# Conflicting Forces: The role of positive and negative mechanisms in shaping educational outcomes

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**Abstract**

Much has been said about cognition's role in influencing children's and young adults' educational outcomes. However, there is a gap in the existing research regarding understanding the significance of noncognitive skills and behaviors in shaping these outcomes, especially when considering the variable gender. Although conclusions have been mixed, it is clear that multiple factors contribute to students' performance on standardized tests, educational achievements, and future job success. Using data from the Growing Up In Ireland longitudinal study, I find that cognition is still the most important predictor of academic performance even when we consider both measures of noncognition (SDQ and TIPI scores) and after controlling for SES and school characteristics, with different magnitude effects for males and females. I then perform an Oaxaca-Blinder decomposition and find that while all cognitive measures are responsible, to different degrees, for closing the gender grade gap, noncognitive variables mostly widen the gap. I also find a high degree of heterogeneity when I break down cognition between genders for different scores of noncognitive variables, which implies that having high or low scores in noncognition can be positive or detrimental depending on how individuals score in cognition. I then perform a simulation exercise aiming at closing the gender grade gap by varying the noncognitive scores by one standard deviation.

# 1 Introduction

In the modern world, a student's learning journey is closely tied to their environment, where various non-academic factors play a critical role in shaping their academic success. These factors include attitudes and behaviors, which are developed through the various spheres of influence surrounding students, such as their household and school settings. It is essential to tackle the challenge of catering to culturally diverse students with varying abilities and motivations for learning in the 21st century. We cannot merely enroll children in schools; we must ensure they complete the secondary cycle, gaining the knowledge and skills necessary for personal well-being and national development.

To understand the factors that determine educational outcomes, we can divide them into two blocks. The first block concerns a student's family background, including socio-economic status, family structure, family resources, and parental involvement, significantly influencing cognitive and noncognitive performance dimensions. The second block of factors relates to school factors and teaching practices. However, research has revealed that only a few studies examine the impact of a student's noncognitive skills on test scores, considering the many relationships a student has with their parents, teachers, and peers. Therefore, it is vital to investigate how noncognitive skills affect a child's educational outcome while considering these relationships.

We must recognize the significance of noncognitive factors in a student's academic success. By understanding the complex interplay between a student's environment, family background, and school factors, we can ensure that every child acquires the knowledge and skills necessary for their personal well-being and national development, regardless of their cultural background, abilities or motivations for learning.

# 2 Literature review

## 2.1 Education and skills

Education is, in its nature, multidimensional. It happens in a feedback loop across time and space and concerns many agents and institutions. Because resources such as money and labor are often

scarce, they must be allocated wisely. Therefore, it is essential to find ways to get the most significant outcome, given the resources available. One way of measuring the effectiveness of resources that go into educating students is through completed levels and years of education (quantity, educational attainment) and test scores (quality, educational achievement). More properly qualified and skillful students lead better lives and participate more actively in civic duties and the labor market (Oreopoulos and Salvanes (2011)). Kautz et al. (2014) note that a growing body of empirical research shows that noncognitive skills rival IQ in predicting educational attainment, labor market success, health, and criminality. Not only that, but the literature on the returns to education, both private and social, is vast and almost unanimous on the importance of improving educational outcomes for students. However, to improve educational outcomes, we must understand what goes into these outcomes, what factors contribute to more years of education and better test scores, and how we can measure such factors.

From an economics perspective, a skill is a form of human capital that increases productivity while the market defines its value, and education is perceived as an essential investment in skills development (Zhou (2017)). The literature divides skills into two categories: cognitive and noncognitive skills (a term first coined by James Heckman). According to Brunello and Schlotter (2011), there is evidence that high cognitive test scores are likely to result not only from high cognitive skills but also from high motivation and adequate personality traits, which can be considered noncognitive skills in some sense. More persistent students learn more and better. Borghans et al. (2008) state that a relationship between noncognitive skills and test scores can exist for two reasons: when there are sufficient rewards involved, people with favorable behavioral or labor-market outcomes might have an attitude to put in effort, and when rewards are not necessary in place, people who are motivated to perform well and who have a positive attitude toward work might be more inclined to do their best at tests. While cognition can be proxied by test scores, test scores do not simply reflect cognitive ability. Duckworth and Seligman (2006), in a study about the difference in test scores between girls and boys, concluded that because girls had better final grades than boys, even after controlling for measured IQ, they were significantly better at exercising self-discipline during the academic year. Balart et al. (2018) used performance decline in PISA test scores as a measure of effort. They found that the effect of cognitive skills was reduced by approximately forty percent in models that control for noncognitive factors.

Noncognitive skills enable people; consequently, greater skill levels foster social inclusion and promote economic and social mobility (Kautz et al. (2014)). Perseverance, dependability, and consistency are some of the most important predictors of grades in school, according to Bowles and Gintis (2011), and they are noncognitive skills. Noncognitive skills can be learned, and they can change over the life cycle. Skill development is a dynamic process in which the early years lay the foundation for successful investment in later years (Kautz et al. (2014)). Both cognitive and noncognitive skills have different levels of malleability depending on a child's developmental stage; they can change with age and with instruction: cognitive and noncognitive skills are highly malleable in the early years of a child's life, while noncognitive skills are more malleable than cognitive skills later on, during her adolescent years (Kautz et al. (2014)).

## 2.2 Defining and measuring skills

Suárez Pandiello et al. (2016) attest that social groups and public authorities ignore noncognitive abilities because of the lack of objective evaluation metrics and the difficulty in establishing standard definitions for the relevant social values. Humphries and Kosse (2017) note that the definition of noncognitive skills varies widely across fields such as Sociology, Psychology, and Economics and within fields of study. Labor economists see non-recognition skills as a second dimension of individual heterogeneity (next to cognitive skills); Education economists broadly categorize those as skills that are not captured by standardized tests (soft skills), and that can be measured by observing behavior. Behavioral economists are divided into two groups: one that sees noncognitive skills as a super-ordinate concept summarizing various specific concepts (i.e., economic preferences such as time and risk preferences), and the other that views them as personality measures (such as the Big Five). This divisiveness is challenging when comparing outcomes due to the different measurement instruments used. Humphries and Kosse (2017) reiterate that the construction of a noncognitive factor greatly influences what is being measured, and consensus on outcome interpretation is difficult due to the different motives behind the research and available data sources.

Noncognitive skills and abilities, unlike cognition, are challenging to define and measure. Currently, there is no systematic global measure of noncognitive skills. However, fortunately, the field has expanded enough, and a wide variety of instruments aimed at assessing these skills and abili-

ties have been created. Using measured behaviors to capture noncognitive skills, for example, is a promising, empirically practical approach, according to Kautz et al. (2014). Personality traits represent relatively persistent dimensions of the overall personality, and some play an important role in increasing productivity-enhancing skills. More broadly, economists often use the term noncognitive skills to account for traits specifically related to human capital outcomes (such as educational and labor market achievements), and in Psychology, personality traits are measured using psychometric constructs (Thiel and Thomsen (2013)). Therefore, economists and other social scientists can adapt such constructs to their respective fields of study to measure noncognitive skills.

# 3 Methodology

## 3.1 Data

### 3.1.1 Growing Up in Ireland

The data used in the analysis come from the first three waves of the Child Cohort ('98) of the Growing Up in Ireland (GUI) survey. The GUI is a national longitudinal study of children and young people that has been running since 2006. The study followed the progress of two groups of children: 8,568 9-year-olds (Cohort' 98), representing approximately 14% of all 9-year-olds in Ireland, and 10,000 9-month-olds (Cohort' 08), for the last fifteen years. Subsequent waves of the '98 cohort saw some drop-off in participation: 7,525 children (87.9%) in the second wave (2011-2012), 6,216 young adults (72.5%) in the third wave (2015-2016), and 5,190 young adults (60%) in the fourth wave (2018-2019). The survey stands out for its large, nationally representative sample and longitudinal nature. The first cohort sample was selected from clustering at the school level, and the second cohort was sampled randomly from the Child Benefit records. The members of Cohort '98 are now 25-26 years old and were interviewed and had their cognitive abilities tested in every wave so far:

- Wave one at nine years of age in 2007/2008,
- Wave two at 13 years of age in 2011/2012, and
- Wave three at 17/18 years of age in 2015/2016.

### 3.1.2 Cognitive and noncognitive instrument measurements

In wave 1, the children were tested in person in Reading and Maths (using the Drumcondra Reading and Maths tests, a set of curriculum-based standardized assessments of reading and maths achievement for primary school pupils in Ireland) and evaluated by their teacher at the school level. In wave 2, their verbal reasoning and numerical abilities were tested (using the Drumcondra Verbal Reasoning and Numerical Ability tests and the British Ability Scales, one of the leading standardized batteries in the UK for assessing a child's cognitive ability and educational achievement). In wave three, they were asked about Junior and Leaving Cert (if they already sat) results and asked for permission to link to the Central Admissions Office database in the future (if they still need to sit). Their cognitive ability was determined through a composite score derived from three assessments: a verbal fluency test, a vocabulary test, and a set of numeracy tests. The verbal fluency test encompassed two aspects: the FAS score, measuring the number of words generated beginning with F, A, or S in one minute, and the Animal Naming score, gauging the number of animal species named in one minute. The vocabulary test consisted of 20 items, each followed by a list of five words, requiring the selection of the word most closely related in meaning. The numeracy test evaluated performance in basic arithmetic through three mathematical calculations. These measures were combined through principal component analysis, yielding a single component representing cognitive ability, where higher scores indicated more remarkable ability.

Academic achievement at the third wave was assessed via the Junior Certificate Examination, a national exam taken by most Irish children around ages 15-16. Mandatory subjects are Irish, English, Maths, and History, and students can choose up to 10 subjects (with at least four mandatory plus two optional) in the areas of Arts and Humanities, Modern Languages, Sciences, and Applied Sciences. Before 2017 (when the survey took place), grades were given on a scale of A to F across different levels of the exam (Higher, Ordinary, Foundation). The Junior Certificate Examination in Ireland marks the end of three years of studying various subjects. It typically spans two to three weeks of individual subject exams at the end of the school year in June, and a student cannot fail the examination. Regardless of their examination results, all students progress to the next year of education if they wish to do so. The time frame between the Junior Certificate (ages 15-16) and

the age range of the GUI wave 3 participants (16-18) was relatively close. Because the Junior Cert syllabus and exam content are predetermined three years in advance, achieving success reflects the culmination of a structured curriculum and learning process. Given this foresight, one would anticipate that specific noncognitive skills are pivotal in shaping outcomes. These skills may include effective planning, adept time management, the ability to prioritize long-term goals over immediate gratification (such as opting to study for an exam well in advance rather than indulging in leisure activities), proficient organization and upkeep of study materials, and judicious allocation of time across a diverse array of academic subjects.

In all four waves of the Child Cohort, the study-child filled out a sixty-item questionnaire on their concept of self based on the Piers-Harris Children's Self-Concept Scale. One of the questionnaire domains (Intellectual and School Status - INT) concerns the child's evaluation of his or her abilities in terms of intellectual and academic tasks. Albeit indirect due to its self-reported nature, it also measures cognitive ability. About personality, one of the dimensions where non-cognition manifests itself (others being through behavioral problems, social skills, communication, self-esteem, persistence, locus of control, empathy, and impulsivity), the study-child was assessed utilizing the Ten Item Personality Inventory (TIPI), a brief instrument designed to assess the five-factor model (FFM) personality dimensions (Thørrisen and Sadeghi (2023)). Parent One (primary caregiver - PCG) and Parent Two (secondary caregiver - SCG) completed the scale regarding the study-child in wave three (PCG completed in all three waves). In wave three, the study child also filled out the scale, offering an external and self-assessed gauge of the study child's personality and ensuring consistency. This scale comprises ten items gauging five personality facets: Openness to Experience, Agreeableness, Conscientious, Extraversion, and Neuroticism. Each of the ten items was evaluated on a seven-point scale, from strongly disagree to strongly agree. Each dimension of personality included two statements with two descriptors each. The scores for each measure were derived by summing up both responses and dividing by two according to common practice in the literature (Gosling et al. (2003)). This was done by the GUI researchers and the final score for each item can be found in the AMF files of all waves.

Another dimensionality of emotional characteristics and behaviors covered in the GUI was measured using the Strengths and Difficulties Questionnaire (SDQ), which provides a measure of emotional and behavioral regulation, the Short Mood and Feelings Questionnaire (SMFQ), and the Centre for Epidemiological Studies Depression Scale (CES-D). Apart from all measures of cognitive and noncognitive skills, the questionnaires throughout all waves contain information on a wide range of aspects pertaining to household and school settings, parental incomes and educational achievements, relationships between the study child, her parents (Parenting Style), teachers and peers, attitudes, expectation for the future, health status and hobbies.

### 3.1.3 AMF versus RMF

The questionnaire data for all waves of the GUI is divided into two different types according to their availability: the Anonymised Microdata Files (AMF), which are available upon a Data Request Form for Research Purposes through the Irish Social Science Data Archive (ISSDA) website, and the Researcher Microdata Files (RMF), which can only be accessed after a lengthy application process (around eight weeks after the researcher gets appropriately vetted by the Central Statistics Office). Because both the study-child/young adult and her primary and secondary caregiver were assessed on many private topics (such as substance use and mental health disorders), getting access to certain parts of the data requires discretion and a reasonable justification from the researcher. Unfortunately, many other answers were also omitted from the AMF data, such as the exact response to the ten items that compose the TIPI scale and all the items in the SDQ for waves one, two, and three; the choice of college major (by code) and the exact number of points at the Leaving Cert in waves three and four, and the cultural background of the study-child (cultural differences are an essential determinant of noncognitive skills). Many other questions reflect how the study-child's environment contributes to the development of her cognitive and noncognitive skills, such as the level of parental involvement in the study-child's academic life, how far they expect the study-child to go in terms of formal education, how the teacher perceives study-child's skills and abilities in the classroom, the study-child's attitudes towards school, how much time she spends in cognitive-enabling activities such as reading for pleasure. Gathering relevant information and accessing the RMF is still a work in progress.

# 4 Model

The model I estimate is a multivariate multiple regression model: the response variables are Junior Cert scores in Maths and English, and the explanatory variables are two sets of non-cognitive measures: SDQ and TIPI scores. There are two vectors of control variables: one containing socioeconomic status characteristics such as gender, parental education (I only consider cases where I have data on both Primary and Secondary caregivers' education levels), and income quantile (equivalized), and one for school characteristics containing indicators for mixed school, DEIS school, and fee-paying school.

$$\text{Points\_JC}_{i,w,l} = \beta_0 + \mathbf{Cognition}_{i,w} + \mathbf{NonCognition}_{i,w,k}$$
$$+\mathbf{Cognition}_{i,w} * \mathbf{NonCognition}_{i,w,k} + \mathbf{Controls}_{i,w,n} + \epsilon_{i,w,l,k,n}$$

With

$$\mathbf{Cognition}_{i,w} = \begin{bmatrix}\beta_1 & \beta_2 & \beta_3\end{bmatrix} \begin{bmatrix}\text{Naming ability}_{i,w} \\ \text{Maths ability}_{i,w} \\ \text{Vocabulary ability}_{i,w}\end{bmatrix}$$

and

$$\mathbf{NonCognition}_{i,w,k} = \begin{bmatrix}\beta_3 & \beta_4 & \beta_5 & \beta_6\end{bmatrix} \begin{bmatrix}\text{SDQ - Emotional symptons}_{i,w,k} \\ \text{SDQ - Conduct}_{i,w,k} \\ \text{SDQ - Hyperactivity/inattention}_{i,w,k} \\ \text{SDQ - Peer-relationship problems}_{i,w,k}\end{bmatrix}$$

or

$$\mathbf{NonCognition}_{i,w,k} = \begin{bmatrix}\beta_4 & \beta_5 & \beta_6 & \beta_7 & \beta_8\end{bmatrix} \begin{bmatrix}\text{TIPI - Agreeable}_{i,w,k} \\ \text{TIPI - Conscientious}_{i,w,k} \\ \text{TIPI - Emotional Stability/Neuroticism}_{i,w,k} \\ \text{TIPI - Extravert}_{i,w,k} \\ \text{TIPI - Openness}_{i,w,k}\end{bmatrix}$$

for $i$ = individual observation, $l$ = Maths, English, $w$ = Wave 2 (W2) for explanatory variables and = Wave 3 (W3) for the dependent variable, and $k$ = primary caregiver (PCG), and $n$ represents vectors of socio-economic status and school characteristics.

The Strength and Difficulties Questionnaire (SDQ) measures two distinct dimensions of non-cognitive skills: behavioral skills and emotional skills. Twenty items of the SDQ comprise a total scale made up of four sub-scales, each containing five items. These sub-scales tap into emotional symptoms (e.g. often unhappy, downhearted, or tearful); conduct problems (e.g. often fights with other children or bullies them); hyperactivity/inattention (e.g. restless, overactive, cannot stay still for long); and peer relationship problems (e.g. picked on or bullied by other children). Scores on each sub-scale can range from 0 to 10, where 10 indicates a high degree of difficulty and 0 the absence of any problems in the relevant domain. I inverted the scales so that 10 is better and 0 is worse. The SDQ was completed by both the child's primary caregiver and teacher in wave 1, and by the child's primary caregiver in wave 2.

One of the dimensions where non-cognition manifests itself (others being through behavioral problems, social skills, communication, self-esteem, persistence, locus of control, empathy, and impulsivity), the study-child was assessed utilizing the Ten Item Personality Inventory (TIPI), a brief instrument designed to assess the five-factor model (FFM) personality dimensions. PCG and Parent Two (secondary caregiver - SCG) completed the scale regarding the study-child in wave 3 (PCG completed in waves 2 and 3). In wave 3, the study child (YP) also filled out the scale, offering an external and self-assessed gauge of the study child's personality and ensuring consistency. This scale comprises ten items gauging five personality facets: Openness to Experience, Agreeableness, Conscientious, Extraversion, and Neuroticism (Emotional stability). Each of the ten items was evaluated on a seven-point scale, from strongly disagree to strongly agree. Each dimension of personality included two statements with two descriptors each. The scores for each measure were derived by summing up both responses and dividing by two according to common practice in the literature. This was done by the GUI researchers and the final score for each item can be found in the AMF files.

# 5 Regression results - non-standardized (baseline + controls)

### 5.0.1 SDQ PCG, Wave 2

1) Cognitive and noncognitive indicators (SDQ according to PCG in W2) on **Maths** and **English** scores in the Junior Cert (W3).

