Circuit Theory and Electronics Fundamentals

Integrated Masters in Aerospace Engennering, Técnico, University of Lisbon

Laboratory Report 2- Group 28

Beatriz Pedroso 95773, Teresa Gonçalves 95826, Tiago Escalda 95851

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Contents

1 Introduction

The aim of this laboratory assignment is to analyse a RC circuit, which contains a sinusoidal voltage source v_s and a capacitor C. The other components present in this four mesh circuit are 7 resistors (from R_1 to R_7) and a linearly dependent current I_b and voltage V_d sources.

The voltage controlled current source depends on the constant K_d and the current controlled voltage source has a linear dependence on the constant K_b.

The voltage source varies in time as it follows:

$$v_s(t) = V_s u(-t) + sin(2\pi f t)u(t) \tag{1}$$

where

$$u(t) = e\begin{cases} 0 & \text{t<0} \\ 1 & \text{t} \geq 0 \end{cases} \tag{2}$$

The data generated automatically by the Python script is given in the table below.

Name	Value
R1	1.041113e+03
R2	2.099452e+03
R3	3.131091e+03
R4	4.119470e+03
R5	3.115588e+03
R6	2.047994e+03
R7	1.027544e+03
Vs	5.068716e+00
C	1.041275e-06
Kb	7.287471e-03
Kd	8.115684e+03

Table 1: Initial data generated by Python script. All variables are expressed in Ohms, V, F, S or A.

The nodes (from V_1 to V_8) are displayed as it shows in Figure 1. The fourth node is considered to be the ground one.

The report is subdivided in three major sections.

The first, the theoretical analysis(2). This section adresses, sequencially, the nodal analysis, then the calculus of R_eq. With these results, the natural response of the circuit, the forced response are computed, and then superimposed. To finish, it is made a study of the frequency response of the circuit.

The second, the simulation analysis(3). In order to validate the results obtained in section 2, operating point, transient and then frequency analysis are made.

The study of the circuit is concluded with a careful comparison between the results obtained using the different tools, in section 4. The conclusions of this study are outlined in the final section (**??**).

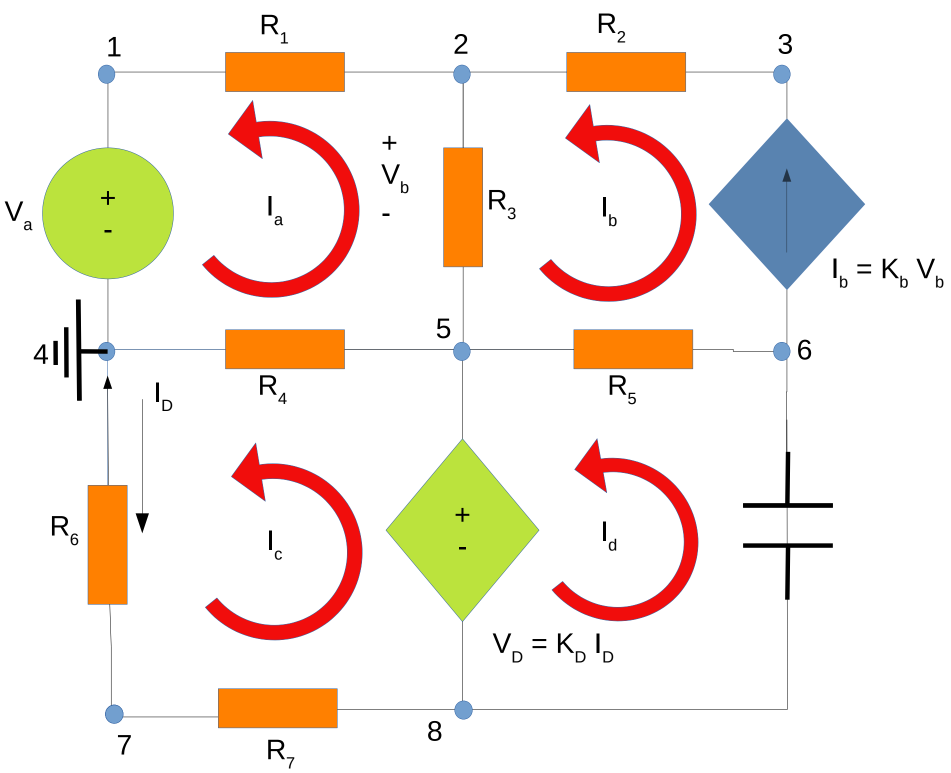

Figure 1: RC Circuit analysed.

