An Artificial Decision Maker for Comparing Reference Point Based Interactive Evolutionary Multiobjective Optimization Methods

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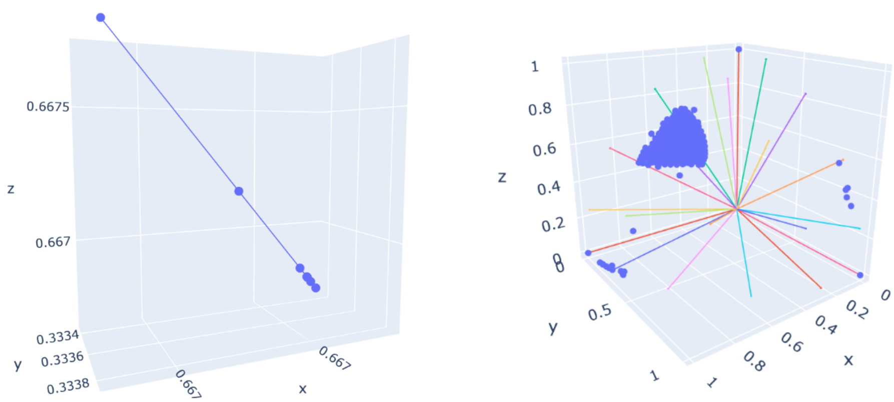
Supplementary Document

Because of the page limitation of the main paper, in this supplementary document, we describe additional results of the computational experiments.



(a) Generated reference points. (b) Composite front.

Fig. S1: Search behaviour of the ADM in the decision phase.

In the main paper, we showed the behaviour of the ADM in the learning phase in Section 4.1. Here we visualize the behaviour in the decision phase. The ADM was run with iRVEA and iNSGA-III using 50 generations per iteration for both algorithms on the three-objective DTLZ2 problem. Figure S1a shows the generated reference points as dots and the search path taken by the ADM as a continuous line. As seen in Figure S1a, the ADM converged in the ROI explored to refine the solutions there. Figure S1b shows the composite front obtained by the two algorithms after 20 iterations.

In Section 4.3, we described part of the results obtained by applying the ADM with iRVEA and iNSGA-III. To complete this, Tables S1 and S2 depict the results for all numbers of objectives in DTLZ1-4 problems using 50 and 100 generations per iteration, respectively. As can be observed, as the number of generations increased per iteration, iRVEA performed significantly better than iNSGA-III in most of the instances of the DTLZ1 problem, at both the learning and the decision phases. For the DTLZ2 and DTLZ4 problems, the results for 50 and 100 generations indicate that, in general, iNSGA-III reached better cumulative metric values than iRVEA for all numbers of objectives, regardless the phase of the interactive solution process. In the DTLZ3 problem, our ADM enabled us to see that, when assigning a higher number of generations per iteration, iNSGA-III seemed to win for the instances with a smaller number of objectives, while iRVEA got better results as the number of objectives increased.

We can say that the fact that our ADM assigns the same computation resources to all algorithms compared enables detecting distinct performances of the algorithms. Depending on the needs and the purpose of the experiment, the ADM can be correspondingly configured as discussed in the main paper.

One should note that the findings of the analysis should not be generalized. The objective of this consideration was to demonstrate how the ADM can be applied, not to find any winner among the algorithms compared.

