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ORIGINAL ARTICLE

Influence of mass on the speed of wheelchair racing

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Abstract Reducing the mass of a racing wheelchair improves the winning time even for relatively small mass reductions over short distances (100 m). In this study, the improvement of speed and winning time on mass reduction was modelled mathematically, the speed profiles of 100 m wheelchair races as well as the rolling resistance and drag area of the athlete–wheelchair system was determined experimentally, and the improvement of the winning time through mass reduction was calculated from the mathematical model and experimental data. For winning times of 30 s, the relative improvement of the winning time is at least equal to the relative reduction of mass. For winning times of 15 s, the relative improvement of the winning time is at least half the relative reduction of mass. A reduction by 1 kg results in an overall improvement of the winning time of about 1–2.3% for winning times of 15–30 s (100-m race). In absolute terms, a mass saving of 1 and 5 kg on a 60-kg system would reduce a 15-s winning time for the 100 m by 0.132 and 0.66 s, respectively, and a 30-s winning time by 0.63 and 2.86 s, respectively. The mass of a wheelchair is the most important parameter for improving the winning time. The influence of the mass on the winning time is 4 or 1.5 times greater than the influence of the rolling friction coefficient, and 4 or 5.5 times greater than the influence of the drag area in a 15 or 30-s race, respectively. These principles should be considered when designing a racing wheelchair.

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1 Introduction

1.1 Wheelchair racing and classification

Wheelchair racing is an integral part of the paralympic sport and covers distances from sprint over middle to long distances (100, 200, 400 and 800 m, 1.5, 5 and 10 km, half marathon and marathon [1]). Athletes of four different classes participate in this sport: class T51, T52, T53 and T54. These classes are defined as equivalent to complete cord injury at cord level C5–6, C7–8, T1–7 and T8–S4, respectively [2]. The loss of various arm and trunk muscle groups due to cord injuries at different levels is reflected in the records of sprint disciplines (Fig. 1, [3–6]). Although wheelchair racing is an athletic (track) discipline, it can be compared to cycling, when considering the design of the equipment.

1.2 Dynamics of wheelchair racing

The sum of all drag forces applied to an engine- or motor-powered vehicle incorporates three terms: the rolling resistance and aerodynamic drag, both of which are parabolic functions of the vehicle speed, and drive-line power loss, which is a linear function [7]. The drive-line power loss does not apply to muscle-powered wheeled sports equipment such as wheelchairs and bicycles.

The free body diagram shown in Fig. 2, and used in the subsequent analysis, comprises the mobile system (athlete + wheelchair) with respect to the environment. Depending on the phase (stroke or recovery) three to four

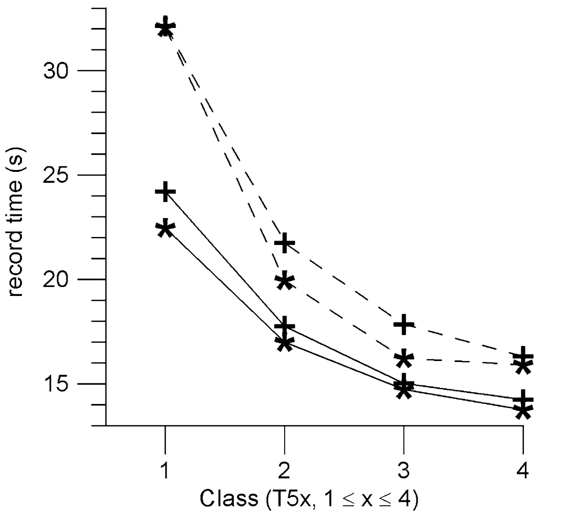

Fig. 1 Athletics records, 100-m wheelchair racing (data from [3–6]) for classes T51, T52, T53, T54; *plus* world championship records, *asterisks* world records, *solid lines* male records, *dashed lines* female records

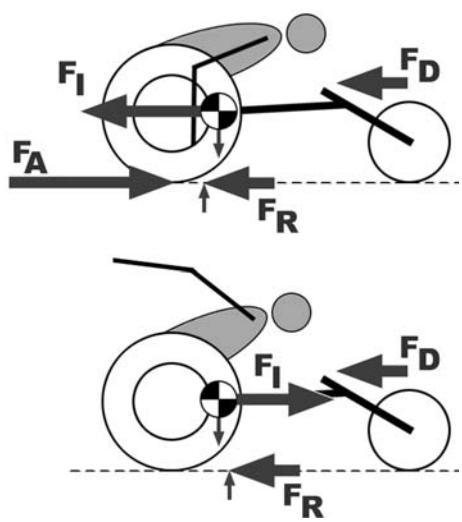

Fig. 2 Free body diagrams of stroke and recovery phase

forces are in equilibrium in the horizontal direction (Fig. 2):

- F_A, the force applied by the ground to the rear wheels;
- inertial (F_I), drag (F_D) and rolling friction (F_R) forces.

The rolling friction is referred to as speed independent or dependent:

(1) Speed independent

The rolling resistance in bicycles on horizontal surfaces is described as

$$F_R = \mu_R mg \tag{1}$$

where μ_R is the coefficient of rolling friction, and m and g are the mass (of the cyclist and the bicycle combined) and the gravitational acceleration, respectively [8–15]. Woude et al. [16] applied the same principle to wheelchairs.

(2) Linear speed dependency

According to Kyle et al. [17] and Kyle and Burke [18], F_R can be described by

$$F_R = c_a + c_b v \tag{2}$$

where v is the cyclist's speed in m s^{-1}, and c_a and c_b are the coefficients of Eq. 2, which are functions of m. According to Grappe et al. [12], however, the second term of Eq. 2, $c_b v$, is insignificant and can thus be neglected, thereby reducing Eq. 2 to Eq. 1.

(3) Non-linear speed dependency

According to several literature sources [19–21], rolling friction calculated from visco-elastic models is generally non-linear. In the specific case of vehicles, as mentioned above, rolling friction is a parabolic function of speed [7, 22]:

$$F_R = c_a + c_b v^2 \tag{3}$$

or

$$F_R = \mu_R mg + k_f mgv^2 \tag{4}$$

where k_f is the coefficient of speed influence on the rolling resistance [7].

In the case of car tyres, $\mu_R = 0.009 - 0.014$, and $k_f = 1 \times 10^{-6} - 1 \times 10^{-5}$ s^2 m^{-2} [7], which are functions of tyre inflation pressure and temperature. The data were obtained from indoor vehicle coast-down tests on a wooden floor [7]. Applying these data to wheelchairs by assuming that $\mu_R = 0.01$, $k_f = 5 \times 10^{-6}$ s^2 m^{-2}, $v = 10$ m s^{-1} (estimated mean speed at the end of a 100-m race at a winning time of 13.76 s; world record of male T54 racers as of 2008, Fig. 1), and a total mass of 80 kg, the first and second terms of Eqs. 3 and 4 are 7.85 and 0.39 N, respectively, where the second term is 4.7% of F_R. 5% is not as insignificant as suggested by Grappe et al. [12] for the second term of Eq. 2.

As visco-elastic models [19–21] deliver non-linear equations, and as car tyres behave non-linearly [7, 22], the rolling friction will be treated as non-linearly depending on the speed throughout this study, according to Eqs. 3 and 4.

