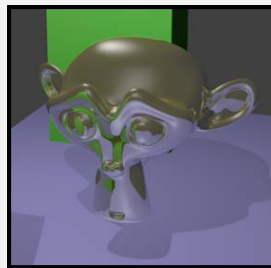
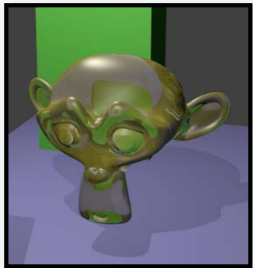
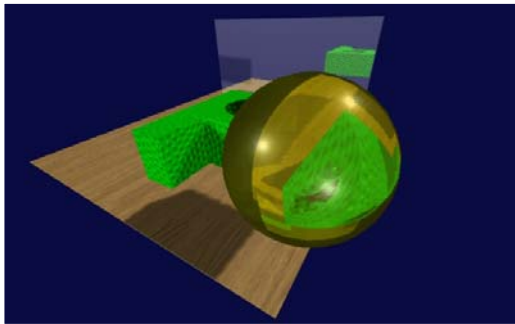


# Rendering



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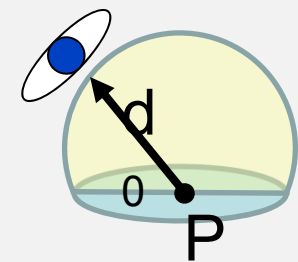
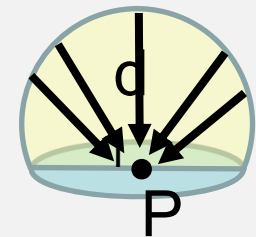
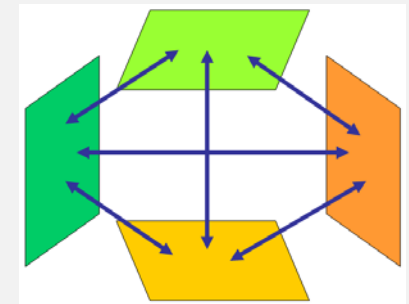
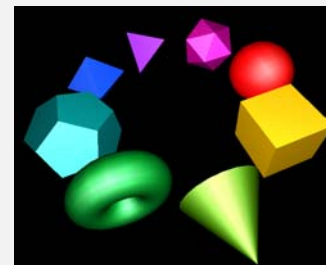


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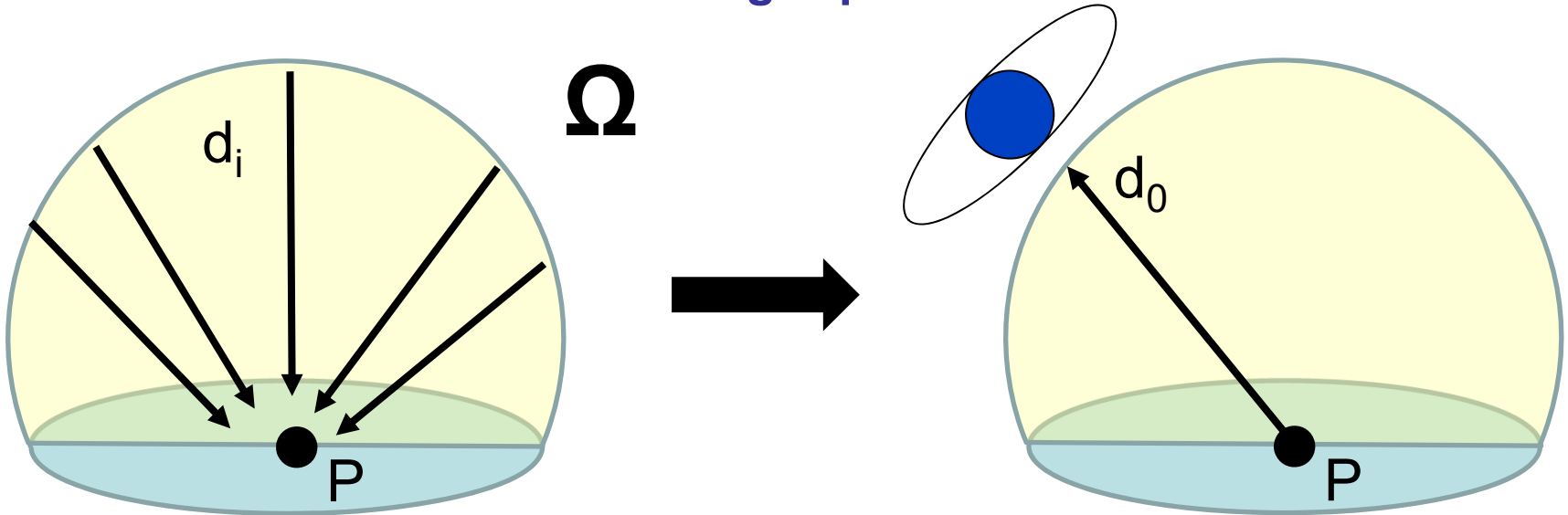
**Oregon State**  
University  
**Mike Bailey**

mjb@cs.oregonstate.edu



# The Rendering Equation

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$$B(P, d_0, \lambda) = E(P, d_0, \lambda) + \int_{\Omega} B(P, d_i, \lambda) f(\lambda, d_i, d_0) (d_i \cdot \hat{n}) d\Omega$$

This is the true rendering situation. Essentially, it is an energy balance:

Light Shining from a point  $P$  =  
Light emitted by that point +  
Reflectivity \*  $\Sigma$ (Light arriving from all other points)

But, this is time-consuming to solve “exactly”.

So, we need to know **how much of an approximation we need**

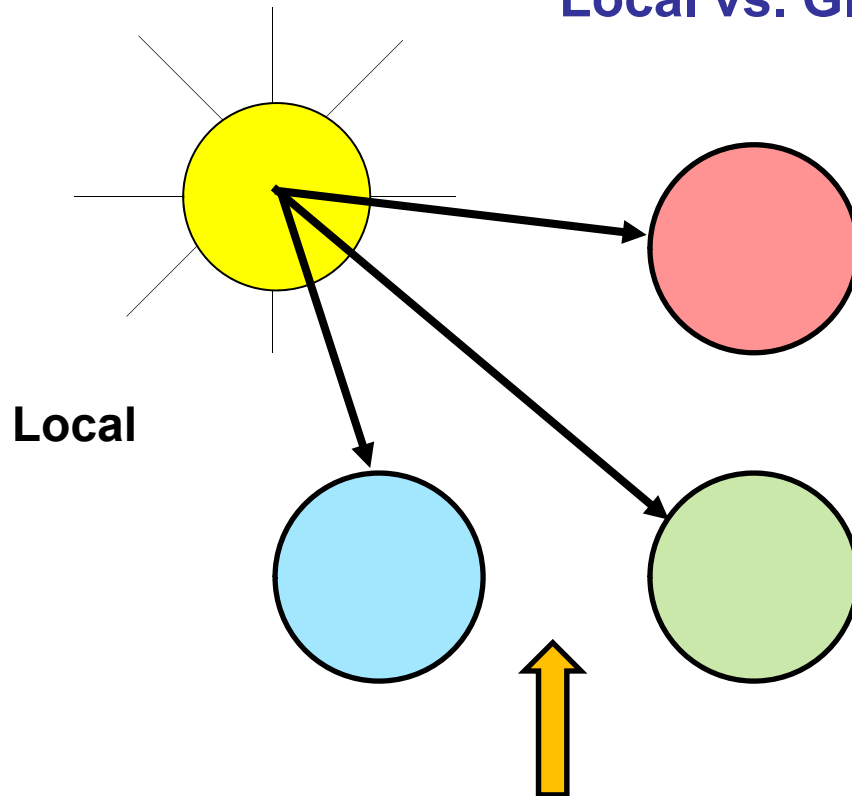
## Rendering

Rendering is the process of creating an image of a geometric model. There are questions you need to ask:

- For what purpose am I doing this?
- How realistic do I need this image to be?
- How much compute time do I have to create this image?
- Do I need to take lighting into account?
- Does the illumination need to be global or will local do?
- Do I need to create shadows?
- Do I need to create reflections and refractions?
- How good do the reflections and refractions need to be?

