Big Data and Economics

Regression analysis in R

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Today's lecture is about the bread-and-butter tool of applied econometrics and data science: regression analysis. My goal is to give you a whirlwind tour of the key functions and packages. I'm going to assume that you already know all of the necessary theoretical background on causal inference, asymptotics, etc. This lecture will *not* cover any of theoretical concepts or seek to justify a particular statistical model. Indeed, most of the models that we're going to run today are pretty silly. We also won't be able to cover some important topics. For example, we won't cover a Bayesian regression model and I won't touch times series analysis at all. (Although, I will provide links for further reading at the bottom of this document.) These disclaimers aside, let's proceed...

Software requirements

R packages

It's important to note that "base" R already provides all of the tools we need for basic regression analysis. However, we'll be using several additional packages today, because they will make our lives easier and offer increased power for some more sophisticated analyses.

- New: fixest, estimatr, ivreg, sandwich, Imtest, mfx, margins, broom, modelsummary, vtable, rstanarm
- Already used: tidyverse, hrbrthemes, listviewer

A convenient way to install (if necessary) and load everything is by running the below code chunk.

While we've already loaded all of the required packages for today, I'll try to be as explicit about where a particular function is coming from, whenever I use it below.

Something else that I want to mention up front is that we'll mostly be working with the starwars data frame that we've already seen from previous lectures. Here's a quick reminder of what it looks like to refresh your memory.

starwars

```
## # A tibble: 87 x 14
                                                                               gender
##
      name
               height mass hair_color skin_color eye_color birth_year sex
##
      <chr>
                <int> <dbl> <chr>
                                        <chr>
                                                   <chr>
                                                                  <dbl> <chr> <chr>
##
   1 Luke Sk~
                         77 blond
                                        fair
                                                                   19
                  172
                                                   blue
                                                                        male
                                                                              mascu~
                                        gold
##
   2 C-3PO
                  167
                         75 <NA>
                                                   yellow
                                                                  112
                                                                        none
                                                                              mascu~
   3 R2-D2
                         32 <NA>
                                       white, bl~ red
##
                   96
                                                                   33
                                                                        none
                                                                              mascu~
##
   4 Darth V~
                  202
                        136 none
                                       white
                                                                   41.9 male mascu~
                                                   yellow
##
  5 Leia Or~
                  150
                         49 brown
                                        light
                                                   brown
                                                                   19
                                                                         fema~ femin~
##
  6 Owen La~
                  178
                        120 brown, gr~ light
                                                                   52
                                                                        male mascu~
                                                   blue
## 7 Beru Wh~
                  165
                         75 brown
                                                                   47
                                        light
                                                   blue
                                                                         fema~ femin~
## 8 R5-D4
                         32 <NA>
                                                                   NA
                   97
                                        white, red red
                                                                        none
                                                                              mascu~
## 9 Biggs D~
                  183
                         84 black
                                       light
                                                   brown
                                                                   24
                                                                        male
                                                                              mascu~
## 10 Obi-Wan~
                  182
                         77 auburn, w~ fair
                                                                   57
                                                                        male mascu~
                                                   blue-gray
## # i 77 more rows
## # i 5 more variables: homeworld <chr>, species <chr>, films <list>,
       vehicles <list>, starships <list>
```

Regression basics

The lm() function

R's workhorse command for running regression models is the built-in lm() function. The "lm" stands for "linear models" and the syntax is very intuitive.

```
lm(y \sim x1 + x2 + x3 + ..., data = df)
```

You'll note that the lm() call includes a reference to the data source (in this case, a hypothetical data frame called df). We covered this in our earlier lecture on R language basics and object-orientated programming, but the reason is that many objects (e.g. data frames) can exist in your R environment at the same time. So we need to be specific about where our regression variables are coming from — even if df is the only data frame in our global environment at the time.

Let's run a simple bivariate regression of mass on height using our dataset of starwars characters.

```
ols1 = lm(mass ~ height, data = starwars)
ols1

##
## Call:
## lm(formula = mass ~ height, data = starwars)
##
## Coefficients:
## (Intercept) height
## -13.8103     0.6386
```

The resulting object is pretty terse, but that's only because it buries most of its valuable information — of which there is a lot — within its internal list structure. If you're in RStudio, you can inspect this structure by typing View(ols1) or simply clicking on the "ols1" object in your environment pane. Doing so will prompt an interactive panel to pop up for you to play around with. That approach won't work for this knitted R Markdown document, however, so I'll use the listviewer::jsonedit() function that we saw in the previous lecture instead.

```
# View(ols1) ## Run this instead if you're in a live session
listviewer::jsonedit(ols1, mode="view") ## Better for R Markdown
```

As we can see, this ols1 object has a bunch of important slots... containing everything from the regression coefficients, to vectors of the residuals and fitted (i.e. predicted) values, to the rank of the design matrix, to the input data, etc. etc. To summarise the key pieces of information, we can use the — wait for it — generic summary() function. This will look pretty similar to the default regression output from Stata that many of you will be used to.

summary(ols1)

```
##
## Call:
## lm(formula = mass ~ height, data = starwars)
##
## Residuals:
##
       Min
                1Q Median
                                3Q
                                        Max
##
   -61.43
           -30.03
                   -21.13
                            -17.73 1260.06
##
## Coefficients:
##
               Estimate Std. Error t value Pr(>|t|)
## (Intercept) -13.8103
                                    -0.124
                                               0.902
                          111.1545
## height
                 0.6386
                            0.6261
                                      1.020
                                               0.312
##
## Residual standard error: 169.4 on 57 degrees of freedom
##
     (28 observations deleted due to missingness)
## Multiple R-squared: 0.01792,
                                    Adjusted R-squared:
## F-statistic: 1.04 on 1 and 57 DF, p-value: 0.312
```

We can then dig down further by extracting a summary of the regression coefficients:

summary(ols1)\$coefficients

```
## Estimate Std. Error t value Pr(>|t|)
## (Intercept) -13.810314 111.1545260 -0.1242443 0.9015590
## height 0.638571 0.6260583 1.0199865 0.3120447
```

Get "tidy" regression coefficients with the broom package

While it's easy to extract regression coefficients via the summary() function, in practice I always use the **broom** package (link) to do so. **broom** has a bunch of neat features to convert regression (and other statistical) objects into "tidy" data frames. This is especially useful because regression output is so often used as an input to something else, e.g. a plot of coefficients or marginal effects. Here, I'll use broom::tidy(..., conf.int = TRUE) to coerce the ols1 regression object into a tidy data frame of coefficient values and key statistics.

```
# library(broom) ## Already loaded
tidy(ols1, conf.int = TRUE)
## # A tibble: 2 x 7
##
                  estimate std.error statistic p.value conf.low conf.high
     term
##
     <chr>
                     <dbl>
                                <dbl>
                                          <dbl>
                                                   <dbl>
                                                             <dbl>
                                                                       <dbl>
## 1 (Intercept)
                   -13.8
                              111.
                                         -0.124
                                                   0.902 - 236.
                                                                      209.
## 2 height
                     0.639
                                0.626
                                          1.02
                                                   0.312
                                                           -0.615
                                                                        1.89
```

Again, I could now pipe this tidied coefficients data frame to a **ggplot2** call, using saying geom_pointrange() to plot the error bars. Feel free to practice doing this yourself now, but we'll get to some explicit examples further below.

broom has a couple of other useful functions too. For example, broom::glance() summarises the model "meta" data (R2, AIC, etc.) in a data frame.