The energy losses of bicycles and wheelchairs are due to air drag and rolling friction. According to Grappe et al. [12], it is necessary to minimise the drag area to reach a high speed and to maintain it during long periods. Speeds in cycling are far higher than in wheelchair racing and thus the reduction of the area drag is first priority. Furthermore, the mass of bicycles is limited by rule to be at least 6.8 kg [23]. In contrast to that, the maximal speed in wheelchair racing is up to approximately 10 m s^{-1}, and the IPC rules [24] do not (as of 2008) regulate the mass of wheelchairs.

This is a major difference between cycling and wheelchair racing: general considerations of reducing air drag by optimising the rider's position and the equipment aerodynamics, as well as reducing the rolling friction coefficient apply to both sports. In wheelchair racing, however, the rolling friction force can still be further influenced by reducing the mass of the equipment. Due to mass reduction, the decreasing rolling friction force and larger accelerations generate higher velocities, which in turn increase the drag force. Yet, it is generally accepted that reducing the mass improves the speed.

The aim of this study was thus to

- mathematically model the improvement of speed and winning time on mass reduction (with modelling confined to a straight line without involving cornering),
- determine the speed profiles of 100-m wheelchair racing as well as the rolling resistance and drag area of the athlete–wheelchair system experimentally, and
- calculate the improvement in winning time due to mass reduction from the mathematical model and experimental data.

2 Mathematical modelling

2.1 Energy and force equilibrium of the athlete–wheelchair system

The output energy, or kinetic energy, of the system equals the input energy (produced and supplied by the athlete) minus the non-conservative energy (taken out by air drag and rolling friction).

$$E_{\mathrm{kin}} = E_{\mathrm{input}} - \left(E_{\mathrm{drag}} + E_{\mathrm{friction}}\right). \tag{5}$$

The total kinetic energy E_{kin} in Eq. 5 corresponds to the sum of the kinetic energy of all mobile parts with a mean speed of >0, i.e. the translational energy of the athlete–wheelchair system and the rotational energy of the three wheels. The mobile body segments, e.g. ante- and retroverting trunk and upper arm, extension and flexion of the forearm, undergo oscillation with a mean speed of zero rather than continuous unidirectional motion and thus do not add to the kinetic energy of the system.

Thus,

$$E_{\mathrm{kin}} = \frac{mv^2 + \sum_{i=1}^{3} I_i \omega_i^2}{2} \tag{6}$$

where I_i and ω_i are the moment of inertia and the angular velocity of wheel i, respectively.

The kinetic energy E_{kin} equals the inertial force F_{I} integrated with distance.

$$E_{\mathrm{kin}} = \int_{x_1}^{x_2} F_{\mathrm{I}} \mathrm{d}x. \tag{7}$$

Combining Eqs. 6 and 7, considering that $v = \omega r_i$ and $a = \alpha r_i$ (where r_i is the radius of wheel i), and solving for F_{I}:

$$F_{\mathrm{I}} = a\left(m + \sum_{i=1}^{3} \frac{I_i}{r_i^2}\right). \tag{8}$$

Note that m is the mass of the athlete–wheelchair system, whereas I is the mass of one wheel times its radius of gyration squared. The equivalent mass M causing F_{I} when accelerated is:

$$M = m + \sum_{i=1}^{3} \frac{I_i}{r_i^2}. \tag{9}$$

The force equilibrium of the stroke phase (acceleration, Fig. 2a) is:

$$F_{\mathrm{A}} + F_{\mathrm{I}} + F_{\mathrm{D}} + F_{\mathrm{R}} = 0, \tag{10}$$

$$F_{\mathrm{A}} + Ma + C_{\mathrm{D}} v^2 + mg\mu_{\mathrm{R}} + mgk_f v^2 = 0 \tag{11}$$

where C_{D} is the lumped coefficient of drag, and v and a are the instantaneous velocity (dx/dt) and acceleration (d^2x/dt^2), respectively. The lumped coefficient of drag C_{D} corresponds to 0.5 $\rho c_{\mathrm{D}} A$, the product of air density ρ, coefficient of drag c_{D} and the frontal area A. F_{R} is expressed according to Eq. 4. Eqs. 10 and 11 apply to wheelchairs on a straight course and do not incorporate cornering forces.

Eq. 11 can be re-written as

$$F_{\mathrm{A}} + Ma + c_1 v^2 + c_2 = 0 \tag{12}$$

where

$$c_1 = C_{\mathrm{D}} + mgk_f \tag{13}$$

and

$$c_2 = mg\mu_{\mathrm{R}}. \tag{14}$$

F_{D} and F_{R} are responsible for the energy losses of the athlete–wheelchair system. In a loss-free environment, both F_{D} and F_{R} are zero.

2.2 Simulation of two races at the same input energy, but at different masses

The key to calculating the improvement of the winning time due to mass reduction is to simulate two races, which have exactly the same input energy profile, i.e. the same input energy is supplied to the system in race 1 and 2 at any stage of the race, but the mass of the athlete–wheelchair system is smaller in race 2. This principle can only be achieved by simulation rather than experimentally, as an athlete is unable to perform two races at exactly the same input energy profile.

The same input energy before and after changing the mass results from keeping the distance (x, dx, d^2x) and the sum of inertial and dissipative forces, and thus keeping the integral of force with distance, the work, the same. The time derivatives of the distance, however, change with the mass m: the lighter the wheelchair, the faster is the athlete (larger velocity v and acceleration a). Thus, dt changes with m, since dx and d^2x remain constant.

2.3 Environment with drag and friction

Comparable to the approach applied in a loss-free environment (see above), two races with identical input energy of the athlete, but with different masses of the athlete–wheelchair system were simulated according to Eq. 10. Thereby, the distance (x, dx, d^2x) and the sum of inertial and dissipative forces were kept the same, leaving the integral of force with distance, the work, unchanged:

$$-F_{\mathrm{A}} = F_{\mathrm{I1}} + F_{\mathrm{D1}} + F_{\mathrm{R1}}, \tag{15}$$

$$-F_{\mathrm{A}} = F_{\mathrm{I2}} + F_{\mathrm{D2}} + F_{\mathrm{R2}}, \tag{16}$$

$$F_{\mathrm{I1}} + F_{\mathrm{D1}} + F_{\mathrm{R1}} = F_{\mathrm{I2}} + F_{\mathrm{D2}} + F_{\mathrm{R2}}. \tag{17}$$

The input, generated by the athlete, equals the output (kinetic energy) plus the non-conservative energy, lost through drag and friction. The frame of the wheelchair is considered sufficiently stiff such that the non-recoverable energy, which goes into the deformation of the frame, is negligible.

Comparable to the approach applied in a loss-free environment, dt changes with m and M, since dx is a constant.

