Re-Entry Series

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Contents

1 Review

This is for things to remember from compressible flow...

2 Hyper Sonic Flow

Why do we care??

1. It is physically different from the Super Sonic Regime.

2. It is flow that will dictate many of the new vehicle designs in the 21st century.

What is it?

1. fore brevity, we will say $M > 5$

2.1 Thin shock layers

For some deflection angle δ, the density ρ increases across the shock wave as M increases.

With the larger ρ, the post-shock flow can more easily squeeze into smaller areas. To put it simply: a big burrito becomes a small burrito, and a small burrito takes up less space.

Using oblique shock theory from supersonic flow, lets find the shock angle of hypersonic flow.

For $M = 36$ and $\gamma = 1.4$

$$tan(\theta) = 2cot(\beta)\frac{M_1^2 sin^2(\beta) - 1}{M_1^2\left(\gamma + cos(2\beta)\right) + 2}$$

Assumptions for this equation:

1. Steady flow ($\frac{\partial}{\partial t} = 0$).
2. Adiabatic flow (q = 0).
3. No body forces ($\vec{F} = 0$).
4. Inviscid flow (no friction/viscous stresses acting on the CV boundaries).

Finding shock angle β

For $M = 36$ we can take the limit as $M_1 \to \infty$:

$$\lim_{M_1 \Rightarrow \infty} tan(\theta) \approx 2cot(\beta)\frac{M_1^2 sin^2(\beta) - \cancelto{0}{1}}{M_1^2\left(\gamma + cos(2\beta)\right) + \cancelto{0}{2}}$$
$$\approx 2cot(\beta)\frac{M_1^2 sin^2(\beta)}{M_1^2\left(\gamma + cos(2\beta)\right)}$$

with some trig $cot(\beta)sin^2(\beta) = \frac{1}{2}sin(2\beta)$

$$tan(\theta) \approx \frac{sin(2\beta)}{\gamma + cos(2\beta)}$$

We can see that β is still trapped from the function. This will still require iteration. We have better tools for that though.

Finding shock angle β through iteration

$$tan(\theta) = 2cot(\beta)\frac{M_1^2 sin^2(\beta) - 1}{M_1^2\left(\gamma + cos(2\beta)\right) + 2}$$
$$f(\beta) = tan(\theta_{calculated}) - \tan(\theta_{given})$$
$$= 2cot(\beta)\frac{M_1^2 sin^2(\beta) - 1}{M_1^2\left(\gamma + cos(2\beta)\right) + 2} - tan(\theta_{given})$$

we know we have found appropriate β, when $f(\beta) = tan(\theta_{calculated}) - \tan(\theta_{given}) \rightarrow 0$

So what should we be seeing right now?

1. We can notice that β is trapped "transendental".

2. the function is smooth.

3. We have a really good guess already since we know the lower and upper bound for any guess.

   β must be steeper than Mach Angle "μ" $= sin^{-1}\frac{1}{M}$

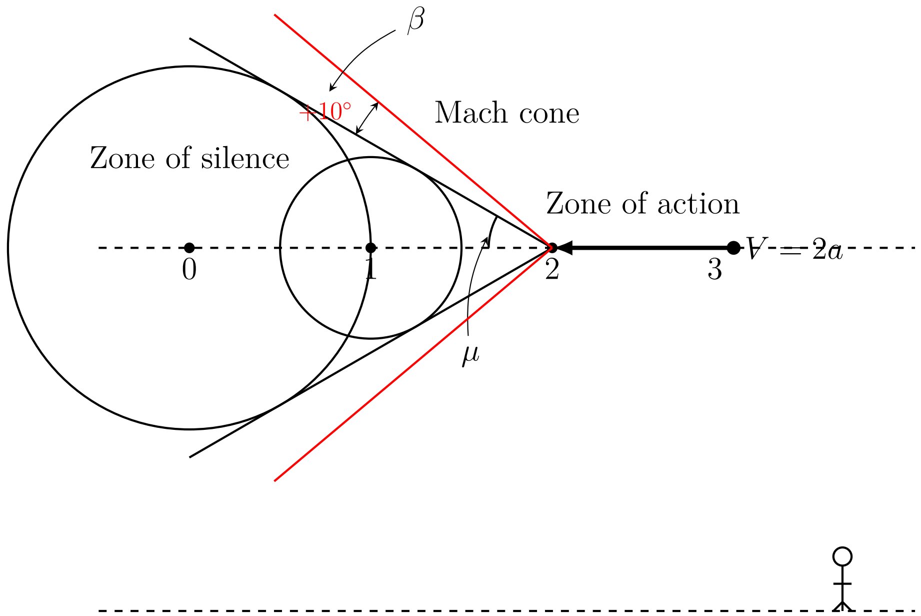

Figure 1: Wave fronts from a supersonic disturbance ($M = 2$).

Python Script: Mach Calculator

```python
# This is now highlighted using Python syntax rules
def get_beta(M, theta):
    return M * np.sin(theta)
```

MATLAB: Alternative Solver

```matlab
% This uses Matlab comment and keyword styles
M = 36;
theta = deg2rad(10);
```