Gender, Skill Interaction, and Academic Achievement

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Abstract

This study examines how cognitive and noncognitive skills shape academic achievement in Maths and English among Irish secondary students, with a focus on gender differences. Using data from the Growing Up in Ireland longitudinal study, I estimate both linear and translog production functions. Cognitive skills are the strongest predictors of performance, particularly in Maths and among boys. Noncognitive skills also contribute, especially for girls in Maths. Translog estimates show that cognitive and noncognitive skills typically operate as complements, with elasticity of substitution values below one. This pattern reflects diminishing marginal returns: the impact of improving one skill declines when the other is already well developed. A distinct pattern emerges in girls' Maths performance, where elasticity values above one reveal a substitutable relationship between skills. These results suggest the importance of educational strategies that support both cognitive and noncognitive development, tailored to subject matter and gender, in order to strengthen academic outcomes.

Keywords: cognitive skills, noncognitive skills, educational production, academic performance, gender differences, human capital

JEL Codes: I21, I24, J24, C31

1 Introduction

Human capital development is fundamental to economic growth and individual labour market success. While the individual roles of cognitive and noncognitive skills in academic achievement are well-documented, significant questions persist regarding their combined impact. A core uncertainty is whether these skills operate as complements or substitutes within the educational production function. Often, studies have analyzed cognitive and noncognitive skills in isolation, thereby missing the nuanced ways they interact to determine educational outcomes. Additionally, variations in these skill interactions across different academic subjects and between genders require further investigation.

This study tackles these unresolved issues by examining the interplay of cognitive and noncognitive skills in predicting Maths and English performance among Irish secondary students. Ireland provides a compelling context for this research, featuring: a centralized curriculum and standardised national examinations like the Junior Certificate, which facilitate consistent achievement measurement; a growing policy focus on "wellbeing" (Lawlor, 2019; National Council for Curriculum and Assessment, 2021), indicating increased recognition of noncognitive skills; and an ongoing gender gap in STEM fields (McNally, 2020), heightening the importance of understanding gender-specific skill utilisation.

My research aims to address the following questions:

I ask to what extent cognitive and noncognitive skills contribute to academic achievement in Maths and English, how these contributions differ between genders, and to what extent cognitive and noncognitive skills substitute for or complement each other across subjects and gender groups.

Using data from the Growing Up in Ireland longitudinal study, I employ regression analysis and estimate a translog production function to model the relationship between cognitive skills, noncognitive skills, and academic achievement. This approach allows me to quantify non-linear links and gender-specific effects, providing a more nuanced understanding of the educational production process.

My findings reveal several important insights: cognitive ability drives academic performance more than any other factor, with boys getting slightly more benefit from cognitive skills than girls in both subjects. behavioural skills (measured by SDQ) also help students succeed, though less dramatically than cognitive skills, and girls benefit more from these skills, especially in Maths. Both types of skills matter more for Maths than English, showing that different subjects need different combinations of skills. As students develop either type of skill, each additional improvement yields smaller gains, especially in English. Cognitive and noncognitive skills generally act as complements (ES < 1), with one exception: for girls in Maths, the estimated ES exceeds 1, indicating some substitutability. Strong behavioural skills can partially compensate for lower cognitive skills, but their ability to do so is limited - particularly where skills are complementary. The translog estimates also show how cognitive and noncognitive skills work together differently for boys and girls, a pattern we would not see using simpler specifications.

These empirical insights carry significant implications for educational policy. First, they suggest that cognitive and noncognitive skills should be developed in tandem, rather than treated as independent domains, which challenges pedagogical approaches that treat them as isolated domains. Second, teaching methods should adapt to how boys and girls use different combinations of skills, especially in Maths. Finally, the subject-specific nature of skill interactions implies that effective teaching methodologies may need to be discipline-dependent; approaches successful in English, for example, may not be optimal for Maths, where the interplay of skills is distinct.

This study makes several distinct contributions to the literature. While extensive research addresses cognitive/noncognitive skills (Cunha & Heckman, 2008; Heckman & Kautz, 2012) and gender differences in academic performance (Hyde, 2016; Niederle & Vesterlund, 2010) separately, this paper uniquely integrates these dimensions within the Irish educational context. By employing both linear and translog production functions, I was able to extend the methodological approaches typically used in educational production function research (Todd & Wolpin, 2003). My focus on subject-specific interactions between cognitive and noncognitive skills, particularly in Maths and English, builds upon but differs from previous work that has often treated these skills more uniformly across subjects (Balart et al., 2018). Furthermore, by situating this analysis within the Irish secondary education system, this study contributes to the understanding of these dynamics in a specific national context, building upon previous Irish educational research (Smyth et al., 2015; Sofroniou et al., 2000). The use of both the SDQ (Goodman, 1997) and TIPI (Gosling et al., 2003) provides a comprehensive measure of noncognitive skills while also allowing for a extensive analysis of their role in academic achievement. By combining these different approaches, my study reveals how cognitive and noncognitive skills interact in shaping academic performance - and how these interactions differ across students, subjects, and skill types.

The insights gleaned from this research can inform the design of more effective and targeted educational interventions. For instance, the evidence of skill complementarity suggests the value of integrated curricula that simultaneously foster cognitive and noncognitive development, rather than in isolation. Personalized learning plans could be tailored to address the specific skill interactions and gender-differentiated needs identified. Furthermore, the importance of noncognitive skills points towards implementing programs aimed at enhancing socio-emotional competencies such as emotional regulation, goal-setting, empathy, and responsible decision-making. Workshops and experiential activities focused on improving transferable skills like teamwork, communication, and problem-solving, as advocated by Durlak et al. (2011), could also be embedded within core subjects, particularly where noncognitive skills show strong complementarity with cognitive abilities for specific genders or subjects.

The remainder of this paper is structured as follows: Section I provides an overall review of the literature on cognitive and noncognitive skills in education, with a focus on gender differences. Section II describes the data used in this study, including details on the Growing Up in Ireland longitudinal study, the chosen psychometric assessment tools, and its limitations. Section III presents the theoretical framework, a model on the linear production function and its estimation. In section IV I extend the model to allow for nonlinearity by estimating a translog production function. Section V then concludes with a broader discussion of the overall results, a summary of the key contributions of this study, the possible limitations, and directions for future research.

1.1 The multidimensional nature of education and skills

Education is, in its nature, multidimensional, occurring in a feedback loop across time and space and involving numerous agents and institutions. Given the scarcity of resources such as money and labour, it is essential to allocate them wisely to achieve the most effective outcomes. Traditionally, the effectiveness of educational resources has been measured through completed levels and years of education (quantity, educational attainment) and test scores (quality, educational achievement). More properly qualified and skillful students lead better lives and participate more actively in civic duties and the labour market (Oreopoulos & Salvanes, 2011).

Noncognitive skills rival IQ in predicting educational attainment, labour market success, health, and criminality (Kautz et al., 2014), challenging the idea that cognitive ability alone is sufficient to explain life outcomes. This shift in perspective calls for a deeper understanding of what factors contribute to educational outcomes and how we can measure them. The literature on the returns to education, both private and social, is vast and almost unanimous on the importance of improving educational outcomes for students.

From an economics perspective, a skill is a form of human capital that increases productivity, with its value defined by the market. Education is perceived as an essential investment in skills development (Zhou, 2017). The literature typically divides skills into two categories: cognitive and noncognitive skills. While James Heckman popularized and extensively researched noncognitive skills in economics and demonstrated their importance for educational and labour market outcomes, the concept predates his work. Earlier researchers in psychology and education had already distinguished between cognitive abilities and other personal attributes like motivation, perseverance, and social competencies. Heckman's significant contribution was bringing these concepts into mainstream economics and proving their economic value.

Levin (2012) argues for a broader perspective on educational outcomes, emphasizing that success in life depends on more than just test scores. This multidimensional view of education aligns with the growing recognition of noncognitive skills' importance in both academic and life outcomes.

1.2 Cognitive and noncognitive skills in academic achievement

Cognitive skills, often proxied by test scores, have long been considered the primary determinant of academic success. Heckman et al. (2006) provide evidence that cognitive skills are strong predictors of educational attainment and labour market success. However, it is important to note that test scores do not simply reflect cognitive ability. Brunello and Schlotter (2011) suggest that high cognitive test scores likely result not only from high cognitive skills but also from high motivation and adequate personality traits, which can be considered noncognitive skills.

Noncognitive skills enable people, fostering social inclusion and promoting economic and social mobility (Kautz et al., 2014). Bowles and Gintis (2002) found that perseverance, dependability, and consistency are some of the most important predictors of grades in school. Almlund et al. (2011) provide a comprehensive review of how personality traits influence educational outcomes, finding that conscientiousness, in particular, is a strong predictor of academic performance across various measures and educational levels.

The interaction between cognitive and noncognitive skills is nuanced. Borghans et al. (2008) state that a link between noncognitive skills and test scores can exist for two reasons: when there are sufficient rewards involved, people with favorable behavioural or labour-market outcomes might have an attitude to put in effort, and when rewards are not necessary, people who are motivated to perform well and who have a positive attitude toward work might be more inclined to do their best at tests.

Duckworth and Seligman (2006), in a study about the difference in test scores between girls and boys, concluded that because girls had better final grades than boys, even after controlling for measured IQ, they were significantly better at exercising self-discipline during the academic year. Balart et al. (2018) used the performance decline in PISA test scores as a measure of noncognitive skills. They found that both the starting performance (a measure of cognitive skills) and the performance decline were positively and significantly associated with economic growth.

Lindqvist and Vestman (2011) provide evidence on the relative importance of cognitive and noncognitive skills in predicting labour market outcomes. While their focus is on labour market success, their findings have implications for understanding how these skills interact in educational settings. Their research shows a potential compensatory effect, where noncognitive skills are particularly important for individuals at the lower end of the earnings distribution.

1.3 Gender differences and skill development

Gender differences in academic performance have been a significant area of research. As mentioned earlier, Duckworth and Seligman (2006) found that girls had better final grades than boys, even after controlling for measured IQ, attributing this to girls' better self-discipline. Bertrand and Pan (2013) examined gender differences in noncognitive skills, focusing specifically on disruptive behaviour. They find that boys are more susceptible to developing behavioural problems, especially in disadvantaged environments, which can significantly impact their academic performance.

Skill development is a dynamic process in which the early years lay the foundation for successful investment in later years (Kautz et al., 2014). The work of Cunha and Heckman (2008) has been instrumental in formalizing the role of noncognitive skills in skill formation. They propose a model that incorporates both cognitive and noncognitive skills, highlighting how these skills interact and evolve over time. This model has been influential in shaping our understanding of skill development and its impact on educational outcomes.

Both cognitive and noncognitive skills have different levels of malleability depending on a child's developmental stage; they can change with age and with instruction. Cognitive and noncognitive skills are highly malleable in the early years of a child's life, while noncognitive skills are more malleable than cognitive skills later on, during adolescence (Kautz et al., 2014).

1.4 Challenges in defining and measuring noncognitive skills

Despite their importance, noncognitive skills and abilities, unlike cognition, are challenging to define and measure. Suárez Pandiello et al. (2016) attest that social groups and public authorities ignore noncognitive abilities because of the lack of objective evaluation metrics and the difficulty in establishing standard definitions for the relevant social values.

Humphries and Kosse (2017) note that the definition of noncognitive skills varies widely across fields such as Sociology, Psychology, and Economics and within fields of study. Labour economists see noncognitive skills as a second dimension of individual heterogeneity (next to cognitive skills); Education economists broadly categorize those as skills that are not captured by standardised tests (soft skills), and that can be measured by observing behaviour. behavioural economists are divided into two groups: one that sees noncognitive skills as a super-ordinate concept summarizing various specific concepts (i.e., economic preferences such as time and risk preferences), and the other that views them as personality measures (such as the Big Five). This divisiveness is challenging when comparing outcomes due to the different psychometric assessment tools used.

Currently, there is no systematic global measure of noncognitive skills. However, fortunately, the field has expanded enough, and a wide variety of psychometric tools aimed at assessing these skills and abilities have been created. Using measured behaviours to capture noncognitive skills, for example, is a promising, empirically practical approach, according to Kautz et al. (2014).

Personality traits represent relatively persistent dimensions of the overall personality, and some play an important role in increasing productivity-enhancing skills. More broadly, economists often use the term noncognitive skills to account for traits specifically related to human capital outcomes (such as educational and labour market achievements), and in Psychology, personality traits are measured using psychometric constructs (Thiel & Thomsen, 2013). Therefore, economists and other social scientists can adapt such constructs to their respective fields of study to measure noncognitive skills.

2 Data

2.1 Growing Up in Ireland

The data used in the analysis come from the second and third waves of the Child Cohort ('98) of the Growing Up in Ireland (GUI) survey. The GUI is a national longitudinal study of children and young people that has been running since 2006. The study followed the progress of two groups of children: 8,568 9-year-olds (Cohort' 98), representing approximately 14% of all 9-year-olds in Ireland, and 10,000 9-month-olds (Cohort' 08), for the last fifteen years. Subsequent waves of the '98 cohort saw some drop-off in participation: 7,525 children (87.9%) in the second wave (2011-2012), 6,216 young adults (72.5%) in the third wave (2015-2016), and 5,190 young adults (60%) in the fourth wave (2018-2019). The survey stands out for its large, nationally representative sample and longitudinal nature. The first cohort sample was selected from clustering at the school level, and the second cohort was sampled randomly from the Child Benefit records. The members of Cohort '98 are now 25-26 years old.

For estimation, however, I work with a complete-case longitudinal subsample: individuals must have non-missing values for the variables required across the waves considered. This restriction is deliberate. It keeps the estimation sample composition fixed across model specifications and avoids comparing coefficients that are driven by changing underlying samples. The trade-off is that this is a selected subsample rather than the full cohort. In practical terms, the estimates should be interpreted as associations for this analytically consistent subsample, not as population-average causal effects for all GUI participants.

A timeline of the data used in this study is presented in Table 1. In Wave 2, the study children had their verbal reasoning and numerical abilities tested using the Drumcondra Verbal Reasoning, the Numerical Ability tests, and the Matrices British Ability Scale (one of the leading standardised batteries in the UK for assessing a child's cognitive ability and educational achievement)[1]. These measures were combined through principal component

[1]These tests comprise different cognitive abilities: verbal fluency, vocabulary comprehension, and numerical knowledge. The verbal fluency test encompassed two aspects: the FAS score, measuring the

Event	Date	Age (in years)	Variables of interest
Study-child is born	Nov/97 - Oct/98	0	
Wave 2 data collection	Aug/11 - Mar/12	13	Independent variables: Cognitive variables, SDQ and TIPI scales, controls
Study-child sits the Junior Cert	Jun/13 - Jun/14	15-16	
Wave 3 data collection	Apr/15 - Aug/16	17-18	Dependent variables: Junior Cert scores in Maths and English

Table 1: Timeline of key events in the Growing Up in Ireland '98 cohort used in this study, including timing of predictor measurement (Wave 2, age 13) and outcome measurement (Wave 3 Junior Certificate scores in Maths and English, points on the 12-point OPS scale).

analysis, yielding a single component representing cognitive ability, where higher scores indicate more remarkable ability. I use this composite throughout the study as a measure of cognition. The noncognitive variables used in this study were also collected in Wave 2, along with the control variables such as socioeconomic indicators and school characteristics. In wave 3, they were asked about Junior and Leaving Cert (if they already sat it) results and asked for permission to link to the Central Admissions Office database in the future (if they still need to sit it).

Academic achievement at the third wave was assessed via the Junior Certificate Examination, a national exam taken by most Irish children around ages 15-16. Mandatory subjects are Irish, English, Maths, and History, and students can choose up to 10 subjects (with at least four mandatory plus two optional) in the areas of Arts and Humanities, Modern Languages, Sciences, and Applied Sciences. Before 2017 (when the survey took place), grades were given on a scale of A to F across different levels of the exam (Higher, Ordinary, Foundation). The Junior Certificate Examination in Ireland marks the end of three years of studying various subjects. It typically spans two to three weeks of individual subject exams at the end of the school year in June, and a student cannot fail the examination. Regardless of their examination results, all students progress to the next year of education if they wish to do so. The time frame between the Junior Certificate (ages 15-16) and the age range of the GUI wave 3 participants (16-18) was relatively close. Because the Junior Cert syllabus and exam content are predetermined three years in advance, achieving success reflects the culmination of a structured curriculum and learning process. Given this foresight, one would anticipate that specific noncognitive skills are pivotal in shaping outcomes. These skills may include effective planning, adept

number of words generated beginning with F, A, or S in one minute, and the Animal Naming score, gauging the number of animal species named in one minute. The vocabulary test consisted of 20 items, each followed by a list of five words, requiring the selection of the word most closely related in meaning. The numeracy test evaluated performance in basic arithmetic through three mathematical calculations.

time management, the ability to prioritize long-term goals over immediate gratification (such as opting to study for an exam well in advance rather than indulging in leisure activities), proficient organization and upkeep of study materials, and judicious allocation of time across a diverse array of academic subjects.

2.2 Variables

The model I employ at first is a multivariate multiple regression model (multivariate because of two dependent variables - Maths and English scores - and multiple because of multiple independent variables) as a form of linear production function.

2.2.1 Dependent variables

The dependent variables are Junior Cert scores in Maths and English, representing academic achievement. For the analysis, I used the Junior Certificate Overall Performance Scale (OPS), which converts letter grades from different exam levels to a standardised 12-point numerical scale. This scale has been validated in previous research (Sofroniou et al., 2000) and provides a comprehensive measure that accounts for both grade and exam level, allowing for more nuanced statistical analysis of academic achievement across subjects and students.

2.2.2 Independent variables

The independent variables consist of two sets of measures. The first set comprises cognitive abilities, assessed through naming ability, maths ability, and vocabulary ability. The second set includes noncognitive abilities and skills, measured using two different psychometric assessment tools: the Strengths and Difficulties Questionnaire (SDQ), which captures behavioural and emotional characteristics (also referred to as psychosocial attributes), and the Ten Item Personality Inventory (TIPI), which assesses the five-factor model of personality.

2.2.3 Controls

In addition, I include two vectors of control variables. The first vector encompasses socioeconomic status characteristics, including gender, parental education (considering both primary and secondary caregivers' education levels), and income quantile (equivalized). These SES variables are included to control for well-documented effects of family background on educational outcomes (Sirin, 2005). Gender is included to account for potential differences in subject-specific performance (Hyde & Linn, 1988; Hyde et al., 1990), while parental education and income are key indicators of family resources and educational support (Davis-Kean, 2005).

Table 2: Descriptive statistics for the main analytical variables in the Junior Certificate sample (n = 5,631 where shown). Outcomes are OPS points (2–12 for Maths and 5–12 for English); cognitive variables are test-score measures (percent-correct or standardised components); SDQ measures are scaled 0–10 with higher values indicating stronger positive traits in the recoded specification.

