_H, U_ϕ, U_θ and U_ψ are the input variables used to manipulate altitude, roll, pitch, and yaw movement of the quad-copter respectively.

The quad-copter rigid body dynamics are derived based on Newton-Euler formalism, considering small rotary inertia, negligible friction terms, and the symmetric structure of the quad-copter UAV, and can be expressed as in equation (4)

$$\left.\begin{aligned} \ddot{x} &= U_H(\sin\psi\sin\varphi + \cos\psi\sin\theta\cos\varphi)/m \\ \ddot{y} &= U_H(\sin\psi\sin\theta\cos\varphi - \cos\psi\sin\varphi)/m \\ \ddot{z} &= U_H\cos\varphi\cos\theta/m - g \\ \ddot{\varphi} &= U_\phi/Ix \\ \ddot{\theta} &= U_\theta/Iy \\ \ddot{\psi} &= U_\psi/Iz \end{aligned}\right\} \qquad (4)$$

Where, Ix, Iy, and Iz are rotary moment of inertia about **x**, **y**, and **z** axes respectively, m is quad-copter mass and g is gravitational acceleration [8].

III. Open Loop Model Validation Study

The popularly used quad-copter model, which is modified, discussed and investigated in our recent work [8], and briefly described above for a quick reference, is simulated using matlab-simulink framework. In this paper, to validate the above modified open-loop model simulation, the actual quad-copter machine is assembled and flown. As shown in Fig.3, on the quad-copter, an electronic payload board containing the pressure sensor, micro-controller and EEPROM is also integrated. This board is used to log real-time pressure data, which subsequently is used to calculate the differential altitude (m) using standard atmospheric data table. The preliminary validation study of the open-loop quad-copter model (currently only for the hovering condition) is carried out using this quad-copter machine. The validation results are presented and discussed in section-V.

IV. Quad-copter Control Study

Typical quad-copter scheme is shown in Fig. 4. In this work, two different control algorithms, PID and back-stepping control, are studied and compared through closed loop simulations for the proposed variable air-density model and discussed in this paper.

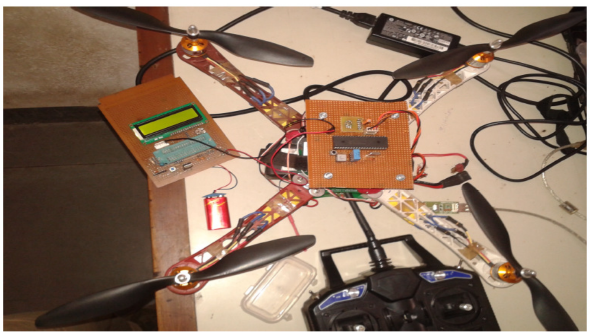
Fig. 3. Our quad-copter machine with payload electronic board

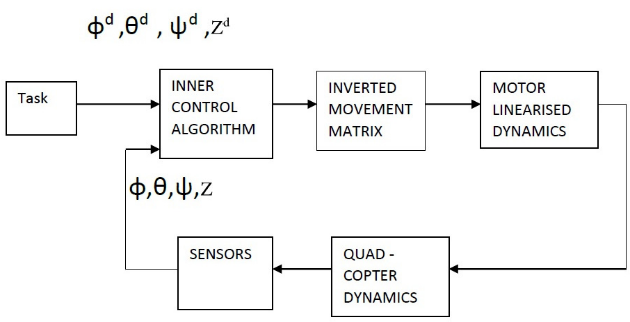

Fig. 4. Typical Quad-copter Control Schematic [12]

A. PID control

The most used liner regulators in industry are PID controllers. The reasons of its success are mainly its simple structure, good performance-cum-robustness for several processes, reasonable knowledge and ease of tuning even with the low accuracy knowledge of the system model, etc. The PD and PID control techniques are used for the attitude and the altitude control in [9], [10], and [11] mainly to achieve stable operation. Hence, in our current work a simulation based investigations are carried out regarding use of PD and PID controllers for our quad-copter machine.

In our work, the performance of these control schemes are compared using error criteria such as ISE and IAE using our quad-copter simulation framework. Also, for PID based altitude-control scheme, performance of different tuning methods, such as Cohen-coon and λ-tuning, are compared and it is shown that λ-tuning method provides a better performance compared to Cohn-coon tuning. The results of our investigations regarding the closed loop control of our quad-copter machine are presentedand discussed in section V.

