



**TÉCNICO**  
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## **Circuit Theory and Electronics Fundamentals**

Integrated Masters in Aerospace Engineering, Técnico, University of Lisbon

Laboratory Report 5- Bandpass Filter, Group 28

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**Contents**

<b>1</b>	<b>Introduction</b>	<b>3</b>
<b>2</b>	<b>Theoretical Analysis</b>	<b>4</b>
2.1	Description and Important Mathematical considerations . . . . .	4
<b>3</b>	<b>Simulation Analysis</b>	<b>7</b>
<b>4</b>	<b>Comparison</b>	<b>9</b>
<b>5</b>	<b>Conclusion</b>	<b>10</b>

# 1 Introduction

The aim of this laboratory assignment was to dimension and implement a Band Pass Filter (BPF) using an OpAmp (Operational Amplifier) with a central frequency of 1KHz and a gain at central frequency of 40dB. As one should bear in mind, an OP-AMP is a transistor-based amplifier with main features, such as: high gain, high input impedance, low output impedance, differential input. The group was given a finite number of components to build this circuit, which is presented in the figure below. Moreover, it is important to highlight that Ngspice was used to simulate the behaviour of the circuit, allowing us to measure the output voltage gain in the passband, the central frequency, and the input and output impedances at this frequency. On top of that, the theoretical analysis, using the Octave tool, enabled us to compute the frequency response  $V_o(f)/V_i(f)$  and the gain, input and output impedances at the central frequency. Octave and Ngspice results were compared side by side.

The quality of the filter is evaluated by the following expression:

$$MERIT = \frac{1}{cost * (gaindeviation + centralfrequencydeviation + 10^{-6})} \quad (1)$$

The circuit is shown below as well as the values associated to each component (in V, Ohm and Farads).

Name	Value
C1	2.200000e-07
C2	1.100000e-07
R1	1.000000e+03
R2	1.000000e+03
R3	1.500000e+05
R4	1.000000e+03
Vcc	1.000000e+01

Table 1: Used Values of each component: Resistors[Ohm]; Capacitors[F]; Voltages[V]

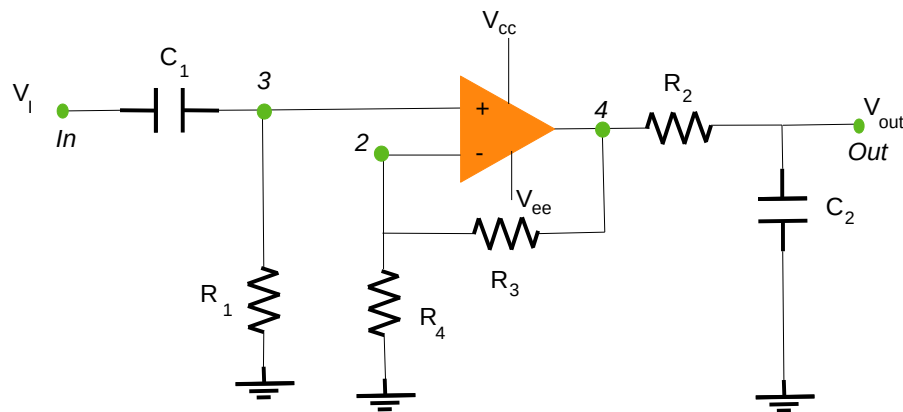


Figure 1: Circuit in analysis.

## 2 Theoretical Analysis

In this section, a theoretical analysis of the circuit shown in section 1 was conducted.

### 2.1 Description and Important Mathematical considerations

A theoretical approach of the circuit shown in section 1 was conducted. In fact the Op-Amp used was considered ideal, which means that the internal impedance between  $v_+$  and  $v_-$  is infinite and that there is no current flowing through it ( $v_+ = v_-$ ). Hence, a band pass filter was built by connecting a capacitor (C1) in series with the input voltage, which will function as a high pass filter. On top of that, in the final stage of the circuit, another capacitor (C2) was connected in parallel with the output voltage, functioning as a low pass filter. All in all, this circuit consists of a high pass filter, a signal amplifier and a low pass filter in series.

