Optimal Room Lighting

Group Number: 5
Subsystem 1: Light Distribution (Paolo Rüegg), Subsystem 2: Cost (Beatriz Lopez)

Abstract

Students living in campus-based accommodation often need to work in the evening and during the night to meet deadlines or prepare for exams. During these study periods, adequate and balanced room lighting is crucial to create an effective work environment. In this report, an optimisation study that determines optimal positions for placing lamps within a student room is presented. The system was decomposed into two subsystems. The former represented the light distribution subsystem with the objective of maximising the global minimum light intensity within the room. An unconstrained Nelder-Mead approach was chosen for initial problem space exploration and results from it were subsequently fed into a constrained optimiser [1][2]. The latter, a trust-region based method, revealed that some of the constraints imposed were active, and that even the darkest spot (756 Lux) was significantly above the recommended Lux threshold for libraries (500 Lux) [3]. The second subsystem aimed to reduce the cost of installing and running the lights. This took into account the trade-off between the initial and running investment depending on the efficiency and position of the lamps. Unconstrained optimization with a Nelder-Mead method achieved the most optimised solution. However, linear and nonlinear constraints were later implemented using a trust-region method, which yielded a minimum cost of £173.02 - 13,5% below the budget. Finally, a system-level study examined how the trade-off between balanced lighting and total cost could be balanced. This study reveals that weighing the light subsystem at 0.2 yields the best trade-off and achieves a minimum light intensity of 733 Lux at a cost £177.

1. Introduction

Effective lighting in student accomodation is crucial, because students' living and work cycles are very individual. This report is concerned with achieving the most balanced and cost-effective light distribution in a student accommodation room by optimising the location of the lamps within the room. A four by three metres sized room was considered, containing three lamps of different types; a bedside lamp, a desk lamp and a floor lamp. A furnished student room poses various constraints in terms of where a lamp can be placed. There is usually a desk, a single bed, cupboard space, as well as the door [4]. The idealised room below (Fig. 1A) shows the location of the furniture considered for this report. In Fig. 1B, the constraints for placing the lamps are illustrated, each of which needs to be at least a lamp radius away from the walls. Bedside and desk lamp could either hang from the ceiling or be placed on the floor. As a consequence, they were allowed to be placed over the bed or desk as well.

The goal of this study is to generate a framework for students to see how they can place their lamps within their rooms. This study lays the basis for a future app into which a prospective student could input lamps available to him or her, as well as the student room geometry including furniture and plugs. The code running on the back-end is presented on GitHub (approx. 1,000 lines) and determines the optimum positions for the placing the lamps. Optimisation was performed using the scipy library for Python 3 [5]. Reasons for that choice were the open-source nature of Python and ease of integration into a web app.

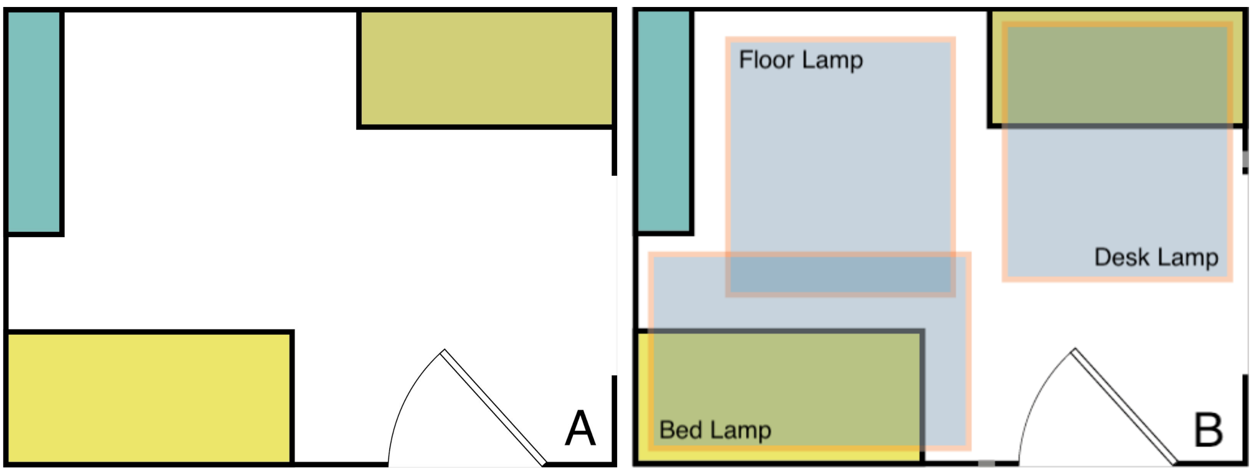

Fig 1: Graphical representation of the room environment without (A) and with constraints (B).