Table S1: Numerical results with 50 generations per iteration

Problem	k	Phase	iRVEA		iNSGA-III	
			mean	std. dev.	mean	std. dev.
DTLZ1	3	learning	2.69E+00	3.22E-01	**2.47E+00**	1.52E-01
		decision	1.89E+00	3.25E-01	**1.76E+00**	1.59E-01
	4	learning	2.74E+00	3.49E-01	**2.62E+00**	1.66E-01
		decision	2.03E+00	3.76E-01	**1.98E+00**	2.69E-01
	5	learning	3.16E+00	5.90E-01	**2.67E+00**	2.11E-01
		decision	**2.02E+00**	1.98E-01	2.27E+00	3.62E-01
	6	learning	3.12E+00	3.92E-01	**2.66E+00**	1.50E-01
		decision	**2.06E+00**	2.64E-01	2.29E+00	4.45E-01
	7	learning	3.30E+00	4.49E-01	**2.73E+00**	2.13E-01
		decision	**2.27E+00**	3.01E-01	2.38E+00	3.89E-01
	8	learning	3.04E+00	1.87E-01	**2.82E+00**	2.60E-01
		decision	**2.27E+00**	3.61E-01	2.50E+00	6.59E-01
	9	learning	3.07E+00	2.59E-01	**2.80E+00**	2.76E-01
		decision	**2.16E+00**	2.99E-01	2.46E+00	6.23E-01
DTLZ2	3	learning	2.08E-01	1.85E-01	**1.08E-01**	1.26E-01
		decision	5.48E-01	6.60E-01	**1.02E-01**	6.47E-02
	4	learning	4.71E-01	3.43E-01	**2.21E-01**	1.21E-01
		decision	5.73E-01	7.67E-01	**2.41E-01**	2.64E-01
	5	learning	3.20E-01	2.93E-01	**1.98E-01**	1.19E-01
		decision	5.51E-01	9.77E-01	**2.46E-01**	3.41E-01
	6	learning	5.60E-01	3.27E-01	**3.92E-01**	1.89E-01
		decision	7.04E-01	9.31E-01	**4.66E-01**	3.53E-01
	7	learning	8.28E-01	5.28E-01	**4.15E-01**	2.10E-01
		decision	1.47E+00	1.57E+00	**6.10E-01**	5.00E-01
	8	learning	**5.25E-01**	2.49E-01	5.44E-01	6.75E-01
		decision	7.16E-01	1.16E+00	**3.47E-01**	3.19E-01
	9	learning	9.68E-01	5.32E-01	**4.93E-01**	1.78E-01
		decision	1.23E+00	1.53E+00	**5.01E-01**	3.36E-01
DTLZ3	3	learning	2.78E-01	4.55E-01	**2.25E-01**	1.11E-01
		decision	1.32E-01	1.78E-01	**7.02E-02**	5.39E-02
	4	learning	**4.47E-01**	3.09E-01	5.66E-01	2.31E-01
		decision	1.42E-01	9.69E-02	**9.20E-02**	5.82E-02
	5	learning	**6.94E-01**	6.06E-01	8.81E-01	4.17E-01
		decision	2.37E-01	1.87E-01	**1.45E-01**	3.74E-02
	6	learning	**5.79E-01**	3.55E-01	8.05E-01	3.99E-01
		decision	**2.72E-01**	9.92E-02	4.67E-01	4.01E-01
	7	learning	8.91E-01	5.14E-01	**8.26E-01**	2.95E-01
		decision	**3.43E-01**	1.39E-01	7.50E-01	5.05E-01
	8	learning	**7.45E-01**	3.79E-01	9.43E-01	4.20E-01
		decision	**3.92E-01**	1.11E-01	7.74E-01	4.81E-01
	9	learning	**7.03E-01**	3.34E-01	1.02E+00	3.70E-01
		decision	**3.75E-01**	2.20E-01	9.87E-01	7.22E-01
DTLZ4	3	learning	2.54E-01	6.55E-01	**7.69E-02**	3.56E-02
		decision	1.88E-01	4.77E-01	**7.37E-02**	5.30E-02
	4	learning	3.60E-01	4.43E-01	**2.19E-01**	1.92E-01
		decision	3.68E-01	3.91E-01	**3.43E-01**	3.95E-01
	5	learning	4.79E-01	3.73E-01	**3.92E-01**	2.45E-01
		decision	5.32E-01	4.24E-01	**5.23E-01**	4.39E-01
	6	learning	7.83E-01	6.94E-01	**6.09E-01**	2.92E-01
		decision	6.11E-01	3.63E-01	**5.99E-01**	3.86E-01
	7	learning	3.01E+00	2.19E+00	**7.29E-01**	3.70E-01
		decision	1.18E+00	6.74E-01	**6.69E-01**	4.48E-01
	8	learning	1.31E+00	1.05E+00	**7.38E-01**	2.24E-01
		decision	9.71E-01	4.95E-01	**7.62E-01**	4.11E-01
	9	learning	8.25E-01	3.11E-01	**7.60E-01**	2.83E-01
		decision	8.26E-01	4.48E-01	**6.90E-01**	4.20E-01

