UNIVERSITY OF
Waterloo

Department of Mechanical and Mechatronics Engineering

MTE 119: Statics

Project 2 Report: Design of a Robotic Manipulator With Final Torque of 36.592 N · m

A Report Prepared For:
Prof. Naveen Chandrashekar, PhD, PEng
Department of Mechanical and Mechatronics Engineering,
The University of Waterloo
200 University Ave. W.,
Waterloo, ON, N2L 3G1

Prepared By:
Group 6
Ben Boguslavsky (21010767)
Richard Wang (21007917)

April 10, 2023

Table of Contents

1. Introduction

Robotic manipulators are widely used in many industries, such as manufacturing, automation, surgery, and even space exploration. As technology develops and these industries become more reliant on automation with this technology, it is becoming increasingly important to use the technology in the most efficient way possible. Reducing energy consumption and increasing the lifespan of the motors are some of the many methods to make robotic manipulators more efficient. One way to implement these solutions is minimizing the torque required for a manipulator to operate, as there is less stress on the motors (therefore making them last longer) and less energy is required to create a higher force. Minimizing torque can also lead to other benefits, such as increased safety and higher precision. Thus, when creating a robotic manipulator, minimizing torque based on the device's applications is a key component in the design process.

This project involves the design of a three-degree-of-freedom (DOF) robotic manipulator that will be used at three main positions, with the goal in mind being to minimize the torque about the base of the arm (the origin). The first position is where the third section of the arm is at x = 0.75m and y = 0.1m with the arm being -60 degrees with respect to the x-axis. The second position is having the third arm at x = 0.5m and y = 0.5m, with the arm at a 0-degree angle with the x-axis. Finally, the last position is where the third arm is at x = 0.2m and y = 0.6m, with the arm making a 45-degree angle with the x-axis. The first arm of the manipulator weighs 4 kg/m, the second part weighs 2 kg/m, and the third section weighs 1 kg/m. In addition to these weights, the third arm will always have a 5-kg load attached to it. Using these givens, the most optimal length and angles for the arms were selected to minimize torque for all three positions, which is important for the reasons stated previously.

1.1. Objective

The objective of this report is to investigate and demonstrate the process of experimenting with different length and angle combinations to identify the most optimal values. The report discusses the progression and application of different approaches and ideologies in the design process while considering specific constraints to ultimately reach the most optimized values.

2. Design Methodology

Across all positions, the key constraint was the fixed angle of the third arm. This constraint severely affects the robot's work envelope, forcing the team to develop workarounds, such as increasing/decreasing the length of each arm to ensure the robot can reach the target. Additionally, no arm could cross below the x-axis (floor), and no arms could intersect/overlap. A few key assumptions were made when initially designing the manipulator to achieve the best possible torque about the base point. These can be thought of as "guiding ideas" in which each set of attempted designs was based on a specific idea.

Design Philosophy 1:

This approach was based on the assumption that minimizing each arm length would reduce the overall moment about the base point (sum of all arm lengths = 1). This assumption came from the observation that the moment equation is given by:

$$M = F \cdot d$$

Where F is the force and d is the perpendicular distance between the point of interest to the force's line of action. By reducing each arm's length, the distance from the center of gravity of each arm to the base point would be reduced as well as the weight of each arm. In theory, this would minimize the overall moment about the base point. However, it was soon discovered that this was not the best approach as a significant improvement could be made.

Design Philosophy 2:

Building on this insight, the team developed a revised approach. When attempting designs, it was observed that a large majority of the moment about the base point came from the gripper. Knowing this, the idea to use the weight of each of the manipulator's arms to counteract the moment caused by the gripper weight was implemented. To do this, the arms must be positioned in a way where their centers of gravity were often in the negative x-axis, while the gripper was in the positive x-axis. This could be achieved by going against Design Philosophy 1 and increasing the lengths of the arms. Using this approach, the lengths of each arm were optimized to achieve a minimum torque and the team created a final design that met all requirements while minimizing the total moment about the base point.