2. System-level Problem and Subsystem Breakdown

The objective of this optimisation study is to **maximise the minimum light intensity in a room with minimal cost**. To effectively approach a solution for this problem, a clear definition of the system was essential. A decomposition approach with a hierarchy level division into two subsystems was chosen in order to break down the problem. The diagram below (Fig. 2) shows the system concept. The environment for this study was a the student room, with a cost and light distribution subsystem. The inputs include the cost of installing and running the lamps, while light and heat were outputs. Fig. 3 represents the system diagram with the hierarchy levels, where the subsystems have been further divided using aspect decomposition.

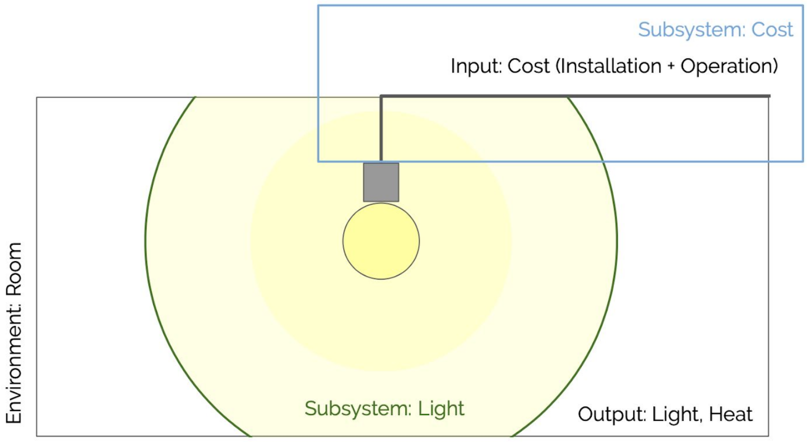

Fig 2: System concept

Once an understanding of the problem environment was achieved, the variables and parameters for the optimisation study were defined (refer to *Appendix A*) . At this point, assumptions had to be made in order for the problem to be manageable. The room was assumed to be rectangular, with a size of 4 by 3 metres, the average size of a room at Imperial College halls [4]. The room includes a bed, a desk, storage space as well as two plugs. The number of lamps was restricted to three lamps; a desk lamp, a bedside lamp and a floor lamp, where the height of each lamp was kept constant at 1.6 metres. This corresponds to the line of sight for the average UK male and female [6], at which a 2D model is most representative of the perceived illuminance. The shared design variables between the subsystems are the x-, and y-positions of the lamps, whereas lamp efficiency is only a variable for the cost subsystem.

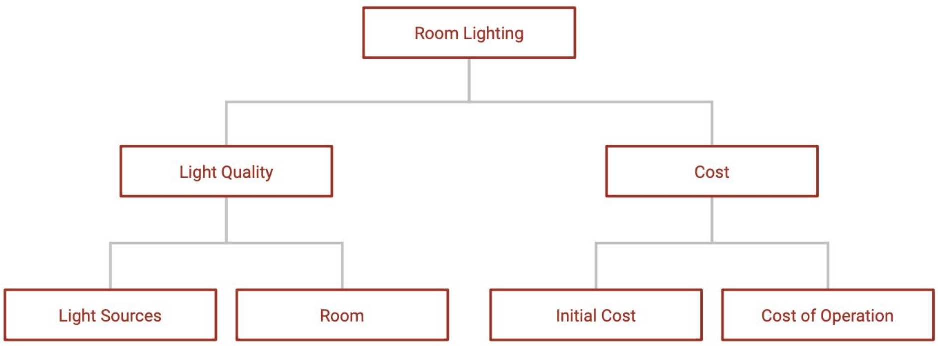

Fig 3: System decomposition

The rest of the parameters include cost of power, work and cable as well as average hours of use per year [7][8]. Lamp radii were set related to average values for their lamp types. The investment factor reflects the justification of an initial investment for the installation of the lamps while this is lower than the running costs after a set number of years. This was set at 3 years, which relates to the length of a Bachelor's degree in the UK. The power of the lamps was set to 120 W for the floor lamp and 50 W for the desk and bedside lamp. These powers are representative of bulbs available at UK supermarkets and take into account that floor lamps are generally more powerful than bedside or desk lamps. Finally, the albedo (reflectance) of the wall was set at a value of 0.5, a standard value for interior walls [9].