Table 1: Cognitive and noncognitive indicators (SDQ according to PCG in W2) on Maths score in the Junior Cert (W3).

| Dependent Variable: | Maths | | | |
|---|---|---|---|---|
| Model: | (1) | (2) | (3) | (4) |
| *Variables* | | | | |
| Constant | 3.556*** | 3.066*** | 3.987*** | 3.371*** |
| | (0.1703) | (0.2097) | (0.1800) | (0.2193) |
| Vocal reasoning | 0.0186*** | 0.0140*** | 0.0173*** | 0.0139*** |
| | (0.0009) | (0.0011) | (0.0010) | (0.0011) |
| Numerical ability | 0.0304*** | 0.0271*** | 0.0300*** | 0.0264*** |
| | (0.0009) | (0.0011) | (0.0010) | (0.0011) |
| Matrices | 0.0127*** | 0.0126*** | 0.0125*** | 0.0125*** |
| | (0.0010) | (0.0012) | (0.0011) | (0.0012) |
| Emotional symptoms | 0.0231** | 0.0283** | 0.0279*** | 0.0300** |
| | (0.0100) | (0.0116) | (0.0104) | (0.0119) |
| Conduct problems | 0.0673*** | 0.0585*** | 0.0615*** | 0.0543*** |
| | (0.0146) | (0.0171) | (0.0149) | (0.0174) |
| Hyperactivity/Inattention | 0.1005*** | 0.0912*** | 0.0944*** | 0.0943*** |
| | (0.0086) | (0.0100) | (0.0089) | (0.0102) |
| Peer relationship problems | 0.0142 | 0.0015 | 0.0055 | -0.0045 |
| | (0.0128) | (0.0146) | (0.0132) | (0.0150) |
| Male | | -0.1146*** | -0.1308*** | -0.1075*** |
| | | (0.0389) | (0.0358) | (0.0401) |
| *Controls* | | | | |
| SES | No | Yes | No | Yes |
| School | No | No | Yes | Yes |
| *Fit statistics* | | | | |
| Observations | 5,664 | 4,047 | 5,185 | 3,805 |
| $R^2$ | 0.48971 | 0.50747 | 0.50680 | 0.51194 |
| Adjusted $R^2$ | 0.48908 | 0.50613 | 0.50575 | 0.51014 |

*IID standard-errors in parentheses*
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 2: Cognitive and noncognitive indicators (SDQ according to PCG in W2) on English score in the Junior Cert (W3).

| Dependent Variable: | English | | | |
|---|---|---|---|---|
| Model: | (1) | (2) | (3) | (4) |
| *Variables* | | | | |
| Constant | 6.402*** | 6.604*** | 6.922*** | 6.825*** |
| | (0.1500) | (0.1805) | (0.1561) | (0.1878) |
| Vocal reasoning | 0.0215*** | 0.0194*** | 0.0210*** | 0.0190*** |
| | (0.0008) | (0.0009) | (0.0008) | (0.0009) |
| Numerical ability | 0.0102*** | 0.0106*** | 0.0115*** | 0.0106*** |
| | (0.0008) | (0.0009) | (0.0008) | (0.0009) |
| Matrices | 0.0047*** | 0.0030*** | 0.0037*** | 0.0028*** |
| | (0.0009) | (0.0011) | (0.0009) | (0.0011) |
| Emotional symptoms | -0.0189** | 0.0023 | -0.0026 | 0.0035 |
| | (0.0088) | (0.0099) | (0.0090) | (0.0101) |
| Conduct problems | 0.0060 | -0.0102 | 0.0078 | -0.0123 |
| | (0.0128) | (0.0146) | (0.0129) | (0.0149) |
| Hyperactivity/Inattention | 0.1034*** | 0.0780*** | 0.0790*** | 0.0785*** |
| | (0.0075) | (0.0086) | (0.0077) | (0.0087) |
| Peer relationship problems | 0.0627*** | 0.0508*** | 0.0454*** | 0.0437*** |
| | (0.0112) | (0.0126) | (0.0114) | (0.0129) |
| Male | | -0.4327*** | -0.4231*** | -0.4163*** |
| | | (0.0334) | (0.0309) | (0.0343) |
| *Controls* | | | | |
| SES | No | Yes | No | Yes |
| School | No | No | Yes | Yes |
| *Fit statistics* | | | | |
| Observations | 5,647 | 4,033 | 5,168 | 3,791 |
| $R^2$ | 0.33273 | 0.36447 | 0.37022 | 0.36829 |
| Adjusted $R^2$ | 0.33190 | 0.36274 | 0.36888 | 0.36594 |

*IID standard-errors in parentheses*
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

### 5.0.2 TIPI PCG, Wave 2

1) Cognitive and noncognitive indicators (Study-child's TIPI according to their PCG in W2) on **Maths** and **English** scores in the Junior Cert (W3).

Table 3: Cognitive and noncognitive indicators (TIPI according to PCG in W2) on Maths score in the Junior Cert (W3).

| Dependent Variable: | Maths | | | |
|---|---|---|---|---|
| Model: | (1) | (2) | (3) | (4) |
| *Variables* | | | | |
| Constant | 4.648*** | 3.953*** | 5.040*** | 4.175*** |
| | (0.1242) | (0.1561) | (0.1320) | (0.1653) |
| Vocal reasoning | 0.0203*** | 0.0155*** | 0.0189*** | 0.0155*** |
| | (0.0009) | (0.0011) | (0.0010) | (0.0011) |
| Numerical ability | 0.0317*** | 0.0283*** | 0.0315*** | 0.0278*** |
| | (0.0009) | (0.0011) | (0.0010) | (0.0011) |
| Matrices | 0.0136*** | 0.0133*** | 0.0131*** | 0.0130*** |
| | (0.0010) | (0.0012) | (0.0011) | (0.0013) |
| Agreeableness | 0.0073 | 0.0136 | -0.0003 | 0.0158 |
| | (0.0092) | (0.0104) | (0.0094) | (0.0106) |
| Conscientiouness | 0.0707*** | 0.0610*** | 0.0661*** | 0.0580*** |
| | (0.0086) | (0.0098) | (0.0089) | (0.0100) |
| Emotional stability | 0.0330*** | 0.0225** | 0.0356*** | 0.0243** |
| | (0.0091) | (0.0102) | (0.0094) | (0.0105) |
| Extraversion | -0.0076 | -0.0089 | -0.0113 | -0.0108 |
| | (0.0088) | (0.0099) | (0.0090) | (0.0102) |
| Openness | -0.0342*** | -0.0138 | -0.0278*** | -0.0073 |
| | (0.0097) | (0.0109) | (0.0100) | (0.0112) |
| Male | | -0.1588*** | -0.1891*** | -0.1512*** |
| | | (0.0387) | (0.0355) | (0.0399) |
| *Controls* | | | | |
| SES | No | Yes | No | Yes |
| School | No | No | Yes | Yes |
| *Fit statistics* | | | | |
| Observations | 5,654 | 4,043 | 5,177 | 3,801 |
| $R^2$ | 0.47234 | 0.49309 | 0.49200 | 0.49696 |
| Adjusted $R^2$ | 0.47159 | 0.49158 | 0.49082 | 0.49497 |

*IID standard-errors in parentheses*
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 4: Cognitive and noncognitive indicators (TIPI according to PCG in W2) on English score in the Junior Cert (W3).

| Dependent Variable: | English | | | |
|---|---|---|---|---|
| Model: | (1) | (2) | (3) | (4) |
| *Variables* | | | | |
| Constant | 7.057*** | 7.094*** | 7.550*** | 7.242*** |
| | (0.1100) | (0.1343) | (0.1145) | (0.1414) |
| Vocal reasoning | 0.0226*** | 0.0203*** | 0.0219*** | 0.0199*** |
| | (0.0008) | (0.0009) | (0.0008) | (0.0009) |
| Numerical ability | 0.0115*** | 0.0117*** | 0.0129*** | 0.0117*** |
| | (0.0008) | (0.0009) | (0.0008) | (0.0009) |
| Matrices | 0.0051*** | 0.0032*** | 0.0038*** | 0.0030*** |
| | (0.0009) | (0.0011) | (0.0009) | (0.0011) |
| Agreeableness | 0.0160** | 0.0190** | 0.0108 | 0.0198** |
| | (0.0081) | (0.0089) | (0.0081) | (0.0091) |
| Conscientiouness | 0.0480*** | 0.0368*** | 0.0354*** | 0.0389*** |
| | (0.0076) | (0.0084) | (0.0077) | (0.0086) |
| Emotional stability | 0.0052 | 0.0007 | 0.0095 | 0.0008 |
| | (0.0081) | (0.0087) | (0.0081) | (0.0089) |
| Extraversion | 0.0122 | 0.0102 | 0.0095 | 0.0099 |
| | (0.0078) | (0.0085) | (0.0078) | (0.0087) |
| Openness | 0.0070 | 0.0097 | 0.0032 | 0.0078 |
| | (0.0086) | (0.0094) | (0.0086) | (0.0096) |
| Male | | -0.4751*** | -0.4786*** | -0.4570*** |
| | | (0.0331) | (0.0307) | (0.0340) |
| *Controls* | | | | |
| SES | No | Yes | No | Yes |
| School | No | No | Yes | Yes |
| *Fit statistics* | | | | |
| Observations | 5,637 | 4,029 | 5,160 | 3,787 |
| $R^2$ | 0.30600 | 0.35012 | 0.35434 | 0.35501 |
| Adjusted $R^2$ | 0.30501 | 0.34818 | 0.35283 | 0.35244 |

*IID standard-errors in parentheses*
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

# 6 Regression results - standardized + PCA + controls + interaction

### 6.0.1 SDQ PCG, Wave 2

1) Cognitive and noncognitive indicators (SDQ according to PCG in W2) on **Maths** and **English** scores in the Junior Cert (W3).

Table 5: Cognitive and noncognitive indicators (SDQ according to PCG in W2) on Maths score in the Junior Cert (W3).

| Dependent Variable: | Maths | | |
|---|---|---|---|
| Model: | (1) | (2) | (3) |
| *Variables* | | | |
| Constant | 9.570*** | 9.705*** | 9.729*** |
| | (0.0168) | (0.0352) | (0.0355) |
| Principal component (PC) | 0.7830*** | 0.6689*** | 0.6703*** |
| | (0.0131) | (0.0164) | (0.0164) |
| Emotional symptoms (std) | 0.0499*** | 0.0565** | 0.0462** |
| | (0.0192) | (0.0226) | (0.0227) |
| Conduct problems (std) | 0.0865*** | 0.0716*** | 0.0740*** |
| | (0.0196) | (0.0233) | (0.0234) |
| Hyperactivity/Inattention (std) | 0.2415*** | 0.2267*** | 0.2166*** |
| | (0.0198) | (0.0235) | (0.0236) |
| Peer relationship problems (std) | 0.0179 | -0.0031 | 0.0014 |
| | (0.0184) | (0.0215) | (0.0218) |
| Male | | -0.0702* | -0.0693* |
| | | (0.0402) | (0.0402) |
| Principal component (PC) × Emotional symptoms (std) | | | -0.0316** |
| | | | (0.0161) |
| Principal component (PC) × Conduct problems (std) | | | 0.0126 |
| | | | (0.0181) |
| Principal component (PC) × Hyperactivity/Inattention (std) | | | -0.0565*** |
| | | | (0.0171) |
| Principal component (PC) × Peer relationship problems (std) | | | -0.0134 |
| | | | (0.0148) |
| *Controls* | | | |
| SES | No | Yes | Yes |
| School | No | Yes | Yes |
| *Fit statistics* | | | |
| Observations | 5,664 | 3,805 | 3,805 |
| $R^2$ | 0.47612 | 0.50254 | 0.50572 |
| Adjusted $R^2$ | 0.47566 | 0.50096 | 0.50363 |

*IID standard-errors in parentheses*
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 6: Cognitive and noncognitive indicators (SDQ according to PCG in W2) on English score in the Junior Cert (W3).

| Dependent Variable: | English | | |
|---|---|---|---|
| Model: | (1) | (2) | (3) |
| *Variables* | | | |
| Constant | 10.13*** | 10.39*** | 10.41*** |
| | (0.0148) | (0.0303) | (0.0305) |
| Principal component (PC) | 0.4641*** | 0.4161*** | 0.4170*** |
| | (0.0115) | (0.0141) | (0.0141) |
| Emotional symptoms (std) | -0.0274 | 0.0120 | 0.0021 |
| | (0.0169) | (0.0194) | (0.0194) |
| Conduct problems (std) | 0.0042 | -0.0220 | -0.0183 |
| | (0.0172) | (0.0201) | (0.0201) |
| Hyperactivity/Inattention (std) | 0.2440*** | 0.1902*** | 0.1795*** |
| | (0.0174) | (0.0203) | (0.0202) |
| Peer relationship problems (std) | 0.0736*** | 0.0503*** | 0.0573*** |
| | (0.0162) | (0.0185) | (0.0188) |
| Male | | -0.4029*** | -0.4007*** |
| | | (0.0346) | (0.0345) |
| Principal component (PC) × Emotional symptoms (std) | | | -0.0176 |
| | | | (0.0139) |
| Principal component (PC) × Conduct problems (std) | | | 0.0266* |
| | | | (0.0157) |
| Principal component (PC) × Hyperactivity/Inattention (std) | | | -0.0735*** |
| | | | (0.0148) |
| Principal component (PC) × Peer relationship problems (std) | | | -0.0264** |
| | | | (0.0128) |
| *Controls* | | | |
| SES | No | Yes | Yes |
| School | No | Yes | Yes |
| *Fit statistics* | | | |
| Observations | 5,647 | 3,791 | 3,791 |
| $R^2$ | 0.31011 | 0.34688 | 0.35392 |
| Adjusted $R^2$ | 0.30950 | 0.34480 | 0.35118 |

*IID standard-errors in parentheses*
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

### 6.0.2 TIPI PCG, Wave 2

1) Cognitive and noncognitive indicators (Study-child's TIPI according to their PCG in W2) on **Maths** and **English** scores in the Junior Cert (W3).

Table 7: Cognitive and noncognitive indicators (TIPI according to PCG in W2) on Maths score in the Junior Cert (W3).

| Dependent Variable: | Maths | | |
|---|---|---|---|
| Model: | (1) | (2) | (3) |
| *Variables* | | | |
| Constant | 9.571*** | 9.737*** | 9.751*** |
| | (0.0171) | (0.0355) | (0.0356) |
| Principal component (PC) | 0.8351*** | 0.7154*** | 0.7176*** |
| | (0.0127) | (0.0161) | (0.0161) |
| Agreeableness (std) | 0.0092 | 0.0290 | 0.0309 |
| | (0.0181) | (0.0210) | (0.0210) |
| Conscientiouness (std) | 0.1580*** | 0.1325*** | 0.1371*** |
| | (0.0181) | (0.0210) | (0.0211) |
| Emotional stability (std) | 0.0677*** | 0.0474** | 0.0580*** |
| | (0.0184) | (0.0210) | (0.0211) |
| Extraversion (std) | -0.0191 | -0.0200 | -0.0181 |
| | (0.0177) | (0.0203) | (0.0204) |
| Openness (std) | -0.0725*** | -0.0211 | -0.0219 |
| | (0.0179) | (0.0207) | (0.0208) |
| Male | | -0.1148*** | -0.1167*** |
| | | (0.0401) | (0.0400) |
| Principal component (PC) × Agreeableness (std) | | | -0.0173 |
| | | | (0.0161) |
| Principal component (PC) × Conscientiouness (std) | | | -0.0136 |
| | | | (0.0161) |
| Principal component (PC) × Emotional stability (std) | | | -0.0678*** |
| | | | (0.0156) |
| Principal component (PC) × Extraversion (std) | | | 0.0021 |
| | | | (0.0149) |
| Principal component (PC) × Openness (std) | | | 0.0047 |
| | | | (0.0154) |
| *Controls* | | | |
| SES | No | Yes | Yes |
| School | No | Yes | Yes |
| *Fit statistics* | | | |
| Observations | 5,654 | 3,801 | 3,801 |
| $R^2$ | 0.45811 | 0.48680 | 0.49051 |
| Adjusted $R^2$ | 0.45753 | 0.48504 | 0.48809 |

*IID standard-errors in parentheses*
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

Table 8: Cognitive and noncognitive indicators (TIPI according to PCG in W2) on English score in the Junior Cert (W3).

| Dependent Variable: | English | | |
|---|---|---|---|
| Model: | (1) | (2) | (3) |
| *Variables* | | | |
| Constant | 10.13*** | 10.41*** | 10.42*** |
| | (0.0151) | (0.0305) | (0.0306) |
| Principal component (PC) | 0.5039*** | 0.4464*** | 0.4483*** |
| | (0.0113) | (0.0138) | (0.0138) |
| Agreeableness (std) | 0.0271* | 0.0349* | 0.0369** |
| | (0.0160) | (0.0180) | (0.0181) |
| Conscientiouness (std) | 0.0912*** | 0.0755*** | 0.0801*** |
| | (0.0160) | (0.0180) | (0.0181) |
| Emotional stability (std) | 0.0131 | 0.0045 | 0.0091 |
| | (0.0163) | (0.0180) | (0.0182) |
| Extraversion (std) | 0.0181 | 0.0124 | 0.0147 |
| | (0.0156) | (0.0174) | (0.0175) |
| Openness (std) | 0.0065 | 0.0106 | 0.0138 |
| | (0.0159) | (0.0178) | (0.0179) |
| Male | | -0.4452*** | -0.4443*** |
| | | (0.0344) | (0.0343) |
| Principal component (PC) × Agreeableness (std) | | | -0.0120 |
| | | | (0.0140) |
| Principal component (PC) × Conscientiouness (std) | | | -0.0185 |
| | | | (0.0138) |
| Principal component (PC) × Emotional stability (std) | | | -0.0262* |
| | | | (0.0134) |
| Principal component (PC) × Extraversion (std) | | | -0.0074 |
| | | | (0.0129) |
| Principal component (PC) × Openness (std) | | | -0.0234* |
| | | | (0.0134) |
| *Controls* | | | |
| SES | No | Yes | Yes |
| School | No | Yes | Yes |
| *Fit statistics* | | | |
| Observations | 5,637 | 3,787 | 3,787 |
| $R^2$ | 0.28163 | 0.33165 | 0.33481 |
| Adjusted $R^2$ | 0.28086 | 0.32935 | 0.33163 |

*IID standard-errors in parentheses*
*Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*

## 6.1 Discussion

I ran a basic OLS with non-standardized variables for the first part of the analysis. I kept all three cognitive indicators (Verbal reasoning, Numerical ability, and Matrices - the percentage of correct answers) as they were originally in the dataset, so the scale is preserved. Because of how the SDQ questionnaire was structured (where a bigger value meant a more negative response) I inverted the scale to interpret the other non-cognitive variable indicators more cohesively. In the second part of the analysis, I ran PCA for the cognitive indicator variables and used the first component (which explained 63% of the variance) as a measure of cognition. Because PCA standardizes the variables, I also standardized (mean zero, standard variation of one) the other non-cognitive indicator variables and the controls. Since I would interact the cognitive principal component with the non-cognitive indicators, it made sense to work with all the variables on the same scale. It also facilitated cross-regression comparisons with different predictors.

Overall we see that cognition is still the most important predictor of academic performance even when we consider both measures of non-cognition (SDQ and TIPI scores) and after controlling for SES and school characteristics. The effects are bigger in magnitude for Maths than for English, and we see consistent differences in the profile of students who score higher in Maths and English, albeit different from each other. The difference is higher in the control variables, where being male is four times more detrimental to English scores than Maths scores, and parents' highest education level and equivalized income are twice more relevant in Maths scores than in English.

In terms of the difference between cognitive and non-cognitive indicator variables, as we can see in Tables 1 and 5 the higher a student scores on the SDQ Hyperactivity/Inattention scale (the less hyperactive they are), the higher is their score in Maths, with higher scores in the SDQ Conduct and Emotional scales also being associated with higher scores in Maths (albeit smaller in magnitude and significance); while all cognitive indicator variables are highly significant to the Maths score, a student's numerical ability is the most important predictor in magnitude. For English, Tables 2 and 6, both SDQ Hyperactivity/Inattention and Peer-relationship are highly significant (with the former having a similar magnitude to the coefficient when the dependent variable is Maths score). The SDQ Hyperactivity/Inattention coefficient is almost half the magnitude of the cognitive principal component (Table 6) when I include control and interaction terms (for Maths score it is a third). The coefficient for Verbal reasoning (cognitive indicator) is the biggest in magnitude and the most significant when the dependent variable is the English score.

The Male variable always has a negative and highly significant coefficient, and its magnitude is more than four times bigger for English than for Maths (in module), regardless of non-cognitive indicators and added controls. The primary caregiver's education level (the mother's) matters more than the secondary caregiver's (the father's), and both are twice as big in magnitude for Maths when compared to English. The most important variable control is the DEIS indicator: it is highly significant and negative in all regressions, and its absolute magnitude is two times the size of the second-biggest control, the primary caregiver's education level, even after adding all controls.