Equating inertial, drag and friction forces at two different masses, m_1 and m_2, and M_1 and M_2: (corresponding to the actual and equivalent mass of the athlete–wheelchair system in race 1 and 2, respectively), according to Eq. 17:

$$\begin{aligned} M_1\frac{\mathrm{d}^2x}{\mathrm{d}t_1^2} + \left(C_{\mathrm{D}} + m_1 g k_f\right)\frac{\mathrm{d}x^2}{\mathrm{d}t_1^2} + m_1\mu_{\mathrm{R}}g \\ = M_2\frac{\mathrm{d}^2x}{\mathrm{d}t_2^2} + \left(C_{\mathrm{D}} + m_2 g k_f\right)\frac{\mathrm{d}x^2}{\mathrm{d}t_2^2} + m_2\mu_{\mathrm{R}}g. \end{aligned} \tag{18}$$

Solving for dt_2 yields

$$\mathrm{d}t_2 = \sqrt{\frac{\mathrm{d}t_1^2\left[M_2\mathrm{d}^2x + \left(C_{\mathrm{D}} + m_2 g k_f\right)\mathrm{d}x^2\right]}{M_1\mathrm{d}^2x + \left(C_{\mathrm{D}} + m_1 g k_f\right)\mathrm{d}x^2 + \mathrm{d}t_1^2\mu_{\mathrm{R}}g(m_1 - m_2)}}. \tag{19}$$

Numerical integration of Eq. 19 delivers t_2 as a function of primarily m_1 and m_2 (and M_1 and M_2), as well as secondarily of t_1 and distance x.

The distance x remains the same in both races, independent of the mass. Thus, dx and d^2x are not influenced by the mass either; however, they are a function of the instantaneous velocity and acceleration.

The variables in Eq. 19 are:

Dependent variable: dt_2 (which is a non-linear function of the constant dt_1)
Independent variables: m_1 and m_2, and M_1 and M_2 (which are constant throughout race 1 and 2, respectively).

The constants in Eq. 19 are: dt_1, C_{D}, μ_{R}, k_f, g, and dx, dx^2, d^2x.

dx, dx^2 and d^2x, however, are functions of velocity v and acceleration a, which change throughout the race:

$$\mathrm{d}x = v_1\mathrm{d}t_1, \tag{20}$$

$$\mathrm{d}x^2 = (v_1\mathrm{d}t_1)^2, \tag{21}$$

$$\mathrm{d}^2x = a_1\mathrm{d}t_1^2. \tag{22}$$

According to Eq. 22, d^2x is negative when decelerating. In such cases, the solution of Eq. 19, dt_2, may become infinite and finally imaginary, if the denominator of Eq. 19 is zero or negative. This case, however, will never occur as long as the winning time actually improves when reducing the mass.

If v_1 is constant (steady state), $a_1 = 0$ and Eq. 19 becomes

$$\mathrm{d}t_2 = \sqrt{\frac{\mathrm{d}t_1^2(C_{\mathrm{D}} + m_2 g k_f)\mathrm{d}x^2}{(C_{\mathrm{D}} + m_1 g k_f)\mathrm{d}x^2 + \mathrm{d}t_1^2\mu_{\mathrm{R}}g(m_1 - m_2)}}. \tag{23}$$

3 Experimental

3.1 Instrumentation

To obtain data for simulating the influence of mass, the wheelchairs of three athletes were instrumented.

An instrumentation method, comparable to the one developed by Moss et al. [25], was applied. However, instead of using an optical encoder, a ferrite motor (ACC337, Maxx Products International, Lake Zurich, IL 60047, USA) was selected and equipped with a toy wheel (diameter 5 cm), which can be connected to the frame of the wheelchair in various ways (Fig. 3). The motor is driven by the rear wheels of the racing chair through the toy wheel, and the motor's back emf (electromotive force) is measured and stored by a data logger (DI-710-ULS, DATAQ Instruments, Akron, OH, USA) at 1.2 kHz. The energy losses in the small ferrite motor are negligible compared with those due to aerodynamic drag and rolling resistance.

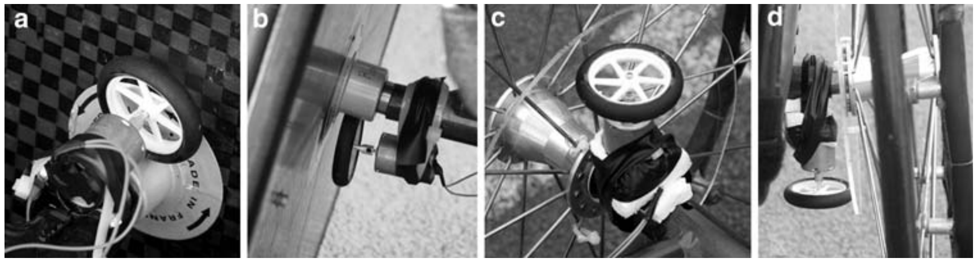

Fig. 3 Instrumentation. **a**, **b** wheelchair of athlete 1, **c**, **d** wheelchair of athletes 2 and 3

3.2 Calibration

The data stored in the data logger were uploaded with WINDAQ Waveform Browser (DATAQ Instruments, Akron, OH, USA). The first calibration of all wheelchairs was carried out on a conventional treadmill for exercising at velocities of up to 2 m s^{-1}, to test the linearity of the signal. The second calibration test was performed on the track by covering a distance of exactly 100 m at various speeds (2–6 m s^{-1}). As the frequency of the motor's noise is a function of the speed, and as the maximal push frequency is close to the noise at lower speeds, a second-order Savitzky-Golay filter was applied thrice at a window width of 501 data points in Matlab (by MathWorks, Natick MA, USA). The coefficient for converting the back emf to speed in m s^{-1} resulted from integrating the signal of the second calibration tests, which must deliver exactly 100 m when multiplied by the coefficient. The resulting mean coefficient produced an error of approximately 1.21% in the unfiltered signal, and about 0.09% in the filtered one.

3.3 Athletes and wheelchairs

The wheelchairs of three athletes were instrumented as described above. The athletes performed their regular training programme, preparing for the Beijing 2008 Paralympic Games or for qualification for the said games.

The data of the athletes, wheelchairs and athlete–wheelchair system are shown in Table 1.

3.4 Racing

After warming up, all three athletes performed two races each over a distance of 100 m. The athletes were instructed to consider each race as a competition rather than as a training race. All races including the subsequent coast-down tests were performed on a granulated rubber track on a windless day (no noticeable air movement).

3.5 Coasting down

The non-conservative energy of the system and the dissipative forces (drag and rolling resistance) were analysed by coast-down tests, with the athlete in the typical aerodynamic racing position and the wheelchair accelerated to a certain speed, then allowed to coast down until it came to a standstill.

Coasting down experiment is a standard procedure for assessing the dissipative forces and their coefficients, and was applied by several authors to bicycles [14], wheelchairs [15, 30], automobiles [7, 31] and skiers [32].

The wheelchair coast-down experiments were performed at different speeds (2.5–6 m s^{-1}) in opposite directions on the track. The two opposite directions were required to eliminate any possible influence of track inclination and unidirectional air movement. All three athletes performed seven coast downs each, after racing. The data of coasting were fitted by a function, derived from the following non-linear differential equation:

$$M\frac{\mathrm{d}v}{\mathrm{d}t} + c_1 v^2 + c_2 = 0 \tag{24}$$

where c_1 and c_2 are defined in Eqs. 13 and 14, respectively. The solution of Eq. 24 is

$$v_t = \sqrt{\frac{c_2}{c_1}}\tan\left[\tan^{-1}\left(v_0\sqrt{\frac{c_1}{c_2}}\right) - t\frac{\sqrt{c_1 c_2}}{M}\right] \tag{25}$$

where v_t is the instantaneous velocity decreasing with time when coasting down, and v_0 is the initial condition at impending deceleration. Equation 25 returns the coefficients c_1 and c_2, when fitting the data in Matlab (by MathWorks, Natick MA, USA). The coefficient c_2 equals the speed-independent rolling resistance $mg\mu_{\mathrm{R}}$, whereas the coefficient c_1 combines the lumped drag coefficient C_{D} and the speed-dependent rolling resistance mgk_f. If the overall rolling resistance is speed independent, k_f becomes zero and c_2 equals C_{D}. In the absence of data on k_f of wheelchair tyres, k_f was assumed to be between 0 and 1 $\times$ 10^{-5} s^2 m^{-2} (data of automobile tyres [7]), included in the sensitivity analysis.