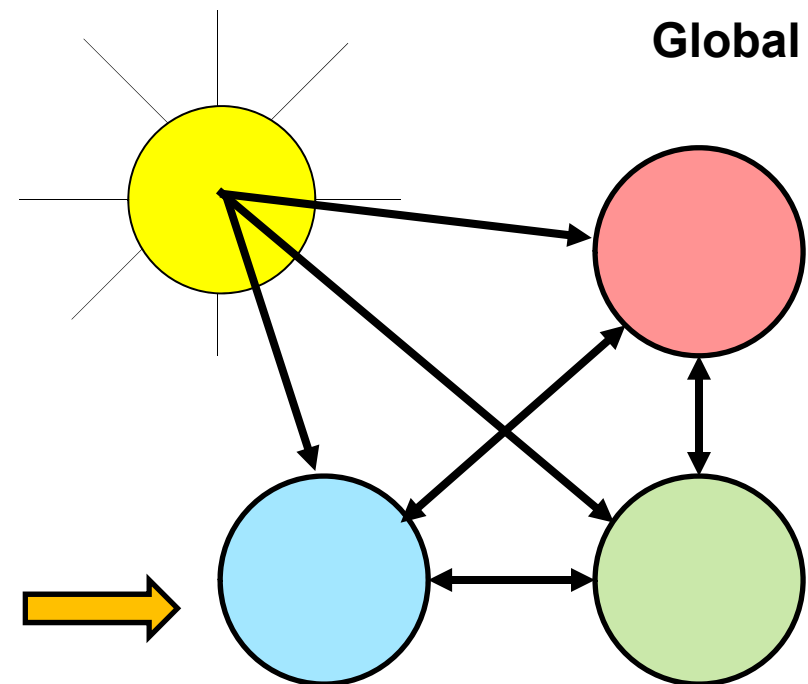


## Local vs. Global Illumination

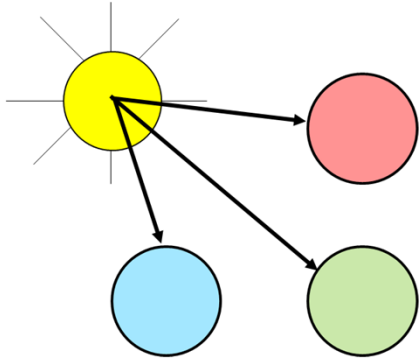


If the appearance of an object is only affected by its own characteristics and the characteristics of the light sources, then you have **Local Illumination**.

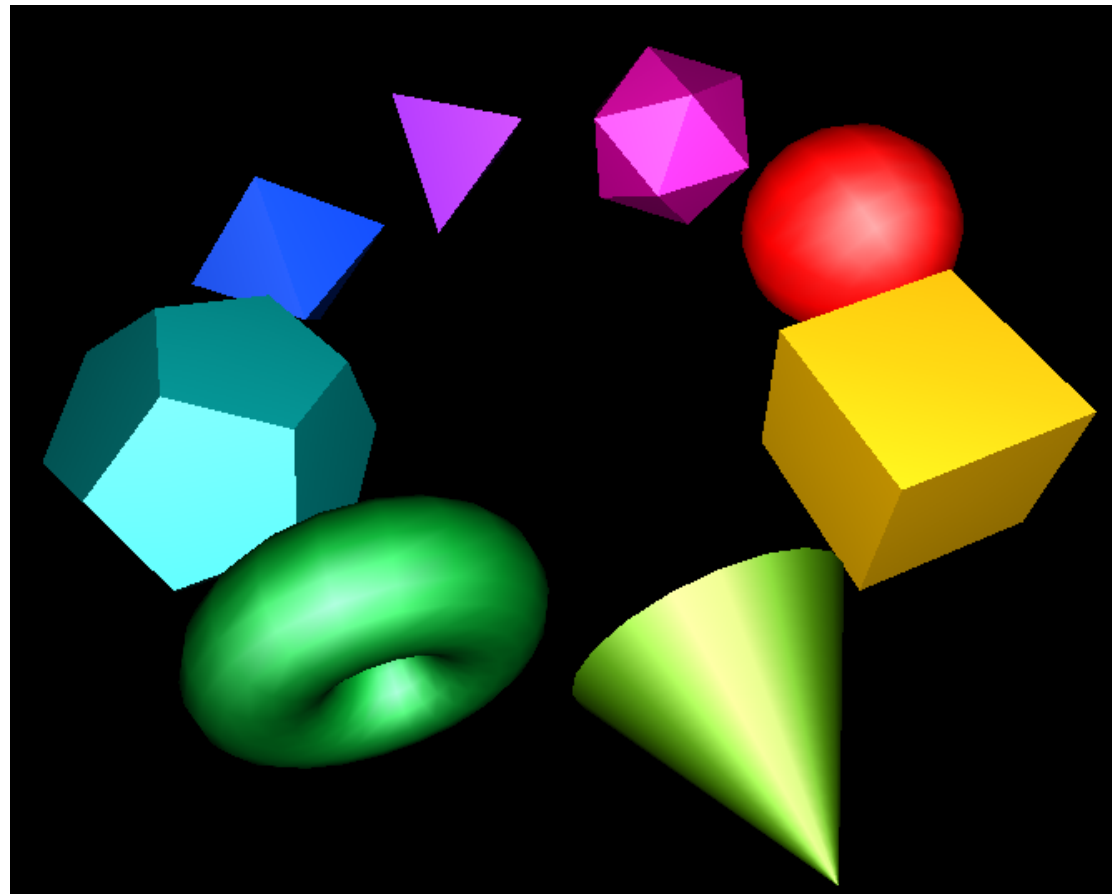
If the appearance of an object is also affected by the appearances of other objects, then you have **Global Illumination**.



## Local Illumination at Work

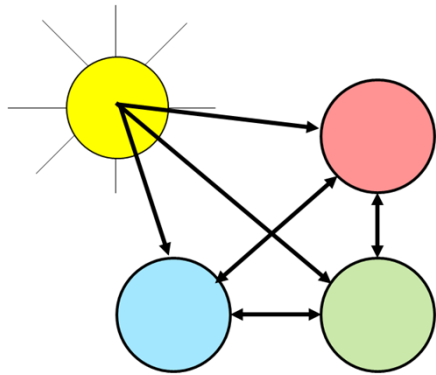


“If the appearance of an object is only affected by its own characteristics and the characteristics of the light sources, then you have **Local Illumination**.”