When the indicators for non-cognition are the TIPI scores in Agreeableness, Conscientiousness, Extraversion, Emotional Stability, and Openness, we get similar coefficients (in significance and magnitude within a 10-15% margin) for the cognitive indicator variables in their original form and also the principal component as we did for the SDQ indicators. This means that Numerical ability is the most important cognitive predictor for Maths scores and Verbal reasoning is the most important cognitive predictor for English scores. In terms of non-cognitive indicators, the highest a student scores in Conscientiousness and Emotional stability, the higher they score in Maths (with the former having a coefficient twice as large in magnitude as the latter's coefficient in Table 3 and three times larger in Table 7). For English, the most significant non-cognitive indicator coefficients are Conscientiousness and Agreeableness (again with the former having a coefficient twice as large in magnitude as the latter's - Tables 4 and 8).

Concerning the interaction terms, when they are significant, they are negative and small in magnitude in comparison to all the other variables. The only highly significant ones are Hyperactivity/Inattention (Maths and English SDQ) and Emotional stability (Maths TIPI). This negative relationship is interesting and I further study it in the next section. The highest Adjusted $R^2$ are found in Tables 1 and 5 (0.51014 and 0.50363), and they are higher when the dependent variable is Maths score.

In summary, some non-cognitive variables contribute a lot to higher grades in Maths and English (like Hyperactivity/Inattention, Peer-relationship problems, Conscientiousness, and Emotional stability) for both genders and when we consider all controls, but none matter as much as the cognitive ones, especially Numerical ability for both subjects and Vocal reasoning for English.

# 7 KOB decomposition - Explaining the grade-gap

Table 9: **Maths SDQ** Results - Threefold decomposition

| | **Maths Points** | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| Female (Group 1) | 9.685*** | 9.685*** | 9.685*** | 9.685*** |
| Male (Group 2) | 9.803*** | 9.803*** | 9.803*** | 9.803*** |
| Difference | -0.118* | -0.118* | -0.118* | -0.118* |
| Endowments | -0.264*** | -0.255*** | -0.247*** | -0.244*** |
| Coefficients | 0.101* | 0.096* | 0.083 | 0.082 |
| Interaction | 0.045 | 0.041 | 0.046 | 0.044 |
| **Endowments** | | | | |
| Vocal reasoning | -0.071*** | -0.057*** | -0.064*** | -0.055*** |
| Numerical ability | -0.210*** | -0.192*** | -0.200*** | -0.187*** |
| Matrices | -0.014* | -0.014* | -0.015* | -0.015* |
| Emotional symptoms | -0.019* | -0.016* | -0.017* | -0.015 |
| Conduct problems | 0.001 | 0.001 | 0.001 | 0.001 |
| Hyperactivity/Inattention | 0.049*** | 0.050*** | 0.051*** | 0.051*** |
| Peer relationship problems | 0.001 | 0.000 | 0.000 | 0.000 |
| **Coefficients** | | | | |
| Vocal reasoning | 0.016 | -0.023 | 0.037 | -0.010 |
| Numerical ability | -0.235 | -0.155 | -0.153 | -0.115 |
| Matrices | 0.268 | 0.115 | 0.128 | 0.045 |
| Emotional symptoms | -0.043 | -0.129 | -0.030 | -0.109 |
| Conduct problems | 0.561 | 0.662* | 0.664* | 0.706* |
| Hyperactivity/Inattention | 0.221 | 0.199 | 0.213 | 0.191 |
| Peer relationship problems | -0.119 | -0.099 | -0.158 | -0.117 |
| Constant | -0.567 | -0.570 | -0.519 | -0.549 |
| **Interaction** | | | | |
| Vocal reasoning | -0.001 | 0.001 | -0.002 | 0.001 |
| Numerical ability | 0.027 | 0.018 | 0.018 | 0.013 |
| Matrices | -0.003 | -0.001 | -0.001 | -0.000 |
| Emotional symptoms | 0.002 | 0.007 | 0.002 | 0.006 |
| Conduct problems | 0.002 | 0.003 | 0.003 | 0.003 |
| Hyperactivity/Inattention | 0.018 | 0.017 | 0.018 | 0.016 |
| Peer relationship problems | -0.001 | -0.001 | -0.002 | -0.001 |
| **SES Controls:** | No | Yes | No | Yes |
| **School Controls:** | No | No | Yes | Yes |
| Observations | 3,783 | 3,783 | 3,783 | 3,783 |

Table 10: **English SDQ** Results - Threefold decomposition

| | English Points | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| Female (Group 1) | 10.402*** | 10.402*** | 10.402*** | 10.402*** |
| Male (Group 2) | 10.091*** | 10.091*** | 10.091*** | 10.091*** |
| Difference | 0.310*** | 0.310*** | 0.310*** | 0.310*** |
| Endowments | -0.123*** | -0.123*** | -0.107*** | -0.104*** |
| Coefficients | 0.427*** | 0.427*** | 0.420*** | 0.419*** |
| Interaction | 0.007 | 0.007 | -0.002 | -0.005 |
| **Endowments** | | | | |
| Vocal reasoning | -0.085*** | -0.085*** | -0.082*** | -0.079*** |
| Numerical ability | -0.085*** | -0.085*** | -0.079*** | -0.074*** |
| Matrices | -0.003 | -0.003 | -0.004 | -0.003 |
| Emotional symptoms | -0.004 | -0.004 | -0.003 | -0.002 |
| Conduct problems | -0.000 | -0.000 | -0.001 | -0.001 |
| Hyperactivity/Inattention | 0.050*** | 0.050*** | 0.051*** | 0.052*** |
| Peer relationship problems | 0.005 | 0.005 | 0.005 | 0.005 |
| **Coefficients** | | | | |
| Vocal reasoning | -0.114 | -0.114 | -0.100 | -0.142 |
| Numerical ability | -0.066 | -0.066 | -0.029 | -0.022 |
| Matrices | 0.070 | 0.070 | 0.022 | -0.013 |
| Emotional symptoms | 0.006 | 0.006 | 0.014 | -0.024 |
| Conduct problems | 0.076 | 0.076 | 0.156 | 0.176 |
| Hyperactivity/Inattention | -0.084 | -0.084 | -0.099 | -0.113 |
| Peer relationship problems | 0.038 | 0.038 | 0.012 | 0.035 |
| Constant | 0.501 | 0.501 | 0.399 | 0.295 |
| **Interaction** | | | | |
| Vocal reasoning | 0.006 | 0.006 | 0.006 | 0.008 |
| Numerical ability | 0.008 | 0.008 | 0.003 | 0.003 |
| Matrices | -0.001 | -0.001 | -0.000 | 0.000 |
| Emotional symptoms | -0.000 | -0.000 | -0.001 | 0.001 |
| Conduct problems | 0.000 | 0.000 | 0.001 | 0.001 |
| Hyperactivity/Inattention | -0.007 | -0.007 | -0.008 | -0.009 |
| Peer relationship problems | 0.000 | 0.000 | 0.000 | 0.000 |
| **SES Controls:** | No | Yes | No | Yes |
| **School Controls:** | No | No | Yes | Yes |
| Observations | 3,783 | 3,783 | 3,783 | 3,783 |

Table 11: **Maths TIPI** Results - Threefold decomposition

| | **Maths Points** | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| Female (Group 1) | 9.685*** | 9.685*** | 9.685*** | 9.685*** |
| Male (Group 2) | 9.803*** | 9.803*** | 9.803*** | 9.803*** |
| Difference | -0.118* | -0.118* | -0.118* | -0.118* |
| Endowments | -0.302*** | -0.289*** | -0.284*** | -0.279*** |
| Coefficients | 0.157*** | 0.154*** | 0.139** | 0.139*** |
| Interaction | 0.027 | 0.017 | 0.027 | 0.022 |
| **Endowments** | | | | |
| Vocal reasoning | -0.076*** | -0.062*** | -0.070*** | -0.060*** |
| Numerical ability | -0.221*** | -0.201*** | -0.210*** | -0.196*** |
| Matrices | -0.015* | -0.015* | -0.015* | -0.015* |
| Agreeableness | 0.003 | 0.003 | 0.003 | 0.003 |
| Conscientiousness | 0.015* | 0.018** | 0.016** | 0.018** |
| Emotional stability | -0.004 | -0.003 | -0.004 | -0.003 |
| Extraversion | -0.001 | -0.001 | -0.001 | -0.001 |
| Openness | -0.003 | -0.001 | -0.002 | -0.000 |
| **Coefficients** | | | | |
| Vocal reasoning | 0.042 | -0.002 | 0.070 | 0.014 |
| Numerical ability | -0.216 | -0.136 | -0.132 | -0.096 |
| Matrices | 0.305 | 0.156 | 0.178 | 0.091 |
| Agreeableness | -0.029 | 0.027 | -0.021 | 0.026 |
| Conscientiousness | 0.130 | 0.072 | 0.116 | 0.068 |
| Emotional stability | -0.018 | 0.007 | 0.000 | 0.015 |
| Extraversion | -0.012 | 0.010 | -0.012 | 0.009 |
| Openness | -0.051 | -0.056 | -0.072 | -0.066 |
| Constant | 0.006 | -0.014 | 0.116 | 0.033 |
| **Interaction** | | | | |
| Vocal reasoning | -0.002 | 0.000 | -0.004 | -0.001 |
| Numerical ability | 0.025 | 0.016 | 0.015 | 0.011 |
| Matrices | -0.003 | -0.002 | -0.002 | -0.001 |
| Agreeableness | -0.002 | 0.001 | -0.001 | 0.001 |
| Conscientiousness | 0.011 | 0.006 | 0.010 | 0.006 |
| Emotional stability | 0.001 | -0.000 | -0.000 | -0.000 |
| Extraversion | -0.000 | 0.000 | -0.000 | 0.000 |
| Openness | -0.002 | -0.002 | -0.003 | -0.003 |
| **SES Controls:** | No | Yes | No | Yes |
| **School Controls:** | No | No | Yes | Yes |
| Observations | 3,783 | 3,783 | 3,783 | 3,783 |

Table 12: **English TIPI** Results - Threefold decomposition

| | **English Points** | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| Female (Group 1) | 10.402*** | 10.402*** | 10.402*** | 10.402*** |
| Male (Group 2) | 10.091*** | 10.091*** | 10.091*** | 10.091*** |
| Difference | 0.310*** | 0.310*** | 0.310*** | 0.310*** |
| Endowments | -0.170*** | -0.163*** | -0.154*** | -0.150*** |
| Coefficients | 0.462*** | 0.461*** | 0.454*** | 0.454*** |
| Interaction | 0.018 | 0.012 | 0.010 | 0.006 |
| **Endowments** | | | | |
| Vocal reasoning | -0.089*** | -0.083*** | -0.086*** | -0.082*** |
| Numerical ability | -0.095*** | -0.087*** | -0.088*** | -0.083*** |
| Matrices | -0.004 | -0.003 | -0.004 | -0.004 |
| Agreeableness | 0.006 | 0.006 | 0.007 | 0.007 |
| Conscientiousness | 0.012* | 0.013** | 0.013* | 0.014** |
| Emotional stability | -0.001 | -0.000 | -0.001 | -0.000 |
| Extraversion | 0.000 | 0.000 | 0.000 | 0.000 |
| Openness | -0.000 | 0.001 | 0.001 | 0.002 |
| **Coefficients** | | | | |
| Vocal reasoning | -0.114 | -0.157 | -0.097 | -0.145 |
| Numerical ability | -0.104 | -0.077 | -0.066 | -0.060 |
| Matrices | 0.060 | -0.006 | 0.020 | -0.017 |
| Agreeableness | -0.094 | -0.062 | -0.082 | -0.054 |
| Conscientiousness | 0.035 | 0.009 | 0.028 | 0.006 |
| Emotional stability | -0.024 | -0.012 | -0.022 | -0.015 |
| Extraversion | 0.018 | 0.027 | 0.021 | 0.030 |
| Openness | 0.025 | 0.020 | 0.006 | 0.006 |
| Constant | 0.660* | 0.583* | 0.605* | 0.477 |
| **Interaction** | | | | |
| Vocal reasoning | 0.007 | 0.009 | 0.006 | 0.008 |
| Numerical ability | 0.012 | 0.009 | 0.008 | 0.007 |
| Matrices | -0.001 | 0.000 | -0.000 | 0.000 |
| Agreeableness | -0.005 | -0.003 | -0.004 | -0.003 |
| Conscientiousness | 0.003 | 0.001 | 0.002 | 0.001 |
| Emotional stability | 0.001 | 0.000 | 0.001 | 0.000 |
| Extraversion | 0.000 | 0.000 | 0.000 | 0.001 |
| Openness | 0.001 | 0.001 | 0.000 | 0.000 |
| **SES Controls:** | No | Yes | No | Yes |
| **School Controls:** | No | No | Yes | Yes |
| Observations | 3,783 | 3,783 | 3,783 | 3,783 |

Table 13: **Maths SDQ** Results - Twofold decomposition

| | **Maths points** | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| Female (Group 1) | 9.685*** | 9.685*** | 9.685*** | 9.685*** |
| Male (Group 2) | 9.803*** | 9.803*** | 9.803*** | 9.803*** |
| Difference | -0.118* | -0.118* | -0.118* | -0.118* |
| Explained | -0.244*** | -0.237*** | -0.227*** | -0.225*** |
| Unexplained | 0.126** | 0.119** | 0.109** | 0.107** |
| **Explained** | | | | |
| Vocal reasoning | -0.071*** | -0.056*** | -0.066*** | -0.055*** |
| Numerical ability | -0.197*** | -0.183*** | -0.192*** | -0.181*** |
| Matrices | -0.016* | -0.015* | -0.016* | -0.015* |
| Emotional symptoms | -0.018** | -0.013* | -0.017** | -0.013* |
| Conduct problems | 0.002 | 0.002 | 0.002 | 0.002 |
| Hyperactivity/Inattention | 0.056*** | 0.057*** | 0.058*** | 0.058*** |
| Peer relationship problems | 0.000 | 0.000 | -0.000 | -0.000 |
| **Unexplained** | | | | |
| Vocal reasoning | 0.015 | -0.023 | 0.037 | -0.009 |
| Numerical ability | -0.221 | -0.146 | -0.144 | -0.108 |
| Matrices | 0.266 | 0.115 | 0.127 | 0.044 |
| Emotional symptoms | -0.042 | -0.126 | -0.029 | -0.106 |
| Conduct problems | 0.563 | 0.664 | 0.666 | 0.708* |
| Hyperactivity/Inattention | 0.231 | 0.209 | 0.224 | 0.200 |
| Peer relationship problems | -0.120 | -0.100 | -0.160 | -0.118 |
| Constant | -0.567 | -0.570 | -0.519 | -0.549 |
| **SES Controls:** | No | Yes | No | Yes |
| **School Controls:** | No | No | Yes | Yes |
| Observations | 3,783 | 3,783 | 3,783 | 3,783 |

Table 14: **English SDQ** Results - Twofold decomposition

| | **English points** | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| Female (Group 1) | 10.402*** | 10.402*** | 10.402*** | 10.402*** |
| Male (Group 2) | 10.091*** | 10.091*** | 10.091*** | 10.091*** |
| Difference | 0.310*** | 0.310*** | 0.310*** | 0.310*** |
| Explained | -0.119*** | -0.114*** | -0.107*** | -0.105*** |
| Unexplained | 0.429*** | 0.425*** | 0.418*** | 0.416*** |
| **Explained** | | | | |
| Vocal reasoning | -0.081*** | -0.075*** | -0.079*** | -0.074*** |
| Numerical ability | -0.082*** | -0.075*** | -0.078*** | -0.073*** |
| Matrices | -0.004 | -0.003 | -0.004 | -0.003 |
| Emotional symptoms | -0.004 | -0.001 | -0.003 | -0.001 |
| Conduct problems | -0.000 | -0.000 | -0.000 | -0.000 |
| Hyperactivity/Inattention | 0.047*** | 0.048*** | 0.048*** | 0.048*** |
| Peer relationship problems | 0.005* | 0.005 | 0.005 | 0.005 |
| **Unexplained** | | | | |
| Vocal reasoning | -0.111 | -0.149 | -0.098 | -0.139 |
| Numerical ability | -0.062 | -0.038 | -0.027 | -0.020 |
| Matrices | 0.069 | 0.003 | 0.022 | -0.012 |
| Emotional symptoms | 0.006 | -0.033 | 0.013 | -0.023 |
| Conduct problems | 0.076 | 0.123 | 0.156 | 0.177 |
| Hyperactivity/Inattention | -0.088 | -0.100 | -0.104 | -0.119 |
| Peer relationship problems | 0.038 | 0.050 | 0.012 | 0.035 |
| Constant | 0.501 | 0.431 | 0.399 | 0.295 |
| **SES Controls:** | No | Yes | No | Yes |
| **School Controls:** | No | No | Yes | Yes |
| Observations | 3,783 | 3,783 | 3,783 | 3,783 |

Table 15: **Maths TIPI** Results - Twofold decomposition

| | **Maths points** | | | |
|---|---|---|---|---|
| | **I** | **II** | **III** | **IV** |
| Female (Group 1) | 9.685*** | 9.685*** | 9.685*** | 9.685*** |
| Male (Group 2) | 9.803*** | 9.803*** | 9.803*** | 9.803*** |
| Difference | -0.118* | -0.118* | -0.118* | -0.118* |
| Explained | -0.289*** | -0.281*** | -0.272*** | -0.268*** |
| Unexplained | 0.171*** | 0.163*** | 0.153*** | 0.150*** |
| **Explained** | | | | |
| Vocal reasoning | -0.077*** | -0.062*** | -0.072*** | -0.061*** |
| Numerical ability | -0.209*** | -0.193*** | -0.202*** | -0.191*** |
| Matrices | -0.016* | -0.015* | -0.016* | -0.015* |
| Agreeableness | 0.002 | 0.004 | 0.002 | 0.004 |
| Conscientiousness | 0.021*** | 0.021*** | 0.022*** | 0.022*** |
| Emotional stability | -0.004 | -0.003 | -0.004 | -0.003 |
| Extraversion | -0.001 | -0.001 | -0.001 | -0.001 |
| Openness | -0.004 | -0.002 | -0.003 | -0.002 |
| **Unexplained** | | | | |
| Vocal reasoning | 0.041 | -0.002 | 0.068 | 0.014 |
| Numerical ability | -0.204 | -0.128 | -0.125 | -0.091 |
| Matrices | 0.304 | 0.155 | 0.178 | 0.091 |
| Agreeableness | -0.030 | 0.028 | -0.021 | 0.027 |
| Conscientiousness | 0.136 | 0.075 | 0.121 | 0.071 |
| Emotional stability | -0.018 | 0.007 | 0.000 | 0.015 |
| Extraversion | -0.012 | 0.010 | -0.012 | 0.009 |
| Openness | -0.052 | -0.057 | -0.074 | -0.067 |
| Constant | 0.006 | -0.014 | 0.116 | 0.033 |
| **SES Controls:** | No | Yes | No | Yes |
| **School Controls:** | No | No | Yes | Yes |
| Observations | 3,783 | 3,783 | 3,783 | 3,783 |

Table 16: **English TIPI** Results - Twofold decomposition

| | **English Points** | | | |
|---|---|---|---|---|
| | I | II | III | IV |
| Female (Group 1) | 10.402*** | 10.402*** | 10.402*** | 10.402*** |
| Male (Group 2) | 10.091*** | 10.091*** | 10.091*** | 10.091*** |
| Difference | 0.310*** | 0.310*** | 0.310*** | 0.310*** |
| Explained | -0.161*** | -0.156*** | -0.149*** | -0.147*** |
| Unexplained | 0.471*** | 0.466*** | 0.460*** | 0.457*** |
| **Explained** | | | | |
| Vocal reasoning | -0.085*** | -0.078*** | -0.083*** | -0.077*** |
| Numerical ability | -0.090*** | -0.082*** | -0.085*** | -0.080*** |
| Matrices | -0.004 | -0.004 | -0.004 | -0.004 |
| Agreeableness | 0.004 | 0.005 | 0.004 | 0.005 |
| Conscientiousness | 0.014*** | 0.014*** | 0.014*** | 0.014*** |
| Emotional stability | -0.001 | -0.000 | -0.001 | -0.000 |
| Extraversion | 0.001 | 0.001 | 0.001 | 0.001 |
| Openness | 0.000 | 0.002 | 0.001 | 0.002 |
| **Unexplained** | | | | |
| Vocal reasoning | -0.112 | -0.153 | -0.095 | -0.141 |
| Numerical ability | -0.097 | -0.072 | -0.061 | -0.056 |
| Matrices | 0.060 | -0.006 | 0.020 | -0.017 |
| Agreeableness | -0.097 | -0.064 | -0.084 | -0.055 |
| Conscientiousness | 0.037 | 0.009 | 0.030 | 0.006 |
| Emotional stability | -0.024 | -0.012 | -0.021 | -0.015 |
| Extraversion | 0.018 | 0.027 | 0.021 | 0.031 |
| Openness | 0.026 | 0.021 | 0.006 | 0.007 |
| Constant | 0.660* | 0.583* | 0.605* | 0.477 |
| **SES Controls:** | No | Yes | No | Yes |
| **School Controls:** | No | No | Yes | Yes |
| Observations | 3,783 | 3,783 | 3,783 | 3,783 |

## 7.1 Discussion

As seen in the Appendix, boys score higher than girls, on average, in all cognitive measures in all waves, even when subsetting by parents' income and education levels. Girls only outperform boys in Junior Cert English scores (by 0.322 points). Boys also have higher means than girls in all control variables (SES status and school characteristics), even if there are fewer of them represented in the sample. Conversely, girls outperform boys in all non-cognitive indicators except for Emotional Stability (Neuroticism) and Emotional Symptoms. These results ask for a deeper analysis, so I employed the Oaxaca-Blinder decomposition (**?**, **?**), or Kitagawa decomposition (**?**), method to see from where exactly these differences in grades (girls score higher than boys in English, and boys score higher than girls in Maths) arise, if from the cognitive part of the model or the non-cognitive one, and how much these differences matter to the final outcome (Junior Cert grades).