Additionally, one of the popular non-linear control technique namely “back-stepping control” [12] is also investigated. Some background and a derivation of the back-stepping control-law for our quad-copter machine is given below.

B. Back-stepping control

The state space model $\dot{\mathbf{X}} = f(\mathbf{X}, \mathbf{U})$ of the quad-copter is derived using the input vector $\mathbf{U}$ defined by (3), simplified dynamic model in (4), and the state vector $\mathbf{X}$ consisting of 12 state variables as defined below:

$x_1 = \varphi$	$x_7 = z$
$x_2 = \dot{\varphi}$	$x_8 = \dot{x}_7 = \dot{z}$
$x_3 = \theta$	$x_9 = x$
$x_4 = \dot{x}_3 = \dot{\theta}$	$x_{10} = \dot{x}_9 = \dot{x}$
$x_5 = \psi$	$x_{11} = y$
$x_6 = \dot{x}_5 = \dot{\psi}$	$x_{12} = \dot{x}_{11} = \dot{y}$

Using this state space model and back-stepping approach [12], one can synthesize the control law forcing the system to follow some desired trajectory.

First step, consider tracking-error:

$$z_1 = x_{1d} - x_1 \tag{5}$$

Now, applying Lyapunov’s theorem, derive the stabilizing control input which ensures that the time-derivative of Lyapunov function remains negative semi-definite for a positive definite Lyapunov function $V(z_1)$ as following:

$$V(z_1) = \tfrac{1}{2} z_1^2 \tag{6}$$

$$\dot{V}(z_1) = z_1 \dot{z}_1 = z_1 (\dot{x}_{1d} - x_2) \tag{7}$$

The stabilization of z_1 can be obtained by introducing a virtual control input x_2 and some positive constant α_1 –

$$x_2 = \dot{x}_{1d} + \alpha_1 z_1 \qquad \text{with: } \alpha_1 > 0 \tag{8}$$

Then using equation (5) then gives:

$$\dot{V}(z_1) = -\alpha_1 z_1^2 \tag{9}$$

Let us proceed to a variable change by making:

Second step, consider augmented Lyapunov function:

$$V(z_1,z_2) = \tfrac{1}{2}(z_1^2 + z_2^2) \tag{11}$$

And it’s time derivative is then given by:

$$\left.\begin{aligned} \dot{V}(z_1,z_2) = {} & z_2 (a_1 x_4 x_6 + a_2 x_4 \omega_m + b_1 U_\theta) \\ & - z_2 \{\ddot{x}_{1d} - \alpha_1 (z_2 + \alpha_1 z_1)\} \\ & - z_1 z_2 - \alpha_1 z_1^2 \end{aligned}\right\} \tag{12}$$

The control input U_θ is then extracted considering $\ddot{x}_{1d}$, $\ddot{x}_{2d}$, $\ddot{x}_{3d} = 0$, and satisfying $\dot{V}(z_1 z_2) < 0$, which is given by:

$$U_\theta = \{z_1 - \alpha_1 x_4 x_6 - \alpha_2 x_4 \omega_m - \alpha_1 (z_2 + \alpha_1 z_1) - \alpha_2 z_2\}/b_1 \tag{13}$$

The term $\alpha_2 z_2$ with $\alpha_2 > 0$ is added to stabilize z_1.

The same steps are followed to derive U_ϕ and U_ψ as below.

$$U_\phi = \{z_3 - \alpha_3 x_2 x_6 - \alpha_4 x_2 \omega_m - \alpha_3 (z_4 + \alpha_3 z_3) - \alpha_4 z_4\}/b_2 \tag{14}$$

$$U_\psi = \{z_5 - \alpha_5 x_2 x_4 - \alpha_5 (z_6 + \alpha_5 z_5) - \alpha_6 z_6\}]/b_3 \tag{15}$$

with:

$$\left.\begin{aligned} z_3 &= x_{3d} - x_3 \\ z_4 &= z_4 - \dot{x}_{3d} - \alpha_3 z_3 \\ z_5 &= x_{5d} - x_5 \\ z_6 &= x_6 - \dot{x}_{5d} - \alpha_5 z_5 \end{aligned}\right\} \tag{16}$$