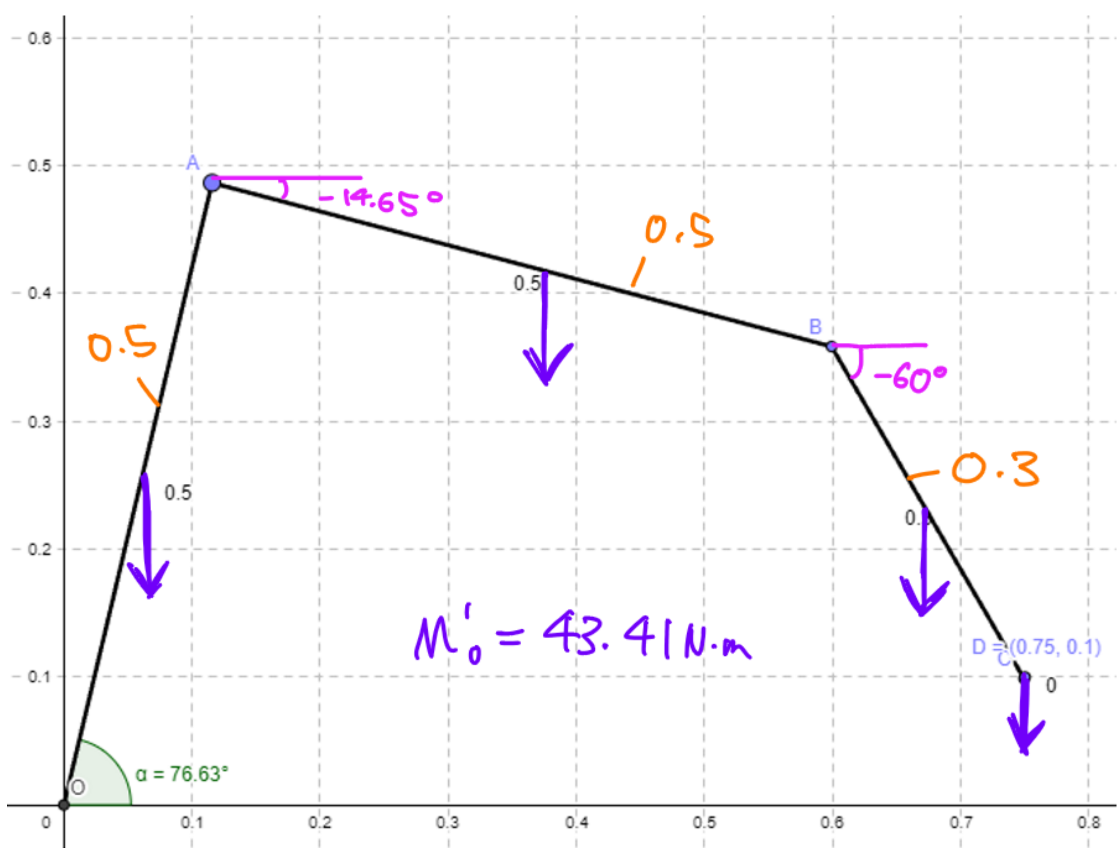
Design Methods and Tools

The design and testing process involved two key components, the first being a calculator software written by the team which would take in 3 arm lengths and determine the moment generated about the base point, as well as the angle of each arm section. The second was a GeoGebra simulation created by Jonathan Powell, which was used to help visualize the arm (Robotic Arm Simulator). These two components helped the design process to progress effectively and efficiently.

