# Big Data and Economics

## Linear Model Selection and Regularization

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# Prologue

# Regressions

- What do we typically do when we run OLS?
- We run a regression with all the variables we think are important
- But what happens when we have more variables than observations?

# Too many variables

- Most of the analysis we have done in this class has focused on the case where we have a small number of variables relative to the number of observations.
- But sometimes you have LONG data
- In this case, you have a large number of variables $J$ relative to the number of observations $n$.
- If you try to use OLS with all the variables, you will run into problems. Why?

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- But sometimes you have LONG data
- In this case, you have a large number of variables $J$ relative to the number of observations $n$.
- If you try to use OLS with all the variables, you will run into problems. Why?
- The number of variables is larger than the number of observations!
- Uh oh

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- What would be the regression tree approach?
- Iteratively split training data using variables that minimize residual sum of squares and use test data to determine the optimal number of leaves
- This is a form of **variable selection**
- But it forces us to turn continuous data binary ($X > c$ vs. $Xgeqc$)
- But what other ways are available?

# Linear Model Selection

# Typical OLS

- Good old-fashioned regression minimizes the residual sum of squares (RSS)

$$\min_{\beta} \sum_{i=1}^{n} \underbrace{(y_i - \beta_0 - \sum_{j=1}^{k} \beta_j x_{ij})^2}_{\text{RSS}}$$

- What does that mean?

# Typical OLS

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- What does that mean?
- We are trying to find the $\beta$s that predict a dependent variable $\boldsymbol{y}$ as a linear combination of the independent variables $\boldsymbol{x}$.

# Adding dimensions with OLS

- Each additional variable $x_j$ adds a new dimension to the problem
  - As in each additional variable is a new axis in $J$-dimensional space where $J$ is the number of variables
  - (You've likely never thought about it that way before, but any regression is a multi-dimensional problem)
- If you have more variables than observations, you have more dimensions than observations
- Why? Well solve this equation:

$$x + y = 5$$

- How many solutions are there? Infinite
- Now solve this system of equations:

$$x + y = 5$$

$$x + 2y = 10$$

- The same logic applies to regression (though it is a bit more complicated)

# Ridge Regression

# Shrinkage

- In OLS, we are trying to minimize the residual sum of squares (RSS)
- In machine learning, there are shrinkage methods that add a penalty term to the RSS
    - These penalize coefficients that are too large

$$\min_{\beta} \sum_{i=1}^{n} \underbrace{\textbf{model fit}}_{\text{RSS}} + \textbf{penalty on size of coefficients}$$

- Why penalize large coefficients?
- Large coefficients are more likely to be overfitting the data since they are more sensitive to small changes in the data
    - By penalizing large coefficients, we are reducing the variance of the model and thus complexity
    - Intuitively, a larger $\beta$ the further your model is from a null hypothesis of $\beta = 0$, which is the simplest model
- What happens if we reduce bias in the data?

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- What happens if we reduce bias in the data?
- We increase variance!

