

1 Review of Differential Equations

General First Order ODE:

$$\frac{dy}{dt} + py = g$$

If p and g are constants we can solve this by separation of variables:

$$\begin{aligned}
 \frac{dy}{dt} + py &= g \\
 \frac{dy}{dt} &= g - py \\
 \frac{1}{g - py} dy &= 1 dt \\
 \int \frac{1}{g - py} dy &= \int 1 dt \\
 u &= g - py \\
 du = -pdy &\rightarrow dy = -\frac{du}{p} \\
 \int \frac{1}{u} \frac{-1}{p} du &= \int dt \\
 -\frac{1}{p} \ln|u| &= t + C \\
 \ln|u| &= (t + C)(-p) \\
 \ln|u| &= -tp + C \\
 u &= e^{-tp+C} \\
 &= e^{-tp} e^C \\
 u &= e^{-tp} \mathbf{C}_1 \\
 g - py &= e^{-tp} C_1 \\
 -py &= e^{-tp} C_1 - g \\
 y &= -\frac{e^{-tp} C_1 - g}{p} \\
 y &= -\frac{e^{-tp} C_1}{p} + \frac{g}{p} \\
 y &= e^{-tp} \mathbf{C}_2 + \frac{g}{p}
 \end{aligned}$$

Here C_1 and C_2 are arbitrary constants. We will use those for Initial Value problems (IVP) in the future.

now