The transfer function of the circuit was determined. Accordingly to what was learned in lectures and in the presentational laboratory class, we reached the following conclusion:

$$T(s) = \frac{V_{out}(s)}{V_{in}(s)} = 1 + \frac{Z_{out}(s)}{Z_{in}(s)} = \frac{R1 * C1 * s}{1 + R1 * C1 * s} * \left(1 + \frac{R3}{R4}\right) * \frac{1}{1 + R2 * C2 * s} \quad (2)$$

In fact, the gain is given by the product of these 3 expressions ( $A_V$ ,  $A_H$ ,  $A_L$ ), where  $A_V$  is the gain that results from the OP-AMP (signal amplifier),  $A_H$  and  $A_L$  are the gains that correspond to the high pass and low pass filters, respectively.

$$A_V = 1 + \frac{R3}{R4} \quad (3)$$

$$A_L = \frac{1}{1 + R2 * C2 * s} \quad (4)$$

$$A_H = \frac{R1 * C1 * s}{1 + R1 * C1 * s} \quad (5)$$

Using the previous calculations we were able to determine the gain , which is shown below.

Name	Value
Central Freq (Hz)	1.023087e+03
Gain Central Freq (dB)	4.005771e+01

Table 2: Central Frequency(Hz) and Respective Gain(dB).

So that it can be better understood, and as it was requested, the gain frequency response was plotted. As one may observe, both low and high frequencies have a reduced gain. On the contrary, the frequencies in the central frequency (near to 1KHz) have a maximum gain associated. As it was expected for a band pass filter. This is due to the fact that, in the first stage, the high pass filter blocks the low frequencies (for low frequencies, the impedance goes to infinity and the capacitor is basically an open circuit) and, in the final stage the low pass filter does not allow the high frequencies to pass (for high impedances, the impedance goes to 0 and the capacitor is basically a short circuit). In fact, at the central frequency, both  $A_L$  and  $A_H$  are approximately 1. Hence  $A_V$  is the one that should be maximized.

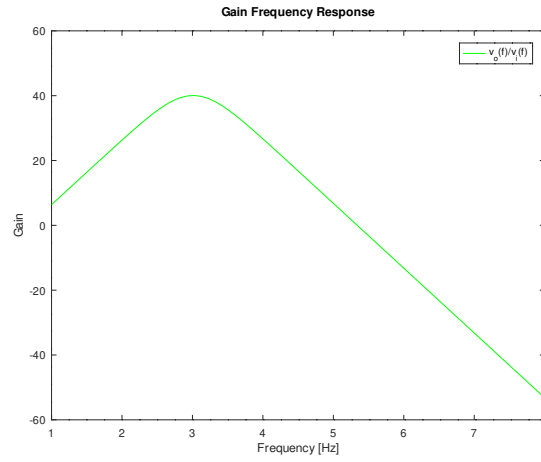


Figure 2: Gain Frequency Response.

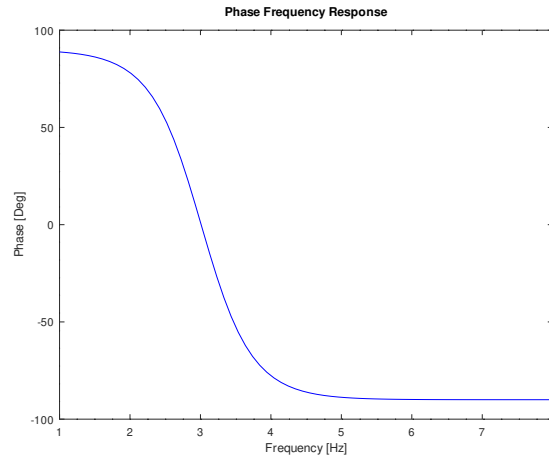


Figure 3: Phase Frequency Response.

We also computed the low cutoff frequency and the high cut off frequency in octave. The central frequency was also calculated. The equations and the results computed are the following:

$$\omega_L = \frac{1}{R1 * C1} \quad (6)$$

$$\omega_H = \frac{1}{R2 * C2} \quad (7)$$

$$\omega_0 = \sqrt{\omega_L * \omega_H} \quad (8)$$

Name	Value
LowFreq BandPass [rad/s]	4.545455e+03
HighFreq BandPass [rad/s]	9.090909e+03
Central Freq [rad/s]	6.428243e+03
Central Freq [Hz]	1.023087e+03

Table 3: Low cut-off frequency, High cut-off frequency, Central Frequency.