# Ridge Regression

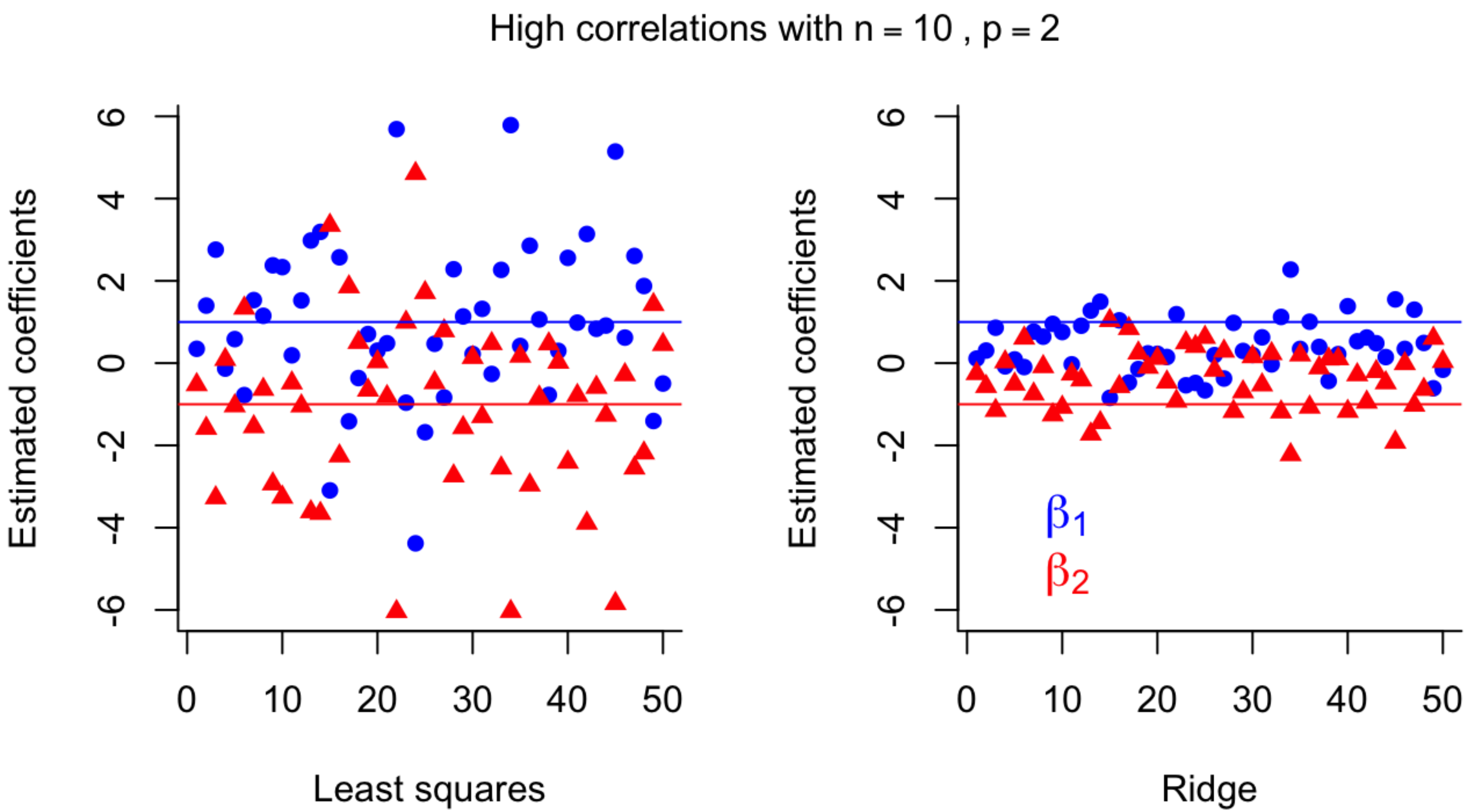
- So what form do these penalties take?
- Well Ridge Regression is one such example
- Ridge regression adds a penalty term to the RSS that is proportional to the sum of the squared coefficients
- Essentially, it adds a constraint to the optimization problem

$$\min \underbrace{\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{J}\beta_j x_{ij})}_{\text{model fit}} + \lambda \underbrace{\sum_{j=1}^{J}\beta_j^2}_{\text{penalty}} = RSS + \lambda \sum_{j=1}^{J}\beta_j^2$$