A thorough initial analysis of the problem was essential to anticipate future problems and recognise the trade-off between maximising the light balance and minimising cost. Both of these presented limitations. The minimum illuminance to be achieved for the student room was set at 500 Lux, corresponding to relevant values for library work spaces [3]. The maximum budget for the initial and operation cost of lightning a room for a year was limited to £200. These limitations are represented in the Fig. 4, where the feasible domain and the relationship between the two subsystems is shown. The line section from A to B depicts the Pareto set.

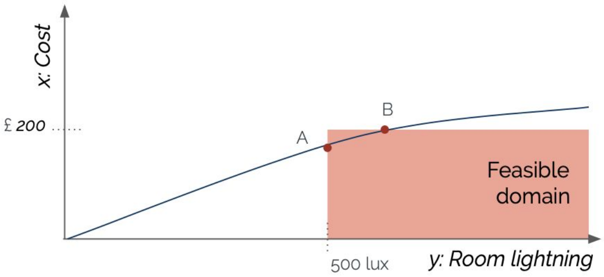

Fig.4: Pareto set

Therefore, our mathematical model can be represented by the following function. Note that the negative of the minimum light intensity is taken in order to maximise it.

$$min_x \ f[x_i, y_i, e] = w_{cost}C(x_i, y_i, e) - w_{light}E_{min}(x_i, y_i)$$

Where E_{min} and C refer to minimum illuminance within the room and cost, respectively. The effect of varying weights for both subsystems are explored in the system-level optimisation. Further mathematical derivations can be found in *Section 3* and *4* of this report as well as on the Github repository.

3. Subsystem 1: Light Distribution (Paolo Rüegg)

The goal of this subsystem optimisation was to find the lamp distribution that maximises the minimum light intensity within the room. This ensures that the illuminance has the least possible range within the room, making it as homogenous as possible, thereby creating a balanced lighting.

3.1 Optimisation formulation

A full derivation of the objective function is lengthy and can be found on the team's GitHub repository in the subdirectory *subsystem_light*. The resulting function determines the minimum light intensity in a 2D room given the x- and y-positions of three isotropic lamps that are placed in it [10]. Since the interim review, the equation has been expanded to include reflections, and involves a variety of parameters as presented below. Please refer to *Appendix A* for nomenclature and to *Section 2* for assumptions.

$$E_{min}(x_1, y_1, x_2, y_2, x_3, y_3) = min\left(\sum_{j=-\infty}^{\infty} \sum_{i=1}^{3} \frac{\alpha^{|2j|}\eta P_i}{4\pi((x-(x_{j,i}-Lj))^2+(y-(y_{j,i}-Wj))^2)} \right)$$

$$\begin{aligned} where \quad x_{j,i} &= \tfrac{1}{2}\left(1+(-1)^j\right)x_i + \tfrac{1}{2}\left(1-(-1)^j\right)(L-x_i) \\ y_{j,i} &= \tfrac{1}{2}\left(1+(-1)^j\right)y_i + \tfrac{1}{2}\left(1-(-1)^j\right)(W-y_i)\,. \end{aligned}$$

The lamp efficiency was fixed as 0.8 for this subsystem, all other values can be retrieved from the source code or *Appendix.* In negative null form, the problem can be formalised as follows.

$$\min_x \quad E_{min}(x_1, y_1, x_2, y_2, x_3, y_3, L, W, \eta, P_1, P_2, P_3, r_1, r_2, r_3, \alpha)$$

$$\begin{aligned} where \quad & x = (x_1, y_1, x_2, y_2, x_3, y_3) \\ & p = (L, W, \eta, P_1, P_2, P_3, r_1, r_2, r_3, \alpha) \end{aligned}$$

$$\begin{array}{llll} s.t. & g_1 : 0.1 - x_1 \leq 0 & g_5 : 0.6 - x_2 \leq 0 & g_9 : 2.4 - x_3 \leq 0 \\ & g_2 : x_1 - 2.2 \leq 0 & g_6 : x_2 - 2.1 \leq 0 & g_{10} : x_3 - 3.9 \leq 0 \\ & g_3 : 0.1 - y_1 \leq 0 & g_7 : 1.1 - y_2 \leq 0 & g_{11} : 1.2 - y_3 \leq 0 \\ & g_4 : y_1 - 1.4 \leq 0 & g_8 : y_2 - 2.8 \leq 0 & g_{12} : y_3 - 2.9 \leq 0 \end{array}$$

