

1 Review of Differential Equations

General First Order ODE:

$$\frac{dy}{dt} + py = g$$

If p and g are constants we can solve this by separation of variables:

$$\begin{aligned}\frac{dy}{dt} + py &= g \\ \frac{dy}{dt} &= g - py \\ \frac{1}{g - py} dy &= 1 dt \\ \int \frac{1}{g - py} dy &= \int 1 dt \\ u &= g - py \\ du &= -p dy \rightarrow dy = -\frac{du}{p} \\ \int \frac{1}{u} \frac{-1}{p} du &= \int dt \\ -\frac{1}{p} \ln|u| &= t + C \\ \ln|u| &= (t + C)(-p) \\ \ln|u| &= -tp + C \\ u &= e^{-tp+C} \\ &= e^{-tp} e^C \\ u &= e^{-tp} \mathbf{C_1} \\ g - py &= e^{-tp} C_1 \\ -py &= e^{-tp} C_1 - g \\ y &= -\frac{e^{-tp} C_1 - g}{p} \\ y &= -\frac{e^{-tp} C_1}{p} + \frac{g}{p} \\ y &= e^{-tp} \mathbf{C_2} + \frac{g}{p}\end{aligned}$$

Here C_1 and C_2 are arbitrary constants. We will use those for Initial Value problems (IVP) in the future.

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