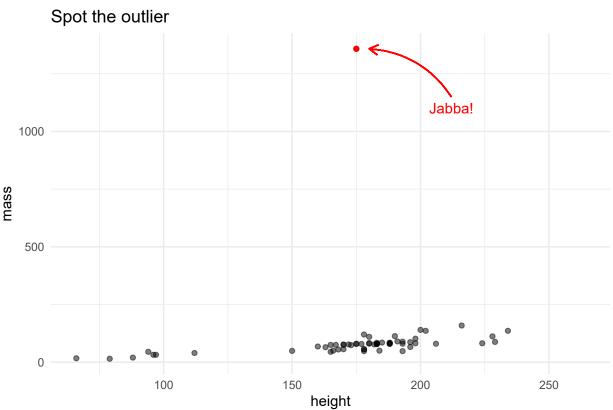
```
glance(ols1)
```

```
## # A tibble: 1 x 12
     r.squared adj.r.squared sigma statistic p.value
                                                          df logLik
                                                                      AIC
##
         <dbl>
                        <dbl> <dbl>
                                        <dbl>
                                                <dbl> <dbl>
                                                              <dbl> <dbl> <dbl>
        0.0179
                                                0.312
## 1
                    0.000696 169.
                                         1.04
                                                              -386.
                                                                     777.
                                                                           783.
                                                           1
## # i 3 more variables: deviance <dbl>, df.residual <int>, nobs <int>
```

By the way, if you're wondering how to export regression results to other formats (e.g. LaTeX tables), don't worry: We'll get to that at the end of the lecture.

Regressing on subsetted data

Our simple model isn't particularly good; the R2 is only 0.018. Different species and homeworlds aside, we may have an extreme outlier in our midst...



Remember: Always plot your data...

Maybe we should exclude Jabba from our regression? You can do this in two ways: 1) Create a new data frame and then regress, or 2) Subset the original data frame directly in the lm() call.

1) Create a new data frame Recall that we can keep multiple objects in memory in R. So we can easily create a new data frame that excludes Jabba using, say, **dplyr** (lecture) or **data.table** (lecture). For these lecture notes, I'll stick with **dplyr** commands since that's where our starwars dataset is coming from. But it would be trivial to switch to **data.table** if you prefer.

```
starwars2 =
starwars %>%
```

```
filter(name != "Jabba Desilijic Tiure")
  # filter(!(qrepl("Jabba", name))) ## Regular expressions also work
ols2 = lm(mass ~ height, data = starwars2)
summary(ols2)
##
## Call:
## lm(formula = mass ~ height, data = starwars2)
## Residuals:
                1Q Median
                                3Q
       Min
                                       Max
## -39.382 -8.212
                     0.211
                             3.846 57.327
##
## Coefficients:
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -32.54076
                           12.56053
                                     -2.591
                                              0.0122 *
## height
                 0.62136
                            0.07073
                                      8.785 4.02e-12 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 19.14 on 56 degrees of freedom
     (28 observations deleted due to missingness)
## Multiple R-squared: 0.5795, Adjusted R-squared: 0.572
## F-statistic: 77.18 on 1 and 56 DF, p-value: 4.018e-12
2) Subset directly in the lm() call Running a regression directly on a subsetted data frame is equally easy.
ols2a = lm(mass ~ height, data = starwars %>% filter(!(grepl("Jabba", name))))
summary(ols2a)
##
## Call:
## lm(formula = mass ~ height, data = starwars %>% filter(!(grepl("Jabba",
##
       name))))
##
## Residuals:
##
                1Q Median
                                3Q
                                       Max
                     0.211
##
  -39.382 -8.212
                             3.846
                                   57.327
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept) -32.54076
                           12.56053 -2.591
                                              0.0122 *
## height
                 0.62136
                            0.07073
                                     8.785 4.02e-12 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 19.14 on 56 degrees of freedom
     (28 observations deleted due to missingness)
## Multiple R-squared: 0.5795, Adjusted R-squared: 0.572
## F-statistic: 77.18 on 1 and 56 DF, p-value: 4.018e-12
```

The overall model fit is much improved by the exclusion of this outlier, with R2 increasing to 0.58. Still, we should be cautious about throwing out data. Another approach is to handle or account for outliers with statistical methods. Which provides a nice segue to nonstandard errors.

Nonstandard errors

Dealing with statistical irregularities (heteroskedasticity, clustering, etc.) is a fact of life for empirical researchers. However, it says something about the economics profession that a random stranger could walk uninvited into a live seminar and ask, "How did you cluster your standard errors?", and it would likely draw approving nods from audience members.

The good news is that there are *lots* of ways to get nonstandard errors in R. For many years, these have been based on the excellent **sandwich** package (link). However, here I'll demonstrate using the **estimatr** package (link), which is both fast and provides convenient aliases for the standard regression functions. Some examples follow below.

Robust standard errors

You can obtain heteroskedasticity-consistent (HC) "robust" standard errors using estimatr::lm_robust(). Let's illustrate by implementing a robust version of the ols1 regression that we ran earlier. Note that **estimatr** models automatically print in pleasing tidied/summary format, although you can certainly pipe them to tidy() too.

```
# library(estimatr) ## Already loaded
ols1 robust = lm robust(mass ~ height, data = starwars)
# tidy(ols1 robust, conf.int = TRUE) ## Could tidy too
ols1 robust
                 Estimate Std. Error
                                         t value
                                                     Pr(>|t|)
                                                                  CI Lower
## (Intercept) -13.810314 23.45557632 -0.5887859 5.583311e-01 -60.7792950
## height
                 0.638571 0.08791977 7.2631109 1.159161e-09
                                                                 0.4625147
##
                 CI Upper DF
## (Intercept) 33.1586678 57
## height
                0.8146273 57
```

The package defaults to using Eicker-Huber-White robust standard errors, commonly referred to as "HC2" standard errors. You can easily specify alternate methods using the se_type = argument. For example, you can specify Stata robust standard errors if you want to replicate code or results from that language. (See here for more details on why this isn't the default and why Stata's robust standard errors differ from those in R and Python.)

estimatr also supports robust instrumental variable (IV) regression. However, I'm going to hold off discussing these until we get to the IV section below.