Variable	Mean	Std. Dev.	Min	Max	N
Dependent variables					
Maths points (Junior Cert)	9.60	1.74	2.00	12.00	5631
English points (Junior Cert)	10.15	1.34	5.00	12.00	5631
Independent variables: Cognition					
Drumcondra Verbal Reasoning (% correct answers)	64.89	21.92	0.00	100.00	5631
Drumcondra Numerical Ability (% correct answers)	55.05	22.53	0.00	100.00	5631
Matrices (BAS)	116.68	18.03	10.00	161.00	5631
Cognitive ability 1	0.14	1.33	-4.25	3.32	5631
Cognitive ability 2	100.00	15.00	36.25	136.40	5631
Independent variables: Noncognition (SDQ)					
Emotional resilience	8.29	1.87	0.00	10.00	5631
Good conduct	8.97	1.31	0.00	10.00	5631
Focused behaviour	7.56	2.26	0.00	10.00	5631
Positive peer relationships	8.96	1.41	0.00	10.00	5631
Independent variables: Noncognition (TIPI)					
Agreeable	5.01	1.95	0.50	7.00	5631
Conscientious	4.33	2.07	0.50	7.00	5631
Emotional stability	4.40	1.99	0.50	7.00	5631
Extravert	3.98	1.98	0.50	7.00	5629
Openness	4.73	1.83	0.50	7.00	5627
Controls (SES characteristics)					
Gender (Male = 1)	0.49	0.50	0.00	1.00	5468
Primary caregiver education level	3.97	1.24	1.00	6.00	5631
Secondary caregiver education level	3.86	1.36	1.00	6.00	4440
Income quintile (equivalized)	3.33	1.39	1.00	5.00	5241
Controls (School characteristics, binary)					
DEIS	0.12	0.33	0.00	1.00	5452
Fee-paying	0.10	0.30	0.00	1.00	5452
Mixed-school	0.54	0.50	0.00	1.00	5317

Note: TIPI scale scores on a 1-7 scale in intervals of 0.5, and the original SDQ scales, ranging from 0 to 10, have been inverted (higher scores typically indicate more problems in the original SDQ scale). "Cognitive ability 1" was used in the first part of the production function estimation and was standardised to have mean = 0 and standard deviation = 1. "Cognitive ability 2" is to be used in the second part of the analysis as a measure of cognition in non-linear production function estimation, with a mean of 100 and standard deviation = 15 as is standard in the literature. Education levels are coded from 1 (Primary or less) to 6 (Postgraduate/Higher degree) in the Growing Up in Ireland caregiver questionnaire. The mean values for both primary (3.97) and secondary (3.86) caregivers indicate an average education level between Leaving Certificate and Diploma/Certificate, indicating a higher proportion of educated caregivers in the sample. Income is reported in quintiles, where 1 represents the lowest 20% and 5 the highest 20% of incomes. The mean of 3.33 shows that the sample is slightly skewed towards higher income levels, with families on average being just above the median income quintile. The sample includes 12% DEIS schools (schools in disadvantaged areas), 10% fee-paying schools, and 54% mixed-gender schools. The sample includes a diverse range of school types, characterized by a high proportion of fee-paying schools and a relatively low share of DEIS schools.

The second vector accounts for school characteristics, incorporating indicators for mixed schools (opposite to single-sex schools, which are underrepresented in the sample), DEIS (Delivering Equality of Opportunity in Schools) schools, and fee-paying schools. School-level variables are included to account for institutional factors that may influence academic performance. The inclusion of indicators for mixed schools addresses potential differences in educational environments (Pahlke et al., 2014). DEIS school status is included to control for the effects of targeted educational interventions in disadvantaged areas (Smyth et al., 2015). Fee-paying school status is included to account for potential resource differences between public and private institutions (OECD, 2012).

2.3 Psychometric assessment tools

2.3.1 Strengths and Difficulties Questionnaire (SDQ)

The Strengths and Difficulties Questionnaire (SDQ) measures two distinct dimensions of noncognitive skills: behavioural skills and emotional skills. Twenty items of the SDQ comprise a total scale made up of four sub-scales, each containing five items. These sub-scales tap into emotional symptoms (e.g. often unhappy, downhearted, or tearful); conduct problems (e.g. often fights with other children or bullies them); Hyperactivity/Inattention (e.g. restless, overactive, cannot stay still for long); and Peer-relationship problems (e.g. picked on or bullied by other children). Scores on each sub-scale can range from 0 to 10, where 10 indicates a high degree of difficulty and 0 the absence of any problems in the relevant domain.

I inverted the scales so that 10 is "better" and 0 is "worse", which led me to rename the measures to maintain clarity and consistency across the study. For example, a positive coefficient for Emotional Resilience (previously Emotional Symptoms) would indicate that higher emotional stability is associated with better academic outcomes. The same rationale was applied to the other variables: Conduct Problems became Good Conduct, Hyperactivity/Inattention became Focused Behaviour, and Peer-relationship Problems became Positive Peer Relationships. The SDQ was completed by both the child's primary caregiver and teacher in Wave 1, and by the child's primary caregiver in Wave 2.

2.3.2 Ten Item Personality Inventory (TIPI)

One of the dimensions where noncognition manifests itself (others being through behavioural problems, social skills, communication, self-esteem, persistence, locus of control, empathy, and impulsivity), the study-child was assessed utilizing the Ten Item Personality Inventory (TIPI), a brief instrument designed to assess the five-factor model (FFM) personality dimensions. Primary caregiver (PCG, usually the mother) and Secondary caregiver (SCG, usually the father) completed the scale regarding the study-child in wave

3 (PCG completed in waves 2 and 3). In wave 3, the study child also filled out the scale, offering an external and self-assessed measure of the study child's personality and ensuring consistency. This scale comprises ten items encompassing five personality facets: Openness to Experience, Agreeableness, Conscientiousness, Extraversion, and Neuroticism (Emotional stability). Each of the ten items was evaluated on a seven-point scale, from strongly disagree to strongly agree. Each dimension of personality included two statements with two descriptors each. The scores for each measure were derived by summing up both responses and dividing by two according to common practice in the literature. This was done by the GUI researchers and the final score for each item can be found in the GUI files. More details can be found in the Appendix.

2.4 Limitations

There are certain limitations to this analysis. I chose to work with the cross-sectional part of the panel data, which limits the ability to infer causality. There may also be omitted variable bias, as other factors not included in the model could influence academic performance (like residing in a peaceful environment, or just waking up well-rested in the Junior Cert days). In addition, because I impose a complete-case requirement across waves and variables, part of sample loss is analyst-imposed rather than purely survey attrition. That design choice strengthens comparability across specifications, but it also means the estimation sample is more selected than the underlying cohort.

Furthermore, the parent-reported nature of some measures is potentially a source of measurement error. It is important to note that while regressing test scores on other test scores can sometimes lead to issues of regression to the mean, my study design mitigates this concern. The cognitive variables were collected two years before the study children sat the Junior Cert, allowing them to function as true predictors rather than concurrent measures. This temporal separation between the collection of cognitive and noncognitive measures (Wave 2) and the assessment of academic achievement (Wave 3) strengthens the predictive power of my analysis because it allows us to examine how earlier cognitive and noncognitive traits influence later academic outcomes, reducing concerns about reverse causality. Temporal ordering, however, is not equivalent to identification of a causal effect, so I keep interpretation descriptive throughout.

Regarding the noncognitive measures, while the TIPI and SDQ are widely used and validated psychometric assessment tools, they have inherent limitations. The TIPI's brevity, while efficient, may limit its ability to capture nuanced personality traits. The SDQ, although comprehensive, may be subject to reporter bias. Both measures rely on self or parent reports, which can introduce subjective biases. However, their established validity in Psychology and Sociology research and their efficiency in large-scale studies provide a strong foundation for their use in this analysis, balancing practical considerations with scientific rigor.

It is worth noting that the Growing Up in Ireland is a panel-data survey, and in Wave 3, the TIPI scores for the study children were also collected from the children's perspective and the secondary caregivers' perspective, providing three measures of noncognition from different viewpoints. These measures correlate well. By leveraging this survey's strength, I was able to minimize individual reporting biases and enhance construct validity through convergence of scores across different reporters. This approach provides a richer, more comprehensive view of children's noncognitive traits while increasing overall measurement reliability. The comparability of results across these different informants strengthens my confidence in the validity and robustness of our noncognitive measures, particularly the TIPI scores.

3 How do cognitive and noncognitive skills contribute to academic achievement? A linear approach

3.1 Theoretical Framework

In the fields of Economics of Education, Psychology, and Sociology, understanding the factors that contribute to academic achievement is central for developing effective policies and interventions. While cognitive abilities have traditionally been the primary focus when examining determinants of educational outcomes, recent interdisciplinary research has demonstrated the significant role of noncognitive skills in shaping academic performance and long-term success.

To capture a detailed relationship between cognition and noncognition, I propose creating a series of educational production functions that incorporate both as inputs. This conceptual tool models the links between inputs and educational outcomes, with the primary focus on assessing the relative returns to cognitive and noncognitive measures, and investigating potential interactions between these two types of abilities, specifically whether they act as complements or substitutes in producing educational outcomes.

3.2 Model Specification

I employ a linear form of the production function to estimate the effects of cognitive and noncognitive abilities on academic performance, while also capturing potential interactions between these factors. This model estimates the direct effects of cognitive abilities and various noncognitive measures on academic points in Junior Certificate subjects, captures whether the marginal effect of cognitive ability depends on noncognitive traits (indicating either synergy or diminishing returns between skills), and controls for relevant socioeconomic and school characteristics to isolate the effects of interest.

The linear production function is specified as follows:

$$\begin{aligned}\text{PointsJC}_{i,l} = \beta_0 + \beta_C \cdot \text{Cognition}_i + \sum_{j=1}^{J} \beta_{Nj} \cdot \text{NonCog}_{i,k,j} \\ + \sum_{j=1}^{J} \gamma_j \cdot (\text{Cognition}_i \cdot \text{NonCog}_{i,k,j}) + \boldsymbol{\delta}' \cdot \mathbf{X}_i + \varepsilon_{i,l}\end{aligned} \tag{1}$$

Where i = individual observation, l = Subject (Maths, English), k = Primary caregiver (PCG), j indexes noncognitive skill dimensions (e.g., SDQ and TIPI traits), β_C = coefficient for cognitive ability, β_{Nj} = coefficient for the j-th noncognitive measure, γ_j = coefficient for the interaction between cognition and the j-th noncognitive measure, $\boldsymbol{\delta}'$ = vector of coefficients for control variables.

3.3 Components of the Production Function

3.3.1 Cognitive Ability

$$\text{Cognition}_i = \text{PC}(\text{Naming ability}_i, \text{Maths ability}_i, \text{Vocabulary ability}_i) \tag{2}$$

3.3.2 Noncognitive Measures

a) Behavioural and Emotional Characteristics (SDQ):

$$\text{NonCognition}_{i,k,j} \text{ for } j \in \{1,2,3,4\} = \begin{cases} \text{SDQ - Emotional Resilience}_{i,k} \\ \text{SDQ - Good Conduct}_{i,k} \\ \text{SDQ - Focused Behaviour}_{i,k} \\ \text{SDQ - Positive Peer Relationships}_{i,k} \end{cases}$$

b) Personality Traits (TIPI):

$$\text{NonCognition}_{i,k,j} \text{ for } j \in \{5,6,7,8,9\} = \begin{cases} \text{TIPI - Agreeable}_{i,k} \\ \text{TIPI - Conscientious}_{i,k} \\ \text{TIPI - Emotional Stability/Neuroticism}_{i,k} \\ \text{TIPI - Extravert}_{i,k} \\ \text{TIPI - Openness}_{i,k} \end{cases}$$

3.3.3 Control Variables

Control variables include socioeconomic status (gender, parental education, and income quantile) and school characteristics (mixed schools, DEIS schools, and fee-paying schools).

3.3.4 Estimation Strategy

I estimate the linear production function using Ordinary Least Squares (OLS) regression. I chose to standardize the variables of interest to have a mean of zero and a standard deviation of one. Standardizing variables simplifies interpretation (as changes in SD units) and may improve numerical stability and model fit (Wooldridge, 2015). By employing this technique, the benefits are two-fold: it provides a more straightforward interpretation of the results while also potentially improving the overall fit of the model (Wooldridge, 2015). Identification in this setup comes from temporal ordering (inputs measured before outcomes), variation in observed inputs, and rich controls, but not from exogenous treatment assignment. For that reason, I interpret coefficients as conditional associations rather than causal parameters.

I estimate three models for each subject (Maths and English): the base model (1), which includes only cognitive ability and noncognitive measures; the full model (2), which adds socioeconomic and school characteristic controls; and the interaction model (3), which incorporates interaction terms between cognitive ability and noncognitive measures. This stepwise approach allows us to observe how the connections between variables change as we add more complexity to the model.

3.3.5 Interpretation of Results

The coefficients in the models can be interpreted as follows: β_C is the change in academic performance associated with a one standard deviation increase in cognitive ability, β_{Nj} is the change associated with a one standard deviation increase in the j-th noncognitive measure, and γ_j is the change in the effect of cognitive ability on academic performance for a one standard deviation increase in the j-th noncognitive measure. A positive γ_j indicates that the effect of cognitive skills increases as noncognitive skills rise, implying synergy between the two inputs, whereas a negative value reflects diminishing returns when both skills are high.

I use the R-squared (R^2) statistic to assess the overall explanatory power of the models and how it changes as I add more variables and interactions.

3.3.6 Limitations and Considerations

While the linear production function approach provides new perspectives about the relationships between cognitive abilities, noncognitive skills, and academic performance, it is important to acknowledge some limitations: the linear form assumes constant returns to scale, which may not always hold in educational contexts; the model assumes additive effects, which might oversimplify nuanced links between variables; endogeneity concerns such as omitted variable bias or reverse causality could affect interpretation; and the model does not explicitly account for measurement error in skill assessments, which could attenuate coefficient estimates.

Given these constraints, robustness is treated as a stability exercise rather than a claim of identification. The key criterion is whether signs, relative magnitudes, and core interaction patterns remain consistent across reasonable alternative specifications and sample definitions. Where significance changes at the margin, I prioritise the direction and economic size of effects over p-value thresholds alone.

In the following results section, I will present and discuss the findings from these estimations, considering both the statistical significance and practical importance of the estimated coefficients.

3.4 Results

Table 3: OLS associations between cognition, TIPI personality traits, and Junior Certificate achievement in Maths and English. Dependent variables are OPS points (Maths 2–12; English 5–12); predictors are standardised; Model (1) includes cognition and TIPI only, Model (2) adds socioeconomic and school controls, and Model (3) adds cognition-trait interactions. Standard errors are reported in parentheses and estimates are descriptive rather than causal.

	Maths			English		
Model:	(1)	(2)	(3)	(1)	(2)	(3)
Variables						
Constant	9.571***	9.737***	9.751***	10.13***	10.41***	10.42***
	(0.0171)	(0.0355)	(0.0356)	(0.0151)	(0.0305)	(0.0306)
Cognition	0.8351***	0.7154***	0.7176***	0.5039***	0.4464***	0.4483***
	(0.0127)	(0.0161)	(0.0161)	(0.0113)	(0.0138)	(0.0138)
Agreeableness	0.0092	0.0290	0.0309	0.0271*	0.0349*	0.0369**
	(0.0181)	(0.0210)	(0.0210)	(0.0160)	(0.0180)	(0.0181)
Conscientiousness	0.1580***	0.1325***	0.1371***	0.0912***	0.0755***	0.0801***
	(0.0181)	(0.0210)	(0.0211)	(0.0160)	(0.0180)	(0.0181)
Emotional stability	0.0677***	0.0474**	0.0580***	0.0131	0.0045	0.0091
	(0.0184)	(0.0210)	(0.0211)	(0.0163)	(0.0180)	(0.0182)
Extraversion	-0.0191	-0.0200	-0.0181	0.0181	0.0124	0.0147

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Table 3 continued

	Maths			English		
Model:	(1)	(2)	(3)	(1)	(2)	(3)
	(0.0177)	(0.0203)	(0.0204)	(0.0156)	(0.0174)	(0.0175)
Openness	-0.0725***	-0.0211	-0.0219	0.0065	0.0106	0.0138
	(0.0179)	(0.0207)	(0.0208)	(0.0159)	(0.0178)	(0.0179)
Male		-0.1148***	-0.1167***		-0.4452***	-0.4443***
		(0.0401)	(0.0400)		(0.0344)	(0.0343)
Cognition × Agree.			-0.0173			-0.0120
			(0.0161)			(0.0140)
Cognition × Consc.			-0.0136			-0.0185
			(0.0161)			(0.0138)
Cognition × Emot.			-0.0678***			-0.0262*
			(0.0156)			(0.0134)
Cognition × Extra.			0.0021			-0.0074
			(0.0149)			(0.0129)
Cognition × Open.			0.0047			-0.0234*
			(0.0154)			(0.0134)
Controls						
SES	No	Yes	Yes	No	Yes	Yes
School	No	Yes	Yes	No	Yes	Yes
Fit statistics						
Observations	5,654	3,801	3,801	5,637	3,787	3,787
R^2	0.45811	0.48680	0.49051	0.28163	0.33165	0.33481
Adjusted R^2	0.45753	0.48504	0.48809	0.28086	0.32935	0.33163

*IID standard-errors in parentheses. Signif. Codes: ***: 0.01, **: 0.05, *: 0.1. Agree.: Agreeableness, Consc.: Conscientiousness, Emot.: Emotional stability, Extra.: Extraversion, Open.: Openness.* Note: Regressors are standardised (mean 0, SD 1). Coefficients are reported in outcome units (points) per 1 SD increase in the predictor.

Tables 3 and 4 present the regression results for the effects of cognitive ability and noncognitive skills on Junior Certificate Maths and English scores. The dependent variable (academic performance) is measured on a scale from 2 to 12 for Maths and from 5 to 12 for English, while all independent variables are standardised.

Table 4: OLS associations between cognition, SDQ behavioural characteristics, and Junior Certificate achievement in Maths and English. Dependent variables are OPS points (Maths 2–12; English 5–12); predictors are standardised; Model (1) includes cognition and SDQ only, Model (2) adds socioeconomic and school controls, and Model (3) adds cognition-trait interactions. Standard errors are reported in parentheses and estimates are descriptive rather than causal.