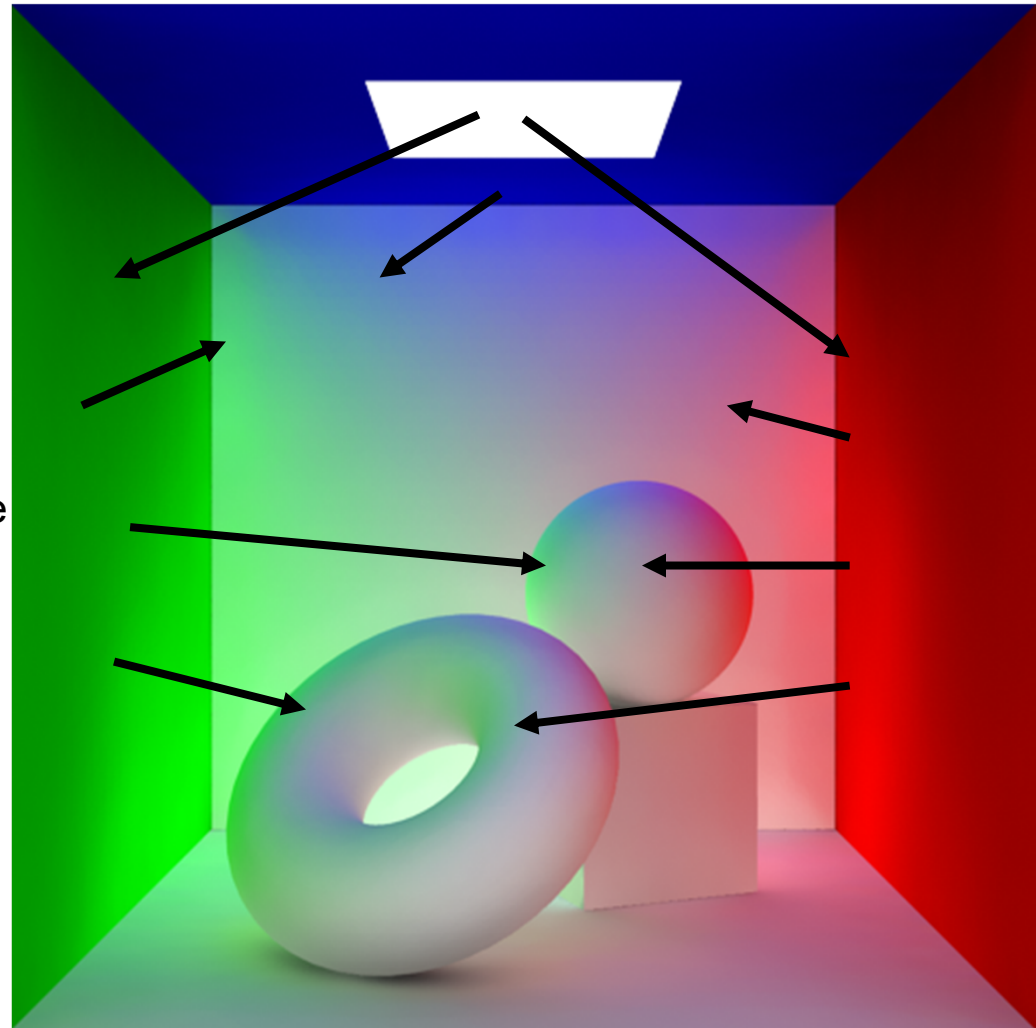
OpenGL rendering uses Local Illumination.

## Global Illumination at Work



- The left wall is green.
- The right wall is red.
- The back wall is white.
- The ceiling is blue with a light source in the middle of it.
- The objects sitting on the floor are white.

“If the appearance of an object is also affected by the appearances of other objects, then you have **Global Illumination.**”



<http://www.swardson.com/unm/tutorials/mentalRay3/>

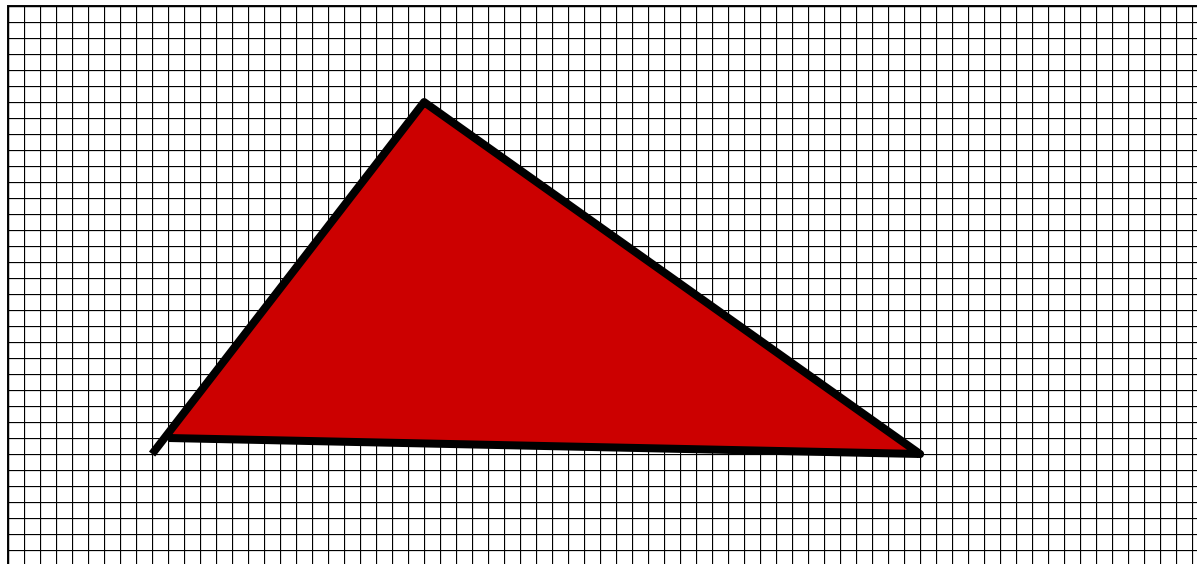


## Two Directions for the Rendering to Happen

1. Starts at the object, works towards the eye
2. Starts at the eye, works towards the object

## Starts at the Object, Works Towards the Eye

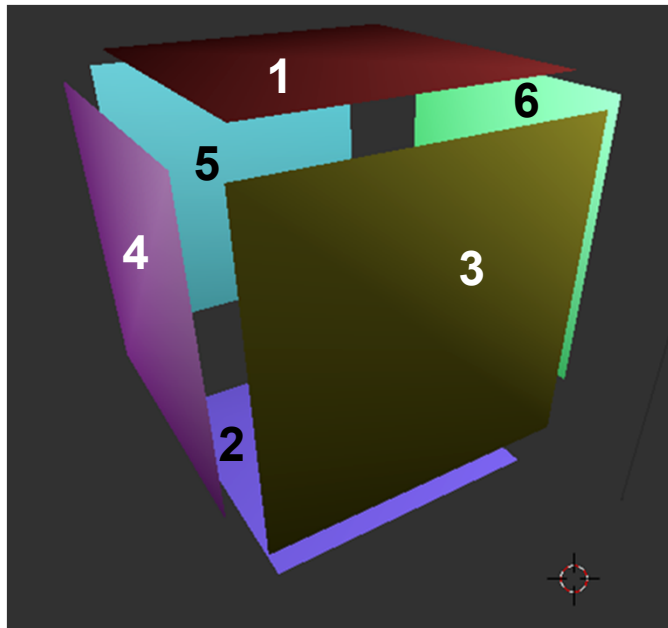
- This is the kind of rendering you get on a graphics card (e.g., OpenGL).
- You have been doing this all along.
- Start with the geometry and project it onto the pixels.





## How do things in front look like they are *really* in front?

Your application might draw this cube's polygons in 1-2-3-4-5-6 order, but 1, 3, and 4 still need to look like they were drawn last:



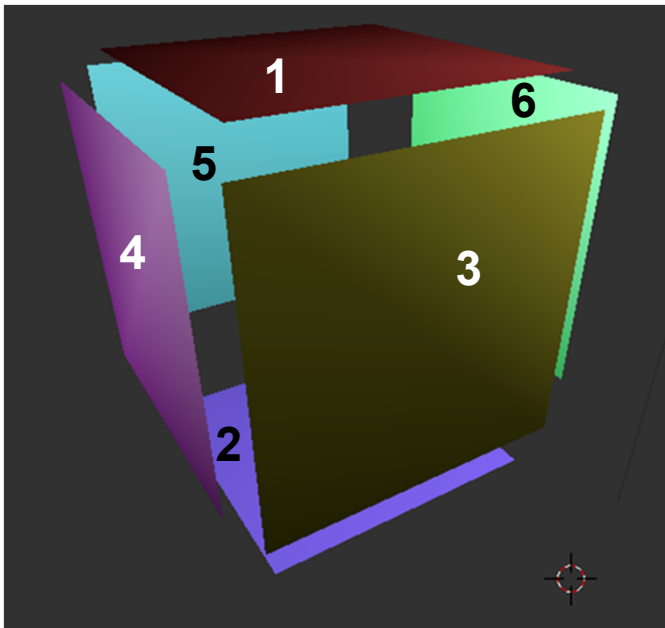
**Solution #1:** Sort your polygons in 3D by depth and draw them back-to-front.

In this case 1-2-3-4-5-6 becomes 5-6-2-4-1-3.