As requested, the input and output impedances were also calculated as follows:

$$Z_{in} = R1 + \frac{1}{j * \omega_0 * C1} \quad (9)$$

$$Z_{out} = \frac{R2}{j * \omega_0 * C2 * (R2 + \frac{1}{j * \omega_0 * C2})} \quad (10)$$

The results of the input and output impedances are presented in the table below. The central frequency previously calculated was considered.

Name	Value
Z in	1.000000e+03 + -7.071068e+02j
Z out	6.666667e+02 + -4.714045e+02j

Table 4: Input and output Impedences [Ohm]

The figure of merit was also computed, following the expression presented in section 1.

Name	Value
Cost	1.405788e+04
Merit	2.952448e-06

Table 5: Cost and Merit.

### 3 Simulation Analysis

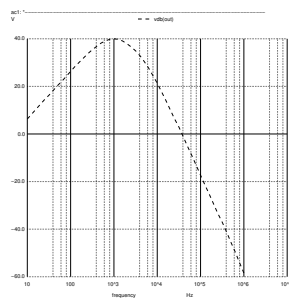
In this section, the several steps taken using ngspice in order to conduct the simulation of the band pass filter using an OP-AMP, as requested, will be described. The main focus of the simulation was to determine and optimize the values of the gain, the central frequency and the output and input impedances. The quality and overall figure of merit will then be analysed. The group proceeded as follows:

1. Design of the circuit, having as a starting point the circuit presented in section 1
2. In the frequency domain, measure of the output voltage gain, using the function .meas as well as the lower and upper cut off frequencies and the central frequency.

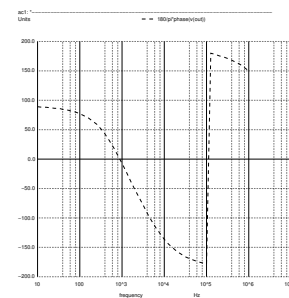
Gain (dB)	39.977
Central Frequency (Hz)	1000
Gain deviation	0.263873
Central frequency deviation	0

Table 6: Results for ngspice

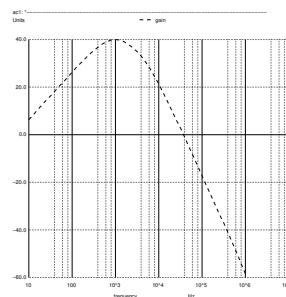
Then, the response of the circuit in dB and the phase were computed. The gain was also determined. In fact, the main goal of the assignment was to design a band pass filter. This means that this filter should cut both low an very high frequencies.



(a) Output Voltage in dB



(b) Output voltage (phase in degrees)



(c) Gain in dB

3. Determination of the input impedance, seen from the input voltage source.

Z <sub>in</sub>	998.999 + -7.2972 j
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Table 7: Input impedance in Ohm

The result obtained for the input impedance, considering the value in Ohm, is high. This is beneficial for the gain, because the voltage in the node In 2 must be as similar to V<sub>in</sub> as possible. Using a voltage divider, the only way to achieve this was to have a very high resistance value.

4. Determination of the output impedance, using a different set up, seen from the load resistance.

Z <sub>o</sub>	0.20916 + -14.4657 j
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Table 8: Output impedance in Ohm

Concerning the input impedance, an opposite deduction to the one made for the output impedance is mandatory. Considering a voltage divider, the output impedance must be as low as possible, in order to the output voltage to be as high as possible. Having said that, an analysing tables 7 8, the difference needed between the two is confirmed.

5. Compute of the cost and figure of merit

To finally understand the efficiency of the amplifier, the cost and figure of merit were calculated

Cost	14057.9
Merit	0.000269578

Table 9: Cost and Figure of merit

Analysing table 9, the results obtained may be considered satisfying.



## 4 Comparison

In this section, a global comparison between Octave and Ngspice results will be made.