Aside on HAC (Newey-West) standard errors On thing I want to flag is that estimatr does not yet offer support for HAC (i.e. heteroskedasticity and autocorrelation consistent) standard errors a la Newey-West. I've submitted a feature request on GitHub — vote up if you would like to see it added sooner! — but you can still obtain these pretty easily using the aforementioned sandwich package. For example, we can use sandwich: :NeweyWest() on our existing ols1 object to obtain HAC SEs for it.

```
# library(sandwich) ## Already loaded

# NeweyWest(ols1) ## Print the HAC VCOV
sqrt(diag(NeweyWest(ols1))) ## Print the HAC SEs
```

¹See the package documentation for a full list of options.

```
## (Intercept) height
## 21.2694130 0.0774265
```

If you plan to use HAC SEs for inference, then I recommend converting the model object with lmtest::coeftest(). This function builds on **sandwich** and provides a convenient way to do on-the-fly hypothesis testing with your model, swapping out a wide variety of alternate variance-covariance (VCOV) matrices. These alternate VCOV matrices could extended way beyond HAC — including HC, clustered, bootstrapped, etc. — but here's how it would work for the present case:

```
# library(lmtest) ## Already loaded
ols1_hac = lmtest::coeftest(ols1, vcov = NeweyWest)
ols1_hac
##
## t test of coefficients:
##
                 Estimate Std. Error t value Pr(>|t|)
## (Intercept) -13.810314
                           21.269413 -0.6493
                                                  0.5187
## height
                 0.638571
                             0.077427 8.2474 2.672e-11 ***
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
Note that its easy to convert coeftest()-adjusted models to tidied broom objects too.
tidy(ols1_hac, conf.int = TRUE)
## # A tibble: 2 x 7
##
                 estimate std.error statistic p.value conf.low conf.high
     term
##
     <chr>
                     <dbl>
                               <dbl>
                                          <dbl>
                                                   <dbl>
                                                            <dbl>
                                                                       <dbl>
                             21.3
                                                          -56.4
                                                                      28.8
## 1 (Intercept)
                  -13.8
                                        -0.649 5.19e- 1
## 2 height
                    0.639
                              0.0774
                                         8.25 2.67e-11
                                                            0.484
                                                                       0.794
```

Clustered standard errors

Clustered standard errors is an issue that most commonly affects panel data. As such, I'm going to hold off discussing clustering until we get to the panel data section below. But here's a quick example of clustering with estimatr::lm_robust() just to illustrate:

Dummy variables and interaction terms

For the next few sections, it will prove convenient to demonstrate using a subsample of the starwars data that comprises only the human characters. Let's quickly create this new dataset before continuing.

```
humans =
  starwars %>%
  filter(species=="Human") %>%
  select(where(Negate(is.list))) ## Drop list columns (optional)
humans
```

```
## # A tibble: 35 x 11
##
                height mass hair_color skin_color eye_color birth_year sex
      name
                                                                                    gender
                 <int> <dbl> <chr>
                                          <chr>>
##
      <chr>
                                                      <chr>
                                                                       <dbl> <chr> <chr>
##
                   172
                           77 blond
                                          fair
                                                      blue
                                                                        19
    1 Luke Sk~
                                                                             \mathtt{male}
                                                                                   mascu~
##
    2 Darth V~
                   202
                          136 none
                                          white
                                                      yellow
                                                                        41.9 male
                                                                                   mascu~
##
    3 Leia Or~
                   150
                           49 brown
                                          light
                                                      brown
                                                                        19
                                                                             fema~ femin~
##
    4 Owen La~
                   178
                          120 brown, gr~ light
                                                      blue
                                                                        52
                                                                             male
                                                                                   mascu~
##
    5 Beru Wh~
                   165
                           75 brown
                                          light
                                                      blue
                                                                        47
                                                                             fema~ femin~
##
    6 Biggs D~
                   183
                           84 black
                                          light
                                                      brown
                                                                        24
                                                                             male
                                                                                   mascu~
##
    7 Obi-Wan~
                   182
                           77 auburn, w~ fair
                                                      blue-gray
                                                                        57
                                                                             male
                                                                                   mascu~
    8 Anakin ~
                   188
                           84 blond
                                          fair
                                                      blue
                                                                        41.9
                                                                             male
                                                                                   mascu~
    9 Wilhuff~
                   180
##
                           NA auburn, g~ fair
                                                      blue
                                                                        64
                                                                             male
                                                                                   mascu~
## 10 Han Solo
                   180
                           80 brown
                                                      brown
                                                                        29
                                                                             male
                                          fair
                                                                                   mascu~
## # i 25 more rows
## # i 2 more variables: homeworld <chr>, species <chr>
```

Dummy variables as factors

Dummy variables are a core component of many regression models. However, these can be a pain to create in some statistical languages, since you first have to tabulate a whole new matrix of binary variables and then append it to the original data frame. In contrast, R has a very convenient framework for creating and evaluating dummy variables in a regression: Simply specify the variable of interest as a factor.²

Here's an example where we explicitly tell R that "gender" is a factor. Since I don't plan on reusing this model, I'm just going to print the results to screen rather than saving it to my global environment.

```
summary(lm(mass ~ height + as.factor(gender), data = humans))
```

```
##
## Call:
## lm(formula = mass ~ height + as.factor(gender), data = humans)
## Residuals:
##
       Min
                1Q
                   Median
                                3Q
                                       Max
##
   -16.068
           -8.130
                    -3.660
                             0.702
                                    37.112
##
## Coefficients:
##
                              Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                              -84.2520
                                                   -1.281
                                          65.7856
                                                    2.156
                                                             0.0441 *
                                0.8787
                                           0.4075
## height
## as.factor(gender)masculine
                              10.7391
                                          13.1968
                                                    0.814
                                                             0.4259
##
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
## Residual standard error: 15.19 on 19 degrees of freedom
     (13 observations deleted due to missingness)
## Multiple R-squared: 0.444, Adjusted R-squared:
## F-statistic: 7.587 on 2 and 19 DF, p-value: 0.003784
```

Okay, I should tell you that I'm actually making things more complicated than they need to be with the heavy-handed emphasis on factors. R is "friendly" and tries to help whenever it thinks you have misspecified a function or variable. While this is something to be aware of, normally It Just WorksTM. A case in point is that we don't actually *need* to specify a string (i.e. character) variable as a factor in a regression. R will automatically do this for you regardless, since it's the only sensible way to include string variables in a regression.

²Factors are variables that have distinct qualitative levels, e.g. "male", "female", "hermaphrodite", etc.

```
## Use the non-factored version of "gender" instead; R knows it must be ordered
## for it to be included as a regression variable
summary(lm(mass ~ height + gender, data = humans))
##
## Call:
## lm(formula = mass ~ height + gender, data = humans)
##
## Residuals:
##
       Min
                1Q
                   Median
                                3Q
                                       Max
                    -3.660
                             0.702
                                    37.112
##
   -16.068
           -8.130
##
## Coefficients:
##
                   Estimate Std. Error t value Pr(>|t|)
                   -84.2520
                               65.7856
                                        -1.281
                                                  0.2157
## (Intercept)
## height
                     0.8787
                                0.4075
                                          2.156
                                                  0.0441 *
## gendermasculine 10.7391
                               13.1968
                                          0.814
                                                  0.4259
##
                   0 '***' 0.001 '**' 0.01 '*' 0.05 '.' 0.1 ' ' 1
## Signif. codes:
##
## Residual standard error: 15.19 on 19 degrees of freedom
     (13 observations deleted due to missingness)
## Multiple R-squared: 0.444, Adjusted R-squared:
## F-statistic: 7.587 on 2 and 19 DF, p-value: 0.003784
```

Interaction effects

##

Like dummy variables, R provides a convenient syntax for specifying interaction terms directly in the regression model without having to create them manually beforehand.³ You can use any of the following expansion operators:

- x1:x2 "crosses" the variables (equivalent to including only the x1 × x2 interaction term)
- x1/x2 "nests" the second variable within the first (equivalent to x1 + x1:x2; more on this later)
- x1*x2 includes all parent and interaction terms (equivalent to x1 + x2 + x1:x2)

As a rule of thumb, if not always, it is generally advisable to include all of the parent terms alongside their interactions. This makes the * option a good default.