2 Theoretical Analysis

2.1 Node analysis for t<0

In this section, a theoretical analysis of the circuit was conducted. The node method was the chosen approach.

The aim of using this method is to determine every node voltage. To do so, the node 4 was considered a reference node. Then, eight independent equations were written in orther to find the remaining unknown node voltage values. Before $t = 0s$, v(s) is constant. Therefore, the capacitor behaves like an open-circuit which means Ix=0.

The equations were then put in the form of the matrix shown below. Octave math tools were used to solve the system.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & 0 & -G3 & 0 & 0 & 0 \\ 0 & -Kb-G2 & G2 & 0 & Kb & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & -G3 & 0 & -G4 & G3+G4+G5 & -G5 & -G7 & G7 \\ 0 & Kb & 0 & 0 & -Kb-G5 & G5 & 0 & 0 \\ 0 & 0 & 0 & -G6 & 0 & 0 & G6+G7 & -G7 \\ 0 & 0 & 0 & Kd*G6 & -1 & 0 & -Kd*G6 & 1 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \end{bmatrix} = \begin{bmatrix} Vs \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

2.2 Calculus of R_{eq}

The purpose of this task was to compute the Req (equivalant resistance) of the circuit.

It is known that:

$$R_{eq} = \frac{V_x}{I_x} \tag{3}$$

In order to determine the variable aforementioned, first it was sugested to calculate it seen from the capacitors terminals. Then, using the Thevenin and Norton theorem, we put the independent voltage source to 0V. V_x is equivalent to Thevenin's Voltage, and I_x to Norton's Current. This is necessary because the depedent voltage short cannot be put equal to 0V(short-circuit) and the independent current source cannot be erased from the circuit.

An ilustration of the circuit analysed is showed in figure 2

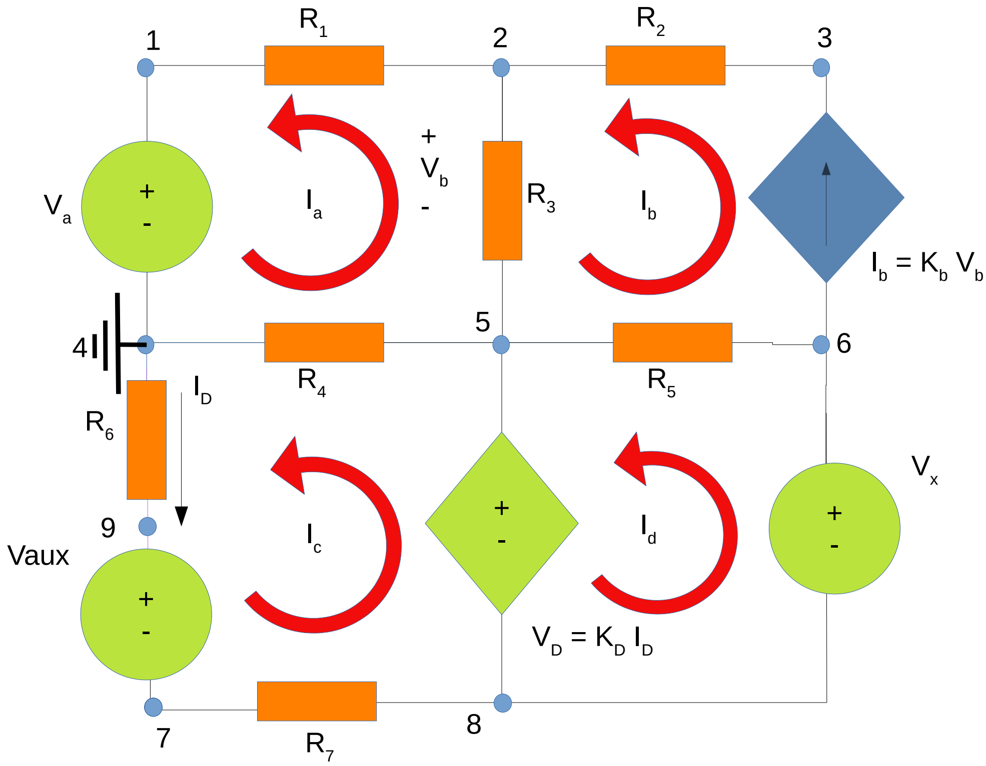

Figure 2: RC Circuit analysed in section 2.