Finally, the altitude control input U_H derived by this back-stepping approach is given by:

$$U_H = \{m/\cos x_1 \cos x_3\}\{z_7 + g - \alpha_7 (z_8 + \alpha_7 z_7) - \alpha_8 z_8\} \tag{17}$$

where

$$\left.\begin{aligned} z_7 &= x_{7d} - x_7 \\ z_8 &= x_8 - \dot{x}_{7d} - \alpha_7 z_7 \end{aligned}\right\} \tag{18}$$

and

$a_1 = (I_y - I_z) / I_x$	$b_1 = L / I_x$
$a_2 = -J_R / I_x$	$b_2 = L / I_y$
$a_3 = (I_z - I_x) / I_y$	$b_3 = 1 / I_z$
$a_4 = J_R / I_y$	
$a_5 = (I_x - I_y) / I_z$	

Given J_R as rotor inertia; and α_i (i=1 to 7) as some appropriately chosen positive real constants.

V. Results and Discussions

Our quad-copter model simulator is configured for the physical parameters of the quad-copter machine available in our laboratory. Table-1 shows various physical parameters of our quad-copter machine. Again, when all the motors supply the same thrust, the quad rotor rises until the total thrust produced exactly balances with the weight of quad rotor (including pay-load).

First, the open-loop model validation results are discussed. Next, the results of closed loop control investigations for PD,

TABLE-1. PARAMETERS OF OUR QUAD-COPTER MACHINE.

Parameter	Value	Description
W[m]	0.1050	Width of hub
D [m]	0.1050	Depth of hub
h [m]	0.0390	Height of hub
Q [kg]	0.0400	Mass of motor
P [kg]	0.1000	Mass of hub
R [m]	0.0130	Radius of motor-body
Hm [m]	0.0150	Height of motor-body
R [m]	0.2475	Motor to hub distance
C_T	0.5000	Thrust Coefficient
M [kg]	1.0000	Total mass

PID, and back-stepping control schemes are discussed including the comparison of control performance metrics like ISE, IAE. Additionally, the closed loop control responses for two popular PID controller tuning methods -- Cohen-coon and λ-tuning – are also compared and discussed in the same section.

A Open loop model validation

As discussed earlier in section-IV, the open loop model response is validated using the actual quad-coptermachine, currently only for the hovering condition. The results are as shown in Fig. 5. Fig. 6, shows the comparison of the actual altitude data measured using the pressure-sensor. and the altitude-data simulated for our quad-copter machine. The validation results are found well in agreement.

B. Model approximation and closed loop simulations

Towards this, first an open-loop non-linear model simulation response is approximated by the FOPDT transfer function model. The maximum transient and steady-state model approximation errors are found within acceptable limits i.e. <6% and <2% as shown in Fig. 7.

Using the derived FOPDT altitude model for our quad-copter machine performance of different PID tuning methods, such as Cohen-coon and λ-tuning method, are compared. Fig. 8, shows the closed loop simulation response for various PID tuning methods using this approximate model. As expected, the Cohen-coon tuning shows oscillatory behaviour with faster rise-time in the altitude response. Whereas λ-tuning shows non-oscillatory response and slower rise-time.

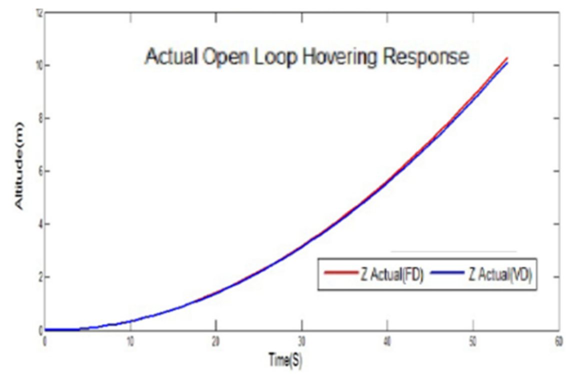

Fig. 5. Comparison of altitude model simulation data: Fixed versus variable air-denisty models

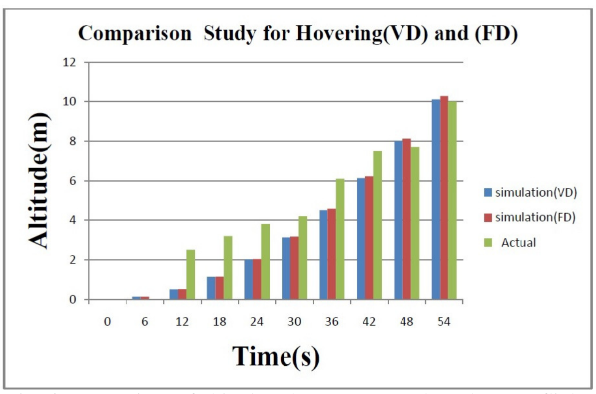

Fig. 6. Comparison of altitude values (m): Actual quad-copter flight data versus fixed and variable air-density model simulation data.