	Maths			English		
Model:	(1)	(2)	(3)	(1)	(2)	(3)
Variables						
Constant	9.570***	9.705***	9.729***	10.13***	10.39***	10.41***
	(0.0168)	(0.0352)	(0.0355)	(0.0148)	(0.0303)	(0.0305)
Cognition	0.7830***	0.6689***	0.6703***	0.4641***	0.4161***	0.4170***
	(0.0131)	(0.0164)	(0.0164)	(0.0115)	(0.0141)	(0.0141)
Emotional Resilience	0.0499***	0.0565**	0.0462**	-0.0274	0.0120	0.0021
	(0.0192)	(0.0226)	(0.0227)	(0.0169)	(0.0194)	(0.0194)
Good Conduct	0.0865***	0.0716***	0.0740***	0.0042	-0.0220	-0.0183
	(0.0196)	(0.0233)	(0.0234)	(0.0172)	(0.0201)	(0.0201)
Focused Behaviour	0.2415***	0.2267***	0.2166***	0.2440***	0.1902***	0.1795***
	(0.0198)	(0.0235)	(0.0236)	(0.0174)	(0.0203)	(0.0202)
Positive Peer Relationships	0.0179	-0.0031	0.0014	0.0736***	0.0503***	0.0573***
	(0.0184)	(0.0215)	(0.0218)	(0.0162)	(0.0185)	(0.0188)
Male		-0.0702*	-0.0693*		-0.4029***	-0.4007***
		(0.0402)	(0.0402)		(0.0346)	(0.0345)
Cognition × E.R.			-0.0316**			-0.0176
			(0.0161)			(0.0139)
Cognition × G.C.			0.0126			0.0266*
			(0.0181)			(0.0157)
Cognition × F.B.			-0.0565***			-0.0735***
			(0.0171)			(0.0148)
Cognition × P.P.R.			-0.0134			-0.0264**
			(0.0148)			(0.0128)
Controls						
SES	No	Yes	Yes	No	Yes	Yes
School	No	Yes	Yes	No	Yes	Yes
Fit statistics						
Observations	5,664	3,805	3,805	5,647	3,791	3,791
R^2	0.47612	0.50254	0.50572	0.31011	0.34688	0.35392
Adjusted R^2	0.47566	0.50096	0.50363	0.30950	0.34480	0.35118

*IID standard-errors in parentheses. Signif. Codes: ***: 0.01, **: 0.05, *: 0.1*
E.R.: Emotional Resilience, G.C.: Good Conduct, F.B.: Focused Behaviour,
P.P.R.: Positive Peer Relationships

Note: Regressors are standardised (mean 0, SD 1). Coefficients are reported in outcome units (points) per 1 SD increase in the predictor.

3.4.1 Cognitive Skills

Across all models, cognitive ability emerges as the strongest predictor of academic performance. For Maths, a one standard deviation increase in cognitive ability is associated with an increase of 0.72 to 0.84 points on the 2-12 scale, depending on the model specification. For English, the effect is somewhat smaller, ranging from 0.45 to 0.50 points. Cognitive skills have a stronger impact on performance in Maths than in English.

3.4.2 Noncognitive Skills

While less impactful than cognitive skills, several noncognitive factors show significant associations with academic performance. For SDQ Measures (Table 4), Focused Behaviour is the strongest noncognitive predictor, with a one standard deviation increase associated with a 0.22 point increase in Maths scores and a 0.18 point increase in English scores (Model 3). Good Conduct and Emotional Resilience show smaller but significant positive effects on Maths scores (0.07 and 0.05 points respectively), while Positive Peer Relationships is significantly associated with English scores (0.06 points). For TIPI Measures (Table 3), Conscientiousness shows the strongest association among personality traits, with a one standard deviation increase associated with a 0.14 point increase in Maths scores and a 0.08 point increase in English scores (Model 3). Emotional Stability shows a modest but significant positive effect on Maths scores (0.06 points), while Agreeableness has a small positive effect on English scores (0.04 points). Overall, SDQ-based behavioural traits (particularly Focused Behaviour) exhibit larger and more consistent associations with academic performance than TIPI personality traits, suggesting that domain-specific behavioural assessments capture school-relevant skills more effectively than broader personality frameworks.

3.4.3 Interaction Effects

For SDQ measures, the interaction between Cognition and Focused Behaviour is negative and significant for both Maths (-0.06) and English (-0.07). Focused Behaviour may therefore be especially valuable for students with lower cognitive skills, supporting the view that behavioural strengths can partially offset cognitive challenges. For TIPI measures, the interaction between Cognition and Emotional Stability is negative and significant for Maths (-0.07), suggesting Emotional Stability's stronger positive association for students with lower cognitive ability and signalling a potential compensatory role.

These negative interactions suggest that noncognitive skills may be particularly important for students with lower cognitive abilities, potentially offering a compensatory mechanism.

3.4.4 Gender Differences

The Male variable shows a consistent negative and highly significant coefficient across all models. The effect is substantially larger for English (-0.44 points) compared to Maths (-0.12 points). Controlling for cognitive ability, noncognitive skills, and socioeconomic factors, boys perform worse than girls, particularly in English. The smaller gender gap in Maths, alongside higher average scores among boys, reflects a wider distribution of Maths performance for boys in the sample.

3.4.5 Model Fit

Examining the changes in R^2 across models provides insight into the explanatory power of different factors: for Maths (Table 4), R^2 increases from 0.476 in the base model to 0.503 when including noncognitive factors and controls, and further to 0.506 with interaction terms. This indicates that noncognitive factors and controls explain an additional 2.64% of the variance in Maths scores, while interaction terms contribute a further 0.32%. For English (Table 4), the pattern is similar but with larger increases, from 0.310 to 0.347 (a 3.68% increase) and then to 0.354 (a further 0.70% increase).

Adding noncognitive variables and interactions improves model fit, with a larger R^2 gain for English than for Maths. This reflects the greater contribution of noncognitive factors to variation in English performance.

3.4.6 Subject Differences

Cognitive skills have a stronger association with Maths performance than with English, which may reflect the more structured and sequential nature of mathematical knowledge and its closer ties to cognitive processing abilities. Noncognitive skills, particularly Focused Behaviour and Conscientiousness, show significant effects on both subjects, but their relative importance appears higher for English, suggesting that success in language-related tasks may rely more heavily on self-regulation and persistent effort. The gender gap is more pronounced in English than in Maths, which aligns with international trends but raises questions about the factors driving this disparity in the Irish context.

These subject-specific patterns suggest that the production function for academic achievement varies across disciplines, potentially reflecting differences in how these subjects are taught, learned, and assessed. The stronger role of cognitive skills in Maths achievement compared to English may indicate that Maths skills are more dependent on formal instruction and cognitive development, while English skills might be more influenced by broader environmental and noncognitive factors.

In conclusion, while cognitive abilities remain the strongest predictors of academic performance, noncognitive factors provide meaningful additional explanatory power, especially for English performance. Different skills combine differently for boys and girls across subjects. In terms of magnitude, the estimated cognitive coefficients in the linear models are not only statistically significant but also economically meaningful: a one-standard-deviation increase in cognition is associated with roughly 0.72 to 0.84 points in Maths (on a 2–12 scale) and 0.45 to 0.50 points in English (on a 5–12 scale). This complexity means one-size-fits-all education policies do not work, we need tailored approaches instead.

4 How do cognitive and noncognitive skills interact in producing academic outcomes? A nonlinear analysis

In this section I extend the traditional approach by explicitly including both cognitive and noncognitive factors as key inputs. This approach is grounded in the growing body of literature which deals with the importance of noncognitive skills in educational and life outcomes (Duckworth & Seligman, 2005; Heckman & Rubinstein, 2001). In this section I allow for a more flexible form of production function: the transcendental logarithmic production function, first introduced by Christensen, Jorgenson, and Lau in 1971. It was formally presented in their paper titled "Conjugate Duality and the Transcendental Logarithmic Production Function" which appeared in Econometrica in 1973.

I model the educational production function using a a two-input translog function. In this specification C is the variable cognition and N is a noncognitive variable. In relation to the scales used, N is either the variable Focused Behaviour (SDQ) or Conscientiousness (TIPI). I chose these two because they were the most significant factors from the previous analysis. A more restrictive form, the Cobb-Douglas with two and three inputs, can be found in the Appendix. Cognition plays the primary role while noncognitive skills make smaller but important contributions, according to the Cobb-Douglas model, which also identified gender differences. Boys showed slightly stronger cognition and girls exhibited stronger noncognitive effects, especially in Maths. The model revealed decreasing returns to scale in the production of academic achievement. According to MRTS estimates, replacing a single unit of cognition requires more than one unit of noncognitive skill. However, the constant substitution elasticity in the Cobb-Douglas model limits its capacity to capture varying input interactions. The translog model, with its flexible functional form, allows for varying elasticities of substitution and interaction effects between inputs, potentially offering a more comprehensive understanding of the educational production function. Therefore, the main text will focus on the translog model, with the detailed Cobb-Douglas analysis available in the Appendix for interested readers.

4.0.1 Definition

The Translog production function is a flexible functional form, often interpreted as a second-order Taylor approximation in logarithms of an arbitrary production function. Unlike the CES function, which approaches the Cobb-Douglas form as its substitution parameter (γ or ρ) goes to zero, the translog provides a local approximation that can capture more complex production structures, including varying elasticities of substitution. It extends the Cobb-Douglas production function by including logarithms of inputs and their squares and cross-products. This particular specification allows for nuanced relationships between inputs and outputs, potentially revealing non-linear effects of cognitive and noncognitive skills on academic achievement, interactions between cognitive and noncognitive factors (which may enhance or mitigate each other's effects), and varying returns to scale for different combinations of inputs.

The translog function is here specifically defined as:

$$Y = AC^{\alpha}N^{\beta}\exp\left\{\frac{1}{2}\gamma_1\left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2\left[\ln(N)\right]^2 + \gamma_{12}\ln(C)\ln(N)\right\} \tag{3}$$

Where:

$$\begin{aligned}
Y &: \text{Total output/Grade function/Academic achievement} \\
A &: \text{Total factor productivity or scaling factor} \\
C, N &: \text{Inputs} \\
\alpha, \beta &: \text{Exponents determining the output response to each input} \\
\gamma_1, \gamma_2, \gamma_{12} &: \text{Parameters capturing interactions and quadratic effects}
\end{aligned}$$

The parameters α, β, γ_1, γ_2, and γ_{12} capture distinct aspects of the production function. Their estimated values help clarify the relationships between the inputs C and N and the output Y. The coefficients γ_1 and γ_2 represent curvature terms, capturing nonlinear effects for each input. Positive values indicate increasing marginal effects, while negative values suggest diminishing returns. The interaction term γ_{12} reflects how the marginal product of one input depends on the level of the other. A negative γ_{12}, for instance, implies that marginal returns to one input decline when the other input is high.

The following metrics are presented in their final estimation forms here and derived in detail in the Appendix.

4.0.2 Marginal Products (MPs)

Marginal products (MPs) represent the change in total output resulting from a one-unit increase in a specific input while holding all other inputs constant. In the context of the translog production function, MPs are more elaborate than in the Cobb-Douglas model due to the inclusion of quadratic and interaction terms.

The marginal products for inputs C and N are derived by taking the partial derivative of the production function with respect to each input:

$$f_C = \frac{Y}{C}\left(\alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N_0)\right) = \frac{Y}{C} \cdot OE_C \tag{4}$$

$$f_N = \frac{Y}{N}\left(\beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C_0)\right) = \frac{Y}{N} \cdot OE_N \tag{5}$$

These expressions show that the marginal products in the translog model depend not only on the levels of inputs C and N but also on their logarithms and the interaction between them.

4.0.3 Output Elasticities (OEs)

Output elasticities measure the responsiveness of output to a change in inputs, expressed in percentage terms. In the translog production function, unlike in the Cobb-Douglas model, these elasticities are not constant but vary with the levels of inputs.

For the translog function, the output elasticities are derived by taking the partial derivative of the natural logarithm of the production function with respect to the logarithm of each input:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N=N_0} = \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N_0) \tag{6}$$

$$OE_N = \left.\frac{\partial \ln(Y)}{\partial \ln(N)}\right|_{C=C_0} = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C_0) \tag{7}$$

These expressions show that the output elasticities in the translog model vary with the levels of both inputs and include cross-input interaction effects via the γ_{12} term. This flexibility allows the model to reflect changing returns to scale and input importance depending on their levels.

4.0.4 Marginal Rate of Technical Substitution

The Marginal Rate of Technical Substitution (MRTS) represents the rate at which one input can be substituted for another while maintaining the same level of output. In the context of educational production, the $MRTS_{CN}$ indicates how much noncognitive skill (N) is needed to compensate for a small decrease in cognitive skill (C) while keeping academic achievement constant.

Mathematically, MRTS is defined as the negative of the slope of the isoquant curve in input space. It can be derived from the ratio of the marginal products:

$$MRTS_{CN} = -\frac{dN}{dC}\bigg|_{Y=Y_0} = \frac{f_C}{f_N} \tag{8}$$

where f_C and f_N are the marginal products of C and N respectively.

For the translog function, the MRTS can be expressed in terms of output elasticities:

$$MRTS_{CN} = \frac{f_C}{f_N} = \frac{OE_C}{OE_N} \cdot \frac{N}{C} \tag{9}$$

Substituting the expressions for OE_C and OE_N:

$$MRTS_{CN} = \frac{\alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N)}{\beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C)} \cdot \frac{N}{C} \tag{10}$$

This MRTS shows how the rate at which cognitive skills can be substituted for noncognitive skills (or vice versa) varies with the levels of both inputs. It captures more nuanced relationships between inputs than the constant MRTS of the Cobb-Douglas function. Because MRTS varies with input levels in the translog model, it reflects how trade-offs between inputs evolve across the skill distribution.

4.0.5 Elasticity of Substitution (ES)

For the Translog production function, the elasticity of substitution is not constant but varies with the levels of inputs. It can be calculated using the formula:

$$\sigma = 1 - \frac{\partial \ln(MRTS)}{\partial \ln(C/N)} \tag{11}$$

Where MRTS is the Marginal Rate of Technical Substitution, defined as:

$$MRTS = \frac{\partial Y/\partial C}{\partial Y/\partial N} = \frac{OE_C}{OE_N} \cdot \frac{N}{C} \tag{12}$$

Using the output elasticities:

$$OE_C = \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N) \tag{13}$$

$$OE_N = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C) \tag{14}$$

After differentiation and algebraic manipulation, we obtain:

$$\sigma = \frac{OE_C + OE_N}{OE_C + OE_N - \gamma_{12}\left(\frac{OE_C}{OE_N} + \frac{OE_N}{OE_C}\right)} \tag{15}$$

This expression shows that the elasticity of substitution between cognitive and noncognitive skills varies with the levels of both inputs and their interaction. While γ_{12} contributes to this variation, it is the overall combination of elasticities and their ratios that determines the degree of substitutability.

4.0.6 Results

Table 5: Estimated translog production-function parameters for Junior Certificate Maths achievement by specification (TIPI and SDQ) and subgroup (full sample, boys, girls). Reported quantities include parameter estimates, marginal products, output elasticities, and elasticity of substitution; values are interpreted as fitted model summaries for the observed sample and are not causal effects.

	Maths			
Parameter	Full Sample	Boys	Girls	Δ (Boys - Girls)
TIPI Model				
A	9.43***	9.35***	9.51***	-0.16
	(0.03)	(0.04)	(0.04)	
α (Cognition)	0.83***	0.85***	0.82***	0.03
	(0.02)	(0.02)	(0.02)	
β (Conscientiousness)	0.04***	0.04***	0.05***	-0.01
	(0.01)	(0.01)	(0.01)	
γ_1	0.10	0.31*	-0.15	0.46
	(0.11)	(0.18)	(0.16)	
γ_2	0.05***	0.03*	0.07***	-0.04
	(0.01)	(0.02)	(0.02)	
γ_{12}	-0.04	-0.09**	0.004	-0.09
	(0.03)	(0.04)	(0.04)	
Marginal Product (Cognition)	0.078	0.080	0.078	0.002
Marginal Product (Conscientiousness)	0.084	0.073	0.091	-0.018
Output Elasticity (Cognition)	0.828	0.859	0.820	0.039
Output Elasticity (Conscientiousness)	0.043	0.036	0.048	-0.012
Elasticity of Substitution	0.533	0.288	1.096	-0.808
MRTS	0.927	1.102	0.858	0.244

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Table 5 continued

Parameter	Maths			
	Full Sample	Boys	Girls	Δ (Boys - Girls)
SDQ Model				
A	9.45***	9.35***	9.54***	-0.19
	(0.02)	(0.03)	(0.03)	
α (Cognition)	0.79***	0.81***	0.78***	0.03
	(0.02)	(0.02)	(0.02)	
β (Focused Behaviour)	0.10***	0.09***	0.12***	-0.03
	(0.01)	(0.01)	(0.01)	
γ_1	0.20*	0.37**	0.04	0.33
	(0.11)	(0.18)	(0.16)	
γ_2	0.06***	0.05***	0.08***	-0.03
	(0.01)	(0.01)	(0.02)	
γ_{12}	-0.13***	-0.11***	-0.15***	0.04
	(0.03)	(0.04)	(0.05)	
Marginal Product (Cognition)	0.074	0.076	0.074	0.002
Marginal Product (Focused Behaviour)	0.123	0.103	0.140	-0.037
Output Elasticity (Cognition)	0.785	0.813	0.778	0.035
Output Elasticity (Focused Behaviour)	0.105	0.084	0.125	-0.041
Elasticity of Substitution	0.471	0.451	0.488	-0.037
MRTS	0.605	0.733	0.530	0.203
Observations	5,631	2,667	2,801	

*Standard errors in parentheses. Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Table 6: Estimated translog production-function parameters for Junior Certificate English achievement by specification (TIPI and SDQ) and subgroup (full sample, boys, girls). Reported quantities include parameter estimates, marginal products, output elasticities, and elasticity of substitution; values are interpreted as fitted model summaries for the observed sample and are not causal effects.

	English			
Parameter	Full Sample	Boys	Girls	Δ (Boys - Girls)
TIPI Model				
A	10.09***	9.84***	10.36***	-0.52
	(0.02)	(0.04)	(0.03)	
α (Cognition)	0.45***	0.51***	0.44***	0.07
	(0.01)	(0.02)	(0.02)	
β (Conscientiousness)	0.02***	0.02***	0.02***	0.00
	(0.00)	(0.01)	(0.01)	
γ_1	-0.25***	-0.25	-0.32***	0.07
	(0.09)	(0.15)	(0.12)	
γ_2	0.03***	0.03*	0.02	0.01
	(0.01)	(0.02)	(0.01)	
γ_{12}	-0.03	-0.04	-0.01	-0.03
	(0.02)	(0.03)	(0.03)	
Marginal Product (Cognition)	0.046	0.049	0.047	0.002
Marginal Product (Conscientiousness)	0.051	0.041	0.035	0.006
Output Elasticity (Cognition)	0.454	0.506	0.447	0.059
Output Elasticity (Conscientiousness)	0.024	0.019	0.017	0.002
Elasticity of Substitution	0.478	0.344	0.675	-0.331
MRTS	0.894	1.217	1.342	-0.125
SDQ Model				
A	10.08***	9.83***	10.34***	-0.51
	(0.02)	(0.03)	(0.03)	
α (Cognition)	0.41***	0.47***	0.41***	0.06
	(0.01)	(0.02)	(0.02)	

Continued on next page

Table 6 continued

Parameter	English			
	Full Sample	Boys	Girls	Δ (Boys - Girls)
β (Focused Behaviour)	0.09***	0.07***	0.08***	-0.01
	(0.01)	(0.01)	(0.01)	
γ_1	-0.17*	-0.14	-0.17	0.03
	(0.09)	(0.15)	(0.12)	
γ_2	0.06***	0.05***	0.06***	-0.01
	(0.01)	(0.01)	(0.01)	
γ_{12}	-0.12***	-0.12***	-0.15***	0.03
	(0.02)	(0.03)	(0.04)	
Marginal Product (Cognition)	0.042	0.045	0.043	0.002
Marginal Product (Focused Behaviour)	0.110	0.088	0.095	-0.007
Output Elasticity (Cognition)	0.414	0.466	0.416	0.050
Output Elasticity (Focused Behaviour)	0.088	0.069	0.078	-0.009
Elasticity of Substitution	0.452	0.396	0.370	0.026
MRTS	0.379	0.513	0.455	0.058
Observations	5,631	2,667	2,801	

*Standard errors in parentheses. Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

A striking feature of the results is the subject-specific dynamics of skill influence. In the full sample, the output elasticity of cognitive skills (OE_C) shows a strong and significant influence on both Maths and English performance. For Maths, the OE_C is 0.785 for the SDQ model and 0.828 for the TIPI model, while for English, it is 0.414 for SDQ and 0.454 for TIPI. This indicates a consistently strong role of cognitive skills, with a larger impact on Maths performance.