Then, the values of current of every branch and the nodal values are obtained using node method. The matrix used in octave is the one that follows.

$$\begin{bmatrix} 1 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & 0 & -G3 & 0 & 0 & 0 & 0 \\ 0 & -G2-Kb & G2 & 0 & Kb & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 & 0 \\ 0 & -G1 & 0 & 0 & -G4 & 0 & -G6 & 0 & 0 \\ 0 & Kb & 0 & 0 & -G5-Kb & G5 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 & G6+G7 & -G7 & 0 \\ 0 & 0 & 0 & 0 & 0 & 1 & 0 & -1 & 0 \\ 0 & 0 & 0 & KdG6 & -1 & 0 & -Kd*G6 & 1 & 0 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \\ I_x \end{bmatrix} = \begin{bmatrix} 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ V_x \\ 0 \end{bmatrix}$$

Theoretically speaking, once the voltage source is equal to 0V, V_4 and V_1 are also to be 0.

2.3 Node analysis for t $\geq$ 0 (Natural solution)

It was proposed to study and determine the natural response of the circuit over time, in node 6. The natural response is what the circuit does including the initial conditions (initial voltage of the capacitor) but with the imput supressed.

Name	Value
V1	0.000000e+00
V2	0.000000e+00
V3	9.496396e-16
V4	0.000000e+00
V5	5.935248e-17
V6	8.553593e+00
V7	-2.967624e-17
V8	0.000000e+00

Table 2: Theoretical nodal voltage results. All variables are expressed in V.

Name	Value
Ix	-2.745419e-03

Table 3: Theoretical Ix. Expressed in A.

In order to calculate the natural solution, we have to eliminate the voltage source, which means vs(t)=0V. Hence, we have an equivalent cicuit described by a voltage source and the capacitor. The current flowing through the capacitor is in fact consumed by it. Therefore, the voltage V8=0V and the amplitude $Vx = V6 - V8 = V6$.The natural solution will have the format $V6n(t) = A * e^{(-t/tau)}$ with $tau = Req * C$ and $A = V6$ obtained in 2.2. As expected, the result is a negative exponencial graph, shown bellow.

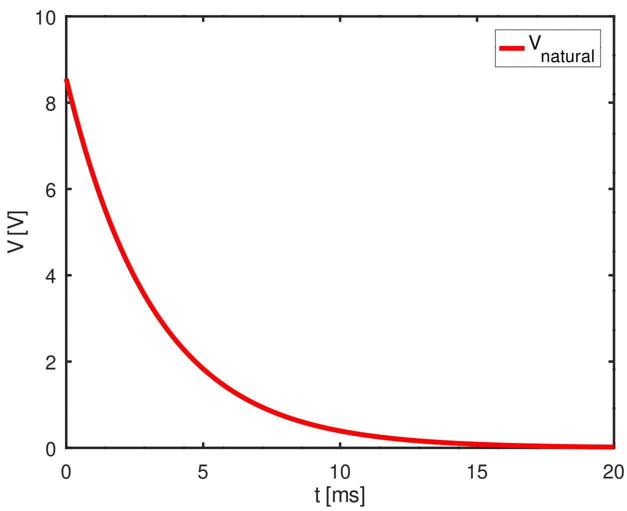

Figure 3: Circuit analysed.

2.4 Node analysis for t $\geq$ 0 (Forced solution)

The forced response of a circuit is calculated with the sources turned on, but with the initial conditions (internal stored energy) set to zero.What is forced response? The forced response is where the output (the voltage on the capacitor) is going to end up in the long run after all stored energy eventually dissipates. The forced response does this by ignoring the presence of energy storage elements (in this case, it ignores the capacitor and its initial voltage). However, the forced response can't tell us what happens at t=0, or during the transition to the final state, because it ignores the stored energy.

Like so, a phasor was used, with $V_s = 1$. The capacitor was replaced by its impiedence Z. Then, the nodal method was run again, with this new variable.

$$w = 2 * pi * f \tag{4}$$

Name	Value
Req	3.115588e+03
tau	3.244185e-03

Table 4: Theoretical Req and tau results. Expressed in Ohm and s.

