MacEwan University **Math 114**

Practice Final Exam Practice

Instructions:

1. Attempt every question.
2. All work must be shown. No marks will be given for answers alone.
3. No marks are given for guess or test methods.
4. No calculators are allowed.
5. The value of individual questions is given in square brackets
6. You have 2 hours to complete this exam.
7. Total maximum score is 43.

Question 1. [12 points]

a) Sketch the graph of a function $f(x)$ such that $f(x)$ is an odd function and is continuous on $\Re$, $\lim_{x\to 0}|f'(x)| = \infty$, $f''(x) < 0 \; on \; (0,5)$, $f''(x) > 0 \; on \; (5,\infty)$.

b) Find the critical numbers of $f(x) = x^2 \cdot \sqrt[3]{2+4x}$.

c) If $\sin(x^2 y) - 5y^3 = \pi^{2/3}$, find $y'(x)$.

d) Find the derivative of the following function. **Do not simplify**.

$$y = \left(\tan\left(\frac{8x^2}{x^3+1}\right)\right)^4 + \frac{4}{\sqrt{\cos(x)}}$$

Question 2. [6 points] Evaluate the following limits:

a) $\lim_{x->-\infty} \frac{2x-3}{\sqrt{3x^2+16}} =$ b) $\lim_{x->2} \frac{\sin(x^2-4)}{x^4-16} =$

Question 3. [3 points] Find *f(x)* if

$f''(x) = -2 + 12\cos(x) - 12x^2$, $f(0) = 4$, $f'(0) = 12$

Question 4. [4 points]

Use the definition of the definite integral to evaluate:

$$\int_{-5}^{0} (x^2 + 6x + 1)dx$$

The following sums are provided for your reference:

$$\sum_{i=1}^{n} i = \frac{n(n+1)}{2}, \quad \sum_{i=1}^{n} i^2 = \frac{n(n+1)(2n+1)}{6}, \quad \sum_{i=1}^{n} i^3 = \frac{n^2(n+1)^2}{4}$$

Question 5. [3 points]

Let $y(x) = \int_0^x \frac{1}{1+t^2} dt$. Determine the interval(s) where the function *y(x)* is concave up.

Question 6. [3 points] Find the dimensions of the rectangle of largest area that can be inscribed in a semicircle of radius 2 cm.

Question 7. [6 points] Evaluate the following integrals:

a. $\int_0^1 \sqrt{x} \cdot (1+4x^5) dx$

b. $\int (x^2+1)\sqrt[5]{x^3+3x}\,dx$

c. $\int_{-1}^{1} \sqrt[3]{x^5+\sin(x)}\,dx$

Question 8. [6 points] Sketch the graph of the function if

$$f(x) = \frac{x^3}{x^2-4}, \quad f'(x) = \frac{x^2(x^2-12)}{(x-2)^2(x+2)^2}, \quad f''(x) = \frac{8x(x^2+12)}{(x-2)^3(x+2)^3}$$

Give a detailed sketch of the graph of $f(x)$ by examining its:
(i) domain, (ii)intercept(s), (iii)asymptotes, (iv)Critical numbers, intervals of increase/decrease, local maximum/minimum, (v)intervals of concavity and inflection points. Don't forget to sketch!