4 Race simulation

Theoretically, the speed profile is expected to approach an asymptote. Initially, the drag force is zero and all energy input to the system can be converted to acceleration;

Table 1 Data of athletes and wheelchairs; wheelchair components mass taken from specifications [26–29] and measurements; moment of inertia calculated from specifications and measurements

Athlete number	1	2	3
Sex, age, and class	Male, 19, T53	Female, 31, T52	Male, 36, T52
Athlete mass (kg)	49.3	45.9	65.1
Mass of athlete–wheelchair system (kg, including the measuring equipment, 1.2 kg)	58.7	53.2	72.2
Wheelchair frame model and mass (kg)	X-Limit (5.7)	X-Limit S (5.2)	X-Limit S (5.2)
Front wheel model, mass (kg) and moment of inertia (kg m^2)	Corima Aero (0.5 kg, 0.08 kg m^2)	Corima Aero (0.5 kg, 0.08 kg m^2)	Sun Components (0.25 kg, 0.051 kg m^2)
Rear wheel model, mass (kg) and moment of inertia (kg m^2)	Corima disc (1 kg, 0.25 kg m^2)	Panaracer Ultima track (0.2 kg, 0.085 kg m^2)	Panaracer Ultima track (0.2 kg, 0.085 kg m^2)
Total moment of inertia (kg m^2)	0.58	0.25	0.221
Wheelchair mass (kg)	8.2	6.1	5.9
Equivalent mass M (kg), according to Eq. 9	60.1	53.9	72.8

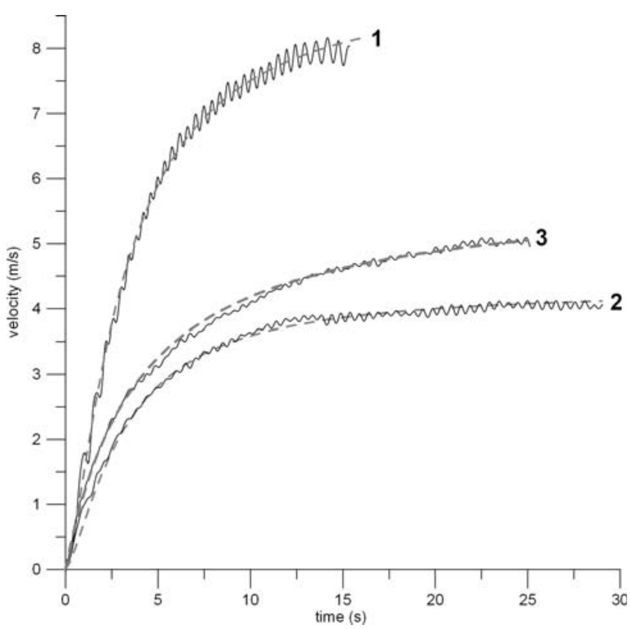

Fig. 4 Speed profiles of the three racers; *dashed curve* fit function according to Eq. 26, *1*, *2*, *3* athlete's number

however, once all energy input is used up to compensate the dissipative forces, the speed remains constant.

In reality, the speed might drop at the end of a 100-m race due to fatigue. The first step of modelling the speed curves is to identify a suitable fit curve.

An ideal fit curve is a reciprocal function of the form

$$v = \frac{dx}{dt} = \frac{A}{B} - \frac{A}{t^C + B} \tag{26}$$

Table 2 Winning times and corresponding coefficients A

Winning time (s)	A, athlete 1 and 2 ($B = 5$, $C = 1.4$)	A, athlete 3 ($B = 4$, $C = 1$)
15	49.65	45.624
20	34.132	31.17
25	25.844	23.425
30	20.7335	18.657

where the coefficients A, B and C are determined numerically by fitting the original speed profile in Matlab. A is a general multiplier.

Athletes 1 and 2 shared the same exponent C (1.4) and coefficient B (5), whereas C and B were different in athlete 3 (1 and 4, respectively). This difference seems to be an expression of the personal racing style and strategy, mirrored in the coefficients B and C, whereas coefficient A, the general multiplier, depends on the maximal speed and thus on the winning time.

The speed profiles and the fit functions are shown in Fig. 4. The coefficient A was calculated for different simulated winning times (15–30 s according to Fig. 1) in the two racing styles (Table 2). The difference in racing style is shown in Fig. 5.

After calculating the speed from Eq. 26 and Table 2, and the acceleration and distance numerically with time, the numerical integration of Eq. 19 delivers the new time axis t_2 after reducing m_1 to m_2 and M_1 to M_2, and thus the new winning time.

The improvement of the winning times was calculated for reductions of the mass of up to 10 kg, for total masses

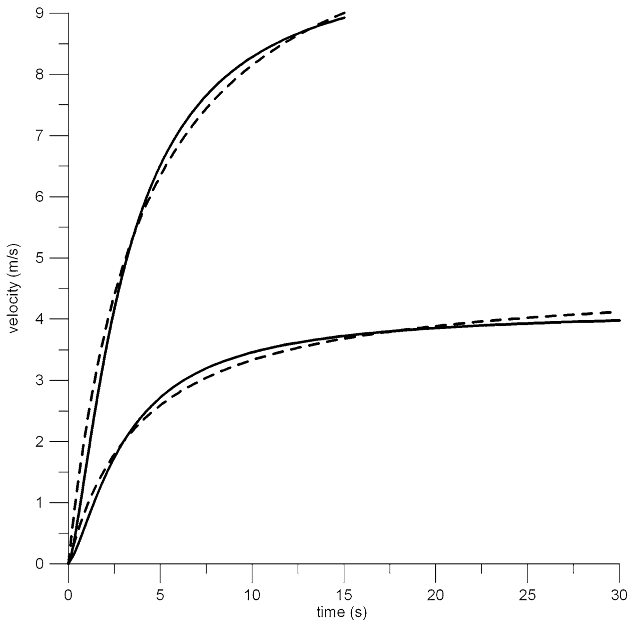

Fig. 5 Differences in racing style for winning times of 15 and 30 s. *Solid lines* $B = 5$, $C = 1.4$, *dashed lines* $B = 4$, $C = 1$, according to Table 2 and Eq. 26

of 50, 60 and 70 kg, and for original winning times of 15, 20, 25 and 30 s.

The kinetic energy of the athlete–wheelchair system and the input energy produced by the athlete were calculated from Eqs. 6 and 5, respectively, resulting in simulated energy profiles of different winning times.