$\lambda$ is the "tuning parameter" that controls the strength of the penalty

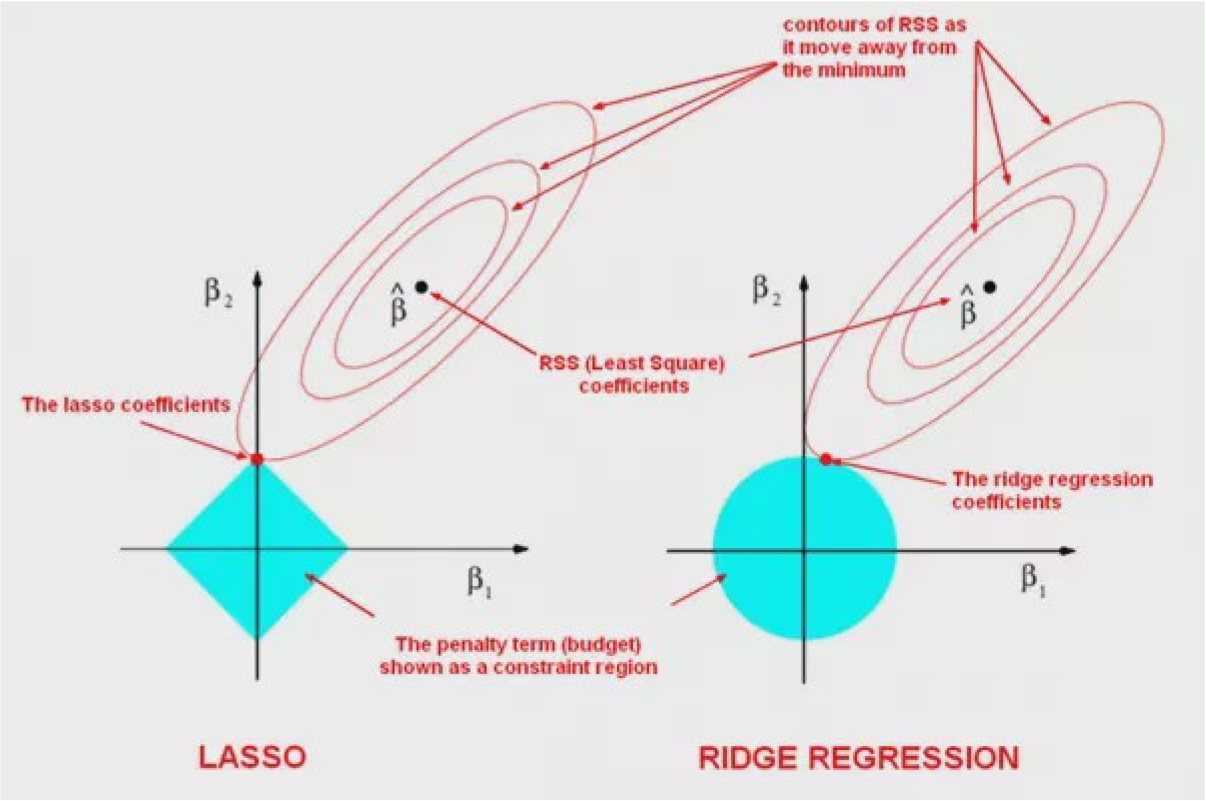
- In order to minimize, we need to find the $\beta$s that minimize the RSS and the penalty
- That means we need smaller $\beta$s -- and necessarily a simpler, less variable model
- Literally, we shrink the $\beta$s towards zero

# Ridge Regression




Example taken from Dr. Samuel E. Jackson online textbook

# Ridge Regression coefficients



# Ridge Regression flaws

- Ridge regression keeps all the variables in the model -- it just shrinks the coefficients
- But what if some variables are just truly noise
  - i.e. they are not correlated with the dependent variable
- Sure, we can check by hand, but shouldn't we just toss them?