For example, we might wonder whether the relationship between a person's body mass and their height is modulated by their gender. That is, we want to run a regression of the form,

$$Mass = \beta_0 + \beta_1 D_{Male} + \beta_2 Height + \beta_3 D_{Male} \times Height$$

To implement this in R, we simply run the following,

```
ols_ie = lm(mass ~ gender * height, data = humans)
summary(ols_ie)

##
## Call:
## lm(formula = mass ~ gender * height, data = humans)
```

^{##} Residuals:
Min 1Q Median 3Q Max

³Although there are very good reasons that you might want to modify your parent variables before doing so (e.g. centering them). As it happens, I'm on record as stating that interaction effects are most widely misunderstood and misapplied concept in econometrics. However, that's a topic for another day.

```
## -16.250 -8.158 -3.684 -0.107 37.193
##
##
  Coefficients:
##
                          Estimate Std. Error t value Pr(>|t|)
##
  (Intercept)
                          -61.0000
                                     204.0565
                                               -0.299
                                                          0.768
## gendermasculine
                          -15.7224
                                               -0.072
                                                          0.944
                                     219.5440
## height
                            0.7333
                                       1.2741
                                                 0.576
                                                          0.572
## gendermasculine:height
                            0.1629
                                       1.3489
                                                 0.121
                                                          0.905
##
## Residual standard error: 15.6 on 18 degrees of freedom
     (13 observations deleted due to missingness)
## Multiple R-squared: 0.4445, Adjusted R-squared:
## F-statistic: 4.801 on 3 and 18 DF, p-value: 0.01254
```

Instrumental variables

As you would have guessed by now, there are a number of ways to run instrumental variable (IV) regressions in R. I'll walk through three different options using the ivreg::ivreg(), estimatr::iv_robust(), and fixest::feols() functions, respectively. These are all going to follow a similar syntax, where the IV first-stage regression is specified in a multi-part formula (i.e. where formula parts are separated by one or more pipes, |). However, there are also some subtle and important differences, which is why I want to go through each of them. After that, I'll let you decide which of the three options is your favourite.

The dataset that we'll be using for this section describes cigarette demand for the 48 continental US states in 1995, and is taken from the **ivreg** package. Here's a quick a peek:

```
data("CigaretteDemand", package = "ivreg")
head(CigaretteDemand)
```

```
##
                  rprice rincome salestax
                                              cigtax packsdiff pricediff
         packs
## AL 101.08543 103.9182 12.91535 0.9216975 26.57481 -0.1418075 0.09010222
  AR 111.04297 115.1854 12.16907 5.4850193 36.41732 -0.1462808 0.19998082
## AZ 71.95417 130.3199 13.53964 6.2057067 42.86964 -0.3733741 0.25576681
      56.85931 138.1264 16.07359 9.0363074 40.02625 -0.5682141 0.32079587
      82.58292 109.8097 16.31556 0.0000000 28.87139 -0.3132622 0.22587189
## CO
      79.47219 143.2287 20.96236 8.1072834 48.55643 -0.3184911 0.18546746
## CT
##
      incomediff salestaxdiff cigtaxdiff
## AL 0.18222144
                    0.1332853 -3.62965832
## AR 0.15055894
                    5.4850193 2.03070663
## AZ 0.05379983
                    1.4004856 14.05923036
## CA 0.02266877
                    3.3634447 15.86267924
## CD 0.13002974
                    0.0000000
                              0.06098283
## CT 0.18404197
                   -0.7062239
                              9.52297455
```

Now, assume that we are interested in regressing the number of cigarettes packs consumed per capita on their average price and people's real incomes. The problem is that the price is endogenous, because it is simultaneously determined by demand and supply. So we need to instrument for it using cigarette sales tax. That is, we want to run the following two-stage IV regression.

$$Price_i = \pi_0 + \pi_1 Sales Tax_i + v_i \qquad \text{(First stage)}$$

$$Packs_i = \beta_0 + \beta_2 \widehat{Price}_i + \beta_1 Real Income_i + u_i \qquad \text{(Second stage)}$$

Option 1: ivreg::ivreg()

I'll start with ivreg() from the **ivreg** package (link).⁴ The ivreg() function supports several syntax options for specifying the IV component. I'm going to use the syntax that I find most natural, namely a formula with a three-part RHS: y - ex | en | in. Implementing our two-stage regression from above may help to illustrate.

```
# library(ivreg) ## Already loaded
## Run the IV regression. Note the three-part formula RHS.
iv =
  ivreg(
   log(packs) ~
                            ## LHS: Dependent variable
                            ## 1st part RHS: Exogenous variable(s)
      log(rincome) |
      log(rprice) |
                            ## 2nd part RHS: Endogenous variable(s)
      salestax,
                            ## 3rd part RHS: Instruments
    data = CigaretteDemand
summary(iv)
##
## Call:
  ivreg(formula = log(packs) ~ log(rincome) | log(rprice) | salestax,
##
       data = CigaretteDemand)
##
##
## Residuals:
                          Median
                                        3Q
##
         Min
                    1Q
                                                 Max
## -0.611000 -0.086072 0.009423 0.106912 0.393159
##
## Coefficients:
##
                Estimate Std. Error t value Pr(>|t|)
## (Intercept)
                  9.4307
                             1.3584
                                      6.943 1.24e-08 ***
## log(rprice)
                 -1.1434
                             0.3595 -3.181 0.00266 **
## log(rincome)
                  0.2145
                             0.2686
                                    0.799 0.42867
##
## Diagnostic tests:
##
                    df1 df2 statistic p-value
                               45.158 2.65e-08 ***
## Weak instruments
                         45
                      1
                         44
                                1.102
                                           0.3
## Wu-Hausman
                      1
## Sargan
                      0
                         NA
                                   NA
                                            NΑ
## ---
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Residual standard error: 0.1896 on 45 degrees of freedom
## Multiple R-Squared: 0.4189, Adjusted R-squared: 0.3931
## Wald test: 6.534 on 2 and 45 DF, p-value: 0.003227
```

ivreg has lot of functionality bundled into it, including cool diagnostic tools and full integration with **sandwich** and co. for swapping in different standard errors on the fly. See the introductory vignette for more.

The only other thing I want to mention briefly is that you may see a number ivreg() tutorials using an alternative formula representation. (Remember me saying that the package allows different formula syntax, right?) Specifically, you'll probably see examples that use an older two-part RHS formula like: $y \sim ex + en \mid ex + in$. Note that here we are writing the ex variables on both sides of the | separator. The equivalent for our cigarette example would be as follows. Run this yourself to confirm the same output as above.