As requested we were able to compute the passband frequency in simulation analysis using the measure function and the central frequency in the theoretical analysis as already explained. Moreover, the POP-AMP model is far more complex in Ngspice than in Octave. The different calculation methods lead to the difference between both methods. The calculated and simulated impedances have the same issues. However, our results were fairly satisfying.

Calculus	Value
Gain (dB)	39.977
Central Frequency (Hz)	1000
Gain deviation	0.263873
Central frequency deviation	0

Table 10: Central frequency [Hz] and respective gain [dB]. (Octave)  
gain [dB]. (Ngspice)

Name	Value
Central Freq (Hz)	1.023087e+03
Gain Central Freq (dB)	4.005771e+01

Table 11: Central frequency [Hz] and respective

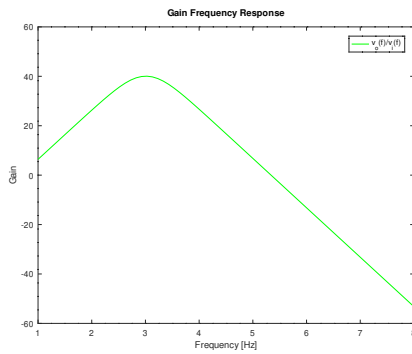


Figure 5: Gain [dB]. Octave

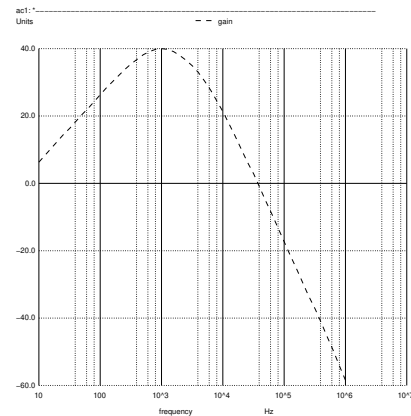


Figure 6: Gain [dB]. Ngspice

Calculus	Value [Ohm]
Zin	998.999 + -7.2972 j

Table 12: Cirtcuit impedances. Variables are expressed in Ohm.(Ngspice)

Name	Value [Ohm]
Z in	1.000000e+03 + -7.071068e+02j
Z out	6.666667e+02 + -4.714045e+02j

Table 13: Cirtcuit impedances. Variables are expressed in Ohm.(Octave)

We must also compare the phase plots. In octave, thanks to the capacitors we have 2 poles and 2 roots. However, in Ngspice we see 4 poles, again due to the complexity of the model used by this software.

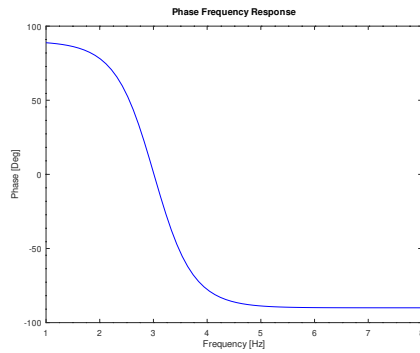


Figure 7: Output voltage (phase). Octave

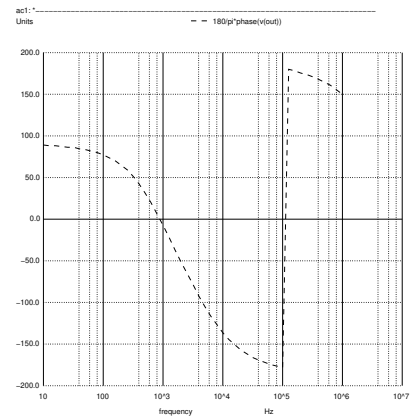


Figure 8: Output voltage (phase). Ngspice

## 5 Conclusion

As discussed in the introduction, the main goal of this assignment was to project a bandpass filter using an OP-AMP. which should allow maximum voltage gain, spending the less possible on the components used, among the ones available.

Despite strong efforts to match the results obtained in the different analysis, it was concluded that, due to the non-linearity of the components of the circuit, particularly the OP-AMP, it was impossible to obtain the exact same quality using both tools. In fact, the complexity of the parameters of the amplifier used in ngspice, are impossible to be replicated in the theoretical analysis. This is believed to be the main reason for the diferrent output of the circuits.

Nevertheless, once the ngspice model is the most similiar to reality, the model used can then be validated.