In the Oaxaca-Blinder decomposition, Female is Group 1 (= 0) and Male is Group 2 (= 1), so whenever we have a negative result in group differences (first panel of the tables), it means that the value from Group 2 is bigger than for Group 1, and the opposite holds. When we get a negative coefficient for the variables considered, it means that the variable is associated with mitigating or reducing the difference in outcomes between the two groups, in our case the variable contributes to reducing the gender grade gap, and the opposite also holds (a positive coefficient contributes to increasing the gender grade gap). The magnitude of the coefficients indicates how much they contribute to the size of the Endowments, Coefficients, and Interaction for the three-fold decomposition, and Explained and Unexplained parts of the two-fold decomposition. Both SDQ and TIPI as indicators of non-cognition yield similar results when analyzing each subject (Maths and English).

1) **Threefold decomposition**:

1.1 **Endowments** (observed covariates): For Maths, cognitive Endowments for SDQ and TIPI (Tables 9 and 11) are consistently more significant and bigger in magnitude (in module) than Endowments for non-cognitive indicators, and both are similar in absolute numbers, directions, and significance when we compare them within each subject. Endowments for cognitive indicators are bigger in absolute numbers when we consider the TIPI non-cognitive indicators than when we use the SDQ non-cognitive ones for both Maths and English (with the difference in Maths being bigger than English). Since Vocal reasoning and Numerical ability are on the same scale, they are directly comparable, and we see that for Maths the Endowment for Numerical ability is more than three times the Endowment for Vocal reasoning, a result that does not hold for English (Tables 10 and 12), where the Endowments for these two variables are almost equal. In terms of non-cognitive indicators' significance, only the Endowments for Hyperactivity/Inattention (SDQ) and Conscientiousness (TIPI) are significant for Maths and English. All significant Endowments for non-cognitive indicators either stay the same or increase in absolute magnitude as we add more controls, whereas the Endowments for cognition, when significant, all decrease in magnitude as we add more controls. Endowments for cognitive variables are all negative for Maths and English when considering both SDQ and TIPI non-cognitive indicators. Endowments for non-cognitive indicators are positive when significant at the 99% and 95% CIs (Conscientiousness and Hyperactivity/Inattention) for both subjects when broken down into variables, and negative and highly significant when considered in total.

1.2. **Coefficients** (returns to Endowments): The Coefficients, when accounted for in total, are not significant for Maths with SDQ non-cognitive indicators (9), but highly significant and positive for Maths and English with TIPI non-cognitive indicators (Tables 11 and 12), and English with SDQ non-cognitive indicators (Table 10). The Coefficients for English are almost four times bigger in magnitude than the Coefficients for Maths. When accounted for individually, only the Coefficient for Conduct problems (Maths SDQ, non-cognition) is significant at the 90% CI, and it increases positively in magnitude as we add more controls.

1.3. **Interaction**: The interaction term accounts for the fact that differences in Endowments and Coefficients exist simultaneously between Groups 1 and 2 (**?**). None of the interaction terms are significant when accounted for individually and in total, and they are positive and very small in magnitude when not equal to zero.

2) **Twofold decomposition**:

2.1. **Explained**: The composition (explained) effect is the difference in grades due to differences in the Endowments of the individuals across the two groups (Popli (2013)). Results are similar within subjects and across SDQ and TIPI decompositions. For both Maths and English, the Explained parts, when accounted for in total, are always highly significant and negative, and the magnitude for Maths is almost twice the magnitude for English for both SDQ (-0.225 versus -0.105) and TIPI (-0.268 versus -0.147) in absolute terms, with the results within subjects across non-cognitive scales being always higher when considering the TIPI than the SDQ scale. In terms of cognitive variables, Vocal reasoning and Numerical ability are always negative and highly significant for both subjects across the two non-cognitive scales, with Numerical ability being three times bigger in absolute terms than Vocal reasoning for Maths and similar in magnitude for English also across non-cognitive

scales. Matrices is negative but only slightly significant for Maths, and only one-fourth in absolute magnitude to Vocal reasoning. All cognitive variables decrease in absolute magnitude as we add more controls. When analyzing the non-cognitive variables, we see that only two are positive and highly significant - SDQ Hyperactivity/Inattention and TIPI Conscientiousness for both Maths and English - and one is slightly significant (at the 90% CI) and negative - SDQ Emotional symptoms for Maths. All non-cognitive significant variables either increase in magnitude or stay the same as we add more controls.

2.2. **Unexplained**: The unexplained component (residual) is here defined as the grade gap associated with some sort of discrimination (probably unintended), unobserved heterogeneity, and omitted (not on purpose) variables (akin to (Popli, 2013)), and is the difference in mean grades due to the difference in returns to individual characteristics (the Coefficients of the threefold decomposition). Results are similar within subjects and across SDQ and TIPI decompositions for the Unplained parts as well. For both Maths and English, the Unexplained parts, when accounted for in total, are always highly significant (except for Maths SDQ with all controls) and positive, with the magnitudes for English being more than four times the magnitudes for Maths for both SDQ (0.416 versus 0.107) and TIPI (0.457 versus 0.150), and the results within subjects across non-cognitive scales are always higher when considering the TIPI than the SDQ scale. In terms of cognitive variables, none is significant individually. The only non-cognitive variable that is positive and highly significant is Conduct problems (for Maths SDQ), and it increases in magnitude as we add more controls.

Maybe the most interesting part of the decomposition is the unexplained part of the two-fold one. It pertains to the residual differences in unmeasured skills or attributes, or discrimination if we are talking about wages or non-blind grading, and it tells about the portion of the gender grade gap that cannot be explained by differences in observable characteristics. The highest coefficient in magnitude is Conduct problems (equal to 0.708, Table 13), but it is only slightly significant (at the 90% CI). Its sign indicates that having more conduct problems is associated with a larger gap in Math scores between the two groups, and its magnitude indicates that it is the greatest contributor to the unexplained part in Table 13. Conduct problems is also the greatest positive contributor to unexplained parts in English scores (Table 14). Higher levels of Numerical ability are always associated with a decrease in the gender grade gap, although it is not independently significant. The signs of other cognitive variables vary, as do their magnitude in terms of contribution to the Unexplained parts.

The Explained part of the Oaxaca-Blinder decomposition tells us how much of the gender grade gap can be attributed to differences in observable characteristics (represented by the explanatory variables) between males and females. Complementary to the results presented in the regression tables, the negative signs of all the cognitive coefficients indicate that the bigger they are, the more they contribute to closing the gender grade gap, with Numerical ability and Vocal Reasoning being always highly significant and also the greatest contributors, in magnitude, to the Explained parts, regardless of what explanatory variables we consider. Some non-cognitive coefficients such as Hyperactivity/Inattention, and Conscientiousness are also highly significant, but positive in sign, which indicates that they contribute to increasing the gender grade gap for both Maths and English scores. On average, then, having higher values of Hyperactivity/Inattention, and Conscientiousness is associated with a larger grade gap between males and females.

# 8 Who is driving the gap?

To account for heterogeneity in the sample, I separate boys and girls according to their cognitive abilities using the principal component variable and plot how each band (average principal component in blue, average principal component plus one standard deviation in green, and average principal component minus one standard deviation in red) performs on test scores (Junior Cert test scores in Maths and English) according to each non-cognitive variable (four for SDQ and five for TIPI). The results from this decomposition are presented below. I find that not all cognition bands of will show the same behavior. While scoring high on a specific noncognitive ability is good for one band (meaning the test score is positively correlated with the noncognitive ability), it can actually be detrimental for the other, and at different rates (equivalent to the inclination of the lines). For example, the higher below-average-cognition females score in Openness, the higher their grades are in English, while the higher above-average-cognition females score in Openness, the lower their grades are in English (Fig. 9, upper right side). For males, on the other hand, scoring high in Openness means scoring high in English for all levels of cognition, while at the same time scoring lower in Maths (Fig. 9, middle plots).

## 8.1 SDQ

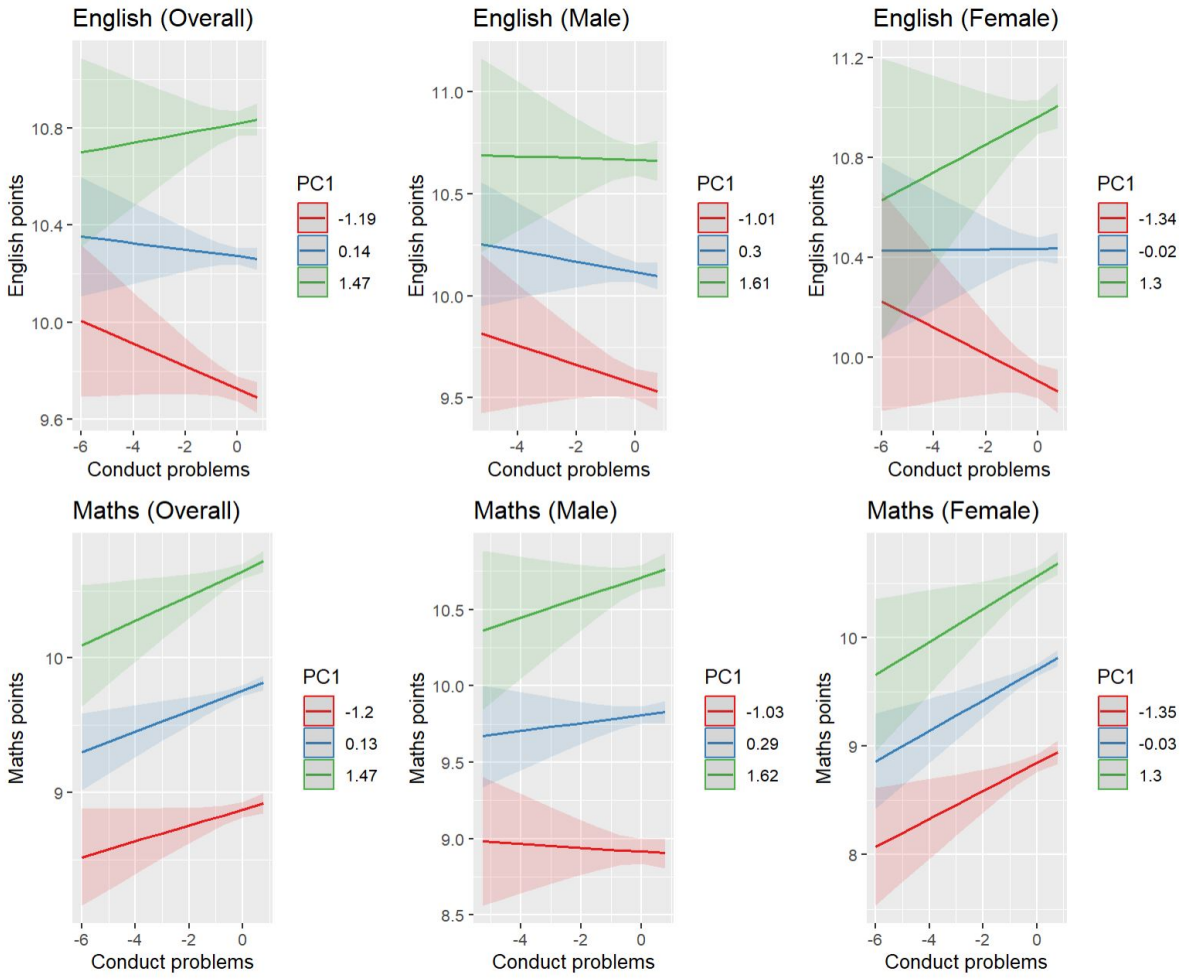



Figure 1

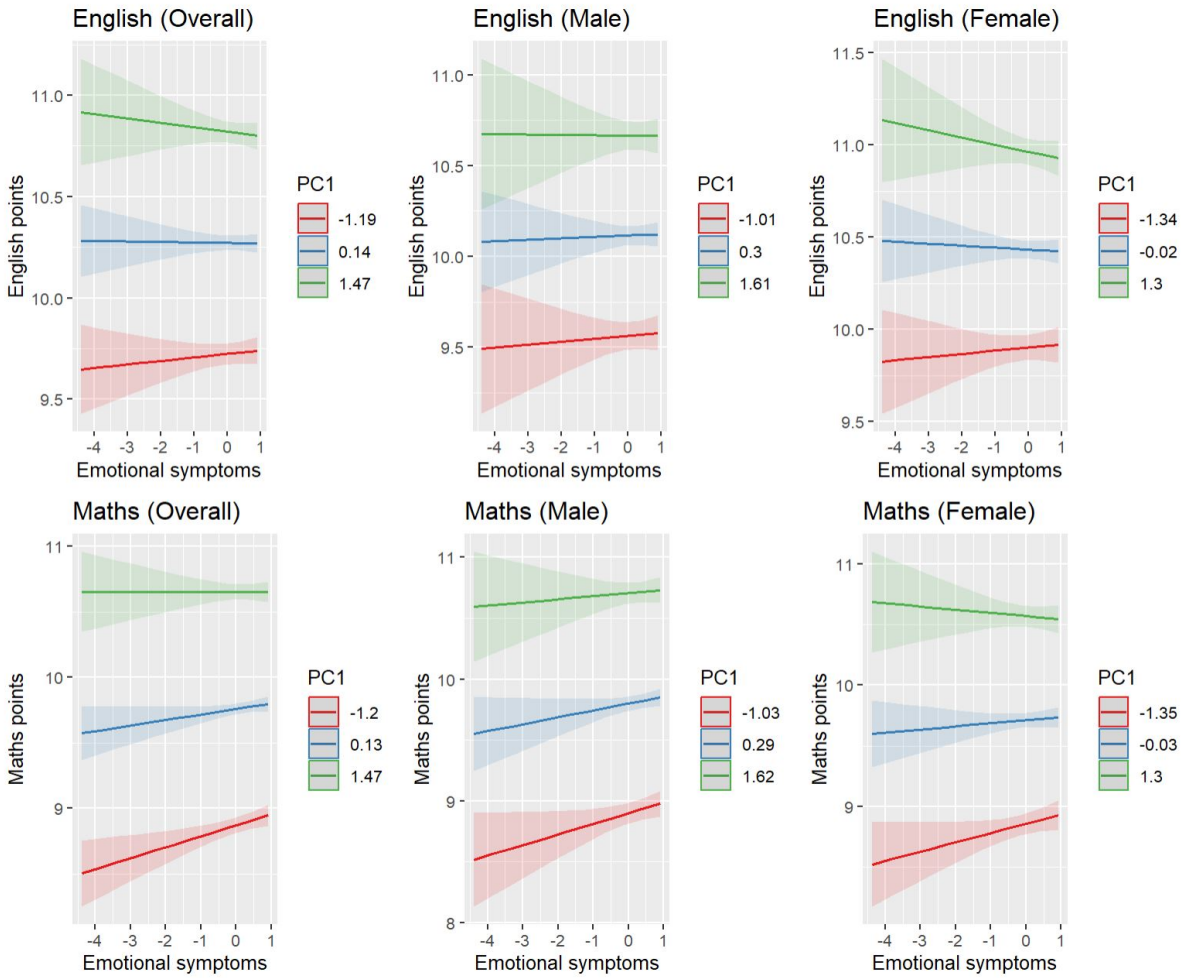

English (Overall)
English (Male)
English (Female)
Maths (Overall)
Maths (Male)
Maths (Female)
English points
Maths points
Emotional symptoms
PC1
-1.19
0.14
1.47
-1.01
0.3
1.61
-1.34
-0.02
1.3
-1.2
0.13
1.47
-1.03
0.29
1.62
-1.35
-0.03
1.3