$$Zc = 1/(j * w * C); \tag{5}$$

The matrix used was the following:

$$\begin{bmatrix} 1 & 0 & 0 & -1 & 0 & 0 & 0 & 0 \\ -G1 & G1+G2+G3 & -G2 & 0 & -G3 & 0 & 0 & 0 \\ 0 & -G2-Kb & G2 & 0 & Kb & 0 & 0 & 0 \\ 0 & 0 & 0 & 1 & 0 & 0 & 0 & 0 \\ 0 & G3 & 0 & G4 & -G3-G5-G4 & G5+1/Zc & G7 & -G7-1/Z_c \\ 0 & Kb & 0 & 0 & -G5-Kb & G5+1/Z_c & 0 & -1/Z_c \\ 0 & 0 & 0 & 0 & 0 & 0 & G6+G7 & -G7 \\ 0 & 0 & 0 & KdG6 & -1 & 0 & -Kd*G6 & 1 \end{bmatrix} \begin{bmatrix} V1 \\ V2 \\ V3 \\ V4 \\ V5 \\ V6 \\ V7 \\ V8 \end{bmatrix} = \begin{bmatrix} 1 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \\ 0 \end{bmatrix}$$

Then, the complex amplitudes with the knowlegde that the amplitude of the forced response is the absolute value of the complex V_6, and the phase is the argument, the forced solution is then given by:

$$V6_f = A * sin(w * t + Ph) \tag{6}$$

The tables below present the values that allows us to compute the comlex amplitude of every node voltage. In fact, it is obtained by the following expression:

$$V_{complex}(i) = V_i exp(-j * phase(i)) \tag{7}$$

Name	Value [V]
V1	1.000000e+00
V2	9.556014e-01
V3	8.619659e-01
V4	2.801649e-17
V5	9.617215e-01
V6	5.886212e-01
V7	3.907822e-01
V8	5.868501e-01

Table 5: Amplitudes of Nodal Voltages

Name	Value [Rad]
Phase1	0.000000e+00
Phase2	-5.722643e-16
Phase3	-1.142921e-15
Phase4	3.141593e+00
Phase5	-5.388346e-16
Phase6	-3.000819e+00
Phase7	3.141593e+00
Phase8	3.141593e+00

Table 6: Phase of Nodal Voltages

2.5 Natural and Forced Superimposed

The total response of a circuit can be teased apart into a forced response plus a natural response. These responses can be combined using the principle of superposition. This principle pressupose the addition of the natural response and the forced response, both calculated in question 3 and 4.

The final solution for $V(6)_{final}$ is then given by:

$$V6_{final} \begin{cases} V_6 - V_8 & \text{t<0} \\ V(6)_n + A * sin(w * t + Ph) & \text{t} \geq 0 \end{cases} \tag{8}$$

The final solution for $V(S)_{final}$ is then given by:

$$VS_{final} \begin{cases} Vs & \text{t<0} \\ e.^{(} - j * (w * t_pos - pi/2)) & \text{t} \geq 0 \end{cases} \tag{9}$$

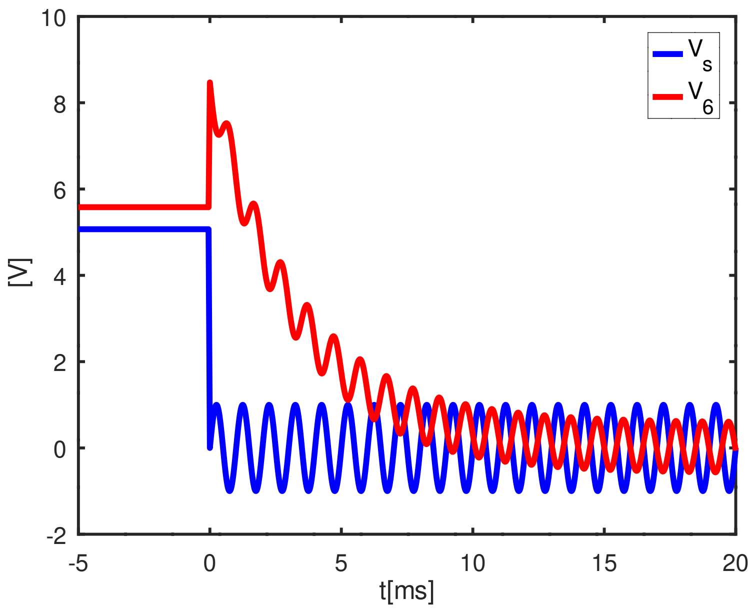

Figure 4: Final response.

It is observed in 4 that $V(6)_{final}$ tends to diminuish until 20ms, the end of the period. By this time, the phase $V(6)_{final}$ and the phase of VS_{final} differ π or 180 degrees.

2.6 Frequency Responses

In this section, both octave and ngspice were used in order to obtain plots of the phase response and of the magnitude response, using logscale. This approach is very useful hence it provides a much better plot fit and, therefore it provides great visualization for users. The magnitude in debicels is of interest for analysis of sound waves, and the analysis of the phase or angle delay is a very interesting way of study another parameter to compare signal. Frequency range in both analysis was from 0.1 Hz to 1MHz. The plots made were v6(f), vs(f) and vc=(v6(f)-v8(f)).