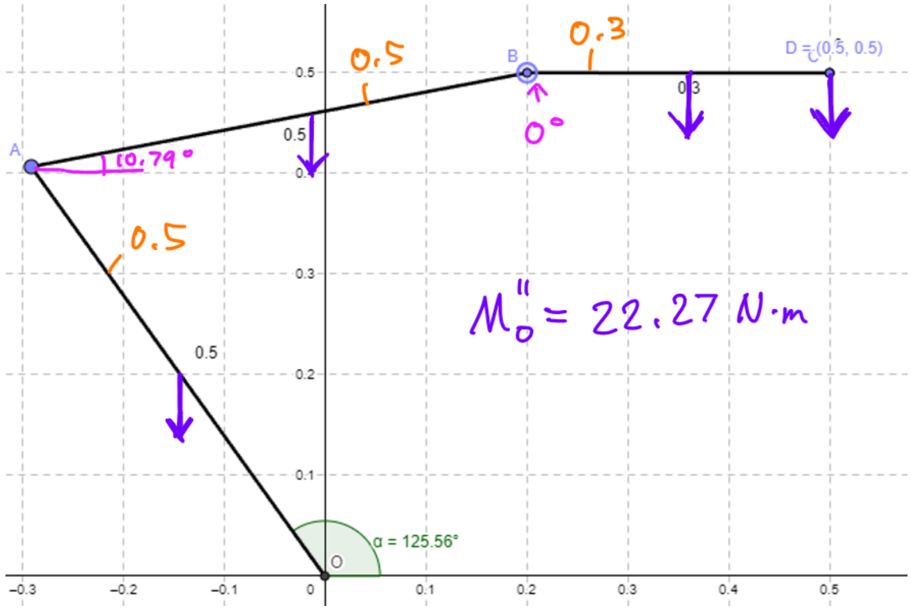
Design Iteration 1

The following design iteration is a mix between Design Philosophy 1 and 2. It represents an intermediate step in the progression from initial tests to final design, and it attempts to minimize length while also moving the center of gravity of some arms into the negative x-axis. The lengths of the arms are 0.5m, 0.5m, and 0.3m (as shown in the diagrams below).

Case 1

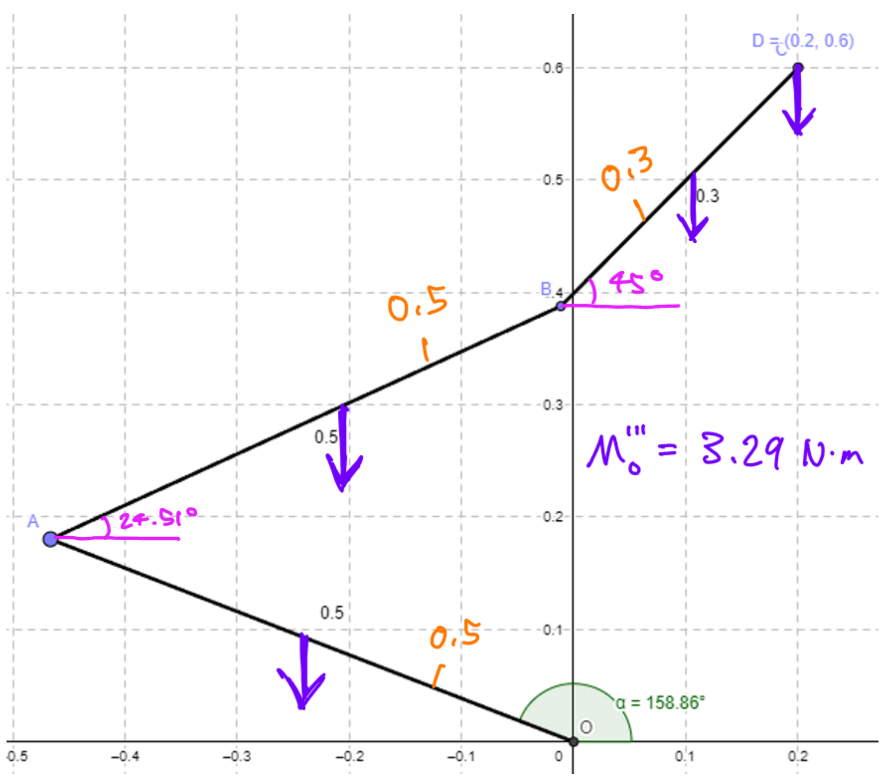


Case 2



0.5
0.3
B
D = (0.5, 0.5)
0.3
0°
0.5
A
10.79°
0.5
0.5
$M_o'' = 22.27$ N·m
α = 125.56°
O

Case 3



D = (0.2, 0.6)
0.3
0.3
B
45°
0.5
0.5
$M_o''' = 3.29$ N·m
A
24.51°
0.5
0.5
α = 158.86°
O

