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CA-3

Solutions :-

Q1.)

Soln :- To obtain the equations of two lines of regression for the following data :-

X	Y	X^2	Y^2	XY
65	67	4225	4489	4355
66	68	4356	4624	4488
67	65	4489	4225	4355
67	68	4489	4624	4556
68	72	4624	5184	4896
69	72	4761	5184	4968
70	69	4900	4761	4830
72	71	5184	5041	5112
$\Sigma X = 544$	$\Sigma Y = 552$	$\Sigma X^2 = 37028$	$\Sigma Y^2 = 38132$	$\Sigma XY = 37560$

No. of term (n) = 8

$$\bar{X} = \frac{\Sigma X}{n} = \frac{544}{8} = 68$$

$$\bar{Y} = \frac{\Sigma Y}{n} = \frac{552}{8} = 69$$

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We know,
Regression lines of
X on Y :-

$$\Rightarrow x - \bar{x} = b_{xy}(y - \bar{y}) \rightarrow (i)$$

and

Y on X :-

$$\Rightarrow y - \bar{y} = b_{yx}(x - \bar{x}) \rightarrow (ii)$$

So, we need find b_{xy} and b_{yx} :-

$$\Rightarrow b_{xy} = \frac{n.\Sigma xy - (\Sigma x)(\Sigma y)}{n.\Sigma y^2 - (\Sigma y)^2}$$

$$= \frac{8 \times 37560 - 544 \times 552}{8 \times 38132 - (552)^2}$$

$$= \frac{300480 - 300288}{305056 - 304704}$$

$$= \frac{192}{352} = 0.54$$

$$\Rightarrow b_{yx} = \frac{n.\Sigma xy - (\Sigma x)(\Sigma y)}{n.\Sigma x^2 - (\Sigma x)^2}$$

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$$= \frac{8 \times 37560 - 544 \times 552}{8 \times 37028 - (544)^2}$$

$$= \frac{300480 - 300288}{296224 - 295936}$$

$$= \frac{192}{288} = 0.66$$

Putting all the values to eqn (i) and (ii), we get

Now By solving eqn (i), we get,

$\Rightarrow X - 68 = 0.54(Y - 69)$

$\Rightarrow X - 68 = 0.54Y - 0.54 \times 69$

$\Rightarrow X - 68 = 0.54Y - 37.26$

$\Rightarrow X = 0.54Y - 37.26 + 68$

$\Rightarrow X = 0.54Y - 30.74 \rightarrow$ (iii)

Now By solving eqn (ii), we get,

$\Rightarrow Y - 69 = 0.66(X - 68)$

$\Rightarrow Y - 69 = 0.66X - 0.66 \times 68$

$\Rightarrow Y - 69 = 0.66X - 44.88$

$\Rightarrow Y = 0.66X - 44.88 + 69$

$\Rightarrow Y = 0.66X - 24.12 \rightarrow$ (iv)

So, The two lines of Regression are $X = 0.54Y - 30.74$ and
$Y = 0.66X - 24.12$

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Now, we need to estimate X for Y when $Y = 70$.
So putting the value of Y in eqn (iii), we get

$$\Rightarrow X = 0.54Y + 30.74$$
$$\Rightarrow X = 0.54 \times 70 + 30.74$$
$$\Rightarrow X = 37.8 + 30.74$$
$$X = 68.54$$

Hence, The value of $X = 68.54$ for X for Y when $Y = 70$.

Q2.)
Soln :- X and Y are two variables.

r_{XY} is the correlation coefficient betwee X and Y.

$$u = \frac{x - a}{h} \quad ; \quad h > 0$$

$$v = \frac{y - b}{k} \quad ; \quad k > 0$$

$$X = a + hu \quad ; \quad Y = b + kv$$
(where a, b, h and k are constants)