The influence of changing the mass, and thus the speed, on the efficiency of the system was assessed as well. Additionally, a sensitivity analysis was carried out to assess the relative change of the improvement of the winning time when changing C_D and μ_R by maximally 10%, and k_f from 0 to 1×10^{-5} s^2 m^{-2} (±100%). This sensitivity analysis addresses how sensitive the model is to changes in the value of the input parameters derived from coast-down experiments. In this analysis, C_D and μ_R are equal in both races, yet different from the data given in Table 3, whereas the mass changes from race 1 to race 2.

Furthermore, the influence of the three key parameters, mass, C_D and μ_R, on the winning time was determined by changing only one of these parameters from race 1 to race 2. For this purpose, Eq. 19 was modified accordingly to account for different C_D and μ_R in both races (C_{D1}, C_{D2}, μ_{R1}, μ_{R2}).

$$\mathrm{d}t_2 = \sqrt{\frac{\mathrm{d}t_1^2[M_2\mathrm{d}^2x + (C_{D2} + m_2gk_f)\mathrm{d}x^2]}{M_1\mathrm{d}^2x + (C_{D1} + m_1gk_f)\mathrm{d}x^2 + \mathrm{d}t_1^2\mu_{R}g(m_1\mu_{R1} - m_2\mu_{R2})}}. \quad (27)$$

5 Results

5.1 Experimental results

The coefficients c_1 and c_2 of Eq. 24 as well as the drag area c_DA (m^2) at different speed-dependent rolling friction coefficients k_f, and the speed-independent rolling friction coefficient μ_R, are listed in Table 3 for all three athletes and wheelchairs.

Figure 4 shows the speed profiles of the three athletes including curve fitting with Eq. 26. The oscillations of the recorded speed are due to the acceleration in the stroke phase and deceleration due to energy loss in the recovery phase (Fig. 2).

Figure 6 shows the speed profile of a typical coast-down experiment as well as the curve fitting with Eq. 25.

5.2 Energy profiles

The energy profiles at a winning time of 15 and 30 s are shown in Fig. 7. The total input energy at the end of the two races is 3,540 and 1,240 J for winning times of 15 and 30 s, respectively. Thus, a wheelchair racer, who finishes the 100 m in 30 s, requires only one-third of the energy of a 15-s sprint. The energy of speed-dependent rolling resistance is minimal and thus negligible. The energy dissipated by the air drag is larger in the 15-s race due to higher speed. The energy lost by the rolling friction is generally higher than the one lost by the air drag, at least on the rubber track where the experiments were conducted.

Table 3 Drag areas and rolling resistance calculated from the coast-down experiments

Athlete number	1	2	3
c_1, from Eq. 25	0.0815 ± 0.0153	0.0867 ± 0.0136	0.0811 ± 0.0162
Drag area $c_D\,A$ (m^2) from Eq. 13 at $k_f = 0$	0.1358	0.1445	0.1352
Drag area $c_D\,A$ (m^2) from Eq. 13 at $k_f = 5 \times 10^{-6}$ s^2 m^{-2}	0.1310	0.1402	0.1293
Drag area $c_D\,A$ (m^2) from Eq. 13 at $k_f = 1 \times 10^{-5}$ s^2 m^{-2}	0.1262	0.1358	0.1234
c_2, from Eq. 25	5.970 ± 0.657	6.028 ± 0.638	8.252 ± 0.844
μ_R, from Eq. 14	0.0104	0.0116	0.0117

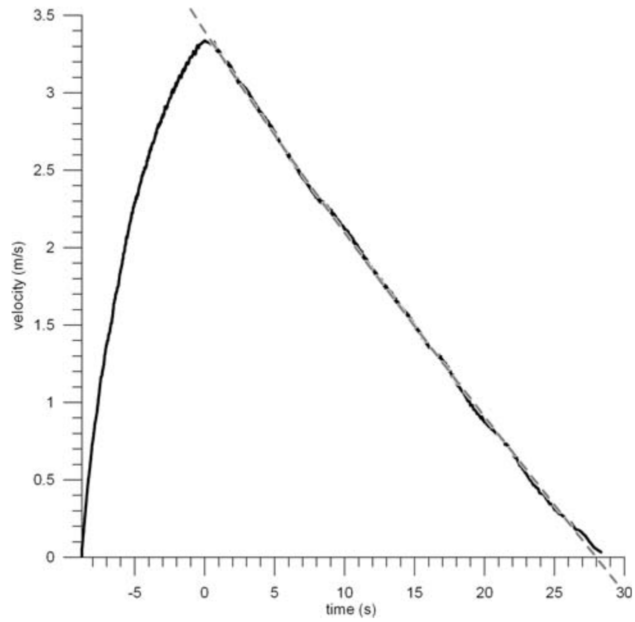

Fig. 6 Coast-down experiment. Initial speed at beginning of deceleration: 3.4 m s^{-1}. *Solid curve* speed against time, *dashed curve* data fitted with Eq. 25

5.3 Results of race simulation

The improvement of the winning time against the mass reduction is shown in Fig. 8.

The rule of the thumb for estimating the relative improvement in winning time Δt from the relative mass reduction is, according to Fig. 8b,

$$\frac{\Delta t}{t_{\mathrm{w}}} \geq \frac{\frac{\Delta M}{M}}{3 - \frac{t_{\mathrm{w}}}{15}} \tag{28}$$

where t_{w} is the winning time. Equation 28 defines the relative improvement of the winning time as a function of both the relative mass reduction and the absolute winning time. Thus, the relative improvement of the winning time is at least half the relative mass reduction if $t_{\mathrm{w}} = 15$ s, and at least equal to the relative mass reduction if $t_{\mathrm{w}} = 30$ s.

In absolute terms, a typical 100-m race of 15, 20, 25 and 30 s duration would be reduced in time by around 0.13, 0.225, 0.375 and 0.61 s, respectively, for each kilogram mass reduction.

5.4 Sensitivity analysis

(1) Sensitivity of the model to changes in the value of the input parameters:

Changing the constants (C_{D}, μ_{R}) by 10% for a mass reduction Δm of 1 or 5 kg resulted in minimal changes (low end) of $\Delta t/t$ at a winning time t_{w} of 15 s, and maximal changes (high end) at $\Delta m = 1$ kg and $t_{\mathrm{w}} = 30$ s (Fig. 9). Increasing the mass reduction reduces the changes of $\Delta t/t$. The relative change in the winning time is generally smaller than the relative change of the constants and thus less sensitive (Fig. 9).