Figure 2

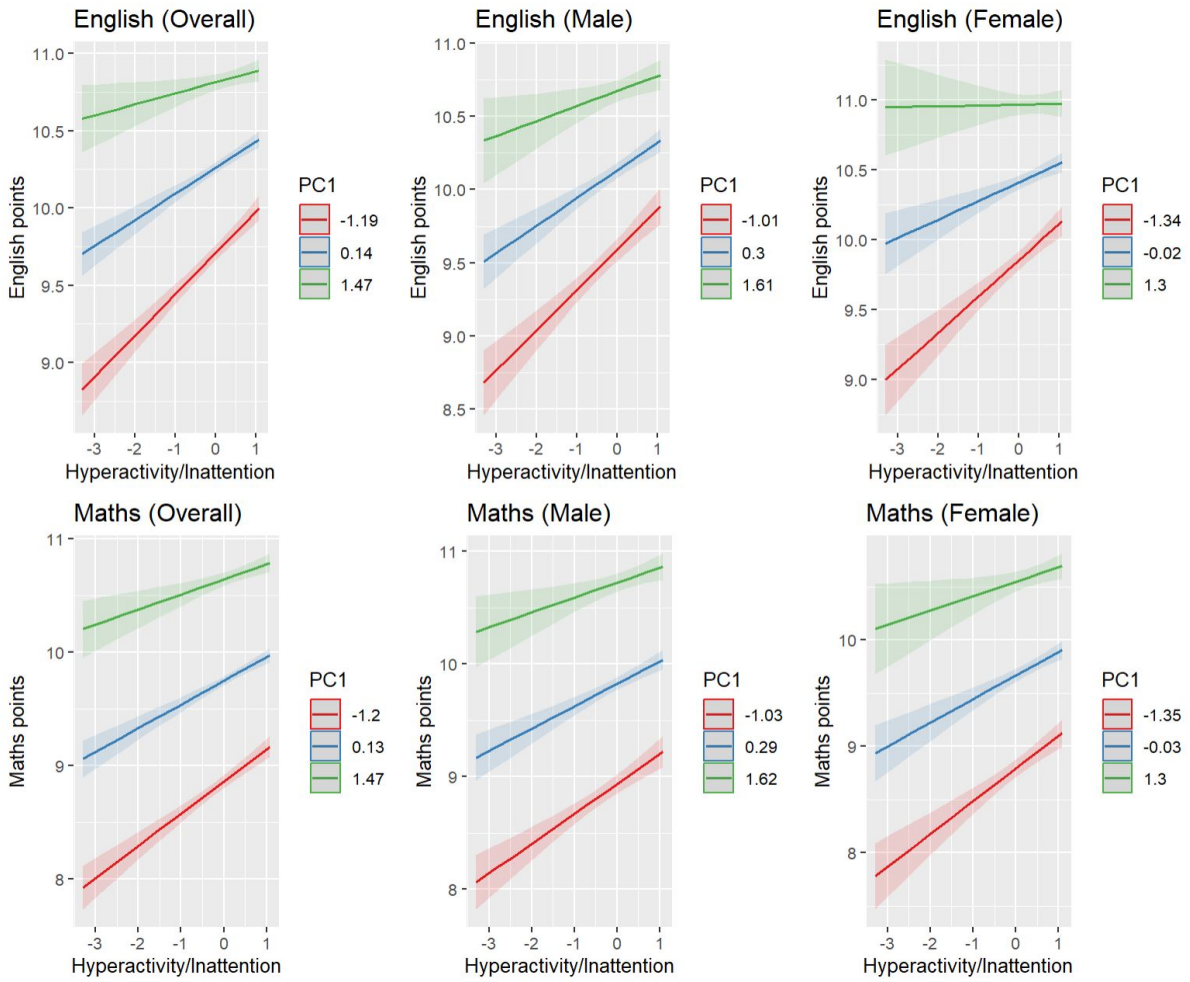

English (Overall)
English (Male)
English (Female)
Maths (Overall)
Maths (Male)
Maths (Female)
English points
Maths points
Hyperactivity/Inattention
PC1
-1.19
0.14
1.47
-1.01
0.3
1.61
-1.34
-0.02
1.3
-1.2
0.13
1.47
-1.03
0.29
1.62
-1.35
-0.03
1.3

Figure 3

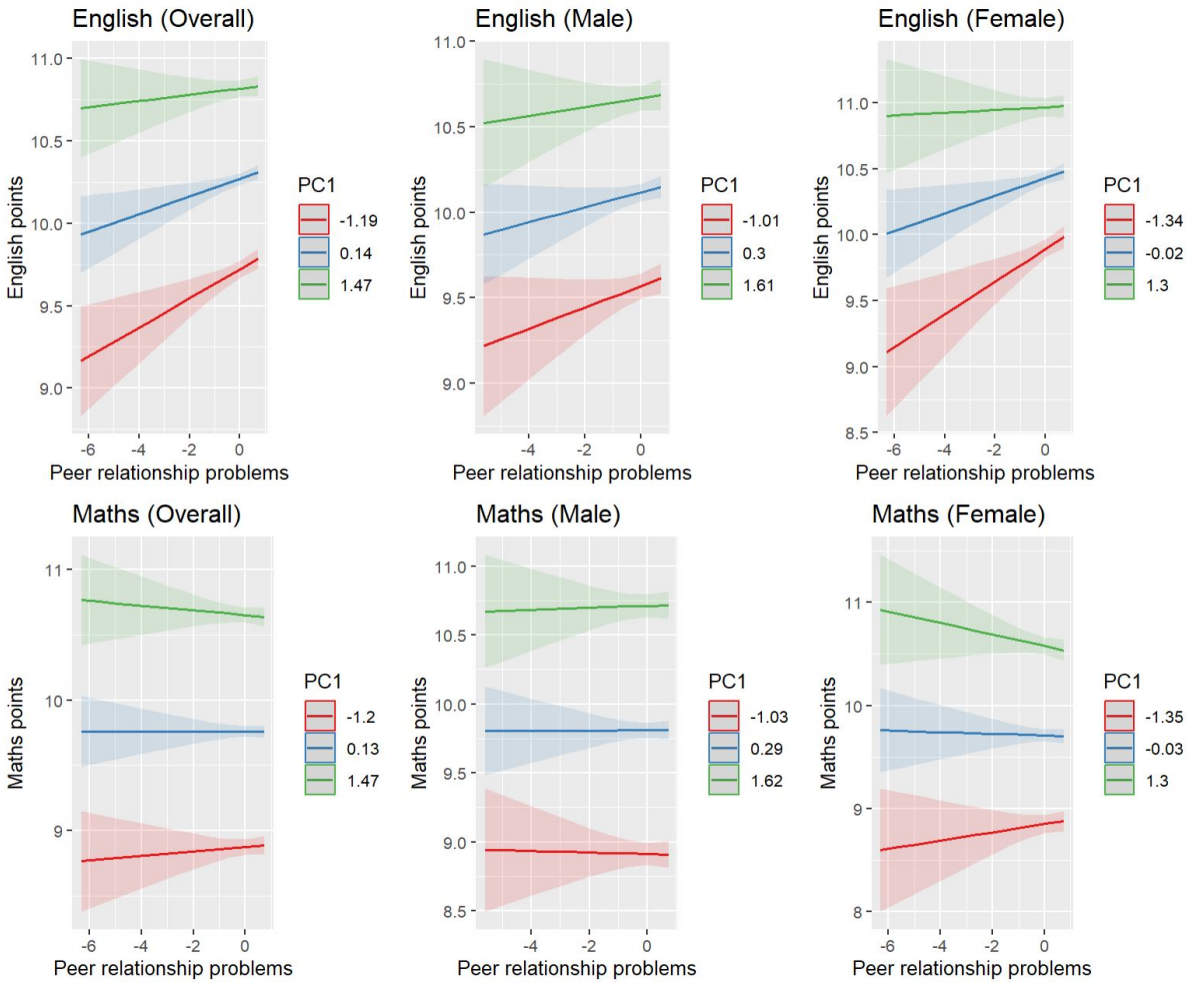

English (Overall)
English (Male)
English (Female)
Maths (Overall)
Maths (Male)
Maths (Female)
English points
Maths points
Peer relationship problems
PC1
-1.19
0.14
1.47
-1.01
0.3
1.61
-1.34
-0.02
1.3
-1.2
0.13
1.47
-1.03
0.29
1.62
-1.35
-0.03
1.3

Figure 4

## 8.2 TIPI

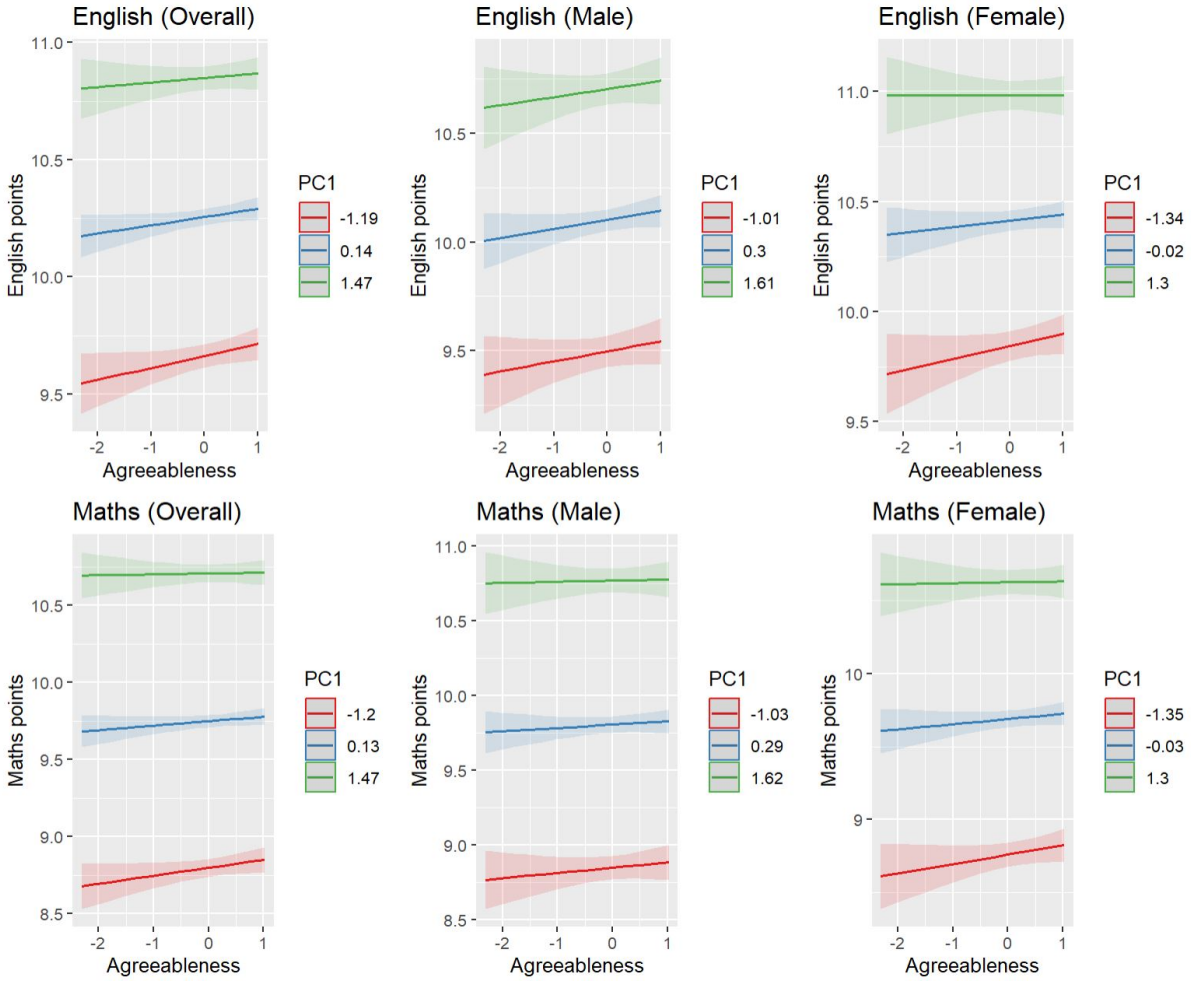



Figure 5

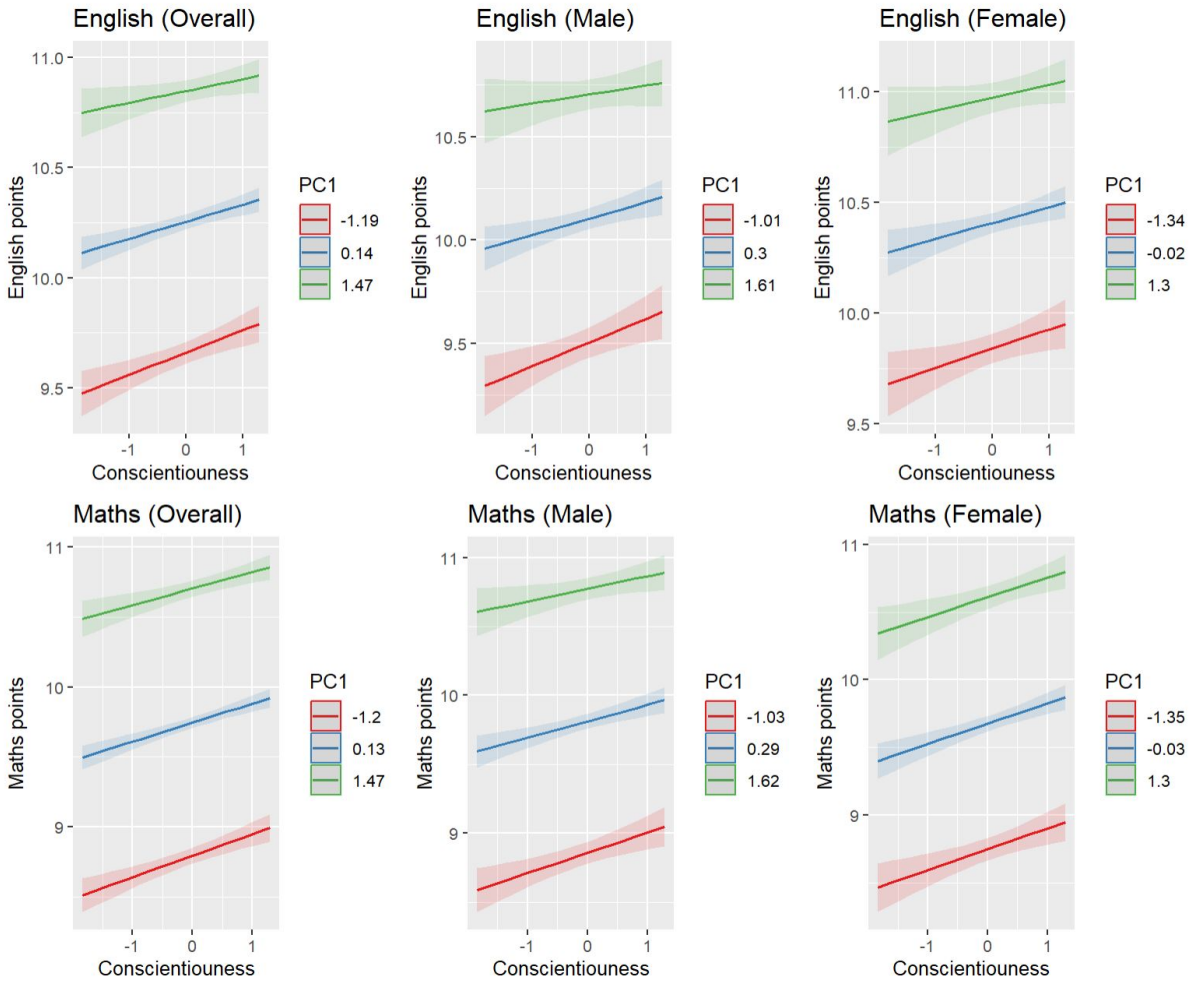

English (Overall)
English (Male)
English (Female)
Maths (Overall)
Maths (Male)
Maths (Female)
English points
Maths points
Conscientiouness
PC1
-1.19
0.14
1.47
-1.01
0.3
1.61
-1.34
-0.02
1.3
-1.2
0.13
1.47
-1.03
0.29
1.62
-1.35
-0.03
1.3

Figure 6

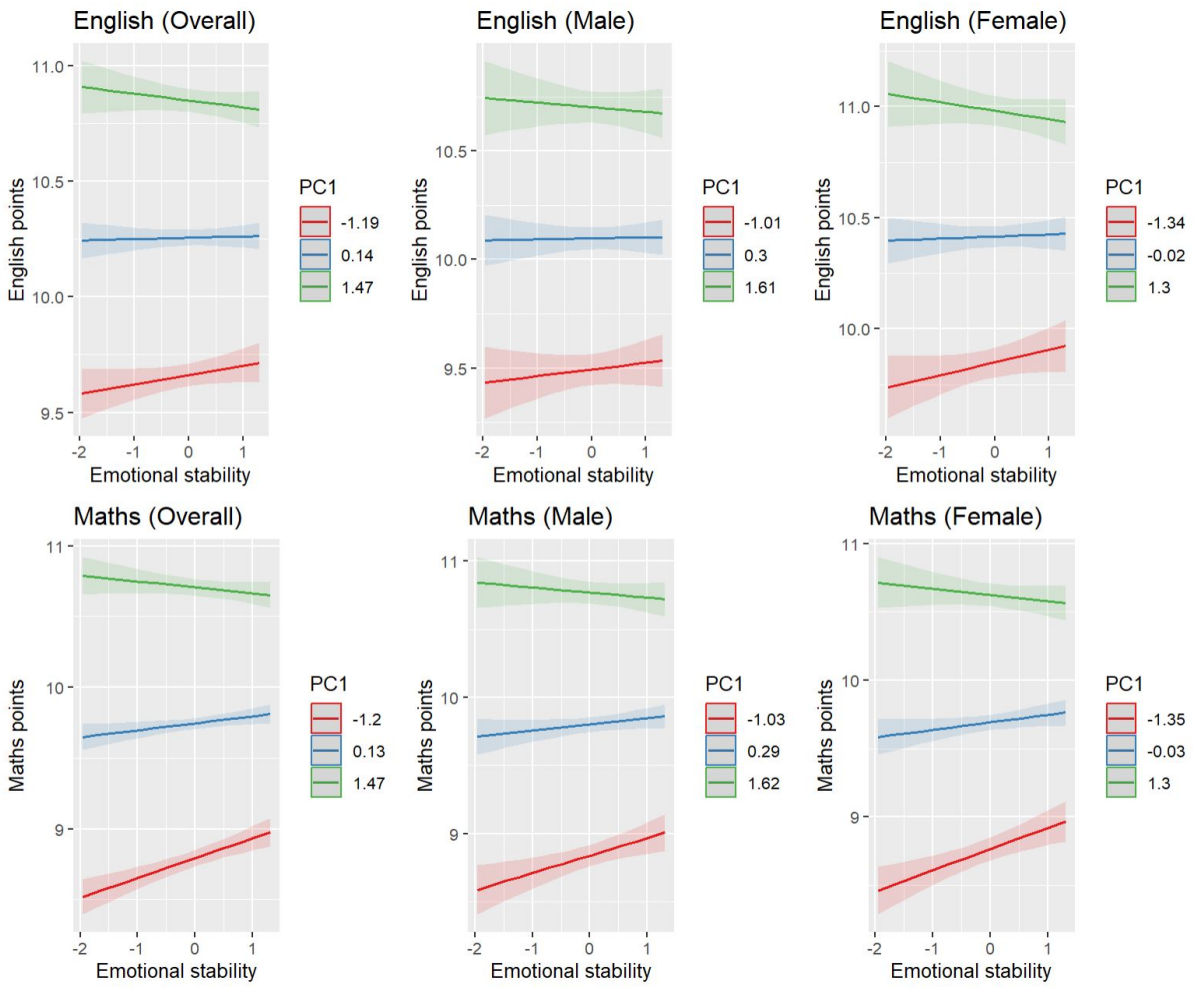

English (Overall)
English (Male)
English (Female)
Maths (Overall)
Maths (Male)
Maths (Female)
English points
Maths points
Emotional stability
PC1
-1.19
0.14
1.47
-1.01
0.3
1.61
-1.34
-0.02
1.3
-1.2
0.13
1.47
-1.03
0.29
1.62
-1.35
-0.03
1.3