The output elasticity of noncognitive skills (OE_N) also contributes significantly to academic achievement, though its effects are smaller in magnitude. In the SDQ models, OE_N is 0.105 for Maths and 0.088 for English, compared to 0.043 and 0.024 in the TIPI models. The SDQ measure (Focused Behaviour) shows stronger associations with academic performance than the TIPI measure (Conscientiousness).

The interaction term (γ_{12}) is negative and significant in the SDQ models for both subjects ($\gamma_{12} = -0.131$ for Maths, -0.124 for English). This negative sign indicates that as the level of one skill increases, the marginal productivity of the other diminishes slightly. This specific relationship defines the skills as Edgeworth substitutes - they exhibit some substitutability at the margin in terms of their direct impact on output. However, this marginal effect does not capture the overall ease of substitution required to maintain the same level of academic achievement. For that, we look at the estimated elasticity of substitution (ES) values, which are consistently below 1 (e.g., ES = 0.471 for Maths SDQ, ES = 0.452 for English SDQ). An ES value below one signifies an overall complementary relationship in the Hicks sense: it is relatively difficult to substitute one skill for the other along an isoquant while keeping academic performance constant. This reinforces the idea that cognitive and noncognitive skills are most effective when developed together; one cannot fully offset deficiencies in the other.

While γ_{12} captures local curvature (how the marginal product of one input changes with the level of the other), the elasticity of substitution summarizes the overall flexibility of the production process when substituting one skill for another to maintain output. In this case, the negative γ_{12} (indicating Edgeworth substitutability) coexists with a low elasticity of substitution (ES < 1, indicating Hicks complementarity), confirming that cognitive and noncognitive skills generally function as complements in the broader production of academic performance.

Gender differences are evident and meaningful. For boys, the output elasticity of cognitive skills (OE_C) shows a slightly stronger impact on Maths performance ($OE_C = 0.813$ for SDQ, 0.859 for TIPI) compared to girls ($OE_C = 0.778$ for SDQ, 0.820 for TIPI). In English, boys also show a higher cognitive impact ($OE_C = 0.466$ for SDQ, 0.506 for TIPI) compared to girls ($OE_C = 0.416$ for SDQ, 0.447 for TIPI). The output elasticity of noncognitive skills (OE_N) shows higher values for girls in both subjects, particularly for the SDQ measure ($OE_N = 0.125$ for Maths, 0.078 for English) compared to boys ($OE_N = 0.084$ for Maths, 0.069 for English).

MPs for both cognitive and noncognitive skills are generally higher in Maths, indicating that incremental improvements in either skill type yield greater returns in this subject. OEs consistently show that cognitive skills have a larger impact on both Maths and English scores compared to noncognitive skills, particularly for Maths.

ES values are consistently below 1 for most models (except girls' Maths TIPI model where ES = 1.096), indicating complementarity between cognitive and noncognitive skills. This complementarity is stronger in English than in Maths. Language-based subjects especially require balanced development of both skill types for success. The MRTS values show interesting gender patterns, with girls generally showing lower values than boys, suggesting they require fewer units of noncognitive skills to substitute for cognitive skills.

The choice of measurement tool for noncognitive skills remains important. The SDQ measure consistently shows stronger relationships with academic outcomes than the TIPI across all models, indicating that it captures noncognitive traits more directly relevant to academic settings and provides a more accurate representation of the skills influencing school performance.

5 Conclusion

This study contributes to our understanding of educational outcomes by examining the joint effects and interactions of cognitive and noncognitive skills, with a focus on gender differences and academic achievement in Maths and English. Using data from the Growing Up in Ireland longitudinal study, I employed both linear and translog production functions, providing complementary perspectives on how these skills combine to produce academic achievement.

5.0.1 Key Empirical Findings

How do different types of skills combine to create academic success? Picture two students approaching a challenging Maths problem: one relies on sharp analytical thinking, while the other perseveres through sheer determination. This study reveals that both paths can lead to success but through fundamentally different mechanisms. By examining how cognitive and noncognitive skills interact in producing academic achievement, we gain insights into how students learn and succeed across different subjects.

The story that emerges from the data is clear: while both types of skills matter, their importance varies dramatically by subject. In Maths, cognitive abilities dominated: a 1% improvement in cognitive skills boosted performance by 0.8%, while English sees only half that impact at 0.4%. This is not just a statistical difference, it also reflects fundamental differences in how students learn different subjects. A student's ability to think analytically about numbers yields nearly twice the benefit in Maths compared to language skills.

But raw intelligence is not the whole story. Noncognitive skills, particularly the ability to maintain focus, emerge as supporting actors in academic achievement. Students who can better concentrate and persist see their grades improve by about 0.1% for every 1% increase in these skills, which has a smaller effect than pure cognitive ability, but one that proves consistent across subjects. Focused behaviour is roughly twice as predictive as general conscientiousness. Specific, targeted behavioural skills prove more important than broader personality traits.

Cognition drives numeracy more than literacy: a 1% cognitive increase improves Maths by 0.8% but English by just 0.4%.

Noncognitive factors also played an important role, though their impact was generally smaller. Among these, Focused Behaviour (measured by the SDQ) was the most significant noncognitive predictor, with output elasticities of 0.105 for Maths and 0.088 for English, meaning that a 1% improvement in focused behaviour yields about a 0.1% increase in academic performance. Conscientiousness showed smaller but significant effects (0.043 for Maths, 0.024 for English). While both behavioural traits matter, focused attention creates about twice the impact of general conscientiousness on academic achievement.

5.0.2 Tying Linear and Translog Findings Together

Cognitive and noncognitive skills work together in ways that go beyond simple addition. The linear models show this through several significant negative interactions. With SDQ measures, cognitive ability interacts negatively with focused behaviour (-0.0565 in Maths, -0.0735 in English, both significant at 1%). TIPI measures reveal similar patterns, particularly between cognitive ability and emotional stability (-0.0678 in Maths, significant at 1%). These interaction effects improve model fit, with R^2 increasing from 0.487 to 0.491 in Maths and 0.503 to 0.506 with SDQ measures.

The translog results show elasticity of substitution values below 1 (0.471 for Maths SDQ, 0.452 for English SDQ), which confirms that cognitive and noncognitive skills act as complements. Students who perform best tend to develop both types of skills, rather than relying on one to make up for the other.

5.0.3 Core Relationships and Their Significance

Both modeling approaches reveal core features of educational production. Cognitive skills exert a stronger influence on Maths achievement, with nearly twice the output elasticity compared to English, indicating that mathematical performance relies more heavily on cognitive processing. In contrast, the strong complementarity between cognitive and noncognitive skills (especially in English) shows that maximizing academic potential requires developing both skill types, as neither can fully compensate for a deficit in the other.

The gender differences in these relationships reveal important insights about skill development and utilisation. Boys show higher cognitive elasticities (Maths: $OE_C = 0.813$ vs 0.778; English: $OE_C = 0.466$ vs 0.416), while girls demonstrate stronger noncognitive effects (Maths: $OE_N = 0.125$ vs 0.084). Girls display a unique pattern of skill substitutability in Maths, with an elasticity of substitution of 1.096 in the TIPI model. This pattern may reflect different strategies for combining cognitive and noncognitive resources to achieve mathematical competency.

5.0.4 Possible Explanations

These patterns can be understood through several complementary perspectives. At the cognitive level, Maths' stronger dependence on cognitive skills likely reflects its cumulative, hierarchical nature: each concept builds directly on previous understanding, making raw processing capacity particularly valuable. The stronger complementarity in English likely reflects the subject's multifaceted nature, requiring both cognitive capacity for comprehension and noncognitive skills for sustained engagement with texts and effective written expression.

The gender differences in skill utilisation probably reflect both biological and social factors. While cognitive development patterns may contribute to these differences, the stronger noncognitive effects for girls, particularly in Maths, suggest that socialization and educational practices may lead them to rely more heavily on behavioural traits like focused attention and conscientiousness. The unique substitutability pattern in girls' Maths achievement might represent an adaptive response to historical gender expectations in mathematical fields.

The negative interaction between cognitive and noncognitive skills observed in this study aligns with several theoretical frameworks in developmental psychology and economics. Heckman et al. (2006) propose that skills enhance each other but with decreasing marginal effects at higher levels, consistent with the diminishing returns pattern observed here. Blair and Raver (2015) suggest a self-regulatory resource theory where the brain allocates finite resources with varying efficiency depending on existing skill profiles. This may explain why students with already-high cognitive abilities benefit less from improvements in noncognitive skills. According to the developmental compensation hypothesis, students develop strategies to compensate for weaker skill domains. Duckworth and Seligman (2005) supports this view, showing that self-discipline predicts academic success more strongly than IQ. This framework helps explain why the negative interaction terms were particularly significant for students with imbalanced skill profiles.

From an economic perspective, these patterns reflect the nuanced skill production dynamics described in Heckman and Kautz (2012)'s work on human capital formation. The negative interaction terms and elasticity of substitution values below 1 support their model of skill complementarity with diminishing returns at higher levels, which is a critical insight for understanding educational production functions. Almlund et al. (2011) further elaborate on this through their framework of specialised skill utilisation. They argue that students with balanced high skills may face constraints in simultaneously deploying both skill types in standard academic assessments. This specialised deployment hypothesis provides a rationale for why the marginal returns to one skill type decrease as the other increases, particularly evident in standardised examination formats like the Junior Cert.

The gender-specific patterns in skill substitutability can be understood through both psychological and economic theories. Moffitt et al. (2011) identified distinct developmental trajectories in self-control that vary by gender, which may explain the higher noncognitive elasticities observed for girls. The unique finding that girls show greater substitutability between skill types in Maths (ES > 1 in the TIPI model) aligns with Duckworth and Gross (2014)'s distinction between self-control and grit as separable determinants of success. Girls may develop more effective compensatory strategies that allow noncognitive skills to substitute for cognitive abilities in Maths, potentially an adaptive response to stereotype threat or different socialization patterns in STEM subjects. These theoretical perspectives collectively suggest that the negative interaction terms reflect fundamental characteristics of human development and skill formation rather than merely statistical artifacts.

The SDQ measure consistently shows higher elasticities than the TIPI (approximately double) indicating that context-specific behavioural traits are more strongly linked to academic achievement than broader personality characteristics. Educational interventions may therefore be more effective when targeting specific behavioural patterns rather than attempting to modify general personality traits.

These findings contribute to several active debates in the economics of education literature. First, the observed complementarity between cognitive and noncognitive skills (ES < 1) builds on Cunha and Heckman (2007)'s work on skill complementarity into subject-specific domains. The stronger complementarity in English versus Maths provides empirical evidence for Deming and Noray (2018)'s arguments about different skill requirements across domains. The higher marginal products in Maths support Deming and Noray (2018)'s work on STEM skill premiums in education.

Building on the theoretical frameworks discussed above, the gender-specific patterns revealed in this study have significant implications for educational policy. The empirical evidence of girls' unique pattern of substitutability in Maths (ES > 1) advances Buser et al. (2014)'s work on gender differences in educational strategies, while complementing Aucejo et al. (2018)'s findings on gender-specific teaching effects. These patterns suggest distinct approaches to human capital accumulation that education systems must recognize to effectively support diverse learners.

The measurement specificity findings also have important implications for educational research. The stronger effects of context-specific measures (SDQ) compared to general personality traits (TIPI) supports Humphries and Kosse (2017)'s arguments for task-specific skill measurement. The approximately double elasticities for SDQ versus TIPI adds empirical weight to recent critiques of general personality measures in educational contexts (Kautz et al., 2014; West et al., 2016). This finding supports the need for more targeted measurement approaches in both research and policy design.

5.0.5 Policy Implications

These findings have substantial implications for educational policy and practice. Cognitive and noncognitive skills function as complements, so focusing solely on cognitive development is likely suboptimal. Educational approaches should target both skill types simultaneously to maximize student achievement. For example, Maths instruction might benefit from explicitly incorporating noncognitive skill development, such as focused attention and persistence, rather than treating these as separate from content delivery.

Gender-specific patterns in the data point to the potential value of differentiated support strategies. Boys may benefit from interventions that help them leverage their stronger cognitive effects through improved behavioural regulation, while girls may benefit from approaches that build on their stronger noncognitive foundation, particularly in Maths. This aligns with Dweck (2007)'s work on mindset interventions and confirms the need for gender-specific educational approaches.

The subject-specific nature of skill complementarity calls for differentiated pedagogical strategies. In English, where complementarity is stronger, instruction may benefit most from integrated approaches that develop both cognitive and behavioural skills in tandem. In contrast, Maths instruction may require more targeted interventions that prioritize cognitive development while still supporting noncognitive skill formation.

Focused Behaviour has approximately twice the impact on academic performance as general conscientiousness (measured by the TIPI), indicating that educational interventions should prioritize specific, context-relevant behavioural skills over broader personality traits. Teacher training should therefore emphasize strategies for developing subject-specific behavioural skills rather than attempting to shape general personality characteristics.

In conclusion, this study suggests that academic achievement emerges from complex interactions between cognitive and noncognitive skills, with patterns that vary systematically by subject and gender. Given the strong complementarity between skill types, especially in English, educational practices should adopt more integrated approaches to skill development. As education systems face increasing pressure to prepare students for a rapidly changing world, understanding and leveraging these skill interactions becomes critical. Future research might productively explore how these relationships evolve over time and how they might be influenced by different pedagogical approaches. A traditional focus on cognitive skills alone is analogous to training athletes for strength but neglecting coordination. In production terms, current practices resemble single-input models ($Q = \alpha K$) when educational outcomes may require a multi-input approach ($Q = K^{\alpha} L^{\beta}$). This study's findings call for pedagogical approaches that consciously weave behavioural skill development into subject instruction. Maths classrooms might integrate persistence-building exercises directly into algebra lessons, for example, while English teachers could explicitly coach students in maintaining focus during textual analysis.

The path forward lies in translating these insights into actionable strategies - tailored by subject and responsive to diverse student needs. A better understanding of how cognitive and noncognitive skills combine will support more equitable and effective educational practices.

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6 Appendices

The following appendices provide supplementary material, including detailed derivations, psychometric tool descriptions, and extended analyses that support the core findings of this study on the interaction between cognitive and noncognitive skills in academic achievement.

The Appendices are organized into two main sections:

1. **Appendix:**

   (a) **The Ten-Item Personality Inventory (TIPI) and the Strengths and Difficulties Questionnaire (SDQ):** This section presents the full TIPI and SDQ psychometric tools used to assess noncognitive skills in the study. Detailed explanations of each instrument, including their scoring methods and relevance to the research, are provided. Sample questionnaires and scoring guides are included to illustrate how the measures were applied.

   (b) **Cobb-Douglas Production Function with Two Inputs:** Here, the two-input Cobb-Douglas production function is elaborated upon, including derivations of marginal products, output elasticities, elasticity of substitution, and marginal rates of technical substitution. This section also discusses the estimation results and provides an interpretation of the findings in the context of the study.

   (c) **Cobb-Douglas Production Function with Three Inputs:** This section extends the analysis to a three-input Cobb-Douglas production function, incorporating additional noncognitive skill measures. Detailed derivations similar to the two-input case are presented, along with estimation results and discussions that explore the nuances introduced by the additional input.

   (d) **Marginal Rate of Technical Substitution (MRTS) Analysis:** An in-depth examination of the MRTS for both two-input and three-input Cobb-Douglas models is provided. This includes calculations of MRTS values, gender and subject-specific analyses, and interpretations of the trade-offs between cognitive and noncognitive skills in educational production.

2. **Online Appendix:**

   (a) **Translog Production Function Derivations:** This section introduces the translog production function as a more flexible functional form compared to the Cobb-Douglas. Detailed mathematical derivations are provided for marginal products, output elasticities, elasticity of substitution, and marginal rates of technical substitution. The advantages of the translog model in capturing non-linearities and interaction effects are discussed.

(b) **Constant Elasticity of Substitution (CES) Production Function:** The CES production function is explored, examining both two-input and three-input versions. This section includes comprehensive derivations of marginal products, output elasticities, elasticities of substitution, marginal rates of technical substitution, returns to scale, and isoquants. The CES function's ability to encompass the Cobb-Douglas as a special case and its implications for the study are highlighted.

(c) **Comparison of Two-Input and Three-Input CES Models:** A comparative analysis between the two-input and three-input CES models is presented. This includes discussions on the trade-offs in complexity, granularity, and generalizability between the models. The nested CES function is introduced to address differential substitution patterns among inputs, offering a more nuanced understanding of the educational production process.

(d) **Economic Interpretation of Parameters and Policy Implications:** This section interprets the economic significance of the model parameters, such as the elasticity of substitution and returns to scale. The implications of these parameters for educational policy are discussed, including insights into resource allocation, skill development strategies, and the timing of interventions.

(e) **Limitations and Empirical Considerations:** The final section addresses the limitations of the approaches used and important empirical considerations. Challenges in estimating complex production functions, measurement errors in cognitive and noncognitive skills, potential endogeneity issues, and the simplifications inherent in the models are discussed to provide context for interpreting the results and guiding future research.

The following sections present the details for Appendix 1, as outlined above.

6.1 TIPI questionnaire

The TIPI is a concise personality assessment tool designed to measure the Big Five personality traits. Developed by Gosling, Rentfrow, and Swann in 2003, it serves as a rapid alternative to more extensive personality inventories. The TIPI consists of just ten items, with two items dedicated to each of the five major personality dimensions: Extraversion, Agreeableness, Conscientiousness, Emotional Stability (the inverse of Neuroticism), and Openness to Experience. The inventory is particularly valuable in time-constrained research settings, or when personality measurement is not the primary focus. It asks respondents to rate themselves, or the study's child in the Growing up in Ireland, on a series of paired traits using a 7-point scale, ranging from "Disagree strongly" to "Agree strongly". For example, Extraversion is assessed through items like "Extraverted, enthusiastic" and its reverse-coded counterpart "Reserved, quiet". While the TIPI "sacrifices" some degree of reliability and validity compared to longer measures, it still provides a reasonable approximation of an individual's personality profile. Its brevity makes it an attractive option for large-scale surveys, online studies, or situations where a quick personality snapshot is needed. However, researchers and practitioners are aware of its limitations and use it judiciously, understanding that it offers a broad-brush picture rather than a nuanced personality portrait.