Reducing C_{D} in addition to the mass increases Δt compared to unchanged C_{D}. Reducing μ_{R} in addition to the mass, however, decreases Δt compared to unchanged μ_{R} (Fig. 9). The latter result is obvious, as the smaller the

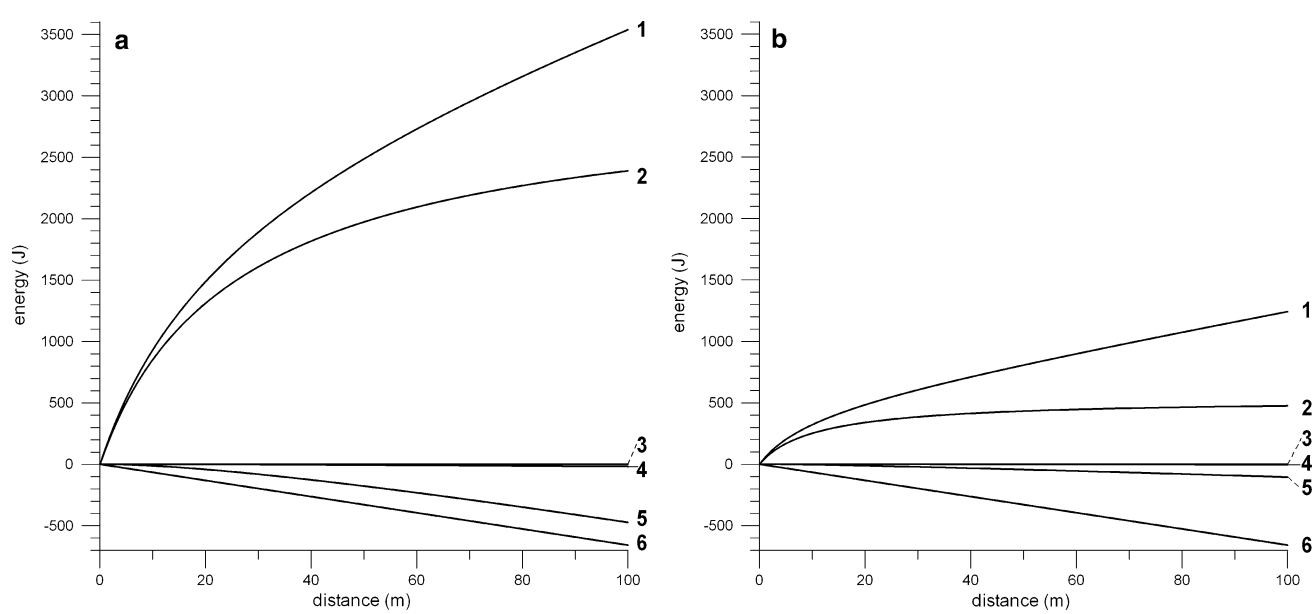

Fig. 7 Energy profiles at a total mass of 60 kg and winning times of 15 s (**a**) and 30 s (**b**) against distance. *1* input energy, *2* output energy (kinetic energy), *3* 0 J, *4* energy of speed-dependent rolling resistance, *5* energy of air drag, *6* energy of speed-independent rolling resistance

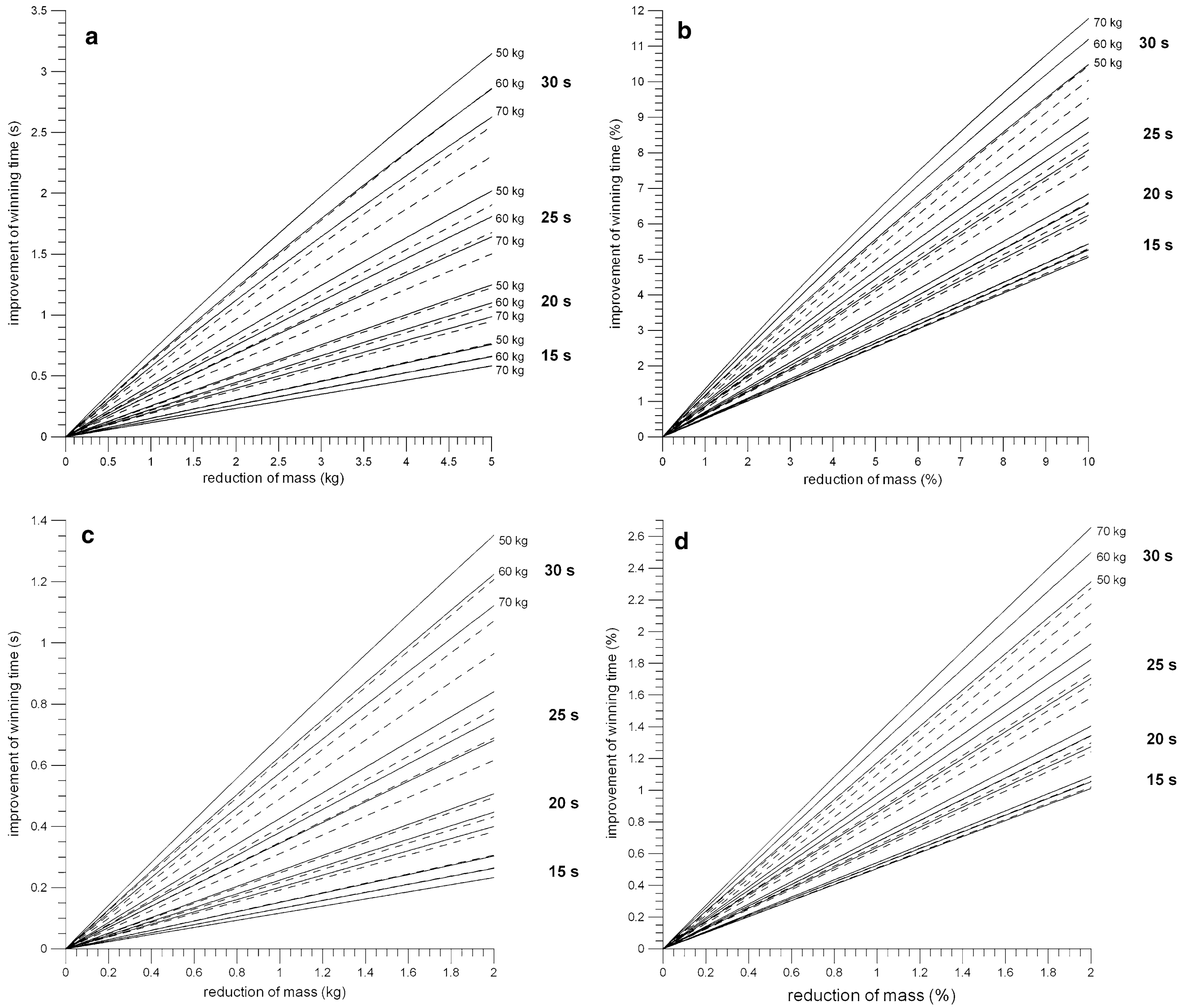

Fig. 8 Improvement of winning time against reduction of mass. **a**, **c** Absolute values, **b**, **d** relative values (in %). **a**, **b** Larger mass reductions, **c**, **d** smaller mass reductions; *solid lines* $B = 5$, $C = 1.4$, *dashed lines* $B = 4$, $C = 1$, according to Table 2 and Eq. 26

friction, the smaller is the effect of mass reduction, or, if the friction were zero, there would be no effect of the mass of a wheelchair.

Changing k_f by $\pm 5\ \mathrm{s}^2\ \mathrm{m}^{-2}$, i.e. by 100%, affects the improvement of the winning time by just 1.3% at 30 s (which is approximately 9 ms) and 0.7% at 15 s, if the mass reduction is 1 kg for a total mass of 50 kg. The effect of the speed-dependent rolling resistance k_f is thus negligible, as seen in the energy profiles (Fig. 7).

