1 Review of Differential Equations

1.1 Seperation of variables

General First Order ODE:

$$\frac{dy}{dt} + py = g$$

If p and g are constants we can solve this by seperation of variables:

$$\begin{aligned}
\frac{dy}{dt} + py &= g \\
\frac{dy}{dt} &= g - py \\
\frac{1}{g-py}dy &= 1dt \\
\int \frac{1}{g-py}dy &= \int 1dt \\
u &= g - py \\
du &= -pdy \rightarrow dy = -\frac{du}{p} \\
\int \frac{1}{u}\frac{-1}{p}du &= \int dt \\
-\frac{1}{p}ln|u| &= t + C \\
ln|u| &= (t+C)(-p) \\
ln|u| &= -tp + C \\
u &= e^{-tp+C} \\
&= e^{-tp}e^{C} \\
u &= e^{-tp}\mathbf{C_1} \\
g - py &= e^{-tp}C_1 \\
-py &= e^{-tp}C_1 - g \\
y &= -\frac{e^{-tp}C_1 - g}{p} \\
y &= -\frac{e^{-tp}C_1}{p} + \frac{g}{p} \\
y &= e^{-tp}\mathbf{C_2} + \frac{g}{p}
\end{aligned}$$

Here C_1 and C_2 are arbitrary constants. We will use those for Initial Value problems (IVP) in the future.

1.2 Method of integrating Factors

$$\frac{dy}{dt} + p(t)y = q(t)$$

Non Homog due to $q(t)$