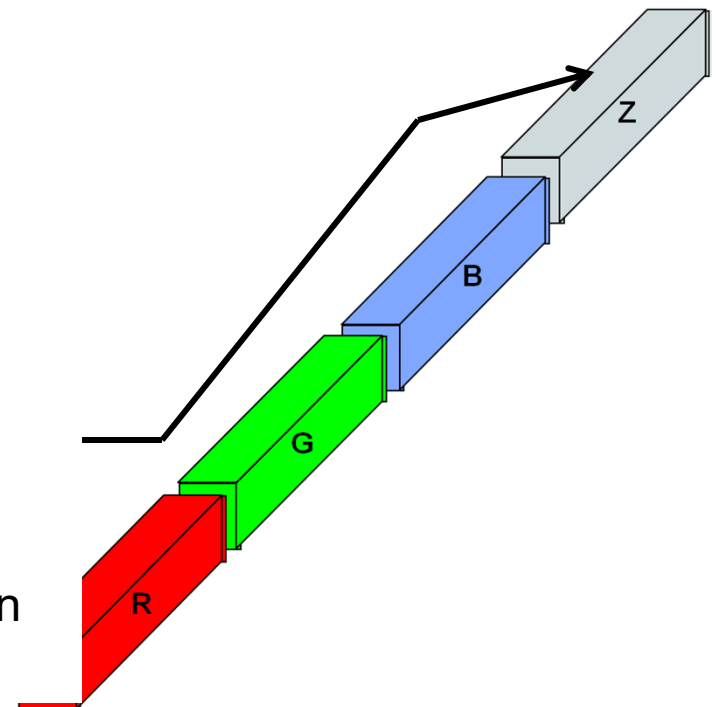
This is called the **Painter's Algorithm**. It sucked to have to do things this way.

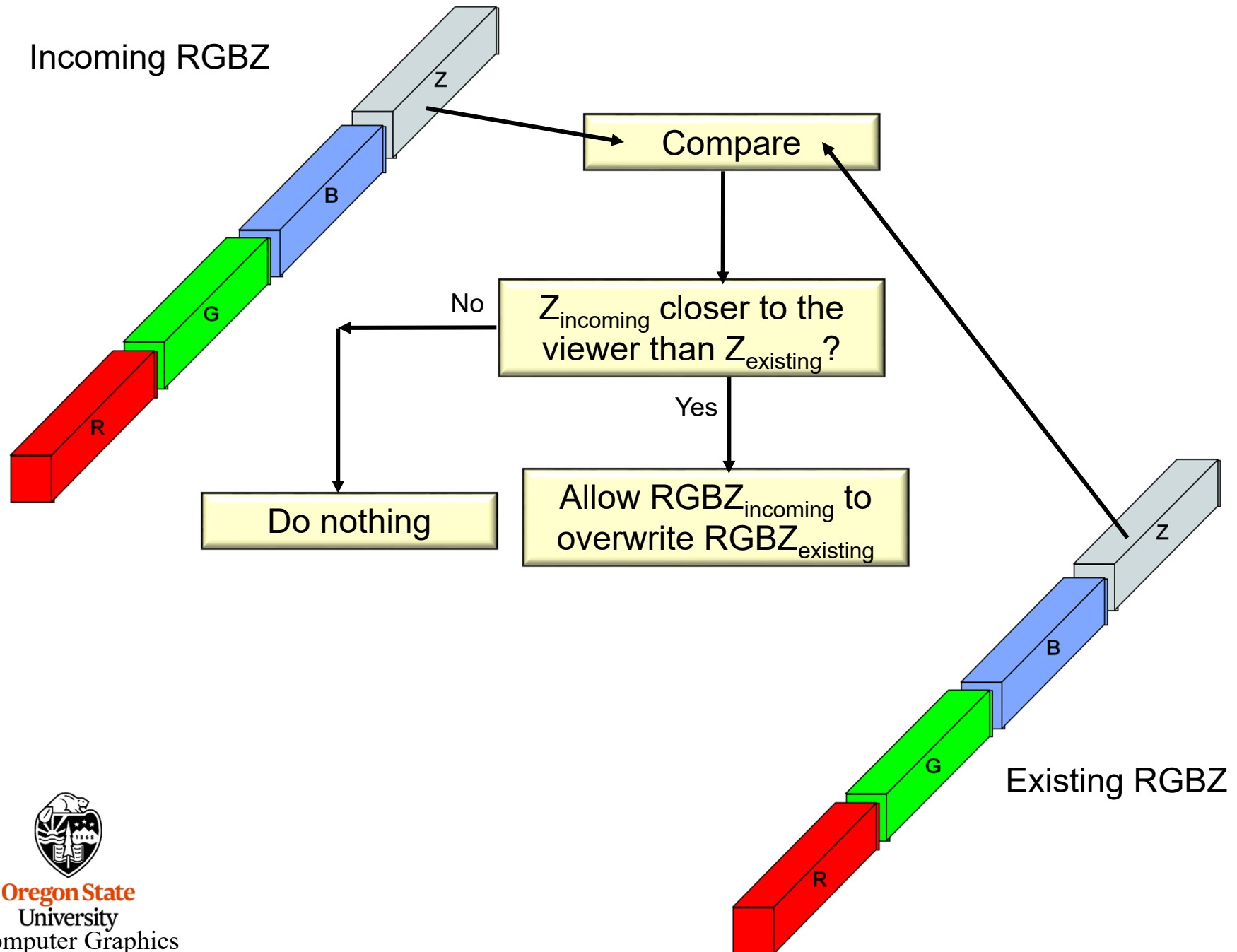
## How do things in front look like they are *really* in front?

Your application might draw this cube's polygons in 1-2-3-4-5-6 order, but 1, 3, and 4 still need to look like they were drawn last:



**Solution #2:** Add an extension to the framebuffer to store the depth of each pixel. This is called a **Depth-buffer** or **Z-buffer**. Only allow pixel stores when the depth of the incoming pixel is closer to the viewer than the pixel that is already there.



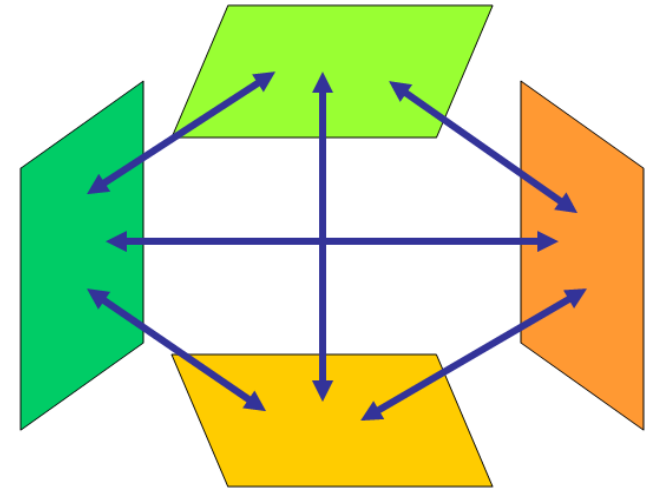


## Another From-the-Object Method -- Radiosity

Based on the idea that all surfaces gather light intensity from all other surfaces

The fundamental radiosity equation is an energy balance that says:

“The light energy leaving surface  $i$  equals the amount of light energy generated by surface  $i$  plus surface  $i$ 's reflectivity times the amount of light energy arriving from all other surfaces”



$$B_i A_i = E_i A_i + \rho_i \sum_j B_j A_j F_{j \rightarrow i}$$

## The Radiosity Equation

$$B_i A_i = E_i A_i + \rho_i \sum_j B_j A_j F_{j \rightarrow i}$$

$B_i$  is the light energy intensity shining from surface element  $i$

$A_i$  is the area of surface element  $i$

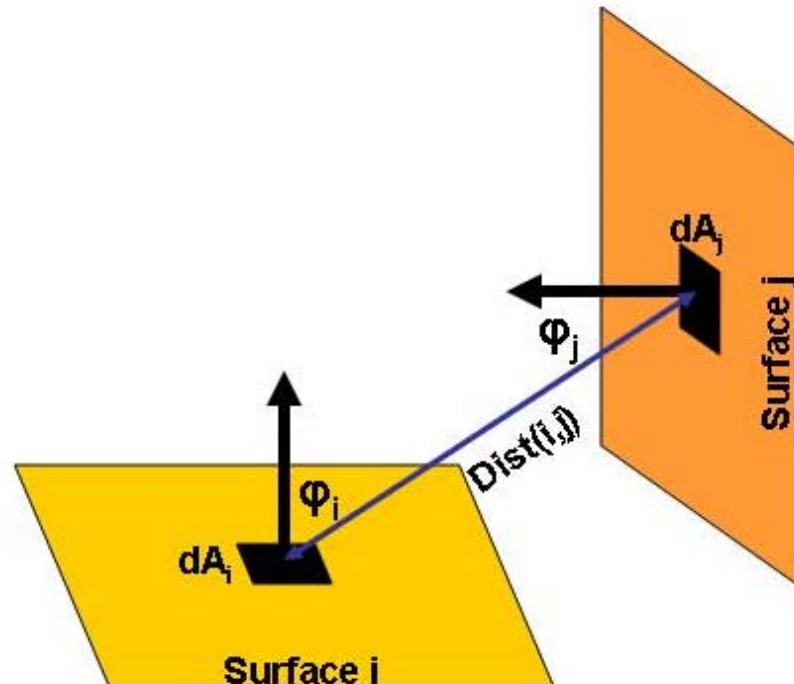
$E_i$  is the internally-generated light energy intensity for surface element  $i$

$\rho_i$  is surface element  $i$ 's reflectivity

$F_{j \rightarrow i}$  is referred to as the **Shape Factor**, and describes what percent of the energy leaving surface element  $j$  arrives at surface element  $i$



## The Radiosity Shape Factor

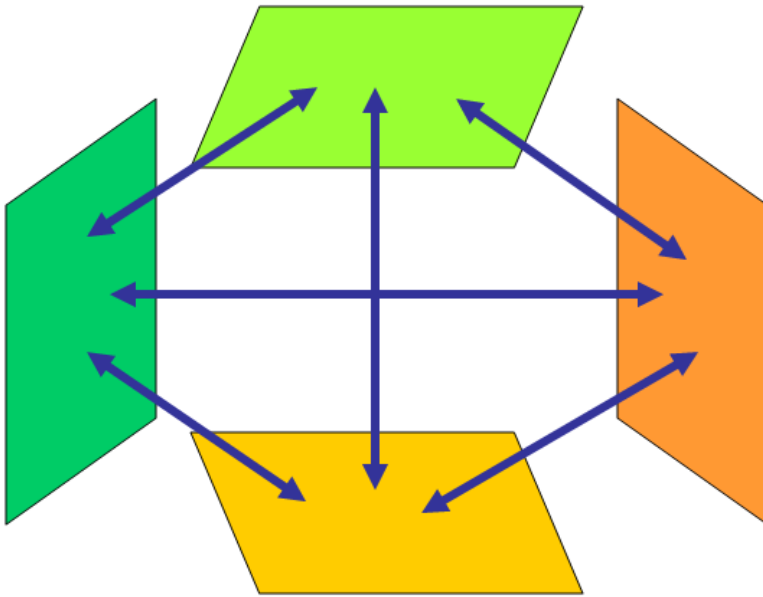


$$F_{j \rightarrow i} = \int_{A_i} \int_{A_j} \text{visibility}(di, dj) \frac{\cos \Theta_i \cos \Theta_j}{\pi \text{Dist}(di, dj)^2} dA_j dA_i$$

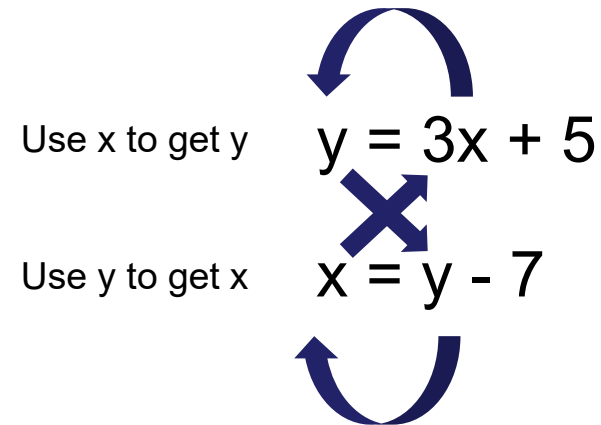


Does it seem to you that the light just keeps propagating  
and you never get an answer?

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To many people, radiosity seems like this:



“x produces y, then y produces x,  
then x produces y, then ...”

Not really – it is simply N equations, N unknowns – you solve for the unique solution

$$\begin{aligned} -3x + y &= 5 \\ x - y &= -7 \end{aligned}$$

$$\begin{aligned} x &= 1 \\ y &= 8 \end{aligned}$$

## The Radiosity Matrix Equation

Expand  $B_i A_i = E_i A_i + \rho_i \sum_j B_j A_j F_{j \rightarrow i}$

For each surface element, and re-arrange to  
**solve for the surface intensities, the  $B$ 's:**

$$\begin{bmatrix} 1 - \rho_1 F_{1 \rightarrow 1} & -\rho_1 F_{1 \rightarrow 2} & \bullet \bullet \bullet & -\rho_1 F_{1 \rightarrow N} \\ -\rho_2 F_{2 \rightarrow 1} & 1 - \rho_2 F_{2 \rightarrow 2} & \bullet \bullet \bullet & -\rho_2 F_{2 \rightarrow N} \\ \bullet \bullet \bullet & \bullet \bullet \bullet & \bullet \bullet \bullet & \bullet \bullet \bullet \\ -\rho_N F_{N \rightarrow 1} & -\rho_N F_{N \rightarrow 2} & \bullet \bullet \bullet & 1 - \rho_N F_{N \rightarrow N} \end{bmatrix} \begin{Bmatrix} B_1 \\ B_2 \\ \bullet \bullet \bullet \\ B_N \end{Bmatrix} = \begin{Bmatrix} E_1 \\ E_2 \\ \bullet \bullet \bullet \\ E_N \end{Bmatrix}$$

This is a lot of equations!