It should be noted that the lamp radii are only used during the computational evaluation of the function and are only included in the parameter set for completeness. The constraints were defined as a set of linear constraints corresponding to Fig. 1B, bounding every design variable from above and below.

3.2 Modelling approach

The light distribution was modelled in Python by discretising the room at a step of 0.01 and calculating the resulting intensity at every point within the grid. This transformed the 4 by 3 metres room into a 400 x 300 meshgrid, where the value of each entry represents the light intensity at that location. The numerical minimum was then found and fed into the optimisation engine, provided the constraints were satisfied. Grid positions within the radius of the lamp were excluded, since intensities approach infinity as the distance to the light source approaches zero, complicating computational evaluation. Also, light intensities within the lamp are irrelevant for room lighting. Maximising the numerical minimum ensures a compact optimisation procedure while the complete light distribution was retained for subsequent contour plotting.

An object-oriented Python implementation was chosen in order to achieve versatility. As a consequence, the code can immediately be reapplied to any two-dimensional room. Albeit lengthier than a MatLab implementation, it can run at the back-end of a web browser, and is therefore a more realistic prototype for a web-based application. The actual optimisation was performed using the scipy.optimize.minimize() function [5]. It calls the objective function and determines new values for the set of design variables.

3.3 Problem Space Exploration

The problem was initially approached in three dimensions, however, it quickly became apparent that such an optimisation was too computationally expensive. One iteration of the Nelder-Mead algorithm in 3D took about 15 seconds, compared to ⅓ of a second in 2D. This was the first major problem space reduction, which was justified with the assumption that all lamps are located at eye level. Nonlinear constraints concerning minimum distances between lamps were also removed, since they were proven inactive early on. Further simpfliciation opportunities were explored using monotonicity analysis. It emerged that the some constraints on the border of the room (bounds) were inactive, but could not be removed, since the solver would otherwise 'try' solutions that would violate how reflections are modelled.

3.4. Optimise

The problem was first approached using an unconstrained Nelder-Mead algorithm to determine good starting value for a subsequent trust-region based constrained optimiser [1]. Comparing this to Fig 1B shows that the constraints are not satisfied, as the desk lamp is out of bounds, The initial values for the constrained optimiser were chosen as the closest feasible ones to this solution.

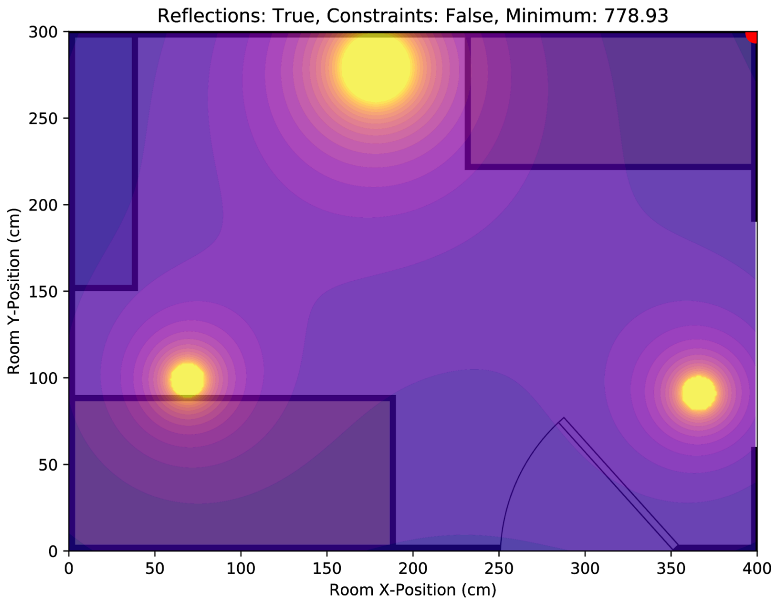

Fig. 5: Unconstrained Nelder-Mead optimum lamp positions. The contour filling represents the relative light intensity, scaled from low (blue) to high (yellow), the red marker locates the global minimum.