⁴Some of you may wondering, but **ivreg** is a dedicated IV-focused package that splits off (and updates) functionality that used to be bundled with the **AER** package.

This two-part syntax is closely linked to the manual implementation of IV, since it requires explicitly stating *all* of your exogenous variables (including instruments) in one slot. However, it requires duplicate typing of the exogenous variables and I personally find it less intuitive to write.⁵ But different strokes for different folks.

Option 2: estimatr::iv_robust()

Our second IV option comes from the **estimatr** package that we saw earlier. This will default to using HC2 robust standard errors although, as before, we could specify other options if we so wished (including clustering). Currently, the function doesn't accept the three-part RHS formula. But the two-part version works exactly the same as it did for ivreg(). All we need to do is change the function call to estimatr::iv_robust().

```
log(rincome) + salestax, data = CigaretteDemand)
##
##
## Standard error type: HC2
##
## Coefficients:
               Estimate Std. Error t value Pr(>|t|) CI Lower CI Upper DF
## (Intercept)
                                                       6.8436 12.0177 45
                 9.4307
                            1.2845
                                   7.342 3.179e-09
## log(rincome)
                 0.2145
                            0.3164
                                    0.678 5.012e-01 -0.4227
                            0.3811 -3.000 4.389e-03 -1.9110 -0.3758 45
## log(rprice)
                -1.1434
## Multiple R-squared: 0.4189,
                                   Adjusted R-squared: 0.3931
## F-statistic: 7.966 on 2 and 45 DF, p-value: 0.001092
```

⁵Note that we didn't specify the endogenous variable (i.e. log(rprice)) directly. Rather, we told R what the *exogenous* variables were. It then figured out which variables were endogenous and needed to be instrumented in the first-stage.

Other models

Generalised linear models (logit, etc.)

To run a generalised linear model (GLM), we use the in-built glm() function and simply assign an appropriate family (which describes the error distribution and corresponding link function). For example, here's a simple logit model.

```
glm_logit = glm(am ~ cyl + hp + wt, data = mtcars, family = binomial)
summary(glm logit)
##
## Call:
## glm(formula = am ~ cyl + hp + wt, family = binomial, data = mtcars)
##
## Coefficients:
               Estimate Std. Error z value Pr(>|z|)
##
## (Intercept) 19.70288
                           8.11637
                                     2.428
                                             0.0152 *
                0.48760
                           1.07162
                                     0.455
                                             0.6491
## cyl
## hp
                0.03259
                           0.01886
                                     1.728
                                             0.0840
               -9.14947
                                             0.0276 *
## wt
                           4.15332
                                    -2.203
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
##
##
  (Dispersion parameter for binomial family taken to be 1)
##
##
       Null deviance: 43.2297
                               on 31 degrees of freedom
                               on 28 degrees of freedom
## Residual deviance: 9.8415
## AIC: 17.841
## Number of Fisher Scoring iterations: 8
```

Alternatively, you may recall me saying earlier that **fixest** supports nonlinear models. So you could (in this case, without fixed-effects) also estimate:

```
feglm(am ~ cyl + hp + wt, data = mtcars, family = binomial)
## GLM estimation, family = binomial, Dep. Var.: am
## Observations: 32
## Standard-errors: IID
                                     t value Pr(>|t|)
##
               Estimate Std. Error
## (Intercept) 19.702883
                          8.540119
                                    2.307097 0.021049 *
                          1.127568 0.432433 0.665427
## cyl
               0.487598
## hp
               0.032592
                          0.019846 1.642249 0.100538
              -9.149471
                          4.370163 -2.093623 0.036294 *
## wt
## Signif. codes: 0 '***' 0.001 '**' 0.05 '.' 0.1 ' ' 1
## Log-Likelihood: -4.92075
                             Adj. Pseudo R2: 0.633551
##
             BIC: 23.7
                               Squared Cor.: 0.803395
```

Remember that the estimates above simply reflect the naive coefficient values, which enter multiplicatively via the link function. We'll get a dedicated section on extracting marginal effects from non-linear models in a moment. But I do want to quickly flag the **mfx** package (link), which provides convenient aliases for obtaining marginal effects from a variety of GLMs. For example,

```
# library(mfx) ## Already loaded
## Be careful: mfx loads the MASS package, which produces a namespace conflict
## with dplyr for select(). You probably want to be explicit about which one you
## want, e.g. `select = dplyr::select`
```

```
## Get marginal effects for the above logit model
# logitmfx(am ~ cyl + hp + wt, atmean = TRUE, data = mtcars) ## Can also estimate directly
logitmfx(glm_logit, atmean = TRUE, data = mtcars)

## Call:
## logitmfx(formula = glm_logit, data = mtcars, atmean = TRUE)

##
## Marginal Effects:
## dF/dx Std. Err. z P>|z|
## cyl 0.0537504 0.1132652 0.4746 0.6351
## hp 0.0035927 0.0029037 1.2373 0.2160
## wt -1.0085932 0.6676628 -1.5106 0.1309
```

Even more models

Of course, there are simply too many other models and other estimation procedures to cover in this lecture. A lot of these other models that you might be thinking of come bundled with the base R installation. But just to highlight a few, mostly new packages that I like a lot for specific estimation procedures:

- Difference-in-differences (with variable timing, etc.): did (link) and DRDID (link)
- Synthetic control: tidysynth (link), gsynth (link) and scul (link)
- Count data (hurdle models, etc.): **pscl** (link)
- Lasso: biglasso (link)
- Causal forests: grf (link)
- etc.

Finally, just a reminder to take a look at the Further Resources links at the bottom of this document to get a sense of where to go for full-length econometrics courses and textbooks.

Marginal effects

Calculating marginal effects in a linear regression model like OLS is perfectly straightforward... just look at the coefficient values. But that quickly goes out the window when you have interaction terms or non-linear models like probit, logit, etc. Luckily, there are various ways to obtain these from R models. For example, we already saw the **mfx** package above for obtaining marginal effects from GLM models. I want to briefly focus on two of my favourite methods for obtaining marginal effects across different model classes: 1) The **margins** package and 2) a shortcut that works particularly well for models with interaction terms.

The margins package

The **margins** package (link), which is modeled on its namesake in Stata, is great for obtaining marginal effects across an entire range of models.⁶ You can read more in the package vignette, but here's a very simple example to illustrate.