Figure 7

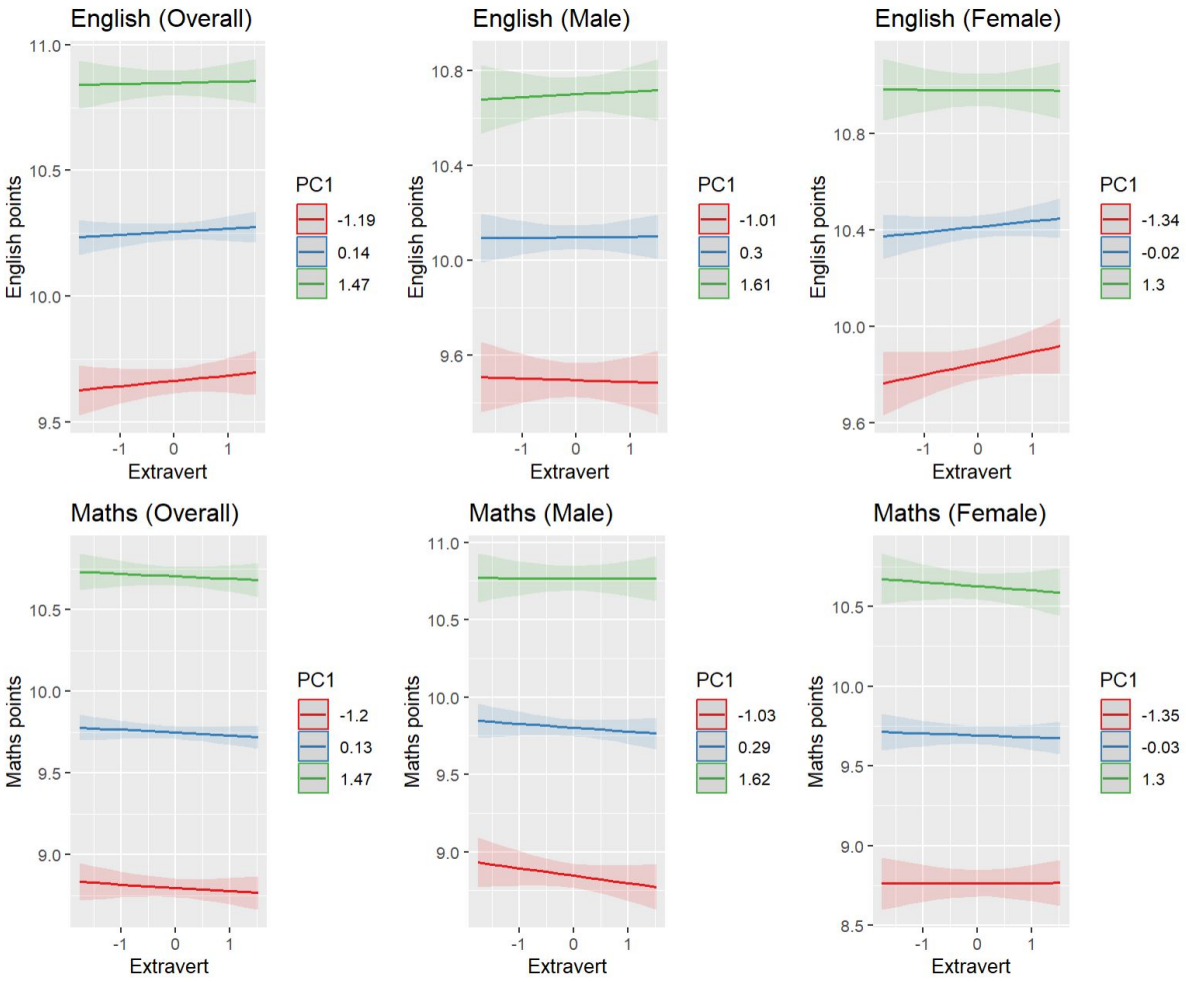

English (Overall)
English points
Extravert
PC1
-1.19
0.14
1.47
English (Male)
English points
Extravert
PC1
-1.01
0.3
1.61
English (Female)
English points
Extravert
PC1
-1.34
-0.02
1.3
Maths (Overall)
Maths points
Extravert
PC1
-1.2
0.13
1.47
Maths (Male)
Maths points
Extravert
PC1
-1.03
0.29
1.62
Maths (Female)
Maths points
Extravert
PC1
-1.35
-0.03
1.3

Figure 8

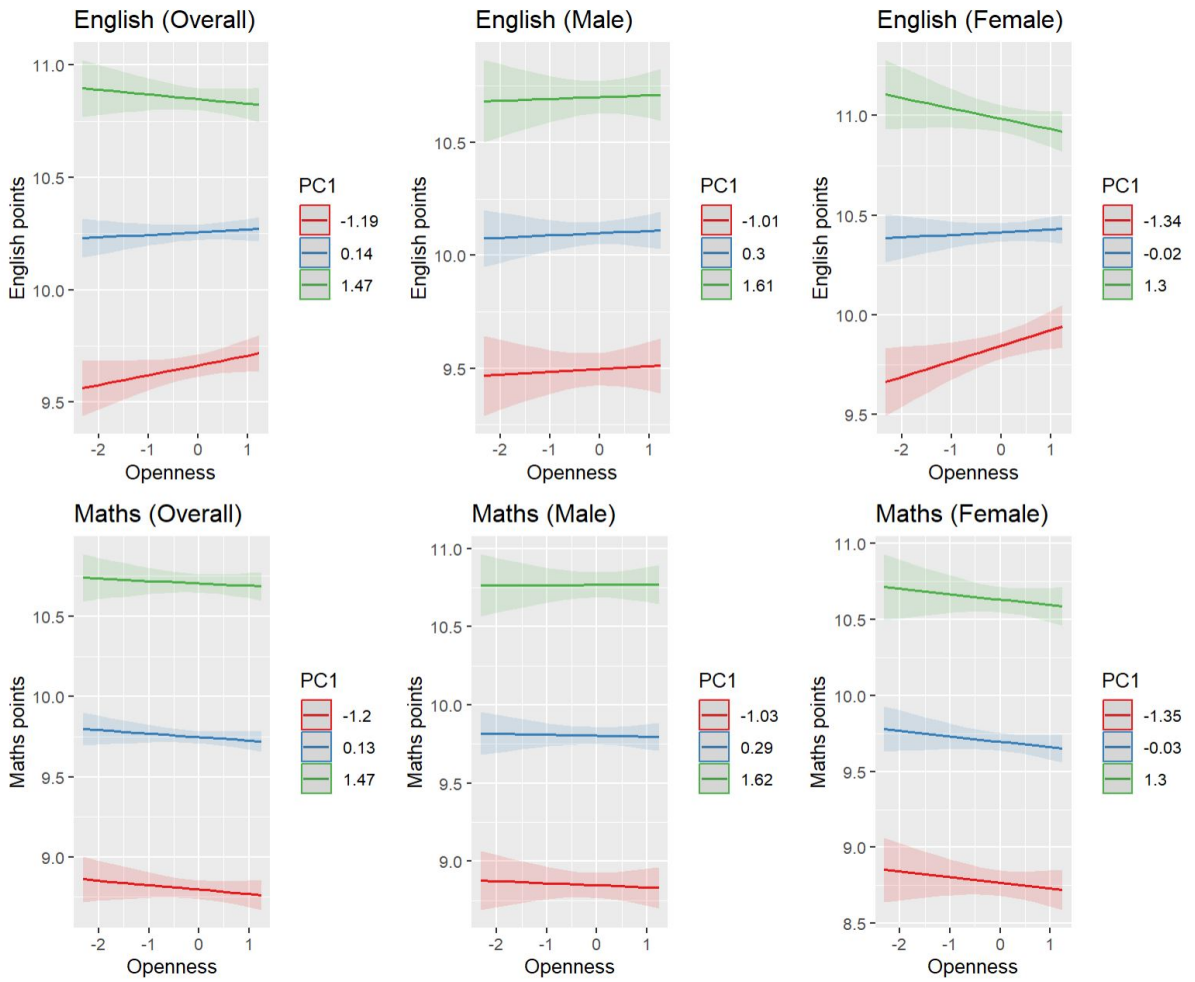

English (Overall)
English (Male)
English (Female)
Maths (Overall)
Maths (Male)
Maths (Female)
English points
Maths points
Openness
PC1
-1.19
0.14
1.47
-1.01
0.3
1.61
-1.34
-0.02
1.3
-1.2
0.13
1.47
-1.03
0.29
1.62
-1.35
-0.03
1.3

Figure 9

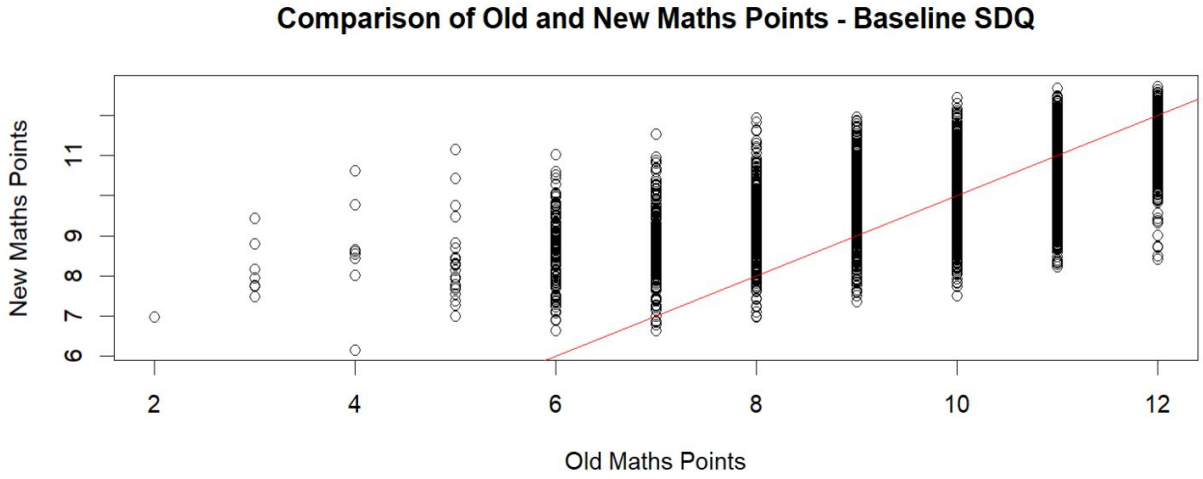
# 9 How do we close the gap?

## 9.1 Simulation of adding one standard deviation to non-cognitive indicators

Are students with greater early cognitive and noncognitive abilities more efficient in later learning of both cognitive and noncognitive skills? What is the "best" period to close the gap?

| Model_Type | Maths_Slope | English_Slope |
|---|---|---|
| Baseline Model - SDQ | 0.46 | 0.32 |
| Model Including All Variables - SDQ | 0.51 | 0.37 |
| Baseline Model - TIPI | 0.45 | 0.30 |
| Model Including All Variables - TIPI | 0.49 | 0.35 |

Table 17: Comparison of Slopes for Maths and English Points



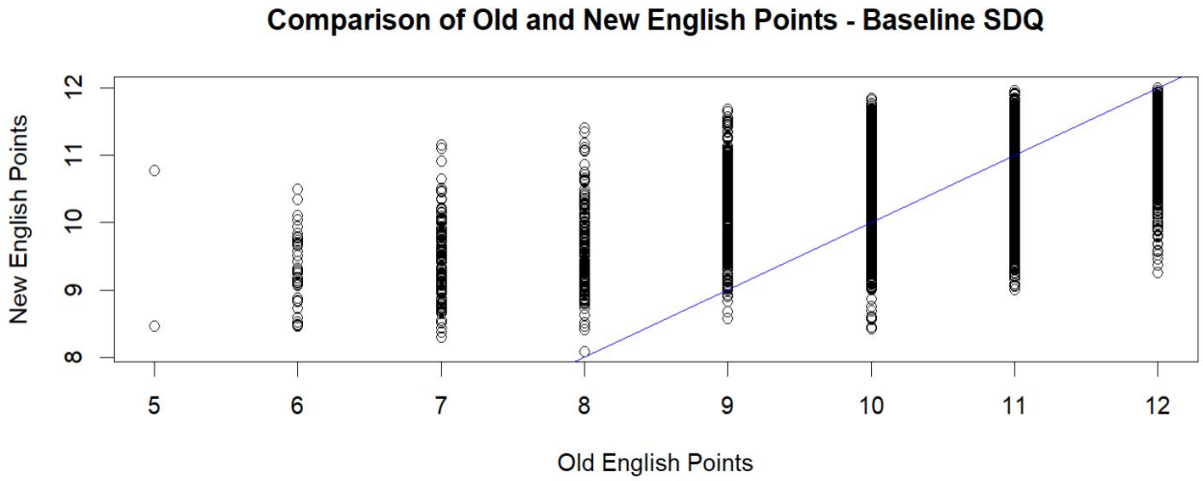
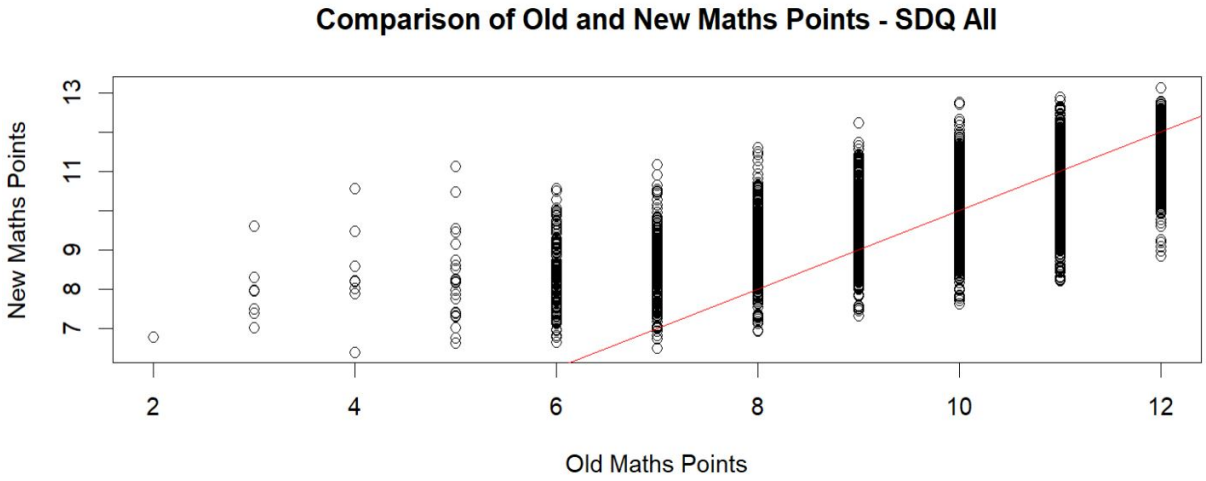




Figure 10


Comparison of Old and New Maths Points - SDQ All
New Maths Points
7
8
9
11
13
2
4
6
8
10
12
Old Maths Points

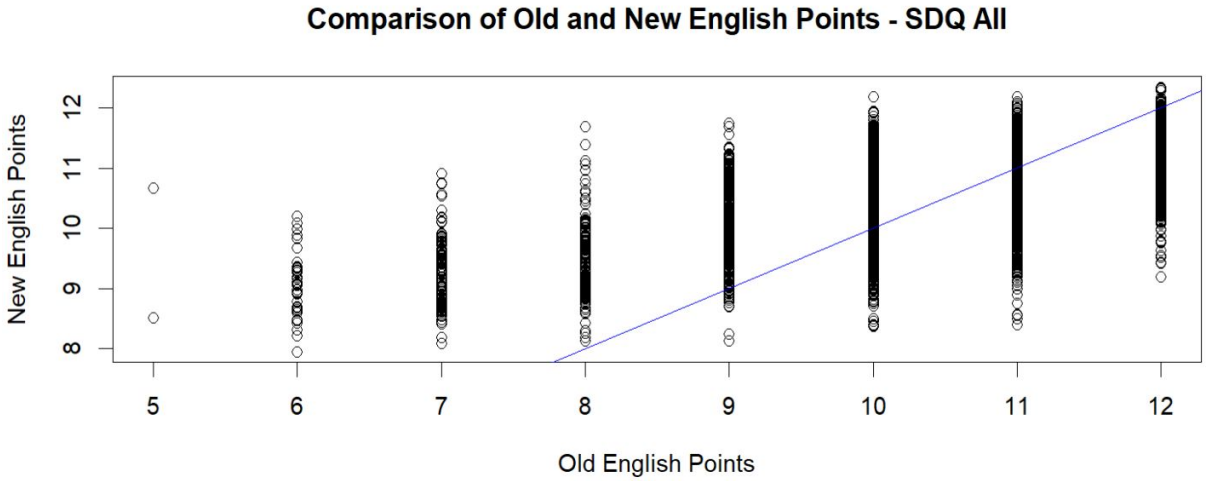
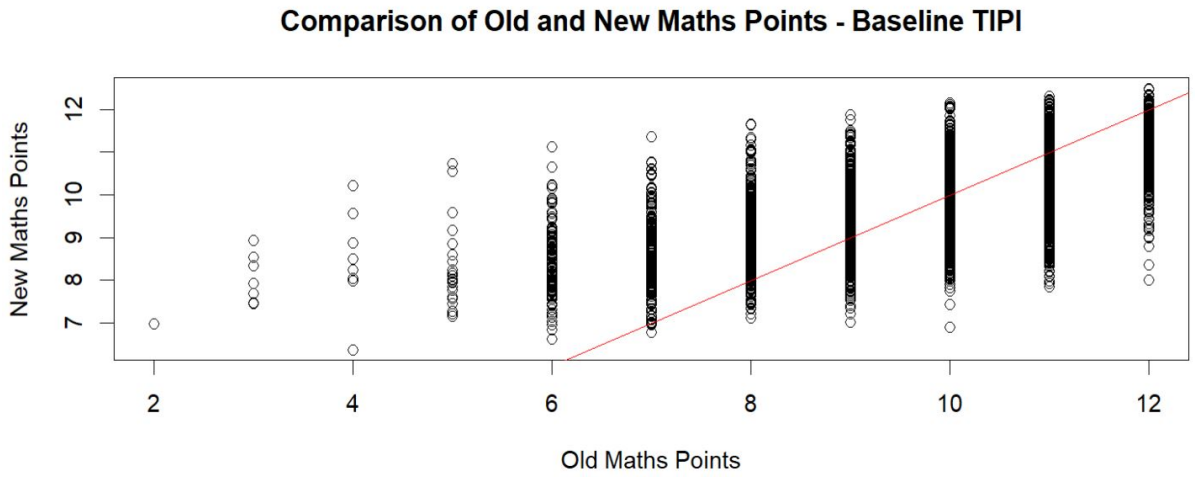

Comparison of Old and New English Points - SDQ All
New English Points
8
9
10
11
12
5
6
7
8
9
10
11
12
Old English Points

Figure 11


Comparison of Old and New Maths Points - Baseline TIPI
New Maths Points
7
8
9
10
12
2
4
6
8
10
12
Old Maths Points