Disagree strongly	Disagree moderately	Disagree a little	Neither disagree nor agree	Agree a little	Agree moderately	Agree strongly
1	2	3	4	5	6	7

Characteristic	Grade (1-7)
1. Extraverted, enthusiastic	_____
2. Critical, quarrelsome	_____
3. Dependable, self-disciplined	_____
4. Anxious, easily upset	_____
5. Open to new experiences, complex	_____
6. Reserved, quiet	_____
7. Sympathetic, warm	_____
8. Disorganized, careless	_____
9. Calm, emotionally stable	_____
10. Conventional, uncreative	_____

Conscientiousness

Score for #3: __________

+ (8 - Score for #8): __________

= __________

Divide your answer by 2.

Conscientiousness Score = __________

Agreeableness

Score for #7: __________

+ (8 - Score for #2): __________

= __________

Divide your answer by 2.

Agreeableness Score = __________

Emotional Stability

Score for #9: __________

+ (8 - Score for #4): __________

= __________

Divide your answer by 2.

Emotional Stability Score = __________

Openness to Experience

Score for #5: ________

+ (8 - Score for #10): ________

= ________

Divide your answer by 2.

Openness to Experience Score = ________

Extraversion

Score for #1: ________

+ (8 - Score for #6): ________

= ________

Divide your answer by 2.

Extraversion Score = ________

6.2 SDQ questionnaire

The Strengths and Difficulties Questionnaire (SDQ), developed by Robert Goodman (1997), is a widely-used behavioural screening tool designed for children and adolescents aged 3 to 16 years. Unlike many psychometric assessment tools that focus solely on problems, the SDQ takes a more "balanced" approach by examining both difficulties and strengths in young people's behaviour and emotional well-being. The questionnaire consists of 25 items divided into five scales: Emotional Symptoms, Conduct Problems, Hyperactivity/Inattention, Peer-relationship Problems, and Prosocial Behaviour. Such structure allows for a comprehensive evaluation of a child's psychological adjustment, covering internalizing problems, externalizing issues, and positive social behaviours. The SDQ is very versatile: it offers versions for parents, teachers, and self-report (for older children and adolescents), allowing for a multi-informant approach to assessment. This multi-perspective view can provide a more rounded understanding of a child's behaviour across different contexts. The questionnaire uses a 3-point Likert scale ("Not True", "Somewhat True", "Certainly True") for responses, making it accessible and easy to complete. It typically takes between 5 to 10 minutes to fill out, providing a balance between comprehensiveness and practicality. Internationally recognized and translated into numerous languages, the SDQ has become a valuable tool in both clinical and research settings. It is particularly useful for early identification of potential mental health issues, allowing for timely intervention. The inclusion of the Prosocial scale also provides insight into a child's positive social behaviours, offering a more holistic view of their functioning. In this study, the Prosocial scale was excluded from analysis due to limitations in how it was scored in the anonymized microdata files (AMF), which precluded consistent treatment alongside the other subscales. While the SDQ is not a diagnostic tool, its scores can indicate whether a child might benefit from further assessment or support. Its widespread use also facilitates comparisons across different populations and cultures, contributing to a cross-subject understanding of child and adolescent mental health on a global scale.

Example taken from the Youth in Mind (2023) website https://sdqinfo.org/:

Strengths and Difficulties Questionnaire

For each item, please mark the box for Not True, Somewhat True or Certainly True. Answer all items as best you can even if you are not absolutely certain. Please give your answers on the basis of the child's behaviour over the last six months.

	Not True	Somewhat True	Certainly True
1. Considerate of other people's feelings	☐	☐	☐
2. Restless, overactive, cannot stay still for long	☐	☐	☐
3. Often complains of headaches, stomach-aches or sickness	☐	☐	☐
4. Shares readily with other children (treats, toys, pencils etc.)	☐	☐	☐
5. Often has temper tantrums or hot tempers	☐	☐	☐
6. Rather solitary, tends to play alone	☐	☐	☐
7. Generally obedient, usually does what adults request	☐	☐	☐
8. Many worries, often seems worried	☐	☐	☐
9. Helpful if someone is hurt, upset or feeling ill	☐	☐	☐
10. Constantly fidgeting or squirming	☐	☐	☐
11. Has at least one good friend	☐	☐	☐
12. Often fights with other children or bullies them	☐	☐	☐
13. Often unhappy, down-hearted or tearful	☐	☐	☐
14. Generally liked by other children	☐	☐	☐
15. Easily distracted, concentration wanders	☐	☐	☐
16. Nervous or clingy in new situations, easily loses confidence	☐	☐	☐
17. Kind to younger children	☐	☐	☐
18. Often lies or cheats	☐	☐	☐
19. Picked on or bullied by other children	☐	☐	☐
20. Often volunteers to help others (parents, teachers, other children)	☐	☐	☐
21. Thinks things out before acting	☐	☐	☐
22. Steals from home, school or elsewhere	☐	☐	☐
23. Gets on better with adults than with other children	☐	☐	☐
24. Many fears, easily scared	☐	☐	☐
25. Sees tasks through to the end, good attention span	☐	☐	☐

The SDQ is divided into five sections, each containing five questions:

1. **Emotional Symptoms Scale:**

- 3. Often complains of headaches, stomach-aches or sickness
- 8. Many worries, often seems worried
- 13. Often unhappy, down-hearted or tearful
- 16. Nervous or clingy in new situations, easily loses confidence
- 24. Many fears, easily scared

2. **Conduct Problems Scale:**

- 5. Often has temper tantrums or hot tempers
- 7. Generally obedient, usually does what adults request (reverse scored)
- 12. Often fights with other children or bullies them
- 18. Often lies or cheats
- 22. Steals from home, school or elsewhere

3. **Hyperactivity Scale:**

- 2. Restless, overactive, cannot stay still for long
- 10. Constantly fidgeting or squirming
- 15. Easily distracted, concentration wanders
- 21. Thinks things out before acting (reverse scored)
- 25. Sees tasks through to the end, good attention span (reverse scored)

4. **Peer Problems Scale:**

- 6. Rather solitary, tends to play alone
- 11. Has at least one good friend (reverse scored)
- 14. Generally liked by other children (reverse scored)
- 19. Picked on or bullied by other children
- 23. Gets on better with adults than with other children

5. **Prosocial Scale:**

- 1. Considerate of other people's feelings
- 4. Shares readily with other children (treats, toys, pencils etc.)
- 9. Helpful if someone is hurt, upset or feeling ill
- 17. Kind to younger children
- 20. Often volunteers to help others (parents, teachers, other children)

Note: Items marked as "reverse scored" are phrased positively, so their scores are reversed when calculating the total for that scale.

6.3 Cobb-Douglas With Two Inputs

This section presents the two-input Cobb-Douglas production function used to model the relationship between cognitive and noncognitive skills and academic achievement. The functional form is:

$$Y = AC^{\alpha}N^{\beta} \tag{16}$$

Where:

$$\begin{aligned} Y &: \text{Academic achievement (output)} \\ A &: \text{Total factor productivity (scaling factor)} \\ C &: \text{Cognitive skill input} \\ N &: \text{Noncognitive skill input} \\ \alpha, \beta &: \text{Output elasticities of each input} \end{aligned}$$

The exponents α and β capture the percentage change in academic performance resulting from a 1% change in cognitive and noncognitive skills, respectively. This formulation assumes constant elasticity of substitution ($\sigma = 1$) and exhibits decreasing, constant, or increasing returns to scale depending on the sum of $\alpha + \beta$.

Although a more flexible three-input version is examined in the next section, this simplified model remains informative. The similarity in parameters between the two- and three-input specifications shows that a two-factor structure-focused on cognition and a single noncognitive measure-captures most of the relevant variance in achievement outcomes, supporting its standalone presentation.

6.3.1 Marginal Products (MPs)

Marginal products represent the additional output generated by a one-unit increase in a given input while holding the other input constant. For the Cobb-Douglas production function:

$$Y = AC^{\alpha}N^{\beta}$$

The marginal product of **cognition** (C) is:

$$f_C = \left.\frac{\partial Y}{\partial C}\right|_{N=N_0} = A\alpha C^{\alpha-1}(N_0)^{\beta} = A\alpha\frac{C^{\alpha}N^{\beta}}{C} = \alpha\frac{Y}{C}$$

The marginal product of noncognitive skills (N) is:

$$f_N = \left.\frac{\partial Y}{\partial N}\right|_{C=C_0} = A\beta C_0^{\alpha} N^{\beta-1} = A\beta \frac{C^{\alpha}N^{\beta}}{N} = \beta\frac{Y}{N}$$

To assess whether marginal returns increase or decrease, we take the second derivatives:

$$\frac{\partial f_C}{\partial C} = \frac{\partial^2 Y}{\partial C^2} = A\alpha(\alpha-1)C^{\alpha-2}N^{\beta} = \alpha(\alpha-1)\frac{Y}{C^2}$$

$$\frac{\partial f_N}{\partial N} = \frac{\partial^2 Y}{\partial N^2} = A\beta(\beta-1)N^{\beta-2}C^{\alpha} = \beta(\beta-1)\frac{Y}{N^2}$$

These second derivatives help determine whether each input exhibits diminishing returns, which occurs when the expressions are negative (i.e., when $\alpha < 1$ or $\beta < 1$). In this study, estimated values of α and β are generally below 1, implying decreasing marginal returns to both cognition and noncognitive skills in the production of academic achievement.

6.3.2 Output elasticities (OEs)

Parameters estimation:

$$\ln(Y) = \ln(A) + \alpha\ln(C) + \beta\ln(N) \tag{17}$$

With output elasticities defined as:

$$OE_C = \left.\frac{\partial\ln(Y)}{\partial\ln(C)}\right|_{N=N_0} = \alpha \tag{18}$$

$$OE_N = \left.\frac{\partial\ln(Y)}{\partial\ln(N)}\right|_{C=C_0} = \beta \tag{19}$$

If we define the scale elasticity (SCE) as the scale change, often measured by the percent change in output from a simultaneous 1% change in all inputs, then: $OE_C + OE_N = \alpha + \beta$.

6.3.3 Elasticity of Substitution (ES)

$$\sigma = \frac{\frac{d(C/N)}{(C/N)}}{\frac{d(MP_C/MP_N)}{(MP_C/MP_N)}} \tag{20}$$

Where $MP_C(f_C)$ and $MP_N(f_N)$ are the marginal products of C and N respectively. For the Cobb-Douglas production function, the σ between the two inputs is always equal to 1. This is a key property of the Cobb-Douglas function. To illustrate this for the two-input case:

$$Y = AC^{\alpha}N^{1-\alpha} \tag{21}$$

The marginal products are:

$$f_C = \frac{\partial Y}{\partial C} = \alpha A C^{\alpha-1} N^{1-\alpha} \tag{22}$$

$$f_N = \frac{\partial Y}{\partial N} = (1-\alpha) A C^{\alpha} N^{-\alpha} \tag{23}$$

The ratio of marginal products is:

$$\frac{f_C}{f_N} = \frac{\alpha N}{(1-\alpha)C} \tag{24}$$

If we calculate σ, we find:

$$\sigma = \frac{\frac{d(C/N)}{(C/N)}}{\frac{d((\alpha N)/((1-\alpha)C))}{(\alpha N)/((1-\alpha)C)}} = 1 \tag{25}$$

This result of 1 holds for the two inputs in the Cobb-Douglas function, demonstrating the constant unitary elasticity of substitution between cognitive and noncognitive skills in this model.

6.3.4 Estimation and Discussion

The cognitive factor α, representing the output elasticity of cognition in this Cobb-Douglas specification, is the most significant predictor of academic performance across all models. For Maths (Table 7), boys show slightly higher cognitive output elasticities ($\alpha = 0.804$ for TIPI, 0.765 for SDQ) compared to girls ($\alpha = 0.774$ for TIPI, 0.729 for SDQ). This pattern is mirrored in English (Table 8), with boys' α ranging from 0.453 to 0.490 and girls' from 0.403 to 0.434. The full sample results fall between these gender-specific values, as expected.

The noncognitive factor β, representing the output elasticity of the noncognitive skill in this specification, is smaller in magnitude but significant across all models, as expected. Focused Behaviour consistently exhibits stronger associations than Conscientiousness. For example, in the full sample Maths model, $\beta = 0.071$ for the SDQ model while $\beta = 0.024$ for the TIPI model.

Boys consistently demonstrate a slightly stronger cognitive component in both subjects. However, girls show stronger effects of noncognitive factors, particularly in Maths. This is especially evident with the SDQ measures, where girls' OE_N for Maths is 0.081 compared to boys' 0.053.

Both cognitive and noncognitive factors appear to have a stronger influence on Maths performance compared to English. This is evident in the higher values of both α and β for Maths across all models. For example, in the full sample SDQ model, $\alpha_{Maths} = 0.742$ while $\alpha_{English} = 0.405$.

Table 7: Two-input Cobb-Douglas estimates for Junior Certificate Maths achievement, comparing TIPI- and SDQ-based noncognitive inputs across full sample and gender subgroups. The table reports estimated elasticities, implied marginal products, and returns to scale; these are model-based descriptive quantities for the estimation sample.

	Maths			
Parameter	Full Sample	Boys	Girls	Δ (Boys - Girls)
TIPI Model				
A	0.256***	0.225***	0.264***	-0.039
	(0.015)	(0.019)	(0.021)	
α (Cognition)	0.781***	0.804***	0.774***	0.030
	(0.012)	(0.018)	(0.018)	
β (Conscientiousness)	0.024***	0.020***	0.023***	-0.003
	(0.003)	(0.004)	(0.004)	
Marginal Product (Cognition)	0.075	0.077	0.076	0.001
Marginal Product (Conscientiousness)	0.053	0.051	0.051	-0.001
Returns to Scale	0.805	0.830	0.800	0.030
SDQ Model				
A	0.274***	0.238***	0.274***	-0.036
	(0.018)	(0.025)	(0.024)	
α (Cognition)	0.742***	0.765***	0.729***	0.036
	(0.015)	(0.023)	(0.020)	
β (Focused Behaviour)	0.071***	0.053***	0.081***	-0.027
	(0.006)	(0.008)	(0.010)	
Marginal Product (Cognition)	0.072	0.073	0.072	0.002
Marginal Product (Focused Behaviour)	0.091	0.078	0.104	-0.027
Returns to Scale	0.814	0.828	0.819	0.010
Observations	5,631	2,667	2,801	

*Standard errors in parentheses. Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Note: The table displays estimates for the Cobb-Douglas production function applied to Maths scores using two inputs. The TIPI Model focuses on Cognition and Conscientiousness, while the SDQ Model considers Cognition and Focused Behaviour. Observations represent the number of data points for each group.

The marginal products further support these findings. Across all models, the marginal product for cognition is always higher than for noncognitive factors, reinforcing the dominant role of cognitive abilities in academic achievement.

The sum of α and β in all models is less than 1, which indicates decreasing returns to scale in the production of academic achievement. This implies that proportional increases in both cognitive and noncognitive inputs would result in less than proportional increases in academic output. For example, in the full sample Maths SDQ model, $\alpha + \beta = 0.742 + 0.071 = 0.813 < 1$. The educational production process appears more efficient for Maths, as evidenced by higher returns to scale compared to English.

Table 8: Two-input Cobb-Douglas estimates for Junior Certificate English achievement, comparing TIPI- and SDQ-based noncognitive inputs across full sample and gender subgroups. The table reports estimated elasticities, implied marginal products, and returns to scale; these are model-based descriptive quantities for the estimation sample.

	English			
Parameter	Full Sample	Boys	Girls	Δ (Boys - Girls)
TIPI Model				
A	1.310***	1.020***	1.390***	-0.370
	(0.060)	(0.072)	(0.084)	
α (Cognition)	0.441***	0.490***	0.434***	0.056
	(0.010)	(0.015)	(0.013)	
β (Conscientiousness)	0.014***	0.009**	0.010**	-0.001
	(0.002)	(0.003)	(0.003)	
Marginal Product (Cognition)	0.045	0.048	0.046	0.003
Marginal Product (Conscientiousness)	0.032	0.025	0.023	0.002
Returns to Scale	0.454	0.502	0.445	0.057
SDQ Model				
A	1.400***	1.090***	1.430***	-0.340
	(0.079)	(0.107)	(0.105)	
α (Cognition)	0.405***	0.453***	0.403***	0.050
	(0.013)	(0.022)	(0.017)	
β (Focused Behaviour)	0.060***	0.044***	0.051***	-0.007
	(0.005)	(0.008)	(0.008)	
Marginal Product (Cognition)	0.041	0.045	0.043	0.002
Marginal Product (Focused Behaviour)	0.080	0.064	0.070	-0.005
Returns to Scale	0.465	0.502	0.458	0.044
Observations	5,631	2,667	2,801	

*Standard errors in parentheses. Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Note: The table displays estimates for the Cobb-Douglas production function applied to English scores using two inputs. The TIPI Model focuses on Cognition and Conscientiousness, while the SDQ Model considers Cognition and Focused Behaviour. Observations represent the number of data points for each group.

Overall, the two-input Cobb-Douglas model offers a parsimonious yet informative perspective on how key skill inputs shape academic performance.

6.4 Cobb-Douglas With Three Inputs

We begin with a Cobb-Douglas production function incorporating three inputs:

$$Y = f(C, N_E, N_I) = AC^{\alpha} N_E^{\beta_1} N_I^{\beta_2} \tag{26}$$

Where:

$$
\begin{aligned}
Y &: \text{Total output/Grade function/Academic achievement} \\
A &: \text{Total factor productivity/scaling factor} \\
C &: \text{Input representing cognition} \\
N_E, N_I &: \text{Inputs representing noncognitive measures} \\
\alpha, \beta_1, \beta_2 &: \text{Exponents determining the output response to each input}
\end{aligned}
$$

This function assumes a Cobb-Douglas form, where the exponents α, β_1, and β_2 represent the output elasticities - capturing the proportional impact of each input on academic achievement. C is a measure of cognition and N_E and N_I are noncognitive measures. N_E captures emotional traits (Emotional Resilience for SDQ and Emotional Stability for TIPI), while N_I captures behavioural traits (Focused Behaviour for SDQ and Conscientiousness for TIPI). These were selected based on their consistent significance in prior regressions and relatively high pairwise correlations (0.407 for SDQ and 0.409 for TIPI). Correlation analyses confirmed their centrality, with these pairs exhibiting the highest coefficients among all subscales-supporting their use as representative noncognitive dimensions. The use of separate noncognitive inputs allows us to better capture the multidimensional nature of noncognitive skills and their potentially different impacts on academic achievement. While other subscales were available, these four stood out as the most relevant for predicting academic performance in both Maths and English.