Consider our interaction effects regression from earlier, where we were interested in how people's mass varied by height and gender. To get the average marginal effect (AME) of these dependent variables, we can just use the margins::margins() function.

```
# library(margins) ## Already loaded

ols_ie_marg = margins(ols_ie)
```

Like a normal regression object, we can get a nice print-out display of the above object by summarising or tidying it.

```
# summary(ols_ie_marg) ## Same effect
tidy(ols_ie_marg, conf.int = TRUE)
```

⁶I do, however, want to flag that it does not yet support **fixest** (or **lfe**) models. But there are workarounds in the meantime.

```
## # A tibble: 2 x 7
##
                      estimate std.error statistic p.value conf.low conf.high
     term
                                                                 <dbl>
##
     <chr>>
                         <dbl>
                                    <dbl>
                                               <dbl>
                                                       <dbl>
                                                                            <dbl>
                        13.5
                                   26.8
                                               0.505
                                                      0.613 -38.9
                                                                           66.0
## 1 gendermasculine
## 2 height
                         0.874
                                    0.420
                                               2.08
                                                      0.0376
                                                                0.0503
                                                                             1.70
```

If we want to compare marginal effects at specific values — e.g. how the AME of height on mass differs across genders — then that's easily done too.

```
ols_ie %>%
  margins (
    variables = "height", ## The main variable we're interested in
    at = list(gender = c("masculine", "feminine")) ## How the main variable is modulated by at specific
    ) %>%
  tidy(conf.int = TRUE) ## Tidy it (optional)
## # A tibble: 2 x 9
##
            at.variable at.value
                                   estimate std.error statistic p.value conf.low
     term
##
     <chr>>
            <chr>>
                         <fct>
                                      <dbl>
                                                 <dbl>
                                                           <dbl>
                                                                   <dbl>
                                                                             <dbl>
```

0.443

1.27

2.02

0.576

0.0431

0.565

0.0278

-1.76

If you're the type of person who prefers visualizations (like me), then you should consider margins::cplot(), which is the package's in-built method for constructing *conditional* effect plots.

0.896

0.733

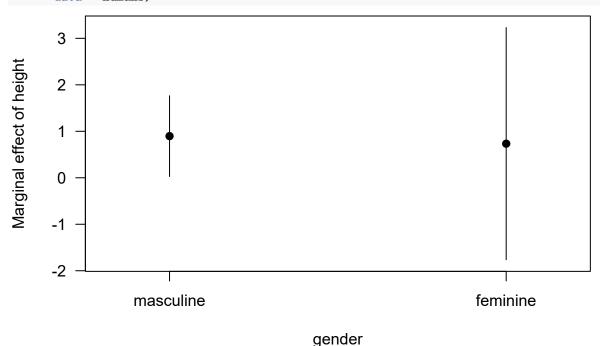
masculine

feminine

i 1 more variable: conf.high <dbl>

1 height gender

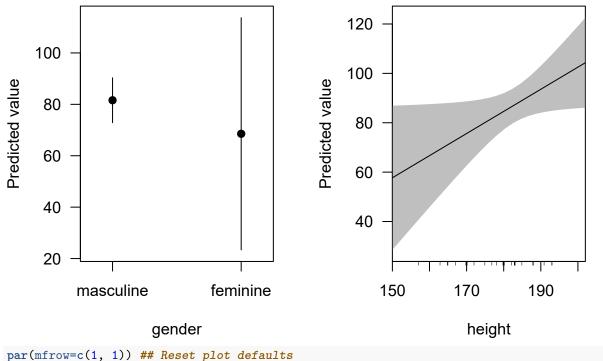
2 height gender



In this case, it doesn't make much sense to read a lot into the larger standard errors on the female group; that's being driven by a very small sub-sample size.

Finally, you can also use cplot() to plot the predicted values of your outcome variable (here: "mass"), conditional on one of your dependent variables. For example:

```
par(mfrow=c(1, 2)) ## Just to plot these next two (base) figures side-by-side
cplot(ols_ie, x = "gender", what = "prediction", data = humans)
cplot(ols_ie, x = "height", what = "prediction", data = humans)
```



Note that cplot() uses the base R plotting method. If you'd prefer **ggplot2** equivalents, take a look at the **marginsplot** package (link).

Finally, I also want to draw your attention to the **emmeans** package (link), which provides very similar functionality to **margins**. I'm not as familiar with it myself, but I know that it has many fans.

Special case: / shortcut for interaction terms

I'll keep this one brief, but I wanted to mention one of my favourite R shortcuts: Obtaining the full marginal effects for interaction terms by using the / expansion operator. I've tweeted about this and even wrote an whole blog post about it too (which you should totally read). But the very short version is that you can switch out the normal f1 * x2 interaction terms syntax for f1 / x2 and it automatically returns the full marginal effects. (The formal way to describe it is that the model variables have been "nested".)

Here's a super simple example, using the same interaction effects model from before.

```
# ols_ie = lm(mass ~ gender * height, data = humans) ## Original model
ols_ie_marg2 = lm(mass ~ gender / height, data = humans)
tidy(ols_ie_marg2, conf.int = TRUE)
## # A tibble: 4 x 7
```

```
##
     term
                              estimate std.error statistic p.value conf.low conf.high
##
     <chr>>
                                 <dbl>
                                           <dbl>
                                                      <dbl>
                                                               <dbl>
                                                                        <dbl>
                                                                                   <dbl>
                                         204.
## 1 (Intercept)
                               -61.0
                                                    -0.299
                                                              0.768
                                                                     -4.90e+2
                                                                                  368.
## 2 gendermasculine
                               -15.7
                                         220.
                                                    -0.0716
                                                             0.944
                                                                     -4.77e+2
                                                                                  446.
## 3 genderfeminine:height
                                           1.27
                                                     0.576
                                                              0.572 -1.94e+0
                                                                                    3.41
                                 0.733
## 4 gendermasculine:height
                                 0.896
                                           0.443
                                                     2.02
                                                              0.0582 -3.46e-2
                                                                                    1.83
```

| | (1) | (2) |
|---------------------------------|-----------|-----------|
| (Intercept) | -13.810 | -61.000 |
| _ | (111.155) | (204.057) |
| height | 0.639 | 0.733 |
| | (0.626) | (1.274) |
| gendermasculine | | -15.722 |
| | | (219.544) |
| $gendermasculine \times height$ | | 0.163 |
| | | (1.349) |
| Num.Obs. | 59 | 22 |
| R2 | 0.018 | 0.444 |
| R2 Adj. | 0.001 | 0.352 |
| AIC | 777.0 | 188.9 |
| BIC | 783.2 | 194.4 |
| Log.Lik. | -385.503 | -89.456 |
| F | 1.040 | |
| RMSE | 166.50 | 14.11 |

Note that the marginal effects on the two gender \times height interactions (i.e. 0.733 and 0.896) are the same as we got with the margins::margins() function above.

Where this approach really shines is when you are estimating interaction terms in large models. The **margins** package relies on a numerical delta method which can be very computationally intensive, whereas using / adds no additional overhead beyond calculating the model itself. Still, that's about as much as say it here. Read my aforementioned blog post if you'd like to learn more.

Presentation

Tables

Regression tables There are loads of different options here. We've already seen the excellent etable() function from **fixest** above. However, my own personal favourite tool or creating and exporting regression tables is the **modelsummary** package (link). It is extremely flexible and handles all manner of models and output formats. **modelsummary** also supports automated coefficient plots and data summary tables, which I'll get back to in a moment. The documentation is outstanding and you should read it, but here is a bare-boned example just to demonstrate.

```
# library(modelsummary) ## Already loaded

## Note: msummary() is an alias for modelsummary()
msummary(list(ols1, ols_ie))
```

One nice thing about **modelsummary** is that it plays very well with R Markdown and will automatically coerce your tables to the format that matches your document output: HTML, LaTeX/PDF, RTF, etc. Of course, you can also specify the output type if you aren't using R Markdown and want to export a table for later use. Finally, you can even specify special table formats like *threepartable* for LaTeX and, provided that you have called the necessary packages in your preamble, it will render correctly (see example here.