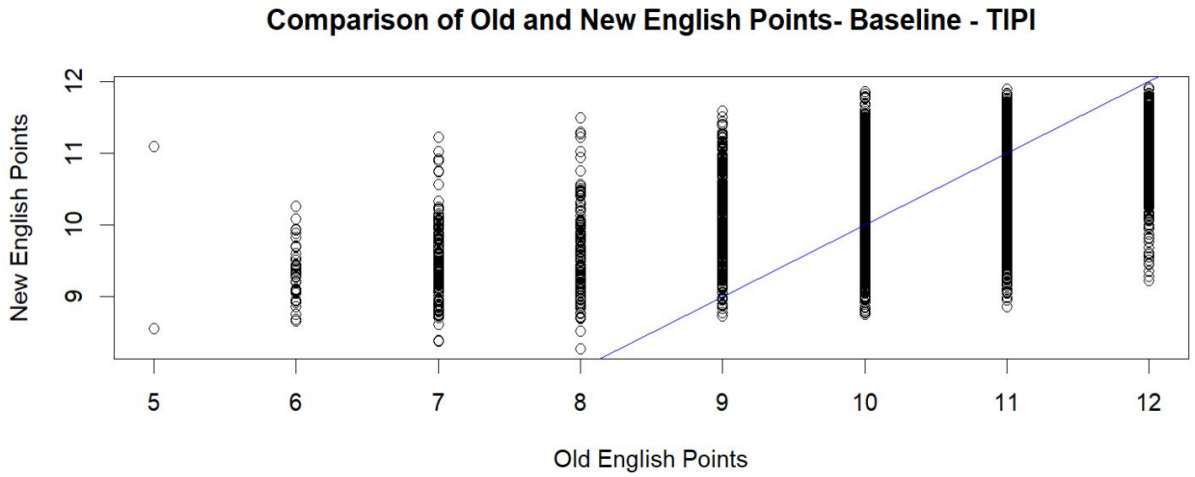
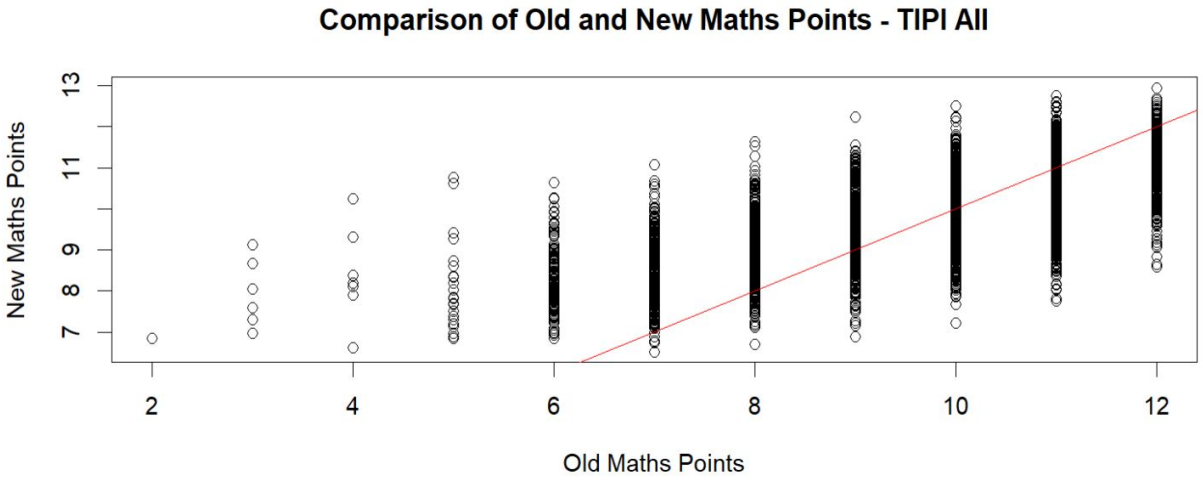

Comparison of Old and New English Points- Baseline - TIPI
New English Points
9
10
11
12
5
6
7
8
9
10
11
12
Old English Points

Figure 12


Comparison of Old and New Maths Points - TIPI All
New Maths Points
7
8
9
11
13
2
4
6
8
10
12
Old Maths Points

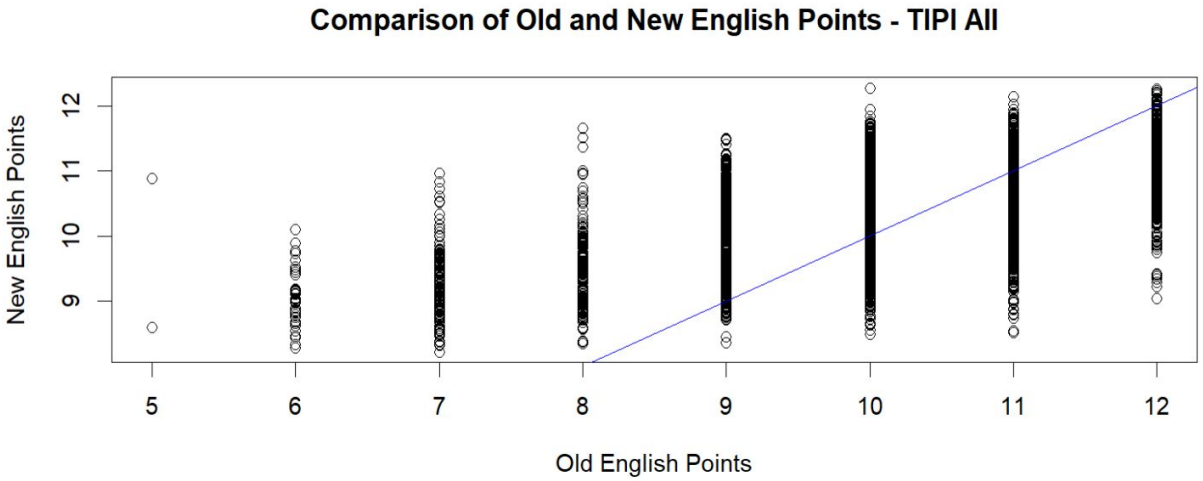

Comparison of Old and New English Points - TIPI All
New English Points
9
10
11
12
5
6
7
8
9
10
11
12
Old English Points

Figure 13

# 10 A production function for test-scores

## 10.1 Cobb-Douglas

### 10.1.1 Three inputs

$$Q_G = A \cdot C^{\alpha} \cdot (N_E)^{\beta_1} \cdot (N_I)^{\beta_2}$$

Where C is a measure of cognition (in this case, the principal component as a composite of three measures of cognition) and $N_E$ is a noncognitive measure that I call External Control and $N_I$ is also a noncognitive measure which I call Internal Control. In relation to the scales used (TIPI and SDQ), Internal Control is a proxy for Hyperactivity/Inattention and Conscientiousness, and External Control is a proxy for Emotional Symptoms and Emotional Stability. These are the four variables that seem to appear to be the most significant ones. I also calculated the correlation matrix for all nine variables (four for the SDQ and five for the TIPI scale), and these pairs had the highest coefficients (0.407 and 0.409, respectively).

Marginal productivities (MP) are defined as:

$$MP_C = \frac{\delta Q_G}{\delta C}\bigg|_{N_E=N_{E0}, N_I=N_{I0}} = A \cdot \alpha \cdot C^{\alpha-1} \cdot (N_{E0})^{\beta_1} \cdot (N_{I0})^{\beta_2}$$

$$MP_{N_E} = \frac{\delta Q_G}{\delta N_E}\bigg|_{C=C_0, N_I=N_{I0}} = A \cdot \alpha \cdot C_0^{\alpha} \cdot \beta_1 \cdot (N_{E0})^{\beta_1-1} \cdot (N_{I0})^{\beta_2}$$

$$MP_{N_I} = \frac{\delta Q_G}{\delta N_I}\bigg|_{C=C_0, N_E=N_{E0}} = A \cdot \alpha \cdot C_0^{\alpha} \cdot (N_{E0})^{\beta_1} \cdot \beta_2 \cdot (N_{I0})^{\beta_2-1}$$

Parameters estimation:

$$\ln(Q_G) = \ln(A) + \alpha \ln(C) + \beta_1 \ln(N_E) + \beta_2 \ln(N_I)$$

With marginal elasticities (which measure the percentage change in $Q_G$ in response to a 1% change in the parameters $C$, $N_E$ and $N_I$) defined as:

$$ME_C = \frac{\partial \ln(Q_G)}{\partial \ln(C)}\bigg|_{N_E=N_{E0}, N_I=N_{I0}} = \alpha$$

$$ME_{N_E} = \frac{\partial \ln(Q_G)}{\partial \ln(N_E)}\bigg|_{C=C_0, N_I=N_{I0}} = \beta_1$$

$$ME_{N_I} = \frac{\partial \ln(Q_G)}{\partial \ln(N_I)}\bigg|_{C=C_0, N_E=N_{E0}} = \beta_2$$

If we define the scale elasticity (SCE) as the scale change, often measured by the percent change in output from a simultaneous 1% change in all inputs, then: $ME_C + ME_{N_E} + ME_{N_I} = \alpha + \beta_1 + \beta_2$.

Output elasticities:

- $\alpha$ measures the impact of cognitive abilities (proxied by PC1) on predicted Maths and English scores. Across both models for Maths, values are 0.462 and 0.481, indicating a strong and positive relationship between cognitive abilities and grades. Improvements in cognitive measures have a substantial impact on predicting academic performance in Maths. For English, the estimate is around half the size, but still highly significant.
- $\beta_1$ measures the impact of External Control (related to Emotional Symptoms and Emotional Stability) on predicted outcomes, while $\beta_2$ measures the impact of Internal Control (related to Hyperactivity/Inattention and Conscientiousness). $\beta_1$ ranges approximately from 0.026 (highest) to 0.003 (lowest), while $\beta_2$ ranges from 0.040 (highest) to 0.008. These values indicate varying degrees of influence of noncognitive factors on Maths and English scores, with $\beta_2$ generally showing a stronger impact than $\beta_1$, but both still not even close to the magnitude of $\alpha$. $\beta_1$ is also mostly not significant.
- Since $\alpha + \beta_1 + \beta_2 < 1$, we observe decreasing returns to scale.

Table 18: Nonlinear Least Squares Model Summaries for Cobb-Douglas Production Function with Three Inputs

| | **TIPI Maths** | **SDQ Maths** | **TIPI English** | **SDQ English** |
|---|---|---|---|---|
| **Parameter** | | | | |
| $A$ | 4.131 | 3.953 | 6.360 | 6.270 |
| | (0.057) | (0.059) | (0.069) | (0.072) |
| t value | 72.447 | 67.395 | 92.507 | 86.665 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| $\alpha$ | 0.481 | 0.462 | 0.268 | 0.255 |
| | (0.008) | (0.008) | (0.006) | (0.006) |
| t value | 62.091 | 59.183 | 43.557 | 41.095 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| $\beta_1$ | 0.007 | 0.026 | 0.003 | 0.004 |
| | (0.002) | (0.006) | (0.002) | (0.004) |
| t value | 3.122 | 4.773 | 1.668 | 0.882 |
| $\Pr(>\|t\|)$ | 0.00181 | 1.86e-06 | 0.0953 | 0.378 |
| $\beta_2$ | 0.013 | 0.040 | 0.008 | 0.031 |
| | (0.002) | (0.003) | (0.001) | (0.003) |
| t value | 7.387 | 11.731 | 5.214 | 11.485 |
| $\Pr(>\|t\|)$ | 1.72e-13 | $<$ 2e-16 | 1.92e-07 | $<$ 2e-16 |
| Residual Std. Error | 1.290 | 1.276 | 1.135 | 1.124 |
| Degrees of Freedom | 5627 | 5627 | 5627 | 5627 |
| Iterations to Convergence | 7 | 7 | 8 | 8 |
| Achieved Convergence Tolerance | 1.49e-08 | 1.49e-08 | 1.49e-08 | 1.49e-08 |

## 10.2 Cobb-Douglas

### 10.2.1 Two inputs

$$Q_G = A \cdot C^{\alpha} \cdot (N)^{\beta}$$

Where C is a measure of cognition (in this case, the principal component as a composite of three measures of cognition) and $N$ is a noncognitive measure that I call Internal Control. In relation to the scales used (TIPI and SDQ), Internal Control is a proxy for Hyperactivity/Inattention and Conscientiousness.

Marginal productivities (MP) are defined as:

$$MP_C = \frac{\delta Q_G}{\delta C}\bigg|_{N=N_0} = A \cdot \alpha \cdot C^{\alpha-1} \cdot (N_0)^{\beta}$$

$$MP_N = \frac{\delta Q_G}{\delta N}\bigg|_{C=C_0} = A \cdot \beta \cdot C_0^{\alpha} \cdot (N)^{\beta-1}$$

Parameters estimation:

$$\ln(Q_G) = \ln(A) + \alpha \ln(C) + \beta \ln(N)$$

With marginal elasticities (which measure the percentage change in $Q_G$ in response to a 1% change in the parameters $C$ and $N$) defined as:

$$ME_C = \frac{\partial \ln(Q_G)}{\partial \ln(C)}\bigg|_{N=N_0} = \alpha$$

$$ME_N = \frac{\partial \ln(Q_G)}{\partial \ln(N)}\bigg|_{C=C_0} = \beta$$

If we define the scale elasticity (SCE) as the scale change, often measured by the percent change in output from a simultaneous 1% change in all inputs, then: $ME_C + ME_N = \alpha + \beta$.

Table 19: Nonlinear Least Squares Model Summaries for Cobb-Douglas Production Function with Two Inputs

| | **TIPI Maths** | **SDQ Maths** | **TIPI English** | **SDQ English** |
|---|---|---|---|---|
| **Parameter** | | | | |
| $A$ | 4.127 | 4.061 | 6.358 | 6.294 |
| | (0.057) | (0.056) | (0.069) | (0.067) |
| t value | 72.394 | 72.990 | 92.500 | 93.430 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| $\alpha$ | 0.483 | 0.467 | 0.269 | 0.256 |
| | (0.008) | (0.008) | (0.006) | (0.006) |
| t value | 62.539 | 60.110 | 43.870 | 41.550 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| $\beta$ | 0.014 | 0.043 | 0.008 | 0.031 |
| | (0.002) | (0.003) | (0.001) | (0.003) |
| t value | 7.829 | 12.650 | 5.458 | 11.790 |
| $\Pr(>\|t\|)$ | 5.82e-15 | $<$ 2e-16 | 5.02e-08 | $<$ 2e-16 |
| Residual Std. Error | 1.291 | 1.279 | 1.135 | 1.124 |
| Degrees of Freedom | 5628 | 5628 | 5628 | 5628 |
| Iterations to Convergence | 6 | 6 | 8 | 8 |
| Achieved Convergence Tolerance | 1.49e-08 | 1.49e-08 | 1.49e-08 | 1.49e-08 |

Working with two inputs instead of three did not lead to major changes in the parameters, which indicates that there are only two significant noncognitive variables (Hyperactivity/Inattention and Conscientiousness).

## 10.3 CES

### 10.3.1 Three inputs

$$Q_G = A\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left((N_E)^{\frac{1}{\alpha}} + (N_I)^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}}$$

Where C is a measure of cognition (in this case, the principal component as a composite of three measures of cognition) and $N_E$ is a noncognitive measure that I call External Control and $N_I$ is also a noncognitive measure which I call Internal Control. In relation to the scales used (TIPI and SDQ), Internal Control is a proxy for Hyperactivity/Inattention and Conscientiousness, and External Control is a proxy for Emotional Symptoms and Emotional Stability. These are the four variables that seem to appear to be the most significant ones. I also calculated the correlation matrix for all nine variables (four for the SDQ and five for the TIPI scale), and these pairs had the highest coefficients (0.407 and 0.409, respectively).

Marginal productivities are defined as:

$$\begin{aligned}
MP_C &= \left.\frac{\partial Q_G}{\partial C}\right|_{N_E=N_{E0},N_I=N_{I0}} \\
&= A\cdot\frac{\alpha^2}{\alpha-1}\cdot C^{\frac{1}{\alpha}-1}\cdot\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_{E0}^{\frac{1}{\alpha}} + N_{I0}^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}-1} \\
MP_{N_E} &= \left.\frac{\partial Q_G}{\partial N_E}\right|_{C=C_0,N_I=N_{I0}} \\
&= A\cdot\frac{\alpha}{\alpha-1}\cdot(1-\alpha)\cdot\frac{1}{\alpha}\cdot\left(N_E^{\frac{1}{\alpha}-1}\right)\cdot\left(N_I^{\frac{1}{\alpha}}\right)\cdot\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_E^{\frac{1}{\alpha}} + N_I^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}-1} \\
MP_{N_I} &= \left.\frac{\partial Q_G}{\partial N_I}\right|_{C=C_0,N_E=N_{E0}} \\
&= A\cdot\frac{\alpha}{\alpha-1}\cdot(1-\alpha)\cdot\frac{1}{\alpha}\cdot\left(N_I^{\frac{1}{\alpha}-1}\right)\cdot\left(N_E^{\frac{1}{\alpha}}\right)\cdot\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_E^{\frac{1}{\alpha}} + N_I^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}-1}
\end{aligned}$$

Parameters estimation:

$$\ln(Q_G) = \ln(A) + \frac{\alpha}{\alpha-1}\ln\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left((N_E)^{\frac{1}{\alpha}} + (N_I)^{\frac{1}{\alpha}}\right)\right]$$

With marginal elasticities (which measure the percentage change in $Q_G$ in response to a 1% change in the parameters $C$ and $N_I$ and $N_E$) defined as:

$$\begin{aligned}
ME_C &= \left.\frac{\partial\ln(Q_G)}{\partial\ln(C)}\right|_{N_E=N_{E0},N_I=N_{I0}} \\
&= \frac{\alpha^2}{\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_{E0}^{\frac{1}{\alpha}} + N_{I0}^{\frac{1}{\alpha}}\right)}
\end{aligned}$$

$$\begin{aligned}
ME_{N_E} &= \left.\frac{\partial\ln(Q_G)}{\partial\ln(N_E)}\right|_{C=C_0,N_I=N_{I0}} \\
&= \frac{\alpha(1-\alpha)(N_E)^{\frac{1}{\alpha}-1}}{\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)\left((N_E)^{\frac{1}{\alpha}} + (N_{I0})^{\frac{1}{\alpha}}\right)}
\end{aligned}$$

$$\begin{aligned}
ME_{N_I} &= \left.\frac{\partial\ln(Q_G)}{\partial\ln(N_I)}\right|_{C=C_0,N_E=N_{E0}} \\
&= \frac{\alpha(1-\alpha)(N_I)^{\frac{1}{\alpha}-1}}{\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)\left((N_{E0})^{\frac{1}{\alpha}} + (N_I)^{\frac{1}{\alpha}}\right)}
\end{aligned}$$

The parameter $\alpha$ determines the degree of substitutability between the inputs: $\alpha = 1$ (inputs are perfect substitutes), $\alpha < 1$ (inputs are imperfect substitutes), and $\alpha > 1$ (inputs are complements). When we have $\alpha = 1$, we need to evaluate the limit because direct substitution leads to an indeterminate form (details in the Appendix).

Table 20: Nonlinear Least Squares Model Summaries for CES Production Function with Three Inputs

| | **TIPI Maths** | **SDQ Maths** | **TIPI English** | **SDQ English** |
|---|---|---|---|---|
| **Parameter** | | | | |
| $A$ | 4.536 | 4.219 | 4.748 | 4.351 |
| | (0.014) | (0.012) | (0.017) | (0.014) |
| t value | 327.4 | 360.6 | 287.4 | 308.8 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| $\alpha$ | 0.977 | 0.907 | 0.975 | 0.880 |
| | (0.002) | (0.003) | (0.002) | (0.004) |
| t value | 600.7 | 264.2 | 525.5 | 212.3 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| Residual Std. Error | 1.656 | 1.576 | 1.973 | 1.860 |
| Degrees of Freedom | 5629 | 5629 | 5629 | 5629 |
| Iterations to Convergence | 8 | 6 | 5 | 8 |
| Achieved Convergence Tolerance | 1.49e-08 | 1.49e-08 | 1.49e-08 | 1.49e-08 |

The values of $\alpha$ suggest that the inputs are imperfect substitutes but exhibit relatively high substitutability. The slightly lower $\alpha$ value in the SDQ English model (0.880) indicates that the inputs are also imperfect substitutes but with a somewhat lower degree of substitutability compared to the other models. While cognitive measures $C$ and noncognitive measures $N_E$ and $N_I$ are substitutable to a some extent in predicting $Q_G$, they are not completely interchangeable.

## 10.4 CES

### 10.4.1 Two inputs

$$Q_G = A\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}}$$

Where C is a measure of cognition (in this case, the principal component as a composite of three measures of cognition) and $N$ is a noncognitive measure that I call Internal Control. In relation to the scales used (TIPI and SDQ), Internal Control is a proxy for Hyperactivity/Inattention and Conscientiousness.