(2) Relative importance of system parameters

The three key parameters affecting the winning time are the mass of the wheelchair, the lumped drag coefficient C_D and the rolling resistance μ_R. Figure 10 shows that the mass is the most important parameter. On average, the influence of the mass on the winning time is 4 times greater than the influence of C_D and μ_R for a winning time of 15 s, and 5.5 or 1.5 times greater than the influence of C_D or μ_R, respectively, for a winning time of 15 s. The least important parameter is the aerodynamic drag, which approximately equals the influence of μ_R for a 15-s race, but which is about four times smaller than the influence of μ_R for a 30-s race. The latter result is obvious, considering the lower speed.

5.5 System efficiency

The efficiency of the racer–wheelchair system, the ratio of energy output (kinetic energy) to energy input (produced by the athlete), is approximately 0.9 at the beginning of the

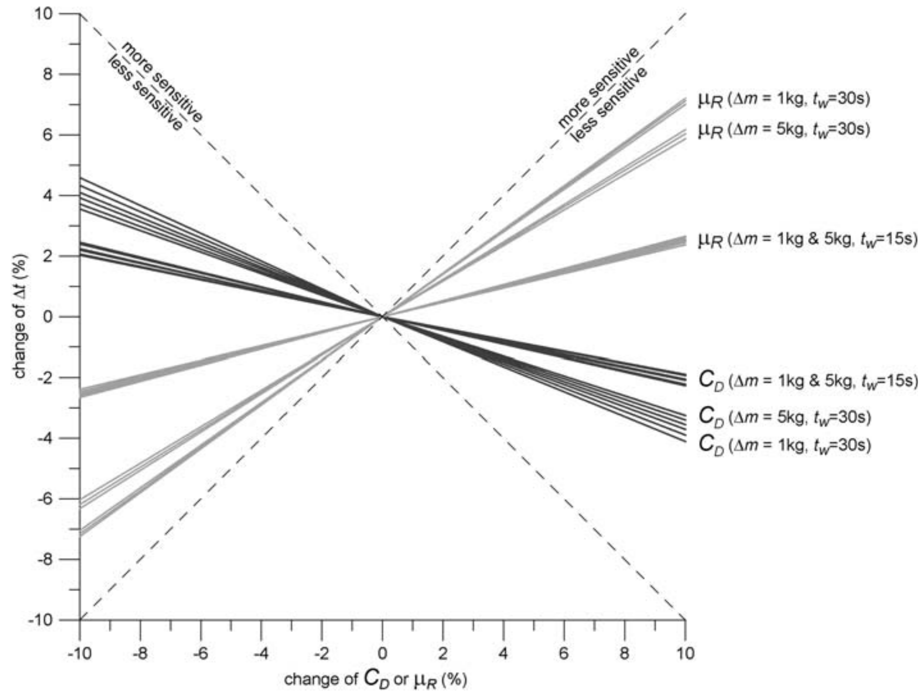

Fig. 9 Sensitivity analysis; change of Δt (%) is less sensitive to change of C_D or μ_R

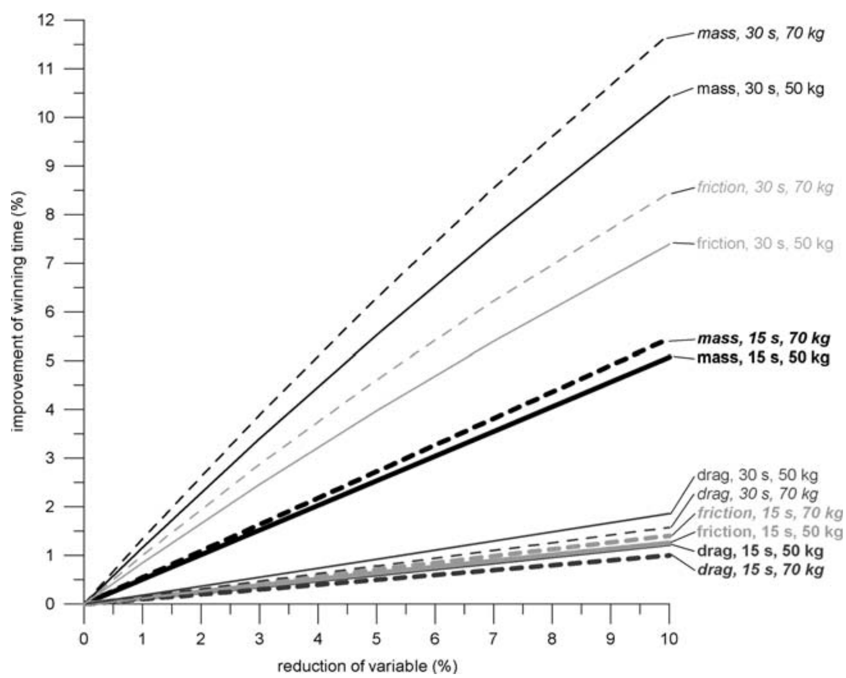

Fig. 10 Relative importance of system parameters; improvement of winning time against reduction of mass, C_D or μ_R

race, and drops to approximately 0.67, 0.56, 0.46 and 0.38 for winning times of 15, 20, 25 and 30 s. Although the average speed and thus the drag force is smaller in a 30-s race than in a 15-s one, the rolling friction force is independent of the speed (as the effect of the speed-dependent rolling resistance k_f is negligible), which explains the small efficiency of 0.38 in a 30-s race (Fig. 7).

As expected, the efficiency of the system improves with mass reduction (Fig. 11), as the energy loss from rolling friction is proportional to the mass. The improvement of the efficiency is about 10% for a 30-s race after reducing the mass by only 2 kg (which is 2.9 and 4% of a total system mass of 70 and 50 kg, respectively). Only for smaller total masses (50 kg), shorter winning times and higher mass reductions does the efficiency decrease slightly (Fig. 11b). For example, the relative change of efficiency is negative for winning times smaller than 15.6 s for any mass reduction, and at 16 and 16.5 s races for mass