# LASSO

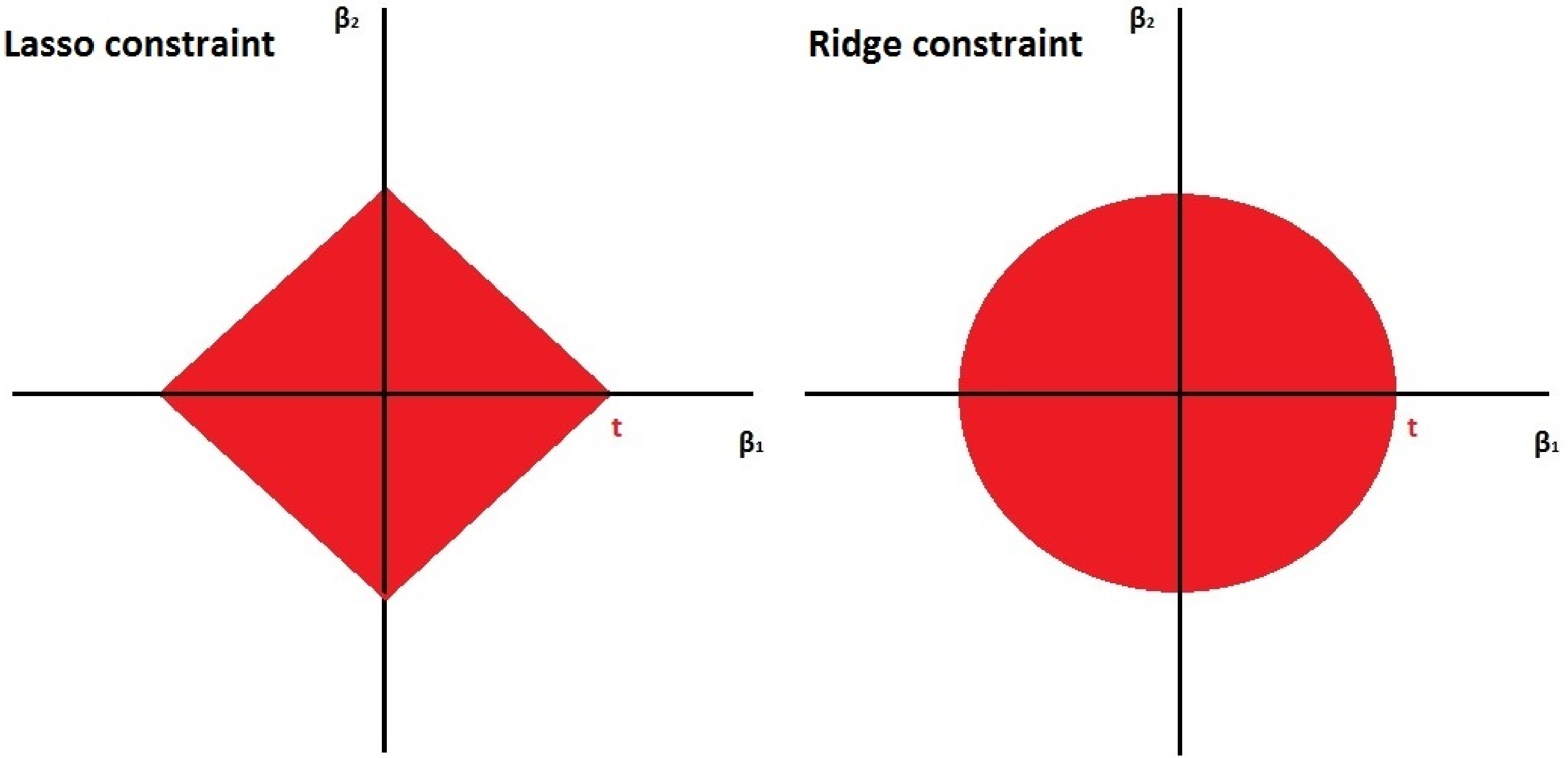
# LASSO

- LASSO stands for Least Absolute Shrinkage and Selection Operator
- It is another shrinkage method that adds a penalty term to the RSS
- But now the penalty term is proportional to the sum of the absolute value of the coefficients

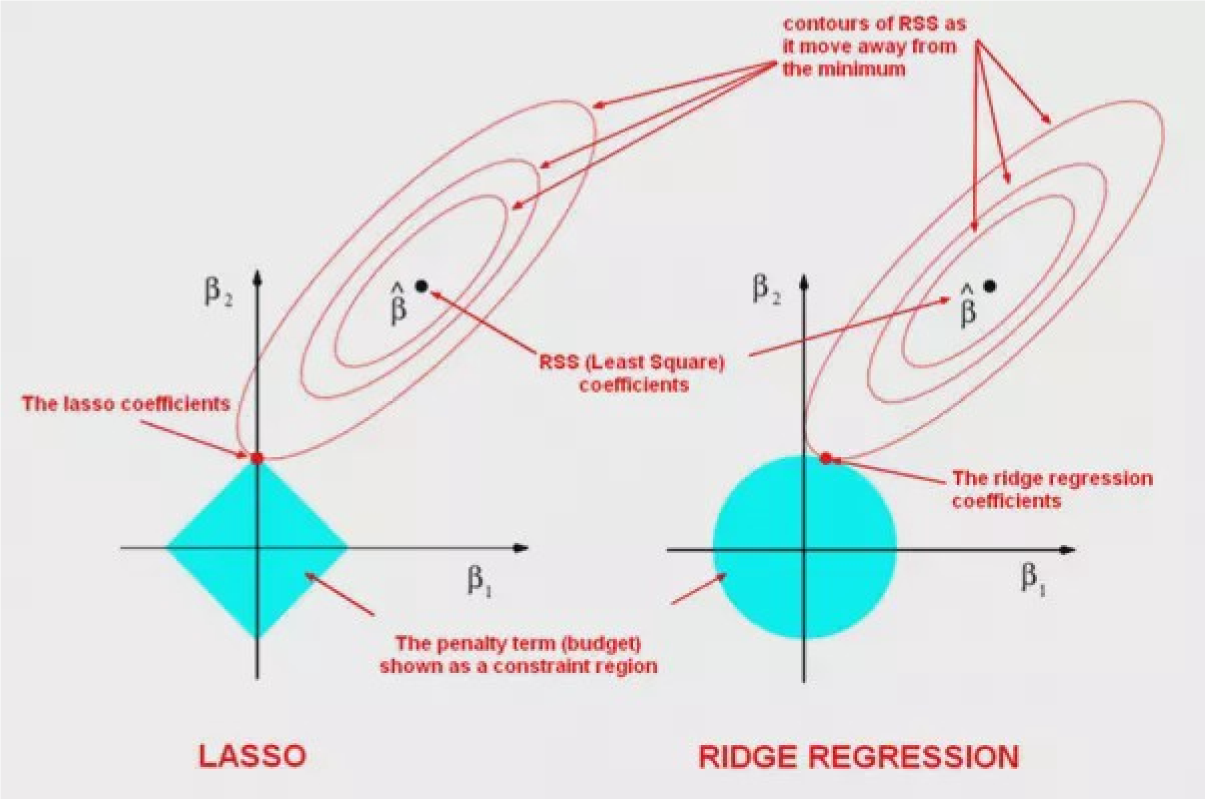
$$\min \underbrace{\sum_{i=1}^{n}(y_i - \beta_0 - \sum_{j=1}^{J}\beta_j x_{ij})}_{\text{model fit}} + \lambda \underbrace{\sum_{j=1}^{J}|\beta_j|}_{\text{penalty}} = RSS + \lambda \sum_{j=1}^{J}|\beta_j|$$