To examine the frequency responses, the system of equations in Section 4 is solved in a loop cycle, what allows us to calculate the V_6, V_c and V_s for each frequence. For each result of these complex vectors, the values were saved.

2.6.1 Frequency Response- Magnitude

To represent the magnitude in dB, the absolute values were converted (X_{dB}=20log_{10}(X)). The frequencies were put in a logarithmic scale.

The magnitude of V_s does not suffer any alteration with the variation of the frequency of the signal. Since its amplitude is 1, as one can observe in the graphics shown at the end of the section, the plot shows a constant horizontal line, with the value zero (0=log_{10}(1)).

On the contrary, as the frequency is increasing, the magnitudes of V_6 and V_c decrease. The value of V_c changes as it is expected in a RC circuit. This is due to the impedance of the capacitor (Z=-j/wC).

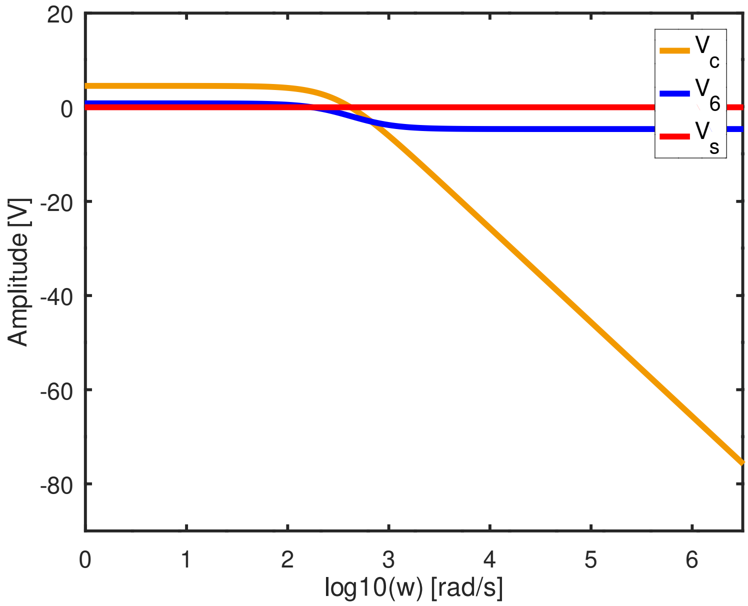

Figure 5: Circuit analysed.

2.6.2 Frequency Response- Phase

The angles of the values saved were transformed from radians to degrees. The phase in V_s is 0, therefore we do not have to do any further calculations. The phase of each voltage corresponds to the exact angles. In the plot shown below, the frequencies were also put in the logarithmic scale. As the frequency increases the phase tend to negative values, which varies as it is expected.

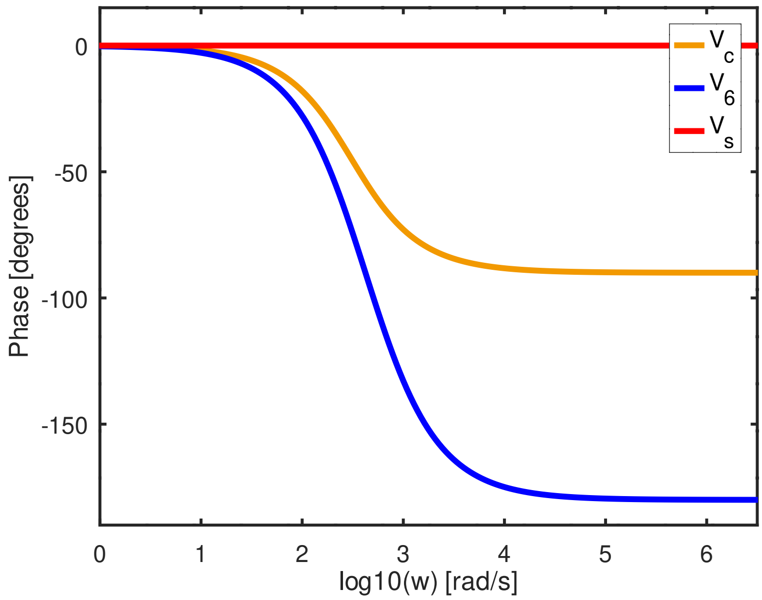

Figure 6: Phase response.