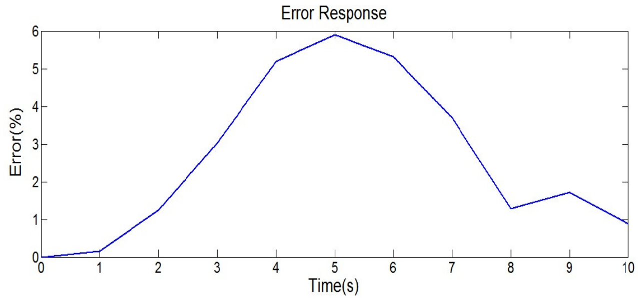

Fig. 7. Error between non-linear and FOPDT approximated model

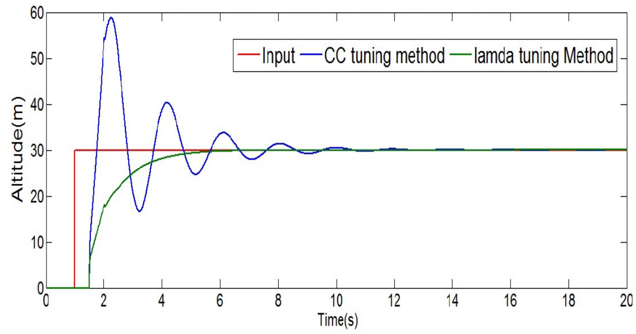

Fig. 8. Comparison of closed loop altitude response simulation data for PID Cohen-coon versus Lambda tuning methods

In next four cases, the altitude and attitude (Roll, Pitch, and Yaw) responses for our quad-copter machine are simulated, wherein approximate FOPTD transfer function model and Cohen-coon PD/PID controller tuning are used for altitude control. Whereas, auto-tuning and trial-error fine-tuning based PD/PID controller is used for the attitude responses.

Case 1: Altitude (z) response:

Fig. 9, shows the altitude response results of PD and PID controllers for altitude-reference tracking and the disturbance rejection. Here, PID controller shows an overshoot and slower settling time as compared to PD controller.

Case 2: Roll Attitude response

Fig. 10, shows the Roll-attitude control response results of PD and PID controllers for set-point tracking and disturbance rejection. In these results, PID controller shows a minor overshoot and faster settling time as compared to PD controller.