Constrained optimisation was performed using the 'trust-constr' algorithm, the most versatile algorithm implemented in SciPy and the most appropriate for large-scale problems [5]. 'It is a trust-region based algorithm that solves inequality constraints by introducing slack variables and solving a sequence of equality-constrained barrier problems for progressively smaller values of the barrier parameter' [2].

Fig. 6: Constrained trust-region based optimum lamp positions. The contour filling represents the relative light intensity, scaled from low (blue) to high (yellow), the red marker locates the global minimum.

3.6 Discussion

Constrained minimisation showed that it makes most sense to place the big floor lamp centrally and move the smaller lamps towards the longitudinal axis of the room. In this setting, the bed lamp could be placed on the cupboard and the desk lamp could be hung from the ceiling. Evidently, unconstrained minimisation delivered a more optimal solution, confirming constraint activity. Constraints were active where the optimum lamp position occurred on an edge of an allowed zone (g_4, g_{10}, g_{11}), whereas all others were inactive. In terms of illuminance, the goal of min. 500 Lux everywhere in the room was surpassed significantly in both unconstrained and constrained optimisation.

Challenges revolve around extending the optimisation to the 3D space. There appears to be a degree of symmetry in the solution space, as optimum arrangements were mostly triangular. This insight could be used for further problem space reduction, lessening computational cost and potentially enabling a 3D implementation. Another limitation is that windows, which have a lower albedo than walls, are not currently modelled, which should be considered in future studies.

4. Cost Optimisation (Bea Lopez-Neira)

This subsystem is focused on optimising the cost to set up lightning in a student room with a limited budget of £200. The final cost is a sum of the initial and operation cost. These are a factor of the lengths of the cable and the efficiency of the lamps, the last one being directly correlated to the operation cost and the cost of the lamp itself. There are set constraints on the position of the lamp which will limit the possibility to reduce the cable length and there are also constraints on the investment factor, referred to the number of years after which the initial investment should be justified in relation to the operation cost. This constraint will affect the trade off between the cost of the lamp and its running cost.

4.1 Optimisation formulation

A full derivation of this subsystem can be found on the team's GitHub repository in the subdirectory *subsystem_cost*. The following function determines the cost of setting up lamps in a room, related to the initial investment and cost of operation. Since the interim review the factor of efficiency as a variable has been included.

$$min\ C_{tot}(x_1, y_1, x_2, y_2, x_3, y_3, e) = min(C_{initial} + C_{operation})$$
$$where\ C_{initial}[x, y, e] = (\sum_{i=1} (x_n + y_n)c_{cable}) + n(e/0.2) + c_{work}ln(n)$$
$$C_{operation}[e] = ((\sum_{i=1}^{n} P_n/e)t)c_{energy}$$

In the null form, the problem can be formulated as the following.

$$min_x \quad C_{min}(x_1, y_1, x_2, y_2, x_3, y_3, c_{cable}, n, e, c_{work}, P_1, P_2, P_3, t, C_{energy})$$
$$where\ x = min_x\ (x_1, y_1, x_2, y_2, x_3, y_3, e)$$
$$p = (n, c_{work}, P_1, P_2, P_3, t, C_{energy})$$

$$s.t. \quad g_1 : 0.1 - x_1 \leq 0$$
$$g_2 : x_1 - 2.2 \leq 0$$
$$g_3 : 0.1 - y_1 \leq 0$$
$$g_4 : y_1 - 1.4 \leq 0$$
$$g_5 : 0.6 - x_2 \leq 0$$
$$g_6 : x_2 - 2.1 \leq 0$$
$$g_7 : 1.1 - y_2 \leq 0$$
$$g_8 : y_2 - 2.8 \leq 0$$
$$g_9 : 2.4 - x_3 \leq 0$$
$$g_{10} : x_3 - 3.9 \leq 0$$
$$g_{11} : 1.2 - y_3 \leq 0$$
$$g_{12} : y_3 - 2.9 \leq 0$$
$$g_{13} : 0.2 - e \leq 0$$
$$g_{14} : e - 1 \leq 0$$
$$g_{15} : C_{initial} - IC_{operation} \leq 0$$