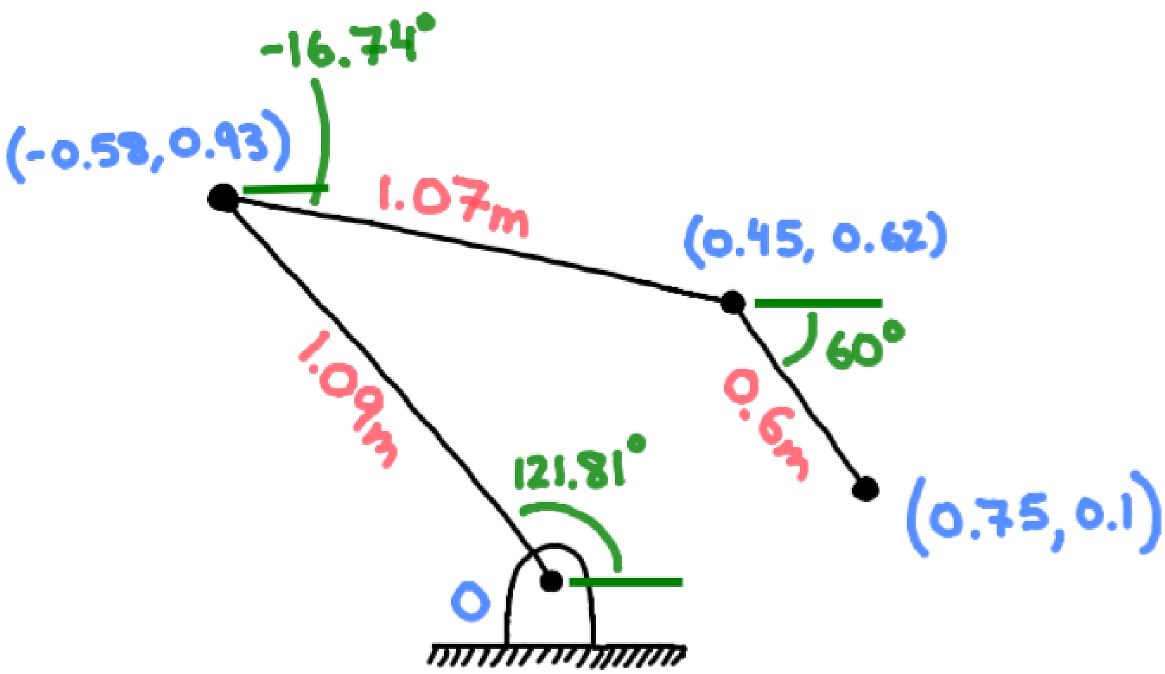
The total moment for this design iteration was 48.9 N·m.

3. Final Design

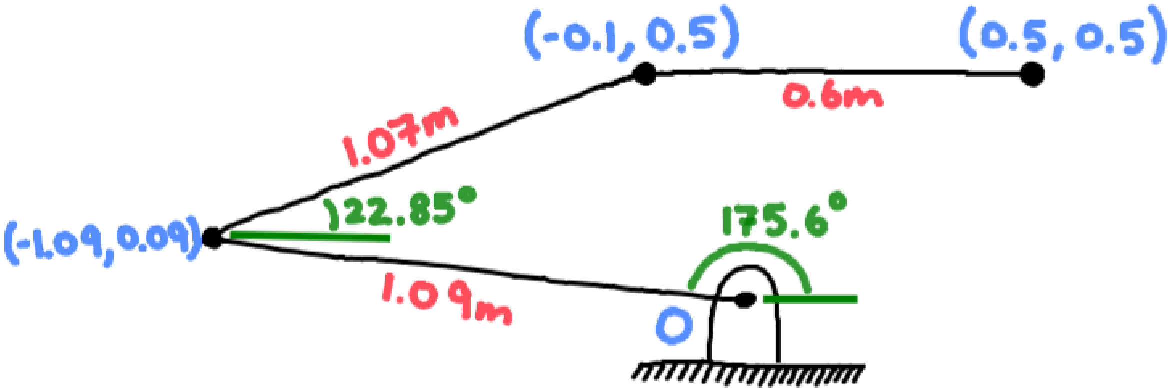
3.1. Overview of Design

As mentioned in Section 2, Design Methodology, the team knew there was a general shape that would allow the weight of the arms to balance out the moment thus resulting in a lower overall moment about the base joint. Building off of this, trial and error was used to obtain the lengths in the final design.