3 Simulation Analysis

3.1 Operating Point Analysis for t<0

First of all, to contextualize the values obtained using the tools in ngspice, it is necessary to state that, as node 0 is connected to ground, its nodal voltage does not appear on the table of results. Furthermore, to be able to describe the voltage flowing in the dependent source, it is necessary to know the current in resistor 6. However, ngspice is not able to compute this value when the depedent source is described. So, in order to do that, an extra dependent voltage source (whose voltage drop is equal to 0 V) was created, and put in series after the resistor 6. This led to the appearence of node 8, that has the same voltage drop as node 6. So, by doing that, ngspice is able to determine the current in this auxiliar independent source, which is exactly the value needed. The circuit with these changes is shown in the drawing below.

3.2 Operating Point Analysis

As requested, an operating point analysis was conducted. The capacitor was replaced by the independent voltage source V_x (2). The values of currents and nodal voltages were then put in a table, as well as the R_{eq}, hence:

$$R_{eq} = \frac{v(6) - v(8)}{vxbranch} \tag{10}$$

3.3 Operating Point Analysis for $\geq$ 0 (Natural Solution)

In this section, a transient analysis was made in order to evaluate the natural response of the circuit, which means, the variation over time. The description of the circuit included the initial values of v(6) and v(8), calculated in question 1. As the ngspice and octave results matched, as observed in **??**, the theorectical values were imported from a .cir file created by octave. The time interval considered was [0,20]ms.

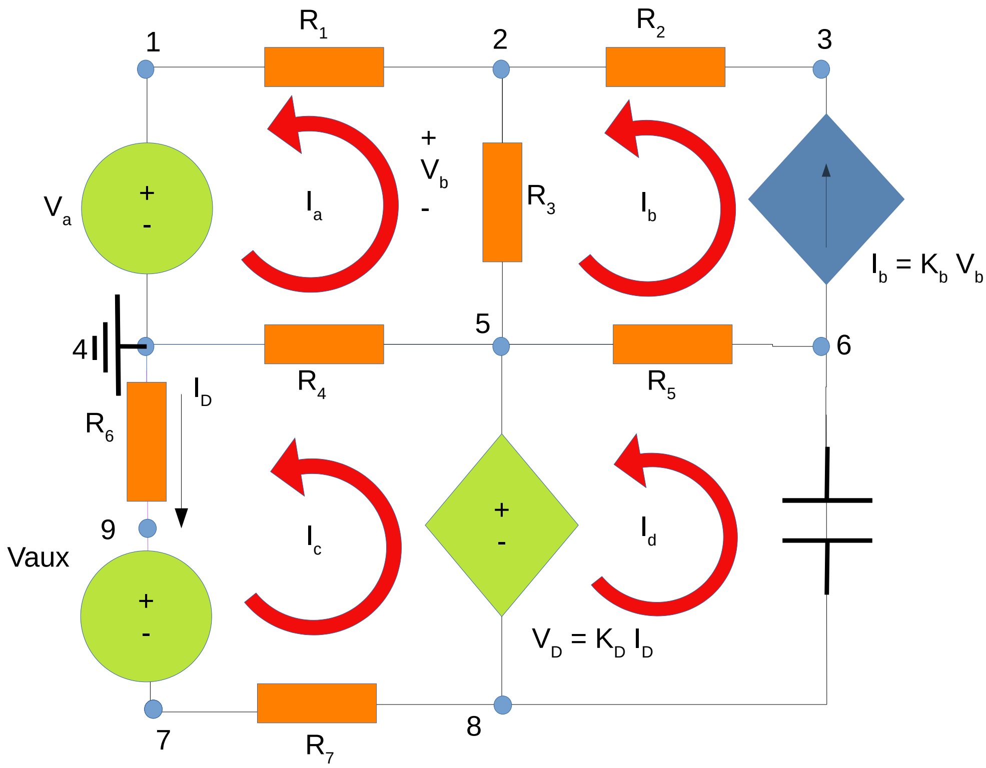

Figure 7: Circuit analysed in ngspice.

3.4 Operating Point Analysis for ≥ 0 (Natural and Forced Solutions)

Once again, a transient analysis was made in order to meet the goal above. The main difference between the analysis in question 3 and this one is that Vs was considered a sinusoidal voltage source. This way, the plot obtained is the sum of both responses.

3.5 Comparison

When observing 9, we conclude that, in the period of time considered, the voltage in the capacitor tends to diminuish until its phase differs π from the phase of the voltage source. The same result has also been accomplish in the theoretical analysis, as seen in 4.