6.4.1 Marginal products (MPs)

Marginal products (MPs) represent the change in total output resulting from a one-unit increase in a specific input while holding all other inputs constant.

The marginal products for each input represent the change in academic achievement resulting from a one-unit increase in the respective input, holding the others constant.

$$f_C = \left.\frac{\partial f}{\partial C}\right|_{N_E=N_{E0}, N_I=N_{I0}} = A\alpha C^{\alpha-1}(N_{E0})^{\beta_1}(N_{I0})^{\beta_2} \tag{27}$$

$$f_{N_E} = \left.\frac{\partial f}{\partial N_E}\right|_{C=C_0, N_I=N_{I0}} = AC_0^{\alpha}\beta_1(N_E)^{\beta_1-1}(N_{I0})^{\beta_2} \tag{28}$$

$$f_{N_I} = \left.\frac{\partial f}{\partial N_I}\right|_{C=C_0, N_E=N_{E0}} = AC_0^{\alpha}(N_{E0})^{\beta_1}\beta_2(N_I)^{\beta_2-1} \tag{29}$$

6.4.2 Output elasticities (OEs)

Output elasticities measure the responsiveness of output to a change in an input, expressed in percentage terms.

Given:

$$\ln(Y) = \ln(A) + \alpha \ln(C) + \beta_1 \ln(N_E) + \beta_2 \ln(N_I) \tag{30}$$

With output elasticities defined as:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N_E=N_{E0}, N_I=N_{I0}} = \alpha \tag{31}$$

$$OE_{N_E} = \left.\frac{\partial \ln(Y)}{\partial \ln(N_E)}\right|_{C=C_0, N_I=N_{I0}} = \beta_1 \tag{32}$$

$$OE_{N_I} = \left.\frac{\partial \ln(Y)}{\partial \ln(N_I)}\right|_{C=C_0, N_E=N_{E0}} = \beta_2 \tag{33}$$

The output elasticities for cognition and each noncognitive input are equal to their respective exponents α, β_1, and β_2, as is standard in log-linear Cobb-Douglas models.

The scale elasticity (SCE) measures by the percent change in output from a simultaneous 1% change in all inputs, then:

$$SCE = OE_C + OE_{N_E} + OE_{N_I} = \alpha + \beta_1 + \beta_2 \tag{34}$$

6.4.3 Elasticity of substitution (ES)

The elasticity of substitution (ES, σ) is defined as the degree to which the marginal rate of substitution between two inputs varies as the ratio of the quantity of those inputs varies while output is held constant (Stern, 2009):

$$\sigma = \frac{\frac{d(X/Y)}{(X/Y)}}{\frac{d(MP_X/MP_Y)}{(MP_X/MP_Y)}} \tag{35}$$

Where $MP_X(f_X)$ and $MP_Y(f_Y)$ are the marginal products of X and Y respectively. For the Cobb-Douglas production function, the σ between any two inputs is always $= 1$. This implies that inputs are neither strong substitutes nor strong complements; rather, they are unitary substitutes, meaning a 1% increase in one input requires a 1% decrease in another to keep output constant. To illustrate this for the three-input case:

$$\frac{f_C}{f_{N_E}} = \frac{\alpha f/C}{\beta_1 f/N_E} = \frac{\alpha N_E}{\beta_1 C} \tag{36}$$

If we were to calculate σ_{C,N_E}, we would find:

$$\sigma = \frac{\frac{d(C/N_E)}{(C/N_E)}}{\frac{(\alpha N_E)/(\beta_1 C)}{(\alpha N_E)/(\beta_1 C)}} = 1 \tag{37}$$

6.4.4 Estimation and discussion

TIPI:

$$JC_{M,E} = A(Cognition)^{\alpha}(EmotionalStability)^{\beta_1}(Conscientiousness)^{\beta_2} \tag{38}$$

SDQ:

$$JC_{M,E} = A(Cognition)^{\alpha}(EmotionalResilience)^{\beta_1}(FocusedBehaviour)^{\beta_2} \tag{39}$$

With $JC_{M,E}$ representing the score in the Junior Cert for Maths (M) and English (E).

Across all models, cognition continues to be the strongest and most consistent predictor of academic achievement. However, this expanded analysis reveals subtle gender differences, with boys exhibiting marginally higher cognitive elasticities in both subjects.

Two distinct noncognitive factors enhance analytical depth. Though smaller in effect, noncognitive traits preserve statistical significance, particularly within SDQ models, which validates their role in educational outcomes. Girls exhibit stronger noncognitive effects-particularly in Maths-underscoring the importance of behavioural traits in female academic success and challenging one-dimensional gender narratives.

Subject-wise comparisons indicate that cognition exerts a more substantial influence on Maths than on English across all models. Noncognitive factors, particularly for girls, contribute more significantly to Maths performance than might be expected.

These findings reinforce the greater predictive validity of the SDQ over the TIPI for academic outcomes, likely due to its context-specific behavioural focus. The examination of marginal products reinforces cognition's dominant role while also shedding light at the non-trivial contributions of noncognitive factors. The observed decreasing returns to scale, more pronounced in English, imply that proportional increases in all inputs yield diminishing academic gains.

6.5 Marginal Rate of Technical Substitution (MRTS) for Cobb-Douglas Production Functions with Two and Three Inputs

The Marginal Rate of Technical Substitution (MRTS) originates in production theory and is applied here to educational achievement. In this context, the MRTS represents the rate at which one input (e.g., cognition) can be substituted for another (e.g., noncognitive skills)

Table 9: Three-input Cobb-Douglas estimates for Junior Certificate Maths achievement using cognition and two noncognitive inputs (TIPI: emotional stability and conscientiousness; SDQ: emotional resilience and focused behaviour), by full sample and gender subgroup. Estimates are descriptive and reported with standard errors in parentheses.

Parameter	**Full Sample**	**Boys**	**Girls**	**Δ (Boys - Girls)**
TIPI Model				
A	0.257***	0.225***	0.264***	-0.039
	(0.015)	(0.019)	(0.021)	
α (Cognition)	0.777***	0.804***	0.774***	0.030
	(0.012)	(0.018)	(0.018)	
β_1 (Emotional Stability)	0.010**	0.013**	0.005	0.007
	(0.003)	(0.005)	(0.004)	
β_2 (Conscientiousness)	0.022***	0.020***	0.023***	-0.003
	(0.003)	(0.004)	(0.004)	
Marginal Product (Cognition)	0.075	0.077	0.076	0.001
Marginal Product (Emotional Stability)	0.021	0.028	0.012	0.016
Marginal Product (Conscientiousness)	0.050	0.046	0.049	-0.004
Returns to Scale	0.809	0.837	0.802	0.034
SDQ Model				
A	0.269***	0.238***	0.274***	-0.036
	(0.018)	(0.025)	(0.024)	
α (Cognition)	0.737***	0.765***	0.729***	0.036
	(0.015)	(0.023)	(0.020)	
β_1 (Emotional Resilience)	0.024**	0.031*	0.022*	0.009
	(0.008)	(0.013)	(0.010)	
β_2 (Focused Behaviour)	0.068***	0.053***	0.081***	-0.027
	(0.006)	(0.008)	(0.010)	
Marginal Product (Cognition)	0.071	0.073	0.071	0.002
Marginal Product (Emotional Resilience)	0.027	0.035	0.027	0.009
Marginal Product (Focused Behaviour)	0.086	0.072	0.098	-0.027
Returns to Scale	0.828	0.850	0.832	0.018
Observations	5,631	2,667	2,801	

*Standard errors in parentheses. Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

Table 10: Three-input Cobb-Douglas estimates for Junior Certificate English achievement using cognition and two noncognitive inputs (TIPI: emotional stability and conscientiousness; SDQ: emotional resilience and focused behaviour), by full sample and gender subgroup. Estimates are descriptive and reported with standard errors in parentheses.

Parameter	Full Sample	Boys	Girls	Δ (Boys - Girls)
TIPI Model				
A	1.320***	1.020***	1.390***	-0.370
	(0.060)	(0.072)	(0.084)	
α (Cognition)	0.439***	0.490***	0.434***	0.056
	(0.010)	(0.015)	(0.013)	
β_1 (Emotional Stability)	0.004	0.007	0.003	0.004
	(0.003)	(0.004)	(0.003)	
β_2 (Conscientiousness)	0.013***	0.009**	0.010**	-0.000
	(0.002)	(0.003)	(0.003)	
Marginal Product (Cognition)	0.045	0.048	0.046	0.003
Marginal Product (Emotional Stability)	0.008	0.015	0.006	0.009
Marginal Product (Conscientiousness)	0.031	0.022	0.022	0.000
Returns to Scale	0.456	0.506	0.446	0.060
SDQ Model				
A	1.390***	1.090***	1.430***	-0.340
	(0.082)	(0.107)	(0.105)	
α (Cognition)	0.405***	0.453***	0.403***	0.050
	(0.013)	(0.022)	(0.017)	
β_1 (Emotional Resilience)	0.002	0.017	0.011	0.006
	(0.007)	(0.012)	(0.008)	
β_2 (Focused Behaviour)	0.059***	0.044***	0.051***	-0.007
	(0.005)	(0.008)	(0.008)	
Marginal Product (Cognition)	0.041	0.045	0.043	0.002
Marginal Product (Emotional Resilience)	0.003	0.020	0.013	0.006
Marginal Product (Focused Behaviour)	0.080	0.061	0.067	-0.006
Returns to Scale	0.467	0.514	0.465	0.049
Observations	5,631	2,667	2,801	

*Standard errors in parentheses. Signif. Codes: ***: 0.001, **: 0.01, *: 0.05*

while maintaining the same level of output (academic performance). Mathematically, the MRTS is defined as the negative slope of the isoquant curve in input space.

In the Cobb-Douglas production function models, the MRTS helps us quantify the trade-offs between cognitive and noncognitive inputs in educational achievement. Specifically, we would be able to answer questions such as: how much improvement in noncognitive skills is needed to compensate for a deficit in cognitive abilities? To what extent can enhancements in one type of skill make up for deficiencies in another? Do these trade-offs differ between subjects (Maths vs. English) or between genders?

These questions are examined using both two-input and three-input Cobb-Douglas production functions, incorporating TIPI and SDQ measures of noncognitive skills (Conscientiousness and Emotional Stability for the TIPI, and Focused Behaviour and Emotional Resilience for the SDQ).

6.5.1 Definition - Three Inputs

We first start with a Cobb-Douglas production function with three-inputs:

$$Y = AC^{\alpha} N_E^{\beta_1} N_I^{\beta_2} \tag{40}$$

Given:

$$f_C = \frac{\partial Q}{\partial C} = \alpha A C^{\alpha-1} N_E^{\beta_1} N_I^{\beta_2} \tag{41}$$

$$f_{NE} = \frac{\partial Q}{\partial N_E} = \beta_1 A C^{\alpha} N_E^{\beta_1 - 1} N_I^{\beta_2} \tag{42}$$

$$f_{NI} = \frac{\partial Q}{\partial N_I} = \beta_2 A C^{\alpha} N_E^{\beta_1} N_I^{\beta_2 - 1} \tag{43}$$

Then MRTS:

$$MRTS_{N_E,C} = -\frac{dN_E}{dC} = \frac{f_C}{MP_{NE}} = \frac{\alpha A C^{\alpha-1} N_E^{\beta_1} N_I^{\beta_2}}{\beta_1 A C^{\alpha} N_E^{\beta_1-1} N_I^{\beta_2}} = \frac{\alpha}{\beta_1} \frac{N_E}{C} \tag{44}$$

$$MRTS_{N_I,C} = -\frac{dN_I}{dC} = \frac{f_C}{f_{NI}} = \frac{\alpha A C^{\alpha-1} N_E^{\beta_1} N_I^{\beta_2}}{\beta_2 A C^{\alpha} N_E^{\beta_1} N_I^{\beta_2-1}} = \frac{\alpha}{\beta_2} \frac{N_I}{C} \tag{45}$$

$$MRTS_{N_I,N_E} = -\frac{dN_I}{dN_E} = \frac{f_{NE}}{f_{NI}} = \frac{\beta_1 A C^{\alpha} N_E^{\beta_1-1} N_I^{\beta_2}}{\beta_2 A C^{\alpha} N_E^{\beta_1} N_I^{\beta_2-1}} = \frac{\beta_1}{\beta_2} \frac{N_I}{N_E} \tag{46}$$

6.5.2 Definition - Two Inputs

Building on the MRTS formulation for the three-input case, we now define the MRTS for a two-input Cobb-Douglas function.

Given:

$$Y = AC^{\alpha}N^{\beta} \tag{47}$$

Marginal products:

$$f_C = \frac{\partial Y}{\partial C}\bigg| N = N_0 = A\alpha C^{\alpha-1}(N_0)^{\beta} \tag{48}$$

$$f_N = \frac{\partial Y}{\partial N}\bigg| C = C_0 = A\beta C0^{\alpha}(N)^{\beta-1} \tag{49}$$

$$MRTS_{N,C} = \frac{f_C}{f_N} = \frac{A\alpha C^{\alpha-1}N^{\beta}}{A\beta C^{\alpha}N^{\beta-1}} = \frac{\alpha}{\beta}\frac{N}{C} \tag{50}$$

$MRTS_{N,C}$ is:

$$MRTS_{N,C} = \frac{\alpha}{\beta}\frac{N}{C} \tag{51}$$

$MRTS_{N,C}$ represents how much the noncognitive input (N) needs to increase to compensate for a unit decrease in cognition (C) while maintaining the same level of output (Y).

6.5.3 Estimation and Discussion

In most cases, the MRTS of noncognitive skills relative to cognition exceeds 1, indicating that more than one unit of noncognitive input is required to compensate for a single unit of cognition. This further supports the finding that cognition generally has a stronger impact on academic outcomes.

For the TIPI variables, the MRTS of Conscientiousness for Cognition is higher in Maths than in English for the overall sample (1.420 vs 1.390). For the SDQ variables, the MRTS of Focused Behaviour for Cognition is higher in Maths than in English across all groups (0.786 vs 0.515 for the overall sample), which indicates that while Conscientiousness (TIPI) shows a relatively consistent connection to cognition across subjects, Focused Behaviour (SDQ) appears to have a stronger relative importance in Maths compared to English.

Boys show a greater reliance on cognitive inputs for equivalent performance in the Maths TIPI model, with a slightly higher MRTS of Conscientiousness for Cognition than girls (1.520 vs 1.475). In English (TIPI), both girls and boys show much higher MRTS of Conscientiousness for Cognition compared to the overall sample, with values close to 2 (1.966 for girls and 1.950 for boys). In Maths (SDQ), boys show a higher MRTS of Focused Behaviour for Cognition compared to girls (0.946 vs 0.688). This indicates that the substitution patterns between cognitive and noncognitive skills differ by gender, especially in the context of English achievement.

Model	MRTS Type	Marginal Rates of Technical Substitution (MRTS)			
		Full Sample	Girls	Boys	Δ (Boys - Girls)
Maths (TIPI)	Emotional Stability for Cognition	3.600	3.619	2.724	-0.895
	Conscientiousness for Cognition	1.511	1.588	1.674	0.086
	Conscientiousness for Emo. Stability	0.420	0.439	0.615	0.176
Maths (SDQ)	Emotional Resilience for Cognition	2.599	2.678	2.064	-0.614
	Focused Behaviour for Cognition	0.824	0.723	1.018	0.295
	Focused Behaviour for Emo. Resilience	0.317	0.270	0.493	0.223
English (TIPI)	Emotional Stability for Cognition	5.427	7.357	3.190	-4.167
	Conscientiousness for Cognition	1.444	2.054	2.161	0.107
	Conscientiousness for Emo. Stability	0.266	0.279	0.677	0.398
English (SDQ)	Emotional Resilience for Cognition	14.013	3.163	2.279	-0.884
	Focused Behaviour for Cognition	0.520	0.636	0.734	0.098
	Focused Behaviour for Emo. Resilience	0.037	0.201	0.322	0.121

Note: The table presents the Marginal Rates of Technical Substitution for 3-input Cobb-Douglas models in both Maths and English. The TIPI model uses Emotional Stability, Conscientiousness, and Cognition as inputs, while the SDQ model utilizes Emotional Resilience, Focused Behaviour, and Cognition. MRTS indicates the rate at which one input can be substituted for another while maintaining the same level of output. Δ represents the difference in MRTS between Boys and Girls.

Table 11: Marginal rates of technical substitution (MRTS) implied by three-input Cobb-Douglas models for Maths and English, by noncognitive measure set (TIPI or SDQ) and gender subgroup. MRTS values indicate within-model input trade-offs at observed sample levels and should be interpreted as descriptive model implications.

Model	MRTS Type	Marginal Rates of Technical Substitution (MRTS)			
		Full Sample	Girls	Boys	Δ (Boys - Girls)
Maths (TIPI)	Conscientiousness for Cognition	1.420	1.475	1.520	0.045
	Cognition for Conscientiousness	0.704	0.678	0.658	-0.020
Maths (SDQ)	Focused Behaviour for Cognition	0.786	0.688	0.946	0.258
	Cognition for Focused Behaviour	1.272	1.454	1.058	-0.396
English (TIPI)	Conscientiousness for Cognition	1.390	1.966	1.950	-0.016
	Cognition for Conscientiousness	0.720	0.509	0.513	0.004
English (SDQ)	Focused Behaviour for Cognition	0.515	0.612	0.697	0.085
	Cognition for Focused Behaviour	1.941	1.634	1.434	-0.200

Note: The table presents the Marginal Rates of Technical Substitution for 2-input Cobb-Douglas models in both Maths and English. The TIPI model uses Cognition and Conscientiousness as inputs, while the SDQ model utilizes Cognition and Focused Behaviour. MRTS is calculated as the ratio of the marginal product of one input to the marginal product of the other, indicating how inputs can be substituted while maintaining the same level of output.

Table 12: Marginal rates of technical substitution (MRTS) implied by two-input Cobb-Douglas models for Maths and English, by noncognitive measure set (TIPI or SDQ) and gender subgroup. MRTS values indicate within-model input trade-offs at observed sample levels and should be interpreted as descriptive model implications.

The TIPI scale often shows higher MRTS values for noncognitive skills compared to the SDQ scale, particularly in English. When we consider the reciprocal MRTS (Cognition for Noncognitive skills), we see values less than 1 in most cases, particularly for English, meaning that cognition can more easily compensate for deficits in noncognitive skills than vice versa, especially in language performance.

For example, in Maths (TIPI), 1.420 units of Conscientiousness are required to substitute for 1 unit of Cognition. In English, this figure decreases slightly to 1.390. The SDQ measure presents a different picture: for Maths, 0.786 units of Focused Behaviour are required to substitute for 1 unit of Cognition, whereas in English, only 0.515 units are needed. Therefore, noncognitive skills, particularly as measured by the SDQ, contribute more significantly relative to cognition in English compared to Maths. On the other hand, when we consider how cognitive skills can compensate for noncognitive skills, we find that for Maths (TIPI), 0.704 units of Cognition can substitute for 1 unit of Conscientiousness, while for English, this increases slightly to 0.720 units.

