* $P(A|B,C) = \frac{P(B|A,C)\,P(A|C)}{P(B|C)}$

$$P(X_{t+1}|Z_{1:t+1}) = P(X_{t+1}|Z_{t+1}, Z_{1:t})$$

$$= \frac{P(Z_{t+1}|X_{t+1}, Z_{1:t})\,P(X_{t+1}|Z_{1:t})}{P(Z_{t+1}|Z_{1:t})}$$

$$= \eta\, P(Z_{t+1}|X_{t+1}, Z_{1:t})\,P(X_{t+1}|Z_{1:t})$$

sensor independence

$$= \eta\, P(Z_{t+1}|X_{t+1})\,P(X_{t+1}|Z_{1:t})$$

Total probability

$$= \eta\, P(Z_{t+1}|X_{t+1}) \sum_{X_t} P(X_{t+1}|X_t, Z_{1:t})\,P(X_t|Z_{1:t})$$

sensor independence

$$= \eta\, P(Z_{t+1}|X_{t+1}) \sum_{X_t} P(X_{t+1}|X_t)\,P(X_t|Z_{1:t})$$

$\Rightarrow$

$$\begin{bmatrix} P(X_{t+1}=1|X_t=1) & P(X_{t+1}=1|X_t=0) \\ P(X_{t+1}=0|X_t=1) & P(X_{t+1}=0, X_t=0) \end{bmatrix} = T^T$$

if $Z=1$: $\begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix}$

else $Z=0$: $\begin{bmatrix} 0.1 \\ 0.7 \end{bmatrix}$

$\Rightarrow$ if $Z=1$

$$P_{t+1} = \eta \begin{bmatrix} 0.9 \\ 0.3 \end{bmatrix} \otimes T^T \cdot P$$

elif $Z=0$

$$P_{t+1} = \eta \begin{bmatrix} 0.1 \\ 0.7 \end{bmatrix} \otimes T^T \cdot P$$

$\#$

$\otimes$ Hadamard Product.