4.2 Modelling approach

In order to formulate the objective function the initial cost was calculated as a combination of the price of the cables and lamps as well as the cost of work. After the interim review, changes were made in order to match a more realistic situation without precluding the system level approach. These changes include the efficiency as a variable and the cost of each lamp as a factor of it. This mathematical relation constitutes the relation of price versus efficiency of a lightbulb. Incandescent light cost can be around £1 and have a 20% efficiency, whereas LEDs efficiency can achieve values higher than 95% efficient and be up to £5 [10]. The cost of operation is represented as a factor of the cost of energy, running time and total power. The time is an average of the hours used in a year, which was estimated to be 2500, around 7 hours per day. The efficiency states the amount of energy that transforms into light. Therefore, the energy required to power a lightning system is the total power, divided by the efficiency, times the hours and price. It was assumed that there would be no maintenance cost in order for the problem to be manageable.

In terms of the code the modeling approach utilised a meshgrid to model the room, similarly to the light subsystem .Python was used to ensure an ease of integration for the system-level optimization and future integration into an app. The optimization was performed using scipy.optimize.minimize() function.

4.3 Problem space exploration

	x1	y1	x2	y2	x3	y3	e		x1	y1	x2	y2	x3	y3	e
f(x)	+	+	+	+	+	+	+		+	+	+	+	+	`+	+
G1	-							G9					-		
G2	+							G10					+		
G3		-						G11						-	
G4		+						G12						+	
G5			-					G13							-
G6			+					G14							+
G7				-				G15	+	+	+	+	`+	+	-
G8				+											

Fig. 7 Monotonicity analysis table

The table above (Fig.7) presents the monotonicity analysis on the fifteen constraints affecting the cost subsystem. Multiple inactive constraints were eliminated during the definition stage, most of them affected variables which then were assumed as parameters. The constraints g1 to g12 are bounds that affect the variables x and y, confining them into a specific area in the room. The constraints g13 and g14 restrict the efficiency values from ranges existent in the market, being 0.2 to 1. All the constraints mentioned above apply to MP1, where all the lower bounds are active, limiting the variables. The upper bound resulted to be inactive. G15 is a nonlinear constraint which seem to be active only for the variable efficiency. However, as efficiency also affects initial cost it can subsequently affect the rest of the variables.

4.4 Optimise

The initial approach with the models TNC and Nelder-Mead resulted in reduced price but not feasible solutions. Both of these models presented unconstrained solutions and therefore minimised the position of the lamps into negative values outside the room. Fig.8 below represents the solution with Nelder-Mead algorithm, this algorithm was chosen for being one of the best known algorithm for multidimensional optimization. In 1098 iterations it lowered the price from £442.8 on the initial guess to £129.53, a reduction by 71% and a lower solution than any other algorithm [1].

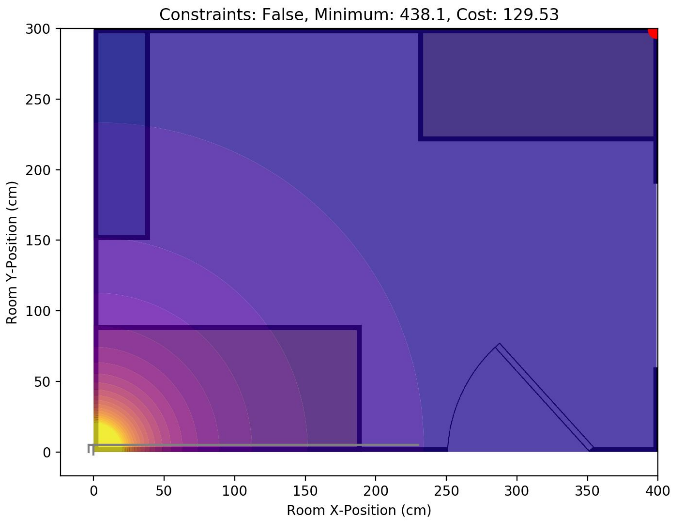

Fig.8 Nelder mead unconstrained optimization - Cost

For the implementations of constrains a trust region method was selected. Trust-constr method could take linear and non linear constraints and therefore was ideal for this subsystem [2][5]. The following figure(Fig.9) presents the solution when trust-contr and the constraints were implemented. The model solves inequality constraints by introducing slack variables and solving. Its execution time was longer than the unconstrained methods, achieving a cost reduction by 60% from the first iteration,