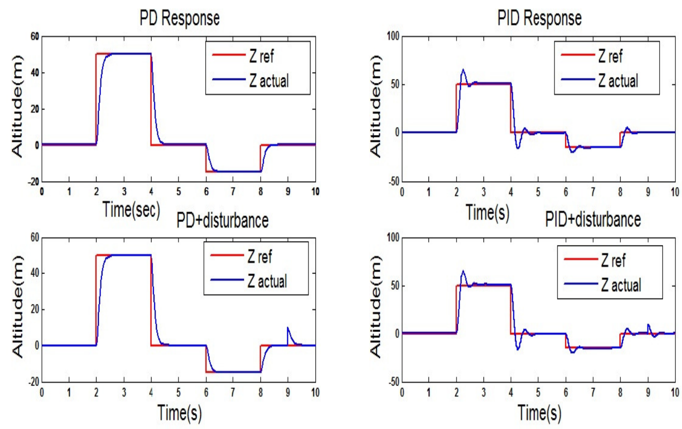

Fig. 9. Altitude Control: Comparison of PD & PID controllers

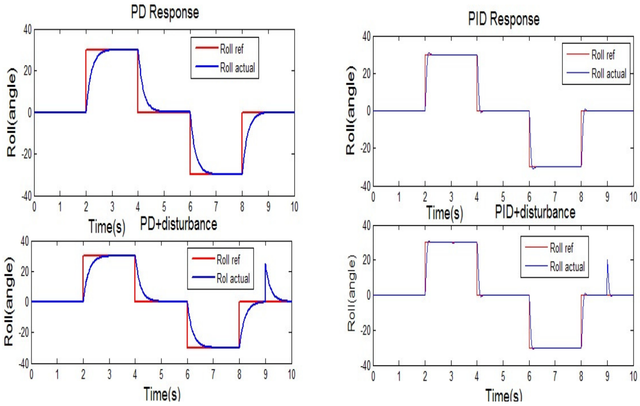

Fig. 10. Attitude Control (Roll): Comparison of PD and PID control

Case 3: Pitch Attitude response

Fig. 11, shows the yaw-attitude control simulation responses for the set-point tracking and disturbance rejection by PD and PID controllers. PID controller shows faster settling but minor overshoot as compared to PD controller for our quad-copter machine.

Case 4: Yaw Attitude Response

Fig. 12, shows the yaw-attitude control simulation responses for the set-point tracking and disturbance rejection by PD and PID controllers. PID controller shows faster settling but minor overshoot as compared to PD controller for our quad-copter machine.

Similarly, Fig. 13 to Fig. 15, show the closed loop altitude and attitude (Roll, Pitch and Yaw) simulation responses for the set-point tracking and disturbance-rejection by the back-stepping control algorithm for our quad-copter machine.

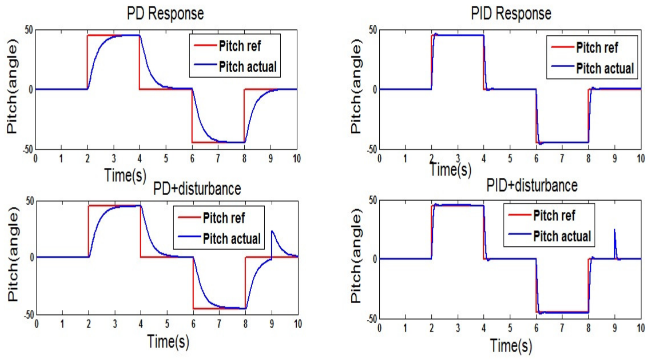

Fig. 11. Attitude Control (Pitch): Comparison of PD and PID control

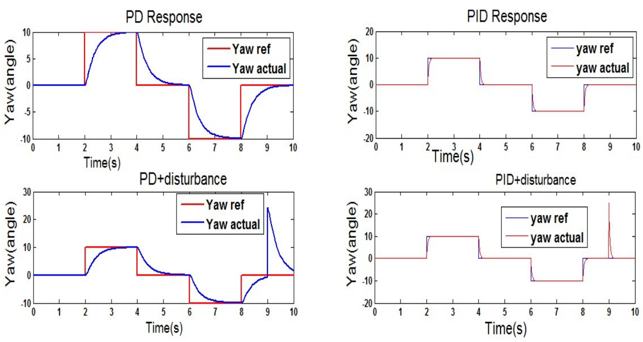

Fig. 12. Attitude Control (Yaw): Comparison of PD and PID control

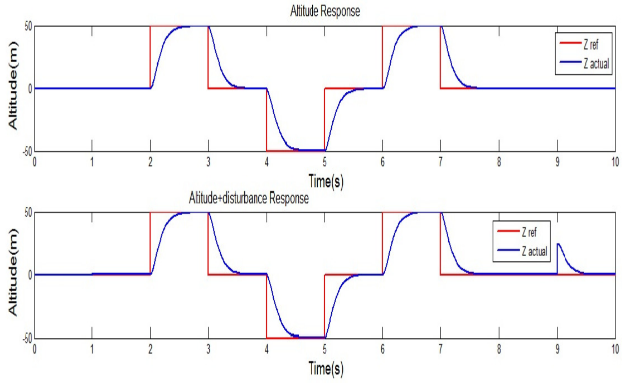

Fig. 13. Back-stepping control for Altitude (m)