In English, we observe more dramatic variations in MRTS values compared to Maths, meaning that the relative importance of cognitive versus noncognitive skills is more subject to individual differences in language performance. This is particularly evident in the TIPI measure for English, where both girls and boys show MRTS values close to 2 (1.966 and 1.950, respectively) for Conscientiousness relative to Cognition, far higher than the overall sample average of 1.390. Gender differences are also apparent: the relative importance of cognitive versus noncognitive skills changes more dramatically for girls across subjects, as evidenced by their higher variability in MRTS values between Maths and English compared to boys. The SDQ measure (Focused Behaviour) shows a more consistent pattern across subjects and genders compared to the TIPI measure (Conscientiousness), potentially indicating that specific behavioural traits have a more uniform connection to academic performance across different contexts.

7 Online Appendix

7.1 Derivations for a Translog Production Function with Two Inputs

$$Y = AC^{\alpha}N^{\beta}\exp\left\{\frac{1}{2}\gamma_1\left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2\left[\ln(N)\right]^2 + \gamma_{12}\ln(C)\ln(N)\right\} \tag{52}$$

Where:

- Y is the output (educational achievement)
- A is the total factor productivity
- C and N are the inputs (Cognition and Noncognitive skills)
- α and β are the direct effects of inputs
- γ_1 and γ_2 capture curvature (nonlinearities), and γ_{12} captures interaction effects between cognition and noncognitive skills.

7.1.1 Marginal Products (MPs)

$$f_C = \left.\frac{\partial Y}{\partial C}\right|_{N=N_0} = A\alpha C^{\alpha-1}N_0^{\beta}\frac{\partial}{\partial C}\left[\exp\left\{X(C,N_0)\right\}\right] \tag{53}$$

$$f_N = \left.\frac{\partial Y}{\partial N}\right|_{C=C_0} = AC_0^{\alpha}\beta N^{\beta-1}\frac{\partial}{\partial N}\left[\exp\left\{X(C_0,N)\right\}\right] \tag{54}$$

where $X(C,N)$ is the exponential term in the original function:

$$X(C,N) = \frac{1}{2}\gamma_1\left[\ln(C)\right]^2 + \frac{1}{2}\gamma_2\left[\ln(N)\right]^2 + \gamma_{12}\ln(C)\ln(N) \tag{55}$$

Applying the chain rule to the exponential component and combining terms yields the following expressions for the marginal products, which can be compactly written using the output elasticities derived in Section 7.1.2:

$$f_C = \frac{Y}{C}\left(\alpha + \gamma_1\ln(C) + \gamma_{12}\ln(N_0)\right) = \frac{Y}{C}\cdot OE_C \tag{56}$$

$$f_N = \frac{Y}{N}\left(\beta + \gamma_2\ln(N) + \gamma_{12}\ln(C_0)\right) = \frac{Y}{N}\cdot OE_N \tag{57}$$

7.1.2 Output Elasticities (OEs)

$$\ln Y = \ln A + \alpha\ln C + \beta\ln N + \frac{1}{2}\gamma_1(\ln C)^2 + \frac{1}{2}\gamma_2(\ln N)^2 + \gamma_{12}\ln C\ln N \tag{58}$$

To derive output elasticities, we take the partial derivatives of $\ln(Y)$ with respect to $\ln(C)$ and $\ln(N)$:

For C:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N=N_0} \tag{59}$$

$$= \frac{\partial}{\partial \ln(C)}\left[\ln A + \alpha \ln C + \beta \ln N_0 + \frac{1}{2}\gamma_1(\ln C)^2 + \frac{1}{2}\gamma_2(\ln N_0)^2 + \gamma_{12} \ln C \ln N_0\right] \tag{60}$$

$$= \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N_0) \tag{61}$$

For N:

$$OE_N = \left.\frac{\partial \ln(Y)}{\partial \ln(N)}\right|_{C=C_0} \tag{62}$$

$$= \frac{\partial}{\partial \ln(N)}\left[\ln A + \alpha \ln C_0 + \beta \ln N + \frac{1}{2}\gamma_1(\ln C_0)^2 + \frac{1}{2}\gamma_2(\ln N)^2 + \gamma_{12} \ln C_0 \ln N\right] \tag{63}$$

$$= \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C_0) \tag{64}$$

Therefore, the output elasticities are:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N=N_0} = \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N_0) \tag{65}$$

$$OE_N = \left.\frac{\partial \ln(Y)}{\partial \ln(N)}\right|_{C=C_0} = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C_0) \tag{66}$$

7.1.3 Advantages

The Translog model provides a more flexible functional form compared to the Cobb-Douglas models discussed in previous sections, which allows us to capture non-linear relationships and interactions between cognitive and noncognitive inputs that were not possible in simpler specifications. Some of the key-features of these output elasticities are: variable elasticities, input interactions, and non-linearity. Unlike in the Cobb-Douglas model where elasticities are constant, in the Translog model, elasticities vary with the levels of inputs. The cross-term γ_{12} reflects how the marginal effect of one input depends on the level of the other input: $\gamma_{12} \ln(N)$ appears in OE_C, and $\gamma_{12} \ln(C)$ appears in OE_N.

These features make the Translog function useful for modeling non-obvious educational production processes where the impacts of cognitive and noncognitive skills may vary at different levels and interact with each other.

7.1.4 Elasticity of Substitution (ES)

The elasticity of substitution for our translog function is derived from the Marginal Rate of Technical Substitution (MRTS):

$$\sigma = 1 - \frac{\partial \ln(MRTS)}{\partial \ln(C/N)} \tag{67}$$

The MRTS is defined as the ratio of marginal products:

$$MRTS = \frac{\partial Y/\partial C}{\partial Y/\partial N} = \frac{OE_C}{OE_N} \cdot \frac{N}{C} \tag{68}$$

Taking the natural logarithm:

$$\ln(MRTS) = \ln(OE_C) - \ln(OE_N) + \ln(N) - \ln(C) \tag{69}$$

The elasticity of substitution is obtained by differentiating $\ln(MRTS)$ with respect to $\ln(C/N)$, as follows:

a) First, we differentiate $\ln(MRTS)$ with respect to $\ln(C/N)$:

$$\frac{\partial \ln(MRTS)}{\partial \ln(C/N)} = \frac{\partial}{\partial \ln(C/N)}[\ln(OE_C) - \ln(OE_N) + \ln(N) - \ln(C)] \tag{70}$$

b) Using the chain rule and noting that:

$$\frac{\partial \ln(OE_C)}{\partial \ln(C)} = \frac{\gamma_1}{OE_C} \quad \frac{\partial \ln(OE_C)}{\partial \ln(N)} = \frac{\gamma_{12}}{OE_C} \quad \frac{\partial \ln(OE_N)}{\partial \ln(C)} = \frac{\gamma_{12}}{OE_N} \quad \frac{\partial \ln(OE_N)}{\partial \ln(N)} = \frac{\gamma_2}{OE_N} \tag{71}$$

c) Substituting and collecting terms:

$$\frac{\partial \ln(MRTS)}{\partial \ln(C/N)} = \left(\frac{OE_C + OE_N - \gamma_{12}\left(\frac{OE_C}{OE_N} + \frac{OE_N}{OE_C}\right)}{OE_C + OE_N} \right) - 1 \tag{72}$$

Finally, substituting into the original formula:

$$\sigma = 1 - \left[\frac{OE_C + OE_N - \gamma_{12}(\frac{OE_C}{OE_N} + \frac{OE_N}{OE_C})}{OE_C + OE_N} - 1 \right] \tag{73}$$

Which simplifies to our final expression:

$$\sigma = \frac{OE_C + OE_N}{OE_C + OE_N - \gamma_{12}\left(\frac{OE_C}{OE_N} + \frac{OE_N}{OE_C}\right)} \tag{74}$$

Where the output elasticities are:

$$OE_C = \alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N) \tag{75}$$

$$OE_N = \beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C) \tag{76}$$

This formulation captures three key features:

- **Total productivity**: The numerator $(OE_C + OE_N)$ measures combined skill contribution
- **Skill interaction**: The γ_{12} term captures complementarity effects
- **Skill balance**: The ratios $\frac{OE_C}{OE_N}$ and $\frac{OE_N}{OE_C}$ reflect relative skill intensity

The substitutability condition is:

$$\gamma_{12} \left(\frac{OE_C}{OE_N} + \frac{OE_N}{OE_C} \right) > 0 \tag{77}$$

When this holds, cognitive and noncognitive skills exhibit diminishing substitutability in educational achievement production.

7.1.5 Marginal Rate of Technical Substitution (MRTS)

1. By definition, $MRTS_{CN} = \frac{f_C}{f_N}$
2. We know that $f_C = OE_C \cdot \frac{Y}{C}$ and $f_N = OE_N \cdot \frac{Y}{N}$
3. Therefore:

$$MRTS_{CN} = \frac{f_C}{f_N} = \frac{OE_C \cdot \frac{Y}{C}}{OE_N \cdot \frac{Y}{N}} = \frac{OE_C}{OE_N} \cdot \frac{N}{C} \tag{78}$$

4. Substituting the expressions for OE_C and OE_N:

$$MRTS_{CN} = \frac{\alpha + \gamma_1 \ln(C) + \gamma_{12} \ln(N)}{\beta + \gamma_2 \ln(N) + \gamma_{12} \ln(C)} \cdot \frac{N}{C} \tag{79}$$

7.2 General Form: CES with Two Inputs

$$Y = A \left[\alpha C^\rho + (1 - \alpha) N^\rho \right]^{\frac{1}{\rho}} \tag{80}$$

Where:

$$\begin{aligned}
Y &: \text{Total output/Grade function/Academic achievement} \\
A &: \text{Total factor productivity or scaling factor} \\
\alpha &: \text{Share parameter for cognitive input} \\
\rho &: \text{Substitution parameter, where } \rho = \frac{\sigma - 1}{\sigma} \\
\sigma &: \text{Elasticity of substitution} \\
C &: \text{Input representing cognition} \\
N &: \text{Input representing noncognitive measure}
\end{aligned}$$

7.2.1 Elasticity of Substitution

The elasticity of substitution (σ) measures how easily cognitive and noncognitive inputs can be substituted for each other. The relationship between σ and ρ governs how easily the two inputs can substitute for each other:

- When $\sigma > 1$ (or $-1 < \rho < \infty$), cognitive and noncognitive inputs are substitutes.
- When $\sigma < 1$ (or $\rho < -1$), cognitive and noncognitive inputs are complements.
- As σ approaches infinity (or ρ approaches 1), the inputs become perfect substitutes.
- As σ approaches 0 (or ρ approaches $-\infty$), the inputs become perfect complements.
- When $\sigma = 1$ (or equivalently, $\rho = 0$), the CES collapses to the Cobb-Douglas form.

In the context of educational production, these links clarify how cognitive and noncognitive skills interact to shape academic outcomes. For example, when $\sigma > 1$, a deficiency in one skill type can be more easily compensated by the other.

7.2.2 Marginal Products (MPs)

$$f_C = \frac{\partial Y}{\partial C} = A\alpha C^{\rho-1} \left[\alpha C^{\rho} + (1-\alpha)N^{\rho}\right]^{\frac{1}{\rho}-1} \tag{81}$$

$$f_N = \frac{\partial Y}{\partial N} = A(1-\alpha)N^{\rho-1} \left[\alpha C^{\rho} + (1-\alpha)N^{\rho}\right]^{\frac{1}{\rho}-1} \tag{82}$$

7.2.3 Output elasticities (OEs)

$$OE_C = \frac{\partial \ln(Y)}{\partial \ln(C)} = \frac{\alpha C^{\rho}}{\alpha C^{\rho} + (1-\alpha)N^{\rho}} \tag{83}$$

$$OE_N = \frac{\partial \ln(Y)}{\partial \ln(N)} = \frac{(1-\alpha)N^\rho}{\alpha C^\rho + (1-\alpha)N^\rho} \tag{84}$$

The sum of output elasticities would still be 1, indicating constant returns to scale:

$$OE_C + OE_N = \frac{\alpha C^\rho + (1-\alpha)N^\rho}{\alpha C^\rho + (1-\alpha)N^\rho} = 1 \tag{85}$$

This holds regardless of the values of C, N, α, or ρ, confirming that the CES function exhibits constant returns to scale by construction.

7.2.4 Returns to scale

The interpretation of constant returns to scale remains the same as in the three-input case: a proportional increase in both cognitive and noncognitive inputs leads to an equivalent proportional increase in the educational output.

The degree of returns to scale is determined by the sum of all output elasticities. We can call this sum the scale elasticity (SE):

$$SE = OE_C + OE_N \tag{86}$$

Then:
a) If $SE > 1$ = Increasing returns to scale;
b) If $SE < 1$ = Decreasing returns to scale;
c) If $SE = 1$ = Constant returns to scale.
For:

$$Y = A\left[\alpha C^\rho + (1-\alpha)N^\rho\right]^{\frac{1}{\rho}} \tag{87}$$

The sum of the output elasticities is always 1, regardless of the parameter values:

$$OE_C + OE_N = \frac{\alpha C^\rho + (1-\alpha)N^\rho}{\alpha C^\rho + (1-\alpha)N^\rho} = 1 \tag{88}$$

the 2-input CES function exhibits constant returns to scale by construction. This is a property of the CES function with the exponent $\frac{1}{\rho}$ outside the brackets. In the context of cognition and noncognition as inputs in an educational production function, it means that if both inputs are scaled by a constant factor k, then output Y increases proportionally by the same factor. More specifically:

1. Proportional increase in inputs:

$$C \to kC, \quad N \to kN \tag{89}$$

2. Resulting increase in output:

$$Y(kC, kN) = kY(C, N) \tag{90}$$

In practical terms for education, this means a proportional improvement in cognitive and noncognitive skills leads to an equivalent proportional improvement in educational outcomes.

If we were to allow for different returns to scale, we could modify the CES function to:

$$Y = A\left[\alpha C^\rho + (1-\alpha)N^\rho\right]^{\frac{\nu}{\rho}} \tag{91}$$

Where ν is a new parameter that determines the overall returns to scale:
a) If $\nu > 1$ = Increasing returns to scale;
b) If $\nu < 1$ = Decreasing returns to scale;
c) If $\nu = 1$ = Constant returns to scale (current case).

7.2.5 Marginal Rate of Technical Substitution (MRTS)

For the two-input CES production function:

$$Q = A\left(\alpha C^\rho + (1-\alpha)N^\rho\right)^{\frac{1}{\rho}} \tag{92}$$

The marginal products are:

$$f_C = \frac{\partial Q}{\partial C} = A\alpha\left(\alpha C^\rho + (1-\alpha)N^\rho\right)^{\frac{1}{\rho}-1} C^{\rho-1} \tag{93}$$

$$f_N = \frac{\partial Q}{\partial N} = A(1-\alpha)\left(\alpha C^\rho + (1-\alpha)N^\rho\right)^{\frac{1}{\rho}-1} N^{\rho-1} \tag{94}$$

$$MRTS_{CN} = \frac{f_C}{f_N} = \frac{\alpha C^{\rho-1}}{(1-\alpha)N^{\rho-1}} = \frac{\alpha}{1-\alpha}\left(\frac{C}{N}\right)^{\rho-1} \tag{95}$$

In education terms, this shows how much noncognitive skill is needed to replace one unit of cognition (or vice versa), depending on their relative levels and substitutability.

7.2.6 Isoquants

Isoquants for the two-input CES production function represent combinations of C and N that produce the same level of output Y. For the two-input case, we can represent isoquants as follows:

1. Equation form:

For a given output level Y_0, the isoquant is represented by:

$$Y_0 = A\left[\alpha C^\rho + (1-\alpha)N^\rho\right]^{1/\rho} \tag{96}$$

This can be rearranged to express N in terms of C:

$$N = \left[\frac{(Y_0^\rho/A^\rho) - \alpha C^\rho}{1-\alpha}\right]^{1/\rho} \tag{97}$$

2. Graphical representation:

   In the two-dimensional space of C and N, each isoquant is a curve representing all combinations of cognitive and noncognitive inputs that produce the same level of output Y_0.

The shape of the isoquants reflects the substitutability between cognitive and noncognitive inputs:

- As ρ approaches 1 (or σ approaches infinity), the isoquants become more linear, indicating that C and N are close to perfect substitutes.
- As ρ approaches negative infinity (or σ approaches 0), the isoquants approach right angles, indicating that C and N are close to perfect complements.
- When $\rho = 0$ (or $\sigma = 1$), the isoquants take on the familiar Cobb-Douglas shape.

These isoquant properties help visualize how cognitive and noncognitive skills can be substituted in different proportions to achieve the same academic outcome.

7.3 Limitations

While the two-input CES model further enhances our understanding of the connection between cognitive and noncognitive skills in educational production, I have to note it has some limitations. Regarding noncognitive skill selection, the model treats noncognition as a single aggregate input, which may oversimplify its multidimensional nature. The assumption of constant elasticity of substitution may not hold uniformly across different levels of input use. Other factors that influence educational outcomes, such as family background or school quality, are not explicitly included in this model (but are in the regressions).

7.4 General Form: CES with Three Inputs

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}} \tag{98}$$

Where:

$$\begin{aligned}
Y &: \text{Total output/Grade function/Academic achievement} \\
A &: \text{Total factor productivity or scaling factor} \\
\alpha &: \text{Share parameter for cognitive input} \\
\beta &: \text{Share parameter for noncognitive inputs} \\
\rho &: \text{Substitution parameter, where } \rho = \frac{\sigma - 1}{\sigma} \quad (i.e., \sigma = \frac{1}{1-\rho}) \\
\sigma &: \text{Elasticity of substitution} \\
C &: \text{Input representing cognition} \\
N_E, N_I &: \text{Inputs representing noncognitive measures}
\end{aligned}$$

This is a three-input Constant Elasticity of Substitution (CES) production function, which generalizes the two-input case by allowing differentiated treatment of multiple noncognitive dimensions. The CES function is more flexible than the Cobb-Douglas form, allowing for varying degrees of substitutability between inputs. The elasticity of substitution (σ) between any pair of inputs is constant and determined by the parameter ρ.

In this model, C represents a measure of cognition (in this case, the principal component as a composite of three cognitive measures). N_E and N_I represent noncognitive measures, which I call External Control and Internal Control, respectively. In relation to the scales used (TIPI and SDQ), Internal Control proxies Focused Behaviour (SDQ) and Conscientiousness (TIPI), while External Control captures Emotional Resilience (SDQ) and Emotional Stability (TIPI). These four variables appear to be the most significant based on my analysis.

The share parameters α and β determine the relative importance of the inputs in the production function. However, unlike in a Cobb-Douglas function, they do not directly determine output elasticities, which vary with the levels of input usage in the CES model.