Summary tables A variety of summary tables — balance, correlation, etc. — can be produced by the companion set of modelsummary::datasummary*() functions. Again, you should read the documentation to see all of the options. But here's an example of a very simple balance table using a subset of our "humans" data frame.

⁷Note that etable() is limited to fixest models only.

| | | femin | ine (N=9) | masculine (N=26) | | | |
|------------|-----------|-------|-----------|------------------|-----------|----------------|------------|
| | | Mean | Std. Dev. | Mean | Std. Dev. | Diff. in Means | Std. Error |
| height | | 160.2 | 7.0 | 182.3 | 8.2 | 22.1 | 3.0 |
| mass | | 56.3 | 16.3 | 87.0 | 16.5 | 30.6 | 10.1 |
| birth_year | | 46.4 | 18.8 | 55.2 | 26.0 | 8.8 | 10.2 |
| | | N | Pct. | N | Pct. | | |
| eye_color | blue | 3 | 33.3 | 9 | 34.6 | | |
| | blue-gray | 0 | 0.0 | 1 | 3.8 | | |
| | brown | 5 | 55.6 | 12 | 46.2 | | |
| | dark | 0 | 0.0 | 1 | 3.8 | | |
| | hazel | 1 | 11.1 | 1 | 3.8 | | |
| | yellow | 0 | 0.0 | 2 | 7.7 | | |

Another package that I like a lot in this regard is **vtable** (link). Not only can it be used to construct descriptive labels like you'd find in Stata's "Variables" pane, but it is also very good at producing the type of "out of the box" summary tables that economists like. For example, here's the equivalent version of the above balance table.

```
# library(vtable) ## Already loaded

## An additional argument just for formatting across different output types of
## this .Rmd document
otype = ifelse(knitr::is_latex_output(), 'return', 'kable')

## vtable::st() is an alias for sumtable()
vtable::st(humans %>% select(height:mass, birth_year, eye_color, gender),
    group = 'gender',
    out = otype)
```

| ## | | Variable | N | Mean | \mathtt{SD} | N | Mean | SD |
|----|----|------------|------------------|------|---------------|-------------------|------|-----|
| ## | 1 | gender | ${\tt feminine}$ | | | ${\tt masculine}$ | | |
| ## | 2 | height | 8 | 160 | 7 | 23 | 182 | 8.2 |
| ## | 3 | mass | 3 | 56 | 16 | 19 | 87 | 17 |
| ## | 4 | birth_year | 5 | 46 | 19 | 20 | 55 | 26 |
| ## | 5 | eye_color | 9 | | | 26 | | |
| ## | 6 | blue | 3 | 33% | | 9 | 35% | |
| ## | 7 | blue-gray | 0 | 0% | | 1 | 4% | |
| ## | 8 | brown | 5 | 56% | | 12 | 46% | |
| ## | 9 | dark | 0 | 0% | | 1 | 4% | |
| ## | 10 | hazel | 1 | 11% | | 1 | 4% | |
| ## | 11 | yellow | 0 | 0% | | 2 | 8% | |

Lastly, Stata users in particular might like the qsu() and descr() functions from the lightning-fast **collapse** package (link).

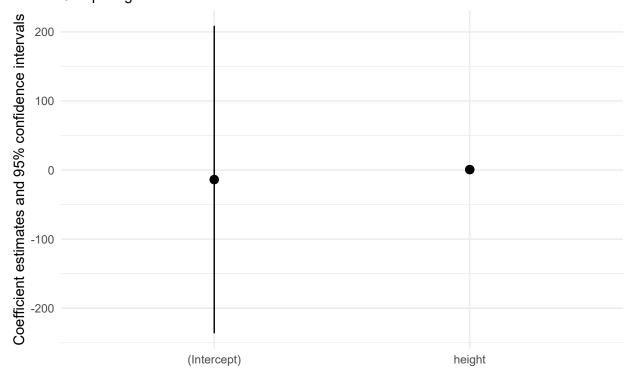
Figures

Coefficient plots We've already worked through an example of how to extract and compare model coefficients here. I use this "manual" approach to visualizing coefficient estimates all the time. However, our focus on **modelsummary** in the preceding section provides a nice segue to another one of the package's features: modelplot(). Consider the following, which shows both the degree to which modelplot() automates everything and the fact that it readily accepts regular ggplot2 syntax.

```
# library(modelsummary) ## Already loaded
mods = list('No clustering' = summary(ols1, se = 'standard'))

modelplot(mods) +
    ## You can further modify with normal ggplot2 commands...
    coord_flip() +
    labs(
        title = "'Effect' of height on mass",
        subtitle = "Comparing fixed effect models"
    )
```

'Effect' of height on mass Comparing fixed effect models

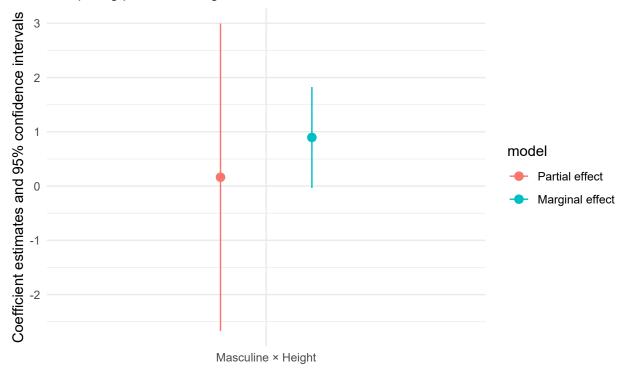


Or, here's another example where we compare the (partial) Masculine × Height coefficient from our earlier interaction model, with the (full) marginal effect that we obtained later on.

```
ie_mods = list('Partial effect' = ols_ie, 'Marginal effect' = ols_ie_marg2)
modelplot(ie_mods, coef_map = c("gendermasculine:height" = "Masculine × Height")) +
coord_flip() +
labs(
   title = "'Effect' of height on mass",
   subtitle = "Comparing partial vs marginal effects"
   )
```

'Effect' of height on mass

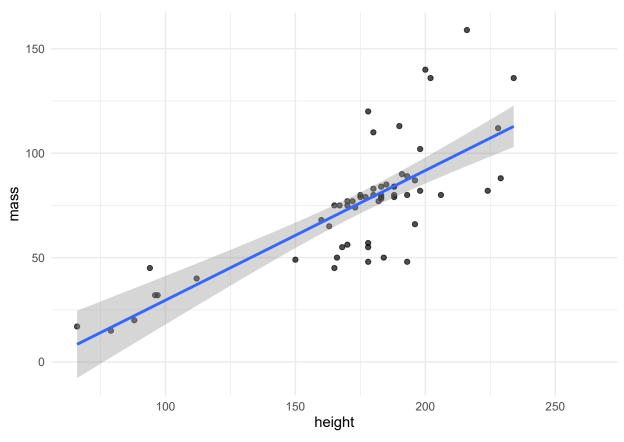
Comparing partial vs marginal effects



Prediction and model validation The easiest way to visually inspect model performance (i.e. validation and prediction) is with **ggplot2**. In particular, you should already be familiar with geom_smooth() from our earlier lectures, which allows you to feed a model type directly in the plot call. For instance, using our starwars2 data frame that excludes that slimy outlier, Jabba the Hutt:

```
ggplot(starwars2, aes(x = height, y = mass)) +
    geom_point(alpha = 0.7) +
    geom_smooth(method = "lm") ## See ?geom_smooth for other methods/options
```

```
## `geom_smooth()` using formula = 'y ~ x'
```



Now, I should say that geom_smooth() isn't particularly helpful when you've already constructed a (potentially complicated) model outside of the plot call. Similarly, it's not useful when you want to use a model for making predictions on a *new* dataset (e.g. evaluating out-of-sample fit).