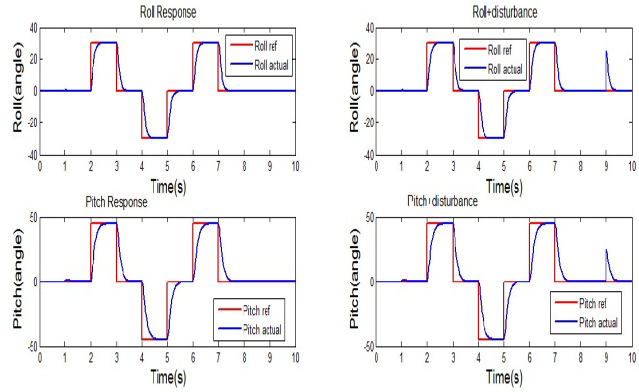

Fig. 14. Back stepping control for Roll and Pitch angle

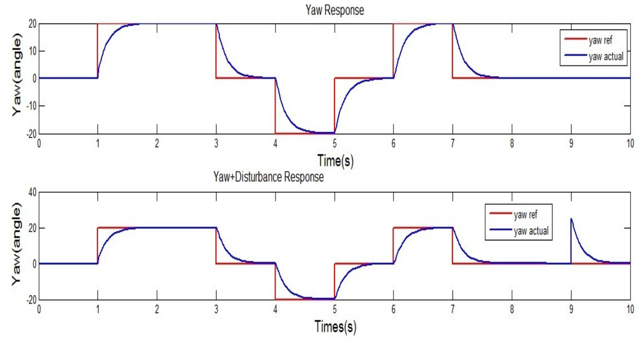

Fig. 15. Back stepping control for Yaw angle

Fig. 16 to Fig. 17, show ISE and IAE performance criteria to compare the performance of PD,PID and back-stepping controllers. Here PID controller shows better ISE and IAE performance as compared to other two controllers.

VI. Conclusion and Future Work

A simulation framework for our modified mathematical model is developed and is configured for the specific quad-copter machine available in our laboratory. A preliminary validation of the altitude model is also carried out using this quad-copter machine. The set-point tracking and disturbance rejection behaviour of PD, PID and the back-stepping controllers are investigated using this simulation framework. The comparison of PD, PID and back-stepping controller performance is done based on ISE and IAE criteria. Also, performances of Cohen-coon and lambda-tuning based PID controllers are compared for the altitude control and found that

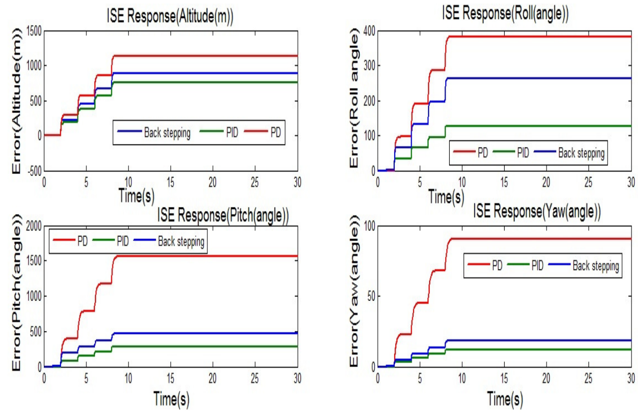

Fig. 16. ISE performance Comparison for PD,PID & back-stepping ontrol

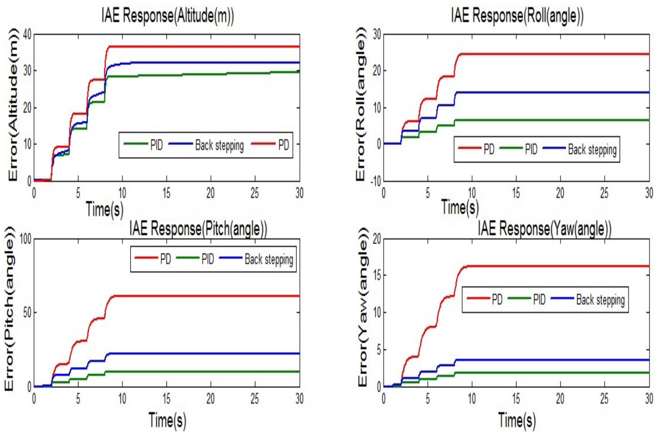

Fig. 17. IAE performace comparison for PD,PID and back-stepping control

λ-tuning method gives better performance as compared to Cohn-coon tuning method.

This work will be extended further through investigations like performance comparison of various other control schemes such as LQR, Model based control etc.; performing rigorous open-loop and closed-loop validation study; etc.

Nomenclature

FOPDT	First order with dead time
Kp, Ki, Kd	Proportional, Integral and Derivative gains
PD	Proportional derivative
PID	Proportional integral derivative
ISE	Integral square error
IAE	Integral absolute error
LQR	Linear Quadratic Regulator
DAS	Data Acquisition System

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