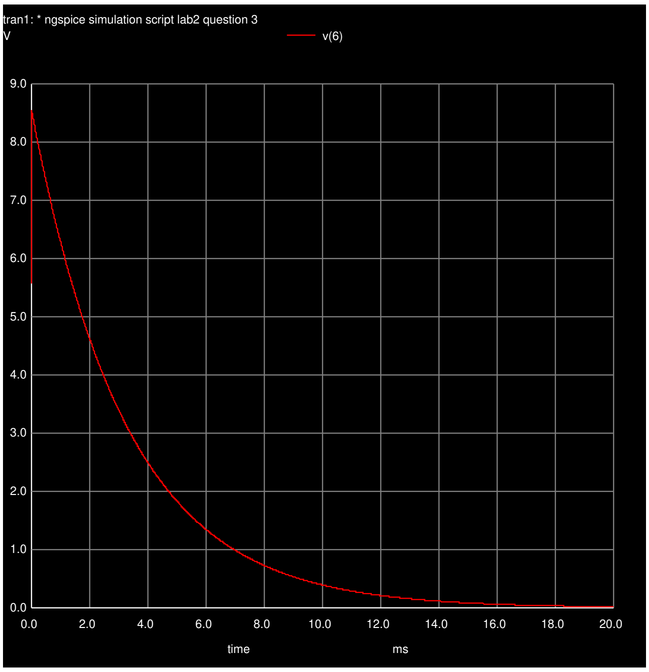

Figure 8: Circuit analysed.

3.6 Frequency Responses

In this part of the assignment, an AC (Small Signal Analysis was conducted, in order to match the goal aforementioned. This type of analysis allows to study the frequency response of the circuit. In other words, there is no frequency variation over time, the so called steady-state analysis.

3.6.1 Frequency Response- Magnitude

3.6.2 Frequency Response- Phase

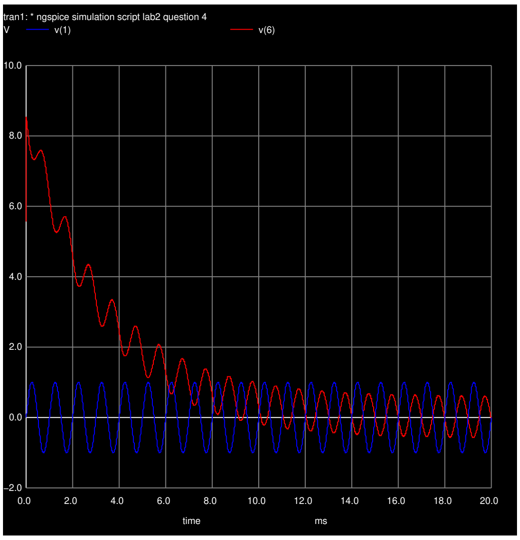

Figure 9: Total Response - Ngspice

After comparing the graphics showed below, it is clear to admit that the results in ngspice and octave match. Any minor difference may be explained by aproximation errors.

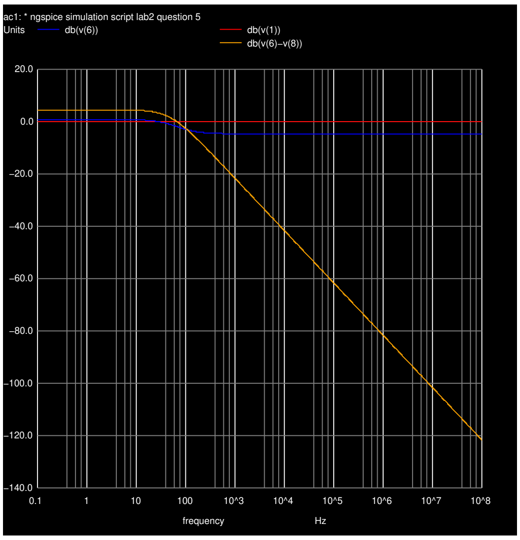

Figure 10: Magnitude Response (in decibels)

4 Octave and Ngspice Comparison

4.1 Analysis for t<0

After running the simulation, the results were put in the table below. Then, a careful analysis of the aforementioned table was conducted.It shows the simulated operating point results for the circuit that is being studied, allowing the group to obtain the current flowing in every risistor, the voltage in the dependent voltage source and even the current flowing in the dependent currrent source.

A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt.