## Radiosity Examples



Cornell University



Cornell University

img - October 27, 2017

## Radiosity Examples

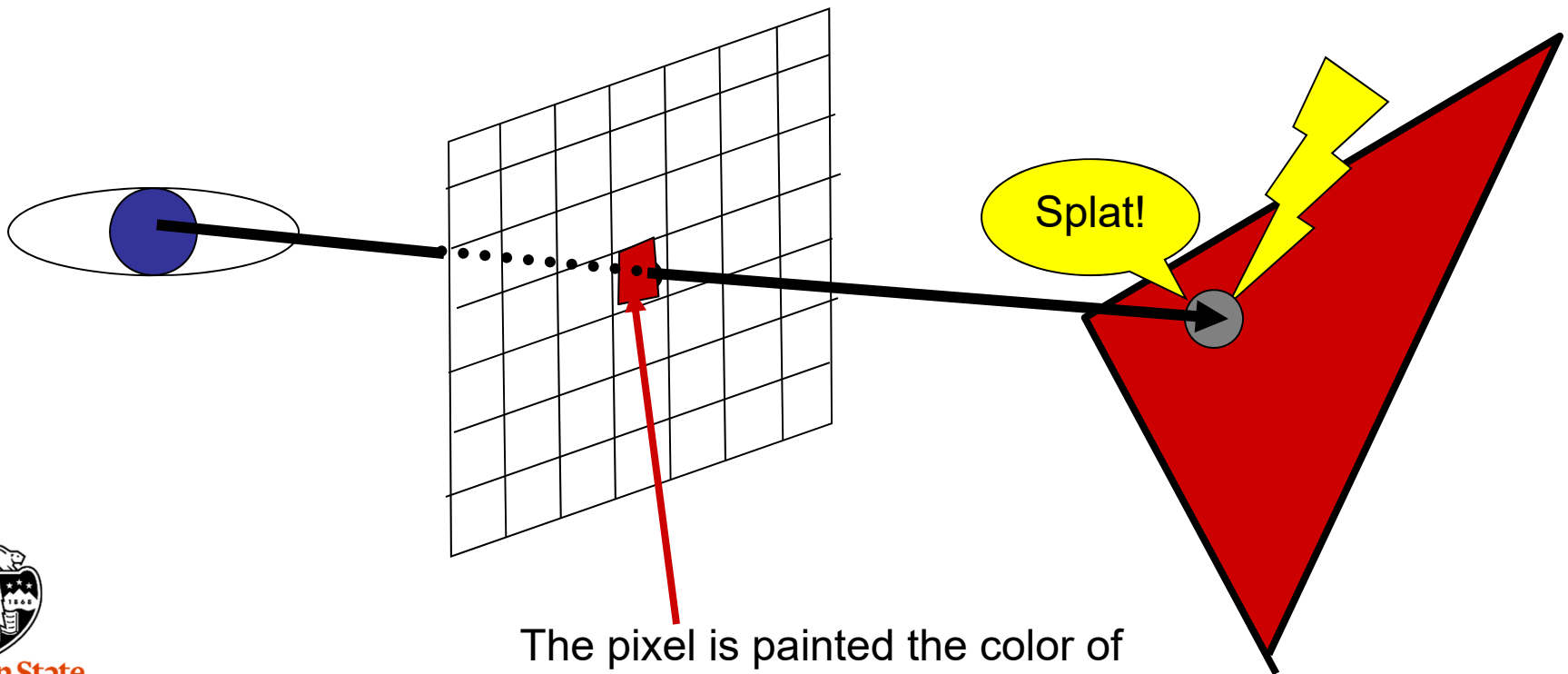


AR Toolkit



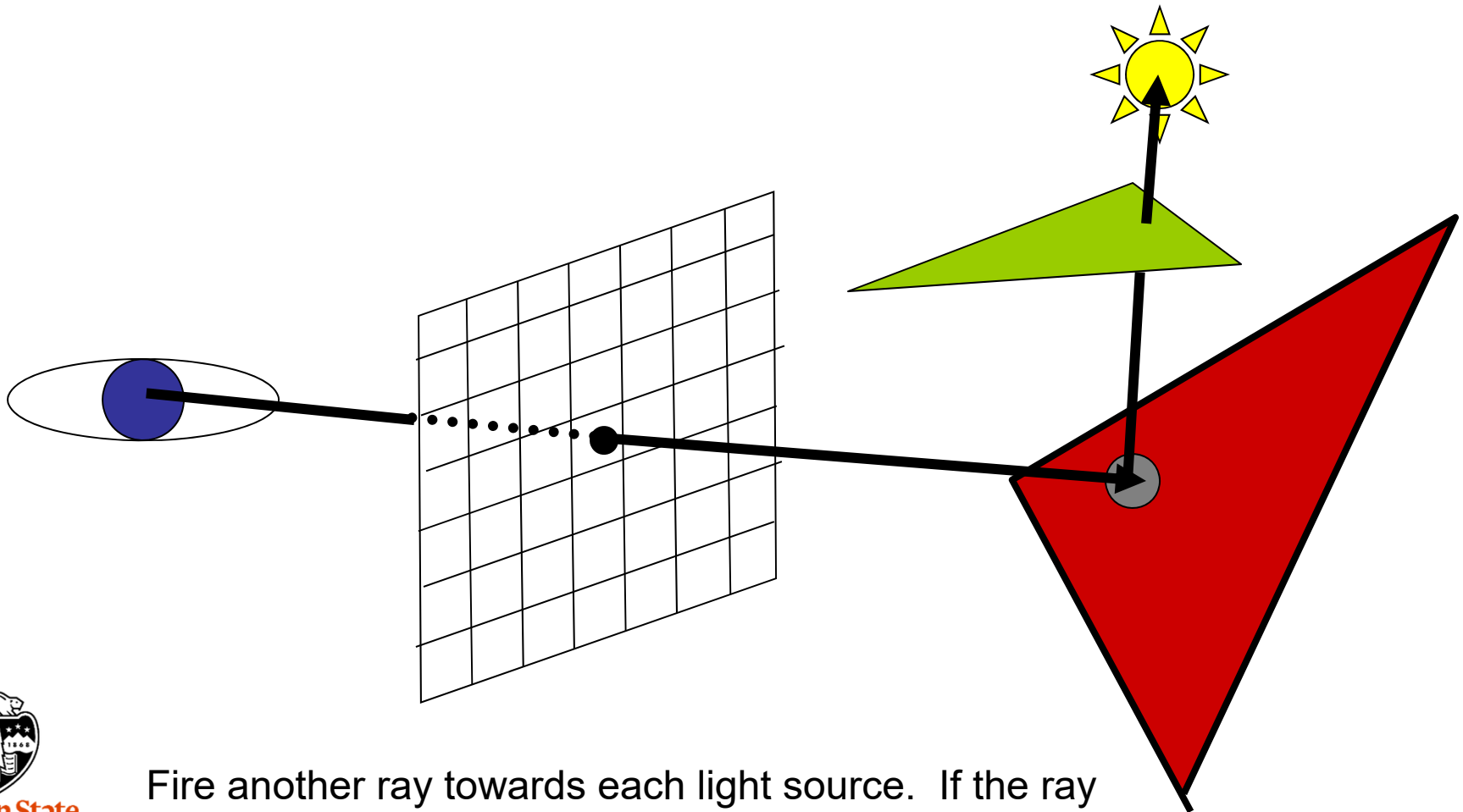
## Starts at the Eye, Works Towards the Objects

The most common approach in this category is **ray-tracing**:



## Starts at the Eye

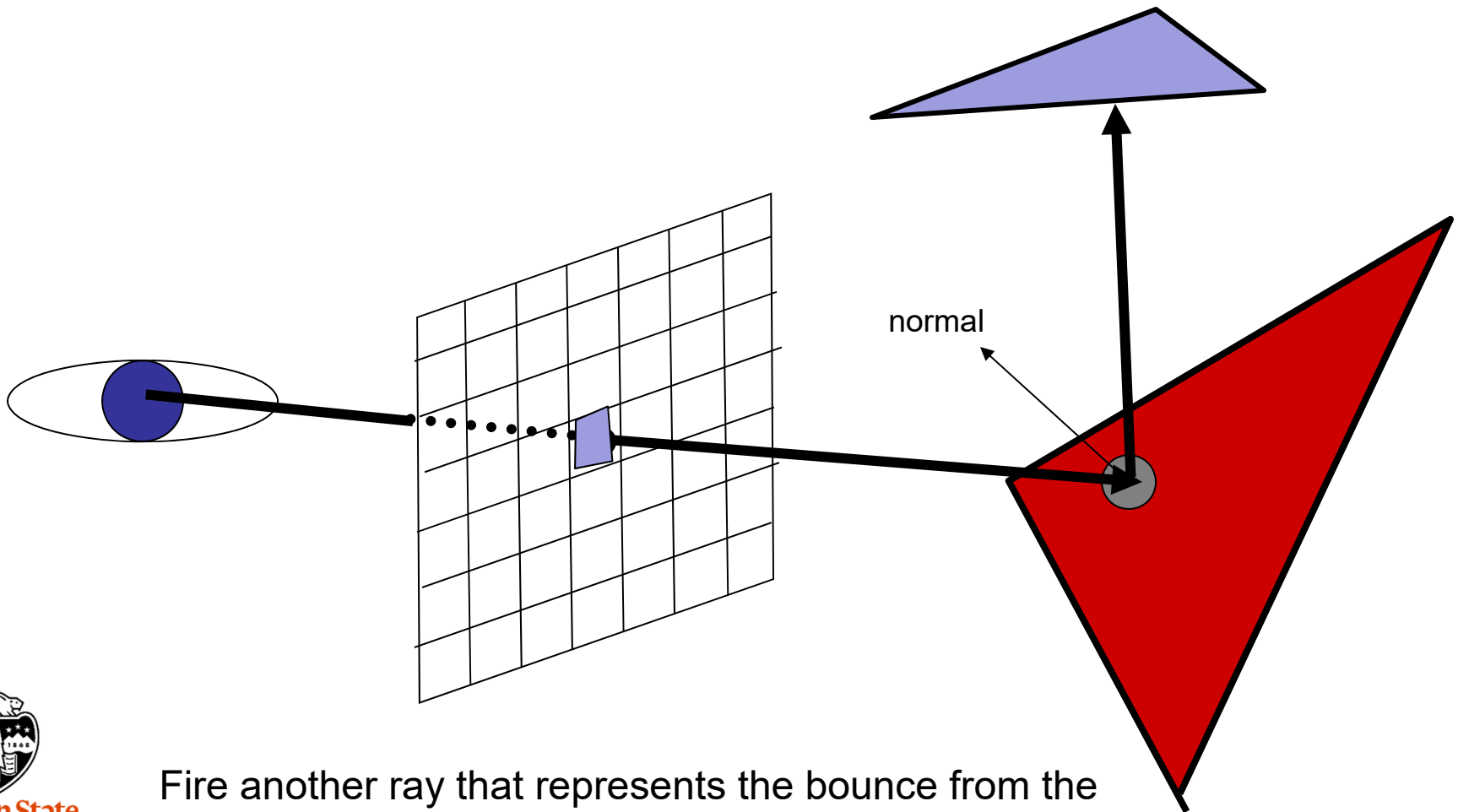
It's also straightforward to see if this point lies in a shadow:



Fire another ray towards each light source. If the ray hits anything, then the point does not receive that light.

## Starts at the Eye

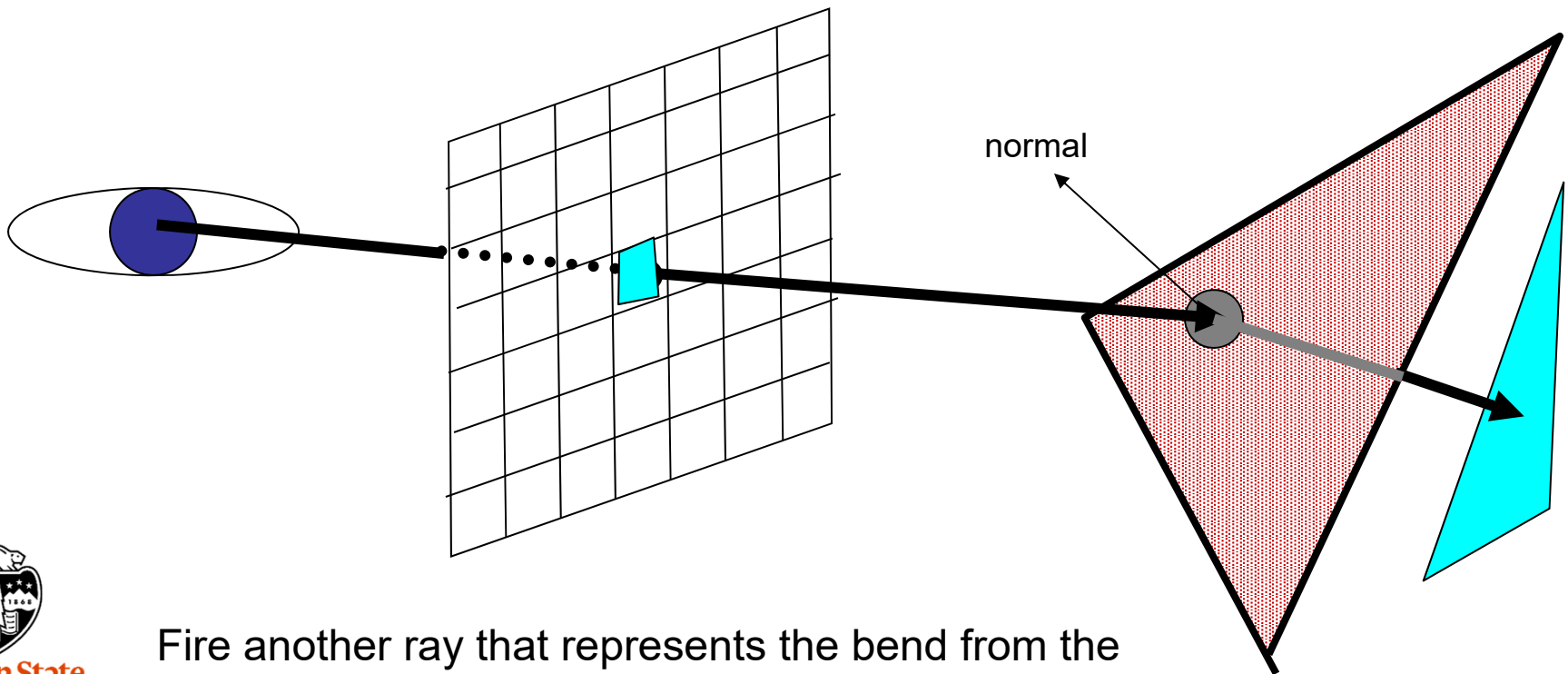
It's also straightforward to handle reflection



Fire another ray that represents the bounce from the reflection. Paint the pixel the color that this ray sees.