Table S2: Numerical results with 100 generations per iteration

Problem	k	Phase	iRVEA		iNSGA-III	
			mean	std. dev.	mean	std. dev.
DTLZ1	3	learning	**2.40E+00**	1.05E-01	2.41E+00	1.12E-01
		decision	1.85E+00	2.04E-01	1.85E+00	2.01E-01
	4	learning	**2.51E+00**	1.55E-01	2.54E+00	1.61E-01
		decision	**1.96E+00**	2.69E-01	2.16E+00	9.28E-01
	5	learning	2.58E+00	2.51E-01	**2.56E+00**	1.92E-01
		decision	**1.89E+00**	2.08E-01	1.94E+00	2.35E-01
	6	learning	**2.54E+00**	1.32E-01	2.70E+00	3.46E-01
		decision	**2.07E+00**	2.47E-01	2.35E+00	3.51E-01
	7	learning	**2.69E+00**	1.87E-01	2.73E+00	4.18E-01
		decision	**2.09E+00**	2.66E-01	2.28E+00	4.18E-01
	8	learning	**2.66E+00**	1.66E-01	2.72E+00	3.04E-01
		decision	**2.13E+00**	3.26E-01	2.43E+00	4.94E-01
	9	learning	**2.64E+00**	1.54E-01	2.75E+00	2.81E-01
		decision	**2.10E+00**	3.01E-01	2.42E+00	7.57E-01
DTLZ2	3	learning	1.89E-01	1.64E-01	**1.87E-01**	3.03E-01
		decision	3.24E-01	4.25E-01	**8.20E-02**	6.09E-02
	4	learning	3.73E-01	3.49E-01	**1.64E-01**	9.28E-02
		decision	8.67E-01	8.08E-01	**2.71E-01**	2.54E-01
	5	learning	7.16E-01	5.23E-01	**2.54E-01**	1.45E-01
		decision	1.17E+00	1.17E+00	**3.83E-01**	4.15E-01
	6	learning	7.43E-01	4.73E-01	**4.89E-01**	2.26E-01
		decision	7.88E-01	6.41E-01	**4.53E-01**	3.42E-01
	7	learning	8.53E-01	4.75E-01	**4.98E-01**	2.19E-01
		decision	1.42E+00	1.34E+00	**7.53E-01**	7.52E-01
	8	learning	8.86E-01	5.73E-01	**5.39E-01**	4.04E-01
		decision	1.16E+00	1.22E+00	**4.38E-01**	3.22E-01
	9	learning	7.35E-01	3.67E-01	**5.43E-01**	3.61E-01
		decision	8.93E-01	1.13E+00	**4.81E-01**	3.32E-01
DTLZ3	3	learning	2.25E-01	3.98E-01	**1.28E-01**	5.53E-02
		decision	1.24E-01	1.82E-01	**8.07E-02**	7.38E-02
	4	learning	2.67E-01	1.65E-01	**2.56E-01**	1.17E-01
		decision	2.71E-01	4.45E-01	**1.78E-01**	2.12E-01
	5	learning	**4.03E-01**	3.64E-01	4.70E-01	2.54E-01
		decision	2.95E-01	4.73E-01	**1.77E-01**	1.67E-01
	6	learning	**4.61E-01**	2.03E-01	5.05E-01	2.07E-01
		decision	**2.92E-01**	7.51E-02	6.03E-01	5.06E-01
	7	learning	**5.22E-01**	1.86E-01	7.94E-01	3.67E-01
		decision	**4.15E-01**	2.09E-01	6.37E-01	6.27E-01
	8	learning	**5.00E-01**	2.04E-01	7.75E-01	3.76E-01
		decision	**2.62E-01**	1.47E-01	8.41E-01	5.74E-01
	9	learning	**4.64E-01**	1.35E-01	9.76E-01	4.96E-01
		decision	**6.07E-01**	6.53E-01	1.18E+00	7.77E-01
DTLZ4	3	learning	1.89E-01	4.48E-01	**5.69E-02**	2.65E-02
		decision	7.93E-02	6.16E-02	**6.64E-02**	4.56E-02
	4	learning	4.20E-01	7.76E-01	**1.85E-01**	1.60E-01
		decision	3.89E-01	3.97E-01	**3.87E-01**	4.07E-01
	5	learning	3.93E-01	2.31E-01	**3.53E-01**	2.64E-01
		decision	**6.35E-01**	4.34E-01	6.59E-01	4.57E-01
	6	learning	7.14E-01	3.00E-01	**6.91E-01**	2.93E-01
		decision	**8.05E-01**	3.95E-01	8.13E-01	4.14E-01
	7	learning	2.89E+00	2.42E+00	**6.67E-01**	3.04E-01
		decision	1.44E+00	1.13E+00	**7.79E-01**	4.44E-01
	8	learning	1.33E+00	1.66E+00	**8.92E-01**	3.60E-01
		decision	1.21E+00	9.71E-01	**9.04E-01**	4.22E-01
	9	learning	8.83E-01	2.61E-01	**8.29E-01**	2.70E-01
		decision	9.67E-01	4.23E-01	**8.13E-01**	4.04E-01