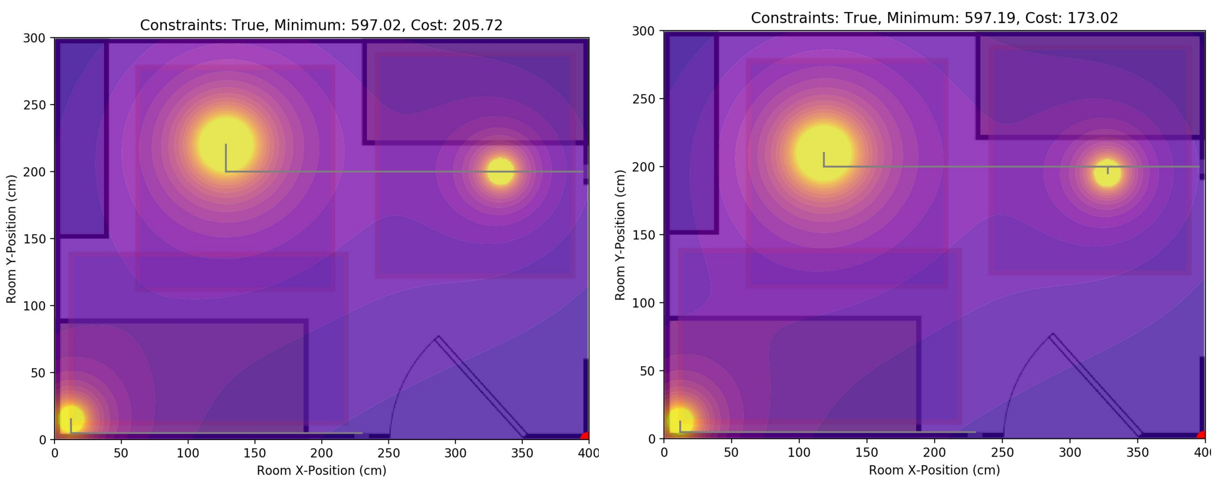

Fig.9 Trust-Constr optimization for cost. a.) Inv.factor = 1 b.) Inv.factor=3

4.6 Discussion

The unconstrained optimization presents the most minimized value. However it takes negative values, giving the efficiency a negative value and placing the lamps outside of the of the room. On the other hand, the trust-optimization does take constraints into account, giving logical values that can be compared and analysed against the light subsystem. Fig.8 a and b compare the solutions when changing the investment factor from one to three years , producing a change in the efficiency, which increases the price of the lamps but decreases the cost of operation from £102 to £66. Therefore, the conclusion is that for a student who is going to reside 3 years in their room it is worth spending more money on the chosen lamps for a higher efficiency in order to reduce operation cost by 15%.

5. System-level Optimisation

It was key to integrate the findings from both subsystems to present a solution that is effective in terms of both light distribution and cost. During a system-level optimisation, the weighted sum of the subsystems was minimised to explore the Pareto set, as formalised in *Section 2.* Fig. 10 shows the Pareto set sampled at nine weights from 0.1 to 0.9 for the light subsystem. It is implied that a light subsystem weight of 0.2 is the best, since the increase in cost thereafter surpasses the increase in light, and is depicted in Fig. 11. Both criteria, a minimum illuminance of 500 Lux and keeping within the budget of £200, were achieved.