Marginal productivities (MP) are defined as:

$$\begin{aligned} MP_C &= \left.\frac{\partial Q_G}{\partial C}\right|_{N=N_0} \\ &= A \cdot \frac{\alpha}{\alpha-1} \cdot \left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_0^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}-1} \cdot C^{\frac{1}{\alpha}-1} \end{aligned}$$

$$\begin{aligned} MP_N &= \left.\frac{\partial Q_G}{\partial N}\right|_{C=C_0} \\ &= A \cdot \frac{\alpha(1-\alpha)}{\alpha-1} \cdot \left[\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)(N^{\frac{1}{\alpha}})\right]^{\frac{\alpha}{\alpha-1}-1} \cdot N^{\frac{1}{\alpha}-1} \end{aligned}$$

Parameters estimation:

$$\ln(Q_G) = \ln(A) + \frac{\alpha}{\alpha-1}\ln\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right]$$

With marginal elasticities (which measure the percentage change in $Q_G$ in response to a 1% change in the parameters $C$ and $N$ defined as:

$$\begin{aligned} ME_C &= \left.\frac{\partial \ln(Q_G)}{\partial \ln(C)}\right|_{N=N_0} \\ &= \frac{1}{\alpha-1} \cdot \frac{C^{\frac{1}{\alpha}-1}}{C^{\frac{1}{\alpha}} + \frac{1-\alpha}{\alpha}\left(N_0^{\frac{1}{\alpha}}\right)} \end{aligned}$$

$$\begin{aligned} ME_N &= \left.\frac{\partial \ln(Q_G)}{\partial \ln(N)}\right|_{C=C_0} \\ &= \frac{1-\alpha}{\alpha(\alpha-1)} \cdot \frac{N^{\frac{1}{\alpha}-1}}{C_0^{\frac{1}{\alpha}} + \frac{1-\alpha}{\alpha}\left(N^{\frac{1}{\alpha}}\right)} \end{aligned}$$

We have similar results to the previous section.

Table 21: Nonlinear Least Squares Model Summaries for CES Production Function with Two Inputs

| | **TIPI Maths** | **SDQ Maths** | **TIPI English** | **SDQ English** |
|---|---|---|---|---|
| **Parameter** | | | | |
| $A$ | 1.676 | 1.704 | 1.749 | 1.793 |
| | (0.006592) | (0.005932) | (0.007861) | (0.007174) |
| t value | 254.2 | 287.4 | 222.5 | 249.9 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| $\alpha$ | 0.977 | 0.926 | 0.978 | 0.916 |
| | (0.002152) | (0.003906) | (0.002461) | (0.004518) |
| t value | 454.0 | 237.0 | 397.2 | 202.7 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| Residual Std. Error | 1.669 | 1.637 | 1.990 | 1.951 |
| Degrees of Freedom | 5629 | 5629 | 5629 | 5629 |
| Iterations to Convergence | 5 | 6 | 5 | 6 |
| Achieved Convergence Tolerance | 1.49e-08 | 1.49e-08 | 1.49e-08 | 1.49e-08 |

## 10.5 Translog with interaction terms

### 10.5.1 Two inputs

The translog production function is a flexible form that extends the Cobb-Douglas production function by including logarithms of inputs and their squares and cross-products.

$$Q_G = A \cdot C^{\alpha} \cdot N^{\beta} \cdot \exp\left\{\frac{1}{2}\gamma_1 \left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2 \left[\ln(N)\right]^2 + \gamma_{12} \ln(C) \ln(N)\right\}$$

Where C is a measure of cognition (in this case, the principal component as a composite of three measures of cognition) and $N$ is a noncognitive measure that I call Internal Control. In relation to the scales used (TIPI and SDQ), Internal Control is a proxy for Hyperactivity/Inattention and Conscientiousness.

Marginal productivities (MP) are defined as:

$$MP_C = \left.\frac{\partial Q_G}{\partial C}\right|_{N=N_0} = A \cdot \alpha \cdot C^{\alpha-1} \cdot N_0^{\beta} \cdot \frac{\partial}{\partial C}\left[\exp\left\{X(C,N)\right\}\right]$$

$$MP_N = \left.\frac{\partial Q_G}{\partial N}\right|_{C=C_0} = A \cdot C_0^{\alpha} \cdot \beta \cdot N^{\beta-1} \cdot \frac{\partial}{\partial N}\left[\exp\left\{X(C,N)\right\}\right]$$

where

$$X(C,N) = \exp\left\{\frac{1}{2}\gamma_1 \left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2 \left[\ln(N)\right]^2 + \gamma_{12} \ln(C) \ln(N)\right\}$$

We have:

$$MP_C = A{\cdot}\alpha{\cdot}C^{\alpha-1}{\cdot}N_0^{\beta}{\cdot}\exp\left\{\frac{1}{2}\gamma_1 \left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2 \left[\ln(N_0)\right]^2 + \gamma_{12} \ln(C) \ln(N_0)\right\}{\cdot}\left[\gamma_1 \ln(C) \cdot \frac{1}{C} + \gamma_{12} \cdot \frac{\ln(N_0)}{C}\right]$$

$$MP_N = A{\cdot}\beta{\cdot}C_0^{\alpha}{\cdot}N^{\beta-1}{\cdot}\exp\left\{\frac{1}{2}\gamma_1 \left[\ln(C_0)\right]^2 + \frac{1}{2}\gamma_2 \left[\ln(N)\right]^2 + \gamma_{12} \ln(C_0) \ln(N)\right\}{\cdot}\left[\gamma_2 \ln(N) \cdot \frac{1}{N} + \gamma_{12} \cdot \frac{\ln(C_0)}{N}\right]$$

Parameters estimation:

$$\ln Q_G = \ln A + \alpha \ln C + \beta \ln N + \frac{1}{2}\gamma_1 (\ln C)^2 + \frac{1}{2}\gamma_2 (\ln N)^2 + \gamma_{12} \ln C \ln N$$

With marginal elasticities (which measure the percentage change in $Q_G$ in response to a 1% change in the parameters $C$ and $N$ defined as:

$$ME_C = \left.\frac{\partial \ln(Q_G)}{\partial \ln(C)}\right|_{N=N_0} = \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N_0)$$

$$ME_N = \left.\frac{\partial \ln(Q_G)}{\partial \ln(N)}\right|_{C=C_0} = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C_0)$$

The parameters $\alpha$, $\beta$, $\gamma_1$, $\gamma_2$ and $\gamma_{12}$ represent specific aspects of the production function, and their values estimated below give insights into the estimated relationships between the inputs $C$ and $N$ and output $Q_G$ in the production process. $\gamma_1$, $\gamma_2$ and $\gamma_{12}$ represent the second-order terms and interaction terms that capture non-linearities and interactions between inputs. A positive $\gamma_i$ indicates increasing returns to the input at an increasing rate, while a negative $\gamma_i$ indicates diminishing returns to the input. We see that $\gamma_1$ and $\gamma_2$ are both positive and highly significant, while $\gamma_{12}$ is negative, but not significant in three out of four outputs. These results are interesting because they seem to corroborate the previous results regarding heterogeneity in the interaction between cognitive and noncognitive variables.

Table 22: Nonlinear Least Squares Model Summaries for Translog Production Function

| | **TIPI Maths** | **SDQ Maths** | **TIPI English** | **SDQ English** |
|---|---|---|---|---|
| **Parameter** | | | | |
| $A$ | 4.820 | 4.349 | 6.791 | 6.137 |
| | (0.082) | (0.102) | (0.093) | (0.114) |
| t value | 58.519 | 42.728 | 73.402 | 53.843 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| $\alpha$ | 0.240 | 0.283 | 0.148 | 0.187 |
| | (0.020) | (0.017) | (0.016) | (0.014) |
| t value | 12.267 | 16.609 | 9.491 | 13.805 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| $\beta$ | 0.045 | 0.101 | 0.024 | 0.103 |
| | (0.012) | (0.018) | (0.010) | (0.015) |
| t value | 3.707 | 5.536 | 2.447 | 7.117 |
| $\Pr(>\|t\|)$ | 0.0002 | $<$ 2e-16 | 0.0144 | $<$ 2e-16 |
| $\gamma_1$ | 0.162 | 0.131 | 0.086 | 0.067 |
| | (0.013) | (0.008) | (0.011) | (0.006) |
| t value | 12.135 | 16.167 | 8.094 | 10.414 |
| $\Pr(>\|t\|)$ | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 | $<$ 2e-16 |
| $\gamma_2$ | 0.008 | 0.026 | 0.004 | 0.024 |
| | (0.002) | (0.003) | (0.001) | (0.002) |
| t value | 4.112 | 9.595 | 2.755 | 11.064 |
| $\Pr(>\|t\|)$ | 0.00004 | $<$ 2e-16 | 0.0059 | $<$ 2e-16 |
| $\gamma_{12}$ | -0.013 | -0.024 | -0.007 | -0.033 |
| | (0.007) | (0.011) | (0.006) | (0.009) |
| t value | -1.936 | -2.171 | -1.213 | -3.759 |
| $\Pr(>\|t\|)$ | 0.0529 | 0.0300 | 0.2252 | 0.0002 |
| Residual Std. Error | 0.154 | 0.151 | 0.123 | 0.120 |
| Degrees of Freedom | 5625 | 5625 | 5625 | 5625 |
| Iterations to Convergence | 6 | 6 | 7 | 9 |
| Achieved Convergence Tolerance | 1.49e-08 | 1.49e-08 | 1.49e-08 | 1.49e-08 |

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# 11 Appendix

## 11.1 Derivation of Marginal Elasticities for a CES production function with three inputs

Given

$$\ln(Q_G) = \ln(A) + \frac{\alpha}{\alpha - 1} \ln\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left((N_E)^{\frac{1}{\alpha}} + (N_I)^{\frac{1}{\alpha}}\right)\right]$$

Marginal elasticities are:

$$\begin{aligned}
ME_C &= \left.\frac{\partial \ln(Q_G)}{\partial \ln(C)}\right|_{N_E = N_{E0}, N_I = N_{I0}} \\
&= \frac{\partial}{\partial \ln(C)}\left[\ln(A) + \frac{\alpha}{\alpha-1}\ln\left(\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_{E0}^{\frac{1}{\alpha}} + N_{I0}^{\frac{1}{\alpha}}\right)\right)\right] \\
&= \frac{\alpha}{\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_{E0}^{\frac{1}{\alpha}} + N_{I0}^{\frac{1}{\alpha}}\right)} \cdot \frac{\alpha}{C} \\
&= \frac{\alpha^2}{\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_{E0}^{\frac{1}{\alpha}} + N_{I0}^{\frac{1}{\alpha}}\right)} \square
\end{aligned}$$

$$\begin{aligned}
ME_{N_E} &= \left.\frac{\partial \ln(Q_G)}{\partial \ln(N_E)}\right|_{C = C_0, N_I = N_{I0}} \\
&= \frac{\partial}{\partial \ln(N_E)}\left[\ln(A) + \frac{\alpha}{\alpha-1}\ln\left(\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)\left((N_E)^{\frac{1}{\alpha}} + (N_{I0})^{\frac{1}{\alpha}}\right)\right)\right] \\
&= \frac{\alpha(1-\alpha)(N_E)^{\frac{1}{\alpha}-1}}{\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)\left((N_E)^{\frac{1}{\alpha}} + (N_{I0})^{\frac{1}{\alpha}}\right)} \square
\end{aligned}$$

$$\begin{aligned}
ME_{N_I} &= \left.\frac{\partial \ln(Q_G)}{\partial \ln(N_I)}\right|_{C = C_0, N_E = N_{E0}} \\
&= \frac{\partial}{\partial \ln(N_I)}\left[\ln(A) + \frac{\alpha}{\alpha-1}\ln\left(\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)\left((N_{E0})^{\frac{1}{\alpha}} + (N_I)^{\frac{1}{\alpha}}\right)\right)\right] \\
&= \frac{\alpha(1-\alpha)(N_I)^{\frac{1}{\alpha}-1}}{\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)\left((N_{E0})^{\frac{1}{\alpha}} + (N_I)^{\frac{1}{\alpha}}\right)} \square
\end{aligned}$$

We then evaluate the limit when $\alpha \to 1$:

$$\begin{aligned}
ME_C &= \lim_{\alpha \to 1} \frac{\alpha^2}{\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_{E0}^{\frac{1}{\alpha}} + N_{I0}^{\frac{1}{\alpha}}\right)} \\
&= \lim_{\alpha \to 1} \frac{1}{C} \\
&= \frac{1}{C}
\end{aligned}$$

$$\begin{aligned}
ME_{N_E} &= \lim_{\alpha \to 1} \frac{\alpha(1-\alpha)(N_E)^{\frac{1}{\alpha}-1}}{\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)\left((N_E)^{\frac{1}{\alpha}} + (N_{I0})^{\frac{1}{\alpha}}\right)} \\
&= \lim_{\alpha \to 1} 0 \\
&= 0
\end{aligned}$$

$$\begin{aligned}
ME_{N_I} &= \lim_{\alpha \to 1} \frac{\alpha(1-\alpha)(N_I)^{\frac{1}{\alpha}-1}}{\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)\left((N_{E0})^{\frac{1}{\alpha}} + (N_I)^{\frac{1}{\alpha}}\right)} \\
&= \lim_{\alpha \to 1} 0 \\
&= 0
\end{aligned}$$

## 11.2 Derivation of Marginal Productivities for a CES production function with two inputs

Given the production function:

$$Q_G = A\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}}$$

To find $MP_C = \frac{\partial Q_G}{\partial C}\Big|_{N=N_0}$:

1. First we define:
$$X(C,N) = \alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)$$
2. The we express $Q_G$ in terms of $X(C,N)$:
$$Q_G = AX^{\frac{\alpha}{\alpha-1}}$$
3. Now we differentiate $Q_G$ with respect to $C$:
$$\frac{\partial Q_G}{\partial C} = A \cdot \frac{\alpha}{\alpha-1} \cdot X^{\frac{\alpha}{\alpha-1}-1} \cdot \frac{\partial X(C,N)}{\partial C}$$
4. And find $\frac{\partial X(C,N)}{\partial C}$:
$$\frac{\partial X(C,N)}{\partial C} = \alpha C^{\frac{1}{\alpha}-1}$$
5. We then substitute $\frac{\partial X(C,N)}{\partial C}$ into the derivative:
$$\frac{\partial Q_G}{\partial C} = A \cdot \frac{\alpha}{\alpha-1} \cdot X^{\frac{\alpha}{\alpha-1}-1} \cdot \alpha C^{\frac{1}{\alpha}-1}$$
6. Simplifying the expression:
$$MP_C = A \cdot \frac{\alpha}{\alpha-1} \cdot \left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}-1} \cdot C^{\frac{1}{\alpha}-1}$$

Therefore, the marginal productivity of $C$, $MP_C$, is given by:

$$MP_C = A \cdot \frac{\alpha}{\alpha-1} \cdot \left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_0^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}-1} \cdot C^{\frac{1}{\alpha}-1}\square$$

We follow similar steps to find $MP_N = \frac{\partial Q_G}{\partial N}\Big|_{C=C_0}$:

1. We define $X(C,N)$:
$$X(C,N) = \alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)N^{\frac{1}{\alpha}}$$
2. The we express $Q_G$ in terms of $X(C,N)$:
$$Q_G = AX^{\frac{\alpha}{\alpha-1}}$$
3. Now we differentiate $Q_G$ with respect to $N$:
$$\frac{\partial Q_G}{\partial N} = A \cdot \frac{\alpha}{\alpha-1} \cdot X^{\frac{\alpha}{\alpha-1}-1} \cdot \frac{\partial X(C,N)}{\partial N}$$
4. And find $\frac{\partial X(C,N)}{\partial N}$:
$$\frac{\partial X(C,N)}{\partial N} = (1-\alpha) \cdot \frac{1}{\alpha} \cdot N^{\frac{1}{\alpha}-1}$$
5. We then substitute $\frac{\partial X(C,N)}{\partial N}$ into the derivative:
$$\frac{\partial Q_G}{\partial N} = A \cdot \frac{\alpha}{\alpha-1} \cdot X^{\frac{\alpha}{\alpha-1}-1} \cdot (1-\alpha) \cdot \frac{1}{\alpha} \cdot N^{\frac{1}{\alpha}-1}$$
6. Simplifying the expression:
$$MP_N = A \cdot \frac{\alpha(1-\alpha)}{\alpha-1} \cdot \left[\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)N^{\frac{1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}-1} \cdot N^{\frac{1}{\alpha}-1}$$

Therefore, the marginal productivity with respect to $N$, $MP_N$, is given by:

$$MP_N = A \cdot \frac{\alpha(1-\alpha)}{\alpha-1} \cdot \left[\alpha C_0^{\frac{1}{\alpha}} + (1-\alpha)N^{\frac{1}{\alpha}}\right]^{\frac{\alpha}{\alpha-1}-1} \cdot N^{\frac{1}{\alpha}-1}\square$$

## 11.3 Derivation of Marginal Elasticities for a CES production function with two inputs

Given the CES (Constant Elasticity of Substitution) production function:

$$Q_G = A\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right]^{\frac{\alpha}{\alpha-1}}$$

To find the marginal elasticities we:
1. Take the natural logarithm on both sides:

$$\ln(Q_G) = \ln(A) + \frac{\alpha}{\alpha-1}\ln\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right]$$

2. Then the marginal elasticity with respect to $C$ ($ME_C$) is:

$$ME_C = \left.\frac{\partial \ln(Q_G)}{\partial \ln(C)}\right|_{N=N_0} = \frac{\partial \ln(Q_G)}{\partial \ln(C)} = \frac{\alpha}{\alpha-1}\cdot\frac{\partial}{\partial \ln(C)}\ln\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right]$$

Since $\frac{\partial \ln(C)}{\partial \ln(C)} = 1$, we can focus on the derivative inside the logarithm:

$$\frac{\partial}{\partial \ln(C)}\ln\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right] = \frac{1}{\alpha}\cdot\frac{\alpha C^{\frac{1}{\alpha}-1}}{\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_0^{\frac{1}{\alpha}}\right)}$$

Therefore,

$$ME_C = \frac{\alpha}{\alpha-1}\cdot\frac{1}{\alpha}\cdot\frac{\alpha C^{\frac{1}{\alpha}-1}}{\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N_0^{\frac{1}{\alpha}}\right)}$$

$$ME_C = \frac{1}{\alpha-1}\cdot\frac{C^{\frac{1}{\alpha}-1}}{C^{\frac{1}{\alpha}} + \frac{1-\alpha}{\alpha}\left(N_0^{\frac{1}{\alpha}}\right)}\square$$

3. Now, we find the marginal elasticity with respect to $N$ ($ME_N$):

$$ME_N = \left.\frac{\partial \ln(Q_G)}{\partial \ln(N)}\right|_{C=C_0} = \frac{\partial \ln(Q_G)}{\partial \ln(N)} = \frac{\alpha}{\alpha-1}\cdot\frac{\partial}{\partial \ln(N)}\ln\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right]$$

Since $\frac{\partial \ln(N)}{\partial \ln(N)} = 1$, we now focus on the derivative inside the logarithm:

$$\frac{\partial}{\partial \ln(N)}\ln\left[\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)\right] = \frac{1}{\alpha}\cdot\frac{(1-\alpha)N^{\frac{1}{\alpha}-1}}{\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)}$$

Therefore,

$$ME_N = \frac{\alpha}{\alpha-1}\cdot\frac{1}{\alpha}\cdot\frac{(1-\alpha)N^{\frac{1}{\alpha}-1}}{\alpha C^{\frac{1}{\alpha}} + (1-\alpha)\left(N^{\frac{1}{\alpha}}\right)}$$

$$ME_N = \frac{1-\alpha}{\alpha(\alpha-1)}\cdot\frac{N^{\frac{1}{\alpha}-1}}{C_0^{\frac{1}{\alpha}} + \frac{1-\alpha}{\alpha}\left(N^{\frac{1}{\alpha}}\right)}\square$$