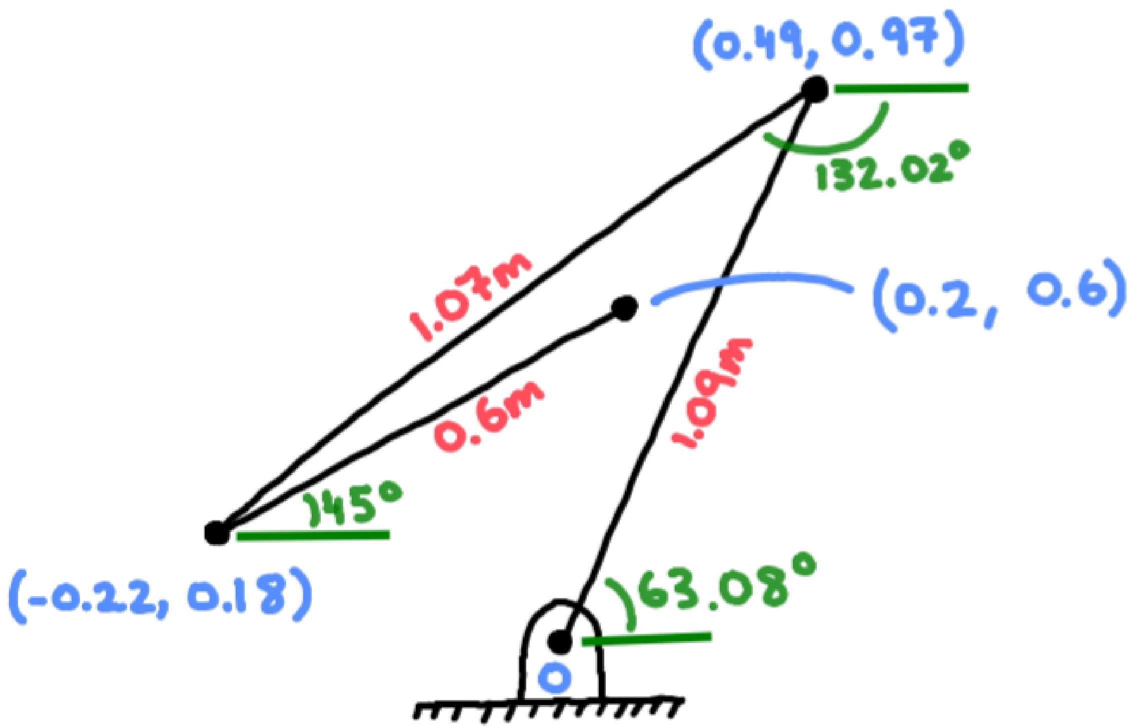
This design is the most optimized as it ensures that for all cases, at least one of the centres of mass for the arms lay on the negative x-axis. This allows for balancing to occur regardless of the case, which greatly helped in reducing the overall moment.



Case 1 Final Design



Case 2 Final Design



Case 3 Final Design

Table 1: Final design link lengths

Link 1 length, l_1 [m]	Link 2, l_2 [m]	Link 3, l_3 [m]
1.09	1.07	0.6

Table 2: Final design link angles for each position

Position Number	Link 1 Angle, q_1 [deg]	Link 2 Angle, q_2 [deg]	Link 3 Angle, q_3 [deg]
1	121.8	-16.74	-60
2	175.6	22.85	0
3	63.56	-132.02	45

3.2. Force Analysis

3.2.1. Weight Calculations

The weight of each section of the manipulator can be determined with the following formula:

$$W = \alpha \cdot L$$

where W is the weight in Newtons, α is the geometry constant (4 kg/m for the first section, 2 kg/m for the second section, and 1 kg/m for the third), and L is the length in meters. It is multiplied by 9.81, the gravitational constant, to determine the force from each segment.