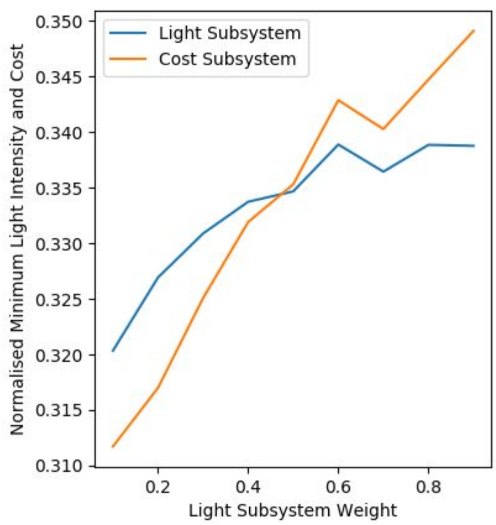

Fig 10: The Pareto Set

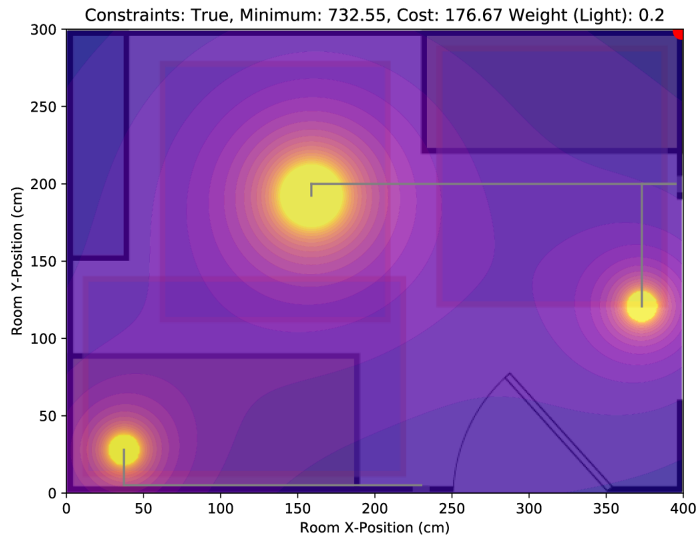

Fig 11: Constrained system-level analysis.
The red marker depicts the minimum light intensity

6. Conclusion

With the chosen parameters for the optimisation study, illuminance and cost targets could be achieved on a subsystem and system-level scale. The constrained light subsystem optimisation determined lamp positions that maximise the minimum light intensity to about 750 Lux. This ensured appropriate light conditions throughout the room for work purposes, indicating that even late night work is possible in the chosen environment. For the cost subsystem, a feasible solution under the budget of £200 was achieved. The investment factor implementation was successful, lowering the price by 15% from 205 to 173 pounds. Therefore, a student that was to stay for three years in the room would be recommended to invest into efficient LED lights for longterm cost savings. This study benefited from a clear system definition, which should by all means be continued in future studies. Challenges included justifying a reduction to 2D space and developing the subsystems individually whilst ensuring downstream comparability.

From a system-level perspective, giving a weight of 0.2 to the light subsystem resulted in the most effective solution. It is implied that some sort of symmetry occurs in the light subsystem, which should be analysed in a further study. These problem space reductions could lead to a massive decrease in computational cost, enabling a 3D implementation and a potential web-based application rollout. Further work should also focus on validating the results presented here with empirical data. This data could be used to create deep neural networks that could potentially run faster than the analytic methods derived here. In terms of the cost, further considerations could be analysed, such as the maintenance cost and return on investment.

7. References

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Appendix A. Nomenclature

Table 1: Design variables

Variable	Definition	Unit	Bounds
x_1	Position of the first lamp along the x axis	metres	$0.1 \le x_1 \le 2.2$
x_2	Position of the second lamp along the x axis	metres	$0.6 \le x_2 \le 2.1$
x_3	Position of the second lamp along the x axis	metres	$2.4 \le x_3 \le 3.9$
y_1	Position of the first lamp along the y axis	metres	$0.1 \le y_1 \le 1.4$
y_2	Position of the second lamp along the y axis	metres	$1.1 \le y_2 \le 2.8$
y_3	Position of the third lamp along the y axis	metres	$1.2 \le y_3 \le 2.9$
E_{min}	Minimum illuminance in the room	Lux	$500 \le E_{min}$
η	Lamp efficiency	-	$0.2 \le \eta \le 1.0$
C_{lamp}	Cost of the lamp	£	$1.0 \le C_{lamp} \le 5.0$

Table 2: Parameters

Parameter	Description	Unit	Value
L	Room length	metres	4
W	Room width	metres	3
S_{first_plug}	Position of first plug	metres	$[2.3, 0.05]^T$
S_{second_plug}	Position of second plug	metres	$[3.95, 2]^T$
c_{power}	Cost of power	£/kWh	0.12
c_{work}	Cost of work	£/hour	40
c_{cable}	Cost of cable	£/metre	2
t	Average hour of use per year	hours	2500
I	Investment factor	years	3
n	Number of lamps	-	3
r_1, r_3	Bedside and desk lamp radius	metres	0.15
r_2	Floor lamp radius	metres	0.3
P_1, P_3	Bedside and desk lamp power	W	50
P_2	Floor lamp power	W	120
a	Albedo (Wall reflectance)	-	0.5
j	Wall Bounces	-	3