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$$\bar{X} = a + h\bar{u} \;;\; \bar{Y} = b + k\bar{V}$$

$$X - \bar{X} = h(u - \bar{u}) \;;\; (Y - \bar{Y}) = k(v - \bar{v})$$

$$r_{XY} = \frac{Cov(X,Y)}{\sigma_X \, \sigma_Y} = \frac{E((X-\bar{X})(Y-\bar{Y}))}{\sqrt{E(X-\bar{X})^2}\sqrt{E(Y-\bar{Y})^2}}$$

$$= \frac{E(h(u-\bar{u}) \cdot k(v-\bar{v}))}{\sqrt{E\,h^2(u-\bar{u})^2} \cdot \sqrt{E\,k^2(v-v)^2}}$$

$$= \frac{hk\,E(u-\bar{u})(v-\bar{v})}{\sqrt{h^2k^2 \cdot E(u-\bar{u})^2 \cdot E(v-\bar{v})^2}}$$

$$= \frac{\cancel{hk}}{\cancel{hk}}\, r_{uv}$$

$$\therefore\; r_{XY} = r_{uv}$$

Hence, The Correlation Coefficient is independent of the change in origin and scale.

For Example :-

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For Example :-
(To suggest that the correlation coefficient is independent of the change in origin and scale.)

X	Y	XY	X^2	Y^2
82	70	5740	6724	4000
78	74	5772	6084	5476
75	78	5850	5625	6084
80	75	6000	6400	5625
95	80	7600	9025	6400
$\Sigma X = 410$	$\Sigma Y = 377$	$\Sigma XY = 30962$	$\Sigma X^2 = 33850$	$\Sigma Y^2 = 28485$

Coefficient Correlation $(r_{XY}) = \dfrac{Cov(X,Y)}{\sigma X \times \sigma Y}$

$$r_{XY} = \frac{\Sigma XY - \dfrac{(\Sigma X)(\Sigma Y)}{n}}{\sqrt{\left(\Sigma X^2 - \dfrac{(\Sigma X)^2}{n}\right)\left(\Sigma Y^2 - \dfrac{(\Sigma Y)^2}{n}\right)}}$$

$$= \frac{30962 - \dfrac{(410)(377)}{5}}{\sqrt{\left(33,858 - \dfrac{(410)^2}{5}\right)\left(28485 - \dfrac{(377)^2}{5}\right)}}$$

$$= \frac{(30962 - 30914)}{\sqrt{(33858 - 33620)\ (28485 - 28425.8)}}$$

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$$= \frac{.48}{\sqrt{(238)(59.2)}} = 0.40$$

Now,

$$U_x = \frac{X - A}{h} = \frac{X - 80}{h};$$

Here $A = 80$

$$V_y = \frac{Y - B}{k} = \frac{Y - 70}{k}; \quad k = h = 2$$

Here $B = 70$

U_x	V_y	$U_x V_y$	U_x^2	V_y^2
1	0	0	1	0
-1	2	-2	1	4
-2.5	4	-10	6.25	16
0	2.5	0	0	6.25
7.5	5	37.5	56.25	25
$\Sigma U_x = 5$	$\Sigma V_y = 13.5$	$\Sigma U_x V_y = 25.5$	$\Sigma U_x^2 = 64.5$	$\Sigma V_y^2 = 51.25$

$$r_{U_x V_y} = \frac{25.5 - \frac{(5)(13.5)}{5}}{\sqrt{(64.5 - 5)(51.25 - 36.45)}}$$

$$= \frac{12}{\sqrt{(59.5)(14.8)}} = \frac{12}{\sqrt{880.6}}$$

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$$r_{U_x V_y} = \frac{12}{29.6749}$$

$$= 0.40$$

$\therefore \cancel{r_{xy} = r_{U_x V_y}}$

$$\therefore \quad r_{xy} = r_{U_x V_y}$$

Q3>
Soln :- Give $E(X_i) = \mu$ and
$Var(X_i) = \sigma^2$

$\cancel{T_1 = 2X_1 + 4X_2 - 3X_3}$
$\cancel{T_2 = 2X_1 - 2X_2 + 3X_3}$
$\cancel{T_3 = 2X_1 -}$

$$T_1 = \frac{2X_1 + 4X_2 - 3X_3}{2}$$

$$T_2 = 2X_1 - 2X_2 + 3X_3$$

$$T_3 = \frac{2X_1 - 1X_2 + X_3}{3}$$

i> To find T_1 and T_2 are unbiased estimator of mean :-

$$E(T_1) = E\left[\frac{2X_1 + 4X_2 - 3X_3}{2}\right]$$

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$$= \frac{1}{2} \sum_{i=1}^{3} [2X_1 + 4X_2 - 3X_3]$$

$$= \frac{1}{2} [2\mu + 4\mu - 3\mu]$$

$$= \frac{3\mu}{2}$$

$\therefore$ T_1 is not an unbiased estimator of mean.