$W_1 = 4 \cdot 1.09 \cdot 9.81 = 42.77$ N
$W_2 = 2 \cdot 1.07 \cdot 9.81 = 20.99$ N
$W_3 = 1 \cdot 0.6 \cdot 9.81 = 5.89$ N

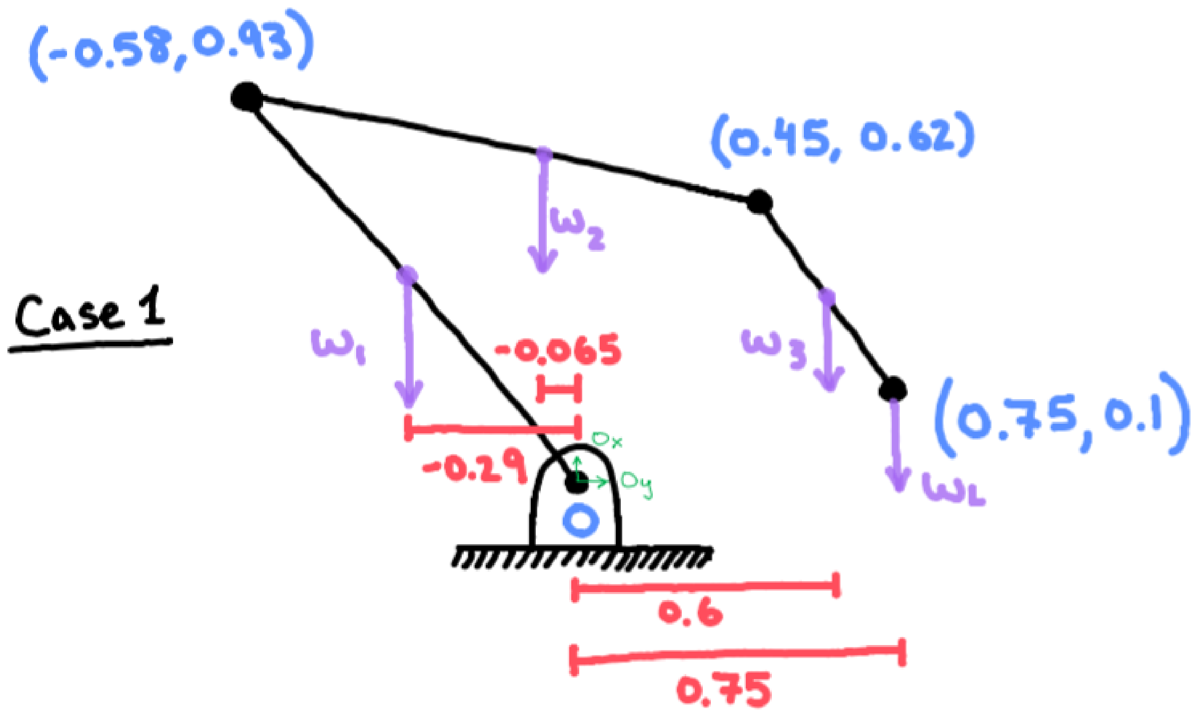
The weight of the 5 kg load at the end of the third section can be calculated by multiplying its mass by the gravitational constant as well.

$W_L = 5 \cdot 9.81 = 49.05$ N

Table 3: Final design weight values

Weight 1, W_1 [N]	Weight 2, W_2 [N]	Weight 3, W_3 [N]	Load Weight, W_L [N]
42.77	20.99	5.89	49.05