- So now instead of the squared penalty on the coefficient size, you have the absolute value of the penalty
- The magic of the absolute value is that it can shrink coefficients to zero with a sufficiently large $\lambda$
  - This means that LASSO can be used for variable selection -- a zero means that variable is not in the model
  - **Intuition**: The absolute value has a "sharp" corner at zero, so it can "cut" coefficients to zero, Ridge is a circle, so it can only shrink coefficients to the edge of the circle
- Selection is a big advantage over Ridge Regression
  - Of course, that can also be a disadvantage if you want to keep all the variables in the model
  - It leads to more bias

# LASSO visualization



# Ridge Regression coefficients



# Other details on Regularization

# K-fold cross-validation: How to pick $\lambda$

- The $\lambda$ in both of the examples above is a "tuning parameter," which controls the strength of the penalty

- You need to do $K$-fold cross-validation:

1. Choose the number of "folds" or groups, $K$ (usually 5 or 10)
2. Randomly split the data into $K$ folds
3. Create a grid of feasible $\lambda$ values to check
4. For each value of $\lambda$:
   - Run Ridge or LASSO on the $K-1$ folds
   - Calculate the $MSE_k$ on the remaining $k$-fold
5. Calculate the average $MSE_k$ for each $\lambda$ $$ MSE{CV}(\lambda) = \frac{1}{K} \sum{k=1}^K MSE_k(\lambda) $$
6. Pick the $\lambda$ with the lowest MSE

- You know what's neat? You can do this in R with the **glmnet** library!

- It will even plot the results for you, so you can see the optimal $\lambda$

# Drawbacks of LASSO and Ridge

- Regularization/coefficient shinkage are useful for reducing variance and overfitting
- But they can also lead to bias
- The more you shrink the coefficients, the more bias you introduce
- You are no longer finding the best linear unbiased estimator (BLUE) that you find with OLS
- Instead, you get the best linear biased estimator (BLBE) because you trade some bias for less variance
- Sometimes you're okay with that!

# Why are you okay with bias?

- Sometimes you don't mind being a little off in your predictions
- For example, if you are predicting the number of people who will show up to a party, you don't care if you are a little off
- You do care if someone tells you that between 0 and 100 people will show up, but 50 on average -- that's not helpful
- It is even less helpful if they tell you that to make am accurate prediction they need to know:
    - The number of invites
    - The weather
    - The day of the week
    - The time of day
    - The number of people who have already RSVP'd
    - The variety of chips you're serving
    - What is on TV that night
    - etc.
- You'd rather just have a simple model that is a little off then a complicated model that is super sensitive

# Warning

- Regularization is a useful tool for reducing variance and overfitting
- But just cause you can run a regression techniques doesn't mean you should
- You should always think about the problem you are trying to solve and the data you have
- Is it worth trying a technique?
- Will this technique help you solve your problem?
- Will it help you understand your data?
- Or are you just trying to seem flashy?

# Conclusion

- Regularization is a useful tool for reducing variance and overfitting
- It recognizes that sometimes you are okay with a little bias if it means you get less variance
- It relies on a tuning parameter $\lambda$ that controls the strength of the penalty from adding more complexity to a regression model
- LASSO can be used to select variables, while Ridge just reduces the magnitude of the coefficients

# What next?

- Try an activity: ISLR lab using tidymodels
- Before class: work through the lab sections on Ridge and LASSO in a .Rmd file that you create
- Write up short answers to the following questions:
    1. What are the coefficients in the Ridge and LASSO regressions when the penalty is zero? Why?
    2. How does tidymodels pick the optimal $\lambda$ in each method?
    3. What is the optimal $\lambda$ in Ridge and LASSO?

# Next lecture: Regular expressions and word clouds