The good news is that the generic predict() function in base R has you covered. For example, let's say that we want to re-estimate our simple bivariate regression of mass on height from earlier. This time, however, we'll estimate our model on a training dataset that only consists of the first 30 characters ranked by height. Here's how you would do it.

```
## Estimate a model on a training sample of the data (shortest 30 characters)
ols1_train = lm(mass ~ height, data = starwars %>% filter(rank(height) <=30))

## Use our model to predict the mass for all starwars characters (excl. Jabba).
## Note that I'm including a 95% prediction interval. See ?predict.lm for other
## intervals and options.
predict(ols1_train, newdata = starwars2, interval = "prediction") %>%
    head(5) ## Just print the first few rows
```

```
## fit lwr upr
## 1 68.00019 46.307267 89.69311
## 2 65.55178 43.966301 87.13725
## 3 30.78434 8.791601 52.77708
## 4 82.69065 60.001764 105.37954
## 5 57.22718 35.874679 78.57968
```

Hopefully, you can already see how the above data frame could easily be combined with the original data in a **ggplot2** call. (I encourage you to try it yourself before continuing.) At the same time, it is perhaps a minor annoyance to have to combine the original and predicted datasets before plotting. If this describes your thinking, then there's even more good

⁸I'm sticking to a bivariate regression model for these examples because we're going to be evaluating a 2D plot below.

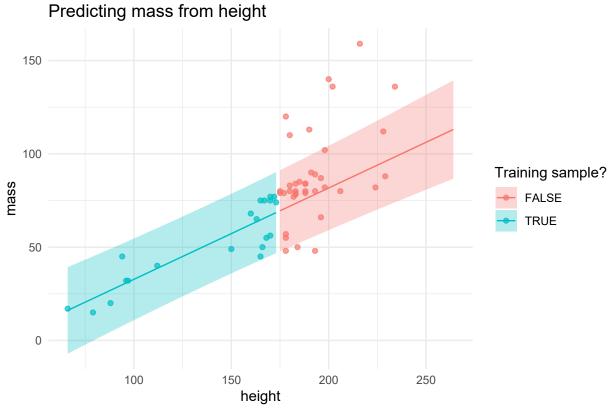
news because the **broom** package does more than tidy statistical models. It also ships the augment() function, which provides a convenient way to append model predictions to your dataset. Note that augment() accepts exactly the same arguments as predict(), although the appended variable names are slightly different.⁹

```
## Alternative to predict(): Use augment() to add .fitted and .resid, as well as
## .conf.low and .conf.high prediction interval variables to the data.
starwars2 = augment(ols1_train, newdata = starwars2, interval = "prediction")
## Show the new variables (all have a "." prefix)
starwars2 %>% select(contains("."), everything()) %>% head()
## # A tibble: 6 x 18
##
     .fitted .lower .upper .resid name
                                                 height mass hair color skin color
       <dbl> <dbl> <dbl> <dbl> <chr>
                                                  <int> <dbl> <chr>
##
                                                                         <chr>>
                                                           77 blond
## 1
        68.0 46.3
                      89.7
                             9.00 Luke Skywalker
                                                    172
                                                                         fair
## 2
        65.6 44.0
                      87.1
                             9.45 C-3PO
                                                           75 <NA>
                                                                         gold
                                                    167
## 3
        30.8
              8.79
                      52.8
                           1.22 R2-D2
                                                     96
                                                           32 <NA>
                                                                         white, bl~
## 4
       82.7 60.0
                     105.
                            53.3 Darth Vader
                                                    202
                                                          136 none
                                                                         white
## 5
        57.2 35.9
                      78.6 -8.23 Leia Organa
                                                           49 brown
                                                    150
                                                                         light
       70.9 49.1
## 6
                      92.8 49.1 Owen Lars
                                                    178
                                                          120 brown, gr~ light
## # i 9 more variables: eye_color <chr>, birth_year <dbl>, sex <chr>,
       gender <chr>, homeworld <chr>, species <chr>, films <list>,
## #
       vehicles <list>, starships <list>
```

We can now see how well our model — again, only estimated on the shortest 30 characters — performs against all of the data.

```
starwars2 %>%
  ggplot(aes(x = height, y = mass, col = rank(height)<=30, fill = rank(height)<=30)) +
  geom_point(alpha = 0.7) +
  geom_line(aes(y = .fitted)) +
  geom_ribbon(aes(ymin = .lower, ymax = .upper), alpha = 0.3, col = NA) +
  scale_color_discrete(name = "Training sample?", aesthetics = c("colour", "fill")) +
  labs(
    title = "Predicting mass from height",
    caption = "Line of best fit, with shaded regions denoting 95% prediction interval."
  )</pre>
```

⁹Specifically, we're adding ".fitted", ".resid", ".lower", and ".upper" columns to our data frame. The convention adopted by augment() is to always prefix added variables with a "." to avoid overwriting existing variables.



Line of best fit, with shaded regions denoting 95% prediction interval.

Further resources

- Ed Rubin has outstanding teaching notes for econometrics with R on his website. This includes both undergradand graduate-level courses. Seriously, check them out.
- Several introductory texts are freely available, including *Introduction to Econometrics with R* (Christoph Hanck *et al.*), *Using R for Introductory Econometrics* (Florian Heiss), and *Modern Dive* (Chester Ismay and Albert Kim).
- Tyler Ransom has a nice cheat sheet for common regression tasks and specifications.
- Itamar Caspi has written a neat unofficial appendix to this lecture, *recipes for Dummies*. The title might be a little inscrutable if you haven't heard of the recipes package before, but basically it handles "tidy" data preprocessing, which is an especially important topic for machine learning methods. We'll get to that later in course, but check out Itamar's post for a good introduction.
- I promised to provide some links to time series analysis. The good news is that R's support for time series is very, very good. The Time Series Analysis task view on CRAN offers an excellent overview of available packages and their functionality.
- Lastly, for more on visualizing regression output, I highly encourage you to look over Chapter 6 of Kieran Healy's *Data Visualization: A Practical Guide*. Not only will learn how to produce beautiful and effective model visualizations, but you'll also pick up a variety of technical tips.