In order to validate the results obtained in NGSPICE, relative errors between the theoretical values, obtained in octave and the ones obtained in ngspice, were calculated. These were put in the table below.

After running the node analysis in octave and the simulation in ngspice, the results were put in the tables below. Then, a careful analysis of the aforementioned tables was conducted.

A variable preceded by @ is of type *current* and expressed in Ampere; other variables are of type *voltage* and expressed in Volt. R_{eq} is presented as in 10 in Ohm.

After the careful evaluation of the results, several observations need to be made. Firstly, in ngspice, there are some node voltage results different from 0V, with values in the order of magnitude of $10^-15, 10^-16$ V. After questioning the professor, these will be considered 0V.

On the other hand, octave results have equally those discrepancies. That said, the procedure was the same and those values were considered 0. We believe the main reason for this situation is that, when the data.txt file is read from the datagen.py in octave, a rounding happens, making it impossible for the nodal voltages to be 0.

However, and despite the small erros above explaned, the values of R_{eq}, V_6 and I_x match perfectly, which led us to fully validate the theorical procedure and the results obtained.

4.2 Natural Solution for t>0

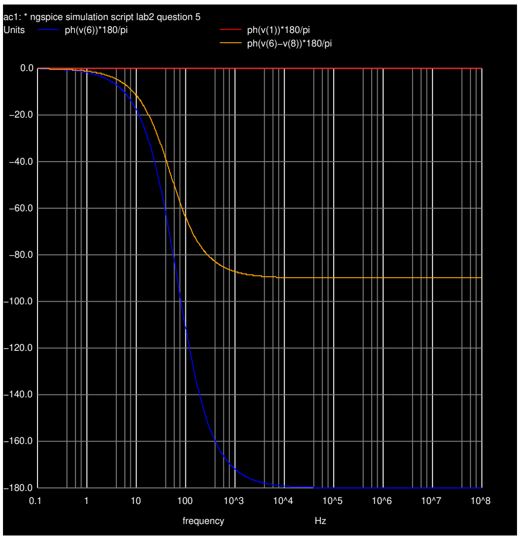

Figure 11: Phase Response (in degrees)

5 Conclusion

It was agreed by the members of the group that the main goal of the task proposed was achieved. As presented, both theoretical and simulation results(obtained using Octave tools and ngpsice simulator, in all its variants(operating, transient and frequency), respectively) matched, reaching total accuracy. Despite the initial belief that the considerable number of components of the circuit, and the number of steps needed to get to the final answer(the total solution), could cause some disparity in the results, such did not happened.

To conclude, the model used can then be validated

Name	Value [A or V]
@c1[i]	0.000000e+00
@gb[i]	-2.26065e-04
@r1[i]	2.161572e-04
@r2[i]	-2.26065e-04
@r3[i]	-9.90741e-06
@r4[i]	1.183330e-03
@r5[i]	-2.26065e-04
@r6[i]	-9.67173e-04
@r7[i]	9.671730e-04
v(1)	5.068716e+00
v(2)	4.843672e+00
v(3)	4.369060e+00
v(5)	4.874693e+00
v(6)	5.579017e+00
v(7)	-1.98076e+00
v(8)	-2.97458e+00
v(9)	-1.98076e+00

Table 7: Simulation and Calculus of Req (NgSpice)

Name	Value [A or V]
V1	5.068716e+00
V2	4.843672e+00
V3	4.369060e+00
V4	0.000000e+00
V5	4.874693e+00
V6	5.579017e+00
V7	-1.980764e+00
V8	-2.974577e+00

Table 8: Simulation and Calculus of Req (NgSpice)

Name	Value [A or V]
@gb[i]	-4.13942e-18
@r1[i]	-3.95801e-18
@r2[i]	-4.13942e-18
@r3[i]	1.814124e-19
@r4[i]	8.624201e-19
@r5[i]	-2.74542e-03
@r6[i]	-8.67362e-19
@r7[i]	-1.72875e-18
v(1)	0.000000e+00
v(2)	-4.12073e-15
v(3)	-1.28112e-14
v(5)	-3.55271e-15
v(6)	8.553593e+00
v(7)	1.776352e-15
v(8)	3.552714e-15
v(9)	1.776352e-15

Table 9: Simulation and Calculus of Req (NgSpice)

Name	Value [A or V]
V1	0.000000e+00
V2	0.000000e+00
V3	9.496396e-16
V4	0.000000e+00
V5	5.935248e-17
V6	8.553593e+00
V7	-2.967624e-17
V8	0.000000e+00

Table 10: Calculus of R_{eq}- Octave