3.2.2. Position 1

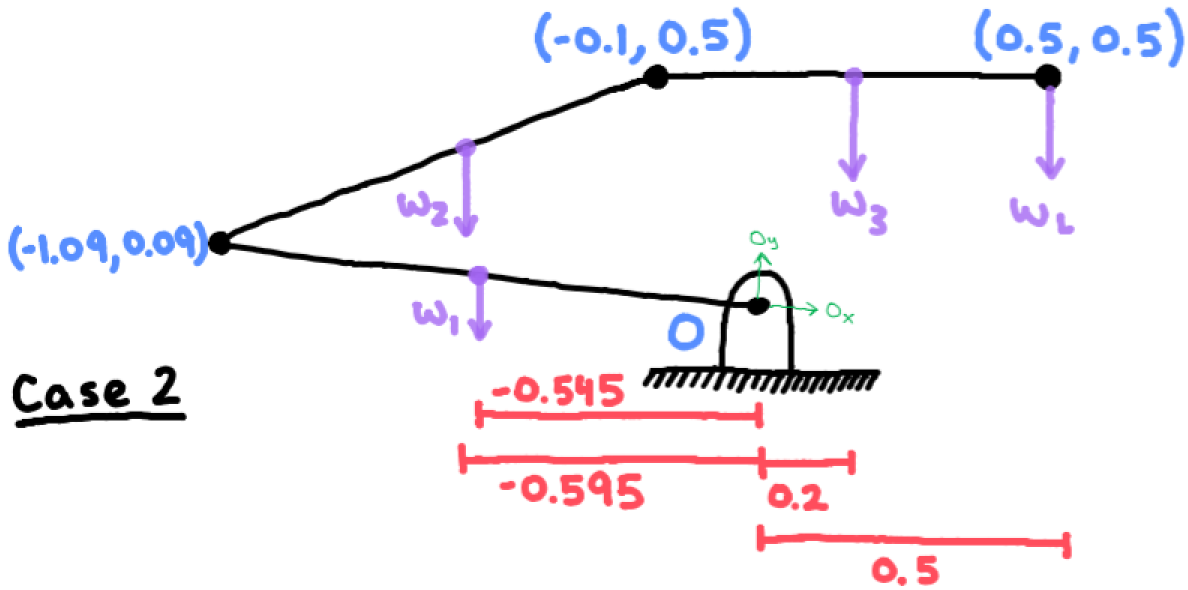


$$\Sigma M_O = T_1 = W_1 \cdot 0.29 + W_2 \cdot 0.065 - W_3 \cdot 0.6 - W_L \cdot 0.75$$

$$\Sigma M_O = T_1 = 42.77 \cdot 0.29 + 20.99 \cdot 0.065 - 5.89 \cdot 0.6 - 49.05 \cdot 0.75$$