$$E(T_2) = E[2X_1 - 2X_2 + 3X_3]$$

$$= \sum_{i=1}^{3} E[2X_1 - 2X_2 + 3X_3]$$

$$= 2\mu - 2\mu + 3\mu$$

$$= 3\mu$$

$\therefore$ T_2 is not an unbiased estimator of mean.

ii> To find λ such that T_3 is unbiased estimator of mean (given) :-

Given $E(T_3) = \mu$

$$\Rightarrow E(T_3) = E\left[\frac{2X_1 - \lambda X_2 + X_3}{3}\right]$$

$$\Rightarrow \mu = E\left[\frac{2X_1 - \lambda X_2 + X_3}{3}\right]$$

$$\Rightarrow 3\mu = E(2X_1) - E(\lambda X_2) + E(X_3)$$

$$\Rightarrow 3\mu = 2\mu - \lambda\mu + \mu$$

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~~$\Rightarrow 3\mu = 3\mu - 3\lambda$~~

$$\Rightarrow 3\mu = 3\mu - \lambda\mu$$
$$\Rightarrow \lambda = 0$$

$\therefore$ The value of $\lambda = 0$

iii) To find the best estimator of mean among T_1, T_2 and T_3. So, we have to find minimum variance :-

$$Var(T_1) = Var\left[\frac{2X_1 + 4X_2 - 3X_3}{2}\right]$$

$$= \frac{1}{4}\left[Var(2X_1) + Var(4X_2) - Var(3X_3)\right]$$

~~$= \frac{1}{4}[4\times Var(X_1) + 16 Var$~~

$$= \frac{1}{4}\left[4\times Var(X_1) + 16\times Var(X_2) - 9\, Var(X_3)\right]$$

$$= \frac{1}{4}\left[4\sigma^2 + 16\sigma^2 - 9\sigma^2\right]$$

$$= \frac{11\sigma^2}{4} = 2.74\sigma^2$$

$$Var(T_2) = Var\left[2X_1 - 2X_2 + 3X_3\right]$$
$$= 4\,Var(X_1) - 4\,Var(X_2) + 9\,Var(X_3)$$
$$= 4\sigma^2 - 4\sigma^2 + 9\sigma^2$$
$$Var(T_2) = 9\sigma^2$$

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$$Var(T_3) = Var\left[\frac{2X_1 - \lambda X_2 + X_3}{3}\right]$$

$$= Var\left(\frac{2X_1}{3}\right) - Var\left(\frac{\lambda X_2}{3}\right) + Var\left(\frac{X_3}{3}\right)$$

$$= \frac{4}{9} Var(X_1) - \frac{\lambda^2}{9} Var(X_2) + \frac{1}{9} Var(X_3)$$

$$= \frac{4}{9} Var(X_1) - 0 + \frac{1}{9} Var(X_3) \quad [\because \lambda = 0]$$

$$= \frac{4}{9}\sigma^2 + \frac{1}{9}\sigma^2$$

$$= \frac{5}{9}\sigma^2 = 0.5\sigma^2$$

$\therefore$ The Most Efficient (Best) Estimator of Mean is T_3.

Q4.)

Soln :- To prove that if T is an unbiased estimator for t, T^2 is a biased estimator for t^2.

Now, let $E(T) = 0$

then $E(T^2) = Var(T) + (E(T))^2$

$= Var(T) + t^2$

if $Var(T) = 0$, then T is constant.

Now, we know $E(T) = 0$

$var(T) > 0$

$E(T^2) - (E(T))^2 > 0$

i.e., $E(T^2) > (E(T))^2$

$E(T^2) > t^2 \; (\because E(T) = t)$

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$\therefore E(T^2) \neq t^2$

Hence, T^2 is a biased estimator of t^2.

—— x —— x —— x ——
—— x —— x —— x ——