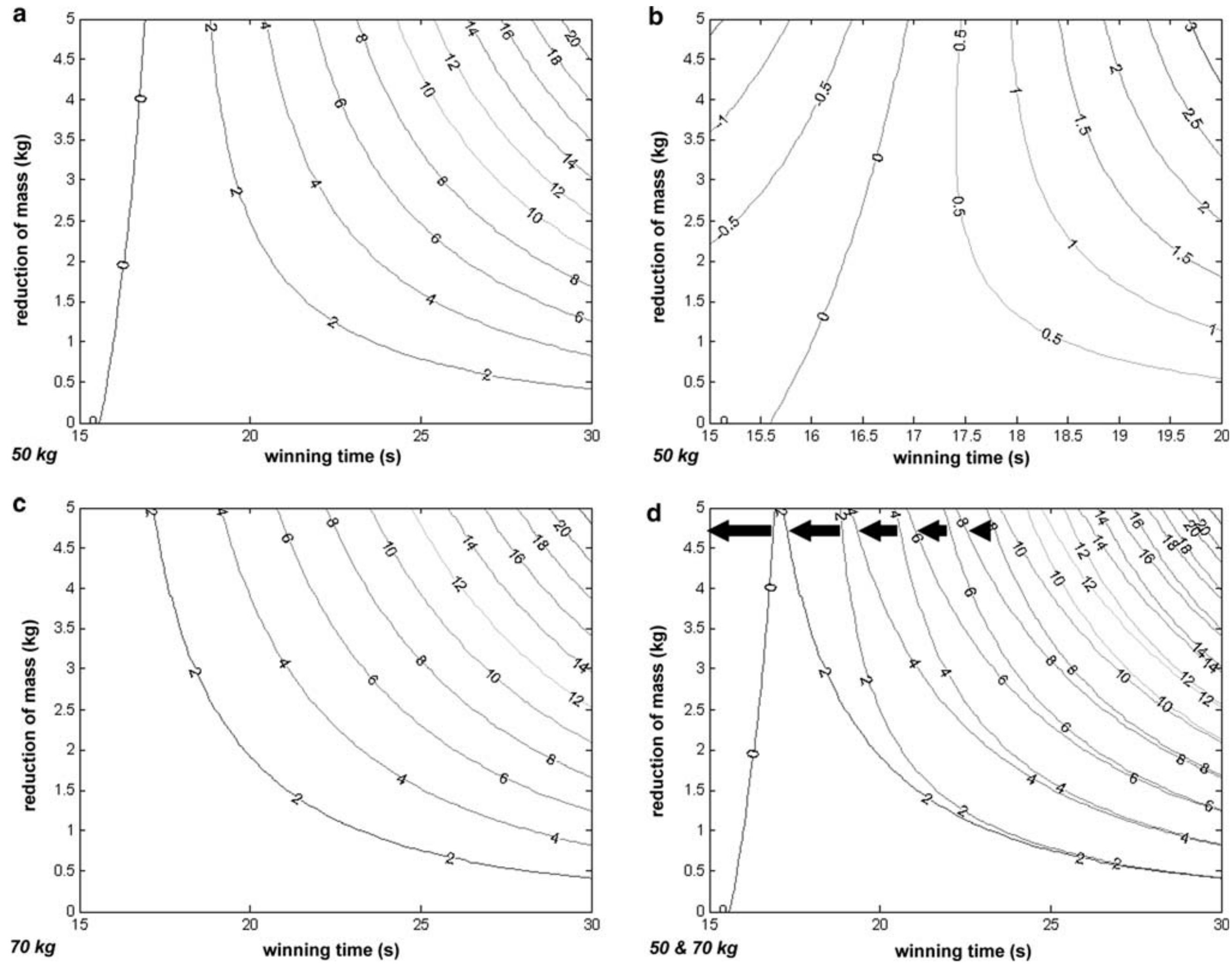

Fig. 11 Contour plot of improvement of efficiency in % (contour labels) as a function of mass reduction and winning time. Negative contour labels indicate that the efficiency drops after reducing the mass. **a** Total mass: 50 kg, winning time from 15 to 30 s; **b** total mass: 50 kg, winning time from 15 to 20 s; **c** total mass: 70 kg, winning time from 15 to 30 s; **d**: comparison of 50 and 70 kg (leftward shift at higher mass reductions when increasing the total mass)

reductions greater than 1 and 2.7 kg, respectively (Fig. 11b). This seems counterintuitive; however, minimising the energy loss through mass reduction allows the athlete to produce higher velocities (at the same energy input). The drag force increases with the velocity squared, and at higher velocities (15 s race), the energy lost by increased air drag outweighs slightly the energy saved from reducing the mass.

6 Discussion

As pointed out in Sec 1, the IPC does not limit the mass of a racing wheelchair (IPC, rule 159 [24]), whereas the UCI [23] restricts the mass of a racing bicycle to $m \geq 6.8$ kg. In the light of the results of this study, reducing the mass of a wheelchair does make sense and provides an edge over heavier wheelchairs. A reduction by 1 kg results in an overall improvement of the winning time of about 1–2.3% for winning times of 15–30 s (100-m race). The mass reduction directly affects F_R and F_I.

The mass of a wheelchair can be reduced easily by changing the material of the frame to aluminium-based alloy [26], carbon fibre composites [33] or MMC (metal matrix composites, e.g. aluminium–boron fibre composites). As the energy that goes into deformation of the frame and cannot be recovered is lost, a stiff frame is preferred. Thus, lower modulus aluminium-based alloy frames require larger cross sections and consequently more mass to increase its stiffness and thereby decrease the

Fig. 12 Typical wheelchair racing position; the *arrow* indicates the instrumentation device

deformation of the frame. However, Siebert [34] reduced the mass of a rehabilitation wheelchair to 5.9 kg by changing the material to carbon fibre composites.

In contrast to racing bicycles, the aerodynamic drag and the rolling resistance of racing wheelchairs, analysed in this study, were smaller and larger, respectively.

The rolling friction coefficient of racing bikes and time trial bikes is 0.0025–0.003 [8, 35, 36] and 0.0057 [14], respectively. The speed-independent rolling friction coefficient of automobiles is 0.009–0.014 [7]. Hoffman et al. [15] reported a rolling friction coefficient of 0.0013 in a racing wheelchair on linoleum.

The rolling friction coefficient μ_R obtained from the present study was 0.01–0.012, which is in the range of car tyres and on average 8.5 times higher than the one reported by Hoffman et al. [15]. This coefficient, however, depends on several factors, such as temperature, inflation pressure, and properties of tyre and surface. The most decisive factor of the diverging rolling resistance of bicycles and wheelchairs is the surface properties, as wheelchair racing is performed on athletic tracks, and, specifically in this study, on a granulated rubber surface. This type of track surface is softer than standard surfaces for various cycling disciplines and thus explains the relatively higher rolling resistance in this study on wheelchairs.

The effective frontal area or drag area $c_D A$ is larger in bicycles, depending on the position, amounting to 0.19–0.39 m^2 [8–10, 12, 14, 35–38].

The drag area obtained in this study was 0.125–0.145 m^2, which is 1.5–3 times smaller than that for a racing bicycle. The smaller frontal area of wheelchair racers arises as the parathlete's legs are constantly flexed (Fig. 12), whereas the legs of a cyclist alternately flex and extend.

Advances in wheelchair technology and the development of lighter wheelchairs might cause the ICP to regulate the minimal mass of a racing wheelchair comparable to bicycles [23]. Regulations such as these are intended to prevent any "unfair" technological advantage over other athletes, when technology for mass reduction although self-limiting is driven by finance, giving an advantage to economically developed nations, which can afford advanced racing wheelchairs instead of relying on self-made equipment [39].

7 Conclusions

Reducing the mass of a racing wheelchair is potentially advantageous even for small mass reductions at short distances (100 m). For winning times of 30 s, the relative improvement of the winning time is at least equal to the relative reduction of mass. For winning times of 15 s, the relative improvement in the winning time is at least half the relative reduction of mass. A reduction by 1 kg results in an overall improvement of the winning time of about 1–2.3% for winning times of 15–30 s (100 m race). In absolute terms, a mass saving of 1 and 5 kg on a 60-kg system would reduce a 15-s winning time for the 100 m by 0.132 and 0.66 s, respectively, and a 30-s winning time by 0.63 s and 2.86, respectively.

The mass of a wheelchair is the most important parameter, of the three studied, for improving the winning time. The influence of the mass on the winning time is 4 or 1.5 times greater than the influence of the rolling friction coefficient and 4 or 5.5 times greater than the influence of the lumped drag coefficient in a 15 or 30-s race, respectively.

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