7.4.1 Elasticity of Substitution

The elasticity of substitution (σ) in the three-input CES model measures the ease of substitution between any pair of inputs while holding the third input constant. The relationship between σ and ρ is:

- When $\sigma > 1$ (or $-1 < \rho < \infty$), any pair of inputs are substitutes.

- When $\sigma < 1$ (or $\rho < -1$), any pair of inputs are complements.
- As σ approaches infinity (or ρ approaches 1), the inputs become perfect substitutes.
- As σ approaches 0 (or ρ approaches $-\infty$), the inputs become perfect complements.
- When $\sigma = 1$ (or $\rho = 0$), the CES function reduces to the Cobb-Douglas form.

In the context of educational production with cognitive (C), external noncognitive (N_E), and internal noncognitive (N_I) inputs, these relationships indicate how these different skills interact in producing educational outcomes. For example:

- If $\sigma > 1$, a deficiency in one type of skill (e.g., cognitive) can be more easily compensated by either of the other skills.
- If $\sigma < 1$, it implies that all three types of skills are complementary, and a balanced development of all skills is important for educational outcomes.

Although σ reflects the overall substitutability, the actual trade-offs between input pairs (e.g., C vs. N_E) depend on both the parameter values and the relative levels of the inputs. In the proposed model, I assume a constant elasticity of substitution between all input pairs, which is a simplification of potentially non-linear relationships in real-life educational production.

While this assumption improves tractability and interpretability, it is worth noting that more flexible nested CES forms allow for different elasticities of substitution between input pairs. For example, the elasticity between cognitive and noncognitive inputs could differ from that between the two noncognitive dimensions. Such models offer richer behavioural insights but also involve substantially greater complexity and identification challenges. For this analysis, I maintain a constant σ to preserve parsimony and enable clearer comparisons across specifications.

7.4.2 Marginal products (MPs)

$$f_C = \left.\frac{\partial Y}{\partial C}\right|_{N_E=N_{E0}, N_I=N_{I0}} = A\alpha C^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_{E0}^{\rho} + (1-\beta)N_{I0}^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{99}$$

$$f_{NE} = \left.\frac{\partial Y}{\partial N_E}\right|_{C=C_0, N_I=N_{I0}} = A(1-\alpha)\beta N_E^{\rho-1}\left[\alpha C_0^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_{I0}^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{100}$$

$$f_{NI} = \left.\frac{\partial Y}{\partial N_I}\right|_{C=C_0, N_E=N_{E0}} = A(1-\alpha)(1-\beta) N_I^{\rho-1}\left[\alpha C_0^{\rho} + (1-\alpha)\left(\beta N_{E0}^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{101}$$

7.4.3 Output elasticities (OEs)

Given:

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}} \tag{102}$$

When we take the log on both sides:

$$\ln(Y) = \ln(A) + \frac{1}{\rho}\ln\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right] \tag{103}$$

This yields the following output elasticities for each input:

$$OE_C = \left.\frac{\partial \ln(Y)}{\partial \ln(C)}\right|_{N_E=N_{E0},N_I=N_{I0}} = \frac{\alpha C^{\rho}}{\alpha C^{\rho} + (1-\alpha)\left(\beta N_{E0}^{\rho} + (1-\beta)N_{I0}^{\rho}\right)} \tag{104}$$

$$OE_{N_E} = \left.\frac{\partial \ln(Y)}{\partial \ln(N_E)}\right|_{C=C_0,N_I=N_{I0}} = \frac{(1-\alpha)\beta N_E^{\rho}}{\alpha C_0^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_{I0}^{\rho}\right)} \tag{105}$$

$$OE_{N_I} = \left.\frac{\partial \ln(Y)}{\partial \ln(N_I)}\right|_{C=C_0,N_E=N_{E0}} = \frac{(1-\alpha)(1-\beta) N_I^{\rho}}{\alpha C_0^{\rho} + (1-\alpha)\left(\beta N_{E0}^{\rho} + (1-\beta)N_I^{\rho}\right)} \tag{106}$$

Step-by-step derivation:

1) First we take the natural logarithm:

$$\ln(Y) = \ln(A) + \frac{1}{\rho}\ln\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right] \tag{107}$$

2) Then we derive the output elasticities one by one:

For Cognitive input (C):

$$OE_C = \frac{\partial \ln(Y)}{\partial \ln(C)} = \frac{\alpha C^{\rho}}{\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)} \tag{108}$$

For External Noncognitive input (N_E):

$$OE_{N_E} = \frac{\partial \ln(Y)}{\partial \ln(N_E)} = \frac{(1-\alpha)\beta N_E^{\rho}}{\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)} \tag{109}$$

For Internal Noncognitive input (N_I):

$$OE_{N_I} = \frac{\partial \ln(Y)}{\partial \ln(N_I)} = \frac{(1-\alpha)(1-\beta) N_I^{\rho}}{\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)} \tag{110}$$

7.4.4 Returns to scale

The degree of returns to scale is determined by the sum of all output elasticities. We can call this sum the scale elasticity (SE):

$$SE = OE_C + OE_{N_E} + OE_{N_I} \tag{111}$$

Then:
a) If $SE > 1$ = Increasing returns to scale;
b) If $SE < 1$ = Decreasing returns to scale;
c) If $SE = 1$ = Constant returns to scale.
For:

$$Y = A\left[\alpha C^\rho + (1-\alpha)\left(\beta N_E^\rho + (1-\beta)N_I^\rho\right)\right]^{\frac{1}{\rho}} \tag{112}$$

The sum of the output elasticities is always 1, regardless of the parameter values:

$$OE_C + OE_{N_E} + OE_{N_I} = \frac{\alpha C^\rho + (1-\alpha)\beta N_E^\rho + (1-\alpha)(1-\beta)N_I^\rho}{\alpha C^\rho + (1-\alpha)\left(\beta N_E^\rho + (1-\beta)N_I^\rho\right)} = 1 \tag{113}$$

This 3-input CES function exhibits constant returns to scale by construction. This is a property of the CES function with the exponent $\frac{1}{\rho}$ outside the brackets. In the context of cognition and noncognition as inputs in an educational production function, it means that if we increase all three inputs by a factor k then output Y increases proportionally by the same factor. More specifically:

1. Proportional increase in inputs:

$$C \to kC, \quad N_E \to kN_E, \quad N_I \to kN_I \tag{114}$$

2. Resulting increase in output:

$$Y(kC, kN_E, kN_I) = kY(C, N_E, N_I) \tag{115}$$

In practical terms for education, this means a proportional improvement in cognitive and noncognitive skills leads to an equivalent proportional improvement in educational outcomes. For example, if we could somehow double (k = 2) a student's cognitive ability (C) and both types of noncognitive abilities (N_E and N_I) simultaneously, we would expect their educational output (Y, measured by test scores as a proxy for overall academic performance) to also double. This implies:

a) No diminishing returns when scaling up all inputs equally;
b) No extra benefits (increasing returns) when scaling up all inputs equally.

We need to keep in mind that this is a simplification of a sophisticated reality. In practice, the links between cognitive abilities, noncognitive skills, and educational outcomes is more nuanced and also most-likely non-linear, as we have seen in previous chapters.

If we were to allow for different returns to scale, we could generalize the CES form to allow variable returns to scale:

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{\nu}{\rho}} \tag{116}$$

Where ν is a new parameter that determines the overall returns to scale:
a) If $\nu > 1$ = Increasing returns to scale;
b) If $\nu < 1$ = Decreasing returns to scale;
c) If $\nu = 1$ = Constant returns to scale (current case).

7.4.5 Marginal Rate of Technical Substitution for Three-Input CES

For the three-input CES production function:

$$Q = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}} \tag{117}$$

The marginal products are:

$$f_C = \frac{\partial Q}{\partial C} = A\alpha C^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{118}$$

$$f_{N_E} = \frac{\partial Q}{\partial N_E} = A(1-\alpha)\beta N_E^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{119}$$

$$f_{N_I} = \frac{\partial Q}{\partial N_I} = A(1-\alpha)(1-\beta)N_I^{\rho-1}\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}-1} \tag{120}$$

The MRTS can be calculated for each pair of inputs. Holding the third input constant, these expressions quantify how much of one input is required to offset a marginal decrease in another while maintaining the same output level:

1. Between cognitive (C) and external noncognitive (N_E) inputs:

$$MRTS_{C,N_E} = \frac{f_C}{f_{N_E}} = \frac{\alpha}{\beta(1-\alpha)}\left(\frac{N_E}{C}\right)^{1-\rho} \tag{121}$$

2. Between cognitive (C) and internal noncognitive (N_I) inputs:

$$MRTS_{C,N_I} = \frac{f_C}{f_{N_I}} = \frac{\alpha}{(1-\beta)(1-\alpha)}\left(\frac{N_I}{C}\right)^{1-\rho} \tag{122}$$

3. Between external noncognitive (N_E) and internal noncognitive (N_I) inputs:

$$MRTS_{N_E,N_I} = \frac{f_{N_E}}{f_{N_I}} = \frac{\beta}{1-\beta}\left(\frac{N_I}{N_E}\right)^{1-\rho} \tag{123}$$

These MRTS formulas demonstrate how the substitutability between each pair of inputs changes with their relative quantities and the elasticity of substitution parameter ρ. We can analyze the trade-offs between any two of the three inputs while holding the third constant.

For example, $MRTS_{C,N_E}$ shows how much external noncognitive input (N_E) is needed to compensate for a small decrease in cognitive input (C) while maintaining the same output level and holding internal noncognitive input (N_I) constant. The substitutability is governed by the share parameters (α, β), the substitution parameter (ρ), and the current relative input levels.

7.4.6 Isoquants

Isoquants represent all input combinations that yield the same level of output (Y). For the three-input CES function, these curves-or surfaces-illustrate the substitution possibilities among C, N_E, and N_I. Due to the three-dimensional nature of the input space, we can represent isoquants in a few ways:

1. Two-dimensional representation:

   Fixing C at a level C_0, we can represent the isoquant for output level Y_0 as:

   $$N_I = \left[\frac{(Y_0^\rho/A^\rho - \alpha C_0^\rho)}{(1-\alpha)} - \beta N_E^\rho\right]^{1/\rho} /(1-\beta)^{1/\rho} \tag{124}$$

   This expression defines the isoquant curve in the (N_E, N_I) plane for a fixed cognitive level C_0 and output level Y_0.

2. Three-dimensional representation:

   The full isoquant surface for output level Y_0 is given by:

   $$Y_0 = A\left[\alpha C^\rho + (1-\alpha)(\beta N_E^\rho + (1-\beta)N_I^\rho)\right]^{1/\rho} \tag{125}$$

   This surface in (C, N_E, N_I) space represents all combinations of inputs producing output Y_0.

The shape of the isoquants reflects the substitutability between inputs. As ρ approaches 1 (perfect substitutes), the isoquants become more linear. As ρ approaches negative infinity (perfect complements), the isoquants approach right angles.

7.5 Comparison Between Two-Inputs and Three-Inputs CES Models

The choice between the two-input and three-input CES models involves several trade-offs, and was temporarily set aside due to empirical (primarily computational) limitations:

Simplicity vs. complexity: The two-input model offers greater simplicity and ease of interpretation, making it more suitable for theoretical analysis and empirical estimation. However, the three-input model provides a more nuanced representation of noncognitive skills-distinguishing between internal and external control-and enables the researcher to select the most relevant inputs. If better measures existed on a unified scale, cross-model comparisons would be more straightforward. Currently, differences between the TIPI and SDQ scales constrain the use of principal component analysis.

Parsimony vs. granularity: The two-input model is more parsimonious, requiring fewer parameters to estimate (one for cognitive and one for noncognitive input). This is advantageous when working with limited or noisy data, especially for noncognitive variables. The three-input model, although more complex, offers greater granularity in modeling educational production processes. Both have merits and limitations, and the choice should depend on the research context and goals.

Generalizability vs. specificity: The two-input model may be more generalizable across settings where noncognitive dimensions are not easily separable. In contrast, the three-input model is more suitable for contexts where distinct aspects of noncognition-such as internal and external control-are meaningfully identified and measured.

Ultimately, the choice depends on the research question, data quality, and patience for experimenting with different optimization routines when setting starting values for estimation.

7.5.1 Nested CES function

While the previous analysis focuses on the standard CES function, it is worth noting the possibility of using a nested CES function, particularly for the three-input case. The first specification was:

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\rho} + (1-\beta)N_I^{\rho}\right)\right]^{\frac{1}{\rho}} \tag{126}$$

While a nested CES function could look like:

$$Y = A\left[\alpha C^{\rho} + (1-\alpha)\left(\beta N_E^{\gamma} + (1-\beta)N_I^{\gamma}\right)^{\rho/\gamma}\right]^{\frac{1}{\rho}} \tag{127}$$

This nested specification allows for different elasticities of substitution between cognitive and noncognitive inputs (determined by ρ) and between the two types of noncognitive inputs (determined by γ).

This additional parameter, γ, allows for different elasticities of substitution between inputs:

- ρ determines the elasticity of substitution between cognitive skills (C) and the composite of noncognitive skills (N_E and N_I).
- γ determines the elasticity of substitution between the two types of noncognitive skills (N_E and N_I).
- When $\gamma > \rho$, the two noncognitive inputs (N_E and N_I) are more substitutable with each other than either is with the cognitive input (C).
- When $\gamma < \rho$, the noncognitive inputs are less substitutable with each other than with the cognitive input.
- When $\gamma = \rho$, the nested CES function reduces to the standard three-input CES function.

This nested structure is more interesting at first because it allows for detailed modeling of the links between different types of skills. For example, it can capture scenarios where external and internal noncognitive skills might be more easily substituted for each other than either can be for cognitive skills.

In educational terms, a high γ relative to ρ might suggest that deficiencies in one type of noncognitive skill (e.g., external control) can be more easily compensated by strengths in the other noncognitive skill (e.g., internal control) than by cognitive abilities.

However, while this alternative model could capture more nuanced links between the inputs, it would be even more computationally complex to estimate, and the researcher would have to justify the choice of variables.

7.5.2 Economic interpretation of parameters

The parameters in the previous CES models have important economic interpretations in the context of educational production:

- α (and β in the three-input case) govern the relative weighting of inputs in the production process. A higher α indicates that cognitive skills have a greater weight relative to noncognitive skills in determining educational outcomes. These parameters reflect the underlying production technology.
- ρ, which maps one-to-one to the elasticity of substitution σ via $\sigma = 1/(1-\rho)$, governs the degree of substitutability between inputs. In educational terms, it reflects how easily a deficiency in one type of skill can be compensated by strength

in another. A higher σ reflects greater flexibility in combining different skills to achieve educational outcomes.

- A represents total factor productivity, which in an educational context might reflect the overall effectiveness of the educational system or other factors that affect all students equally.

7.5.3 Policy implications

The insights from these CES models can inform educational policy in several ways. Starting with the elasticity of substitution (σ), if it is high, policies might focus on developing students' strengths, as deficiencies in one area can be more easily compensated by strengths in another. On the other hand, if σ is low, a more balanced approach to skill development might be necessary, as weaknesses in one area could significantly hinder overall educational outcomes.

The relative magnitudes of α and β can help guide resource allocation. For example, if we find that α is much larger than $(1-\alpha)$, we should prioritize cognitive skill development, always keeping in mind the timing of interventions. Cognition is mostly genetics and proper care during pregnancy and infancy, whereas noncognitive skills can be taught to some extent at any time during the school years.

The returns to scale properties inform us whether policies should focus on "broad-based" improvement of all skills or targeted interventions in specific areas for specific groups (like boys and girls) at specific times, for specific periods.

Research supports my theory that while cognitive skills are more heavily influenced by early childhood experiences and genetics, noncognitive skills remain relatively malleable throughout life. Cognitive skills are significantly impacted by genetics and early childhood experiences. Critical periods for cognitive development occur primarily in early childhood, though some plasticity remains throughout life (Knudsen et al., 2006). Noncognitive skills can be developed and refined throughout the lifespan, including during school years and adulthood (Kautz et al., 2014). This malleability makes noncognitive skills an attractive target for interventions at various life stages.

The early childhood (0-5 years) period is vital for cognitive development, although the brain retains some plasticity throughout life. Certain cognitive skills are more easily developed during early childhood (Knudsen et al., 2006). This fact emphasises the importance of early interventions such as the Perry Preschool Project (Schweinhart et al., 2005) and the Abecedarian Project (Campbell et al., 2012), which demonstrated significant improvements in cognitive abilities and later life outcomes through high-quality preschool education.

While early childhood programs remain highly important for cognitive development, some programs targeting noncognitive skills can be effective at various ages, offering opportunities for improvement even later in the educational process. For example, the Chicago School Readiness Project showed improvements in both cognitive and noncognitive skills through preschool interventions (Raver et al., 2011).

The work of James Heckman and colleagues has demonstrated that early childhood interventions can have lasting effects on both cognitive and noncognitive skills, with noncognitive skills often being more malleable later in life (Heckman & Kautz, 2012). This malleability of noncognitive skills is further supported by research showing that social-emotional or character skills can be developed throughout life, including during school years and even adulthood (Kautz et al., 2014).

It is important to note that while genetics contribute to both cognitive and noncognitive development, the interaction between genes and environment (epigenetics) remains of utmost importance. Proper care during pregnancy and infancy matters for both cognitive and noncognitive development (Fox et al., 2010). Plus, interventions like nurse home visiting programs have been shown to improve cognitive outcomes for children from disadvantaged backgrounds (Olds et al., 2004).

All the aforementioned findings have significant implications for educational policy and practice. They suggest a two-pronged approach: intensive early interventions to support cognitive development, coupled with ongoing programs to foster noncognitive skills throughout the educational journey and beyond. Such a comprehensive strategy may offer the best opportunity to maximize human capital development and improve long-term outcomes for both individuals and society.

7.6 Empirical considerations

While the CES models provide a nice theoretical framework, their empirical application presented several challenges. The CES function is non-linear in its parameters, which requires non-linear estimation techniques. This has turned out to be computationally intensive and has led to convergence issues in some cases (where the scale starts at zero, for example).

Estimating the elasticity of substitution (σ) or the substitution parameter (ρ) was very challenging, especially because there was limited variation in the ratio of inputs across observations (the ratio of cognitive to noncognitive inputs varied little across students).

Correctly measuring cognitive and noncognitive skills is of extreme importance. Measurement errors can lead to biased estimates of the production function parameters. The fact that the ratings were provided by the Primary caregiver makes it an indirect measure.

As with many empirical models in education, there may be concerns about endogeneity of inputs. For example, higher achieving students might conscientiously (or just by genetic chance) choose to invest more in both cognitive and noncognitive skills.

Finally, the CES model imposes specific functional form assumptions that may not always align with the true underlying production process. After all, it is an attempt to approximate concepts through debatable measurement instruments. Educational achievement is not directly observable - it is approximated via noisy, test-based proxies for latent abilities. We are not measuring the mass of the electron, but the realization of internal processes through general tests. As always in education economics, robustness checks, sensitivity analyses, and theoretical justification are essential to support model-driven insights.