$$\Sigma M_O = T_1 = \text{-}26.554 \text{ N} \cdot \text{m}$$

3.2.3. Position 2

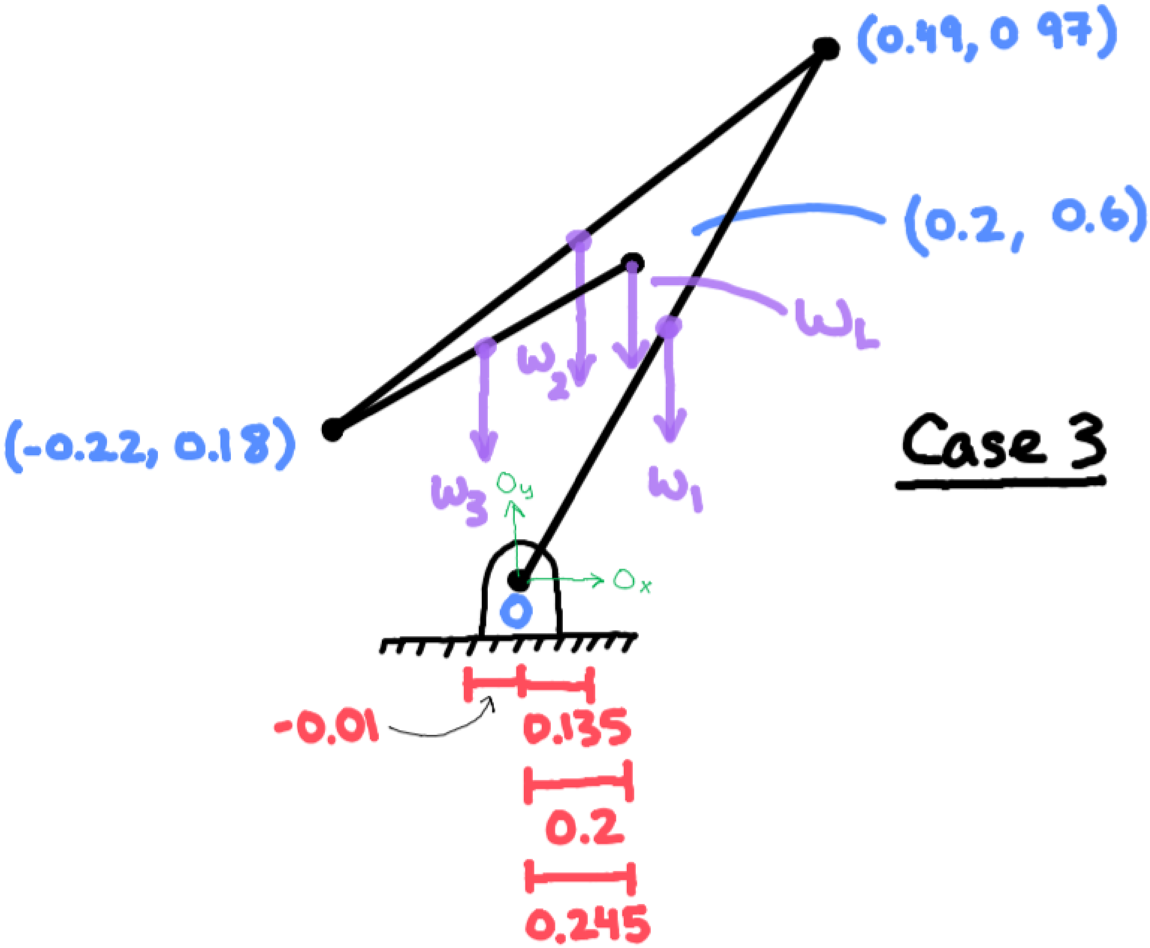


$$\Sigma M_O = T_2 = W_1 \cdot 0.545 + W_2 \cdot 0.595 - W_3 \cdot 0.2 - W_L \cdot 0.5$$

$$\Sigma M_O = T_2 = 42.77 \cdot 0.545 + 20.99 \cdot 0.595 - 5.89 \cdot 0.2 - 49.05 \cdot 0.5$$

$$\Sigma M_O = T_2 = 10.096 \text{ N} \cdot \text{m}$$

3.2.4. Position 3



$$\Sigma M_O = T_3 = W_1 \cdot 0.245 + W_2 \cdot 0.135 - W_3 \cdot 0.01 + W_L \cdot 0.2$$

$$\Sigma M_O = T_3 = 42.77 \cdot 0.245 + 20.99 \cdot 0.135 - 5.89 \cdot 0.01 + 49.05 \cdot 0.2$$

$$\Sigma M_O = T_3 = 23.063 \text{ N} \cdot \text{m}$$

3.2.5. Final Torque

The final torque can be calculated with the following formula:

$$T = \sqrt{T_1^2 + T_2^2 + T_3^2}$$

where T1, T2 and T3 are the total torques about point O in scenarios one, two and three, respectively.

$$T = \sqrt{(-26.554)^2 + (10.096)^2 + (23.063)^2}$$

$$T = 36.592\, N \cdot m$$

4. Summary of Design and Conclusions

The purpose of this project was to design a 3 DOF robotic manipulator that minimizes torque about its base motor at the origin while keeping in mind constraints such as the positioning of the third arm in each case, overlapping of sections, and maintaining the manipulator above the x-axis. It is an important project, as robotic manipulators are vastly used in many industries worldwide. In the end, a final design with a total torque of 36.592 Nm was achieved. This was achieved by applying a strategy where the weight of each section of the arm balances out the weight attached to the gripper. Despite this being the chosen strategy, there were others that were also attempted which proved to not be as efficient, such as minimizing the lengths of each segment. A significant observation to be made from this project is that despite knowing what values affect a certain property (in this case, moment depending on the magnitude of the weight and distance to the origin), other factors still play a major role in the final outcome. It was found that longer arms are easier to arrange in numerous positions, hence making it easier to position the centroid more efficiently and minimizing the moment.

5. Contributions

Name	Contributions
Ben Boguslavsky	Introduction, Objective, Overview of Design, Force Analysis, Drawings in all of section 3, Final Torque, Torque and Angle calculator software, Conclusion
Richard Wang	Design Methodology, Design Iterations and Drawings, Testing and Intermediate Arm Designs (in GeoGebra simulation), Final Arm Design, Optimization of Designs