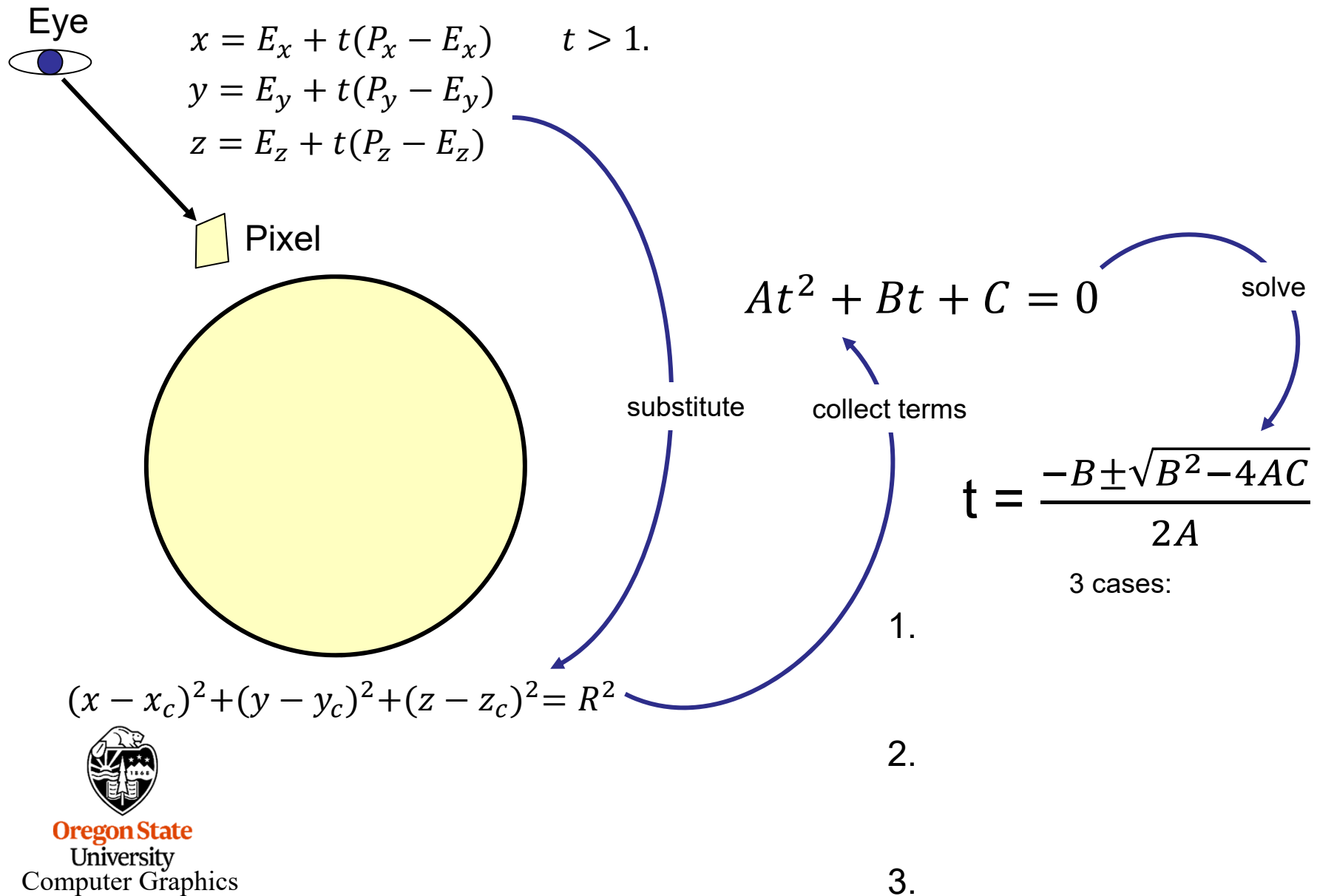
## Starts at the Eye

It's also straightforward to handle refraction

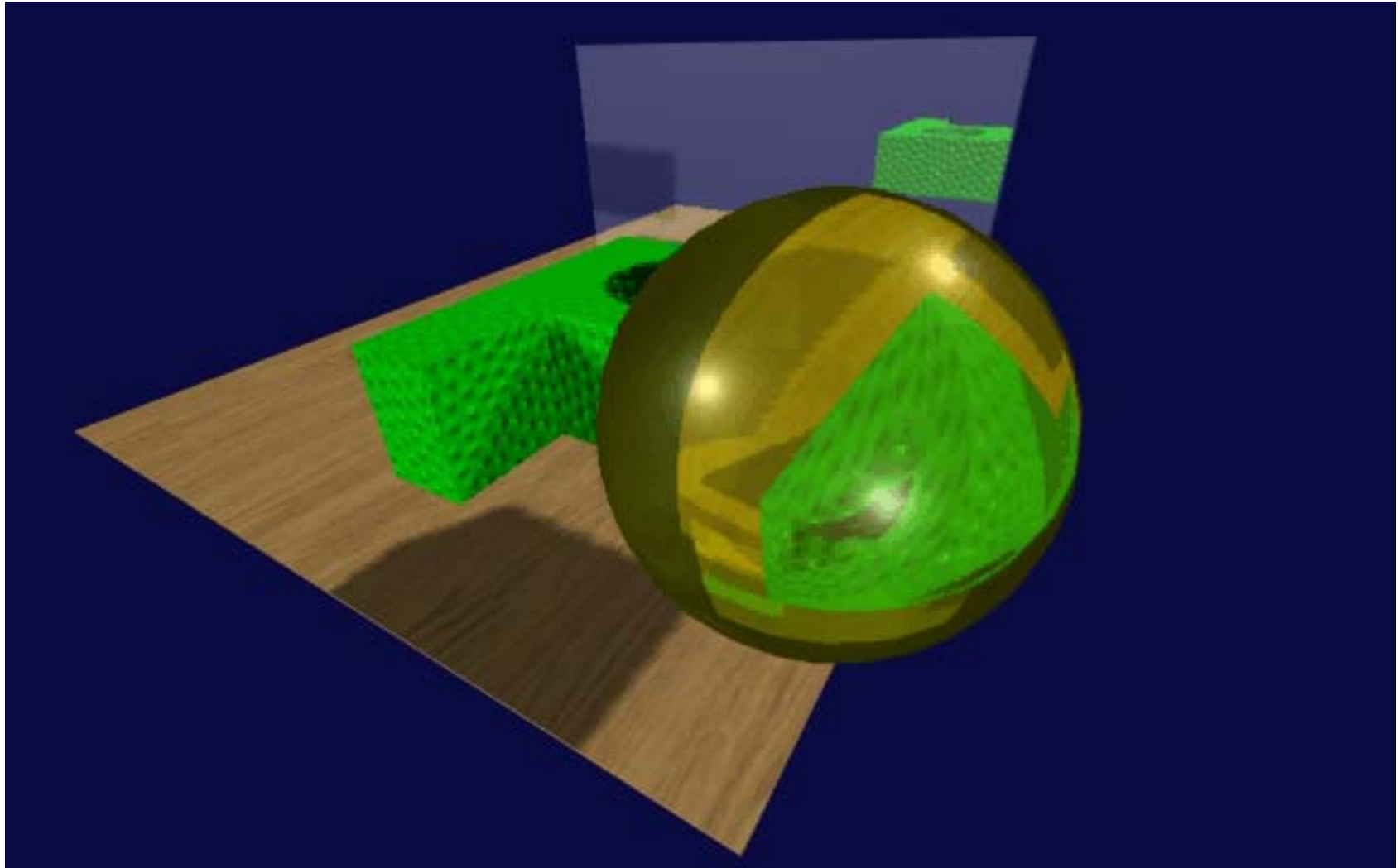


Fire another ray that represents the bend from the refraction. Paint the pixel the color that this ray sees.

## Determining Ray-Shape Intersections



## IronCAD Ray-tracing Example

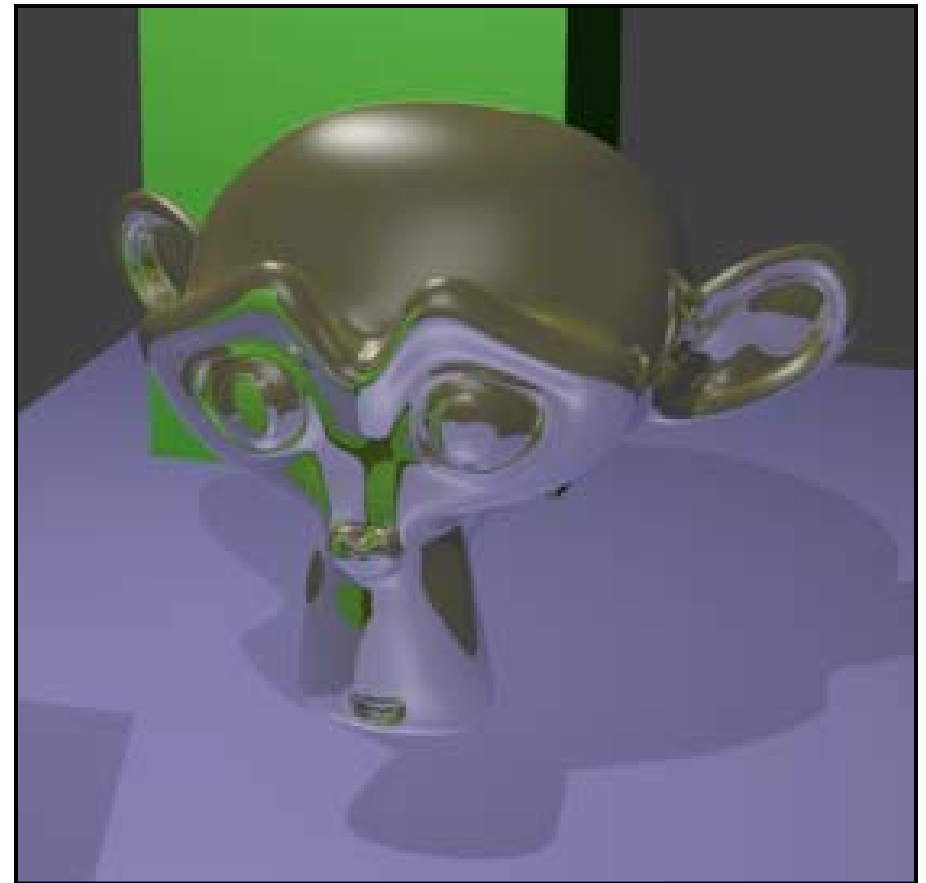




## Blender Ray-tracing Example



Refraction



Reflection

## More Ray-tracing Examples



Oregon State  
University  
Computer Graphics

Quake 4 Ray-Tracing Project

## More Ray-tracing Examples



IBM's Cell Interactive Ray-tracer

## More Ray-tracing Examples



**Bunkspeed**

## Subsurface Scattering

- Subsurface Scattering mathematically models light bouncing around within an object before coming back out.
- This is a good way to render skin, wax, milk, etc.



**Original rendering**

**Subsurface scattering**

